TAKING MULTI-OBJECTIVE GAMES TO THE NEXT LEVEL

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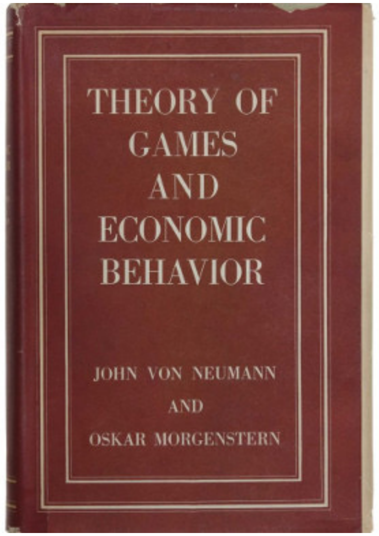
OVERVIEW

- What are multi-objective games?
- The utility-based approach
- Existence guarantees
- Algorithms
- What's next
- Q&A

Multi-objective games present a natural framework for studying ***strategic interactions between rational individuals concerned with more than one objective***.

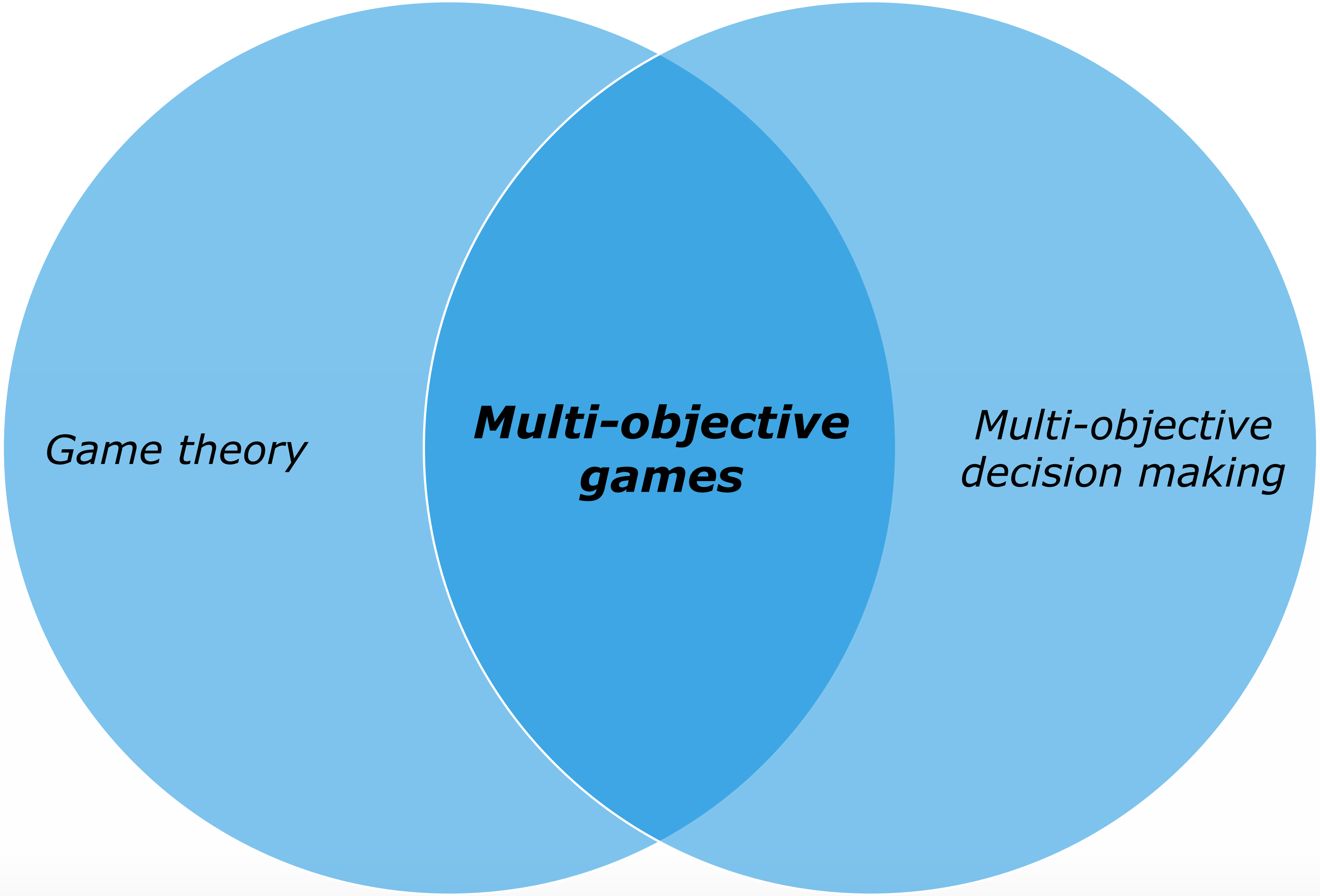
Strategic interactions between rational individuals




Game theory

Rational individuals concerned with more than one objective

Multi-objective decision making


Game theory
Multi-objective games
Multi-objective decision making

Multi-Objective Normal-Form Games (MONFGs)

[Blackwell, 1954]

	A	B
A	(10, 2); (10, 2)	(2, 3); (2, 3)
B	(4, 2); (4, 2)	(6, 3); (6, 3)

Multi-Objective Normal-Form Games (MONFGs)

Called ***actions*** or ***pure strategies***

	A	B
A	(10, 2); (10, 2)	(2, 3); (2, 3)
B	(4, 2); (4, 2)	(6, 3); (6, 3)

Multi-Objective Normal-Form Games (MONFGs)

Called a ***mixed strategy***

	$\frac{1}{2}$	A	B
A	$\frac{1}{2}$	(10, 2); (10, 2)	(2, 3); (2, 3)
B	$\frac{1}{2}$	(4, 2); (4, 2)	(6, 3); (6, 3)

UTILITY-BASED APPROACH

BASICS

It's not obvious how to compare these two outcomes

Option A

(1, 2)

Option B

(2, 1)

But we can still pick our preferred option!

Option A

(1, 2)

Option B

(2, 1)

UTILITY-BASED APPROACH

BASICS

Assume that all decision-makers have a utility function

[Roijers et al., 2013]

$$u_i\colon \mathbb{R}^d \to \mathbb{R}$$

UTILITY-BASED APPROACH

BASICS

$$u_2(p_1, p_2) = p_1 + p_2$$

$$u_1(p_1, p_2) = p_1 \cdot p_2$$

	A	B
A	(10, 2); (10, 2)	(2, 3); (2, 3)
B	(4, 2); (4, 2)	(6, 3); (6, 3)

[Rădulescu et al., 2020]

UTILITY-BASED APPROACH

BASICS

$$u_2(p_1, p_2) = p_1 + p_2$$

$$u_1(p_1, p_2) = p_1 \cdot p_2$$

	A	B
A	(10, 2); (10, 2)	(2, 3); (2, 3)
B	(4, 2); (4, 2)	(6, 3); (6, 3)

How and when to apply this utility function?

[Rădulescu et al., 2020]

OPTIMISATION CRITERIA

EXAMPLE

$\boldsymbol{p} = (4, 2)$

Car

$\boldsymbol{p} = (2, 4)$

Bike

$$u(p_1, p_2) = p_1 \cdot p_2$$

Speed *Eco-friendliness*

What happens when you take the car 50% of the time and the bike 50% of the time?

OPTIMISATION CRITERIA

EXAMPLE

$\boldsymbol{p} = (4, 2)$

Car

$\boldsymbol{p} = (2, 4)$

Bike

$$u(p_1, p_2) = p_1 \cdot p_2$$

Expected Scalarised Returns (ESR)

OPTIMISATION CRITERIA

EXAMPLE

$\boldsymbol{p} = (4, 2)$

Car

$\boldsymbol{p} = (2, 4)$

Bike

$$u(p_1, p_2) = p_1 \cdot p_2$$

ESR

OPTIMISATION CRITERIA

EXAMPLE

$\boldsymbol{p} = (4, 2)$

Car

$\boldsymbol{p} = (2, 4)$

Bike

$$u(p_1, p_2) = p_1 \cdot p_2$$

$u(4, 2) = 8$

ESR

$u(2, 4) = 8$

OPTIMISATION CRITERIA

EXAMPLE

$\boldsymbol{p} = (4, 2)$

Car

$\boldsymbol{p} = (2, 4)$

Bike

$$u(p_1, p_2) = p_1 \cdot p_2$$

ESR

$$u(4, 2) = 8$$

$$u(2, 4) = 8$$

$$\frac{1}{2} \cdot 8 = 4$$

$$\frac{1}{2} \cdot 8 = 4$$

OPTIMISATION CRITERIA

EXAMPLE

$\boldsymbol{p} = (4, 2)$ $\boldsymbol{p} = (2, 4)$

Car *Bike*

$$u(p_1, p_2) = p_1 \cdot p_2$$

$u(4, 2) = 8$

ESR

$u(2, 4) = 8$

$\frac{1}{2} \cdot 8 = 4$ $\xrightarrow{+}$ 8 $\xleftarrow{+}$ $\frac{1}{2} \cdot 8 = 4$

$$\mathbb{E}u(\boldsymbol{p}) = 8$$

OPTIMISATION CRITERIA

EXAMPLE

$\boldsymbol{p} = (4, 2)$

Car

$\boldsymbol{p} = (2, 4)$

Bike

$$u(p_1, p_2) = p_1 \cdot p_2$$

Scalarised Expected Returns (SER)

OPTIMISATION CRITERIA

EXAMPLE

$\boldsymbol{p} = (4, 2)$

Car

$\boldsymbol{p} = (2, 4)$

Bike

$$u(p_1, p_2) = p_1 \cdot p_2$$

SER

OPTIMISATION CRITERIA

EXAMPLE

$\boldsymbol{p} = (4, 2)$

Car

$\boldsymbol{p} = (2, 4)$

Bike

$$u(p_1, p_2) = p_1 \cdot p_2$$

$$\frac{1}{2} \cdot (4, 2) = (2, 1)$$

SER

$$\frac{1}{2} \cdot (2, 4) = (1, 2)$$

OPTIMISATION CRITERIA

EXAMPLE

$\boldsymbol{p} = (4, 2)$

Car

$\boldsymbol{p} = (2, 4)$

Bike

$$u(p_1, p_2) = p_1 \cdot p_2$$

SER

$$\frac{1}{2} \cdot (4, 2) = (2, 1) \xrightarrow{+} (3, 3) \xleftarrow{+} (1, 2) = \frac{1}{2} \cdot (2, 4)$$

OPTIMISATION CRITERIA

EXAMPLE

$\boldsymbol{p} = (4, 2)$

Car

$\boldsymbol{p} = (2, 4)$

Bike

$$u(p_1, p_2) = p_1 \cdot p_2$$

$$\frac{1}{2} \cdot (4, 2) = (2, 1)$$

SER

$$\xrightarrow{+} (3, 3) \xleftarrow{+}$$

$$\frac{1}{2} \cdot (2, 4) = (1, 2)$$

$$u(\mathbb{E}\boldsymbol{p}) = 9$$

OPTIMISATION CRITERIA

EXAMPLE

$\boldsymbol{p} = (4, 2)$

← *Car*

$\boldsymbol{p} = (2, 4)$

→ *Bike*

$$u(p_1, p_2) = p_1 \cdot p_2$$

What happens when you take the car 50% of the time and the bike 50% of the time?

ESR = 8

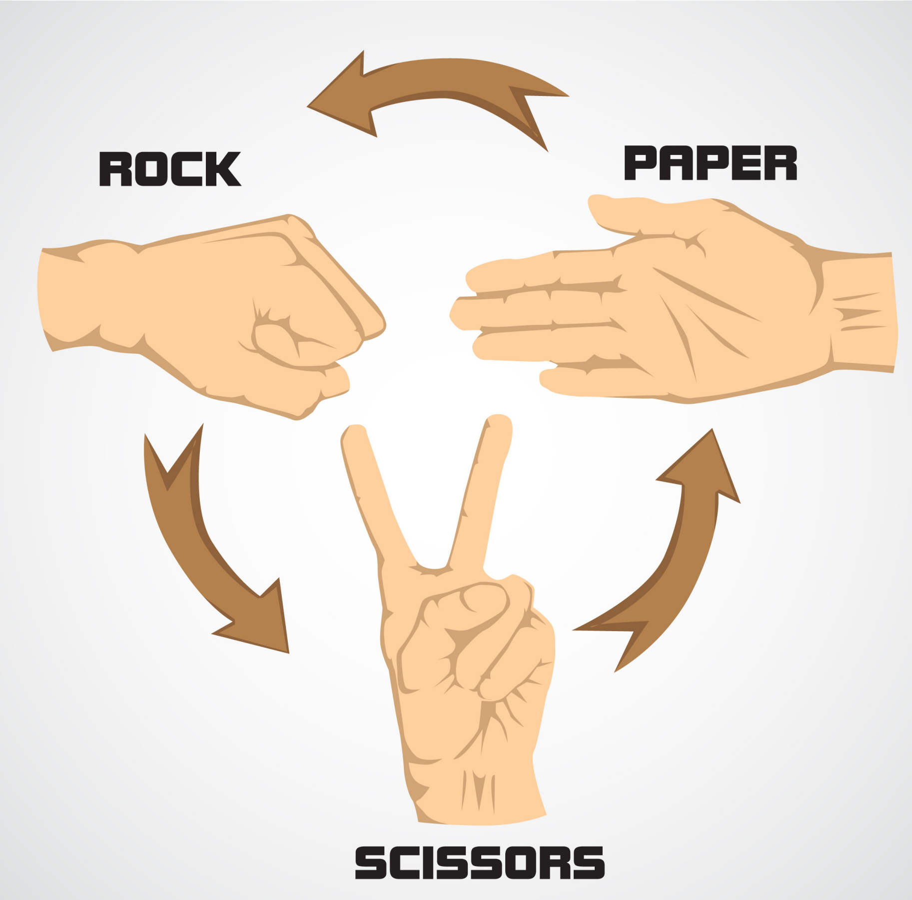
SER = 9

NASH EQUILIBRIUM
BACKGROUND

- Nash equilibrium
  - *No agent can improve their utility by unilatteraly deviating from the joint strategy*

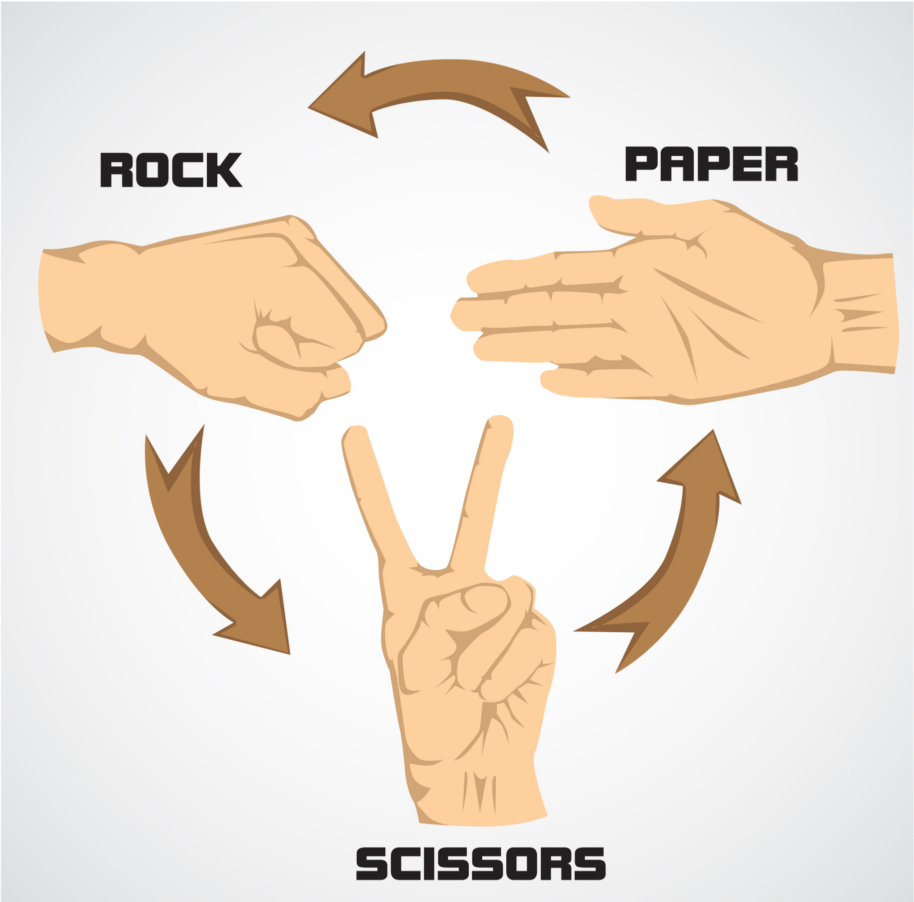
NASH EQUILIBRIUM

BACKGROUND



NASH EQUILIBRIUM

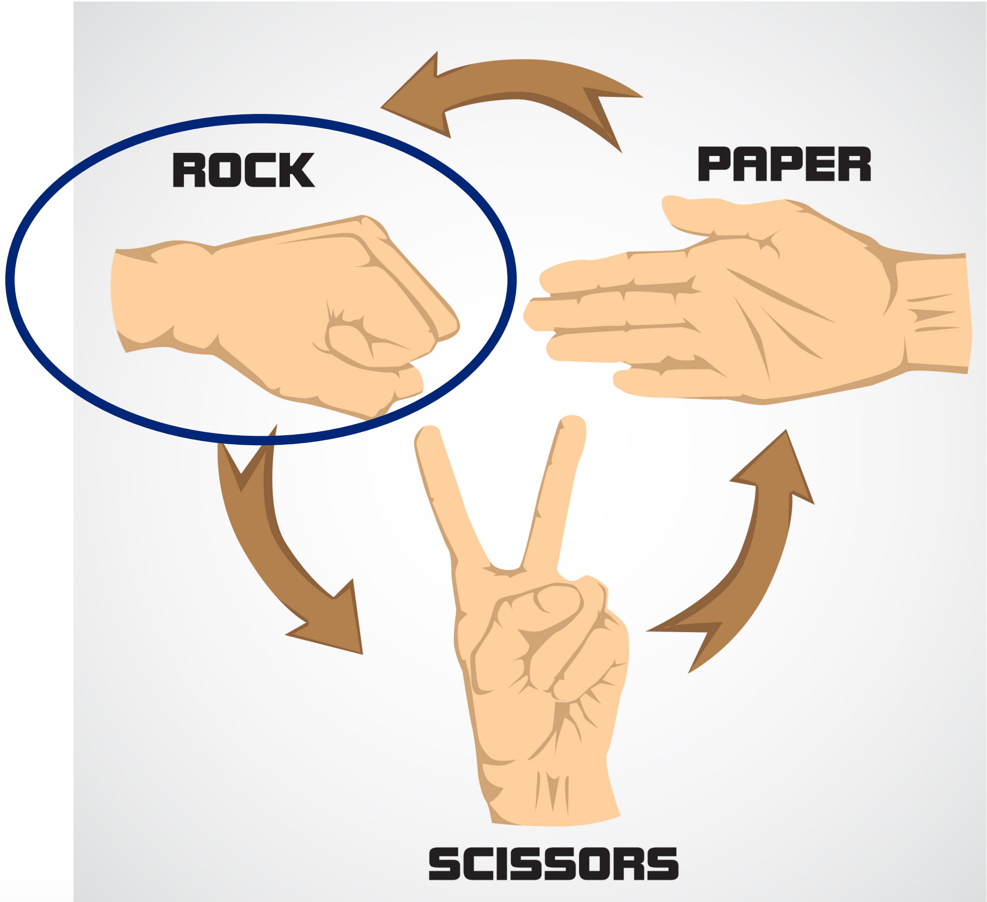
BACKGROUND



We're looking for a joint strategy where noone has an incentive to change their strategy

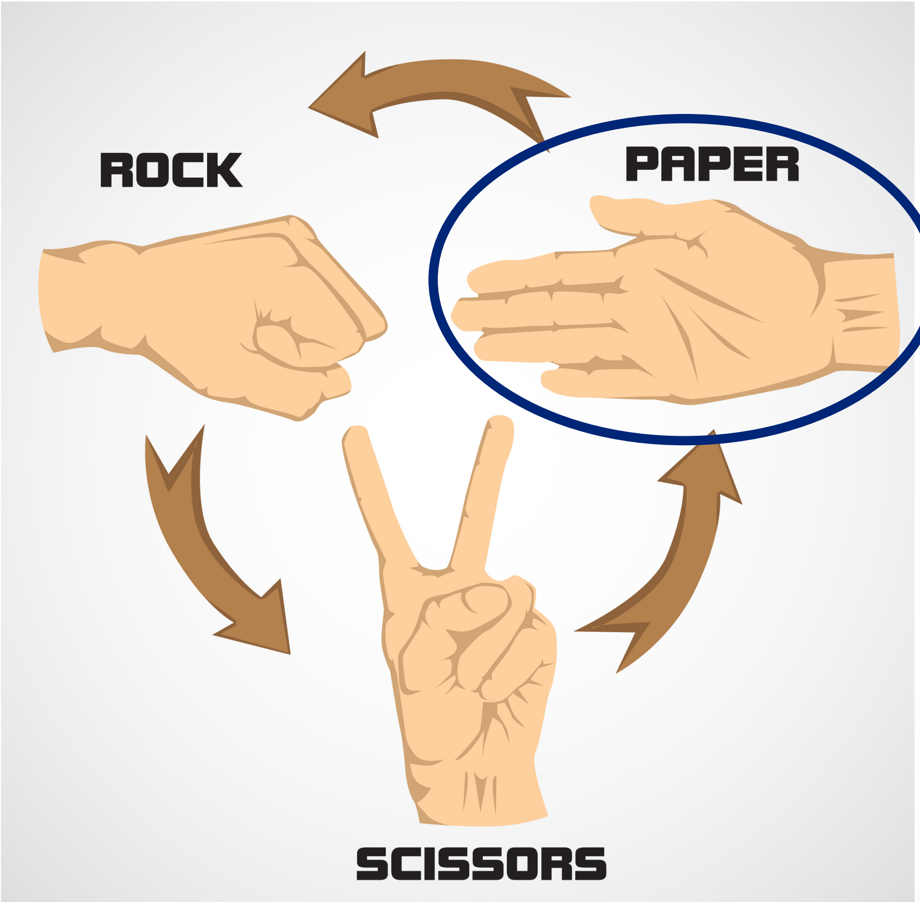
NASH EQUILIBRIUM

BACKGROUND

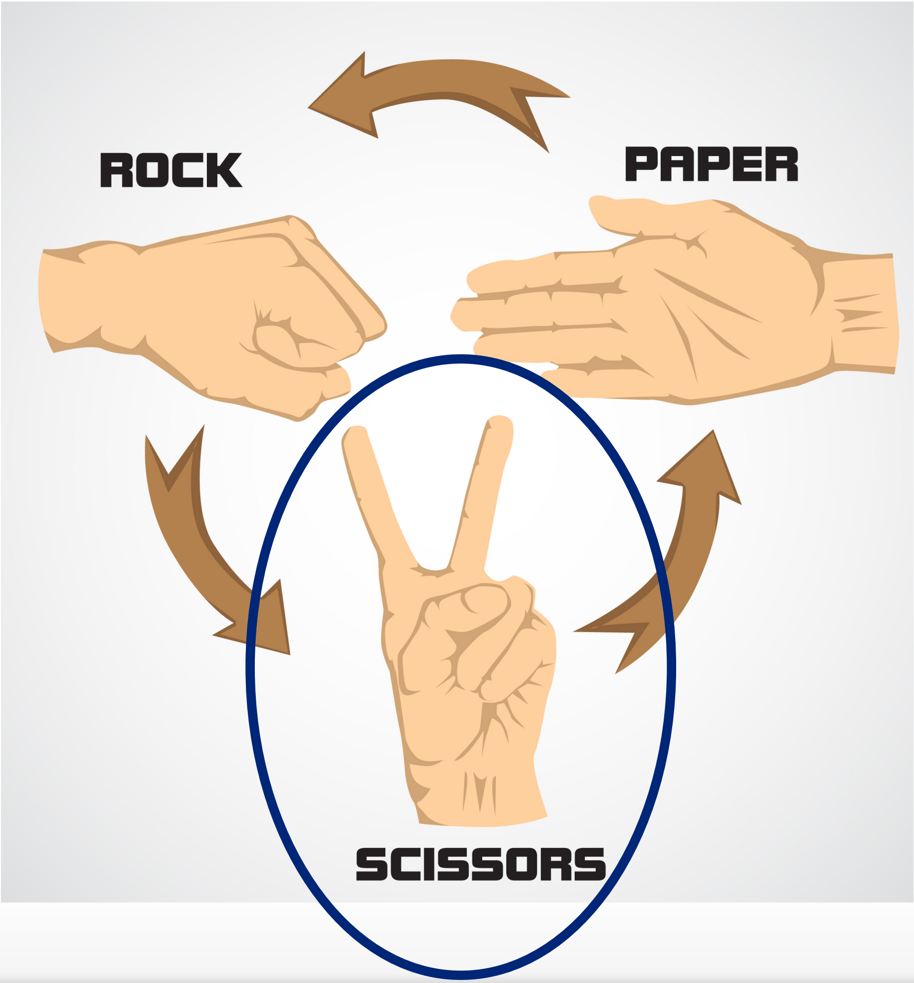


NASH EQUILIBRIUM

BACKGROUND

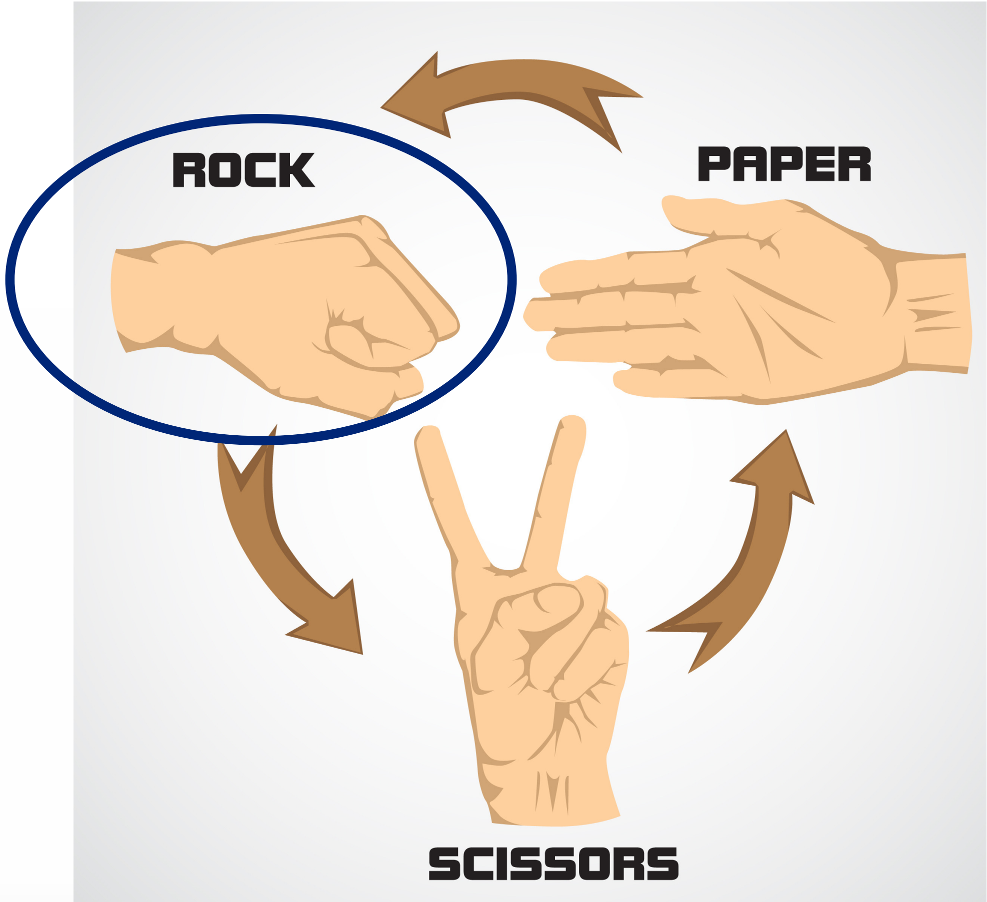


NASH EQUILIBRIUM
BACKGROUND



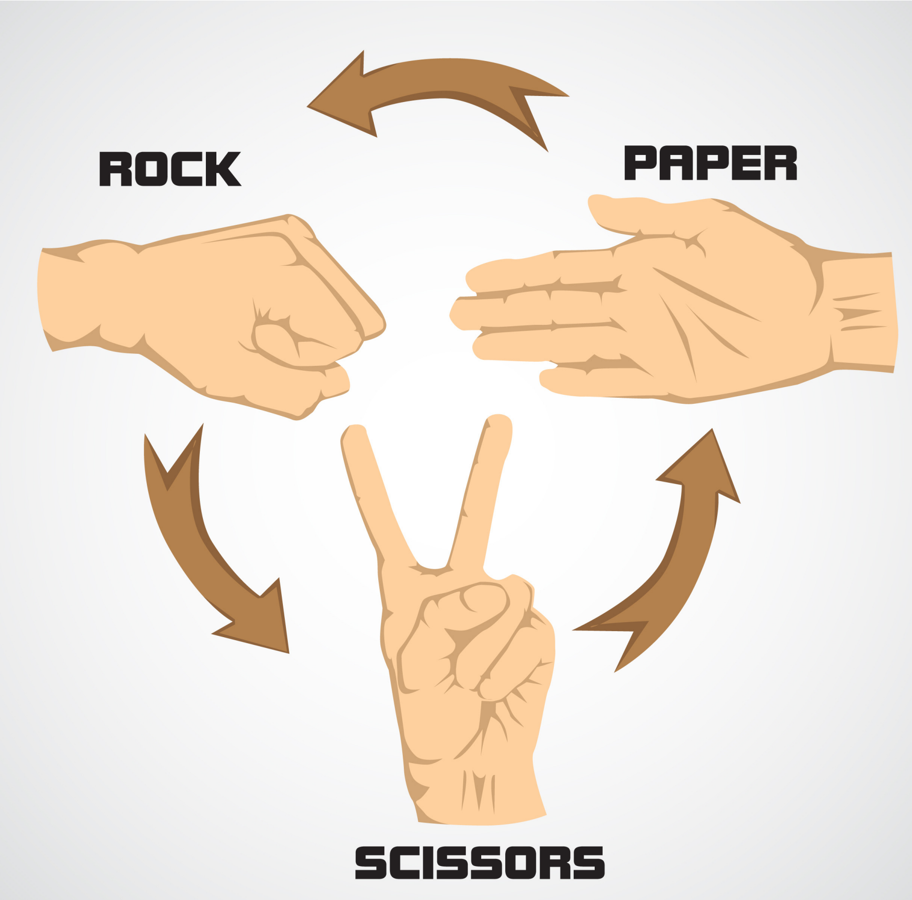
NASH EQUILIBRIUM

BACKGROUND



NASH EQUILIBRIUM

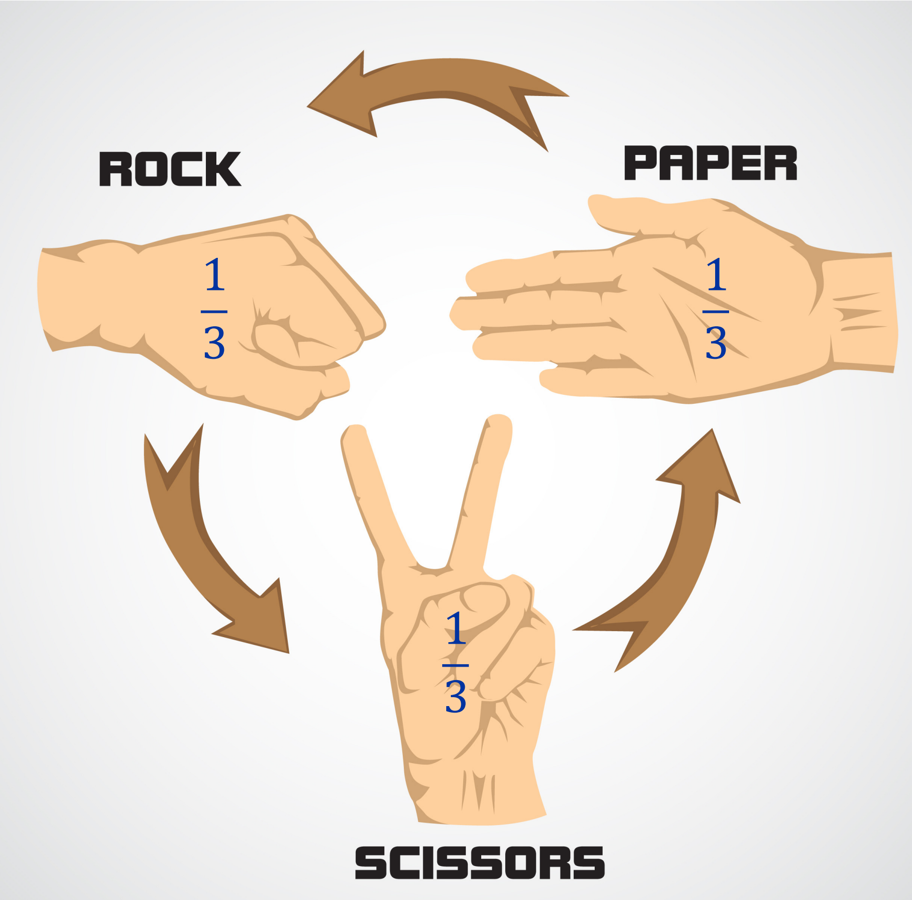
BACKGROUND



We're looking for a joint strategy where noone has an incentive to change their strategy

NASH EQUILIBRIUM

BACKGROUND



Whatever you do in response, you will break even in expectation.

NASH EQUILIBRIUM
BACKGROUND

This is also true for your opponent!

You're playing a
Nash equilibrium

NASH EQUILIBRIUM

BACKGROUND

- Nash equilibria
  - *No agent can improve their utility by unilatteraly deviating from the joint strategy*

$$u_1(p_1, p_2) = u_2(p_1, p_2) = p_1 \cdot p_2$$

	A	B
A	(10, 2); (10, 2)	(0, 0); (0, 0)
B	(0, 0); (0, 0)	(2, 10); (2, 10)

NASH EQUILIBRIUM
BACKGROUND

- Nash equilibria
  - *No agent can improve their utility by unilatteraly deviating from the joint strategy*

$$u_1(p_1, p_2) = u_2(p_1, p_2) = p_1 \cdot p_2$$

Nash equilibrium

$$u_1(10, 2) = 10 \cdot 2 = 20$$
$$u_2(10, 2) = 10 \cdot 2 = 20$$

	A	B
A	(10, 2); (10, 2)	(0, 0); (0, 0)
B	(0, 0); (0, 0)	(2, 10); (2, 10)

NASH EQUILIBRIUM

BACKGROUND

- Nash equilibria
  - *No agent can improve their utility by unilatteraly deviating from the joint strategy*

$$u_1(p_1, p_2) = u_2(p_1, p_2) = p_1 \cdot p_2$$

Nash equilibrium

$u_1(10, 2) = 10 \cdot 2 = 20$
$u_2(10, 2) = 10 \cdot 2 = 20$

	A	B
A	(10, 2); (10, 2)	(0, 0); (0, 0)
B	(0, 0); (0, 0)	(2, 10); (2, 10)

Strictly worse to deviate to B

NASH EQUILIBRIUM
BACKGROUND

- Nash equilibria
  - *No agent can improve their utility by unilatteraly deviating from the joint strategy*

$$u_1(p_1, p_2) = u_2(p_1, p_2) = p_1 \cdot p_2$$

Strictly worse to deviate to B

Nash equilibrium

$u_1(10, 2) = 10 \cdot 2 = 20$
$u_2(10, 2) = 10 \cdot 2 = 20$

	A	**B**
A	(10, 2); (10, 2)	(0, 0); (0, 0)
B	(0, 0); (0, 0)	(2, 10); (2, 10)

NASH EQUILIBRIUM

BACKGROUND

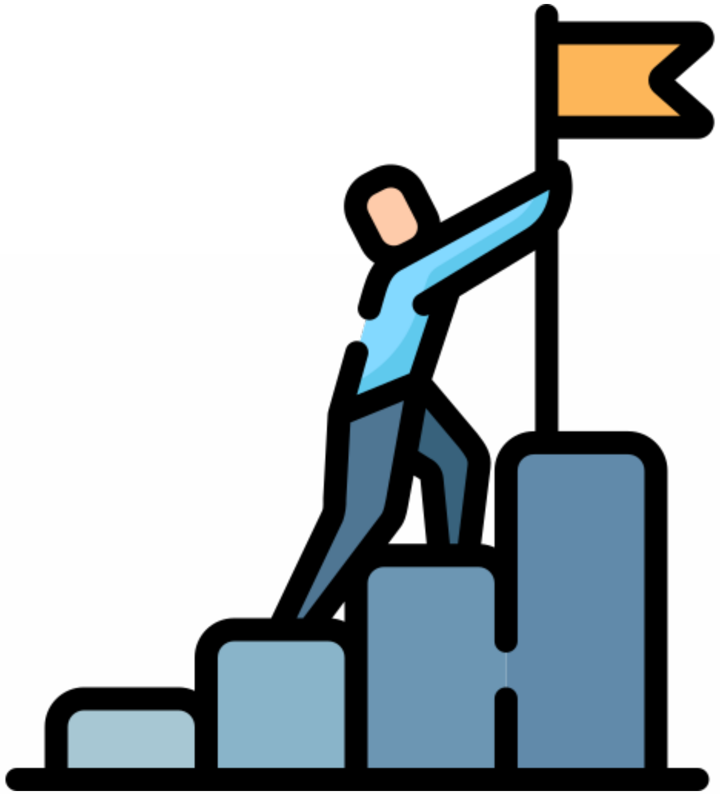
- Nash equilibria
  - *No agent can improve their utility by unilatteraly deviating from the joint strategy*

Not guaranteed to exist in general!

[Rădulescu et al., 2020]

GOAL

- Theoretical
  - Relation with other games
  - Existence or non-existence guarantees
- Algorithms
  - Computing equilibria
  - (Learning equilibria)



WHAT ARE MULTI-OBJECTIVE GAMES?

A NOVEL INTUITION

	A	B	C
A	(4, 1); (4, 1)	(1, 2); (4, 2)	(2, 1); (1, 2)
B	(3, 1); (2, 3)	(3, 2); (6, 3)	(1, 2); (2, 1)
C	(1, 2); (2, 1)	(2, 1); (1, 2)	(1, 3); (1, 3)

[Röpke et al., 2022]

WHAT ARE MULTI-OBJECTIVE GAMES?

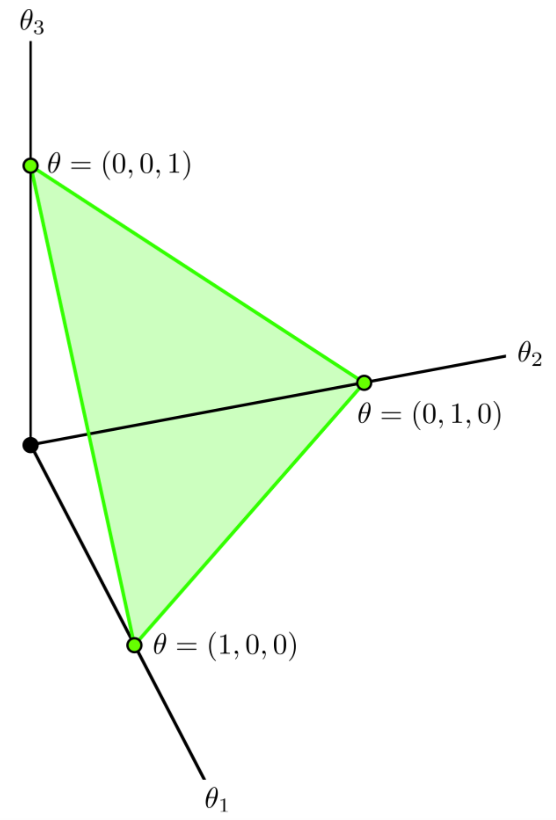
A NOVEL INTUITION

	A	B	C
A	(4, 1); (4, 1)	(1, 2); (4, 2)	(2, 1); (1, 2)
B	(3, 1); (2, 3)	(3, 2); (6, 3)	(1, 2); (2, 1)
C	(1, 2); (2, 1)	(2, 1); (1, 2)	(1, 3); (1, 3)

It turns out we can go from this

WHAT ARE MULTI-OBJECTIVE GAMES?

A NOVEL INTUITION

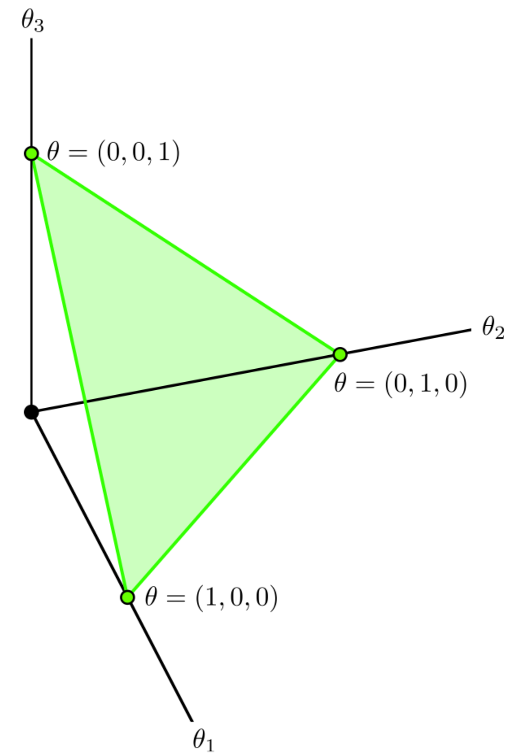


To this

Every MONFG with continuous utility functions can be reduced to a continuous game

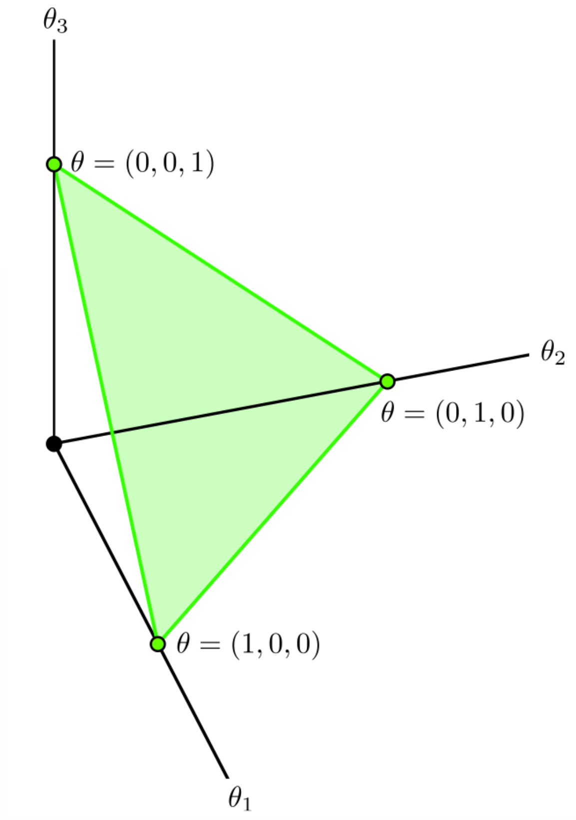
Continuous game

- Single objective
- Infinite number of pure strategies
- Reuse utility functions



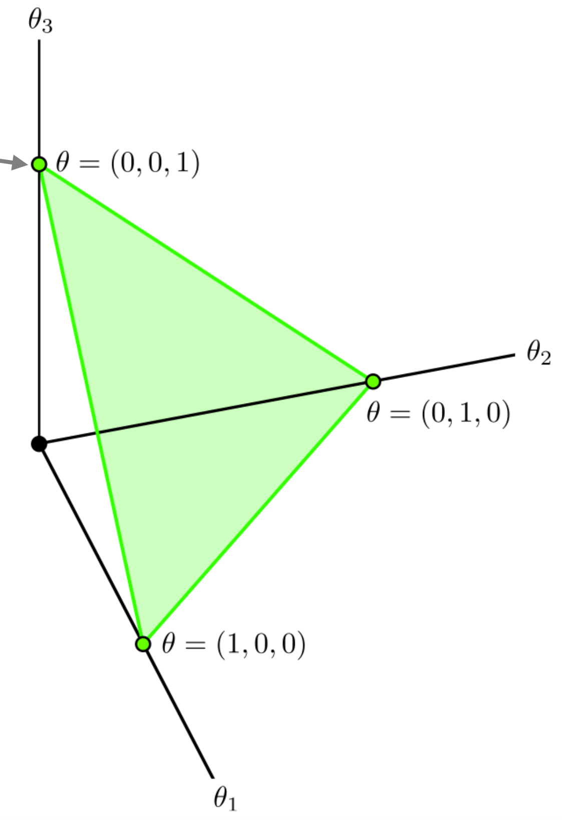
Make every ***mixed strategy*** in the MONFG a ***pure strategy*** in the continuous game

	A	B	C
A	(4, 1); (4, 1)	(1, 2); (4, 2)	(2, 1); (1, 2)
B	(3, 1); (2, 3)	(3, 2); (6, 3)	(1, 2); (2, 1)
C	(1, 2); (2, 1)	(2, 1); (1, 2)	(1, 3); (1, 3)



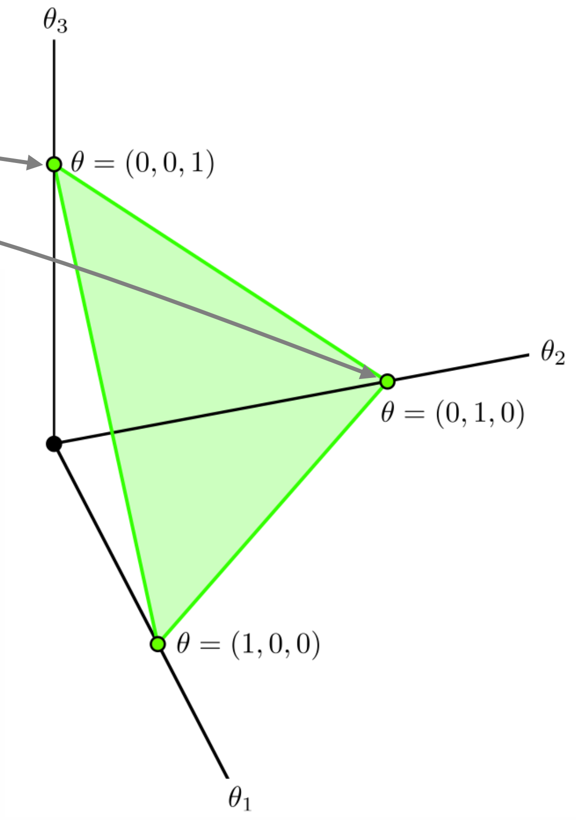
Make every ***mixed strategy*** in the MONFG a ***pure strategy*** in the continuous game

	A	B	C
A	(4, 1); (4, 1)	(1, 2); (4, 2)	(2, 1); (1, 2)
B	(3, 1); (2, 3)	(3, 2); (6, 3)	(1, 2); (2, 1)
C	(1, 2); (2, 1)	(2, 1); (1, 2)	(1, 3); (1, 3)



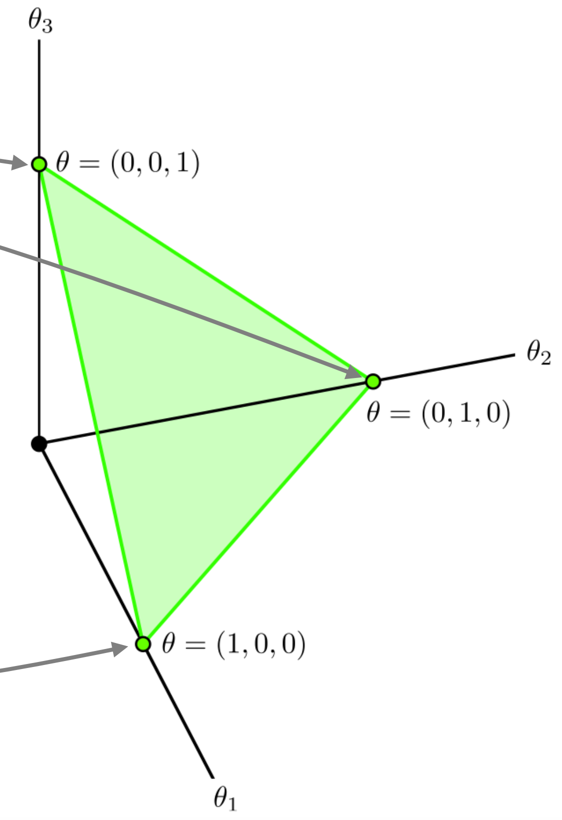
Make every ***mixed strategy*** in the MONFG a ***pure strategy*** in the continuous game

	A	B	C
A	(4, 1); (4, 1)	(1, 2); (4, 2)	(2, 1); (1, 2)
B	(3, 1); (2, 3)	(3, 2); (6, 3)	(1, 2); (2, 1)
C	(1, 2); (2, 1)	(2, 1); (1, 2)	(1, 3); (1, 3)



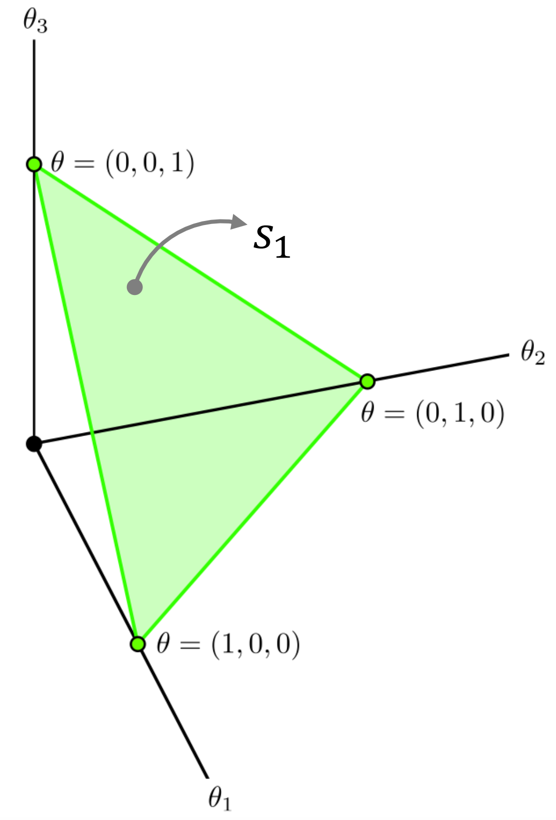
Make every ***mixed strategy*** in the MONFG a ***pure strategy*** in the continuous game

	A	B	C
A	(4, 1); (4, 1)	(1, 2); (4, 2)	(2, 1); (1, 2)
B	(3, 1); (2, 3)	(3, 2); (6, 3)	(1, 2); (2, 1)
C	(1, 2); (2, 1)	(2, 1); (1, 2)	(1, 3); (1, 3)



Make every ***mixed strategy*** in the MONFG a ***pure strategy*** in the continuous game

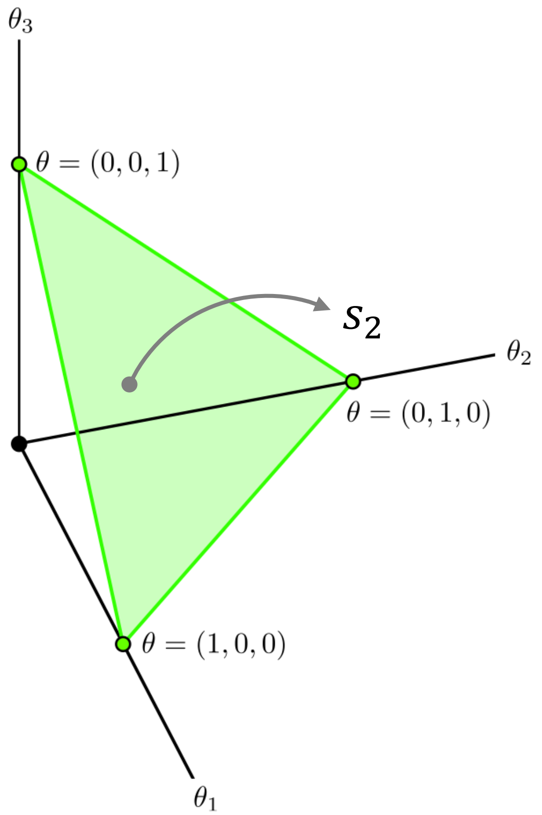
	A	B	C
A	(4, 1); (4, 1)	(1, 2); (4, 2)	(2, 1); (1, 2)
B	(3, 1); (2, 3)	(3, 2); (6, 3)	(1, 2); (2, 1)
C	(1, 2); (2, 1)	(2, 1); (1, 2)	(1, 3); (1, 3)



VRIJE UNIVERSITEIT BRUSSEL

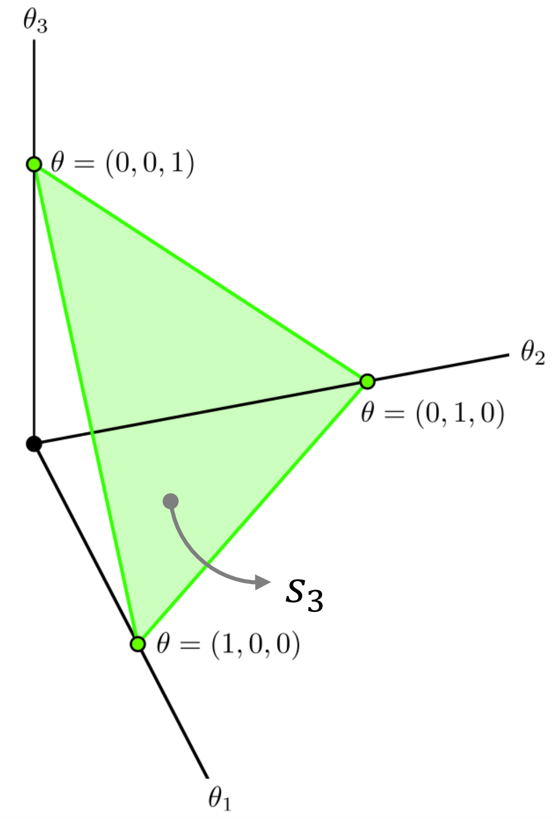
Make every ***mixed strategy*** in the MONFG a ***pure strategy*** in the continuous game

	A	B	C
A	(4, 1); (4, 1)	(1, 2); (4, 2)	(2, 1); (1, 2)
B	(3, 1); (2, 3)	(3, 2); (6, 3)	(1, 2); (2, 1)
C	(1, 2); (2, 1)	(2, 1); (1, 2)	(1, 3); (1, 3)



Make every ***mixed strategy*** in the MONFG a ***pure strategy*** in the continuous game

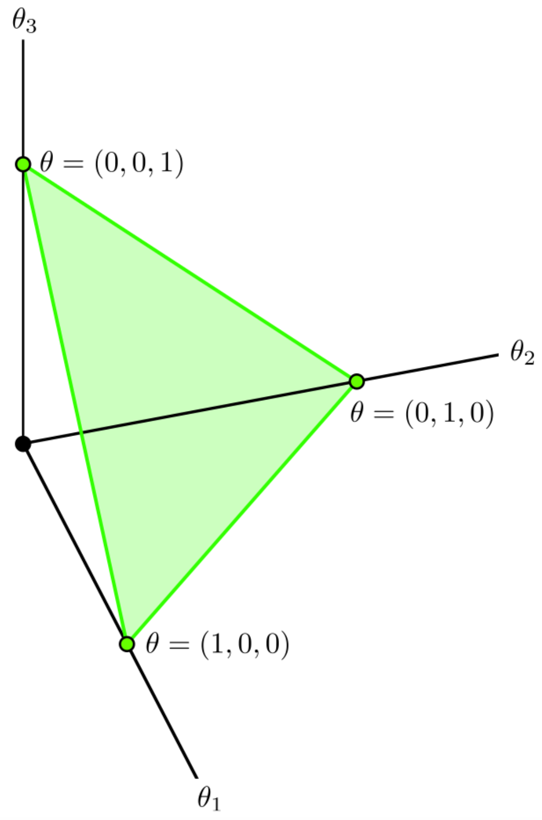
	A	B	C
A	(4, 1); (4, 1)	(1, 2); (4, 2)	(2, 1); (1, 2)
B	(3, 1); (2, 3)	(3, 2); (6, 3)	(1, 2); (2, 1)
C	(1, 2); (2, 1)	(2, 1); (1, 2)	(1, 3); (1, 3)



WHY ARE NASH EQUILIBRIA NOT GUARANTEED?

A NOVEL INTUITION

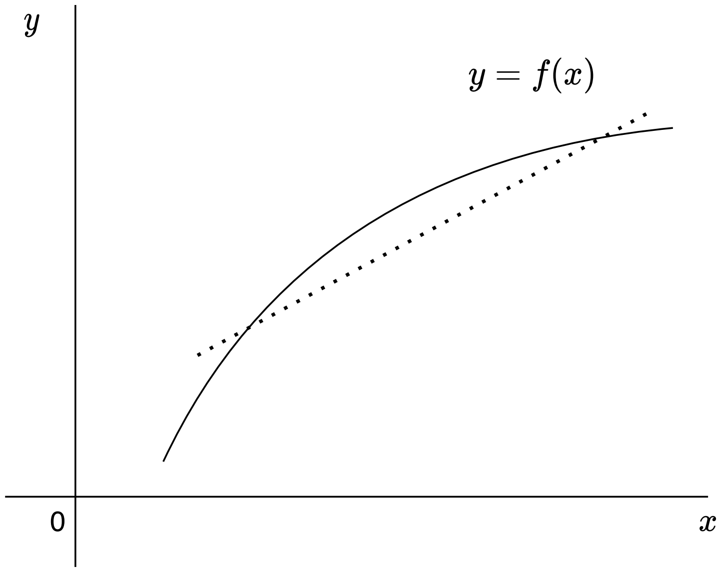
- Nash equilibria are not guaranteed in MONFGs
  - They are guaranteed in single-objective NFGs, so why not here?
- ***Mixed strategy equilibria*** in the MONFG are ***pure strategy equilibria*** in the continuous game
- Continuous games are not guaranteed to have a ***pure strategy*** Nash equilibrium



[Röpke et al., 2022]

EXISTENCE GUARANTEE

- Existence is guaranteed with ***(quasi)concave*** utility functions
  - Used in economics as well
  - Represents “well-behaved” preferences
- Intuition
  - You can reduce an MONFG to a continuous game
  - In this game it is known that a pure strategy Nash equilibrium exists when assuming only quasiconcave utility functions
  - This equilibrium is also an equilibrium in the original MONFG



[Röpke et al., 2022]

NON-EXISTENCE

- We can show that no Nash equilibrium exists in this game
  - With **strict convex** utility functions
- Saving grace
  - Techniques we developped are generally useful
  - Can use it to prove counterexamples for additional possible properties
  - Can use it for an efficient algoritmh (future work)

	A	B
A	(2, 0); (1, 0)	(1, 0); (0, 2)
B	(0, 1); (2, 0)	(0, 2); (0, 1)

$$u_1(p_1, p_2) = u_2(p_1, p_2) = p_1^2 + p_2^2$$

[Röpke et al., 2022]

RELATIONS BETWEEN OPTIMISATION CRITERIA

NASH EQUILIBRIA

- ***No relation*** between both optimisation criteria ***in general***
  - No sharing of number of equilibria or equilibria themselves

	A	B
A	(1, 0); (1, 0)	(0, 1); (0, 1)
B	(0, 1); (0, 1)	(-10, 0); (-10, 0)

Multi-objective reward vectors

	A	B
A	0.1; 0.1	0; 0
B	0; 0	-0.1; -0.1

Scalarised utility for both agents

- ***Relation*** when only considering ***pure strategy*** equilibria
  - Pure strategy equilibrium under SER is also one under ESR
  - Bidirectional when assuming (quasi)convex utility functions

ALGORITHMIC IMPLICATIONS

- Algorithm for calculating ***all pure strategy equilibria*** in a given MONFG ***with quasiconvex utility functions***
- Shown to work because of our theoretical contributions

[Röpke et al., 2022]

NASH EQUILIBRIA

RECENT WORK

Algorithm 1 Computing all PSNE in an MONFG

Input: an MONFG $G = (N, \mathcal{A}, \boldsymbol{p})$ and quasiconvex utility functions $u = (u_1, \cdots, u_n)$

function REDUCE_MONFG(monfg, u)
  $N, \mathcal{A}, \boldsymbol{p} \leftarrow$ monfg
  $u_1, \cdots, u_n \leftarrow$ u
  $f \leftarrow (u_1 \circ \boldsymbol{p}_1, \cdots, u_n \circ \boldsymbol{p}_n)$
  $G' \leftarrow (N, \mathcal{A}, f)$ ▷ An induced normal-form game
  return G'
end function
function COMPUTE_ALL_PSNE(nfg)
  $S = \emptyset$
  for PS **in** nfg **do** ▷ Loop over all pure strategies
    if PS is a PSNE **then** ▷ If it is a PSNE add it to the set
      $S \leftarrow S \cup \{\text{PS}\}$
    end if
  end for
  return S
end function
nfg $\leftarrow$ REDUCE_MONFG(G, u)
PSNE $\leftarrow$ COMPUTE_ALL_PSNE(nfg)

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NASH EQUILIBRIA

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  $N, \mathcal{A}, \boldsymbol{p} \leftarrow$ monfg
  $u_1, \cdots, u_n \leftarrow$ u
  $f \leftarrow (u_1 \circ \boldsymbol{p}_1, \cdots, u_n \circ \boldsymbol{p}_n)$
  $G' \leftarrow (N, \mathcal{A}, f)$ ▷ An induced normal-form game
  return G'
end function
function COMPUTE_ALL_PSNE(nfg)
  $S = \emptyset$
  for PS **in** nfg **do** ▷ Loop over all pure strategies
    if PS is a PSNE **then** ▷ If it is a PSNE add it to the set
      $S \leftarrow S \cup \{\text{PS}\}$
    end if
  end for
  return S
end function
nfg $\leftarrow$ REDUCE_MONFG(G, u)
PSNE $\leftarrow$ COMPUTE_ALL_PSNE(nfg)

Reduce the MONFG

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NASH EQUILIBRIA

RECENT WORK

Algorithm 1 Computing all PSNE in an MONFG

Input: an MONFG $G = (N, \mathcal{A}, \boldsymbol{p})$ and quasiconvex utility functions $u = (u_1, \cdots, u_n)$

function REDUCE_MONFG(monfg, u)
    $N, \mathcal{A}, \boldsymbol{p} \leftarrow$ monfg
    $u_1, \cdots, u_n \leftarrow$ u
    $f \leftarrow (u_1 \circ \boldsymbol{p}_1, \cdots, u_n \circ \boldsymbol{p}_n)$
    $G' \leftarrow (N, \mathcal{A}, f)$ ▷ An induced normal-form game
    return G'
end function
function COMPUTE_ALL_PSNE(nfg)
    $S = \emptyset$
    for PS **in** nfg **do** ▷ Loop over all pure strategies
        if PS is a PSNE **then** ▷ If it is a PSNE add it to the set
            $S \leftarrow S \cup \{\text{PS}\}$
        end if
    end for
    return S
end function
nfg $\leftarrow$ REDUCE_MONFG(G, u)
PSNE $\leftarrow$ COMPUTE_ALL_PSNE(nfg)

Solve the trade-off game

CONCLUSION

- Lots of new theoretical insights
  - Relation to other games opens up a new perspective
  - Equilibrium existence and non-existence
  - Things are simpler when only considering pure strategies

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  - Zero-sum games
  - Exploit continuous game reduction

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  - Things are simpler when only considering pure strategies
- Incorporate everything into a novel algorithm
- Additional guarantees for MONFGs
  - Zero-sum games
  - Exploit continuous game reduction
- More algorithmic work
  - Use theorems to find Nash equilibria efficiently

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BLENDED SETTING
EXAMPLE

- Appartment building gym
  - Treadmills
  - Weightlifting equipment
- Shared between residents
  - One athlete
  - Others are amateurs

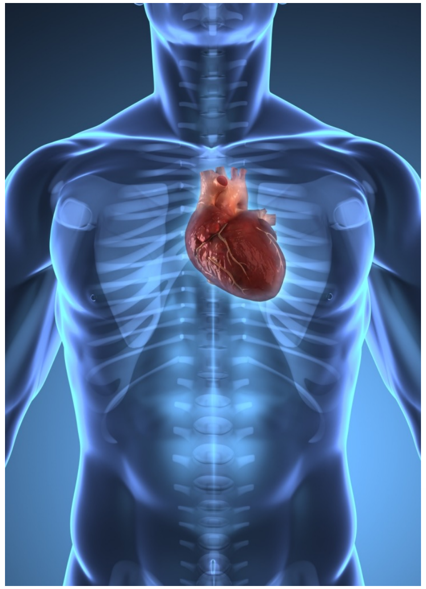
Small appartment building gym

Athlete

Amateur

BLENDED SETTING
EXAMPLE

- Objectives
  - Improve cardiovascular health
  - Improve strength
- Athlete plays a game against another resident
  - Select equipment
  - Selecting the same reduces effectivity



BLENDED SETTING
EXAMPLE

Player 1: Amateur

Maximise utility of each (occasional) workout

$$u_1(p_1, p_2) = p_1^2 + p_2$$

ESR

Player 2: Athlete

Sustain a training schedule

$$u_2(p_1, p_2) = p_1 \cdot p_2$$

SER

BLENDED SETTING
EXAMPLE

	Cardio	Lifting
Cardio	(4, 1); (4, 1)	(5, 1); (1, 4)
Lifting	(1, 4); (5, 1)	(1, 3); (1, 3)

The multi-objective reward vectors.

	Cardio	Lifting
Cardio	17; 4	26; 4
Lifting	5; 5	4; 3

The ESR utilities.

- The ESR player will always go running
  - Dominates weightlifting
- What is the best-response for the SER player?

BLENDED SETTING
EXAMPLE

	Cardio	Lifting
Cardio	(4, 1); (4, 1)	(5, 1); (1, 4)
Lifting	(1, 4); (5, 1)	(1, 3); (1, 3)

The multi-objective reward vectors.

	Cardio	Lifting
Cardio	17; 4	26; 4
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The ESR utilities.

- SER player wants to mix over cardio and lifting
  - Optimal balance
  - Sustainable training program

BLENDED SETTING
EXAMPLE

	Cardio	Lifting
Cardio	(4, 1); (4, 1)	(5, 1); (1, 4)
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The multi-objective reward vectors.

	Cardio	Lifting
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The ESR utilities.

- {(1 Cardio, 0 Lifting), ($\frac{1}{2}$ Cardio, $\frac{1}{2}$ Lifting)} is a Nash equilibrium!
  - ESR player plays a best response
  - SER player plays a best response

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BLENDED SETTING
EXAMPLE

	Cardio	Lifting
Cardio	(4, 1); (4, 1)	(5, 1); (1, 4)
Lifting	(1, 4); (5, 1)	(1, 3); (1, 3)

The multi-objective reward vectors.

	Cardio	Lifting
Cardio	17; 4	26; 4
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The ESR utilities.

- What happens when we dismiss SER?
  - Athlete has a Nash equilibrium at (Cardio, Cardio) or (Cardio, Lifting)
  - Clearly suboptimal

BLENDED SETTING

EXAMPLE

	Cardio	Lifting
Cardio	(4, 1); (4, 1)	(5, 1); (1, 4)
Lifting	(1, 4); (5, 1)	(1, 3); (1, 3)

The multi-objective reward vectors.

	Cardio	Lifting
Cardio	17; 4	26; 4
Lifting	5; 5	4; 3

The ESR utilities.

- Even stronger
  - The best *overall* utility that the athlete can aspire to under ESR
    - (Lifting, Cardio) = 5
  - Simply playing the Nash equilibrium under SER
    - {(1 Cardio, 0 Lifting), ($\frac{1}{2}$ Cardio, $\frac{1}{2}$ Lifting)} = 6.25