Chapter 1

Introduction to Vectors

1.1 Vectors and Linear Combinations

Vectors

1.1.1 Definition

$\forall\ n-$dimensional vector $\mathbf{v}$ where $n \in \mathbb{N}^*$:

$$\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}.$$

$v_1, v_2, \cdots, v_n$ are the 1st, 2nd, $\cdots$, n-th component of $\mathbf{v}$. Every vector is written as a **column**.

1.1.2 Operation

1. Addition

   $\forall\ n-$dimensional vectors $\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_m$ where $m, n \in \mathbb{N}^*$:

$$\mathbf{v}_1 = \begin{bmatrix} v_{11} \\ v_{21} \\ \vdots \\ v_{n1} \end{bmatrix}, \qquad \mathbf{v}_2 = \begin{bmatrix} v_{12} \\ v_{22} \\ \vdots \\ v_{n2} \end{bmatrix}, \qquad \cdots, \qquad \mathbf{v}_m = \begin{bmatrix} v_{1m} \\ v_{2m} \\ \vdots \\ v_{mn} \end{bmatrix}.$$

   The vector addition of $\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_m$ is

$$\mathbf{v}_1 + \mathbf{v}_2 + \cdots + \mathbf{v}_m = \begin{bmatrix} v_{11} + v_{12} + \cdots + v_{1n} \\ v_{21} + v_{22} + \cdots + v_{2n} \\ \vdots \\ v_{m1} + v_{m2} + \cdots + v_{mn} \end{bmatrix}.$$

2. Scalar Multiplication

$\forall$ $n-$dimensional vector $\mathbf{v}$ where $n \in \mathbb{N}^*$ and $\forall$ number c:

$$\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}.$$

The scalar multiplication of c and $\mathbf{v}$ is

$$c\mathbf{v} = \begin{bmatrix} cv_1 \\ cv_2 \\ \vdots \\ cv_n \end{bmatrix}.$$

The number c is called a “scalar”.

1.1.3 Linear Combination

Combine addition with scalar multiplication to produce a “linear combination”. $\forall$ $n-$dimensional vectors $\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_m$ where $m, n \in \mathbb{N}^*$ and $\forall$ number $\alpha_1, \alpha_2, \cdots, \alpha_m$. The sum of $\alpha_1\mathbf{v}_1$, $\alpha_2\mathbf{v}_2$, $\cdots$, $\alpha_m\mathbf{v}_m$ is a linear combination

$$\alpha_1\mathbf{v}_1 + \alpha_2\mathbf{v}_2 + \cdots + \alpha_m\mathbf{v}_m.$$