I $(P^{\dagger})^{-1} = (P^{-1})^{\dagger}$

$$(P^{\dagger})^{-1} \cdot P^{\dagger} = (P^{-1})^{\dagger} P^{\dagger} \; ; \quad M = P^{\dagger}$$

$$M^{-1} \cdot M = (P^{-1} P)^{\dagger}$$

$$\mathbb{I} = \mathbb{I}^{\dagger}$$

$$\mathbb{I} = \mathbb{I}$$

II $(PQ)^{-1} = Q^{-1} P^{-1}$

$$(PQ)^{-1} \cdot PQ = Q^{-1} P^{-1} P Q$$

$$\mathbb{I} = Q^{-1} \mathbb{I} Q = Q^{-1} Q$$

$$\mathbb{I} = \mathbb{I}$$

$\underline{a)}$ III $[P, Q] = PQ - QP = 0$

$$\Rightarrow PQ = QP \Rightarrow Q^{-1} P \overset{\mathbb{I}}{\cancel{Q Q^{-1}}} = \overset{\mathbb{I}}{\cancel{Q^{-1} Q}} P Q^{-1}$$

$$\boxed{Q^{-1} P = P Q^{-1}}$$

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IV $\left(e^{P}\right)^{\dagger} = e^{P^{\dagger}}$

$$\left(\mathbb{I} + P + \frac{P^2}{2} + \cdots\right)^{\dagger} = \mathbb{I}^{\dagger} + P^{\dagger} + \frac{P^{2\dagger}}{2} + \cdots$$

$$= e^{P^{\dagger}}$$

V $P e^{Q} P^{-1} = P\left(\mathbb{I} + Q + \frac{Q^2}{2} + \frac{Q^3}{3!}\right) P^{-1}$

$$= \left(P\mathbb{I} + PQ + \frac{PQ^2}{2} + \frac{PQ^3}{3!}\right) P^{-1}$$

$$= PP^{-1} + PQP^{-1} + \frac{PQ^2P^{-1}}{2} + \frac{PQ^3P^{-1}}{3}$$

$$= \mathbb{I} + PQP^{-1} + \frac{PQP^{-1}PQP^{-1}}{2} + \frac{PQP^{-1}PQP^{-1}PQP^{-1}}{3!} + \cdots$$

$$= \mathbb{I} + PQP^{-1} + \frac{(PQP^{-1})^2}{2} + \frac{(PQP^{-1})^3}{3!} + \cdots$$

$$= e^{PQP^{-1}}.$$

b) $A = A^{\dagger}$

$$\tilde{A} = U^{-1} A U \longrightarrow \text{PREMULTIPLICAR POR } U$$

$$(U\tilde{A})^{\dagger} = (AU)^{\dagger}$$

$$\tilde{A}^{\dagger} U^{\dagger} = U^{\dagger} A^{\dagger} \longrightarrow A = A^{\dagger}; \ U^{\dagger} = U^{-1}$$

$$\tilde{A}^{\dagger} U^{\dagger} = U^{-1} A$$

$$\underline{\tilde{A}^{\dagger} = U^{-1} A U}$$

c) $A = A^{\dagger} \Rightarrow e^{iA}$ ES UNITARIO

$$U = e^{iA} \; ; \; U^{\dagger} = e^{-iA^{\dagger}} = e^{-iA}$$

$$\Rightarrow UU^{\dagger} = e^{iA} e^{-iA} = \mathbb{I}$$

$$\mathbb{I} = U^{\dagger} U = e^{-iA} e^{iA}$$

d) $\tilde{K} = iA$ $\qquad \tilde{K} = U^{-1} K U$

$$(U\tilde{K})^{\dagger} = (KU)^{\dagger}$$

$$\tilde{K}^{\dagger} U^{-1} = U^{-1} K^{\dagger}$$

$$\tilde{K}^{\dagger} = U^{-1} K^{\dagger} U = (iA)^{\dagger} = -iA^{\dagger}$$

$$-iA^{\dagger} = -iA = -\tilde{K} = U^{-1} K^{\dagger} U$$

$$\underline{-\tilde{K} = U^{-1} K^{\dagger} U}$$

e) $A = A^{\dagger}$, $B = B^{\dagger}$

$$AB = (AB)^{\dagger} = B^{\dagger} A^{\dagger} = BA$$

si y sólo si $\underline{AB = BA}$

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f) $S^T = -S$

(I) $(I - S)(I + S) = (I + S)(I - S)$

$$I + \cancel{S} - \cancel{S} - S^2 = I - \cancel{S} + \cancel{S} - S^2$$

$$I - S^2 = I - S^2$$

(II) $(I - S)(I + S) = 1 - S^2 = M$

$$\tilde{S} = \frac{M + M^T}{2} = \frac{1 - S^2 + 1 - (-S)^2}{2} = \frac{2 - 2S^2}{2} = 1 - S^2$$

$$\underline{\tilde{S} = 1 - S^2 = M}$$

$$(I - S)(I + S)^{-1} = (I - S)(I^{-1} + S^{-1}) = (I - S)(I + S^{-1})$$

$$= I + S^T - S - I = S^T - S = 0$$

PORQUE SI A UNA MATRIZ SIMÉTRICA LE QUITAMOS LA I Y LE RESTAMOS ELLA MISMA, QUEDA 0.

g) $I + S = A$

$$(2 - A) \cdot A^{-1} = R = 2A^{-1} - I$$

$$\frac{R + I}{2} = A^{-1}$$

$$\text{INV}\left(\frac{R+I}{2}\right) - I = S = \begin{bmatrix} 0 & -tg\theta \\ tg\theta & 0 \end{bmatrix}$$