

$$2) a) f(t) = 5t + 3t^2 + 4t^3$$

$$f(t) = 5|\pi_2\rangle + 3|\pi_3\rangle + 4|\pi_4\rangle$$

$$|P_1\rangle = |\pi_1\rangle \quad ; \quad |P_2\rangle = |\pi_2\rangle$$

$$|P_3\rangle = |\pi_3\rangle - \frac{1}{3}$$

$$|P_4\rangle = |\pi_4\rangle - \frac{3}{5}|\pi_2\rangle$$

$$|P_5\rangle = |\pi_5\rangle$$

$$- \frac{6}{7}|\pi_3\rangle + \frac{3}{35}|\pi_2\rangle$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1/3 & 0 & 1 & 0 \\ 0 & -3/5 & 0 & 1 \end{bmatrix} \begin{bmatrix} |\pi_1\rangle \\ |\pi_2\rangle \\ |\pi_3\rangle \\ |\pi_4\rangle \end{bmatrix} = \begin{bmatrix} |P_1\rangle \\ |P_2\rangle \\ |P_3\rangle \\ |P_4\rangle \end{bmatrix}$$

\downarrow
 A^T

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1/3 & 0 & 1 & 0 \\ 0 & 3/5 & 0 & 1 \end{bmatrix}^T$$

A^{-1} ES LA
MATRIZ QUE
LLEVA DE
LA BASE 1
A LA DE
POLINOMIOS
DE LEGENDRE.

PARA $n=4$

$$\Rightarrow |\pi_1\rangle = |P_1\rangle \quad ; \quad |\pi_2\rangle = |P_2\rangle$$

$$|\pi_3\rangle = \frac{1}{3}|P_1\rangle + |P_3\rangle$$

$$|\pi_4\rangle = \frac{3}{5}|P_2\rangle + |P_4\rangle$$

$$\text{INV}(A) \cdot |\pi\rangle = |P\rangle$$

$$f(t) = 5 |P_2\rangle + 3 \left(\frac{1}{3} |P_1\rangle + |P_3\rangle \right) + 4 \left(\frac{3}{5} |P_2\rangle + |P_4\rangle \right)$$

$$= |P_1\rangle + 7.4 |P_2\rangle + 3 |P_3\rangle + 4 |P_4\rangle$$

BASE DUAL:

LA FORZAMOS A SER
ORTOGONAL

$$\langle p^i | p_j \rangle = \int_{-1}^1$$

$$\langle p^1 | p_1 \rangle = 1 = \int_{-1}^1 1 \cdot (a + bt + ct^2) dt$$

$$\langle p^1 | p_2 \rangle = 0 = \int_{-1}^1 t \cdot (a + bt + ct^2) dt$$

$$\langle p^1 | p_3 \rangle = 0 = \int_{-1}^1 t^2 \cdot (a + bt + ct^2) dt$$

$$\int_{-1}^1 a + bt + ct^2 dt = at + \frac{b}{2}t^2 + \frac{1}{3}ct^3 \Big|_{-1}^1$$

$$= a + \frac{b}{2} + \frac{c}{3} - \left(-a + \frac{b}{2} - \frac{c}{3}\right)$$

$$= \underline{2a + \frac{2c}{3}}$$

$$\int_{-1}^1 at + bt^2 + ct^3 = \frac{1}{2}at^2 + \frac{1}{3}bt^3 + \frac{1}{4}ct^4 \Big|_{-1}^1$$

$$= \underline{\frac{2}{3}b}$$

$$\int_{-1}^1 at^2 + bt^3 + ct^4 = \frac{1}{3}at^3 + \frac{1}{4}bt^4 + \frac{1}{5}ct^5 \Big|_{-1}^1$$

$$= \underline{\frac{2}{3}a + \frac{2}{5}c}$$

PARA $\angle P^1$

$$\begin{bmatrix} 2 & 0 & 2/3 \\ 0 & 2/3 & 0 \\ 2/3 & 0 & 2/5 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}; \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 9/8 \\ 0 \\ -15/8 \end{bmatrix}$$

$\angle P^2$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 3/2 \\ 0 \end{bmatrix}$$

$\angle P^3$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -15/8 \\ 0 \\ 45/8 \end{bmatrix}$$

PROYECCIONES:

$$\left. \begin{aligned} \langle P^1 | &= 9/8 - 15/8 t^2 \\ \langle P^2 | &= 3/2 t \\ \langle P^3 | &= -\frac{15}{8} + 45/8 t^2 \end{aligned} \right\} P = |P_i\rangle \langle P^i|$$

$$f(t) = 5t + 3t^2 + 4t^3 = |f\rangle_t$$

* BASE $\{1, t, t^2\}$

$$\left. \begin{aligned} \langle P^1 | f \rangle &= 0 \\ \langle P^2 | f \rangle &= 9 \\ \langle P^3 | f \rangle &= 3 \end{aligned} \right\} * |P_i\rangle = \begin{aligned} &0 \\ &9 |P_2\rangle \\ &3 |P_3\rangle \end{aligned}$$

$$f(t) = |P_1\rangle + 7.4 |P_2\rangle + 3 |P_3\rangle + 4 |P_4\rangle$$

* BASE ORTOGONAL (LEGENDRE)

$$\left. \begin{aligned} \langle P^1 | f \rangle &= 1 \\ \langle P^2 | f \rangle &= 7.4 \\ \langle P^3 | f \rangle &= 3 \end{aligned} \right\} * |P_i\rangle = \begin{aligned} &1 |P_1\rangle \\ &7.4 |P_2\rangle \\ &3 |P_3\rangle \\ &\underline{\underline{4 |P_4\rangle}} \end{aligned}$$

ES CLARO QUE LA PROYECCIÓN SOBRE LA BASE ORTOGONAL NOS DA COMO RESULTADO LAS COMPONENTES DEL VECTOR EXPRESADO EN ESA BASE.

SI EL VECTOR ESTÁ EN OTRA BASE, LA PROYECCIÓN SE DEBE CALCULAR UNA A UNA (MUY CARO COMPUTACIONALMENTE)

$$c) \quad \pi = e^{\mathcal{D}} \quad \mathcal{D} = \frac{d}{dt}$$

$$\pi^{-1} = ? \quad \pi^{\dagger} = ?$$

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CONSIDERANDO:

$$e^{\mathcal{D}} = \sum_{n=0}^{\infty} \frac{(\mathcal{D})^n}{n!};$$

$$e^{\mathcal{D}} = I + \mathcal{D} + \frac{\mathcal{D}^2}{2} + \frac{\mathcal{D}^3}{6} + \frac{\mathcal{D}^4}{24}$$

SI TENEMOS UNA FUNCIÓN $|f\rangle$

$$e^{\mathcal{D}} |f\rangle = |f\rangle + \mathcal{D}|f\rangle + \frac{\mathcal{D}^2}{2}|f\rangle + \frac{\mathcal{D}^3}{6}|f\rangle + \frac{\mathcal{D}^4}{24}|f\rangle$$

SI SE LE APLICA A LA BASE:

$$|e_i\rangle = \{1, t, t^2, t^3, t^4\}$$

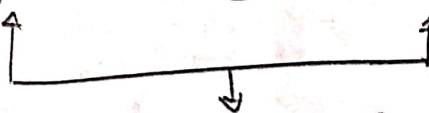
$$e^{\mathcal{D}} |e_1\rangle = 1; \quad e^{\mathcal{D}} |e_2\rangle = t+1; \quad e^{\mathcal{D}} |e_3\rangle = t^2+2t+1$$

$$e^{\mathcal{D}} |e_4\rangle = t^3+3t^2+3t+\frac{1}{2}$$

$$e^{\mathcal{D}} |e_5\rangle = t^4+4t^3+6t^2+2t+\frac{1}{12}$$

↓) EL OPERADOR π APLICA PARA LA
BASE OBLICUA

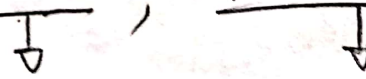
$$\pi \cdot |f\rangle = e^{\mathbb{D}} \cdot |f\rangle$$



 BASE OBLICUA

SI QUEREMOS π_{leg} (LEGENDRE):

$$\frac{-1}{A \cdot \pi} ; \frac{-1}{A \cdot |f\rangle}$$



 $\pi_{\text{leg}} \quad |f\rangle_{\text{leg}}$