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HW7
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1)

a)

$$\begin{aligned}P(A,B|K) &= P(A|B,K) P(B|K) \\P(A,B,K) / P(K) &= P(A,B,K) P(B,K) / (P(B,K) P(K)) \\P(A,B,K) / P(K) &= P(A,B,K) / P(K)\end{aligned}$$

➔ LHS = RHS Proven

b)

$$\begin{aligned}P(A|B,K) &= P(B|A,K)P(A|K)P(B|K) \\P(A,B|K) / P(B|K) &= P(B,A,K)P(A,K)P(B,K)/(P(A,K) P(K) P(K)) \\P(A,B|K) / P(B|K) &= P(B,A,K)P(B,K) / (P(K) P(K)) \\P(A,B,K) P(B,K) / (P(K) P(K)) &= P(B,A,K)P(B,K) / (P(K) P(K))\end{aligned}$$

➔ LHS = RHS Proven

2)

Given:

$$P(\text{Oil}) = 0.5$$

$$P(\text{Gas}) = 0.2$$

$$P(\text{Neither}) = 0.3$$

$$P(+|\text{Oil}) = 0.9$$

$$P(+|\text{Gas}) = 0.3$$

$$P(+|\text{Neither}) = 0.1$$

$$\begin{aligned}P(+) &= P(+|\text{Oil})P(\text{Oil}) + P(+|\text{Gas})P(\text{Gas}) + P(+|\text{Neither})P(\text{Neither}) \\&= 0.5*0.9 + 0.2*0.3 + 0.1*0.3 \\&= 0.54\end{aligned}$$

$$\begin{aligned}P(\text{Oil}|+) &= P(+|\text{Oil}) * P(\text{Oil}) / P(+) \\&= 0.9 * 0.5 / 0.54 \\&= 0.83\end{aligned}$$

➔ 0.83

3)

W	Black	Square	One	P
1	F	F	F	1/13
2	F	F	T	1/13
3	F	T	F	1/13
4	F	T	T	1/13
5	T	F	F	2/13
6	T	F	T	1/13
7	T	T	F	4/13
8	T	T	T	2/13

Ai) $P(\text{Black}) = (2+1+4+2)/13 = 9/13$

Aii) $P(\text{Square}) = (1+1+4+2)/13 = 8/13$

Aiii) $P(\text{Square} \mid \text{Black or One}) = ((1+4+2)/13) / (11/13) = 7/11$

Given gamma = Not black

1) alpha = square, beta = one are independent

2) alpha = square, beta = not one are independent

4)

a)

I (A, 0, {B,E})

I (B, 0, {A,C})

I (C, A, {B,D,E})

I (D, {A,B}, {C,E})

I (E, B, {A,C,D,F,G})

I (F, {C,D}, {A,B,E})

I (G, F, {A,B,C,D,E,H})

I (H, {E,F}, {A,B,C,D,G})

b)

1) d_separated(A, BH, E)

false, because H unblocks the path A-C-F-H-E

2) d_separated(G, D, E)

false, because D unblocks the path A-D-B-E

3) d_separated (AB,F,GH)

true

c) $P(A|B,C,D,E,F,G,H) * P(B|C,D,E,F,G,H) * P(C|D,E,F,G,H) * P(D|E,F,G,H) * P(E|F,G,H) * P(F|G,H) * P(G|H) * P(H)$

d)

a) $P(A = 0, B = 0) = P(A=0) P(B=0)$ since A and B are independent
 $= 0.8 * 0.3$
 $= 0.24$

$$\begin{aligned} \text{b) } P(E=1|A=1) &= P(E=1) \text{ since A and E are independent} \\ &= P(E=1|B=0)P(B=0) + P(E=1|B=1)P(B=1) \\ &= 0.9 * 0.3 + 0.1 * 0.7 \\ &= 0.34 \end{aligned}$$