MATH1131 Mathematics 1A – Algebra

Lecture 1: Geometric Vectors

Lecturer: Sean Gardiner – sean.gardiner@unsw.edu.au

Based on slides by Jonathan Kress

Geometric Vectors

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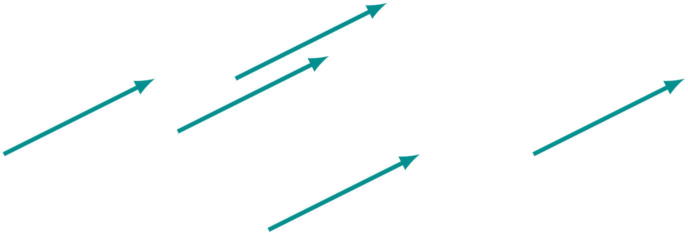

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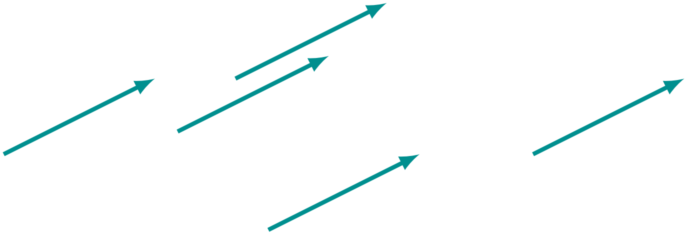
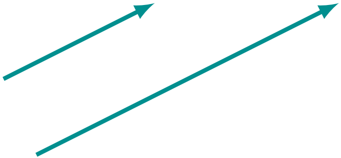


... because they all have the same length and direction.

Geometric Vectors

These vectors are different:



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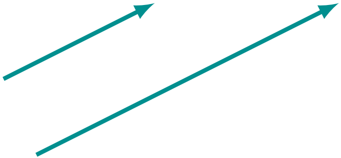

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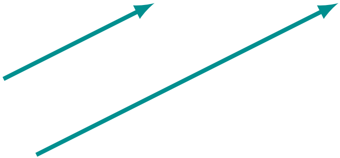

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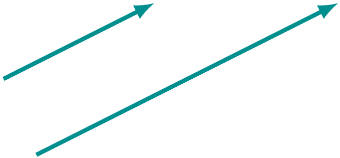

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- Vectors are written as letters in boldface (when typed), or with an arrow drawn over them, or with a tilde drawn under them:

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Other notation:

- The length of a vector is denoted with vertical bars:

$$|\mathbf{u}| = \text{the length of } \mathbf{u}$$

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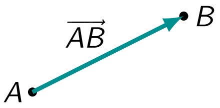

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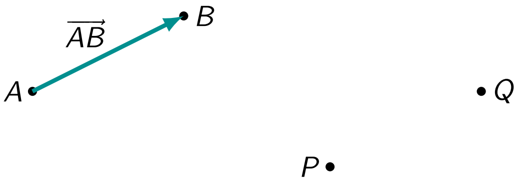

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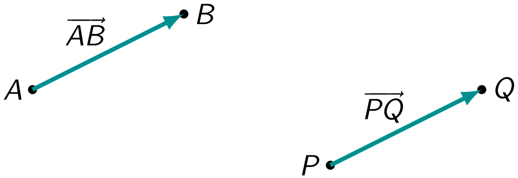

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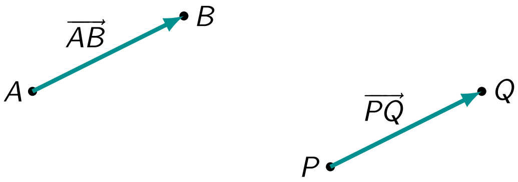

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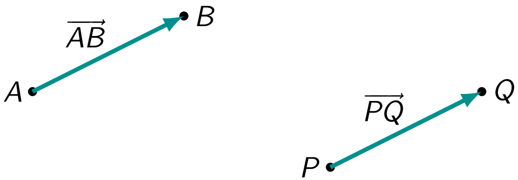
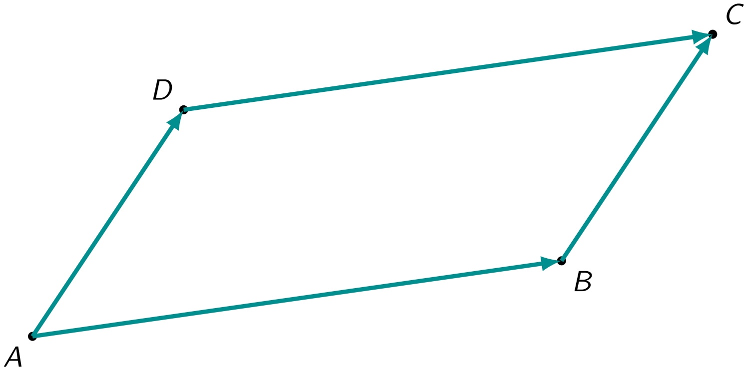


In the picture above, the vector that points from A to B is the same as the vector that points from P to Q.

Since vectors' positions do not matter, we have that $\overrightarrow{AB} = \overrightarrow{PQ}$.

In the parallelogram below, which vectors are equal?



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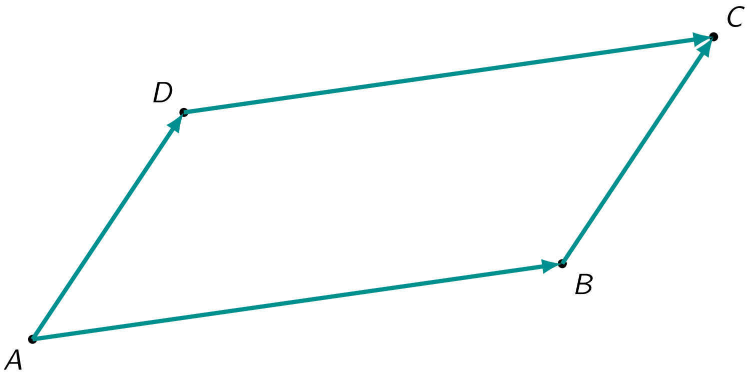

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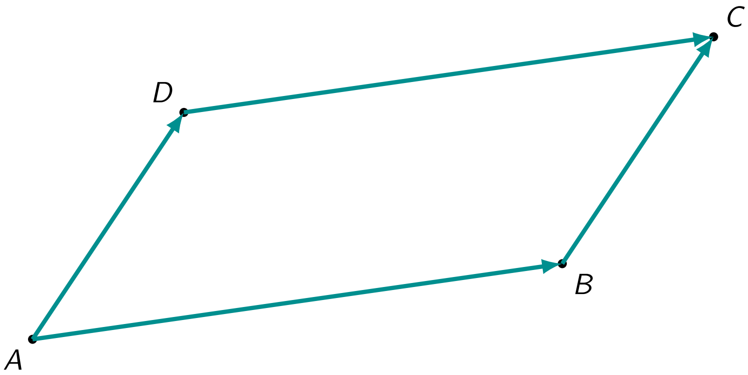

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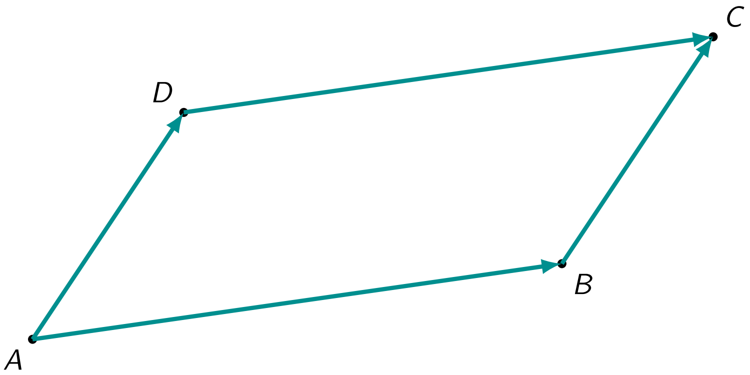
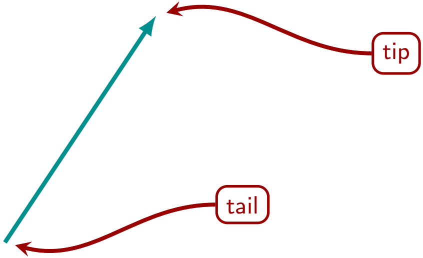


$$\overrightarrow{AB} = \overrightarrow{DC} \qquad \text{and} \qquad \overrightarrow{AD} = \overrightarrow{BC}$$

To geometrically add two vectors, we place them tip to tail



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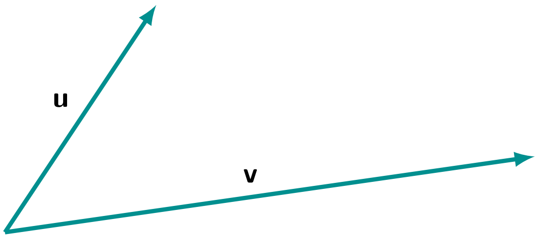

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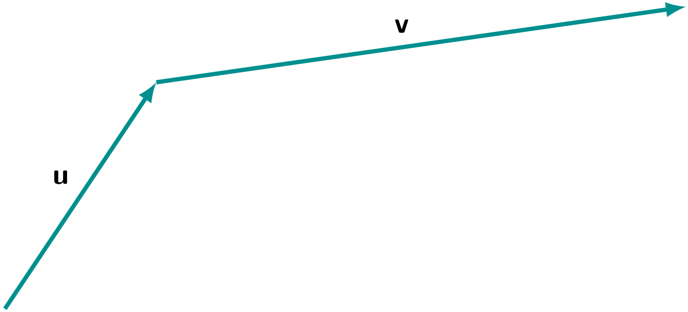

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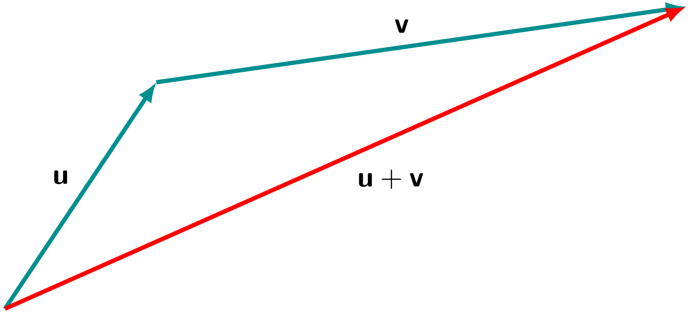
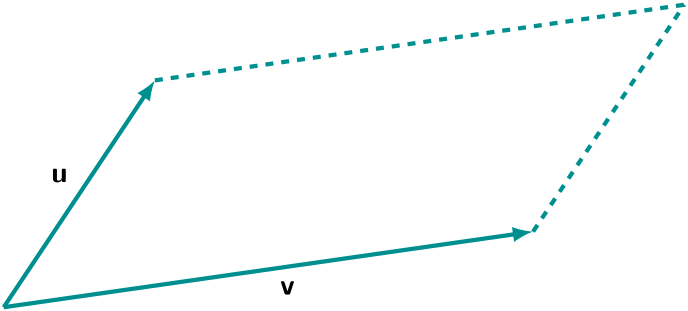


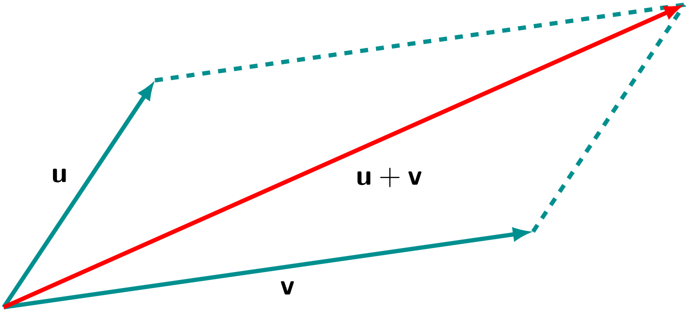
So given $\mathbf{u}$ and $\mathbf{v}$, to find $\mathbf{u}+\mathbf{v}$, we move the tail of $\mathbf{v}$ to the tip of $\mathbf{u}$ and then complete the triangle.

This method of addition is known as the triangle law of addition.

An alternative interpretation of vector addition uses the parallelogram with the two vectors **u** and **v** as adjacent sides.



An alternative interpretation of vector addition uses the parallelogram with the two vectors $\mathbf{u}$ and $\mathbf{v}$ as adjacent sides. Then $\mathbf{u} + \mathbf{v}$ is the vector that points from the tails of the two vectors to the furthest vertex.



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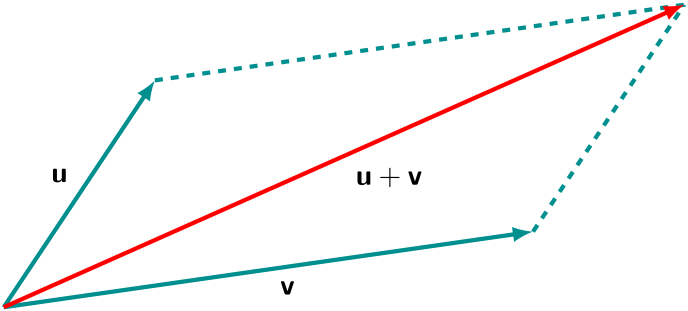

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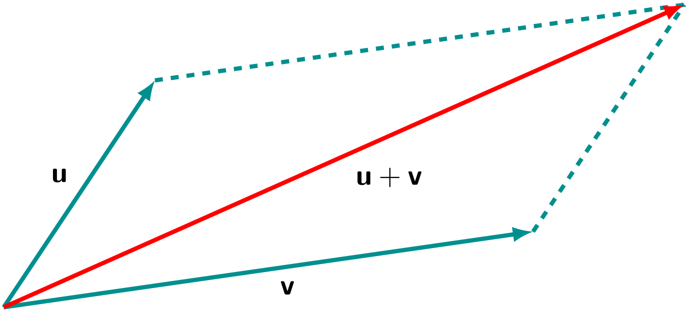

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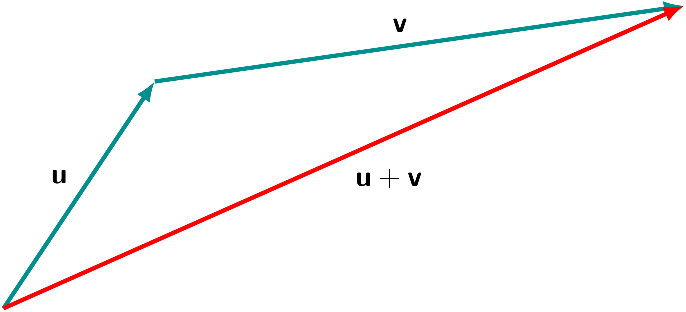
Obviously, the two definitions are equivalent.

Vector addition happens to have many of the properties we take for granted with normal scalar addition.



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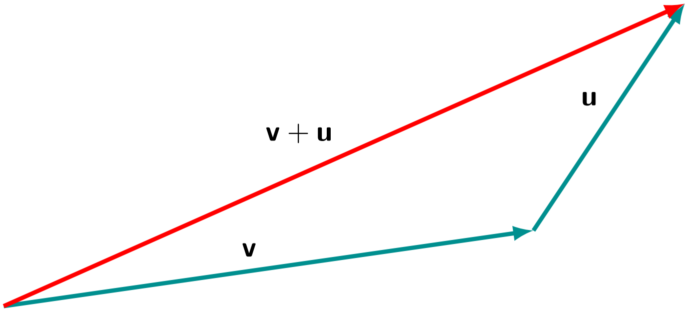

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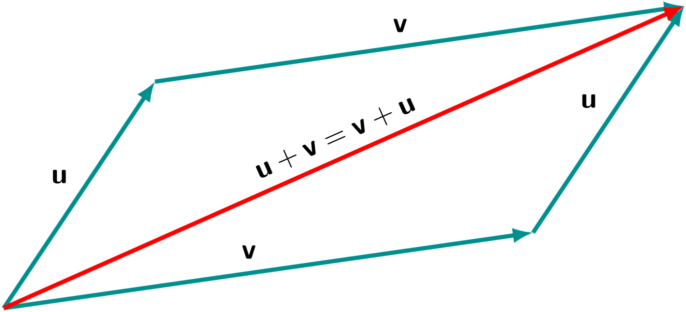


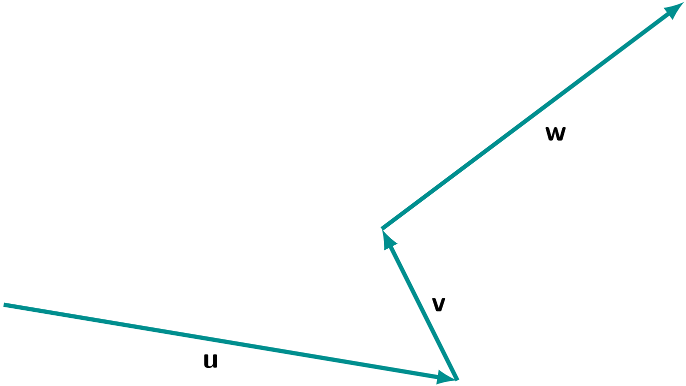
Vector addition is commutative:

$$\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$$

so we don't have to worry about the order in which vectors are added.

Geometric Vectors

Vector addition is associative



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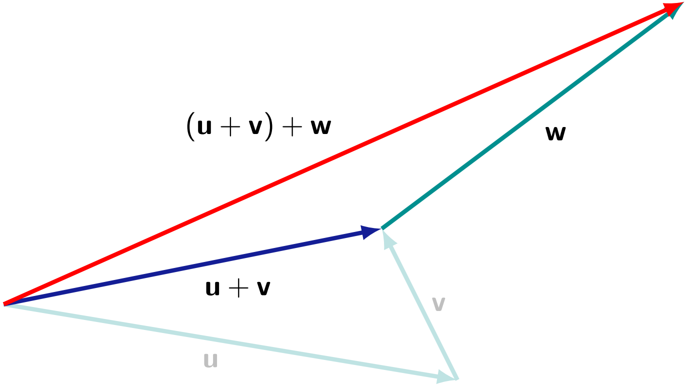

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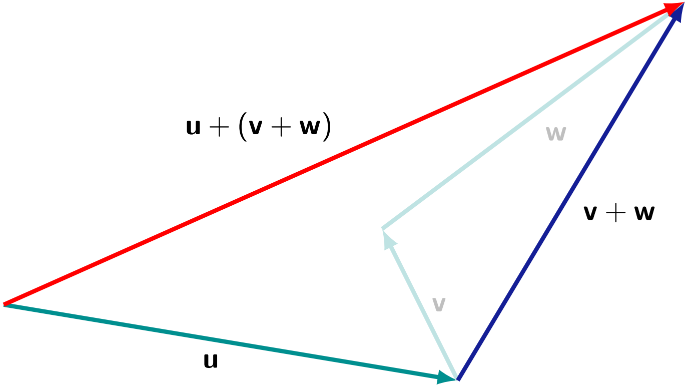

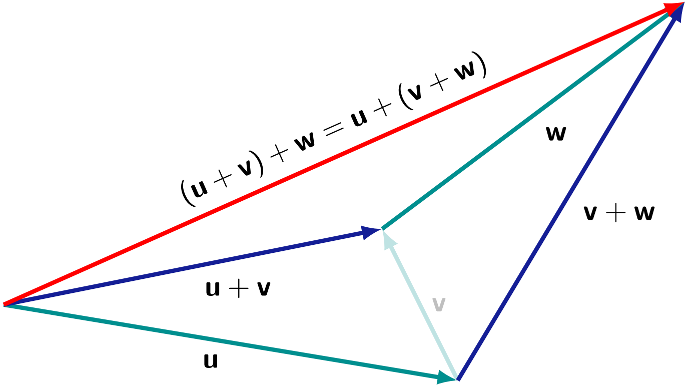

Vector addition is associative:

$$(\mathbf{u}+\mathbf{v})+\mathbf{w}=\mathbf{u}+(\mathbf{v}+\mathbf{w})$$

so it is safe to write $\mathbf{u}+\mathbf{v}+\mathbf{w}$.

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Note: Never write the zero vector simply as 0, or it could be confused with the zero scalar!

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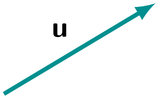

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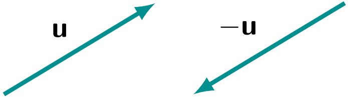

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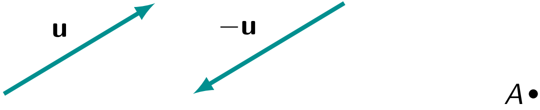

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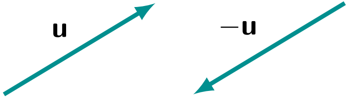

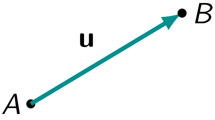

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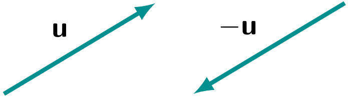

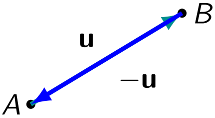

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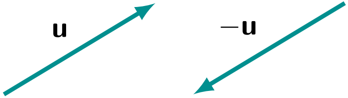

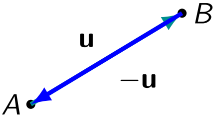

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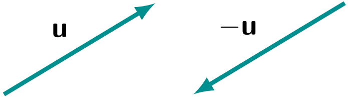

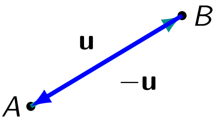

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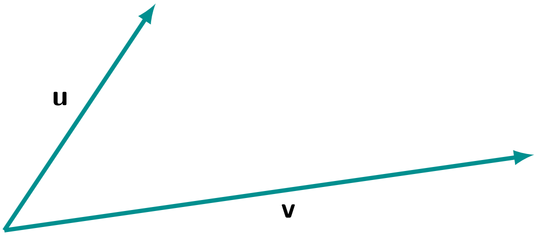
- $\mathbf{u}$ and $-\mathbf{u}$ have the same length (that is, $|\mathbf{u}| = |-\mathbf{u}|$), but opposite directions.
- In general, $\overrightarrow{BA} = -\overrightarrow{AB}$.

To subtract vectors we add the negative:

$$\mathbf{v} - \mathbf{u} = \mathbf{v} + -\mathbf{u}$$



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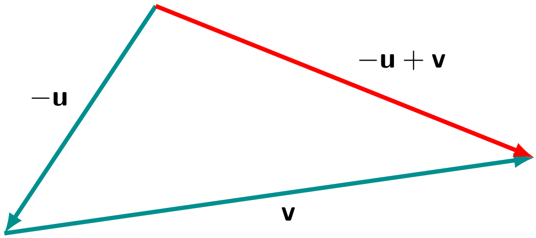

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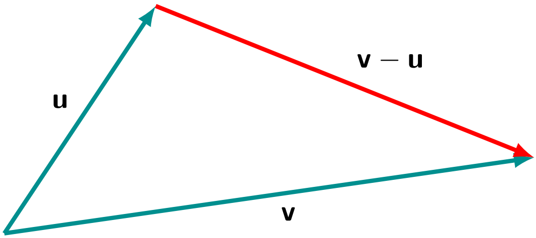
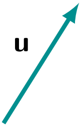


So $\mathbf{v} - \mathbf{u}$ is the vector that points from the tip of $\mathbf{u}$ to the tip of $\mathbf{v}$.

Scalar multiplication is another operation on vectors. Multiplying a vector by a positive scalar changes its length but not its direction. Multiplying a vector by a negative scalar changes its length and reverses its direction.



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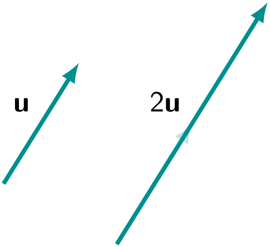

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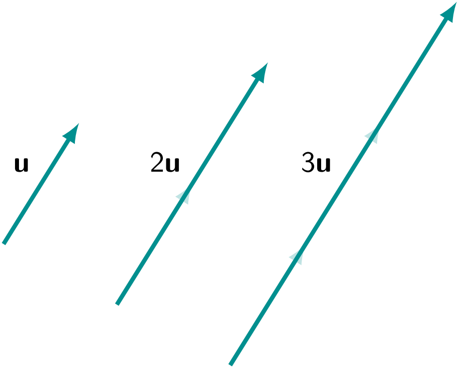

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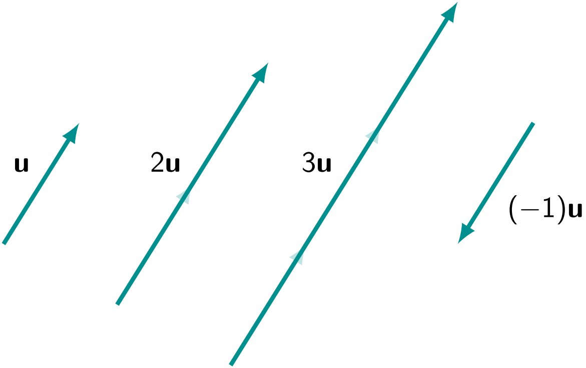

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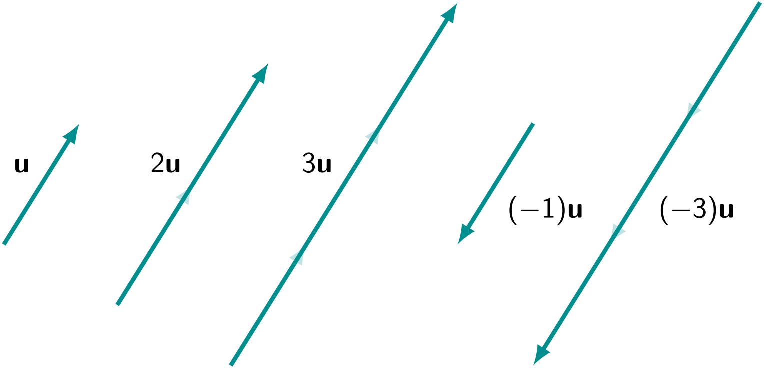

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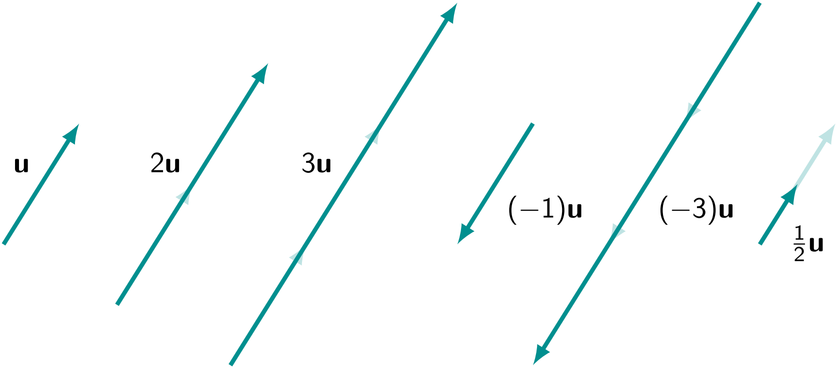

For any scalar λ and vector $\mathbf{v}$, the scalar multiple $\lambda\mathbf{v}$ is the vector that has length $|\lambda||\mathbf{v}|$ and

- the same direction as $\mathbf{v}$ when $\lambda > 0$, or
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Definition

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Conversely, if the two vectors have the same or the opposite direction, then we can write one as a scalar multiple of the other. For example,

$$\mathbf{u} = \begin{cases} \frac{|\mathbf{u}|}{|\mathbf{v}|}\mathbf{v} & \text{if they have the same direction} \\ -\frac{|\mathbf{u}|}{|\mathbf{v}|}\mathbf{v} & \text{if they have the opposite direction} \end{cases}$$

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This means two vectors are parallel if one is a non-zero scalar multiple of the other.

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We can use these laws to simplify vector expressions.

Example

Simplify $3(2\mathbf{u} - \mathbf{v}) + (\mathbf{u} - 2\mathbf{v})$.

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In practice, we simply write

$$3(2\mathbf{u} - \mathbf{v}) + (\mathbf{u} - 2\mathbf{v}) = 6\mathbf{u} - 3\mathbf{v} + \mathbf{u} - 2\mathbf{v} = 7\mathbf{u} - 5\mathbf{v}.$$

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For any three points A, B, and C, what is $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA}$?

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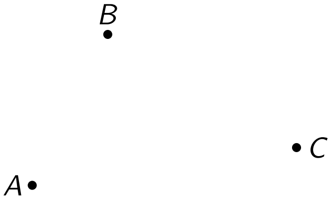

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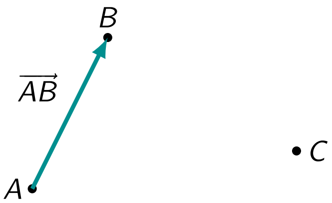

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$\overrightarrow{AB}$ takes us from A to B

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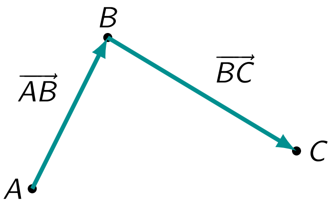

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$\overrightarrow{AB}$ takes us from A to B, then
$\overrightarrow{BC}$ takes us from B to C

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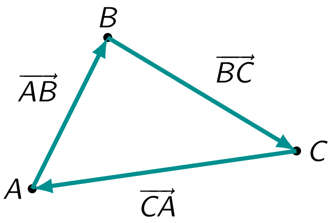

Here are our three points.

$\overrightarrow{AB}$ takes us from A to B, then $\overrightarrow{BC}$ takes us from B to C, and $\overrightarrow{CA}$ takes us from C back to A.

So in total we have not moved at all.

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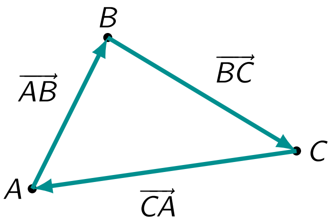

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This means $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \mathbf{0}$.