2) a) $f(t) = 5t + 3t^2 + 4t^3$

$f(t) = 5|\pi_2\rangle + 3|\pi_3\rangle + 4|\pi_4\rangle$

$|P_1\rangle = |\pi_1\rangle$; $|P_2\rangle = |\pi_2\rangle$

$|P_3\rangle = |\pi 3\rangle - \frac{1}{3}$

$|P_4\rangle = |\pi_4\rangle - \frac{3}{5}|\pi_2\rangle$

$|P_5\rangle = |\pi 5\rangle - \frac{6}{7}|\pi_3\rangle + \frac{3}{35}|\pi_1\rangle$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1/3 & 0 & 1 & 0 \\ 0 & -3/5 & 0 & 1 \end{bmatrix}\begin{bmatrix} |\pi_1\rangle \\ |\pi_2\rangle \\ |\pi_3\rangle \\ |\pi_4\rangle \end{bmatrix} = \begin{bmatrix} |P_1\rangle \\ |P_2\rangle \\ |P_3\rangle \\ |P_4\rangle \end{bmatrix}$$

A^T

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1/3 & 0 & 1 & 0 \\ 0 & 3/5 & 0 & 1 \end{bmatrix}^T$$

A^{-1} ES LA MATRIZ QUE LLEVA DE LA BASE 1 A LA DE POLINOMIOS DE LEGENDRE.

PARA $n = 4$

$\Rightarrow |\pi_1\rangle = |P_1\rangle$; $|\pi_2\rangle = |P_2\rangle$

$\boxed{INV(A) \cdot |\pi\rangle = |P\rangle}$

$|\pi_3\rangle = \frac{1}{3}|P_1\rangle + |P_3\rangle$

$|\pi_4\rangle = \frac{3}{5}|P_2\rangle + |P_4\rangle$

$$f(t) = 5|P_2\rangle + 3\left(\frac{1}{3}|P_1\rangle + |P_3\rangle\right) + 4\left(\frac{3}{5}|P_2\rangle + |P_4\rangle\right)$$

$$= |P_1\rangle + 7.4\,|P_2\rangle + 3\,|P_3\rangle + 4\,|P_4\rangle$$

Escaneado con CamScanner

BASE DUAL:

LA FORZAMOS A SER ORTOGONAL

$$\langle p^i | p_j \rangle = \delta^i_j$$

$$\langle p^1 | p_1 \rangle = 1 = \int_{-1}^{1} 1 \cdot (a + bt + ct^2)\, dt$$

$$\langle p^1 | p_2 \rangle = 0 = \int_{-1}^{1} t \cdot (a + bt + ct^2)\, dt$$

$$\langle p^1 | p_3 \rangle = 0 = \int_{-1}^{1} t^2 \cdot (a + bt + ct^2)\, dt$$

$$\int_{-1}^{1} a+bt+ct^2\,dt = at+\frac{b}{2}t^2+\frac{1}{3}ct^3\Big|_{-1}^{1}$$

$$= a+\frac{b}{2}+\frac{c}{3}-\left(-a+\frac{b}{2}-\frac{c}{3}\right)$$

$$= 2a+\frac{2c}{3}$$

$$\int_{-1}^{1} at+bt^2+ct^3 = \frac{1}{2}at^2+\frac{1}{3}bt^3+\frac{1}{4}ct^4\Big|_{-1}^{1}$$

$$= \frac{2}{3}b$$

$$\int_{-1}^{1} at^2+bt^3+ct^4 = \frac{1}{3}at^3+\frac{1}{4}bt^4+\frac{1}{5}ct^5\Big|_{-1}^{1}$$

$$= \frac{2}{3}a+\frac{2}{5}c$$

PARA $\langle P^1|$

$$\begin{bmatrix} 2 & 0 & 2/3 \\ 0 & 2/3 & 0 \\ 2/3 & 0 & 2/5 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} ; \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 9/8 \\ 0 \\ -15/8 \end{bmatrix}$$

$\langle P^2|$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 3/2 \\ 0 \end{bmatrix}$$

$\langle P^3|$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -15/8 \\ 0 \\ 45/8 \end{bmatrix}$$

PROYECCIONES:

$$\left.\begin{aligned} \langle P^1| &= 9/8 - 15/8\, t^2 \\ \langle P^2| &= 3/2\, t \\ \langle P^3| &= -\frac{15}{8} + 45/8\, t^2 \end{aligned}\right\} \quad \mathbb{P} = |P_i\rangle\langle P^i|$$

$$f(t) = 5t + 3t^2 + 4t^3 = |f\rangle_t$$ * BASE $\{1, t, t^2\}$

$$\left.\begin{aligned} \langle P^1|f\rangle &= 0 \\ \langle P^2|f\rangle &= 9 \\ \langle P^3|f\rangle &= 3 \end{aligned}\right\} * |P_i\rangle = \begin{matrix} 0 \\ 9|P_2\rangle \\ 3|P_3\rangle \end{matrix}$$

$$f(t) = |P_1\rangle + 7.4|P_2\rangle + 3|P_3\rangle + 4|P_4\rangle$$ * BASE ORTOGONAL (LEGENDRE)

$$\left.\begin{aligned} \langle P^1|f\rangle &= 1 \\ \langle P^2|f\rangle &= 7.4 \\ \langle P^3|f\rangle &= 3 \end{aligned}\right\} * |P_i\rangle = \begin{matrix} |P_1\rangle \\ 7.4|P_2\rangle \\ 3|P_3\rangle \\ 4|P_4\rangle \end{matrix}$$

ES CLARO QUE LA PROYECCIÓN SOBRE LA BASE ORTOGONAL NOS DA COMO RESULTADO LAS COMPONENTES DEL VECTOR EXPRESADO EN ESA BASE.
SI EL VECTOR ESTÁ EN OTRA BASE, LA PROYECCIÓN SE DEBE CALCULAR UNA A UNA (MUY CARO COMPUTACIONALMENTE)

$\underline{c)}$ $\mathbb{T} = e^{\mathbb{D}}$ $\quad D = \frac{d}{dt}$ $\quad T^{-1} = ?$ $\quad \mathbb{T}^{+} = ?$

CONSIDERANDO:

$$e^{\mathbb{D}} = \sum_{n=0}^{\infty} \frac{(\mathbb{D})^n}{n!};$$

$$e^{\mathbb{D}} = \mathbb{I} + D + \frac{D^2}{2} + \frac{D^3}{6} + \frac{D^4}{24}$$

SI TENEMOS UNA FUNCIÓN $|f\rangle$

$$e^{\mathbb{D}}|f\rangle = |f\rangle + D|f\rangle + \frac{D^2}{2}|f\rangle + \frac{D^3}{6}|f\rangle + \frac{D^4}{24}|f\rangle$$

SI SE LE APLICA A LA BASE:

$$|e_i\rangle = \{1, t, t^2, t^3, t^4\}$$

$$e^{\mathbb{D}}|e_1\rangle = 1 \;;\; e^{\mathbb{D}}|e_2\rangle = t+1 \;;\; e^{\mathbb{D}}|e_3\rangle = t^2 + 2t + 1$$

$$e^{\mathbb{D}}|e_4\rangle = t^3 + 3t^2 + 3t + \frac{1}{2}$$

$$e^{\mathbb{D}}|e_5\rangle = t^4 + 4t^3 + 6t^2 + 2t + \frac{1}{12}$$

$$e^{\mathbb{D}} \cdot |f\rangle = \begin{bmatrix} 1 & 1 & 1 & 1/2 & 1/12 \\ 0 & 1 & 2 & 3 & 2 \\ 0 & 0 & 1 & 3 & 6 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} |f\rangle$$

$$\mathbb{T}$$

$\mathbb{T}$ ES LA REPRESENTACIÓN MATRICIAL DE $e^{\mathbb{D}}$

SI $|f\rangle = 5t + 3t^2 + 4t^3$

$$e^{\mathbb{D}} |f\rangle = \mathbb{T} \cdot \begin{bmatrix} 0 \\ 5 \\ 3 \\ 4 \\ 0 \end{bmatrix} = \begin{bmatrix} 10 \\ 23 \\ 15 \\ 4 \\ 0 \end{bmatrix} = 10 + 23t + 15t^2 + 4t^3$$

$$\mathbb{T}^{-1} = \begin{bmatrix} 1 & -1 & 1 & 0.5 & -2.0833 \\ 0 & 1 & -2 & 3 & -2 \\ 0 & 0 & 1 & -3 & 6 \\ 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} = \mathrm{adj}(\mathbb{T})$$

MATLAB

EL OPERADOR $\mathbb{T}$ ES UNITARIO

d) EL OPERADOR Π APLICA PARA LA BASE OBLICUA

$$\Pi \cdot |f\rangle = e^{\mathbb{D}} \cdot |f\rangle$$

BASE OBLICUA

SI QUEREMOS Π_{LEG} (LEGENDRE):

$$\underbrace{A^{-1} \cdot \Pi}_{\Pi_{LEG}} \; ; \; \underbrace{A^{-1} \cdot |f\rangle}_{|f\rangle_{LEG}}$$