The RoadMapping Code: How to deal with "Real World" Issues in Action-based Dynamical Modelling the Milky Way

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Subject headings: Galaxy: disk — Galaxy: fundamental parameters — Galaxy: kinematics and dynamics — Galaxy: structure

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4 Questions that haven't been covered so far: 25

1. Introduction

[TO DO]

Collection of thoughts for the introduction: *(Text is not yet perfect or concise, but should serve as a starting point to setup a basic structure for the introduction. The text will then have to be shortened, redundant formulations have to be removed, phrasing has to be improved and everything has to be supported with appropriate references.)*

- RoadMapping stands for "Recovery of the Orbit/Action Distribution of Mono-Abundance Populations and Potential Inference Nostrum for our Galaxy".
- **Our modelling method in a nutshell:** We fit simultaneously a model for the Galaxy's gravitational potential and an orbit distribution function (df) to stellar phase-space data. To turn a star's position and velocity into a full orbit, we need the gravitational potential in which the star moves. We assume that we know a family of orbit distribution functions that are close enough to the real distribution of orbits. In this case the stellar orbits calculated within a proposed potential will only follow such a df, if this potential model is close enough to the true potential.
  Or in other words: We need the potential to calculate orbits. At the same time, if we *know* the true orbits, we can deduce the true potential from them. To find the true orbits, we make use of the predictive power of an orbit distribution function.
- **Motivation to use this modelling technique in the Milky Way:** Bovy et al. 2012 .... [TO DO]
- **Introducing orbits and actions:** There are different ways to describe stellar orbits. The most obvious is to give the stars position and velocity vector at each point in time, by evaluating the potential forces that act on the star in each time step. Most orbits in realistic galaxy potentials are however not closed, so we would have to integrate the orbit forever. Another, much more convenient way to describe orbits, are so called integrals of motion. These integrals are functions of the star's time-dependent position and velocity, but are themselves constants in time, i.e. conserved quantities. The most obvious integral in static potentials is the energy of the orbit. Symmetries in potentials frequently allow more than one integral: In spherical potentials all three components of the angular momentum are conserved. In many axisymmetric potentials there is, in addition to the energy E and vertical component of the angular momentum L_z, a third non-classical integral of motion I_3, which has however no easy physical meaning. (Binney & Tremaine, Galactic Dynamics)

Because any function of integrals is an integral of motion itself, it is possible to construct integrals that have both very convenient properties and intuitive physical meanings. One such a set are the so-called actions. In axisymmetric potentials they are frequently called the radial action J_R, the vertical action J_z and the ϕ-action, which is simply the vertical component of the angular momentum, L_z. The radial action and vertical action quantify the amount of oscillation in radial and vertical direction that the orbit exhibits. Actions are constructed in such a way, that they are not only integrals, but also correspond to the momenta in a set of canonical coordinates. The canonical conjugate positions of the actions are the so-called angles, which have the convenient properties, that they increase strictly linearly in time while the star moves along the orbit. They are periodic in 2π and the frequencies by which they change are functions of the actions. In the action-angle coordinate system, the only thing we need to fully describe an orbit in an axisymmetric potential are therefore just three fixed numbers, the actions.

- **Using actions for distribution functions:** Actions are therefore the natural coordinates of orbits and each point in action space corresponds to one specific orbit in a given potential. It is often used in dynamical modelling, e.g. in the Schwarzschild superposition method (source???), to reconstruct a galaxy by superimposing different orbits and populating them with stars. In this way these kind of methods construct orbit distribution functions for galaxies, which are at the same time distribution functions in action space. Because angles increase linearly in time, when a star moves along its orbit, stars are uniformly distributed in angle space. Therefore a orbit distribution function in terms of actions and a uniform distribution of stars in angle-space can be directly mapped to a distribution of stars in canonical configuration phase-space, measurable stellar positions and velocities. While a stellar distribution in configuration space is six-dimensional, the distribution in action-angle space is effectively three-dimensional, because of the uniformity in angles. (Rewrite, too verbose...)

- **Why should we care about actions in realistic galaxies?** In reality galaxies have rarely perfectly static and axisymmetric potentials, which drastically reduces the number of conserved quantites along orbits. In static non-axisymmetric potentials there can still be two integrals of motion, angular momentum however is no longer conserved. The Milky Way's disk might have an overall axisymmetric appearance, but is perturbed by spiral arms. The strongest deviation from axisymmetry in the Galaxy is the bar, which also causes the Galactic potential to vary slowly in time. The stirrs up the stars of the disk and the potential and causes radial migration of the orbits (Reference???), orbits change and with them the actions. One could wonder if, under such non-axisymmetric, non-static potential conditions, the assumption and treatment of globally conserved actions in the Milky Way is still a sensible approach. First of all, actions are the natural way to treat orbits and they can be locally defined, even if they might not be globally conserved. As long as we care about orbits, we should care about actions. An orbit carries information about the star's past, about where the star was born and which tidal processes might have carried it away from its inital orbit. Together with the chemistry of the stars, which determined by their place of birth, their current orbits are valuable diagnostics for the evolution and structure of the Milky Way. Secondly, gravitational processes do only in the most extreme cases completely change the actions. In a slowly changing potential, where orbits adapt adiabatically to those changes, actions are conserved (Binney & Tremaine, Galactic Dynamics). And even during bar-induced radial migration at least the vertical actions are conserved and will continue to carry some amount of information about the stars' inital orbit distribution.

[TO DO] (Maybe cite Potzen 2015, who showed that analysing aspherical systems in spherical actions can still be a powerful tool, when used with care...)

- **Why should we care about an axisymmetric "best fit" model for the Milky Way disk?** One of the key assumptions of our modelling technique is the assumed axisymmetry of the Milky Way's gravitational potential, especially its disk. As we discussed already in the previous paragraph, this assumption is indeed only an approximation to the real disk, which has a much richer structure and more complicated potential, with spiral arms and ring-like structures (like the Monocerros ring), with a warp and a flare in the outer disk (references????). Also the Milky Way's halo has substructure, a multitude of streams (references???) and shell-like overdensities (reference???). The ultimate goal will be to find and identify substructures observationally and describe theoretically the structure and evolution of potential perturbations. Our method and efforts to extract information about the axisymmetric Milky Way potential from disk stars aims to create a reliable and well-constraint basis for these endevours: The best possible axisymmetric approximation to the Milky Way's potential could serve as a realistic equilibrium model from which a description of non-axisymmetric tidal perturbations can be theoretically established by perturbation theory. It will also help a great deal to identify sub-structures, e.g. to find and orbitally connect tidal streams, which in return will then give better constraints on the deviations from axisymmetry. Many modelling and techniques, both purely gravitational, but also chemo-dynamical, can greatly profit from a good axisymmetric model for the galaxy: While we are still far away from knowing the MW's potential all over the place, an axisymmetric model will be the best reference to turn phase-space coordiantes into whole orbits. And orbits are the diagnostics that carry information from everywhere in the galaxy into the solar neighbourhood, where we can hope to exploit them. (Some overlap with section before. How to better structure these two sections and assign the arguments more clearly to "axisymmetric disk" or "actions"?)

- **Previous results with this modelling technique:** Bovy & Rix (2013) ... [TO DO]
  - disk scale length $R_d = 2.15 \pm 0.14$ kpc (Bovy & Rix 2013)
  - disk is maximal (Bovy & Rix 2013)
  - slope of dark matter halo $\alpha < 1.53$ (Bovy & Rix 2013)
- **What do we already know about the axisymmetric MW disk (from other references)?** [TO DO]
  - rotation curve is well-known (reference???)
- **What is there left to learn about the axisymmetric MW disk?** (as Jo asked at the Santa Barbara conference... [TO DO]
  - separation of different MW component is still unclear: individual density profiles, contributions to total pot
  - thin/thick disk vs. continuum of exponential disks
  - dark matter at smaller radii
  - slope & shape of dark matter halo (current state of knowledge?)
- **Other modelling approaches using DF's similar to Binney:**
  - Piffl et al. (2014) used a slightly different DF-based modelling approach to constrain the MW's vertical density profile near the sun. They fitted a superposition of "quasi-isothermal" DFs for thick and thin disk, and a DF for the halo to ˜200,000 giant stars from the RAVE survey (RAdial Velocity Experiment, Steinmetz et al. (2006)). They didn't use any chemical information of the stars. To account for different populations within the thin disk, they weighted the corresponding DF's with an assumed star-formation rate instead. To circumvent the use of RAVE's non-trivial spatial selection function, they separated stars into spatial bins in (R, z) and fitted the velocity distribution predicted by their DF and potential model at the mean (R, z) of each bin to the observed velocities only. They're result for their radial profile of the vertical force within $|z| = 1.1$ kpc and $R > 6.6$ kpc agrees well with the previous results from our method by Bovy & Rix (2013). By not using chemical information and hiding the spatial distribution of stars by binning to circumvent a complicated selection function, Piffl et al. (2014)

is however rejecting a lot of valuable information in the data set. ([TO DO: Look at other useful references in this paper: Bienayme et al. 2014, Zhang et al. 2013, Binney et al. 2014a, Binney 2012b, McMillan & Binney 2013])

- **Motivating this method characterization in anticipation of GAIA:** [TO DO]

2. Dynamical Modelling

2.1. Model

2.1.1. Actions

[TO DO]

2.1.2. Potential models

[TO DO] Mention different ways to calculate actions in different potentials.

2.1.3. Distribution function

The structure of the MW disk is still under debate: While many still support the thin-thick disk dichotomy in the MW disk (references ???), Bovy et al. (2012b) found indications that the MW disk might actually be a super-position of many stellar sub-popluations with a continuous spectrum of scale heights, scale lengths, metallicity and [α/Fe] abundances (dubbed mono-abundance populations (*MAP*s)). Further investigation lead to the findings that *MAP*s in the MW disk have a simple spatial structure that follows an exponential in both radial and vertical direction (Bovy et al. 2012d). The corresponding velocity dispersion profile of the *MAP*s also decreases exponentially with radius and is nearly independent of height above the plane, i.e. quasi-isothermal (Bovy et al. 2012c). The radial decrease in vertical velocity dispersion has, according to Bovy et al. (2012c), a long scale length of $h_{\sigma,z} \sim 7$ kpc for all *MAP*s . Older *MAP*s , which are characterized by lower metallicities and [α/Fe] abundances, have in general shorter density scale lengths, larger scale heights and velocity dispersion (Bovy et al. 2012d). Ting et al. (2013) and Bovy & Rix (2013) finally proposed that these findings could be employed for dynamical modelling techniques using

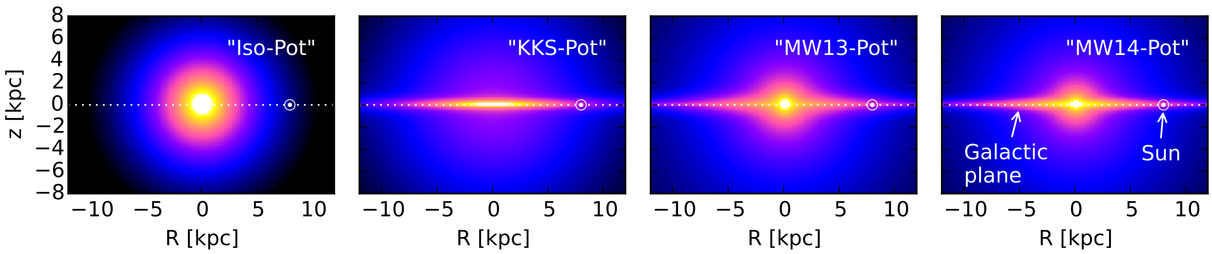

Fig. 1.— [TO DO]

Table 1. Caption [TO DO].

name	potential type	free parameters p_Φ		action calculation	reference
"Iso-Pot"	isochrone potential	circular velocity at the sun isochrone scale length	$v_{\rm circ} = 230$ km s^{-1} $b = 0.9$ kpc	***analytical and exact*** J_r, J_ϑ, L_z; use $J_r \rightarrow J_R, J_\vartheta \rightarrow J_z$ in eq. (???)	Binney & Tremaine (2008)
"KKS-Pot"	2-component Kuzmin-Kutuzov- Stäckel potential	circular velocity at the sun focal distance of coordinate system[a] axis ratio of the coordinate surfaces[a] ... ...of the disk component ...of the halo component relative contribution of the disk mass to the total mass	$v_{\rm circ} = 230$ km s^{-1} $\Delta = 0.3$ $\left(\frac{a}{c}\right)_{\rm Disk} = 20$ $\left(\frac{a}{c}\right)_{\rm Halo} = 1.07$ $k = 0.28$	***exact*** J_R, J_z, L_z using "Stäckel Fudge" (Binney 2012) and interpolation on action grid (**?**)	Batsleer & Dejonghe (1994)
"MW13-Pot"	MW-like potential with fixed Hernquist bulge, 2 exponential disks (stars + gas), spherical power-law halo	circular velocity at the sun stellar disk scale length stellar disk scale height relative halo contribution to $v_{\rm circ}^2(R_\odot)$ "flatness" of rotation curve	$v_{\rm circ} = 230$ km s^{-1} $R_d = 3$ kpc $z_h = 0.4$ kpc $f_h = 0.5$ $\frac{\mathrm{d}\ln(v_{\rm circ}(R_\odot))}{\mathrm{d}\ln(R)} = 0$	***approximate*** J_R, J_z, L_z using "Stäckel Fudge" (Binney 2012) and interpolation on action grid (**?**)	Bovy & Rix (2013)
"MW14-Pot"	MW-like potential with cutoff power-law bulge, Miyamoto-Nagai stellar disk, NFW halo	-	-	see "MW13-Pot"	**?**

[a]The coordinate system of each of the two Stäckel-potential components is $\frac{R^2}{\tau_{i,p}+\alpha_p} + \frac{z^2}{\tau_{i,p}+\gamma_p} = 1$ with $p \in \{\text{Disk}, /\text{Halo}\}$ and $\tau_{i,p} \in \{\lambda_p, \nu_p\}$. Both components have the same focal distance $\Delta = \sqrt{\gamma_p - \alpha_p}$, to make sure that the superposition of the two components itself is still a Stäckel potential. The axis ratio of the coordinate surfaces $\left(\frac{a}{c}\right)_p := \sqrt{\frac{\alpha_p}{\gamma_p}}$ describes the flattness of the corresponding Stäckel component.

action-based distribution functions. An action-based distribution function, that is flexible enough to describe the spectrum of simple phase-space distributions of different *MAP*s , is the quasi-isothermal distribution function (qDF) by Binney & McMillan (2011), as demonstrated by Ting et al. (2013). The qDF by Binney & McMillan (2011) is a function of the actions $\boldsymbol{J} = (J_R, J_z, L_z)$ and has the form

$$\begin{aligned} \mathrm{qDF}(\boldsymbol{J} \mid p_{\mathrm{DF}}) &= f_{\sigma_R}(J_R, L_z \mid p_{\mathrm{DF}}) \times f_{\sigma_z}(J_z, L_z \mid p_{\mathrm{DF}}) \\ \text{with } f_{\sigma_R}(J_R, L_z \mid p_{\mathrm{DF}}) &= n \times \frac{\Omega}{\pi \sigma_R^2(R_g)\kappa} \left[1 + \tanh\left(L_z/L_0\right)\right] \exp\left(-\frac{\kappa J_R}{\sigma_R^2(R_g)}\right) \\ f_{\sigma_z}(J_z, L_z \mid p_{\mathrm{DF}}) &= \frac{\nu}{2\pi\sigma_z^2(R_g)} \exp\left(-\frac{\nu J_z}{\sigma_z^2(R_g)}\right) \end{aligned}$$

Here $R_g \equiv R_g(L_z)$ and $\Omega \equiv \Omega(L_z)$ are the (guidig-center) radius and the circular frequency of the circular orbit with angular momentum L_z in a given potential. $\kappa \equiv \kappa(L_z)$ and $\nu \equiv \nu(L_z)$ are the radial/epicycle (κ) and vertical (ν) frequencies with which the star would oscillate around the circular orbit in R- and z-direction when slightly perturbed (Binney & Tremaine 2008). The term $[1 + \tanh\left(L_z/L_0\right)]$ suppresses counter-rotation for orbits in the disk with $L \gg L_0$ which we set to a random small value ($L_0 = 10 \times R_\odot/8 \times v_{\mathrm{circ}}(R_\odot)/220$).
For this qDF to be able to incorporate the findings by Bovy et al. 2012 about the phase-space structure of *MAP*s summarized above, we set the functions n, σ_R and σ_z, which indirectly set the stellar number density and radial and vertical velocity dispersion profiles,

$$\begin{aligned} n(R_g) &\propto \exp\left(-\frac{R_g}{h_R}\right) \\ \sigma_R(R_g) &= \sigma_{R,0} \times \exp\left(-\frac{R_g - R_\odot}{h_{\sigma_R}}\right) \\ \sigma_z(R_g) &= \sigma_{z,0} \times \exp\left(-\frac{R_g - R_\odot}{h_{\sigma_z}}\right). \end{aligned}$$

The qDF for each *MAP* has therefore a set of five free parameters p_{DF}: the density scale length of the tracers h_R, the radial and vertical velocity dispersion at the solar position $R_\odot$, $\sigma_R, 0$ and $\sigma_z, 0$, and the scale lengths h_{σ_R} and h_{σ_z}, that describe the radial decrease of the velocity dispersion. The *MAP*s we use for illustration through out this work are summarized in table ???.

[TO DO] [To Do here: Also mention how the density is calculated.]

2.1.4. Selection function: observed volume and completeness

[TO DO]

Table 2. Caption [TO DO]. The parameters of the "cooler" ("hotter") *MAP*s were chosen such, that the they have the same σ_R/σ_z ratio as the "hot" ("cool") *MAP* . Hotter populations have shorter tracer scale lengths (Bovy et al. 2012d) and the velocity dispersion scale lengths were fixed according to Bovy et al. (2012c).

name of *MAP*	free parameters $p_{\rm DF}$				
	h_R [kpc]	σ_R [km s^{-1}]	σ_z [km s^{-1}]	h_{σ_R} [kpc]	h_{σ_z} [kpc]
"hot"	2	55	66	8	7
"cool"	3.5	42	32	8	7
"cooler"	2 +50%	55-50%	66-50%	8	7
"hotter"	3.5-50%	42+50%	32+50%	8	7

3. Results

3.1. What if our assumptions on the (in-)completeness of the data set are incorrect?

The selection function of a survey is described by a spatial survey volume and a completeness function, which determines the fraction of stars observed at a given location within the Galaxy with a given brightness, metallicity etc (see §[TO DO CHECK]). The completeness function depends on the characteristics and mode of the survey, can be very complex and is therefore sometimes not perfectly known. We investigate how much an imperfect knowledge of the selection function can affect the recovery of the potential. We model this by creating mock data with varying incompleteness and assuming constant completeness in the analysis. The mock data comes from a sphere of $r_{\mathrm{max}} = 3$ kpc around the sun and an incompleteness function that drops linearly either with distance from the sun (left panels in fig. 2) or with distance from the Galactic plane (right panels in fig. 2). We demonstrate that the potential recovery with *RoadMapping* is very robust against somewhat wrong assumptions about the (in-)completeness of the data (see the tests for the radial incompleteness function in fig. 3 and vertical incompleteness function in 4). A lot of information about the potential comes from the rotation curve measurements in the plane, which is not affected by applying an incompleteness function. The robustness is however still given - at least for small deviations of true and assumed completeness ($\lesssim 10\%$) - if we do not include information about the rotation curve and marginalize the likelihood over the v_T coordinate (see bright grey violins in fig. 3 and 4). For the radial incompleteness function we get better results than for the vertical incompleteness. As the former models the important effect of stars being less likely to be observed the further away they are, this is an encouraging result.

Stuff that needs to be further examined:

- Maybe instead of decreasing completeness with height above the plane, a completeness that INcreases with height above the plan, to model e.g. obscuration due to dust.
- Make similar test as isoSphFlexIncompR, but with KKS potential, to test, if this robustness is a special case for the isochrone potential.

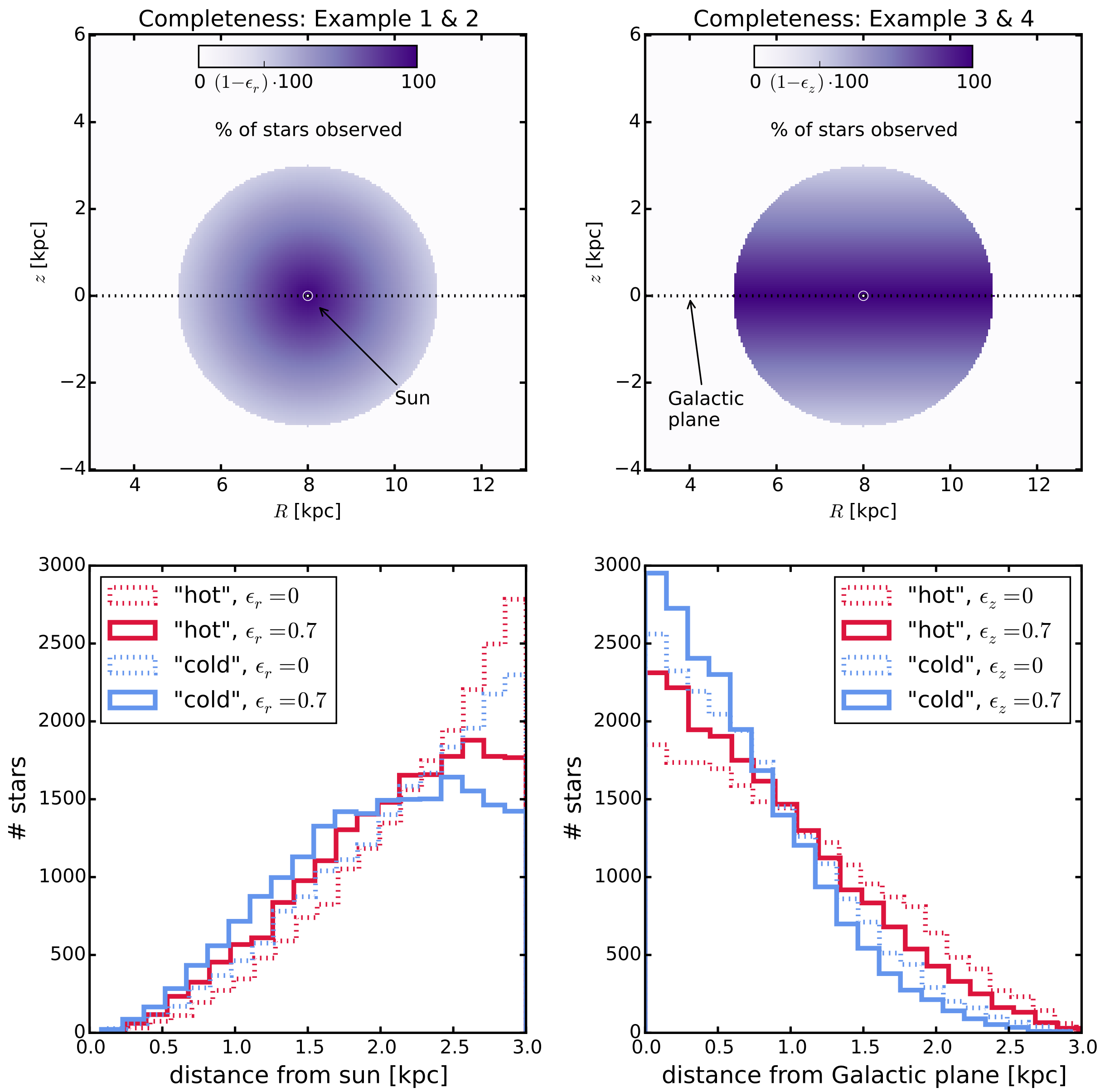

Fig. 2.— Caption [TO DO] ([TO DO] Re-do, if new analyses are in violin plot.)

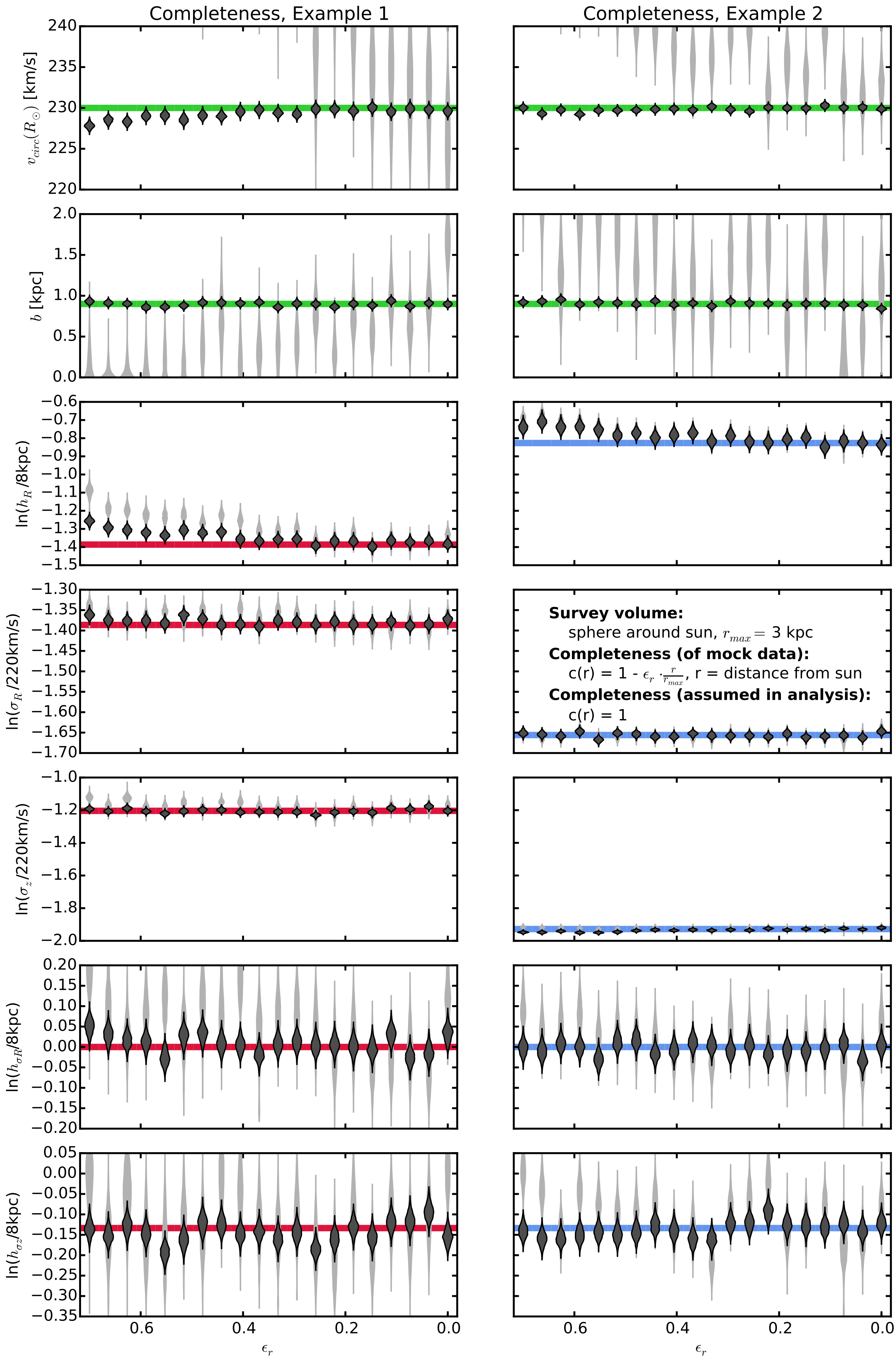

Fig. 3.— Caption [TO DO] (This was done using the current qDF to set the fitting range.

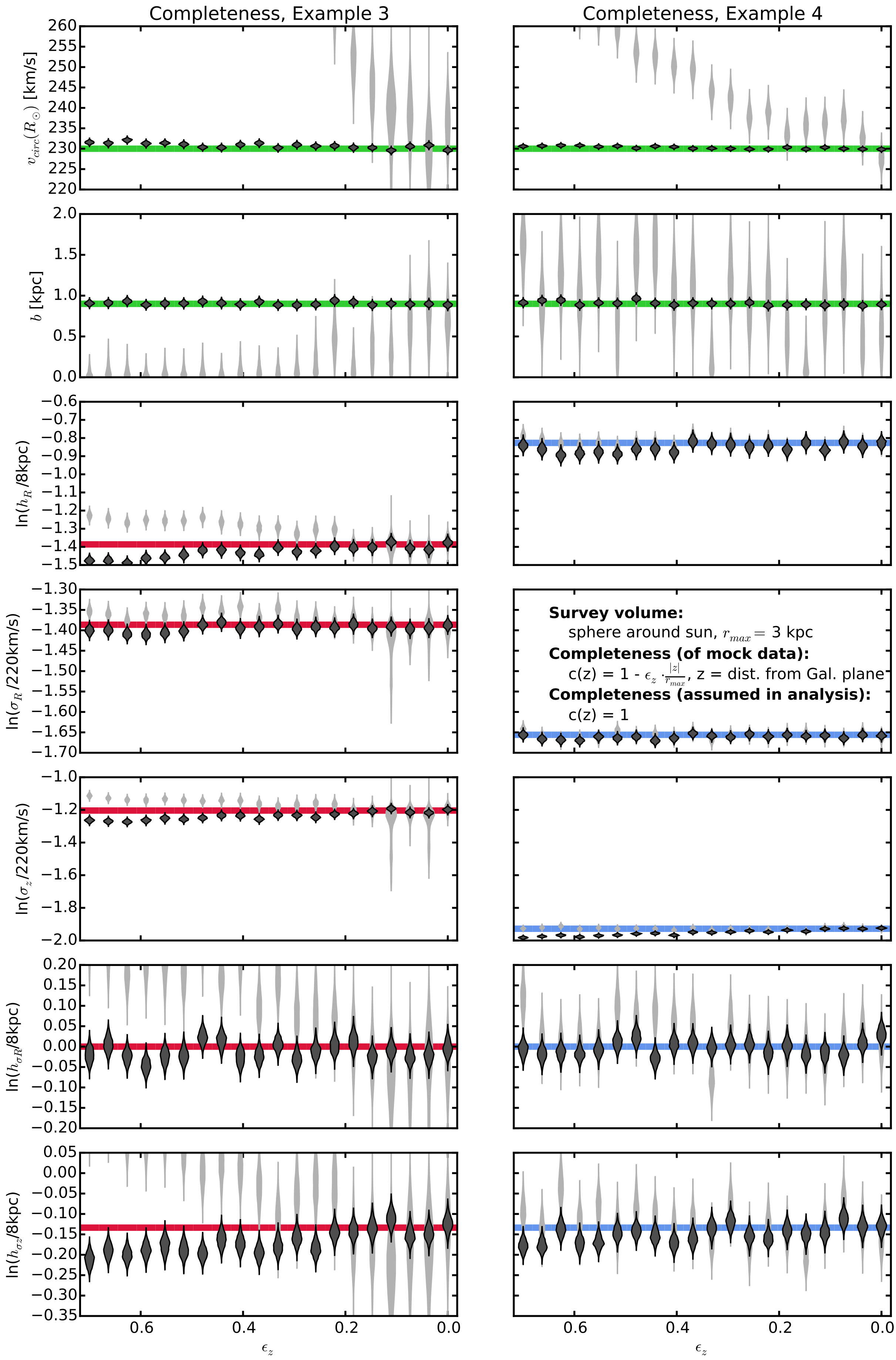

Fig. 4.— Caption [TO DO] (This was done using the current qDF to set the fitting range.

3.2. What if our assumed distribution function differs from the stars' DF?

Sanders & Binney (2015) and ??? develop extended distribution functions (EDFs), that extend action-based DFs to also describe the distribution of the star's metallicities. While a full chemo-dynamical modelling, including metallicity as well as α- and other chemical abundances, is ultimately the right way to go, the form of the EDFs still depends on a lot of additional assumptions. By looking at fig. 6 in Bovy & Rix (2013) (other references???) we doubt that a final version of an EDF will have a simple form in action-metallicity space. Motivated by the findings by Bovy et al. 2012, we therefore resign to the simpler approach outlined in Bovy & Rix (2013) and here, were metallicity and α-abundances are implicitly taken into account by describing each MAP separately by one qDF. This procedure could have two caveats:

First, the binning of the stars according to their abundances could lead to pollution of one MAP, by either choosing the bin sizes too large, or too small compared to the stars' inherent abundance errors.

Second, while Ting et al. (2013) makes us confident that the qDF is indeed a good functional form to describe each MAP, it could very well be, that the stars' true distribution is close to but not exactly of the family of assumed qDFs.

We try to investigate both this issues with the following test: We draw two mock data sets, each from a different qDF, and mix the stars in different fractions together. We then analyse this mixture by assuming all stars sill came from a single qDF. The results are shown in fig. 6 and 7.

In example 1 and 2 (fig. 6) we consider two very different MAPs, a hotter and a cooler one, that are mixed together in different fractions. This test could be understood as the true distribution of stars being a linear combination of two very different qDFs and we investigate how this deviation from a single qDF affects the potential recovery. We find that for a MAP that follows approximately a hot population (polluted by up to $\sim 30\%$ of cooler stars), the potential can still be very well recovered. The analysis of cooler MAPs are much more affected by pollution due to hotter MAP stars.

In example 3 and 4 (fig. 7) it is investigated how different the qDF parameters of two MAPs are allowed to be to be still able to constrain the true potential. This test could be seen as a model scenario for decreasing bin sizes in the metallicity-α plane when sorting stars in different MAPs, assuming that there is a smooth variation of qDF within the metallicity-α plane. We find that differences of 20% in the qDF parameters of two neighbouring MAPs can still give quite good constraints on the potential parameters. We compare this with the relative difference in the qDF parameters in the bins in fig. 6 of Bovy & Rix (2013), which have sizes of $[Fe/H] = 0.1$ dex and $\Delta[\alpha/Fe] = 0.05$ dex. It seems that these bin sizes are large enough to make sure that $\sigma_{R,0}$ and $\sigma_{z,0}$ of neighbouring MAPs do not differ more than

20%. As fig. 6 and 7 suggests especially the tracer scale length h_R needs to be recovered to get the potential right. For this parameter however the bin sizes in fig. 6 of Bovy & Rix (2013) might not yet be small enough to ensure no more than 20% of difference in neighbouring h_R, especially in the low-α ($[\alpha/Fe] \lesssim 0.2$), intermediate-metallicity ($[Fe/H] \sim -0.5$) MAPs.

[TO DO: think, if this might better be two different sections. ???? one for MixDiff about neighbouring MAPS and one for MixCont for difference in DF. ????]

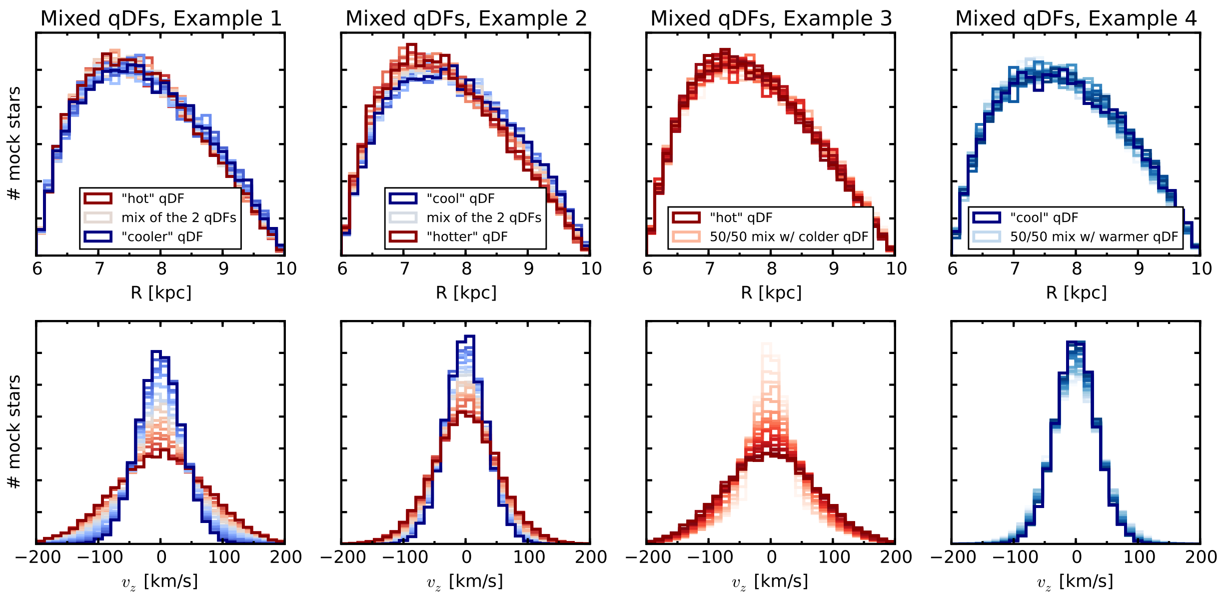

Fig. 5.— [TO DO: Re-write caption] Distribution of mock data in two coordinates (R and v_z), created by mixing different amounts of stars drawn from two different qDFs. The red and blue histograms show data sets drawn from a single qDF only: the "hot" and the "cooler" *MAP*s (Example 1) in the left panels, and the "cool" and "hotter" *MAP*s (Example 2) in the right panels (see table 2). The gray histograms show data drawn from a superpositions of two qDFs. In total there are always 20,000 stars in the data set. The color coding represents the different mixing rates (black - all hot, bright gray - all cool) and is the same as in figure 6, where the corresponding modelling results for each data set are shown. This demonstrates how mixing two qDFs can be used as a test case for adding or removing wings to a pure qDF or slightly changing the radial density profile. The distribution in R is also strongly shaped by the selection function, which is, in this case, a sphere around the sun with $r_{\rm max} = 2$ kpc.

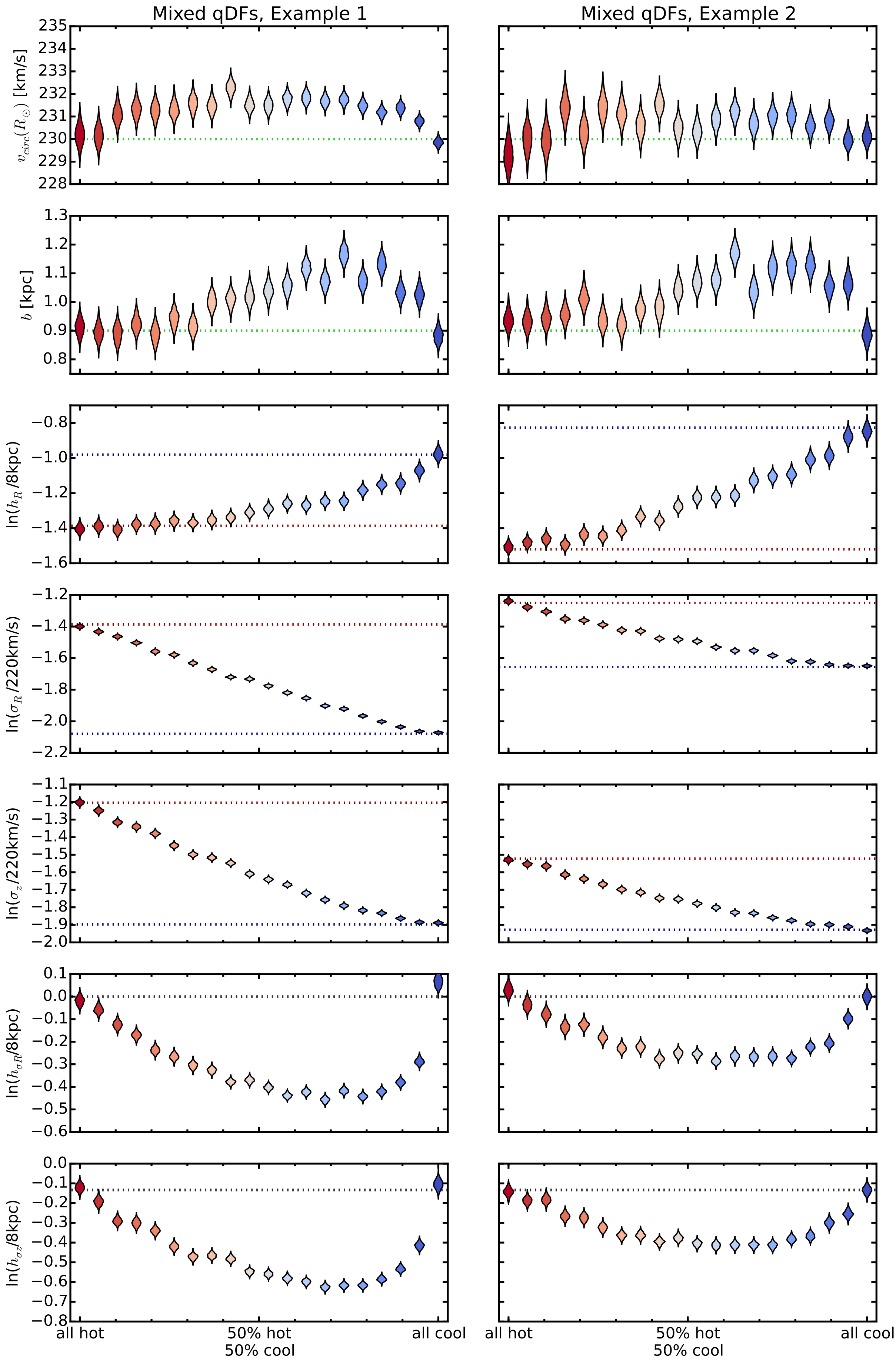

Fig. 6.— (Caption on next page.)

Fig. 6.— (Continued.) [TO DO: Re-write caption] The dependence of the parameter recovery on degree of pollution and 'hotness' of the stellar population. To model the pollution of a hot stellar population by stars coming from a cool population and vice versa, we mix varying amounts of stars from two very different populations, as indicated on the x-axis. (The corresponding mock data sets are shown, in the same gray scales, in fig. 5.) Both populations come from same potential, the isochrone potential "Iso-Pot" from table 1 (true potential parameters are indicated by green lines). The composite data set is then fit with one single qDF. Example 1 (left) mixes the "hot" *MAP* with the "cooler" *MAP* in table 2, while example 2 (right) mixes the "cool" *MAP* with the "hotter" *MAP* . True parameters of the hotter of the two populations are shown as red lines, those of the cooler populations as blue lines. The violines represent the marginalized likelihoods found from the MCMC analysis. [TO DO: This was done using the current qDF to set the fitting range. Nvelocity=24 and Nsigma=5 is high enough (though not perfect). Maybe redo with fiducial qDF to be consistent with MixDiff test. ???]

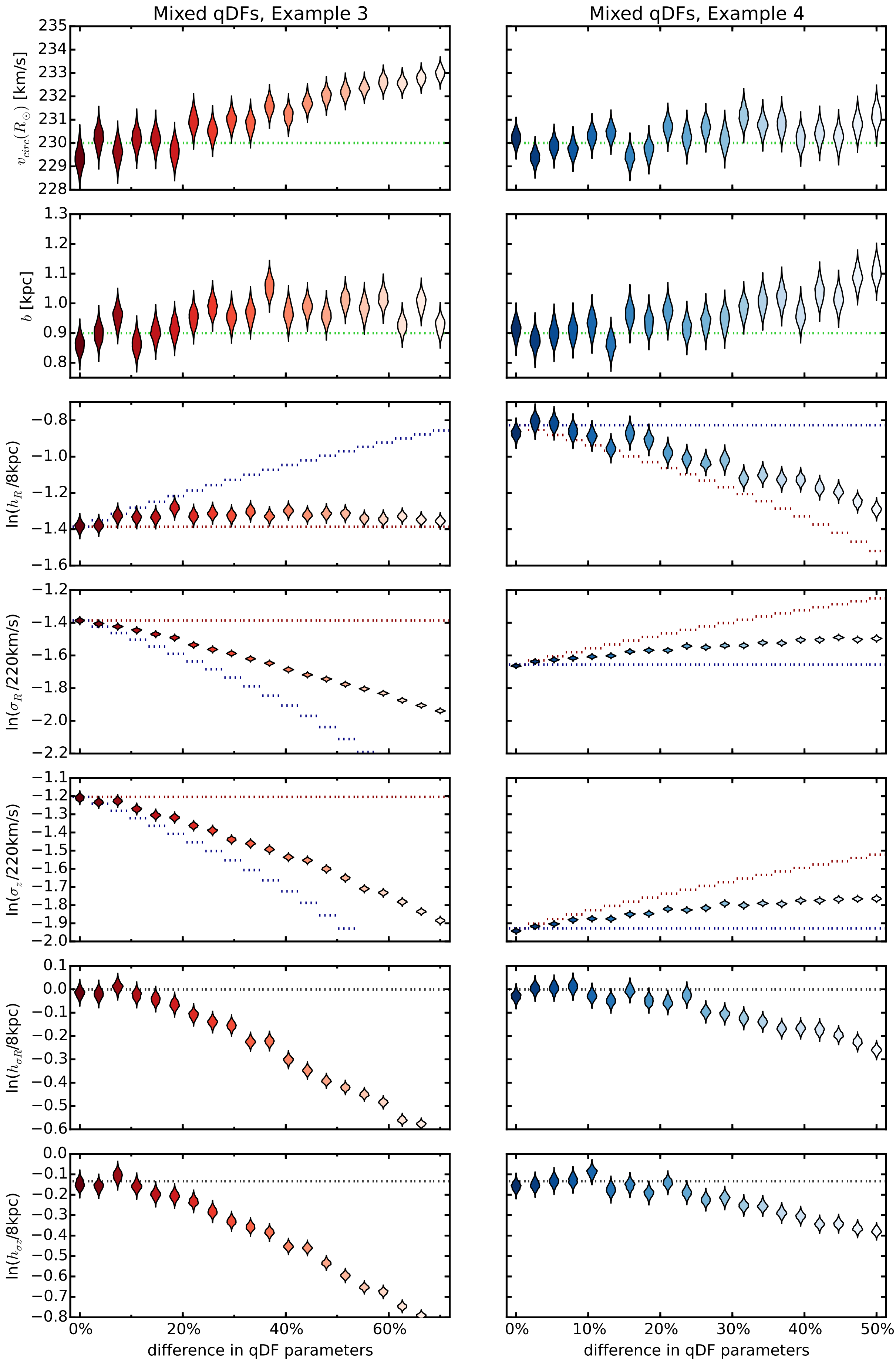

Fig. 7.— (Caption on next page.)

Fig. 7.— (Continued.) The dependence of the parameter recovery on the difference in DF parameters of the mixture of two stellar populations and their 'hotness'. [TO DO], Maybe different/same x-axis??? [TO DO] (This was done using the current qDF to set the fitting range. Nvelocity=24 and Nsigma=5 is not high enough for the largest differences, i.e. grid search and MCMC converge to different values. Redo with fiducial qDF. [TO DO] [TO DO: Add in plot a label, that it is a 50%/50% mix of a hot and a cold population.??])

3.3. What if our assumed potential model differs from the real potential?

Collection of possible tests and plots *Test 1:* Try to recover a Miyamoto-Nagai disk + power-law halo potential by fitting a 2-component Stäckel potential.
Plot 1:

- (R,z)-plane: color coding: difference between true potential's F_R and best fit potential F_R
- (R,z)-plane: color coding: difference between true potential's F_z and best fit potential F_z

  Any idea how to account for the error bars on the best fit potential?

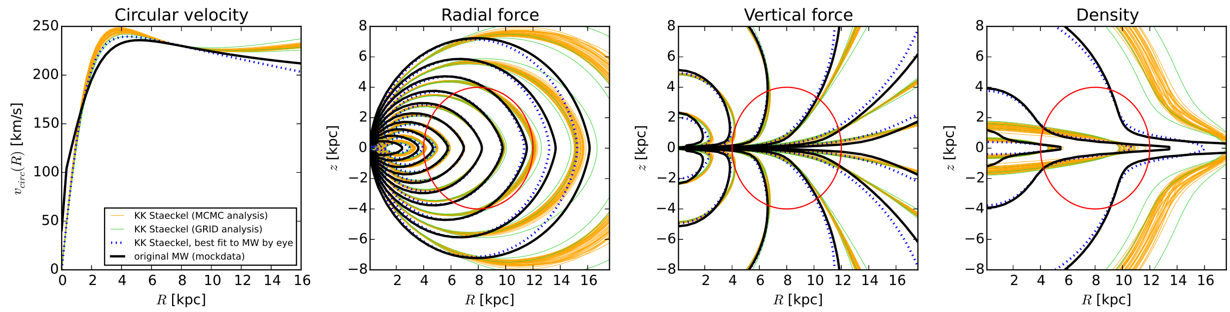

Fig. 8.— [TO DO]

4. Questions that haven't been covered so far:

- What limits the overall code speed?
- What happens, when the errors are not uniform?
- What if errors in distance matter for selection?
- Deviations from axisymmetry: Take numerical simulations.

[TO DO: Check if all references are actually used in paper. ???]

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[TO DO]

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