Costello Divisibility: Exploration of a Comedic Division Algorithm

Sam Thiel and Alexander N. Wilson

The Version of Record of this manuscript has been published and is available in The College Mathematics Journal
.http://www.tandfonline.com/10.1080/07468342.2025.2601489

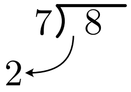
Abbott and Costello are a comedy duo active in the 1940's, who are most famous for their comedy bit "Who's on First?" in which the two comedians discuss baseball players with strange names and explore the depths of semantic ambiguity. However, this is far from their only routine. In another exploration of logical ambiguity and absurdity, their second most famous comedy bit[1] is a routine in which Lou Costello proves to Bud Abbott that $7 \times 13 = 28$ using erroneous versions of long division, multiplication, and addition [1]. We were most interested in the part of the routine that involves long division, which went as follows.

0.1 Costello's First Proof that $7 \times 13 = 28$

Costello sets up his proof as long division of 28 by 7, intending to get a quotient of 13. Costello begins to write his proof on the (conveniently supplied) chalkboard as follows.[2]

$$7\overline{)28}$$

Costello then begins the process of long division as usual, by first asking, "Does seven go into two?" Clearly, seven does not divide two, so Costello sets the two aside for later.



[1] A bit that the first author came across on YouTube while they were *supposed* to be working on their group theory homework.

[2] Costello actually uses a different notation for long division, but we have chosen to use more familiar notation to not add additional confusion for the reader.

Costello, unfazed, then continues the process of long division with the next digit,[3] asking "Does 7 go into 8?" In this case, 7 goes into 8 once, so Costello puts a 1 in our quotient, and subtracts $7 \times 1 = 7$ from 8.

$$\begin{array}{rrr} & & 1 \\ & 7) & \overline{8} \\ & & \underline{7} \\ 2 & & 1 \end{array}$$

With this subtraction complete, Costello then points out that our division is not finished, since we still haven't used that 2 that we put aside earlier. To amend this, Costello very sensibly places the 2 into the remainder alongside the 1 that was put there from our most recent subtraction, which appears as follows.

$$\begin{array}{rr} & 1 \\ 7) & \overline{8} \\ & \underline{7} \\ \hookrightarrow & 21 \end{array}$$

With a newfound 21 in our remainder, Costello continues the process, this time applying the long division process to our remainder rather than any remaining digits in our dividend. Costello asks a third and final time, "Does 7 go into 21?" Which it does, three times. Costello thus puts a 3 into the quotient, and subtracts $7 \times 3 = 21$ from our remainder-turned-dividend.

$$\begin{array}{rr} & 13 \\ 7) & \overline{\ 8} \\ & \underline{7} \\ & 21 \\ & \underline{21} \\ & 0 \end{array}$$

With a 0 in the remainder, and no further digits to divide, Costello concludes the division process with a 13 as the quotient. □

A careful reader will note that Costello's proof is not consistent with modern-day rules of arithmetic: $28 \div 7 = 4$, not 13. Costello's second and third proofs are equally dubious, but each "proves" in some way that $7 \times 13 = 28$.

1 From comedy to mathematical research

This bit by Abbott and Costello isn't just performed in one film; this bit appears in films by the duo as early as 1941—like in the film "In The Navy" [2]—but the

[3] You may notice that this step is poorly justified (nonsensical, even), and that he should instead be considering if 7 goes into 28, rather than just 8.

routine also shows up in later films. However, we noticed something curious: despite the number of different films the routine is performed in, the numbers are never changed. Costello always does this proof with the numbers $7, 13$, and 28. So the question arises: why are these numbers special? What is it about these values makes this process "work" in some sense? What other numbers could Abbott and Costello have used?

In order to address these questions, we must first decide how to appropriately generalize Costello's version of division to other pairs of integers. This is not a simple thing to do, since Costello's example is quite restrictive in the examples we can quickly adapt to. This leaves us with a number of questions with ambiguous answers: What happens if the first digit of our dividend is greater than our divisor? What happens if we have a three or more digit dividend? What happens when there is a zero in our dividend? None of these questions are questions we can answer simply by looking at Costello's proof.

Before reading the next section, make a guess at what you think the answer to these questions should be, and try the following exercise. The given quotients are in agreement with how *we* decide to answer the above questions in the remainder of this paper, but we encourage the reader to compare them with the answers they get under their own chosen conventions. The fact that the answers depend on one's conventions underscores the importance of the careful definitions in the following section.

Exercise. Replicate Costello's division steps to "prove" the following:

(a) $16 \div 4 = 13$

(b) $65 \div 5 = 112$

(c) $396 \div 3 = 132$

(d) $30 \div 2 = 105$

2 From an example to an algorithm

In order to adapt Costello's process into an algorithm, we need to define two operations: concatenation and remainder.

2.1 Concatenation

Informally, concatenation is simply the "sticking" of two numbers together. For example, the concatenation of 1 with 2 is 12. However, for our purposes we will define concatenation more rigorously.

The issue is that the representation of a number in base 10 is not unique. For example, $1 = 01 = 001$. Fortunately, for any natural number, the representation of a number is unique *up to leading zeroes*. This motivates the following definition: Throughout this paper, take $\mathbb{N}$ to be the nonnegative integers (that is, include 0). For $n \in \mathbb{N}$, its **minimal representation** is its unique expression in base 10 with no leading zeros. We will usually write the minimal representation of a number n as a string

$$n = n_1 n_2 \dots n_l \tag{1}$$

where $n_j \in \{0, 1, \ldots, 9\}$ for $1 \leq j \leq l$, and if $n \neq 0$ then $n_1 \neq 0$. Let $\ell(n)$, called the **length** of n be the number of digits in the minimal representation of n. That is, given the minimal representation of n as in (1), we define $\ell(n) := l$.

Additionally, we adopt the convention that 0 has minimal representation consisting of a single zero, and, by consequence, that $\ell(0) = 1$.

Now that we have unique representations of each number, we can rigorously define concatenation as a binary operation on the set $\mathbb{N}$. Given two numbers $a, b \in \mathbb{N}$, we first take their minimal representations:

$$a = a_1 a_2 \ldots a_{l-1} a_l \text{ and } b = b_1 b_2 \ldots b_{m-1} b_m.$$

We then define the **concatenation** of a with b, denoted $a \oplus b$, to be the number c with base 10 expansion

$$c := a_1 a_2 \ldots a_{l-1} a_l b_1 b_2 \ldots b_{m-1} b_m. \tag{2}$$

Note that because we concatenated the unique minimal expressions for a and b, the number $a \oplus b$ is well defined. However, while our inputs were both represented in their minimal representation, the string we obtain in (2) may *not* be the minimal representation for c. Specifically, there may be a leading zero if $a = 0$.

Exercise. Use the above definition to compute the following:

(a.i) $1 \oplus 23$

(a.ii) $23 \oplus 1$

(b.i) $1 \oplus 0$

(b.ii) $0 \oplus 1$

2.2 Remainders

In contrast to the complexity of concatenation, the remainder operation can be expressed much more simply. Given two numbers $a, n \in \mathbb{N}$ with $n \neq 0$, define the **remainder operation (of a by n)**, denoted $a \ \% \ n$, to be the remainder upon dividing a by n.

2.3 Costello division

With these additional operations, we can define an algorithm for Costello division. As discussed earlier, we only have a single example to work from to determine what this algorithm should look like, and so there are a number of reasonable ways of defining this operation that all affirm Costello's first proof.[4]

The following is our choice for the algorithm. The guiding principle we followed was to modify the process of (standard) long division as little as possible, while still accounting for the errors in Costello's original proof. We have chosen to define this algorithm only for single-digit divisors, due to Costello's example following a digit-by-digit process. We discuss divisors having more digits and other potential extensions to the algorithm in our conclusion.

[4] In fact, we began our research by working with a slightly different algorithm, which treated any zeroes in the dividend by simply placing them in the remainder. This resulted in some very strange properties.

Definition (Costello division)**.** Let $m \in \mathbb{N}$ and $n \in \{1, \ldots, 9\}$. We denote the **Costello division** of m by n as $m \oslash n$. The output of Costello division is a pair, (q, r), where we call q the **quotient (under Costello division)**, and r the **remainder (under Costello division)**.

Costello division is defined by first representing m in terms of its minimal representation as $m_1 m_2 \ldots m_l$, then carrying out the following process.

Algorithm 1 Costello division

```
Require: $m \in \mathbb{N}$, $n \in \{1, \ldots, 9\}$
  $q \leftarrow 0$, $r \leftarrow 0$
  for $1 \leq j \leq l$ do
      if $m_j \geq n$ or $m_j = 0$ then    ▷ Componentwise Division Step (on the $j$th digit)
          $q \leftarrow q \oplus \left\lfloor \frac{m_j}{n} \right\rfloor$
          $r \leftarrow r \oplus (m_j \% n)$
      else
          $r \leftarrow r \oplus m_j$
      end if
      if $r \geq n$ then                   ▷ Standard Division Step (on the $j$th digit)
          $q \leftarrow q \oplus \left\lfloor \frac{r}{n} \right\rfloor$
          $r \leftarrow r \% n$            ▷ The $j$th-step remainder.
                                           (See the subsection "Intermediate remainders.")
      end if
  end for
      return $(q, r)$
```

Let's look at $28 \oslash 7$, just to confirm that this algorithm does align with Costello's original proof.

Example ($28 \oslash 7 = (13{,}0)$)**.** Take $m = 28$ and $n = 7$. Set $q = r = 0$. We then start the process in our for-loop. Note that $\ell(28) = 2$, so we only repeat the loop twice.

We consider the componentwise division step on the first digit, 2. Since $2 < 7$ and $2 \neq 0$, we set

$$r \leftarrow r \oplus m_1 = 0 \oplus 2 = 2.$$

Then we consider the standard division step. Since $2 < 7$, we do nothing.

We then return to the top of our for-loop. We consider the componentwise step on the second digit, 8. Since $8 \geq 7$, we set

$$q \leftarrow q \oplus \left\lfloor \frac{m_2}{n} \right\rfloor = 0 \oplus \left\lfloor \frac{8}{7} \right\rfloor = 0 \oplus 1 = 1, \text{ and}$$
$$r \leftarrow r \oplus (m_2 \% n) = 2 \oplus (8 \% 7) = 2 \oplus 1 = 21.$$

Now we consider the standard division step. Since $21 \geq 7$, we set

$$\begin{aligned} q &\leftarrow q \oplus \left\lfloor \frac{r}{n} \right\rfloor = 1 \oplus \left\lfloor \frac{21}{7} \right\rfloor = 1 \oplus 3 = 13, \text{ and} \\ r &\leftarrow r \ \% \ n = 21 \ \% \ 7 = 0. \end{aligned}$$

We then exit our for-loop, and return $(13, 0)$. Therefore 28 Costello-divided by 7 is 13, with a remainder of 0. Indeed, our algorithm does align with Costello's original proof that $7 \times 13 = 28$.

Since this algorithm produces an ordered pair as its output (to account for both the quotient and remainder), it will be useful to have specific notation for when we only want one of the two components. More specifically, we need notation for the remainder under Costello division. So, given $m \in \mathbb{N}, n \in \{1, \ldots, 9\}$, let $m \oslash n = (q, r)$. Then define the **Costello remainder** operation $\textcircled{\%}$ as

$$m \mathbin{\textcircled{\%}} n := r.$$

With these definitions in hand, we can now begin to analyze the algorithm, to see if we can glean any insights about its properties.

3 From an algorithm to an insight

We first looked at a table of values, to see if any patterns emerged. Table 1 is a smaller version of that table, showing the quotients and remainders for Costello division with dividends ranging between 10 and 20 for every possible divisor.

The table of remainders has a very striking pattern: the remainders under Costello division appear to match the remainders under standard division. To prove this, we will need to analyze Costello division, and hence the remainder and concatenation operations, more closely.

In this section, we gather some useful facts about the ways that concatenation and remainders interact, with the ultimate goal of turning this observation into a formal proof.

3.1 Reformulating operations

We will want some slightly easier-to-handle versions of our concatenation and remainder operations. First, note that the concatenation $a \oplus b$ can be thought of as first appending $\ell(b)$ zeros to a, then summing the result with b. This observation leads to the following fact.

Fact 1 (Closed Form of Concatenation)**.** *The concatenation of two numbers, $a, b \in \mathbb{N}$, can be expressed as*

$$a \oplus b = 10^{\ell(b)} a + b.$$

Table 1: Two tables, the bottom of remainders under Costello division and the top of quotients under Costello division. Notice that each row of the remainders table follows a repeating pattern.

q	10	11	12	13	14	15	16	17	18	19	20
1	10	11	12	13	14	15	16	17	18	19	20
2	5	5	15	15	25	25	35	35	45	45	10
3	3	3	4	13	13	14	23	23	24	33	6
4	2	2	3	3	12	12	13	13	22	22	5
5	2	2	2	2	2	12	12	12	12	12	4
6	1	1	2	2	2	2	11	11	12	12	3
7	1	1	1	1	2	2	2	11	11	11	2
8	1	1	1	1	1	1	2	2	11	11	2
9	1	1	1	1	1	1	1	1	2	11	2

r	10	11	12	13	14	15	16	17	18	19	20
1	0	0	0	0	0	0	0	0	0	0	0
2	0	1	0	1	0	1	0	1	0	1	0
3	1	2	0	1	2	0	1	2	0	1	2
4	2	3	0	1	2	3	0	1	2	3	0
5	0	1	2	3	4	0	1	2	3	4	0
6	4	5	0	1	2	3	4	5	0	1	2
7	3	4	5	6	0	1	2	3	4	5	6
8	2	3	4	5	6	7	0	1	2	3	4
9	1	2	3	4	5	6	7	8	0	1	2

An important mathematical tool in studying (standard) division is *modular arithmetic*, and it will also help us here. The following fact is an equivalent characterization to our definition of the remainder operation, this time formulated in the language of modular arithmetic.

Fact 2 (Remainders in Modular Arithmetic)**.** *Given two numbers $a, n \in \mathbb{N}$, the remainder $a \;\%\; n$ is the smallest $a' \in \mathbb{N}$ such that $a \equiv a' \mod n$.*

Now, because addition and multiplication are well defined working modulo n, one can use Facts 1 and 2 to discover the following fact:

Fact 3 (Concatenation mod n)**.** *Let $a, b \in \mathbb{N}$ and $n \in \mathbb{N}$. If $\ell(b \;\%\; n) = \ell(b)$, then*

$$(a \oplus b) \;\%\; n = ((a \;\%\; n) \oplus (b \;\%\; n)) \;\%\; n.$$

In short, so long as the remainder $b \;\%\; n$ has the same length as b, the concatenation and remainder operations play nicely together. In particular, this is true when b and n are both single-digit numbers.

Example. Let $a = 12$, $b = 9$, and $n = 7$. Then

$$(a \oplus b) \ \% \ n = (12 \oplus 9) \ \% \ 7 = 129 \ \% \ 7 = 3,$$

while

$$((a \ \% \ n) \oplus (b \ \% \ n)) = (5 \oplus 2) \ \% \ 7 = 52 \ \% \ 7 = 3.$$

3.2 Intermediate remainders

The last tools we need are characterizations of intermediate steps of our algorithm, specifically with regard to remainders. Let $m \in \mathbb{N}, n \in \{1, \ldots, 9\}$. Additionally, let $1 \leq j \leq l = \ell(m)$. Define $r^{(j)}_{m,n}$ to be the ***j*th step remainder** of $m \oslash n$, where $r^{(j)}_{m,n}$ is the value of our remainder after the standard division step of Algorithm 1 corresponding to the jth digit of m. In other words, this is the value of our remainder after the jth iteration of the for-loop in Algorithm 1 (We have left a comment in the algorithm to point you to the right place).

When the context of our division is clear, we omit the subscript m, n from the jth step remainder notation, instead writing simply $r^{(j)}$.

Additionally, note that when $j = l$, the jth step remainder is precisely the final remainder we produce in our algorithm. That is,

$$r^{(l)}_{m,n} = m \textcircled{\%} n.$$

We also define, in alignment with its initial value at the beginning of our algorithm,

$$r^{(0)} := 0.$$

We will also consider the truncation of a number's minimal representation. This definition may seem a bit strange on its own, but consider this definition as a way to "reverse" the operation of concatenation.[5] Let $m \in \mathbb{N}$, and denote the minimal representation of m as $m_1 \ldots m_l$, Now let $1 \leq j \leq l$. Define the ***j*-truncated representation** of m, denoted $m^{(j)}$, to be the number with base 10 representation

$$m^{(j)} := m_1 m_2 \ldots m_j.$$

That is, $m^{(j)}$ is the number represented by the string of the leftmost j digits of m.

By carefully examining Algorithm 1, one can discover the following fact that relates truncated representations and intermediate remainders.

Fact 4 (Remainders of Truncations)**.** *Let $m \in \mathbb{N}, n \in \{1, \ldots, 9\}$. Then, for all $1 \leq j \leq \ell(m)$, we have that*

$$m^{(j)} \textcircled{\%} n = r^{(j)}_{m,n}.$$

[5] We explicitly avoid using the word "inverse," since concatenation is not a bijective operation. E.g. $1 \oplus 23 = 12 \oplus 3 = 123$.

Finally, one can similarly discover the following fact by carefully considering the steps in Algorithm 1.

Fact 5 (Final-step Remainder)**.** *Let $m \in \mathbb{N}, n \in \{1, \ldots, 9\}$. Denote m in its minimal representation as $m_1 \ldots m_l$. Then,*

$$m \text{\textcircled{\%}} n = (r^{(l-1)} \oplus (m_l \ \% \ n)) \ \% \ n.$$

4 From an insight to a theorem

These five facts are enough to codify our intuition about the remainders under Costello division into a formal theorem. We then provide a proof; we highly encourage the reader to first attempt the proof on their own, taking advantage of Facts 3–5.

Theorem 1 (Costello Remainder Theorem)**.** *Let $m \in \mathbb{N}$ and $n \in \{1, \ldots, 9\}$. Then,*

$$m \text{\textcircled{\%}} n = m \ \% \ n.$$

That is, the remainder under Costello division is precisely the remainder under standard division.

Proof. We prove our result by induction on the length $\ell(m) = l$ of our dividend.

- (Base Case). Let $n \in \{1, \ldots, 9\}$. Let $m \in \mathbb{N}$ such that $\ell(m) = 1$. Then m has minimal representation m_1, where $m_1 = m$. We can now perform our algorithm. Set $q = r = 0$. Since $\ell(m) = 1$, our for-loop reduces to checking the componentwise and standard division steps once.

  Consider the componentwise step on the first digit, m_1. First consider the case where $m_1 \geq n$ or $m_1 = 0$. In this case, set

  $$\begin{aligned} &q \leftarrow q \oplus \left\lfloor \frac{m_1}{n} \right\rfloor = 0 \oplus \left\lfloor \frac{m_1}{n} \right\rfloor = \left\lfloor \frac{m_1}{n} \right\rfloor \text{ and} \\ &r \leftarrow r \oplus (m_1 \ \% \ n) = 0 \oplus (m_1 \ \% \ n) = m_1 \ \% \ n. \end{aligned}$$

  Then we consider the standard division step. Since $m_1 \ \% \ n < n$, nothing happens, and we exit our for-loop, returning $(\lfloor \frac{m_1}{n} \rfloor, m_1 \ \% \ n)$. Since $m_1 = m$, we have that $m \text{\textcircled{\%}} n = m_1 \ \% \ n = m \ \% \ n$.

  Alternatively, consider the case where $0 \neq m_1 < n$. In which case, during the componentwise division step, we set

  $$r \leftarrow r \oplus m_1 = 0 \oplus m_1 = m_1 = m.$$

  Then we consider the standard division step. Since $r = m = m_1 < n$, nothing happens, and we exit our for-loop, returning $(0, m)$. However, note that if $m_1 < n$, then $m < n$. Therefore $m \text{\textcircled{\%}} n = m = m \ \% \ n$. Thus we have proved the base case.

- (Inductive Step). Let $n \in \{1, \dots, 9\}$. Assume that for all $k \in \mathbb{N}$ such that $\ell(k) = l - 1$, we have that

  $$k \textcircled{\%} n = k \; \% \; n.$$

  Now, consider an arbitrary $m \in \mathbb{N}$ such that $\ell(m) = l$. We know by Fact 5 that

  $$m \textcircled{\%} n = (r^{(l-1)} \oplus (m_l \; \% \; n)) \; \% \; n.$$

  We then have by Fact 4 that $r^{(l-1)} = m^{(l-1)} \textcircled{\%} n$, so

  $$m \textcircled{\%} n = ((m^{(l-1)} \textcircled{\%} n) \oplus (m_l \; \% \; n)) \; \% \; n.$$

  Now, noting that $\ell(m^{(l-1)}) = l - 1$, our inductive hypothesis states that $m^{(l-1)} \textcircled{\%} n = m^{(l-1)} \; \% \; n$, which we substitute as well:

  $$m \textcircled{\%} n = ((m^{(l-1)} \; \% \; n) \oplus (m_l \; \% \; n)) \; \% \; n.$$

  We can then apply Fact 3, since $\ell(m_l) = 1 = \ell(m_l \; \% \; n)$. We therefore find that

  $$\begin{aligned} m \textcircled{\%} n &= ((m^{(l-1)} \; \% \; n) \oplus (m_l \; \% \; n)) \; \% \; n \\ &= (m^{(l-1)} \oplus m_l) \; \% \; n \\ &= m \; \% \; n, \end{aligned}$$

  which proves our inductive step, and therefore, our main result.

□

5 From mathematical research to more mathematical research

In spite of its humorous origins, it is clear that there is some level of consistency behind Costello's division mistakes. Theorem 1 is particularly compelling in this regard, since it shows an instance of equivalence between standard division and Costello division—in many ways a surprising result. There is little indication from Costello's original proof (aside from the fact that it, in some sense, "works") that there should be any consistency arising from this example-turned-algorithm.

There are a number of questions still left unanswered. One immediate question that emerges is how one might go about further extending the definition of Costello division to include divisors with more than one digit. Unfortunately, it's quite hard to determine what that should look like from Costello's example, since 7 has length 1. We've considered multiple interpretations: one is to consider "clumps" of digits in the dividend that have the same length as the divisor, and perform the componentwise step on those clumps. Another blunt option is that Costello division should still attempt to consider single digits in the component wise step, but this results in only ever considering the standard division step when the divisor has more than 1 digit.

The third option we considered—which we find the most compelling—is to circumvent the problem of multi-digit divisors by performing the operation in larger bases. For example, in hexadecimal (base 16), the base 10 number 12 has the hexidecimal representation C, which is a single digit. The definitions and theorems in this paper are formulated in base 10 specifically, but many extend very easily if the divisor is of length 1 in some other base. Potential study of Costello division in different bases would involve rigorously defining these operations in other bases, and potentially exploring any options for interpolation between Costello division in different bases. We think bases of the form 2^n may be particularly interesting, and may be of special interest to any readers with a background in computer science, as Theorem 1 may then allow for Costello Division to become a compelling choice of algorithm for finding remainders of a number.

One final and important question left unanswered is the question of inversion, since this would be the resolution of one of the original questions: "Could Costello have used different numbers?" Given a fixed divisor, standard division by that divisor has a well defined inverse operation: simply multiply the quotient by that divisor and add the remainder. However, it is not immediately clear whether or not Costello division by a fixed divisor is invertible. We note that Lou Costello does provide second and third proofs that $7 \times 13 = 28$, with multiplication and addition. However, neither of these operations give (in general) inverses to Costello division. For example, we state without proof that if your fixed divisor is 1, then Costello division produces the true quotient, but Costello multiplication by the fixed divisor 1 does not return the original value. For example, $13 \oslash 1 = (13, 0)$, but $13 \otimes 1 + 0 = 4$, where $\otimes$ represents Costello multiplication.[6] Finding an inverse operation to Costello division by a fixed divisor thus is a nontrivial problem. We conjecture that Costello division by a fixed divisor (other than 1) has an inverse, at least for dividends with specific properties.

Additionally, we were presented the following conjecture by a reviewer:

Conjecture. Let $m \in \mathbb{N}$ and $n \in \{1, \ldots, 9\}$. Suppose that $m \oslash n = (q, r)$. Then,

$$q \equiv \lfloor m/n \rfloor \mod 9. \tag{3}$$

That is, the Costello quotient is equal to the usual quotient mod 9.

Ultimately, we believe these unanswered questions provide directions for interesting (and fun!) further work to be done on this topic.

[6] We omit a full definition of Costello multiplication here. It is highly related to an almost "expected" mistake of those learning multiplication for the first time, where the placing of the digits is ignored and each digit is treated as if it were in the ones place. For more details, take a look at the videos of Costello's original work.

Acknowledgements. The authors thank Susan Jane Colley for her help during the revision process, as well as for her endless encouragement in the pursuit of this and all other projects.

The first author thanks the second author for bearing with them during the conception and production of this paper; math isn't nearly as fun alone. The first author also thanks Pamela E. Harris for being much of the inspiration for this project in the first place—she gave a single piece of advice to "look for math research problems around you" and clearly that was taken to heart.

References

[1] Dieker L. inthenavy 281. YouTube; 2016. Available from: https://www.youtube.com/watch?v=_HvGven4qJ0 (visited on November 16th, 2025).

[2] Lubin A. In The Navy [motion picture]. Universal 8 Studios; 1941.