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1. Introduction

Stellar dynamical modelling is the fundamental tool to infer the gravitational potential of the Milky Way from the positions and motions of its stars (Rix & Bovy 2013; Binney 2011b; Binney & Tremaine 2008) [TO DO: other / better references???]. ~~The o~~observational information on the phase-space coordinates of stars ~~are~~ currently growing at a rapid pace, and will be taken to a whole new level by the upcoming ~~Gaia~~ data. Yet, rigorous and practical modelling tools that turn ~~this information~~ into constraints both on the gravitational potential and on the distribution function (DF) of stellar orbits, are scarce (Rix & Bovy 2013) [TO DO: more references] [TO DO: References that explain that the modelling is scarce, or previous modelling approaches???]

~~Accurately determining~~ the Galactic gravitational potential is fundamental for understanding ~~its~~ dark matter and baryonic structure [TO DO: REF]. ~~Accurately determining~~ the stellar-population dependent orbit distribution function is a ~~fundamental~~ contraint on the

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This reads like the intro to a book, or thesis. The basic elements are there, and should be there. We should just perhaps be a bit more sparing with "far-reaching", "fundamenta;" etc..

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basic

Galaxy's formation history.

Open questions about the MW's potential and structure, on which future modelling attempts will hopefully give more definite answers are: What is the local dark matter density (Zhang et al. 2013; Bovy & Tremaine 2012)? Is the Milky Way's dark matter halo flattened ([TO DO: REF])? Is the MW disk maximal (Sackett 1997) and, to be able to disentangle halo and disk contribution (Dehnen & Binney 1998), what is the disk's overall mass scale length (Bovy & Rix 2013)?

Open questions about the star's distribution within the MW, which dynamical modelling can help to constrain, are: How are stellar kinematics and their chemical abundances are related (Sanders & Binney 2015) [TO DO: REF])? In particular, does the disk have a thin/thick disk dichotomy (Gilmore & Reid 1983) or is it a continuum of many exponential disks (Bovy et al. 2012d)? How does radial migration affect the orbit distribution (Sellwood & Binney 2002; Roškar et al. 2008a,b; Schönrich & Binney 2008; Minchev et al. 2011) [TO DO: These are References from Rix & Bovy 2013 - should I use all of them?]? To address these questions, observed stellar positions and motions need to be turned into full orbits - which stresses again the importance of having a reliable model for the MW's gravitational potential.

In the era of big Galactic surveys all of this could soon be within our reach. Not only will there be full 6D stellar phase-space coordinates for a thousand million of stars measured by Gaia (Perryman et al. 2001) to unprecedented precision by the end of 2016. But already with existing surveys (e.g., SEGUE (Beers et al. 2006), RAVE (Steinmetz et al. 2006), LAMOST (Newberg et al. 2012), APOGEE (Majewski 2012), Gaia-ESO (Gilmore et al. 2012), GALAH (Freeman 2012) [TO DO: I just copied this from ~~Melissas Cannon paper. Should I reference all of them??? Not in reference list yet.]) and sophisticated machine-learning tools (e.g. *The Cannon* by Ness et al. (2015)) to combine them, we will soon have huge data sets at our disposal.~~

In this work we present a rigorous, robust and reliable dynamical modelling machinery, strongly building on previous work by Binney & McMillan (2011); Binney (2012); Bovy & Rix (2013); Bovy (2015) and explicitly developed to exploit and deal with these large data sets in the future.

There is a variety of practical approaches to dynamical modelling of discrete collisionless tracers, such as the stars in the Milky Way (e.g. Jeans modelling: Büdenbender et al. (2015);

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There is a very strong emphasis (so far) on "local" references.

Loebman et al. (2012); action-based DF modelling: Bovy & Rix (2013); Piffl et al. (2014); Sanders & Binney (2015); torus modelling: Made-to-measure modelling: McMillan & Binney (2012, 2013), De Lorenzi et al. (2007); Syer & Tremaine (1996); Bissantz et al. (2004) or Hunt & Kawata (2014); ~~[TO DO: What kind of modelling is Xiangxiang doing?]; Xue et al. (2015))~~. Most of them – explicitly or implicitly – describe the stellar distribution through a distribution function. Actions are good ways to describe orbits, because they are canonical variables with their corresponding angles, have immediate physical meaning, and obey adiabatic invariance (Binney & Tremaine 2008; McMillan & Binney 2008; Binney 2010; Binney & McMillan 2011; Binney 2011b).

Recently, Binney (2012) and Bovy & Rix (2013) [TO DO: are these the correct references???] proposed to combine parametrized axisymmetric potentials with DF's that are simple analytic functions of the three orbital actions to model discrete data. Binney (2010) and Binney & McMillan (2011) had proposed a set of simple action-based (quasi-isothermal) distribution functions (qDF). Ting et al. (2013) and Bovy & Rix (2013) showed that these qDF's may be good descriptions of the Galactic disk, when one only considers so-called mono-abundance populations (*MAP*), i.e. sub-sets of stars with similar [Fe/H] and [α/Fe] (Bovy et al. 2012b,c,d).

Bovy & Rix (2013) implemented a modelling approach that put action-based DF modelling of the Galactic disk in an axisymmetric potential in practice. Given an assumed potential and an assumed DF, they directly calculated the likelihood of the observed $(\vec{x}, \vec{v})$ for each sub-set of *MAP* among SEGUE Gdwarf (Yanny et al. 2009). This modelling also accounted for the complex, but known selection function of the kinematic tracers. For each *MAP* , the modelling resulted in a constraint of its DF, and an independent constraint on the gravitational potential, which members of all *MAP*s feel the same way.
Taken as an ensemble, the individual *MAP* models constrained the disk surface mass density over a wide range of radii ($\sim 4-9$ kpc), and proved a powerful constraint on the disk mass scale length ($\sim$ 2 kpc) and on the disk to dark matter ratio at the Solar radius [TO DO: quote number???].

Yet, these recent models still leave us poorly prepared with the wealth and quality of the existing and upcoming data sets. This is because Bovy & Rix (2013) made a number of quite severe and idealizing assumptions about the potential, the DF and the knowledge of observational effects (such as the selection function). All these idealizations are likely to translate into systematic error on the inferred potential or DF, well above the formal error

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bars of the upcoming data sets.

In this work we present *RoadMapping* ("Recovery of the Orbit Action Distribution of Mono-Abundance Populations and Potential INference for our Galaxy") – an improved and refined version of the original modelling machinery by Bovy & Rix (2013), making extensive use of the *galpy* python package (Bovy 2015). *RoadMapping* relaxes some of the restraining assumptions Bovy & Rix (2013) ~~had to~~ made, is more flexible and more adept in dealing with large data sets. In this paper we set out to explore the robustness of *RoadMapping* against the breakdowns of some of the most important assumptions of DF-based dynamical modelling. What is it about the data, the model and the machinery itself, that limits our recovery of the true gravitational potential?

In the light of Gaia we ~~explicitely~~ analyze how well the modelling machinery behaves in the limit of large data. For a huge number of stars three ~~statistical~~ aspects become important, that ~~are~~ hidden behind Poisson noise for smaller data sets: (i) We have to make sure that our modelling is an un-biased ~~and asymptotically normal estimator~~ (§3.1). (ii) Numerical inaccuracies in the actual modelling machinery ~~start to matter and need to be avoided~~ (§2.5). (iii) ~~P~~arameter estimates become so precise, ~~that we start to be able to distinguish between similar models. We therefore~~ want more flexibility and more free fit parameters in the potential and DF model. The modelling machinery itself needs to be flexible and fast in effectively finding the best fit parameters for a large set of parameters. The improvements made to the machinery used in Bovy & Rix (2013) are presented in §2.6.

~~Different characteristics of the data might influence the success of~~ the parameter recovery. (i) In an era where we can choose data from different MW surveys, it might be worth to explore if different regions within the MW (i.e. differently shaped or positioned survey volumes) are especially diagnostic to recover the potential (§3.2). (ii) What happens if our knowledge about the selection function, specifically the completeness of the data set within the survey volume, is ~~not perfect~~ (§3.3)? (iii) How to account for measurement errors in the modelling (§3.4)?

One of the strongest assumptions is to restrict the dynamical modelling to a certain family of parametrized models. We investigate how well we can we hope to recover the true potential, when our potential and DF models ~~deviate from~~ the true potential and DF. For the DF we specifically investigate two of our assumptions in §3.5: First, what would happen if the stars within *MAP*s do intrinsically not follow a single qDF as assumed by Ting et al.

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We also explore how different aspects of the observational experiment design impact:

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(2013); Bovy & Rix (2013). ~~Second, and assuming *MAPs* do indeed follow the qDF, what would be the effect of pollution of *MAPs* through stars from neighbouring *MAPs* in the ([Fe/H],[α/Fe]) plane due to too big abundance errors or bin sizes.~~ ~~And last but not least~~ we test in §3.6 how well the modelling works, if our assumed potential family deviaties from the true potential.

For all of these aspects we show some plausible and illustrative examples on the basis of investigating mock data. The mock data is generated from galaxy models presented in §2.1-2.3 following the procedure in §2.4, analysed according to the description of the machinery in §2.5-2.6 and the results are presented in §3 and discussed in §4.

The strongest assumption that goes into this kind of dynamical modelling might be the idealization of the Galaxy to be axi-symmetric and being in steady state. We do not investigate this within the scope of this paper but strongly suggest a systematic investigation of this for future work.

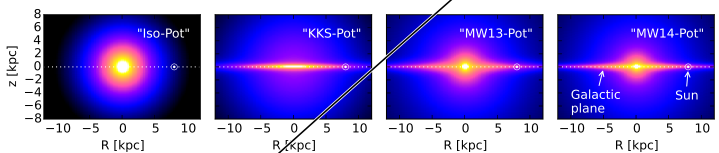

Fig. 1.— Density distribution of the four reference galaxy potentials in Table 1, for illustration purposes. These potentials are used throughout this work for mock data creation and potential recovery. [TO DO: Halo sichtbarer machen, evtl. mit isodensity contours]

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2. Dynamical Modelling

2.1. Actions and Potential Models

Actions. Orbits in axisymmetric potentials are best described and fully specified by the three actions J_R, J_z and $J_\phi = L_z$. They are integrals of motion and generally defined as

$$J_i = \frac{1}{2\pi}\int_{\mathrm{orbit}} p_i(t)\,\mathrm{d}x_i(t) \qquad (1)$$

and depend on the potential via the connection between position x_i and momentum p_i along the orbit. Actions have a clear physical meaning: They quantify the amount of oscillation in each coordinate direction of the full orbit [REF]. The position of a star along the orbit is denoted by a set of angles, which form together with the angles a set of canonical conjugate phase-space coordinates (Binney & Tremaine 2008). Even though actions are the optimal choice as orbit labels and arguments for stellar distribution functions, their computation is very expensive.

Action calculation. The action calculation depends on the choice of potential in which the star moves: The spherical isochrone (Binney & Tremaine 2008) is the only potential for which Equation (1) takes an analytic form. For axisymmetric Stäckel potentials actions can be calculated exactly by the (numerical) evaluation of a single integral. In all other potentials numerically calculated actions will always be approximations, unless Equation (1) is integrated up to infinity. A computational fast way to get actions for arbitrary axisymmetric potentials is the "Stäckel fudge" by Binney (2012), which locally approximates the potential by a Stäckel potential. To speed up the calculation even more, an interpolation grid for J_R and J_z in energy E, angular momentum L_z and [TO DO: what else???] can be build out of these Stäckel fudge actions, as described in Bovy (2015).[1]

Potential models. In our modelling we assume a family of parametrized potentials with a fixed number of free parameters. We use different kinds of potentials: Besides the Milky Way like potential from Bovy & Rix (2013) ("MW13-Pot") with bulge, disk and halo, we also extensively use the spherical isochrone potential ("Iso-Pot") in our test suites to make use of the analytic (and therefore exact and fast) way to calculate actions. In addition we use the 2-component Kuzmin-Kutuzov Stäckel potential by Batsleer & Dejonghe (1994) ("KKS-Pot"), which displays a disk and halo structure and also provides exact actions. Table 1

[1] [TO DO: Write which numerical accuracy I needed for the grid, as the default values were not good enough.]

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