

Lec03: Lines

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Jonathan Kress

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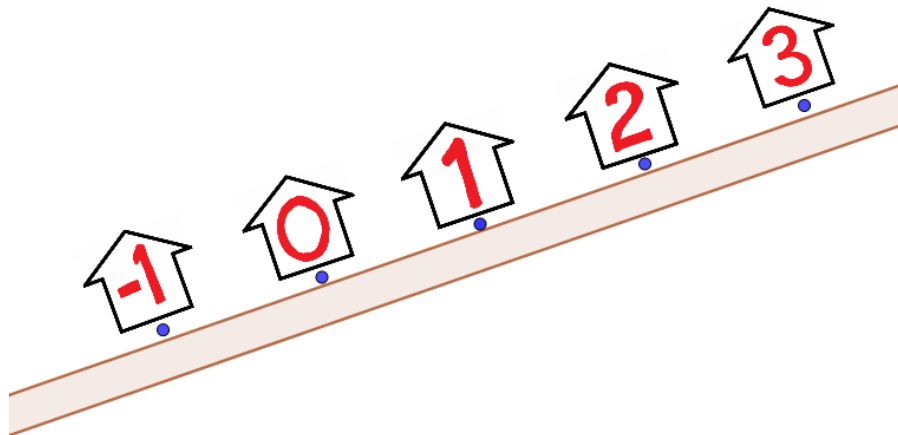
Red-Centre, Room 3090

2019 Term 1

Lines

How do you find a given house on a street?

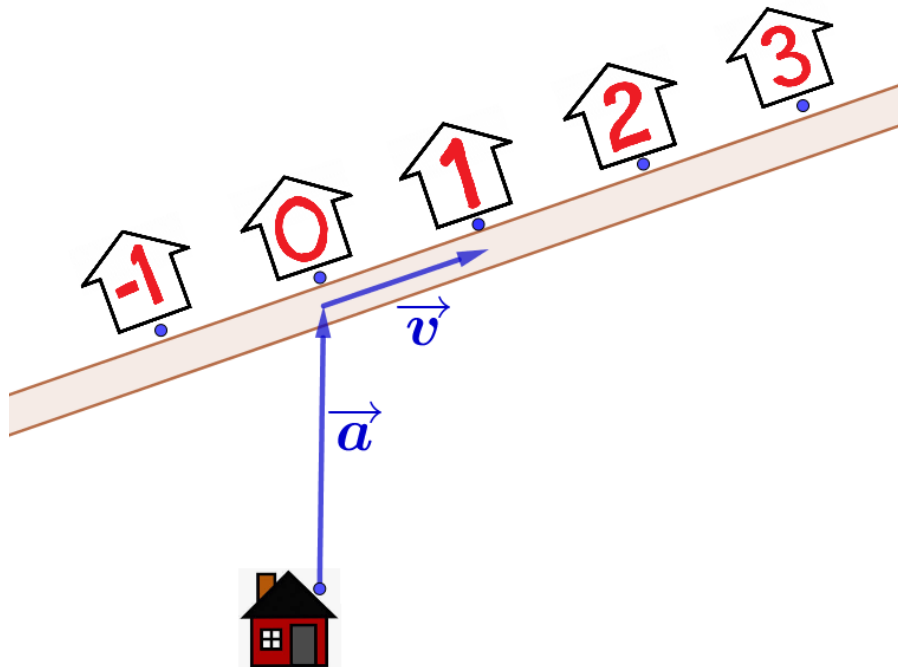
Go to somewhere on the street and then travel in the direction of the street until you reach the correct house.



Lines

How do you find house number 1 on a street?

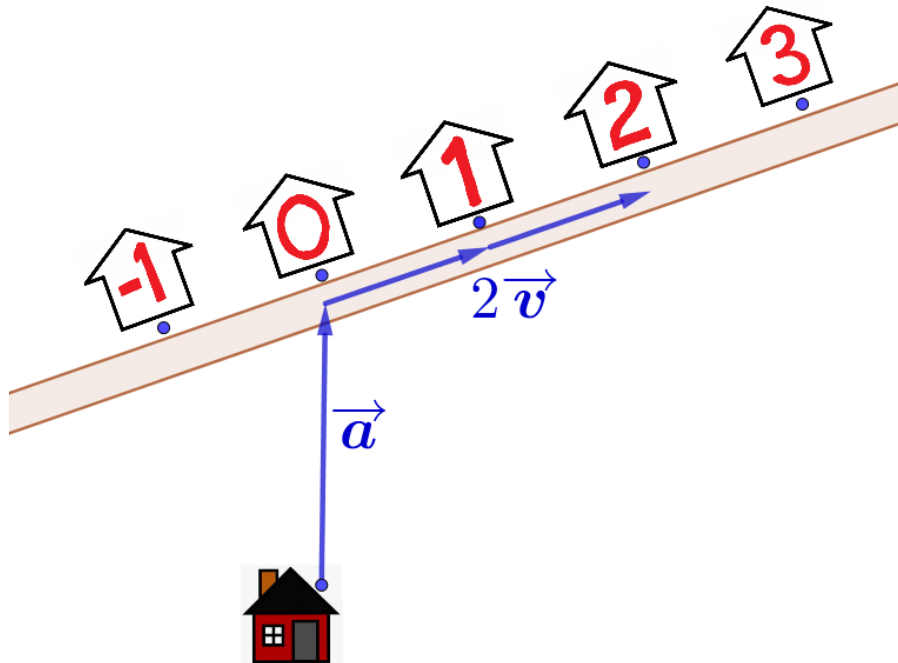
Go to somewhere on the street and then travel in the direction of the street until you reach house number 1.



Lines

How do you find house number 2 on a street?

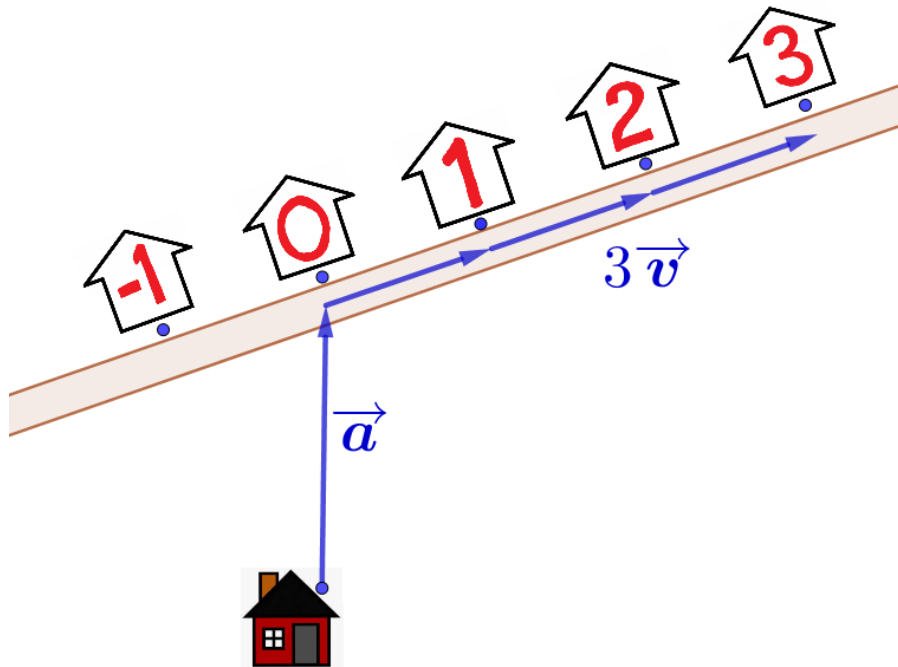
Go to somewhere on the street and then travel in the direction of the street until you reach house number 2.



Lines

How do you find house number 3 on a street?

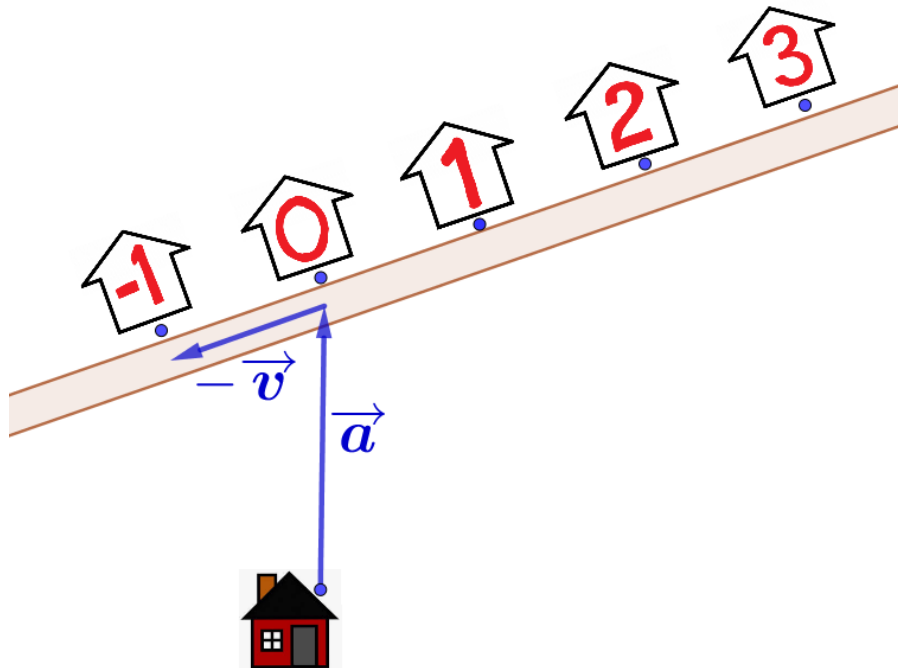
Go to somewhere on the street and then travel in the direction of the street until you reach house number 3.



Lines

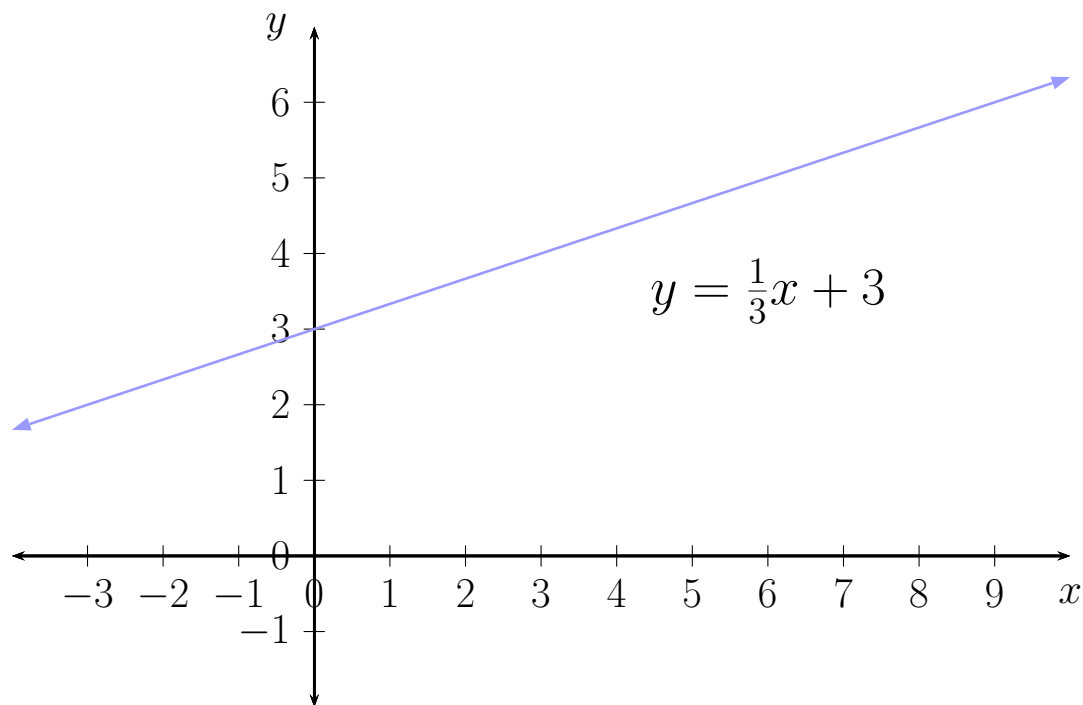
How do you find house number -1 on a street?

Go to somewhere on the street and then travel in the direction of the street until you reach house number -1 .



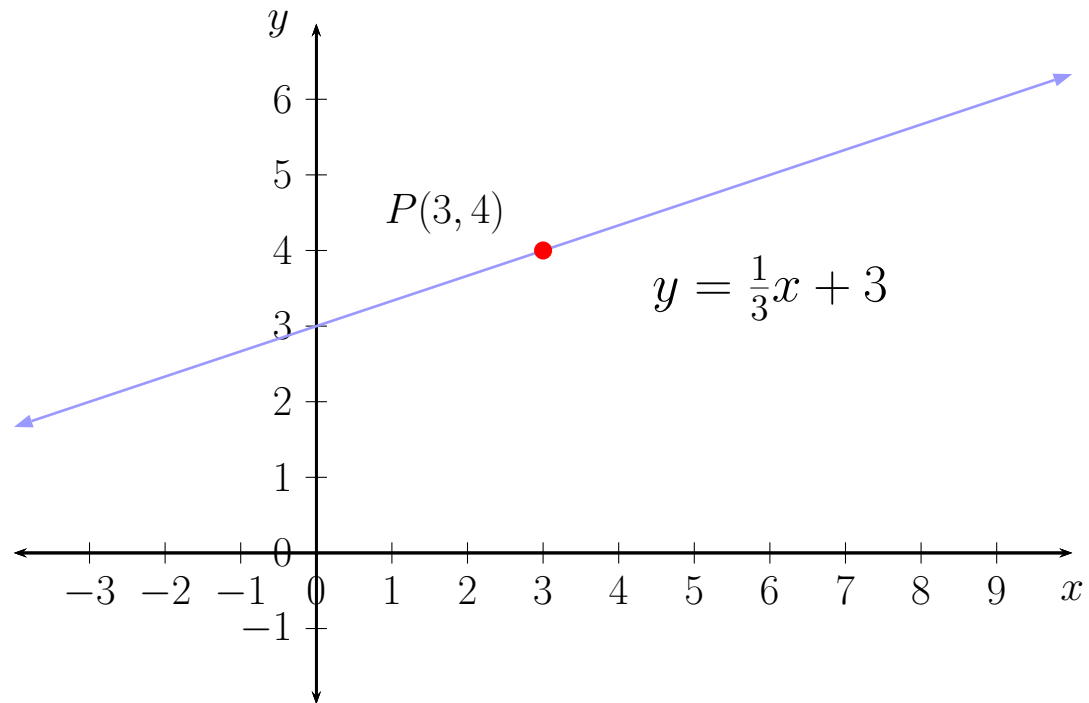
Lines in two dimensions

We can describe a line in 2D using a Cartesian equation.



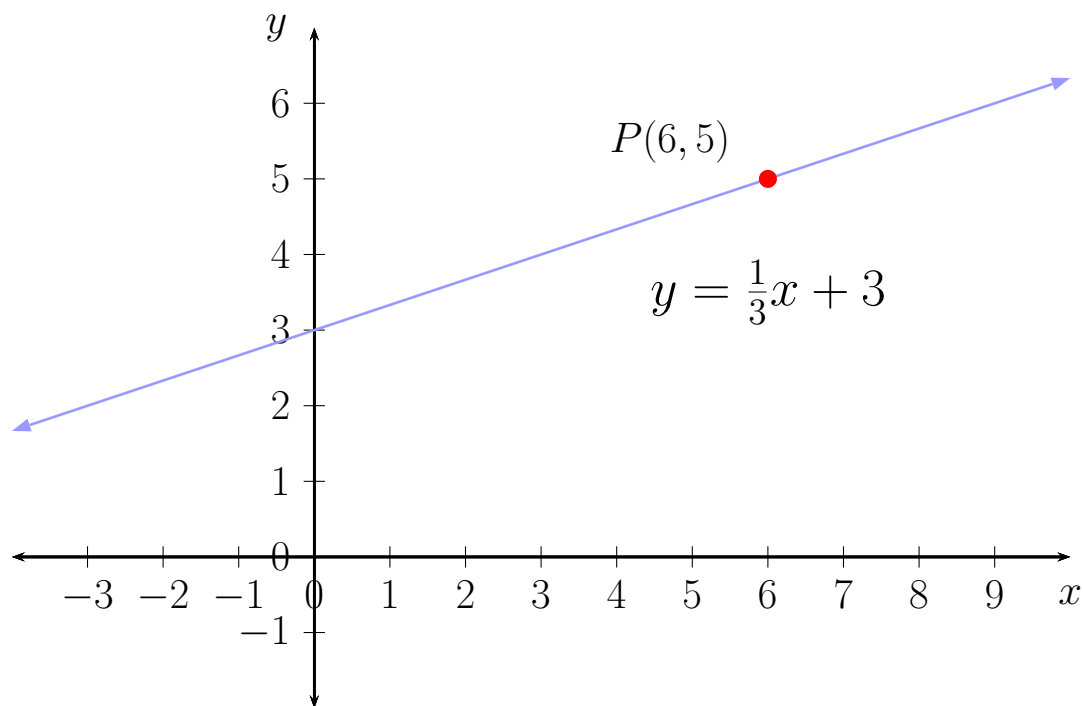
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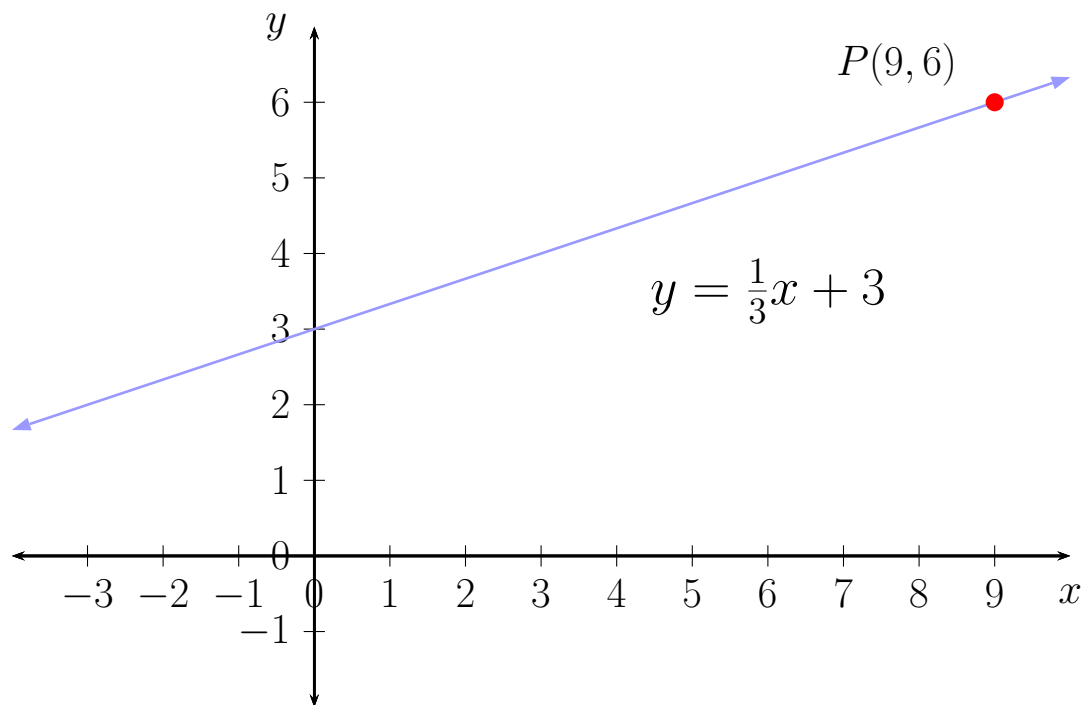
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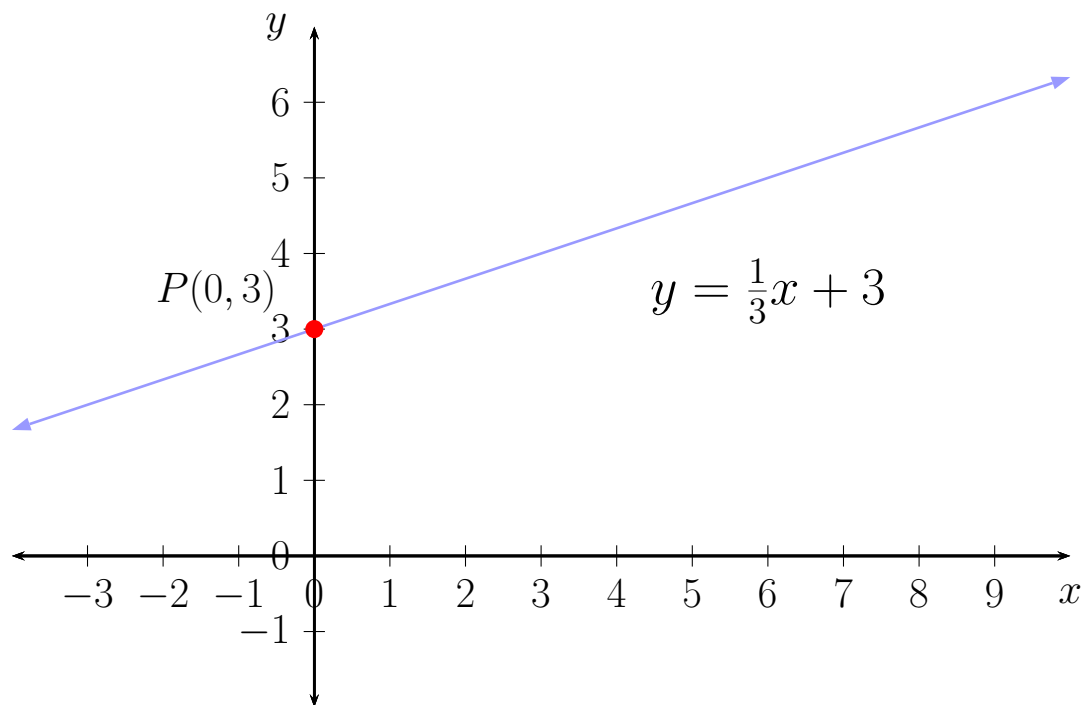
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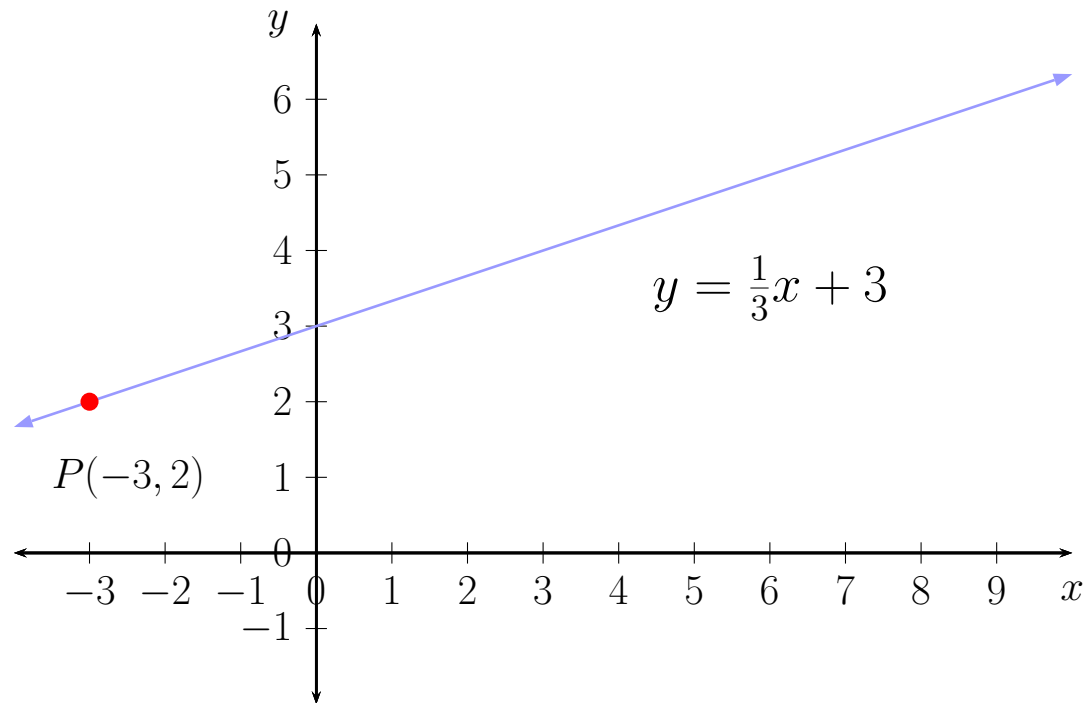
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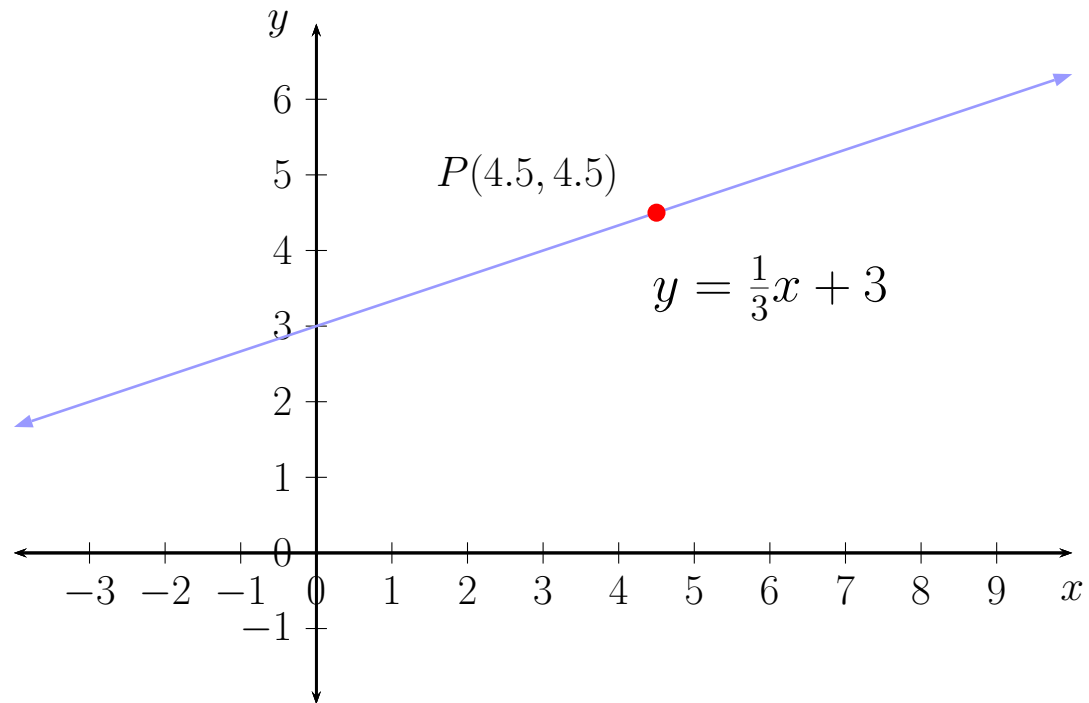
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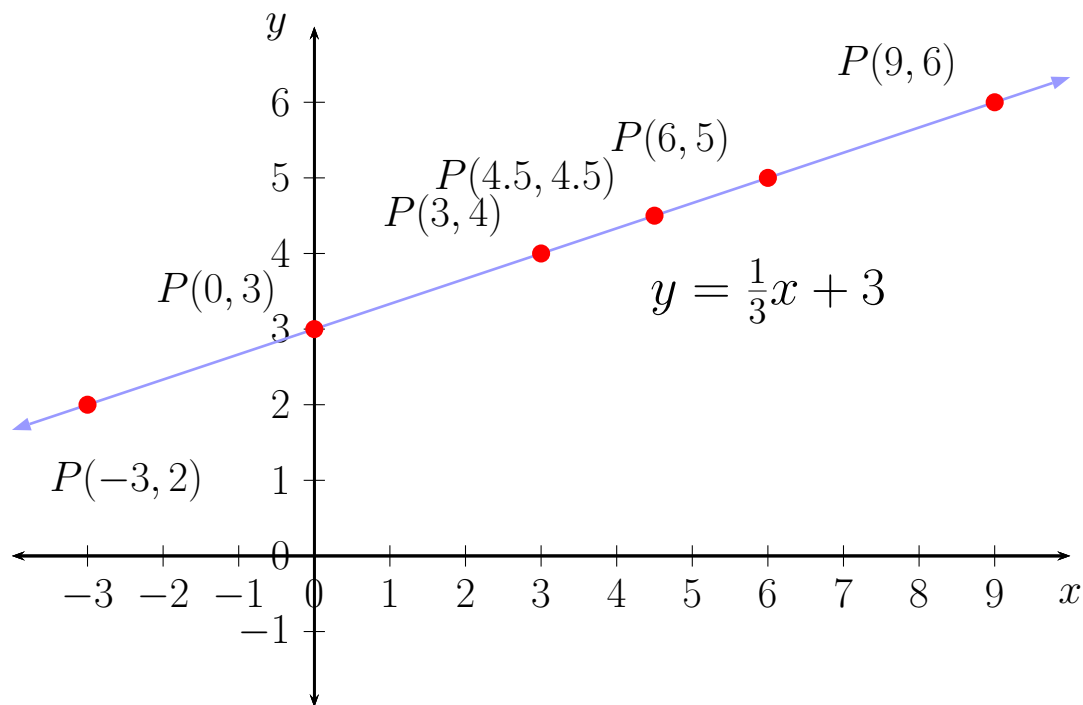
Lines in two dimensions

We can describe a line in 2D using a Cartesian equation.



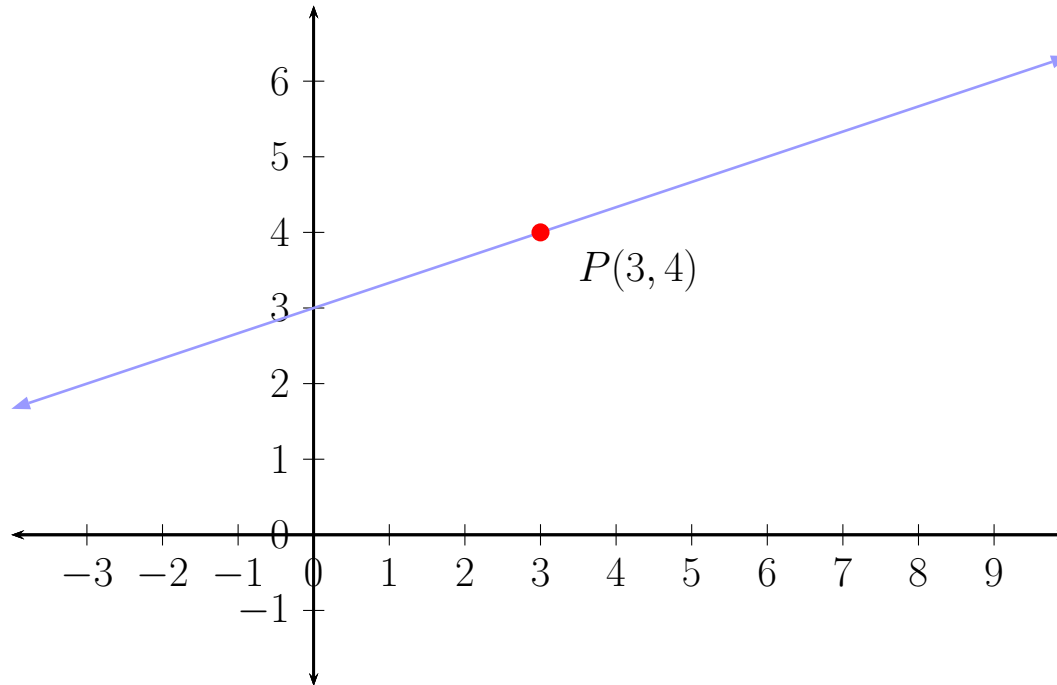
Lines in two dimensions

We can describe a line in 2D using a Cartesian equation.



Lines in two dimensions

Now let's describe this line using vectors.

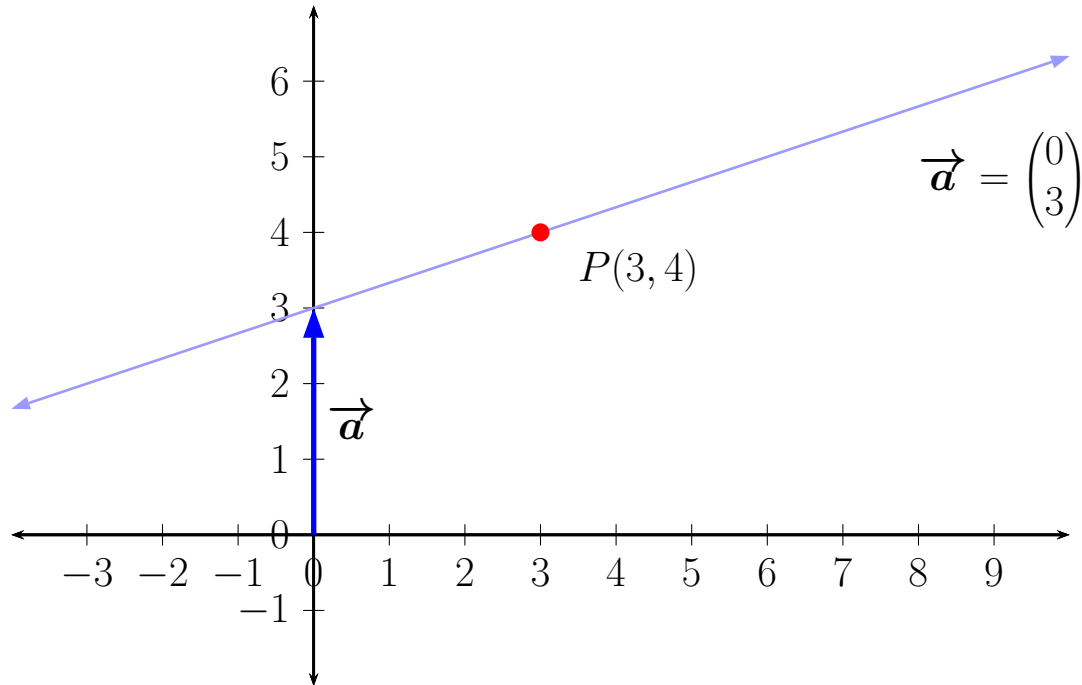


First we pick a vector that takes us to a point on the line.

Then we pick a vector in the direction of the line.

Lines in two dimensions

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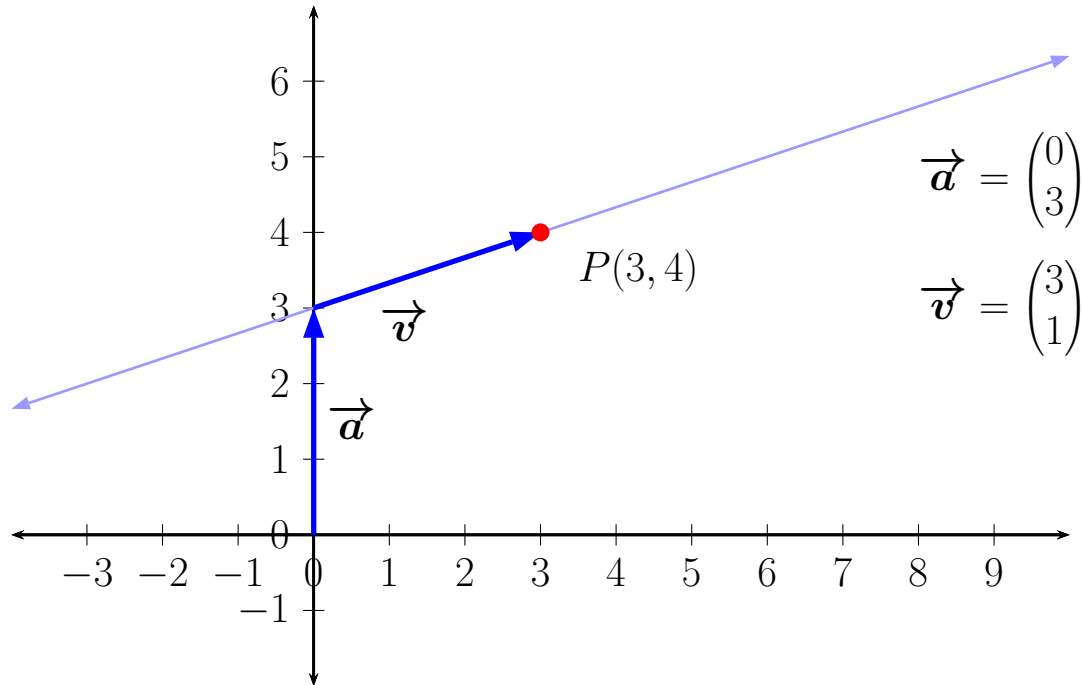


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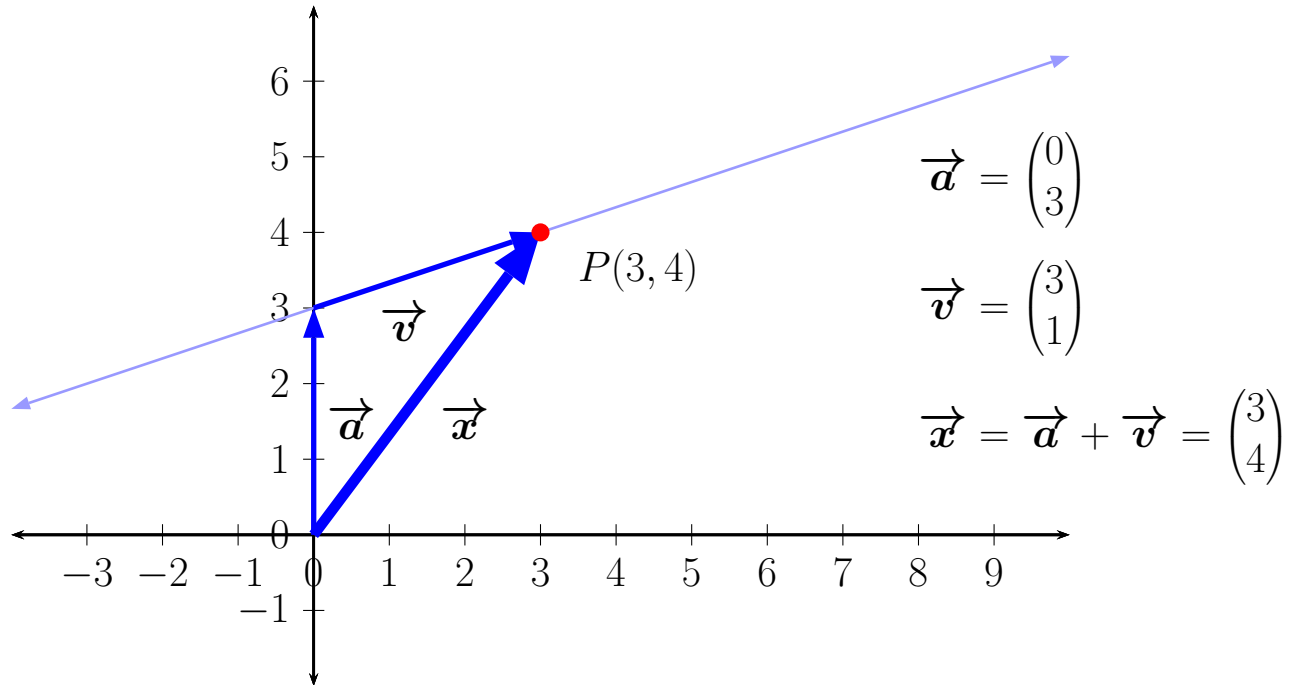


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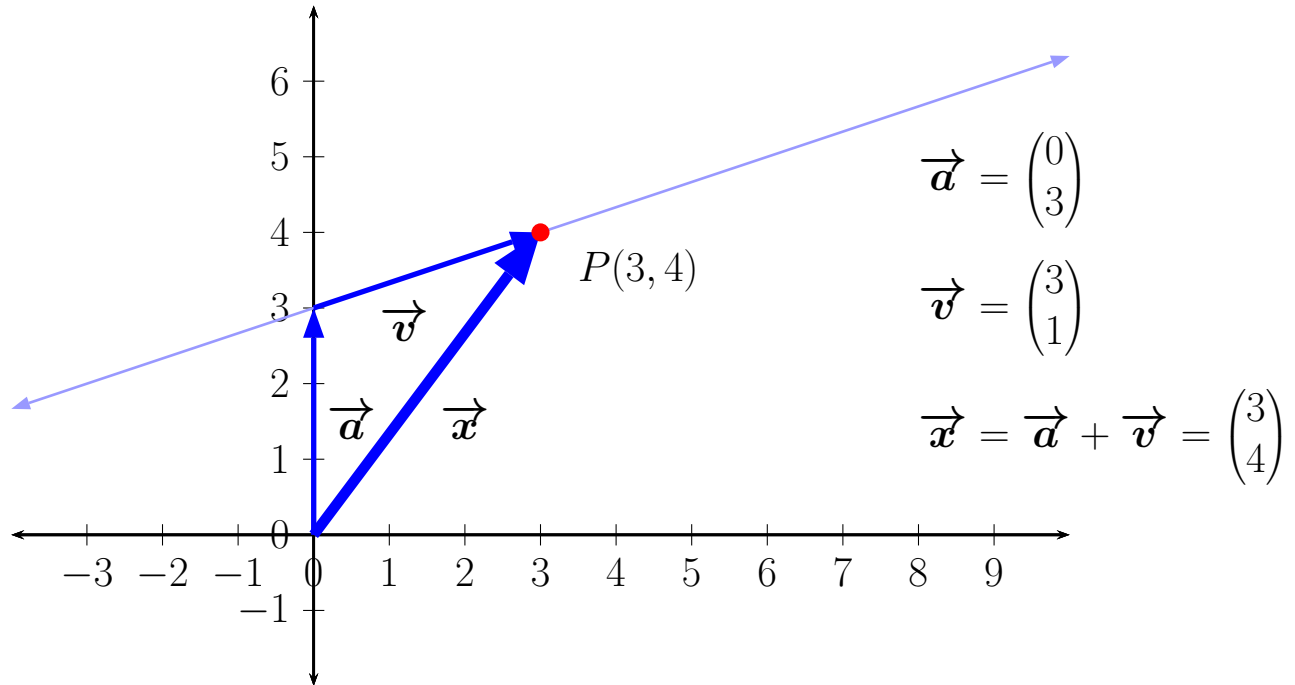


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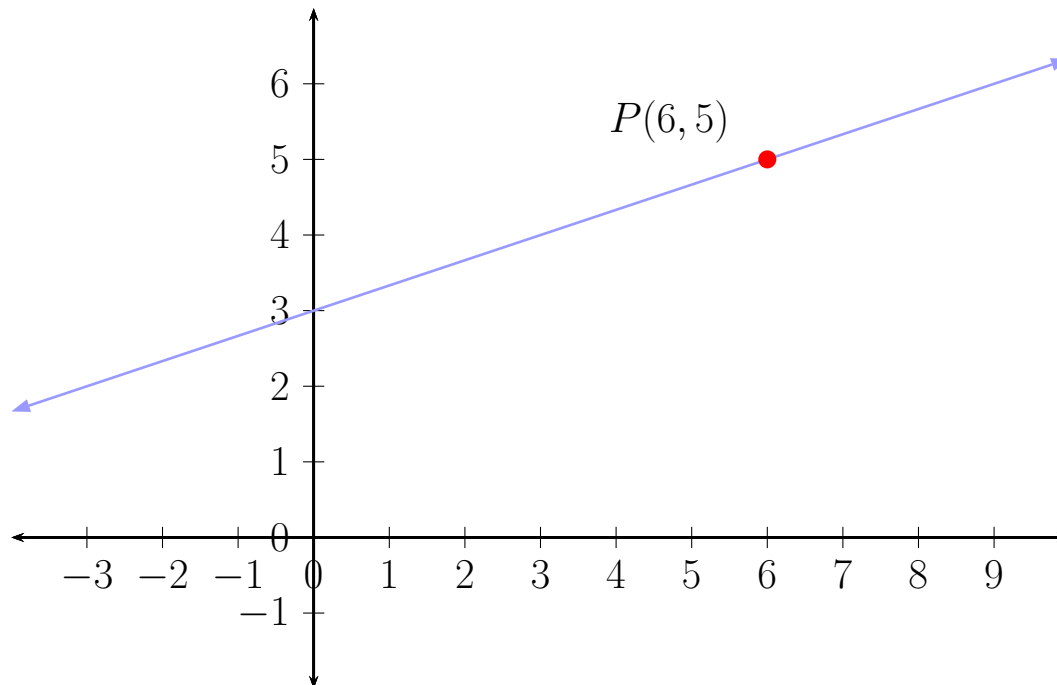


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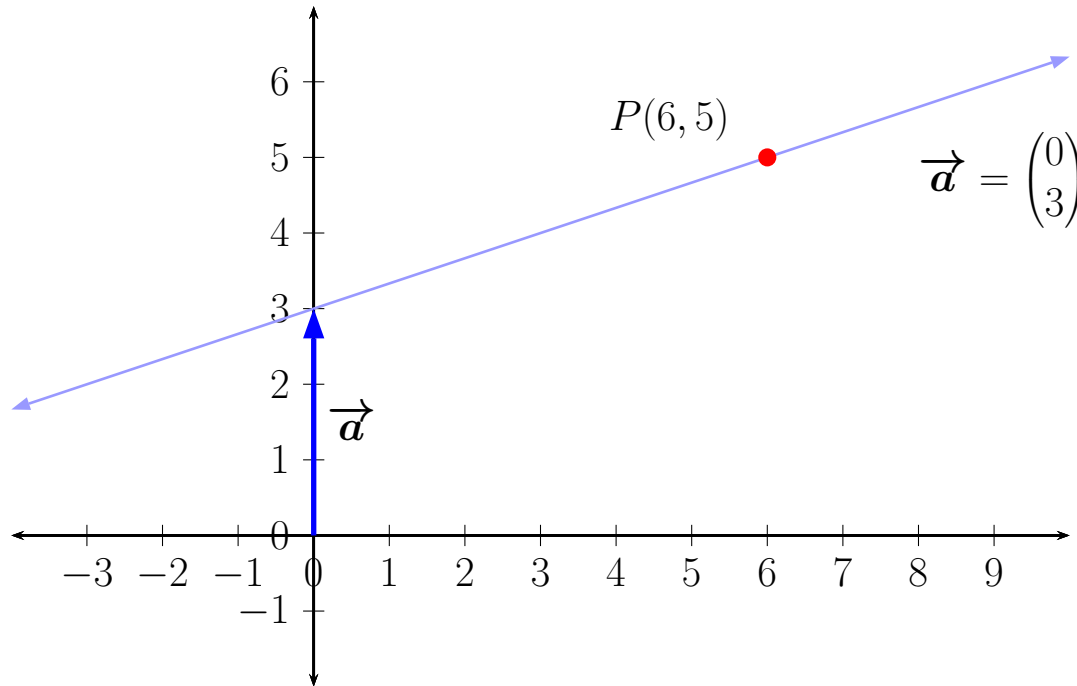


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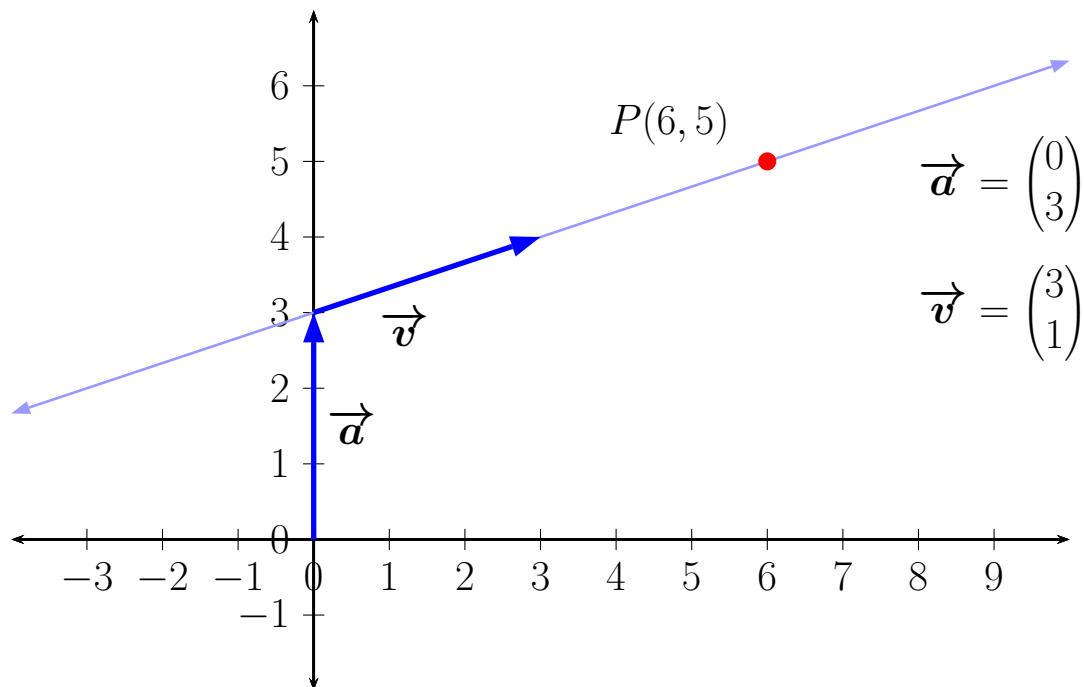


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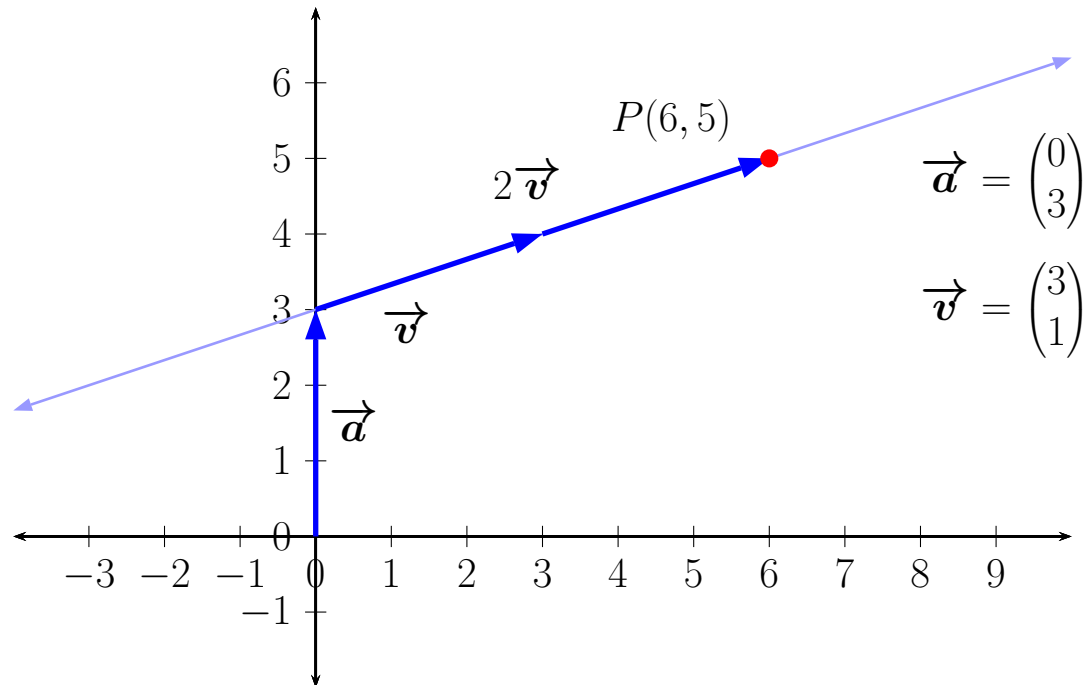


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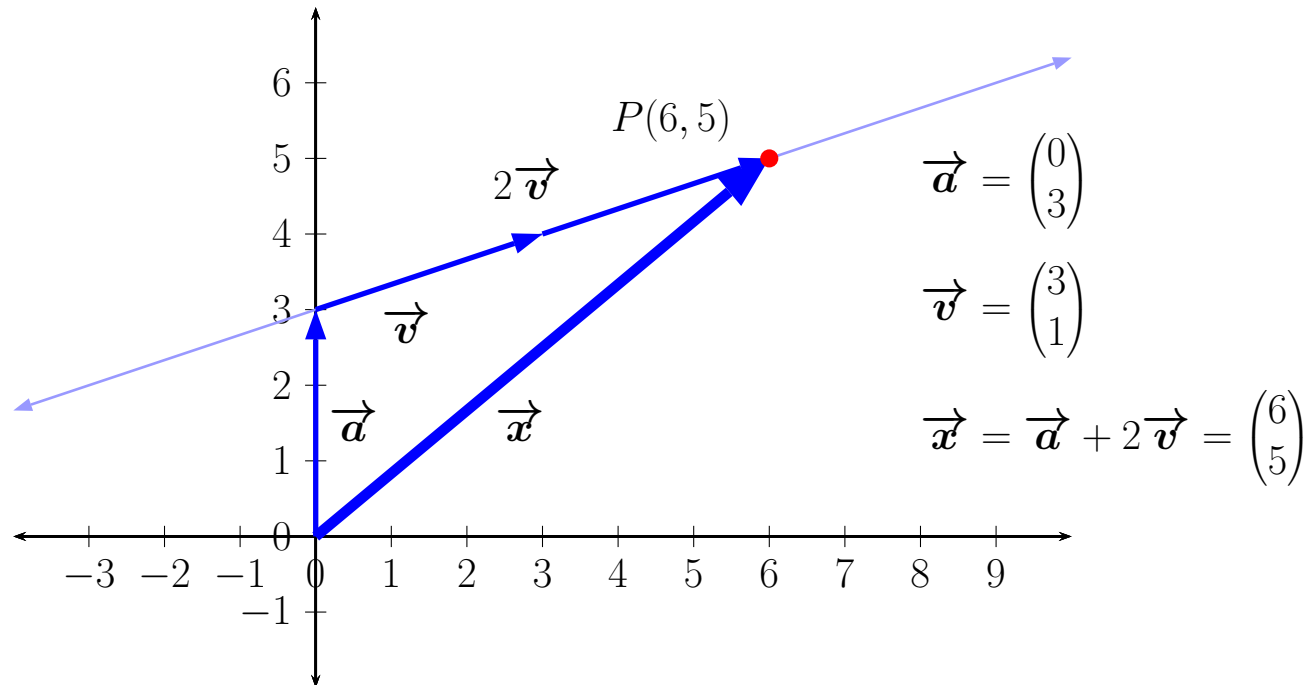


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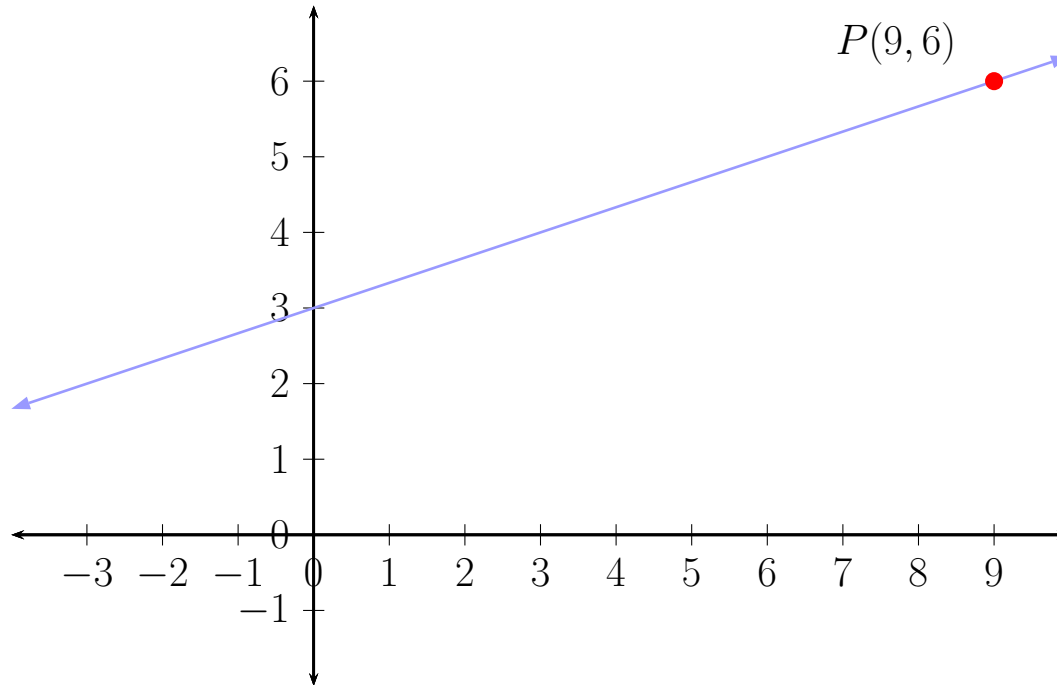


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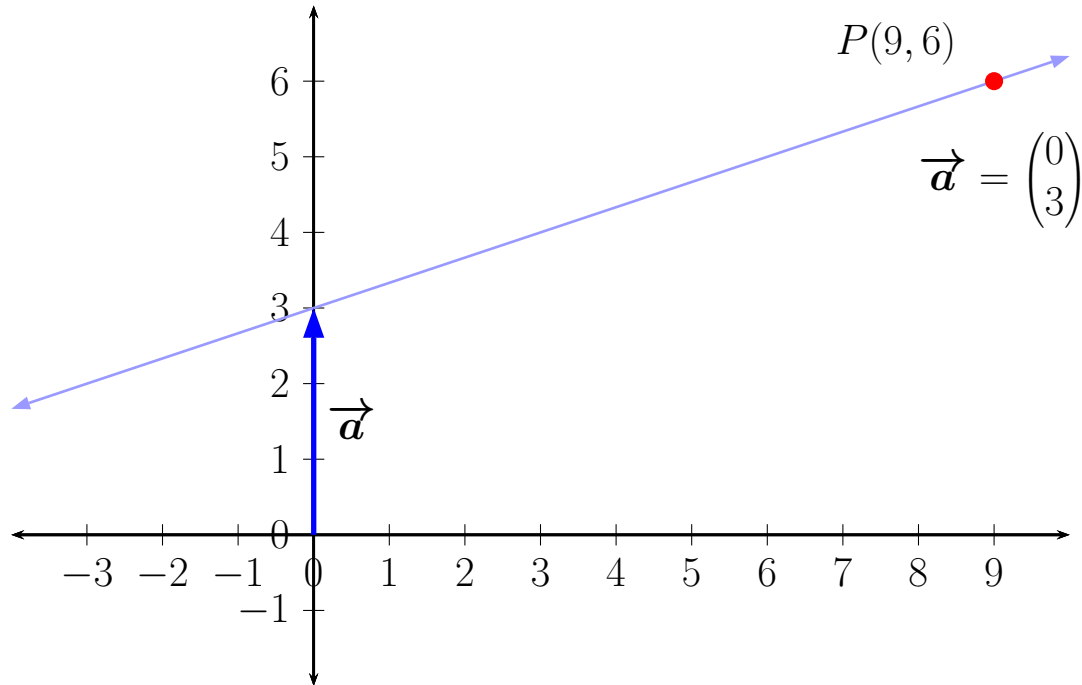


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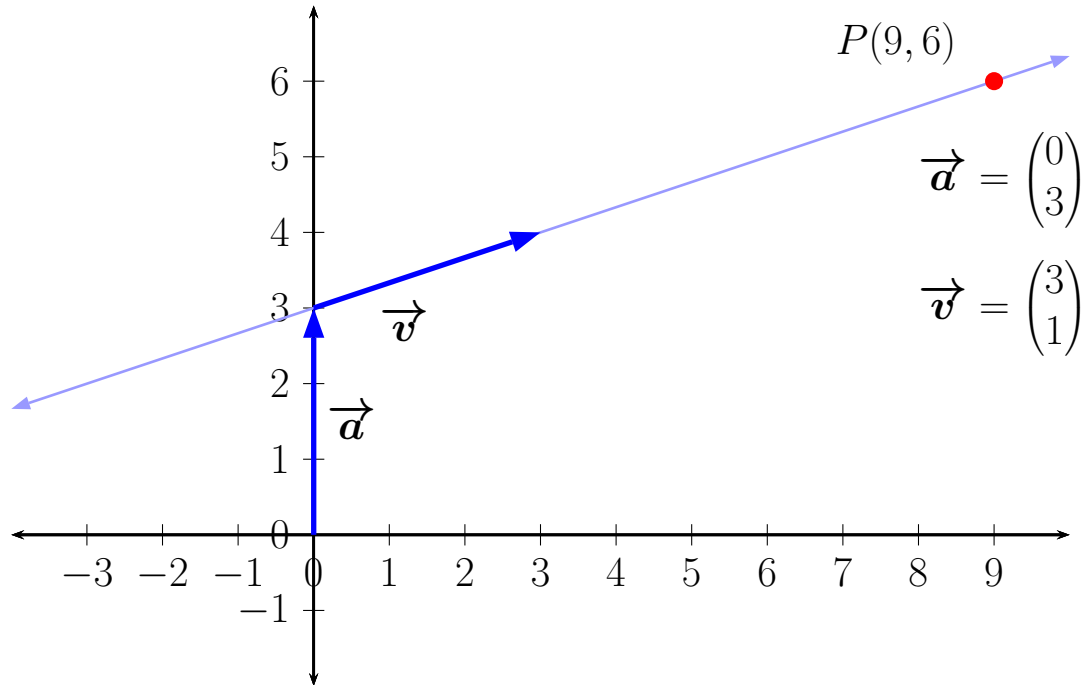


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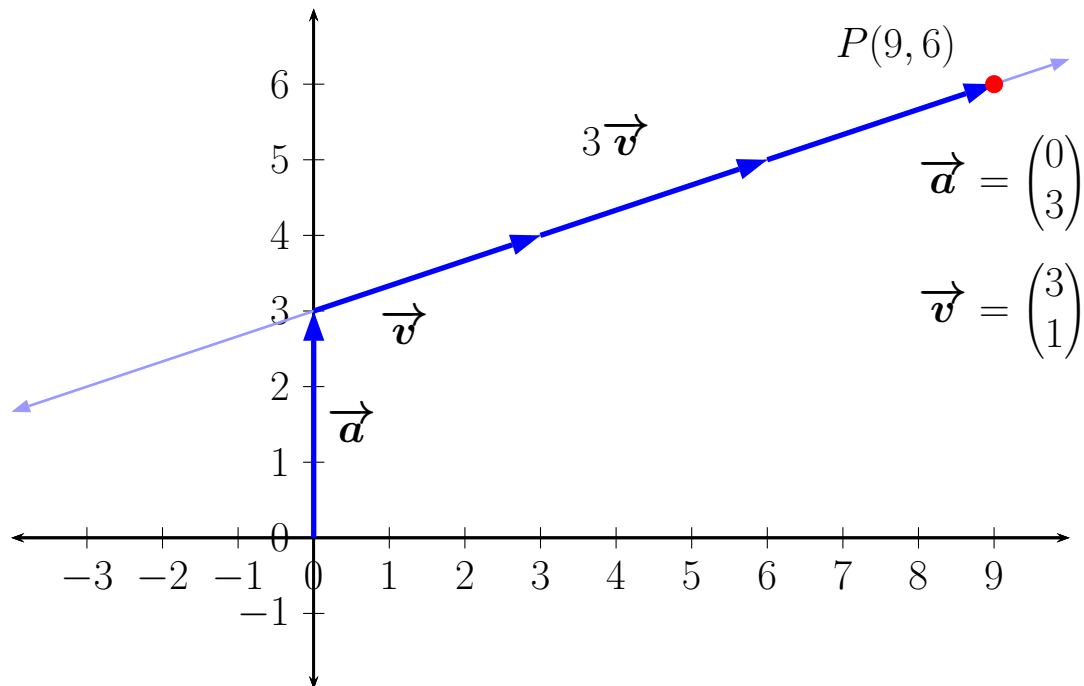


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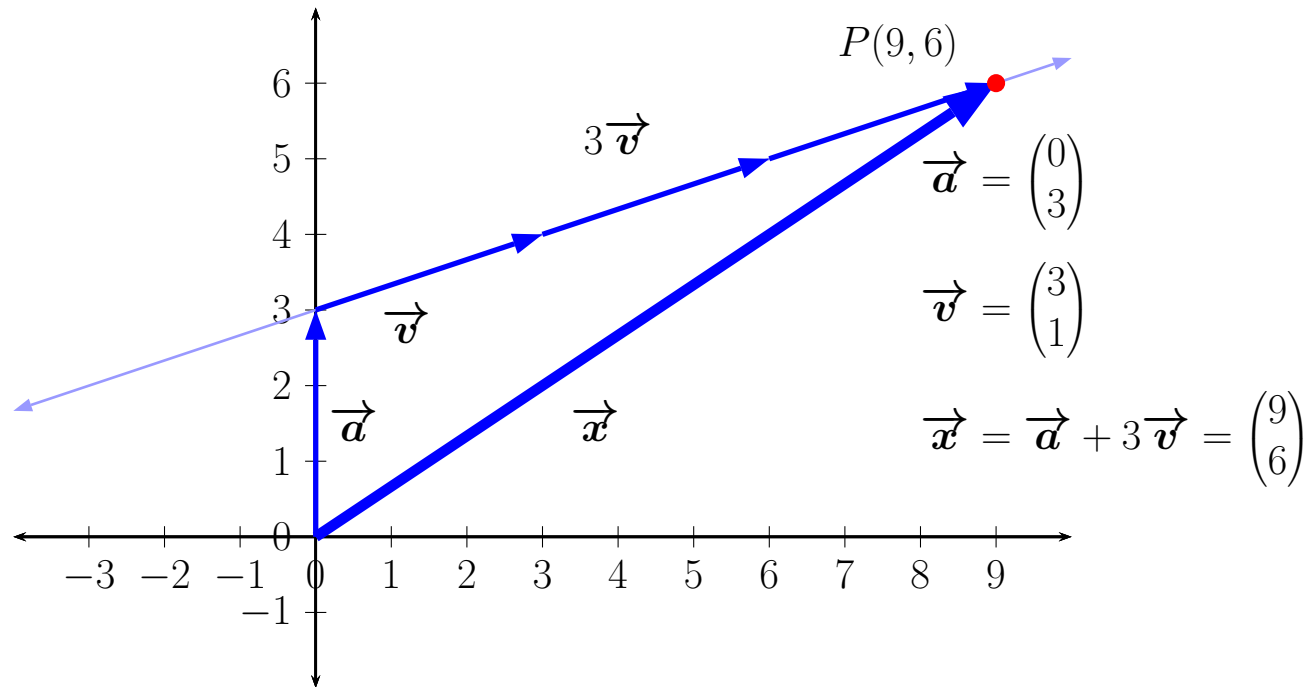


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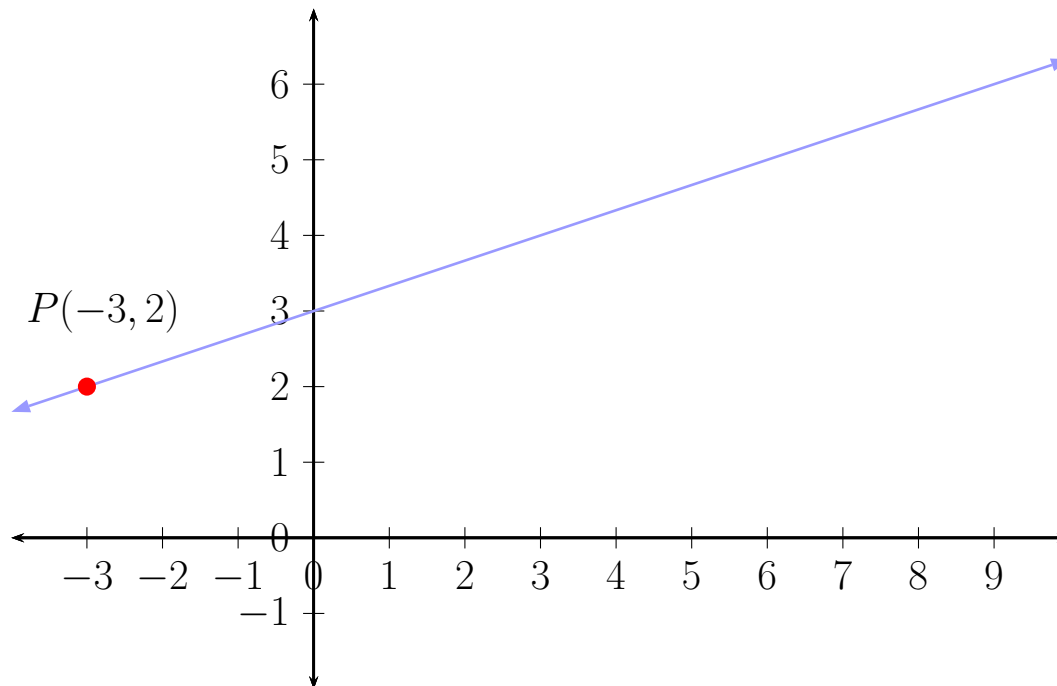


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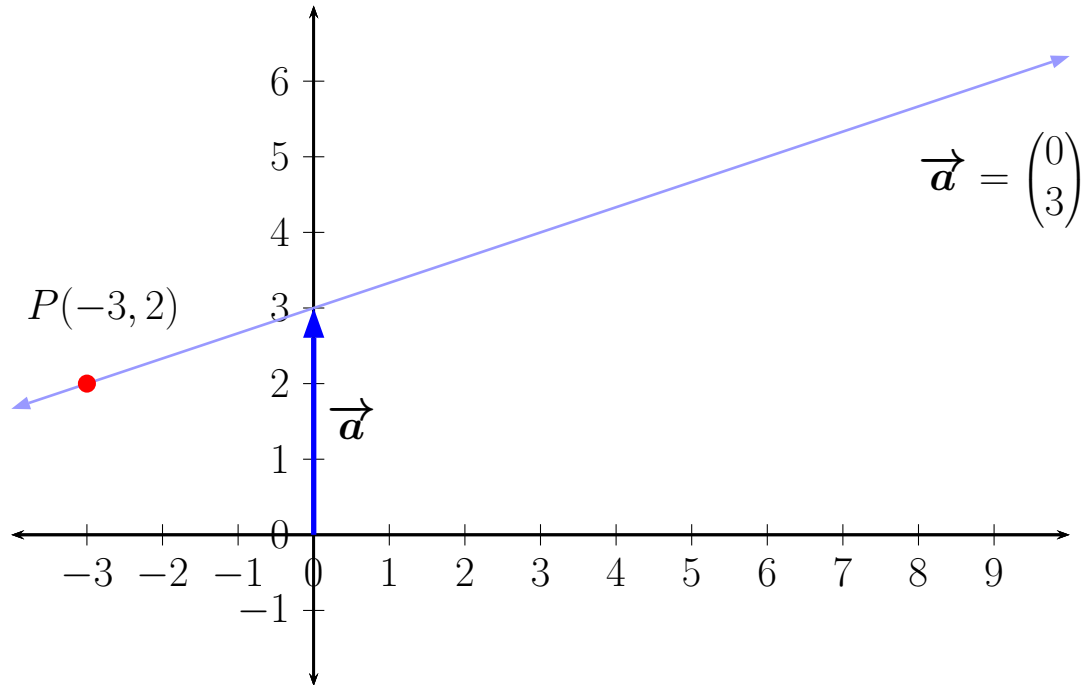


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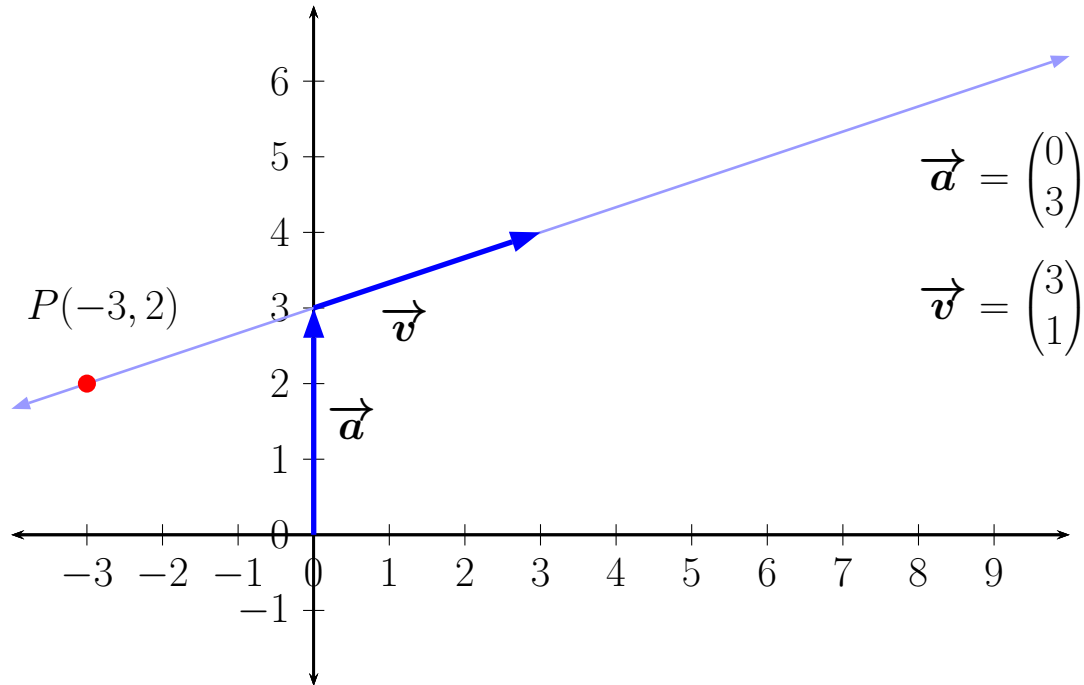


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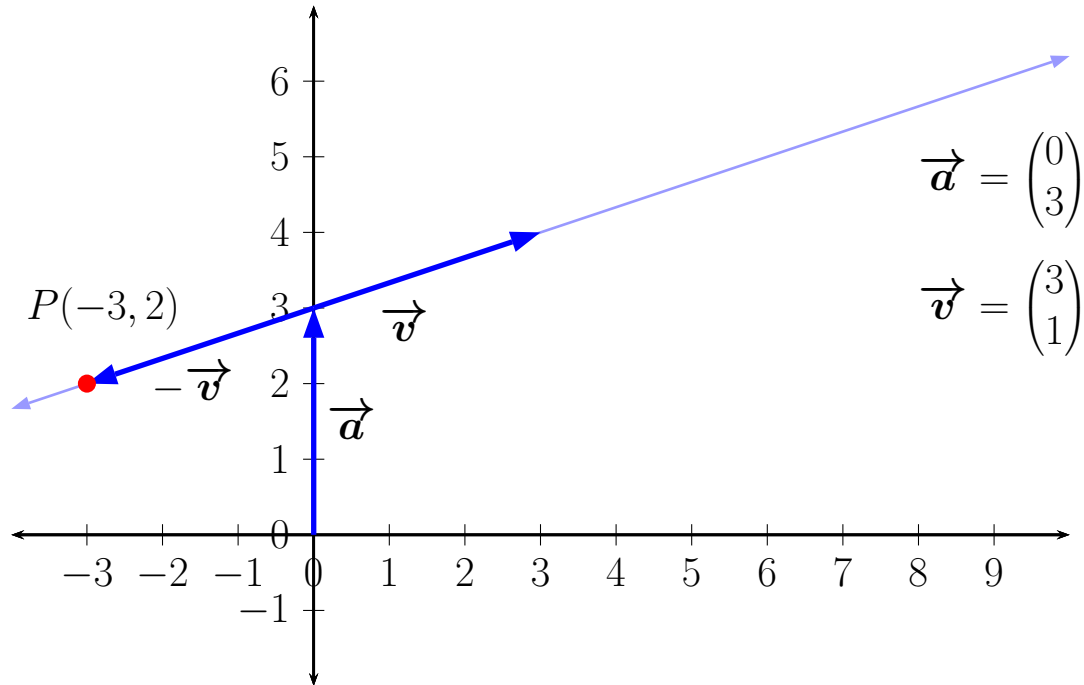


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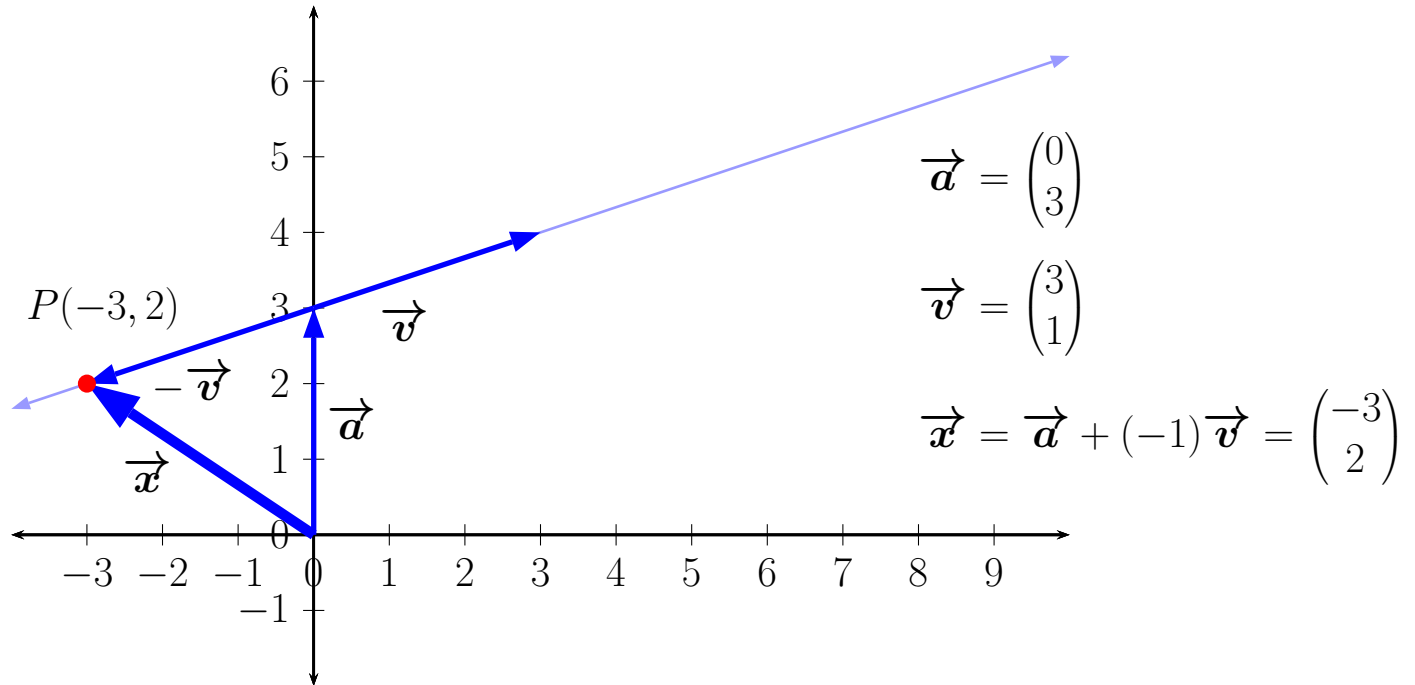


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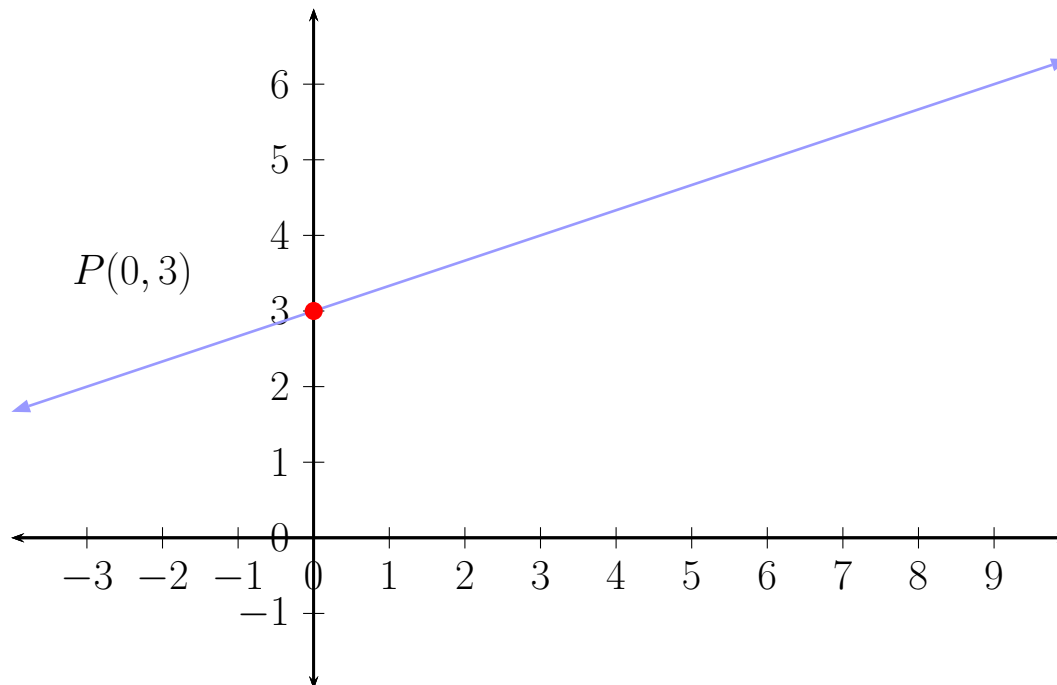


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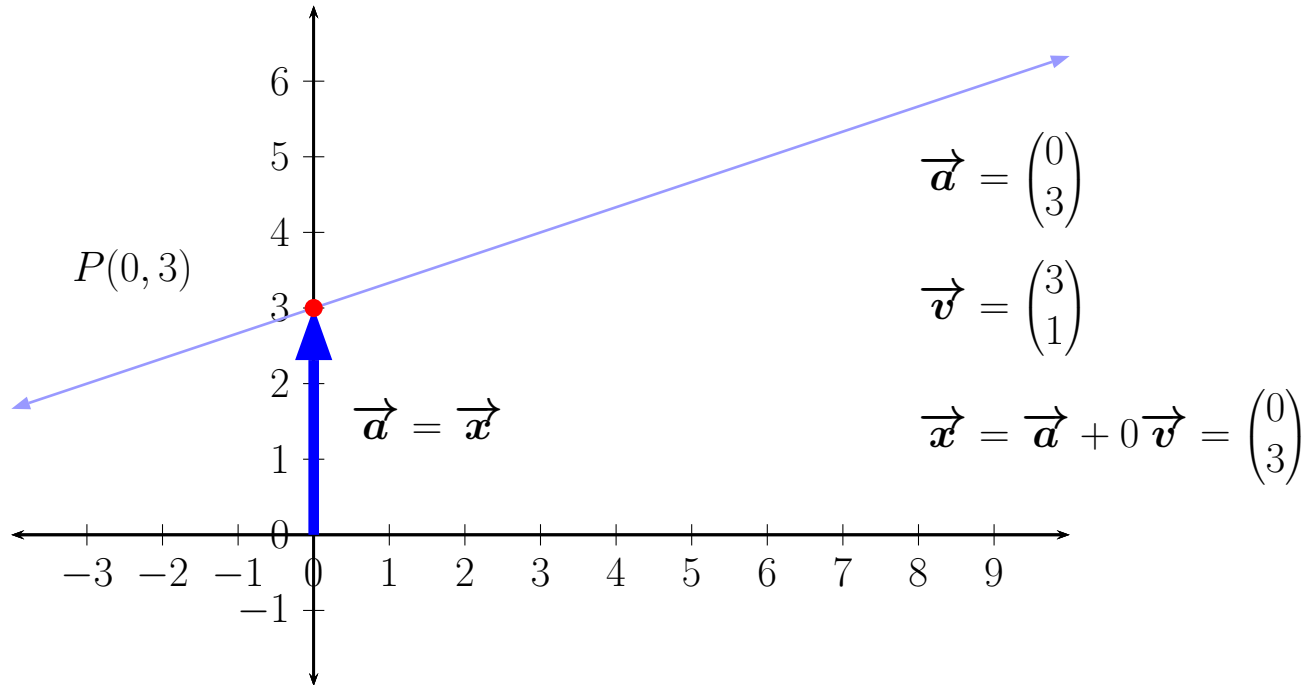


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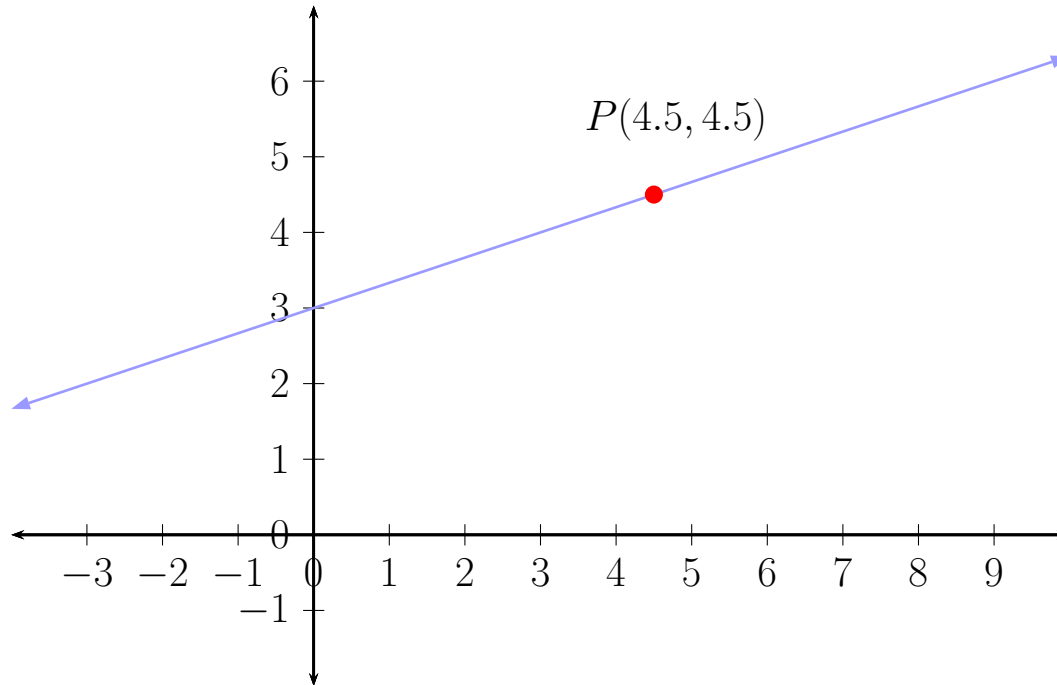


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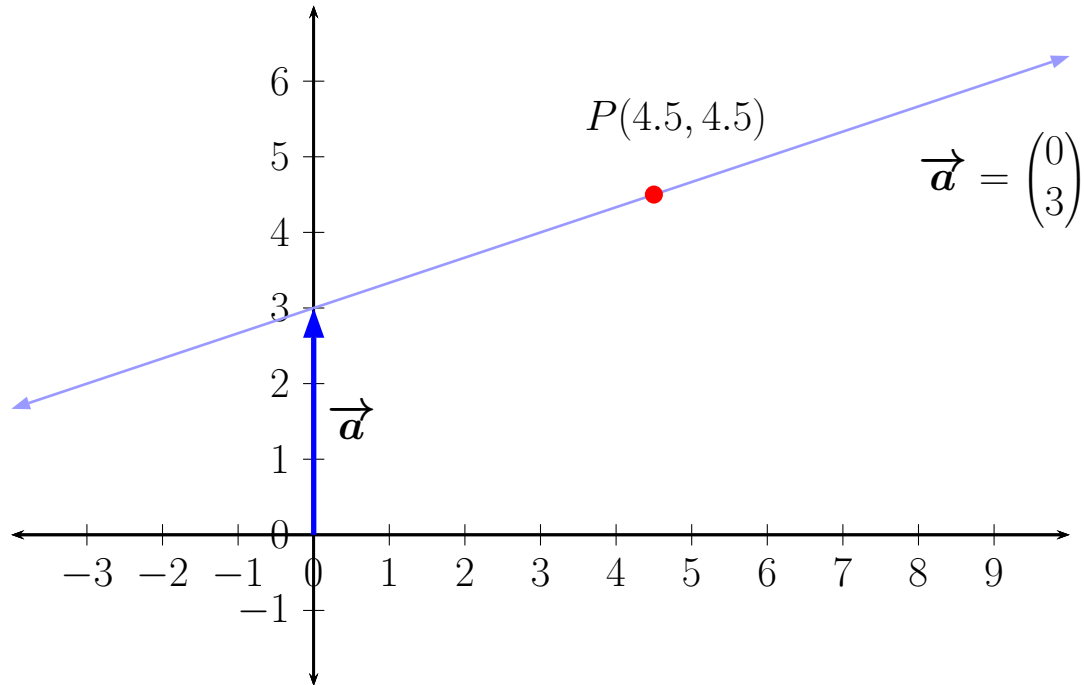


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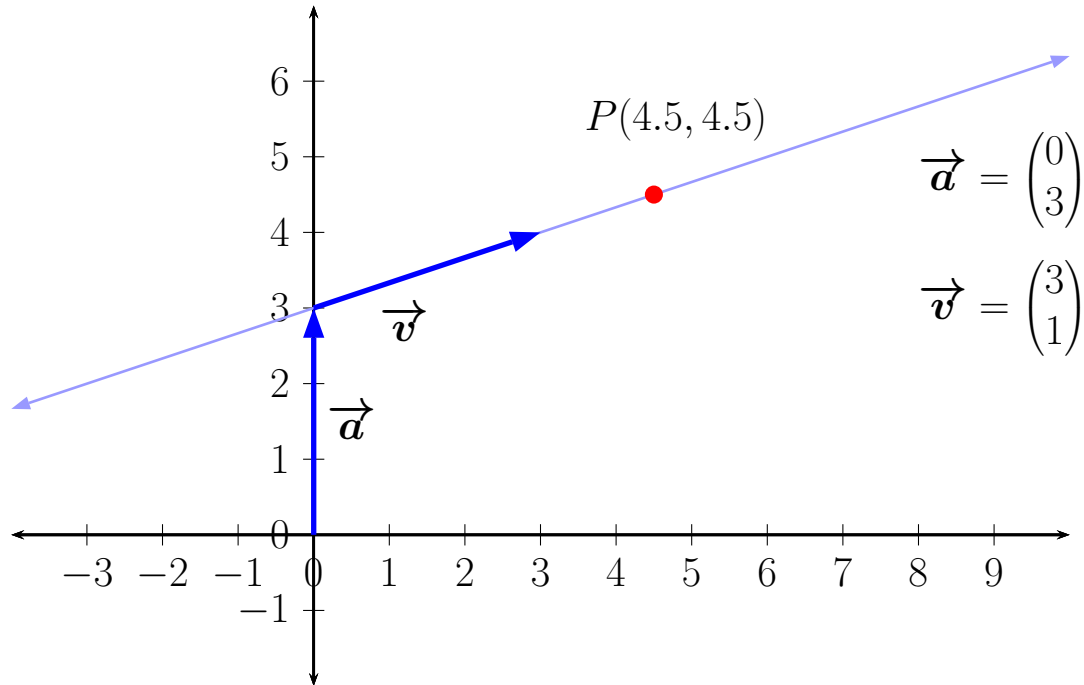


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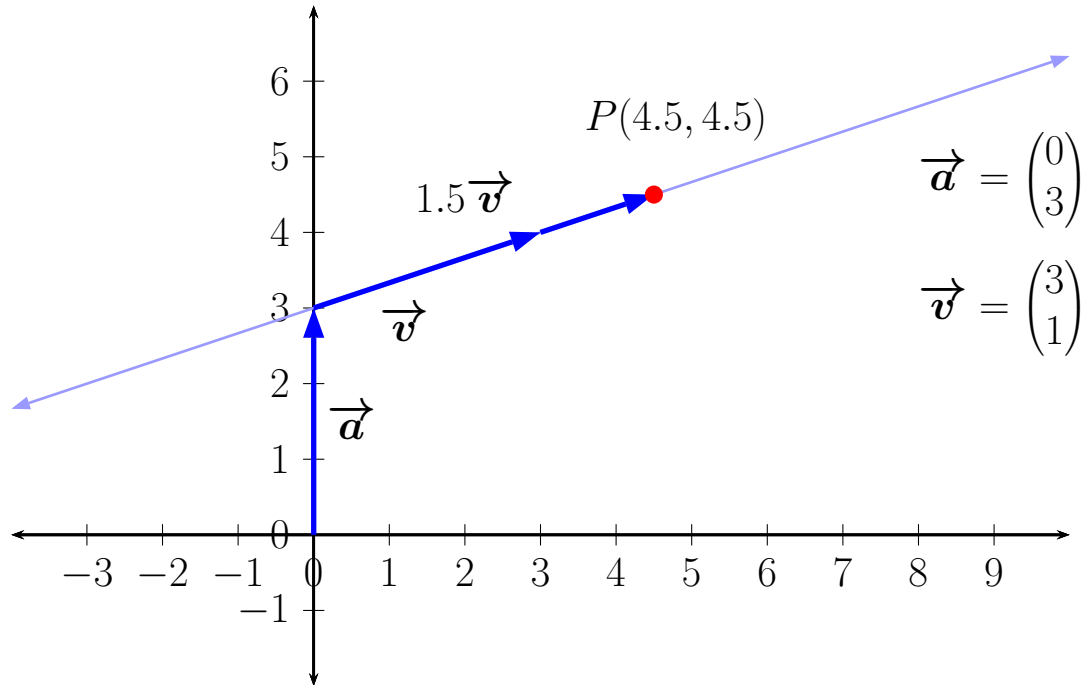


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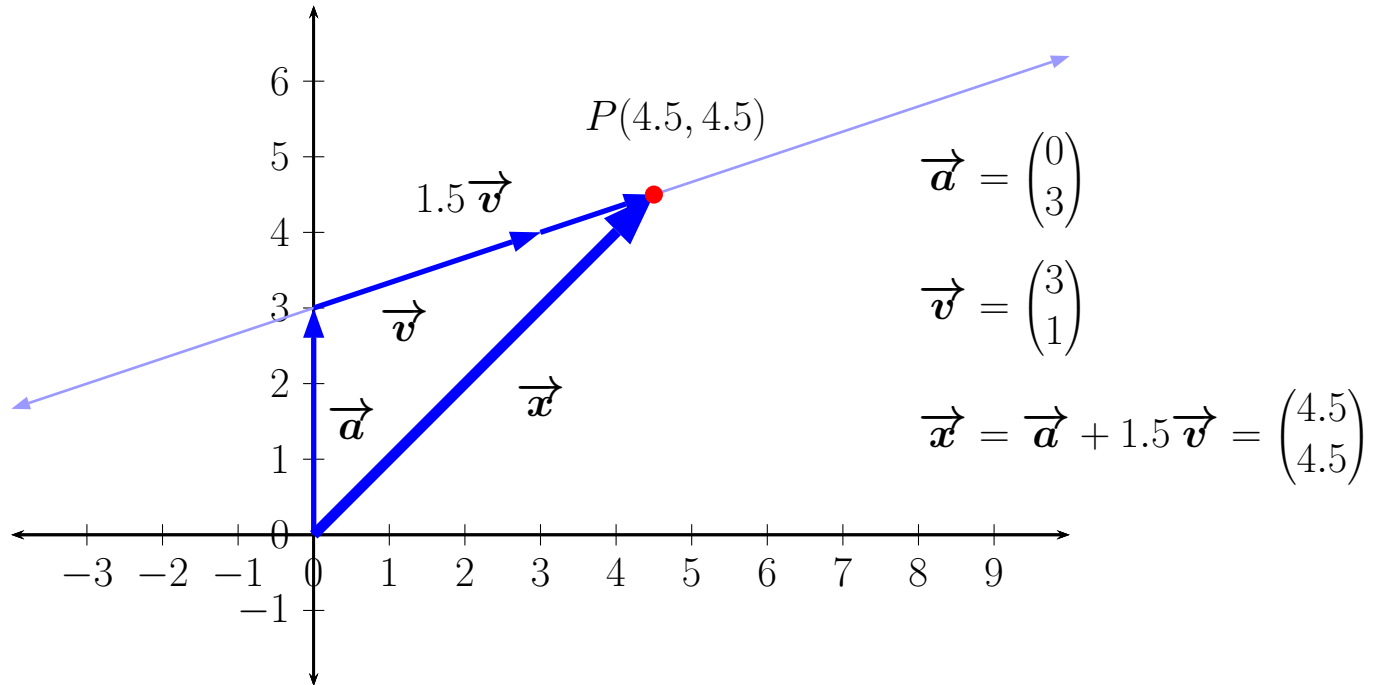


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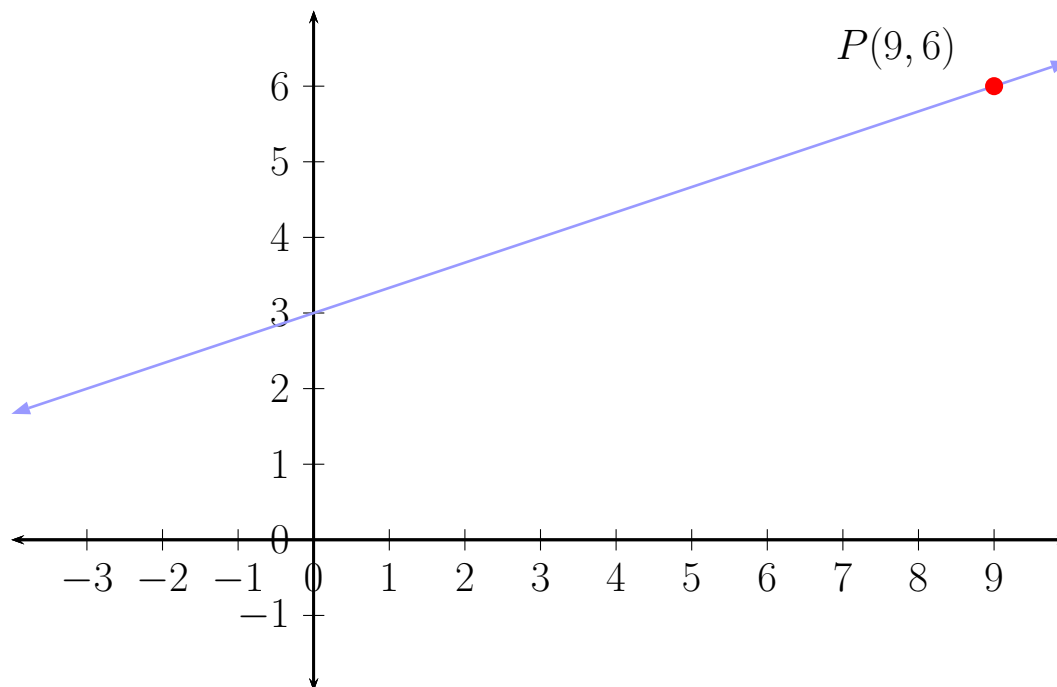


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Lines in two dimensions

We could choose a different starting point on the line.

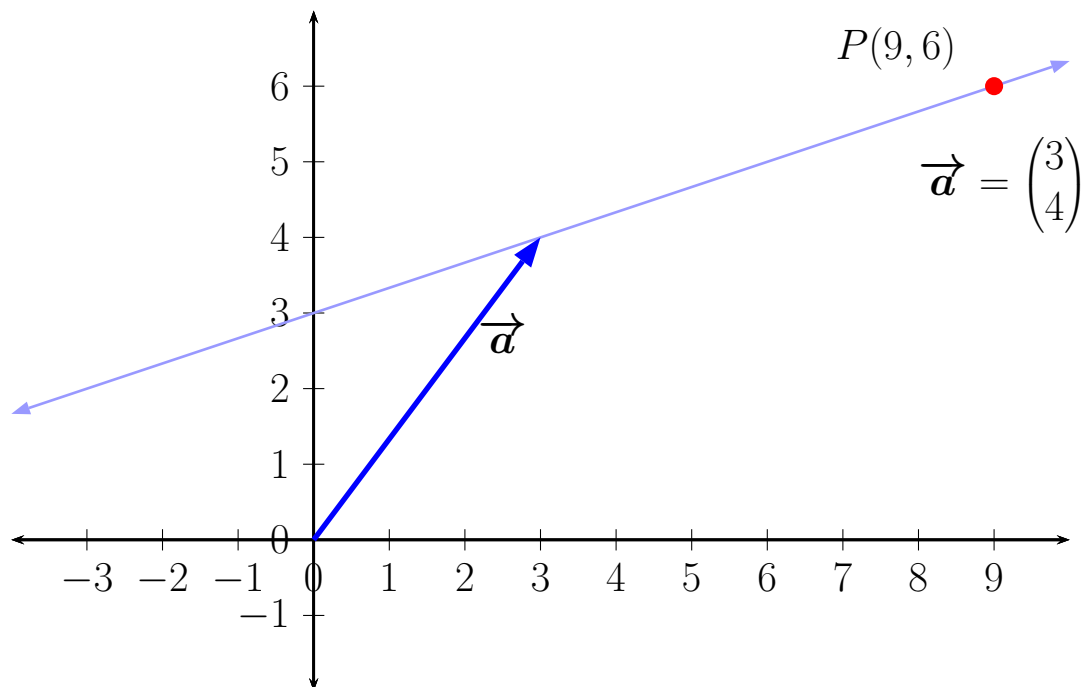


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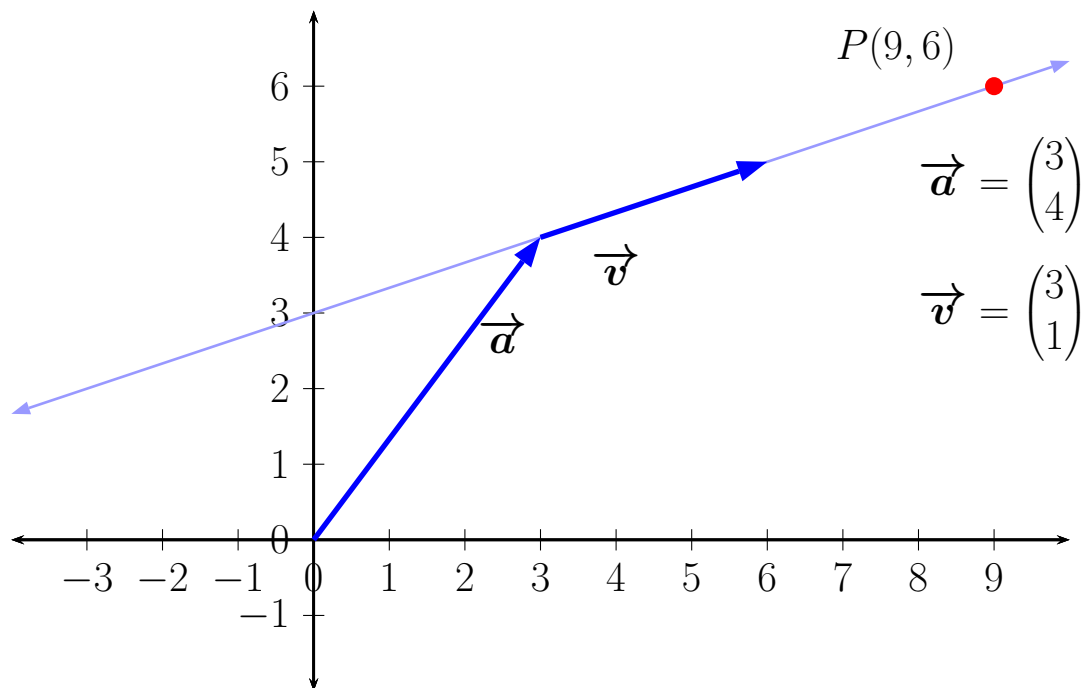


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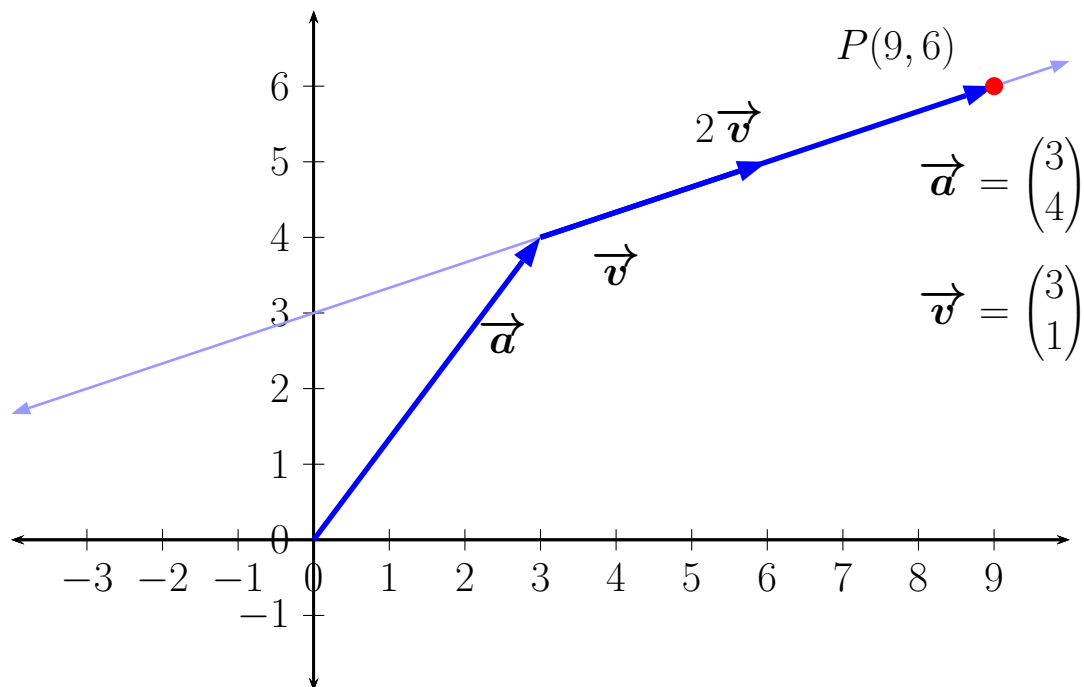


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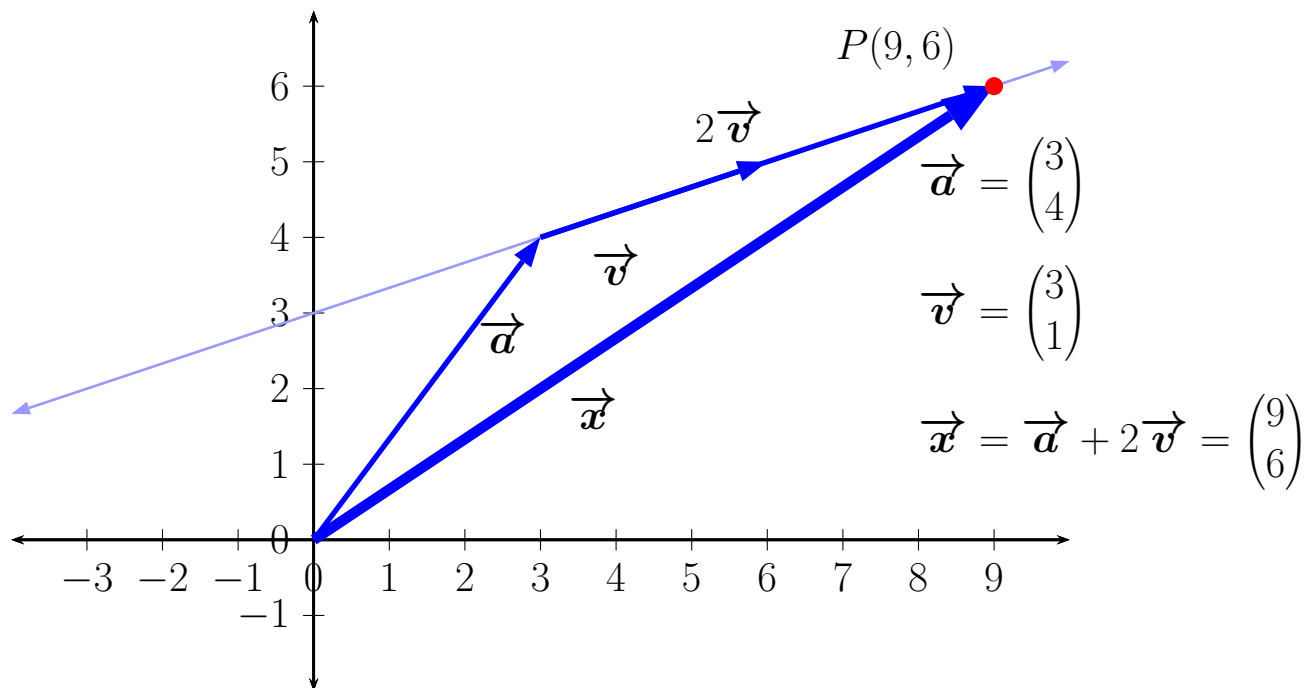


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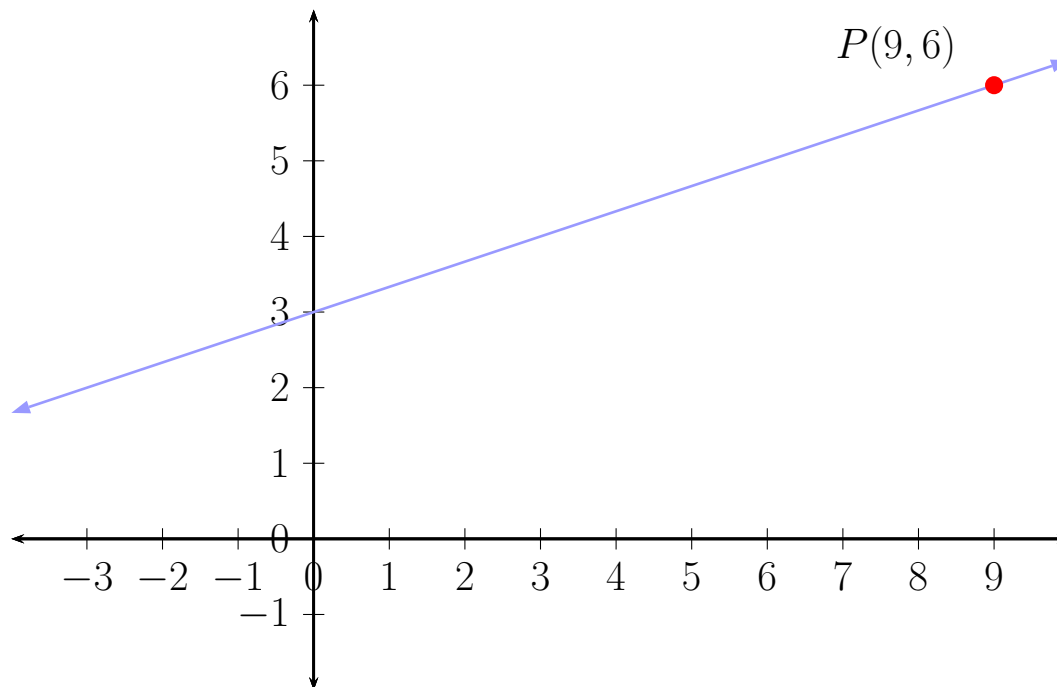


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We could choose a different vector in the direction of the line.

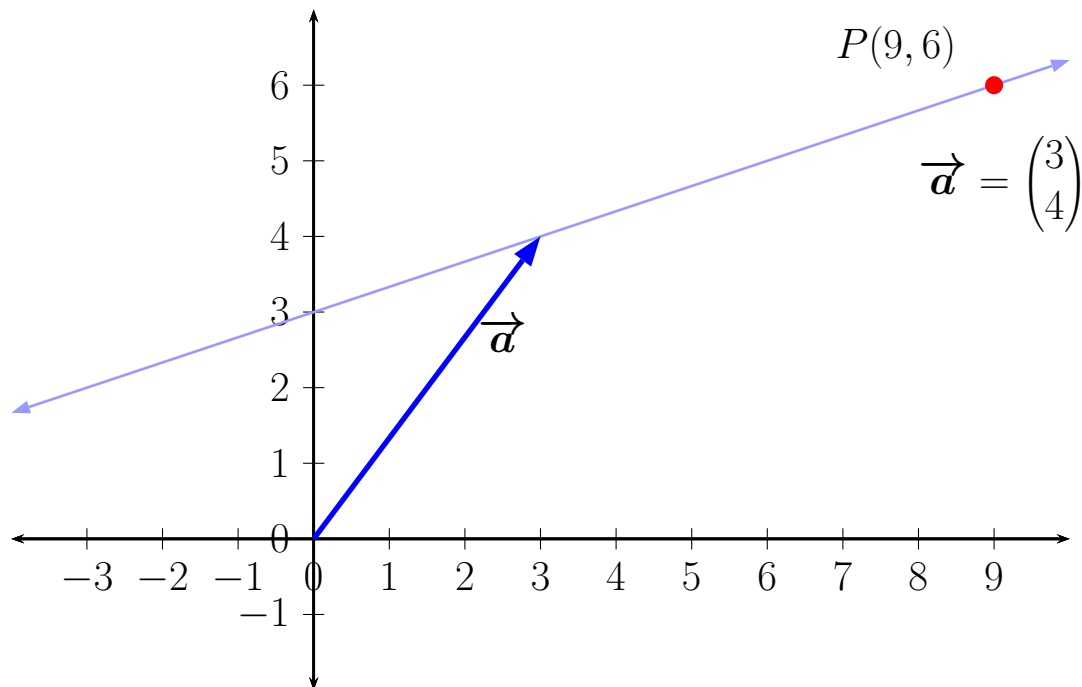


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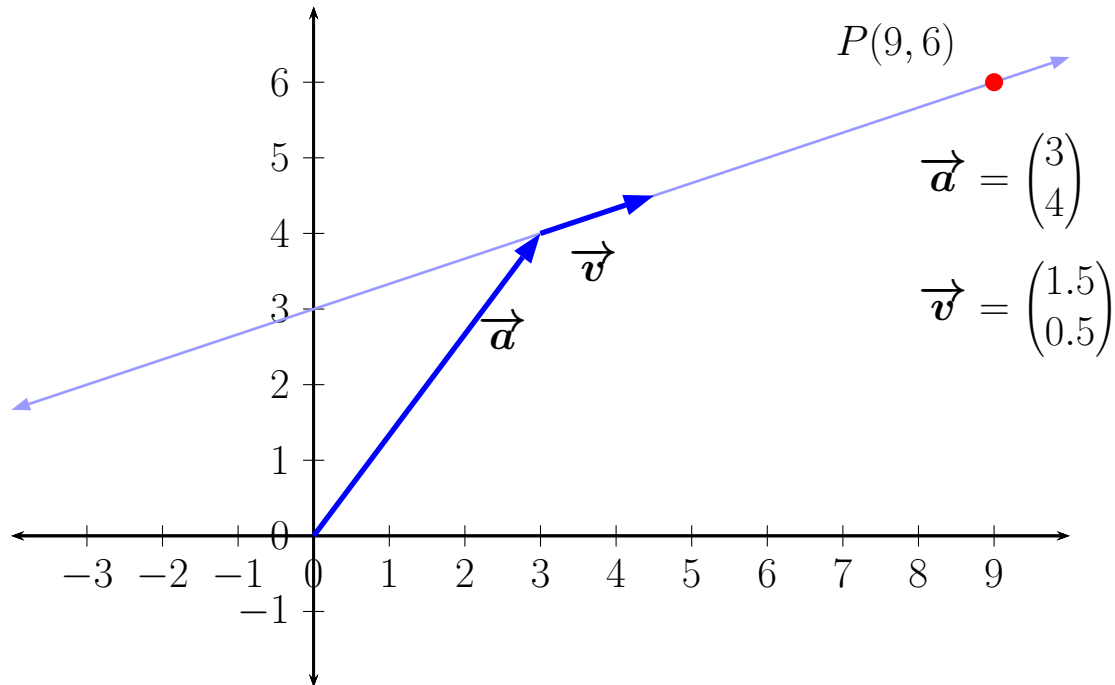


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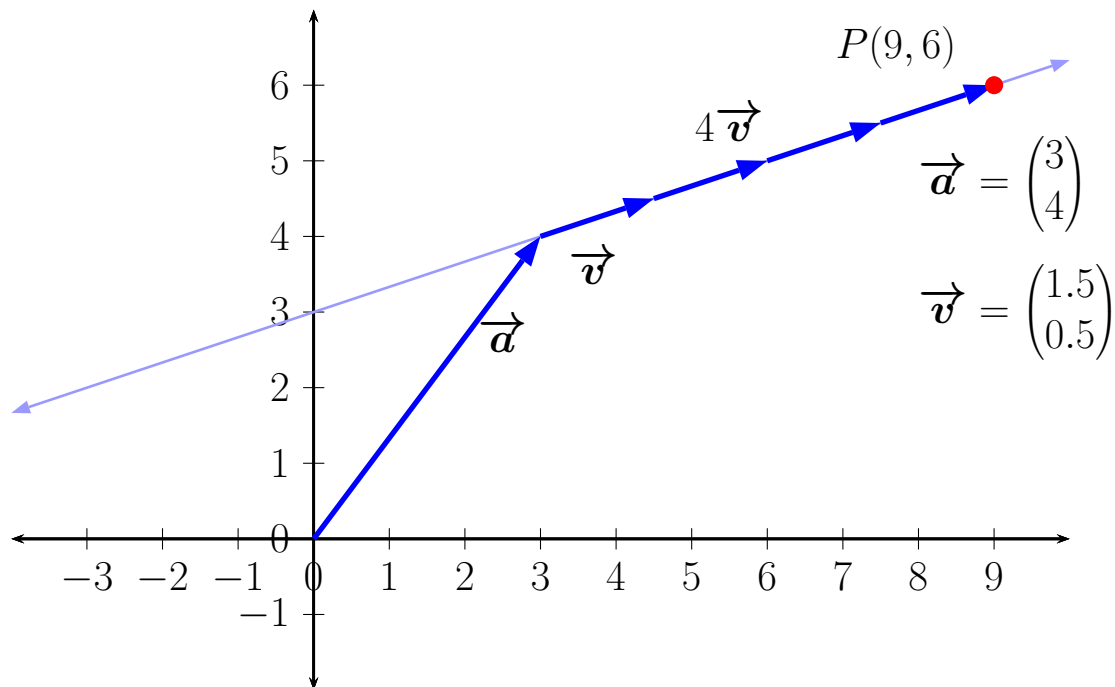


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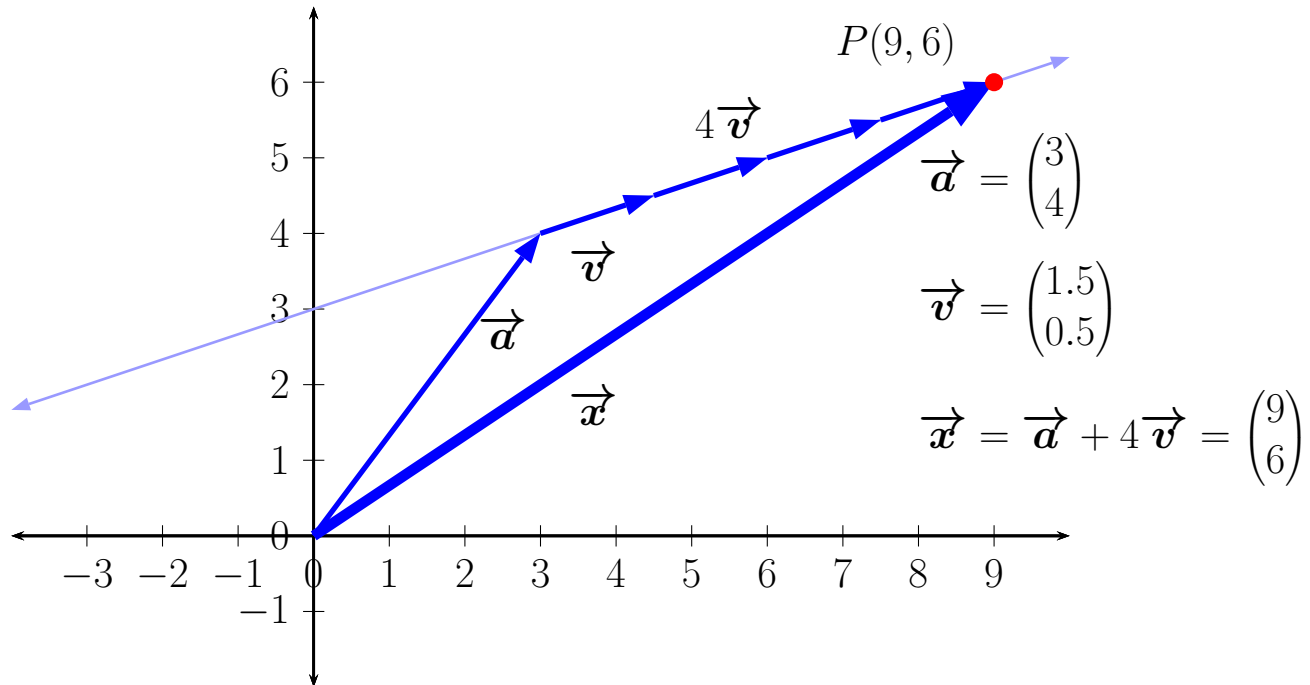


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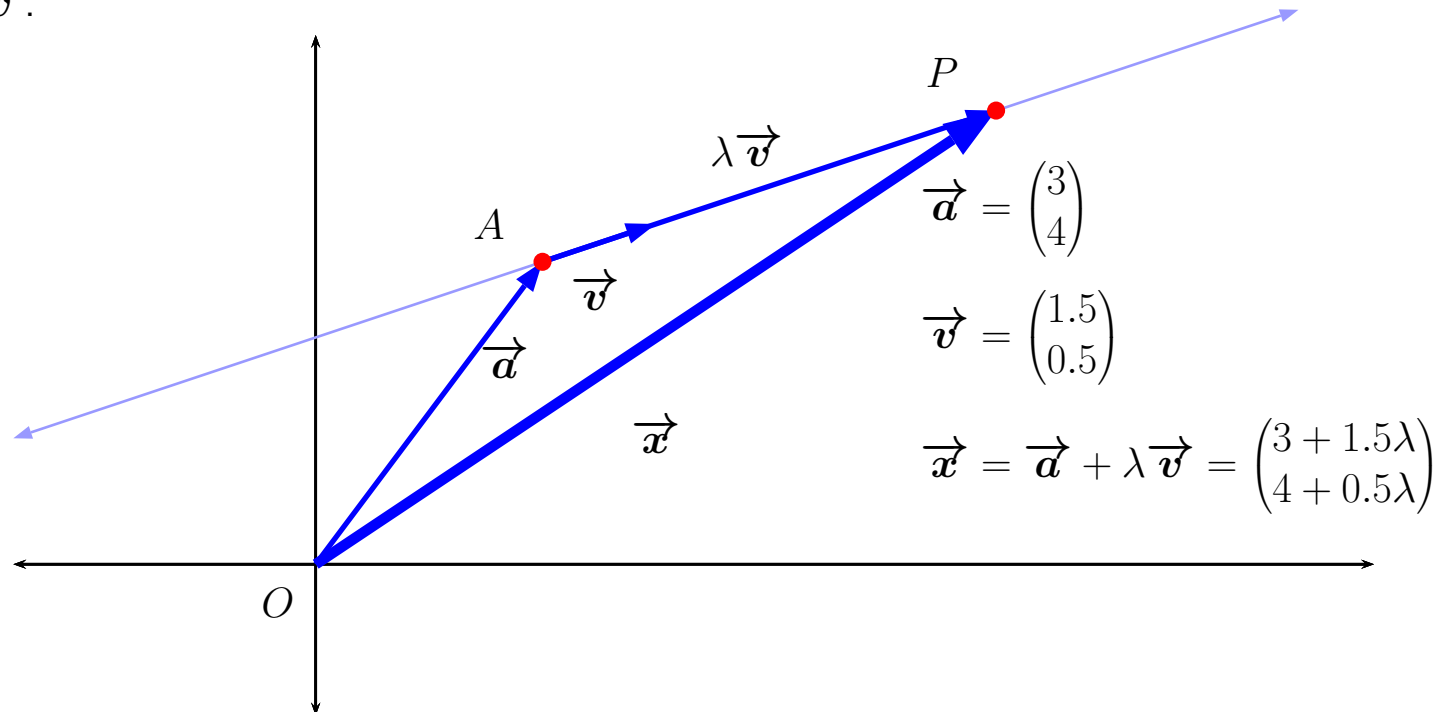


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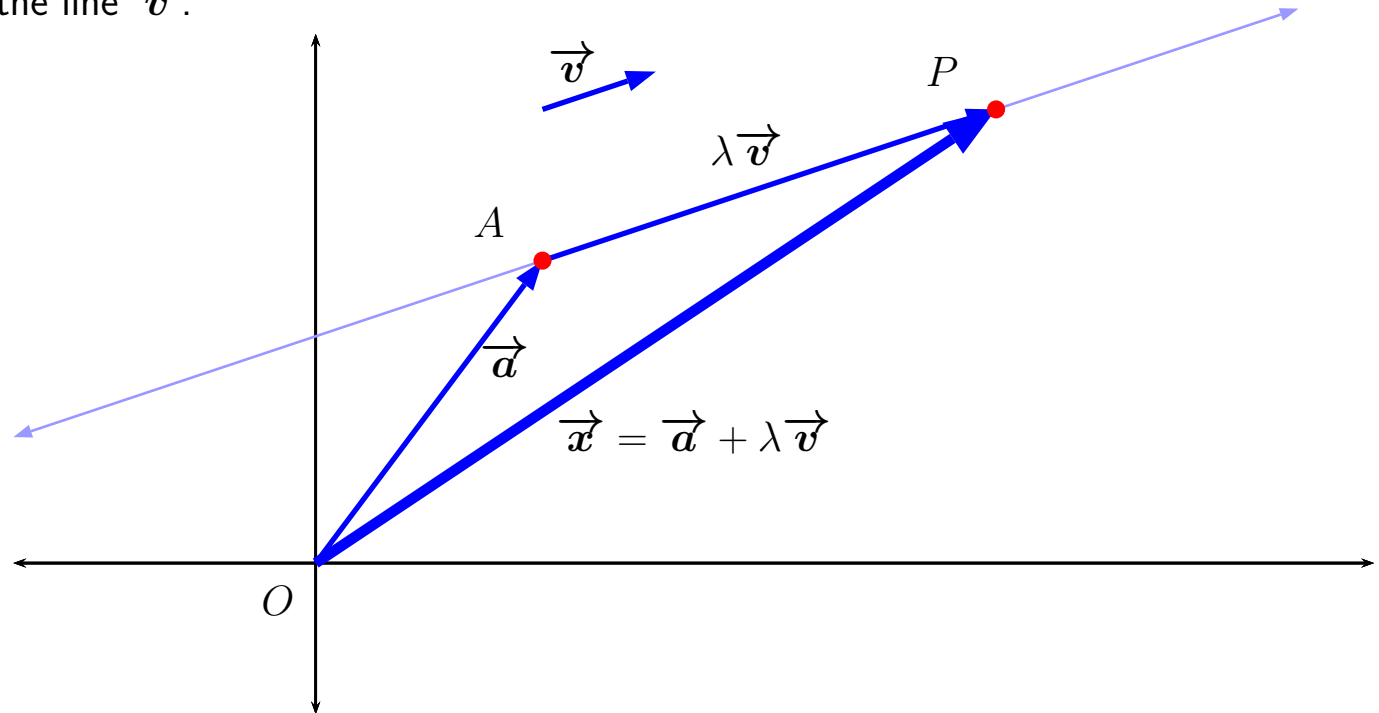
Vector parametric form of a line

We can describe points on a line with a point A on the line and a vector in the direction of the line \vec{v} .



Vector parametric form of a line

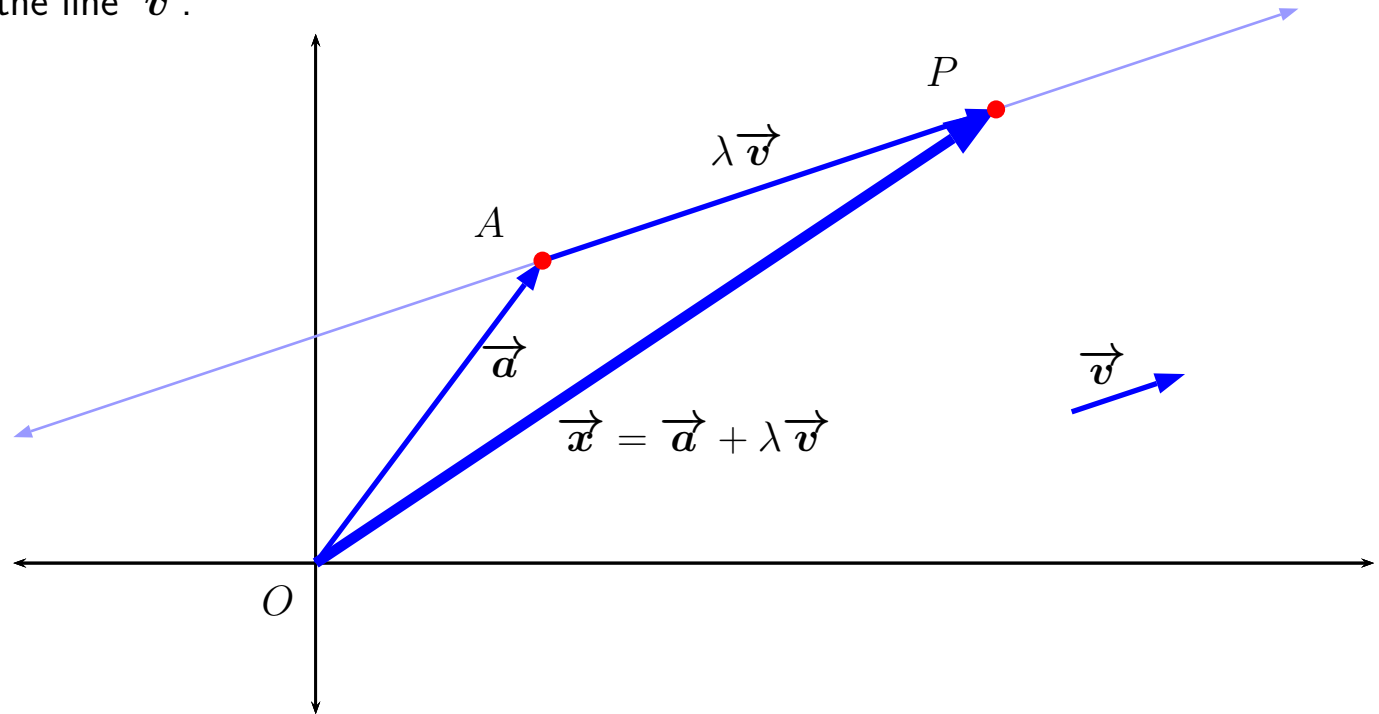
We can describe points on a line by with a point A on the line and a vector in the direction of the line \vec{v} .



Remember that the location of a vector isn't fixed.

Vector parametric form of a line

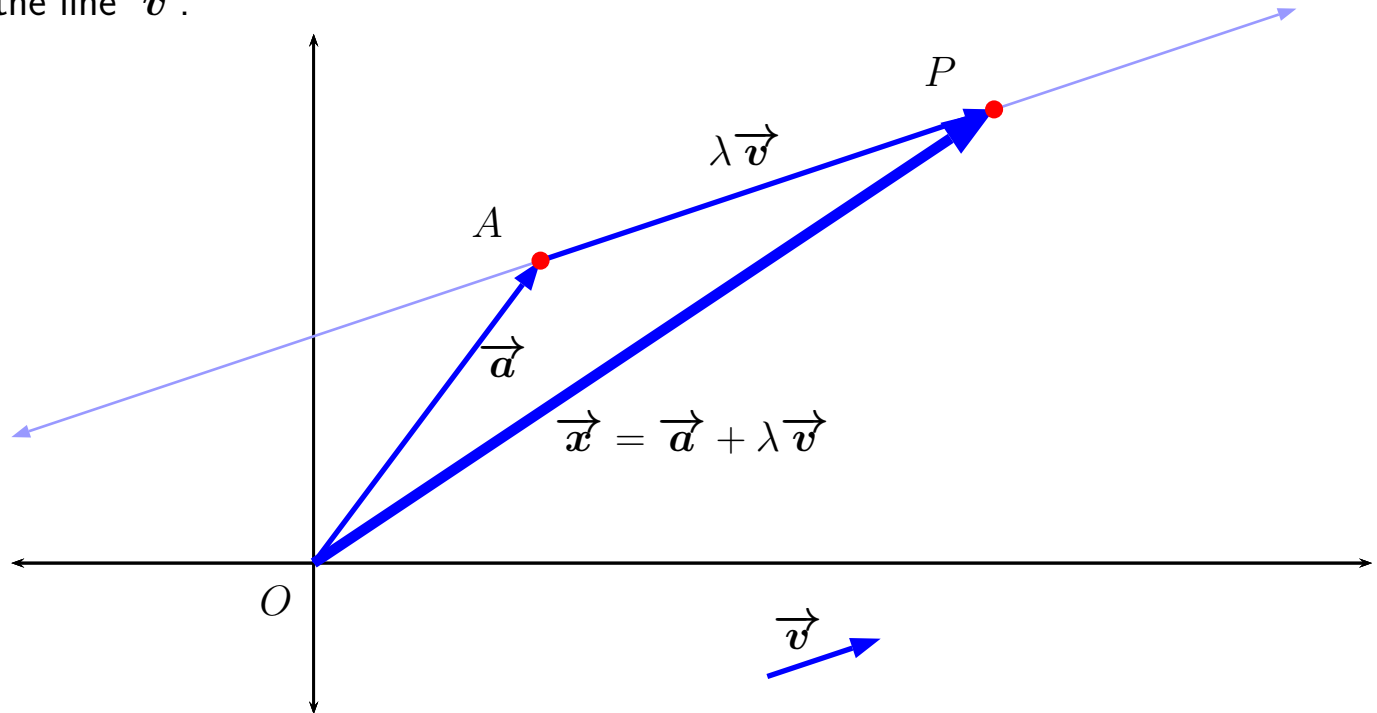
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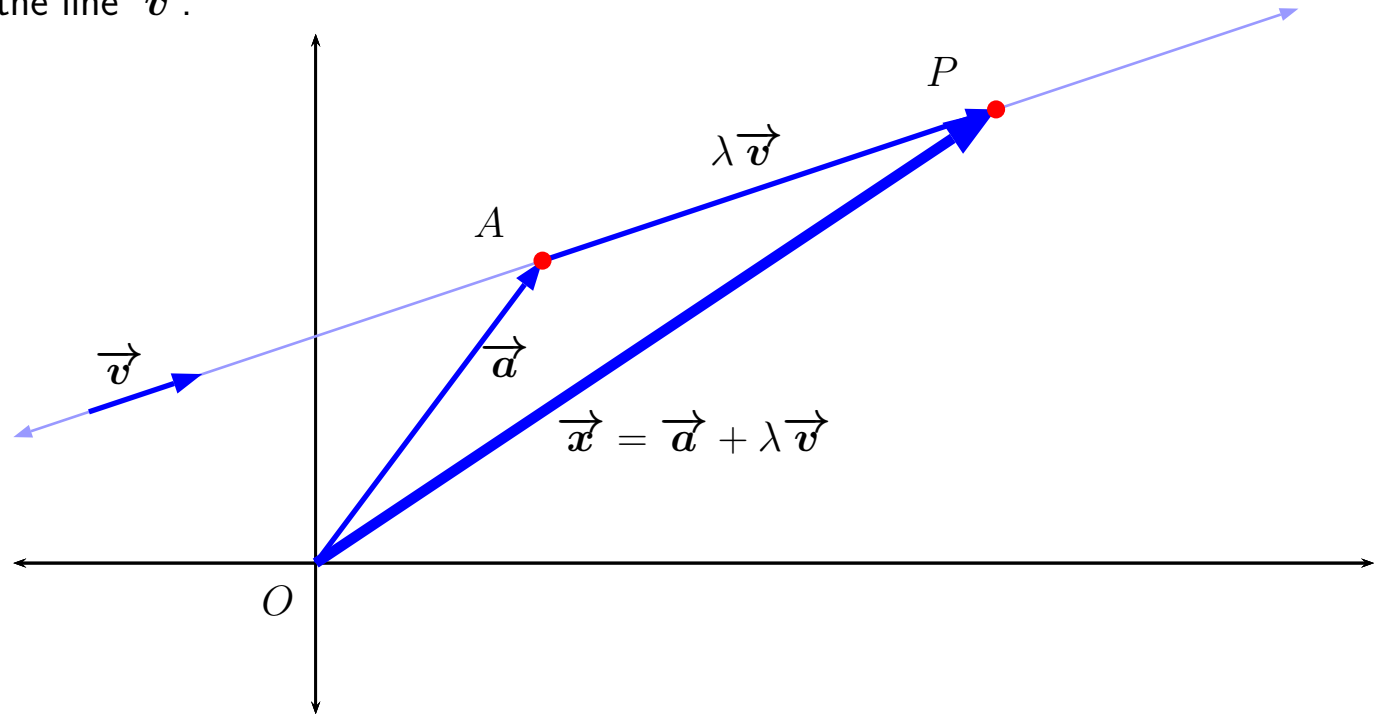
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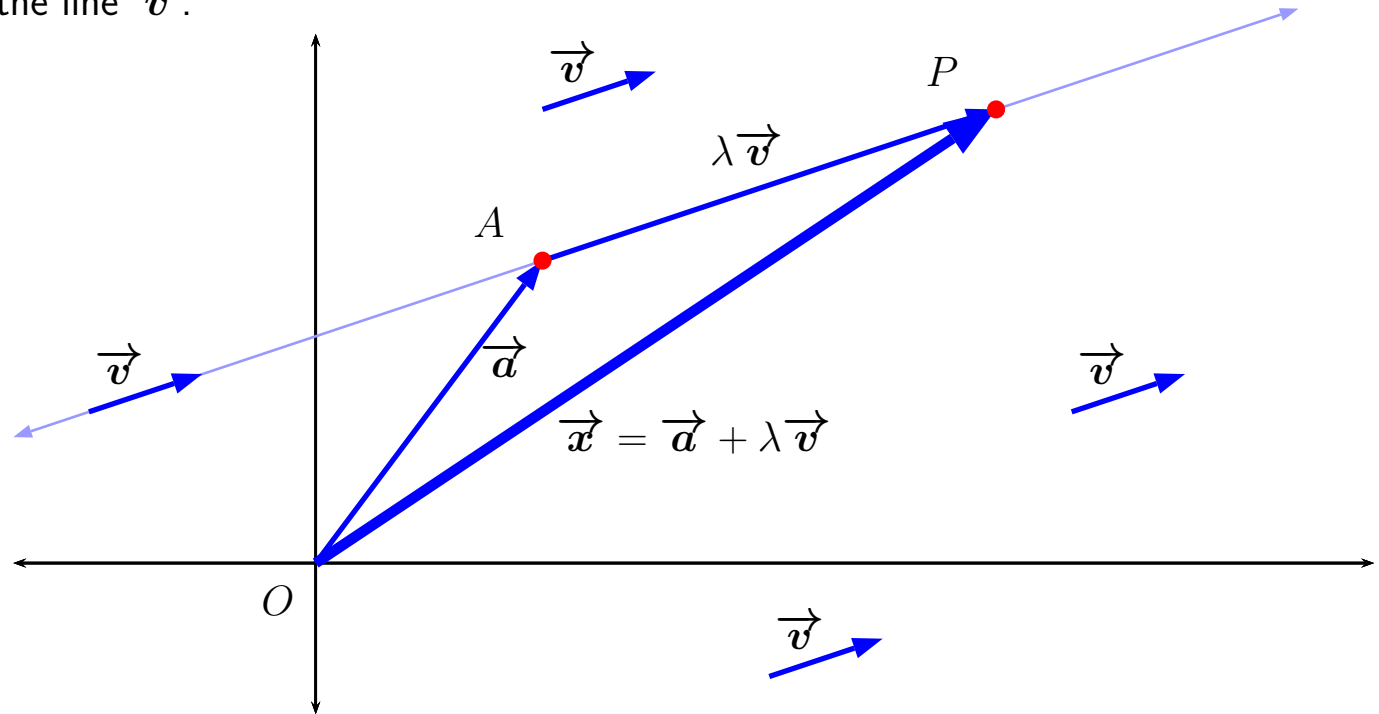
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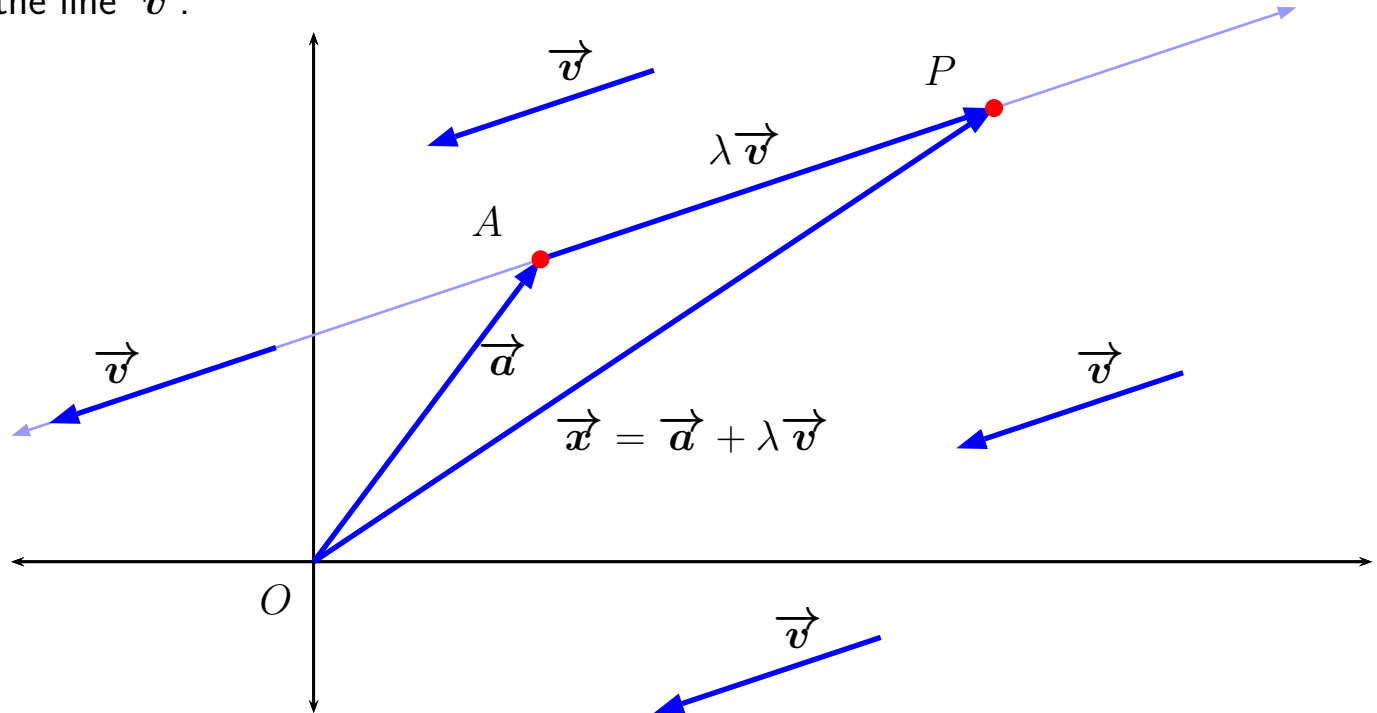
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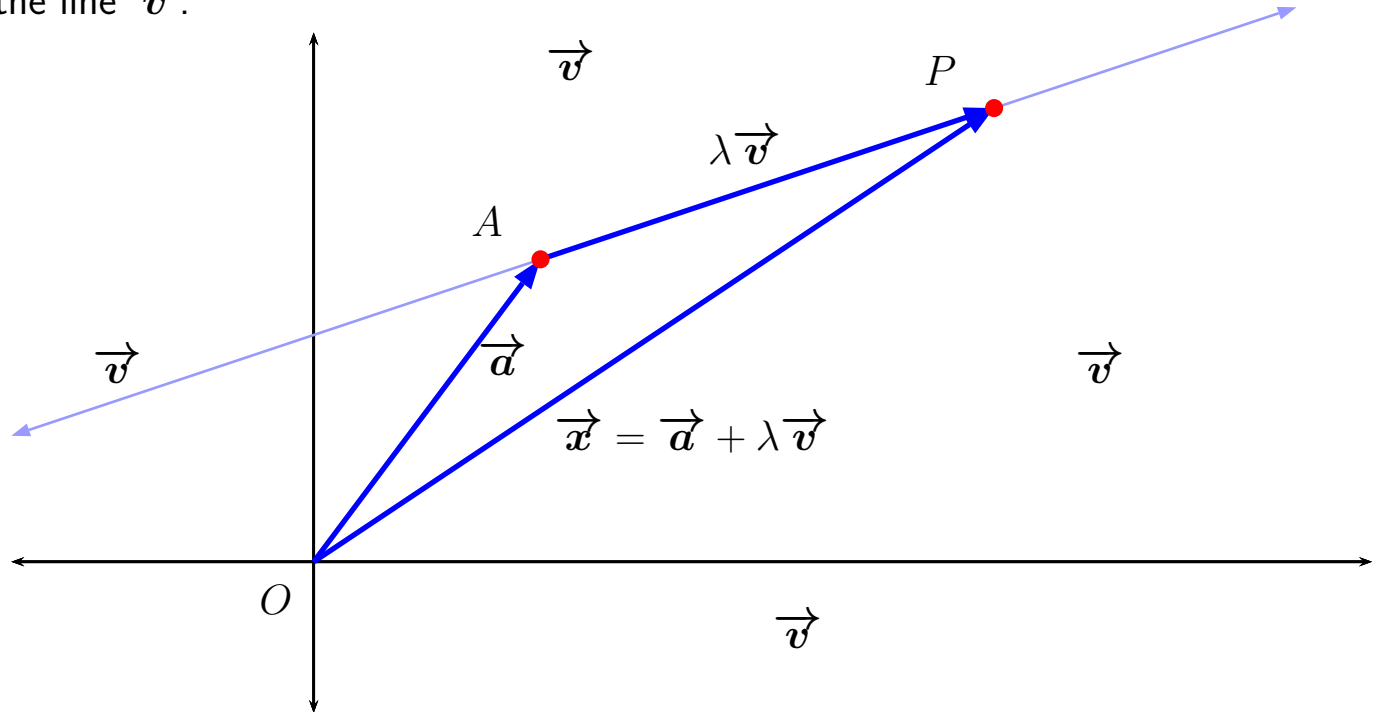
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Any vector in the direction of the line will do.

Vector parametric form of a line

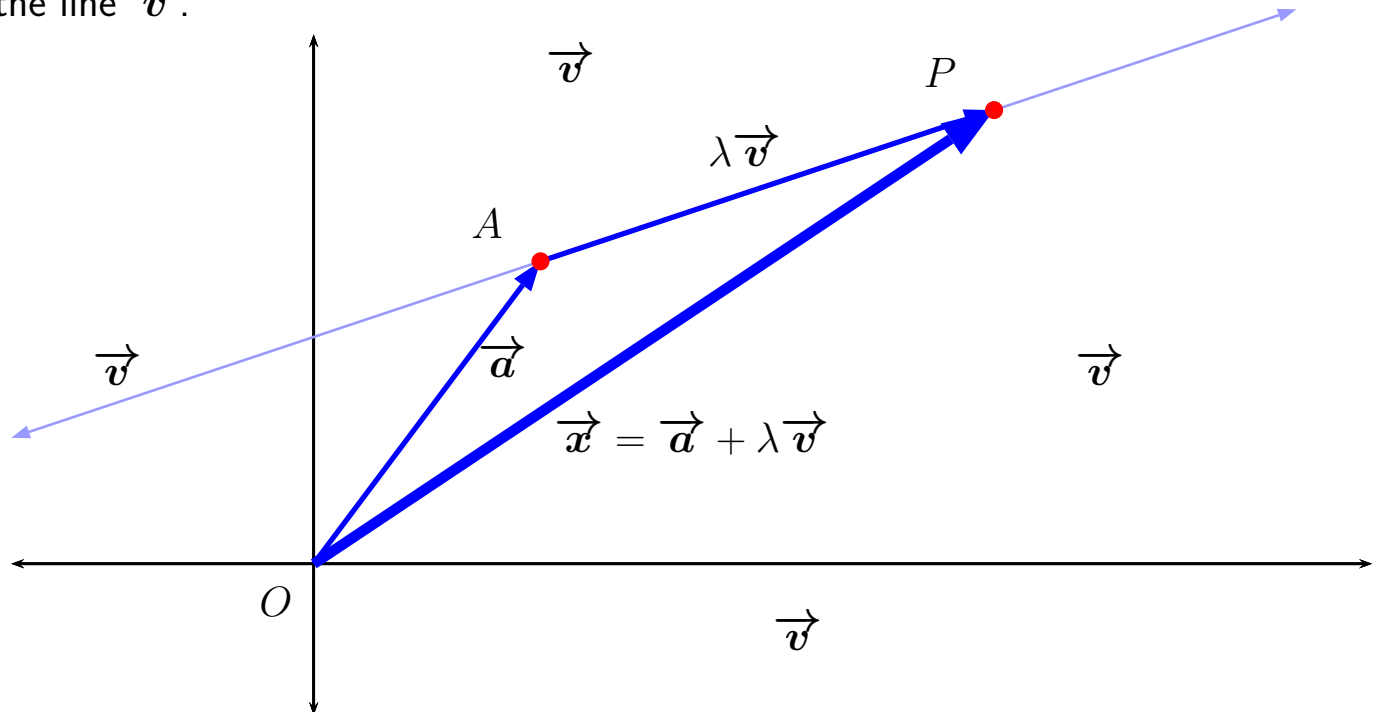
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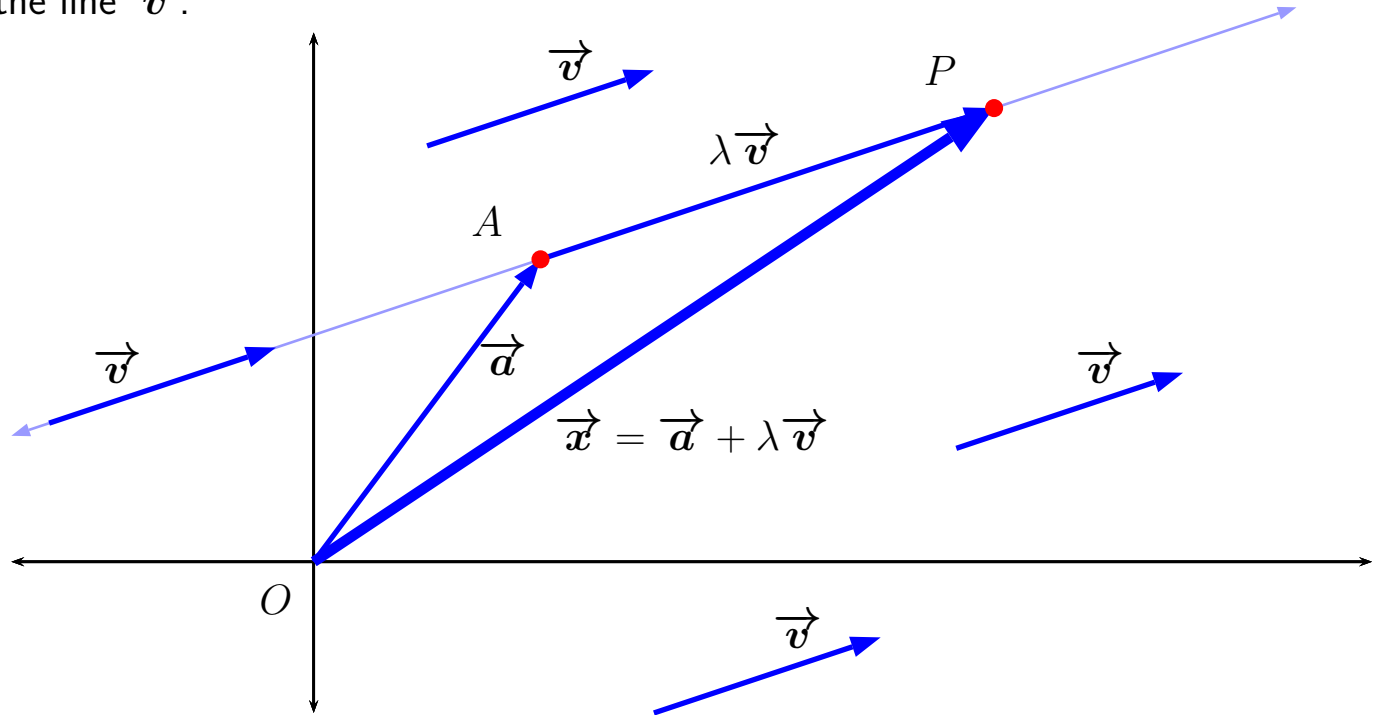
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Vector parametric form of a line

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Vector parametric form of a line

Here's a GeoGebra app showing the vector equation of a line:

<https://www.geogebra.org/m/09FqJ0kn>

SUMMARY: Vector parametric form of a line



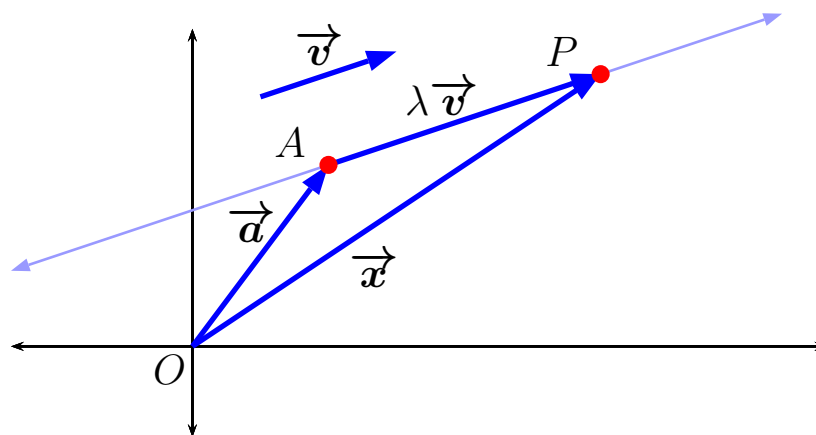
A parametric equation for the points P with position vectors $\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$ on a

line in \mathbb{R}^n passing through a point A with position vector $\vec{a} \in \mathbb{R}^n$ and parallel to the nonzero vector $\vec{v} \in \mathbb{R}^n$ is:

$$\vec{x} = \vec{a} + \lambda \vec{v}$$

i.e. $\vec{OP} = \vec{OA} + \lambda \vec{v}$

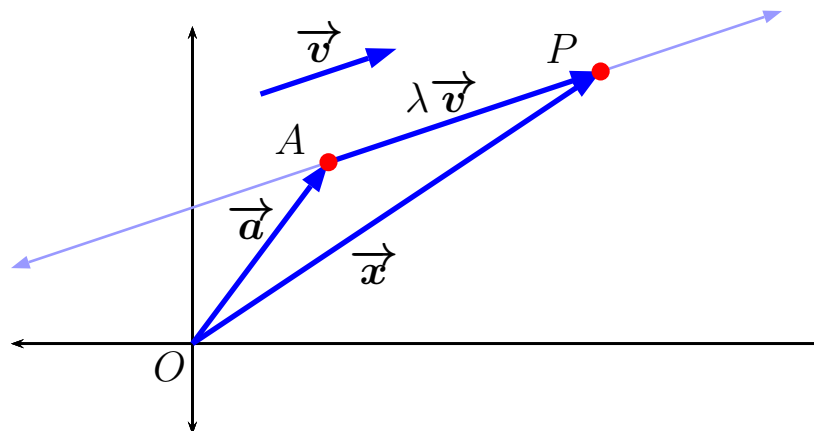
where $\lambda \in \mathbb{R}$.



The scalar λ is called a parameter.

Each point on the line has its own value of λ .

SUMMARY: Vector parametric form of a line



A point P is on the line passing through the point A and parallel to the nonzero vector \vec{v}

if and only if \overrightarrow{AP} is parallel to \vec{v}

i.e. *if and only if* there exist a scalar λ such that $\overrightarrow{AP} = \lambda \vec{v}$

i.e. *if and only if* there exist a scalar λ such that $\overrightarrow{OP} = \overrightarrow{OA} + \lambda \vec{v}$.

Parametric line example

Example 1. Let

$$\vec{x} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}, \quad \lambda \in \mathbb{R}$$

be a line in \mathbb{R}^3 .

1. Give two points that lie on the line.
2. Does $B(9, -2, -5)$ lie on the line?
3. Does $C(-3, 4, 0)$ lie on the line?

Parallel lines

How to tell if two lines are parallel from their parametric forms?

Two lines

$$\begin{aligned}\vec{x} &= \vec{a}_1 + \lambda \vec{v}_1 \\ \vec{x} &= \vec{a}_2 + \lambda \vec{v}_2\end{aligned}$$

are **parallel** if the \vec{v}_1 and \vec{v}_2 are parallel.

Example 2. Let

$$\ell : \vec{x} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}, \quad \lambda \in \mathbb{R}$$

be a line in \mathbb{R}^3 . Write down a parametric vector equation of the line ℓ' through $(1, 2, 3)$ that is parallel to the line ℓ above.

Example

Example 3. Find an equation of the line ℓ in \mathbb{R}^4 which passes through $A(2, -3, -1, 2)$ and $B(-1, 2, 2, 7)$.

Cartesian equation of a line in \mathbb{R}^2

Example 4. Find a Cartesian equation of the line ℓ given parametrically as

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$