Sets and Indices

S: sets of all stations, indexed by i, i', j, j' (ordered)

OD: set of pairs of origin-destination, indexed by (i,j)

K: Set of cabins, indexed by k

C: Set of control classes, indexed by c, c'

C_k: Set of control classes belonging to cabin k

$Leaf_k$ / $Root_k$: Leaf / Root control class in cabin k

$Child_c$: Set of immediate children of class c

$Parent_c$: Set of immediate parent of class c

$S_{(i,j)}$: Set of stations in the route of the OD (i,j)

OD_j: Set of pairs OD passing through station j (where j is neither the origin station, nor the destin)

OD_i^{orig}: Set of pairs OD for which i is the origin-station.

$Routes_{(ij)}$: Set of possible routes to travel from station i to station j (indexed by r). A route is represented as a set of OD pairs.

Parameters

Q_K: Capacity of cabin K

P_{cij}: Price of control class c for the OD (i,j)

d_{ctij}: demand of control class c in period t for the OD (i,j)

F_{ctij}: 1 if $d_{ctij} = 0$; 0 if $d_{ctij} > 0$ (Binary parameter)

Variáveis

X_{ctij}: Number of seats reserved for class c in period t, for OD (i,j)

Y_{cij}: Authorization level for class c and OD (i,j)

A_{iK}: Number of seats availables at station i in cabin K

α_{ctij}: 1 if $X_{ctij} > 0$, $\emptyset$ if $X_{ctij} = \emptyset$

γ_{cij}: 1 if $Y_{cij} > 0$, $\emptyset$ if $Y_{cij} = \emptyset$

β_{cij} : 1 if class c is the cheapest class authorized for OD (i,j)

Constraints

Restrições de fluxo de passageiros.

$$\sum_{(i,j)\in OD_{i'}^{orig}} \sum_{c\in C_k} \sum_{t\in T} x_{ctij} \leq A_{i'k} \quad \forall i'\in S,\ \forall k\in K \quad (1)$$

A soma total de passageiros saindo da estação origem i' não pode exceder o # de assentos vagos nesta estação

$$\underbrace{A_{i'k}}_{\text{Assentos disponíveis na estação precedente}} = A_{i'-1k} - \underbrace{\sum_{(i,j)\in OD_{i'}} \sum_{c\in C_k} \sum_{t\in T} x_{ctij}}_{\text{Passageiros passando pela estação } i' \text{ (Que vão transicionar)}} \quad \forall i'\in S,\ k\in K \quad (2)$$

Linking constraints for assignment and authorizations

$$y_{cit} \geq \sum_{t\in T} x_{ctij} + y_{c'ij} \quad \forall c\in C,\ c'\in Child_c,\ (i,j)\in OD \quad (3)$$

AU is always greater than the AU of the child + Protection/assignment

$$\underbrace{y_{cit} \overset{=}{\leq} Q_k}_{\text{Upper Bound for root c.c. (it can even be =)}} \quad \forall c\in Root\,C_k,\ k\in K \quad (4)$$

Demand Constraints

$$X_{ctij} \leq d_{ctij} \qquad \forall c \in C, t \in T, (i,j) \in OD \qquad (5)$$

fulfillment over periods

$$\alpha_{ctij} \leq X_{ctij} \leq \alpha_{ctij}\, d_{ctij} \qquad \forall c \in C, t \in T, (i,j) \in OD \qquad (6)$$

$$\underbrace{\alpha_{ctij} \leq \alpha_{ct+1ij}}_{} \qquad \forall c \in C, t \in T : t \neq |T|, (i,j) \in OD \qquad (7)$$

For a given period, we can only reserve seats for a given class at a given OD if we reserve the seats demanded in previous periods

if $d_{ctij} = 0 \longrightarrow X_{ctij} = 0$ because demand constraints.

and $\alpha_{ctij} = 0$. We need $\alpha_{ctij} = 1$ if $X_{ctij} = 0$ because $d_{ctij} = 0$

Let F_{ctij} be a parameter $= 1$ if $d_{ctij} = 0$, $= 0$ if $d_{ctij} > 0$. Then constraints (6) can be replaced by:

$$\boxed{\alpha_{ctij} \leq X_{ctij} + F_{ctij} \leq \alpha_{ctij}\, d_{ctij}} \qquad \forall c \in C, t \in T, (i,j) \in OD$$

I also think that your original (2.11) won't be neccesary.

Linking constraints for authorizations

$$r_{cij} \le y_{cij} \le V_{cij} Q_k \qquad \forall k \in K, c \in C_k, (i,j) \in OD$$

$$B_{cij} = r_{cij} \qquad \forall k \in K, c \in leaf_{C_k}, (i,j) \in OD$$

$$\underbrace{B_{cij} = r_{cij} - B_{c'ij}}_{} \qquad \forall k \in K, c \in C_k : c \neq leaf_{C_k}, c' \in Child_c$$

Forces $B_{cij} = 1$ if c is the lowest authorized class for OD (i,j)

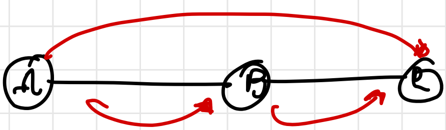
Skip lagging Constraints

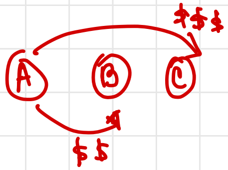
The following 2 set of constraints ensure that the cheapest way to travel for a given OD (i,j) is directly and not via other ODs, no matter the offer available.

The next constraints ensure that tickets avail. for a given OD, example from i, to j, are always cheaper than the sum of tickets for other ODs in the route of (i,j)

- Let $Routes_{(i,j)}$ be the set of possible non-direct routes to travel from estation i to j. A route is defined as a list of at least 2 pairs of OD.

$$\sum_{c \in C_k} B_{cij} P_{cij} \le \sum_{c \in C_k} \sum_{(i',j') \in r} B_{ci'j'} P_{ci'j'} \qquad \forall (i,j) \in OD, r \in Routes_{(i,j)}$$





This constraints ensure that the lowest price available for a given OD (i,j) is cheaper than any other ticket available for an OD departing from the same station and arriven in a further estation than j

$$\sum_{c\in C_k} \beta_{cij}\, P_{cij} \le \sum_{c\in C_k} \beta_{cij'}\, P_{cij'} \qquad \forall k\in K, (i,j)\in OD; j'\in S : j'>j$$

Initializing variables

$$A_{0k} = Q_k \qquad \forall k\in K$$