

$$I (P^t)^{-1} = (P^{-1})^t$$

$$(P^t)^{-1} \cdot P^t = (P^{-1})^t P^t; \quad M = P^t$$

$$M^{-1} \cdot M = (P^{-1} P)^t$$

$$II = II^t$$

$$II = II$$

$$\text{II} \quad (PQ)^{-1} = Q^{-1} P^{-1}$$

$$(PQ)^{-1} \cdot PQ = Q^{-1} P^{-1} P Q$$

$$\text{II} = Q^{-1} \text{II} Q = Q^{-1} Q$$

$$\text{II} = \text{II}$$

$$\underline{a)} \quad \text{III} \quad [P, Q] = PQ - QP = 0$$

$$\Rightarrow PQ = QP \Rightarrow Q^{-1} P Q \overset{\text{II}}{\overset{\nearrow}{=}} Q^{-1} Q P Q^{-1} \overset{\text{II}}{\overset{\nearrow}{=}} P Q^{-1}$$

$$Q^{-1} P = P Q^{-1} \quad |$$

$$\text{IV } (e^P)^T = e^{P^T}$$

$$\bullet \left(I + P + \frac{P^2}{2} + \dots \right)^T = I^T + P^T + \frac{P^2}{2} + \dots$$

$$= e^{P^T}$$

$$\text{V } P e^Q P^{-1} = P \left(I + Q + \frac{Q^2}{2} + \frac{Q^3}{3!} \right) P^{-1}$$

$$= \left(P I + P Q + \frac{Q^2}{2} + \frac{Q^3}{3!} \right) P^{-1}$$

$$= P P^{-1} + P Q P^{-1} + \frac{P Q^2 P^{-1}}{2} + \frac{P Q^3 P^{-1}}{3}$$

$$\bullet \begin{aligned} &= I + P Q P^{-1} + \frac{P Q P^{-1} P Q P^{-1}}{2} + \frac{P Q P^{-1} P Q P^{-1} P Q P^{-1}}{3!} + \dots \\ &= I + P Q P^{-1} + \frac{(P Q P^{-1})^2}{2} + \frac{(P Q P^{-1})^3}{3!} + \dots \\ &= \underline{\underline{e^{P Q P^{-1}}}} \end{aligned}$$

$$\underline{b)} \quad A = A^\dagger$$

$$\tilde{A} = U^{-1} A U \longrightarrow \text{PREMULTIPLICAR POR } U$$

$$(U \tilde{A})^\dagger = (A U)^\dagger$$

$$\tilde{A}^\dagger U^\dagger = U^\dagger A^\dagger \longrightarrow A = A^\dagger; U^\dagger = U^{-1}$$

$$\tilde{A}^\dagger U^\dagger = U^{-1} A$$

$$\underline{\tilde{A}^\dagger = U^{-1} A U}$$

C) $A = A^\dagger \Rightarrow e^{iA}$ ES UNITARIO

$$U = e^{iA}; \quad U^\dagger = e^{-iA^\dagger} = e^{-iA}$$

$$\Rightarrow UU^\dagger = e^{iA} e^{-iA} = \mathbb{I}$$

$$\mathbb{I} = U^\dagger U = e^{-iA} e^{iA}$$

$$\underline{d)} \quad \tilde{K} = j A$$

$$\tilde{K} = U^{-1} K U$$

$$(U \tilde{K})^\dagger = (K U)^\dagger$$

$$\tilde{K}^\dagger U^{-1} = U^{-1} K^\dagger$$

$$\tilde{K}^\dagger = U^{-1} K^\dagger U = (j A)^\dagger = -j A^\dagger$$

$$-j A^\dagger = -j A = -\tilde{K} = U^{-1} K^\dagger U$$

$$\underline{\underline{-K = U^{-1} K^\dagger U}}$$

e) $A = A^{\dagger}, B = B^{\dagger}$

$$AB = (AB)^{\dagger} = B^{\dagger} A^{\dagger} = BA$$

SI Y SÓLO SI $AB = BA$

$$f) \quad S^T = -S$$

$$(I) \quad (I - S)(I + S) = (I + S)(I - S)$$

$$I + \cancel{S} - \cancel{S} - S^2 = I - \cancel{S} + \cancel{S} - S^2$$

$$I - S^2 = I - S^2$$

$$(II) \quad (I - S)(I + S) = I - S^2 = M$$

$$\tilde{S} = \frac{M + M^T}{2} = \frac{I - S^2 + I - (-S)^2}{2} = \frac{2 - 2S^2}{2} = I - S^2$$

$$\underline{\underline{\tilde{S} = I - S^2 = M}}$$

$$(I - S)(I + S)^{-1} = (I - S)(I^{-1} + S^{-1}) = (I - S)(I + S^{-1})$$

$$= I + S^T - S - I = S^T - S = 0$$

POROQUE SI A UNA MATRIZ SIMÉTRICA LE
QUITAMOS LA I Y LE RESTAMOS ELLA MISMA,
QUEDA 0.

$$g) \quad I + S = A$$

$$(2 - A) \cdot A^{-1} = R = 2A^{-1} - I$$

$$\frac{R + I}{2} = A^{-1}$$

$$\text{inv}\left(\frac{R + I}{2}\right) - I = S = \begin{bmatrix} 0 & -\text{tg} \theta \\ \text{tg} \theta & 0 \end{bmatrix}$$