MATH1131 Mathematics 1A – Algebra

Lecture 9: Complex Numbers

Lecturer: Sean Gardiner – sean.gardiner@unsw.edu.au

Based on slides by Jonathan Kress

Numbers

The natural numbers form the set

$$\mathbb{N} = \{0, 1, 2, 3, 4, \ldots\}.$$

These arise naturally from trying to count whole objects.

Numbers

The natural numbers form the set

$$\mathbb{N} = \{0, 1, 2, 3, 4, \ldots\}.$$

These arise naturally from trying to count whole objects.

The integers form the set

$$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, 3, \ldots\}.$$

These arise naturally from trying so solve equations like $x + 2 = 1$.

Numbers

The natural numbers form the set

$$\mathbb{N} = \{0, 1, 2, 3, 4, \ldots\}.$$

These arise naturally from trying to count whole objects.

The integers form the set

$$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, 3, \ldots\}.$$

These arise naturally from trying so solve equations like $x + 2 = 1$.

The rational numbers form the set

$$\mathbb{Q} = \left\{ \frac{p}{q} \mid p, q \in \mathbb{Z},\ q \neq 0 \right\}.$$

These arise naturally from trying to solve equations like $2x = 1$.

Numbers

The natural numbers form the set

$$\mathbb{N} = \{0, 1, 2, 3, 4, \ldots\}.$$

These arise naturally from trying to count whole objects.

The integers form the set

$$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, 3, \ldots\}.$$

These arise naturally from trying so solve equations like $x + 2 = 1$.

The rational numbers form the set

$$\mathbb{Q} = \left\{ \frac{p}{q} \mid p, q \in \mathbb{Z},\ \ q \neq 0 \right\}.$$

These arise naturally from trying to solve equations like $2x = 1$.

The real numbers form the set

$$\mathbb{R} = \{\text{all points on the real number line}\}.$$

(This set includes numbers like $\sqrt{2}$, π, and e.)
These arise naturally from trying to solve equations like $x^2 = 2$.

Fields

A field $\mathbb{F}$ is a set with two operations, $+$ and $\times$, that satisfies the following properties for all $x, y, z \in \mathbb{F}$:

$x + y \in \mathbb{F}$	(Closure under addition)
$xy \in \mathbb{F}$	(Closure under multiplication)
$(x + y) + z = x + (y + z)$	(Associativity under addition)
$x(yz) = (xy)z$	(Associativity under multiplication)
$x + y = y + x$	(Commutativity under addition)
$xy = yx$	(Commutativity under multiplication)
$x(y + z) = xy + xz$ and $(x + y)z = xz + yz$	(Distributivity)
There is a $0 \in \mathbb{F}$ such that $x + 0 = x$ for all x.	(Existence of 0)
There is a $1 \in \mathbb{F}$ such that $1x = x$ for all x.	(Existence of 1)
There is $-x \in \mathbb{F}$ such that $x + (-x) = 0$.	(Existence of negatives)
If $x \neq 0$, there is $x^{-1} \in \mathbb{F}$ such that $x^{-1}x = 1$.	(Existence of inverses)

Fields – Examples

Example

Which of $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$ are fields?

Example

Which of $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$ are fields?

- $\mathbb{N}$ is not a field since it does not satisfy two of the properties: no natural numbers (except 0) have negatives, and no natural numbers (except 1) have inverses.

Fields – Examples

Example

Which of $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$ are fields?

- $\mathbb{N}$ is not a field since it does not satisfy two of the properties: no natural numbers (except 0) have negatives, and no natural numbers (except 1) have inverses.
- $\mathbb{Z}$ is not a field since it does not satisfy one of the properties: no natural numbers (except ± 1) have inverses.

Fields – Examples

Example

Which of $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$ are fields?

- $\mathbb{N}$ is not a field since it does not satisfy two of the properties: no natural numbers (except 0) have negatives, and no natural numbers (except 1) have inverses.
- $\mathbb{Z}$ is not a field since it does not satisfy one of the properties: no natural numbers (except ± 1) have inverses.
- $\mathbb{Q}$ is a field since all the properties hold: for example, the negative of $\frac{p}{q}$ is $-\frac{p}{q}$, and its inverse is $\frac{q}{p}$. We can also check closure by showing $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd} \in \mathbb{Q}$ and $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd} \in \mathbb{Q}$.

Fields – Examples

Example

Which of $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$ are fields?

- $\mathbb{N}$ is not a field since it does not satisfy two of the properties: no natural numbers (except 0) have negatives, and no natural numbers (except 1) have inverses.
- $\mathbb{Z}$ is not a field since it does not satisfy one of the properties: no natural numbers (except ± 1) have inverses.
- $\mathbb{Q}$ is a field since all the properties hold: for example, the negative of $\frac{p}{q}$ is $-\frac{p}{q}$, and its inverse is $\frac{q}{p}$. We can also check closure by showing $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd} \in \mathbb{Q}$ and $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd} \in \mathbb{Q}$.
- $\mathbb{R}$ is a field since all the properties hold: for example, the negative of some $r \in \mathbb{R}$ is simply $-r$, and its inverse is $\frac{1}{r}$. To check closure, note that the sum or product of any two numbers on the real number line is also on the real number line.

Consider the equation

$$x^2 = -1.$$

There is no number in $\mathbb{R}$ that solves this. So we can naturally extend our number system again by including numbers that solve these sorts of equations.

Consider the equation

$$x^2 = -1.$$

There is no number in $\mathbb{R}$ that solves this. So we can naturally extend our number system again by including numbers that solve these sorts of equations.

First we introduce the imaginary unit i which has the property that

$$i^2 = -1.$$

We can think of i as $\sqrt{-1}$.

Consider the equation

$$x^2 = -1.$$

There is no number in $\mathbb{R}$ that solves this. So we can naturally extend our number system again by including numbers that solve these sorts of equations.

First we introduce the imaginary unit i which has the property that

$$i^2 = -1.$$

We can think of i as $\sqrt{-1}$.

The complex numbers form the set

$$\mathbb{C} = \{a + bi \mid a, b \in \mathbb{R}\}.$$

These arise naturally from trying to solve equations like $x^2 = -1$.

Complex numbers – Arithmetic

When performing arithmetic on complex numbers, we treat i like a variable but replace i^2 with -1.

When performing arithmetic on complex numbers, we treat i like a variable but replace i^2 with -1.

Example

Find the sum, the difference, and the product of $3+2i$ and $5-4i$.

Complex numbers – Arithmetic

When performing arithmetic on complex numbers, we treat i like a variable but replace i^2 with -1.

Example

Find the sum, the difference, and the product of $3+2i$ and $5-4i$.

$$(3+2i)+(5-4i) \quad =$$

When performing arithmetic on complex numbers, we treat i like a variable but replace i^2 with -1.

Example

Find the sum, the difference, and the product of $3+2i$ and $5-4i$.

$$(3+2i)+(5-4i) = (3+5)+(2i-4i)$$

When performing arithmetic on complex numbers, we treat i like a variable but replace i^2 with -1.

Example

Find the sum, the difference, and the product of $3 + 2i$ and $5 - 4i$.

$$\begin{aligned}(3+2i)+(5-4i) &= (3+5)+(2i-4i)\\ &= 8-2i\end{aligned}$$

When performing arithmetic on complex numbers, we treat i like a variable but replace i^2 with -1.

Example

Find the sum, the difference, and the product of $3+2i$ and $5-4i$.

$$\begin{aligned}(3+2i)+(5-4i) &= (3+5)+(2i-4i)\\ &= 8-2i\end{aligned}$$

$$(3+2i)-(5-4i) =$$

When performing arithmetic on complex numbers, we treat i like a variable but replace i^2 with -1.

Example

Find the sum, the difference, and the product of $3 + 2i$ and $5 - 4i$.

$$\begin{aligned}(3+2i)+(5-4i) &= (3+5)+(2i-4i)\\ &= 8-2i\end{aligned}$$

$$(3+2i)-(5-4i) = (3-5)+(2i+4i)$$

When performing arithmetic on complex numbers, we treat i like a variable but replace i^2 with -1.

Example

Find the sum, the difference, and the product of $3+2i$ and $5-4i$.

$$\begin{aligned}(3+2i)+(5-4i) &= (3+5)+(2i-4i)\\ &= 8-2i\end{aligned}$$

$$\begin{aligned}(3+2i)-(5-4i) &= (3-5)+(2i+4i)\\ &= -2+6i\end{aligned}$$

When performing arithmetic on complex numbers, we treat i like a variable but replace i^2 with -1.

Example

Find the sum, the difference, and the product of $3+2i$ and $5-4i$.

$$\begin{aligned}(3+2i)+(5-4i) &= (3+5)+(2i-4i)\\ &= 8-2i\end{aligned}$$

$$\begin{aligned}(3+2i)-(5-4i) &= (3-5)+(2i+4i)\\ &= -2+6i\end{aligned}$$

$$(3+2i)(5-4i) =$$

Complex numbers – Arithmetic

When performing arithmetic on complex numbers, we treat i like a variable but replace i^2 with -1.

Example

Find the sum, the difference, and the product of $3+2i$ and $5-4i$.

$$\begin{aligned}(3+2i)+(5-4i) &= (3+5)+(2i-4i)\\ &= 8-2i\end{aligned}$$

$$\begin{aligned}(3+2i)-(5-4i) &= (3-5)+(2i+4i)\\ &= -2+6i\end{aligned}$$

$$(3+2i)(5-4i) = 3\times 5+3\times(-4i)+2\times 5i+2i\times(-4i)$$

When performing arithmetic on complex numbers, we treat i like a variable but replace i^2 with -1.

Example

Find the sum, the difference, and the product of $3+2i$ and $5-4i$.

$$\begin{aligned}(3+2i)+(5-4i) &= (3+5)+(2i-4i)\\ &= 8-2i\end{aligned}$$

$$\begin{aligned}(3+2i)-(5-4i) &= (3-5)+(2i+4i)\\ &= -2+6i\end{aligned}$$

$$\begin{aligned}(3+2i)(5-4i) &= 3\times 5+3\times(-4i)+2\times 5i+2i\times(-4i)\\ &= 15-12i+10i-8i^2\end{aligned}$$

When performing arithmetic on complex numbers, we treat i like a variable but replace i^2 with -1.

Example

Find the sum, the difference, and the product of $3+2i$ and $5-4i$.

$$\begin{aligned}(3+2i)+(5-4i) &= (3+5)+(2i-4i)\\ &= 8-2i\end{aligned}$$

$$\begin{aligned}(3+2i)-(5-4i) &= (3-5)+(2i+4i)\\ &= -2+6i\end{aligned}$$

$$\begin{aligned}(3+2i)(5-4i) &= 3\times 5+3\times(-4i)+2\times 5i+2i\times(-4i)\\ &= 15-12i+10i-8i^2\\ &= 15-2i-8\times(-1)\end{aligned}$$

When performing arithmetic on complex numbers, we treat i like a variable but replace i^2 with -1.

Example

Find the sum, the difference, and the product of $3+2i$ and $5-4i$.

$$\begin{aligned}(3+2i)+(5-4i) &= (3+5)+(2i-4i)\\ &= 8-2i\end{aligned}$$

$$\begin{aligned}(3+2i)-(5-4i) &= (3-5)+(2i+4i)\\ &= -2+6i\end{aligned}$$

$$\begin{aligned}(3+2i)(5-4i) &= 3\times 5+3\times(-4i)+2\times 5i+2i\times(-4i)\\ &= 15-12i+10i-8i^2\\ &= 15-2i-8\times(-1)\\ &= 23-2i\end{aligned}$$

Example

Find the solutions to $x^2 + 9 = 0$.

Example

Find the solutions to $x^2 + 9 = 0$.

If $x^2 = -9$, then

$$x = \pm\sqrt{-9}$$

Example

Find the solutions to $x^2 + 9 = 0$.

If $x^2 = -9$, then

$$\begin{aligned} x &= \pm\sqrt{-9} \\ &= \pm\sqrt{9}\sqrt{-1} \end{aligned}$$

Example

Find the solutions to $x^2 + 9 = 0$.

If $x^2 = -9$, then

$$\begin{aligned} x &= \pm\sqrt{-9} \\ &= \pm\sqrt{9}\sqrt{-1} \\ &= \pm 3i. \end{aligned}$$

Example

Find the solutions to $x^2 + 9 = 0$.

If $x^2 = -9$, then

$$\begin{aligned} x &= \pm\sqrt{-9} \\ &= \pm\sqrt{9}\sqrt{-1} \\ &= \pm 3i. \end{aligned}$$

That is, the solutions are $x = 3i$ and $x = -3i$.

Example

Find the solutions to $x^2 + 2x + 3 = 0$.

Example

Find the solutions to $x^2 + 2x + 3 = 0$.

Using the quadratic formula:

Example

Find the solutions to $x^2 + 2x + 3 = 0$.

Using the quadratic formula:

$$\left(\text{Recall: } ax^2 + bx + c = 0 \iff x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right)$$

Example

Find the solutions to $x^2 + 2x + 3 = 0$.

Using the quadratic formula:

(Recall: $ax^2 + bx + c = 0 \iff x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$)

$$x = \frac{-2 \pm \sqrt{2^2 - 4 \times 1 \times 3}}{2 \times 1}$$

Complex numbers – Solving equations

Example

Find the solutions to $x^2 + 2x + 3 = 0$.

Using the quadratic formula:

(Recall: $ax^2 + bx + c = 0 \iff x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$)

$$
\begin{aligned}
x &= \frac{-2 \pm \sqrt{2^2 - 4 \times 1 \times 3}}{2 \times 1} \\
&= \frac{-2 \pm \sqrt{-8}}{2}
\end{aligned}
$$

Example

Find the solutions to $x^2 + 2x + 3 = 0$.

Using the quadratic formula:

(Recall: $ax^2 + bx + c = 0 \iff x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$)

$$
\begin{aligned}
x &= \frac{-2 \pm \sqrt{2^2 - 4 \times 1 \times 3}}{2 \times 1} \\
&= \frac{-2 \pm \sqrt{-8}}{2} \\
&= \frac{-2 \pm 2\sqrt{2}\sqrt{-1}}{2}
\end{aligned}
$$

Example

Find the solutions to $x^2 + 2x + 3 = 0$.

Using the quadratic formula:

(Recall: $ax^2 + bx + c = 0 \iff x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$)

$$
\begin{aligned}
x &= \frac{-2 \pm \sqrt{2^2 - 4 \times 1 \times 3}}{2 \times 1} \\
&= \frac{-2 \pm \sqrt{-8}}{2} \\
&= \frac{-2 \pm 2\sqrt{2}\sqrt{-1}}{2} \\
&= -1 \pm \sqrt{2}i
\end{aligned}
$$

Complex numbers

Definition

The set $\mathbb{C}$ of complex numbers is given by

$$\mathbb{C} = \{a + bi \mid a, b \in \mathbb{R},\ i^2 = -1\}$$

Complex numbers

Definition

The set $\mathbb{C}$ of complex numbers is given by

$$\mathbb{C} = \{a + bi \mid a, b \in \mathbb{R},\ i^2 = -1\}$$

Theorem

The set $\mathbb{C}$ of complex numbers is a field with respect to the operations $+$ and $\times$ defined by:

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$

and

$$(a + bi)(c + di) = ac - bd + (ad + bc)i.$$

Complex numbers

Definition

The set $\mathbb{C}$ of complex numbers is given by

$$\mathbb{C} = \{a + bi \mid a, b \in \mathbb{R},\ i^2 = -1\}$$

Theorem

The set $\mathbb{C}$ of complex numbers is a field with respect to the operations $+$ and $\times$ defined by:

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$
and
$$(a + bi)(c + di) = ac - bd + (ad + bc)i.$$

Exercise

Check that all the field properties do in fact hold.

More examples

Example

Simplify

$$1, i, i^2, i^3, i^4, i^5, i^6, i^7, i^8, i^9, \ldots$$

More examples

Example

Simplify

$$1, i, i^2, i^3, i^4, i^5, i^6, i^7, i^8, i^9, \ldots$$

Since $i^2 = -1$, we know

$$i^3 = i^2 \times i$$

More examples

Example

Simplify

$$1, i, i^2, i^3, i^4, i^5, i^6, i^7, i^8, i^9, \ldots$$

Since $i^2 = -1$, we know

$$\begin{aligned} i^3 &= i^2 \times i \\ &= (-1) \times i \end{aligned}$$

More examples

Example

Simplify

$$1, i, i^2, i^3, i^4, i^5, i^6, i^7, i^8, i^9, \dots$$

Since $i^2 = -1$, we know

$$\begin{aligned} i^3 &= i^2 \times i \\ &= (-1) \times i \\ &= -i. \end{aligned}$$

More examples

Example

Simplify

$$1, i, i^2, i^3, i^4, i^5, i^6, i^7, i^8, i^9, \dots$$

Since $i^2 = -1$, we know

$$\begin{aligned} i^3 &= i^2 \times i \\ &= (-1) \times i \\ &= -i. \end{aligned}$$

Similarly,

$$i^4 = i^2 \times i^2$$

More examples

Example

Simplify
$$1, i, i^2, i^3, i^4, i^5, i^6, i^7, i^8, i^9, \ldots$$

Since $i^2 = -1$, we know
$$\begin{aligned} i^3 &= i^2 \times i \\ &= (-1) \times i \\ &= -i. \end{aligned}$$

Similarly,
$$\begin{aligned} i^4 &= i^2 \times i^2 \\ &= (-1) \times (-1) \end{aligned}$$

More examples

Example

Simplify

$$1, i, i^2, i^3, i^4, i^5, i^6, i^7, i^8, i^9, \ldots$$

Since $i^2 = -1$, we know

$$\begin{aligned} i^3 &= i^2 \times i \\ &= (-1) \times i \\ &= -i. \end{aligned}$$

Similarly,

$$\begin{aligned} i^4 &= i^2 \times i^2 \\ &= (-1) \times (-1) \\ &= 1. \end{aligned}$$

More examples

Example

Simplify
$$1, i, i^2, i^3, i^4, i^5, i^6, i^7, i^8, i^9, \ldots$$

Since $i^2 = -1$, we know

$$\begin{aligned} i^3 &= i^2 \times i \\ &= (-1) \times i \\ &= -i. \end{aligned}$$

Similarly,

$$\begin{aligned} i^4 &= i^2 \times i^2 \\ &= (-1) \times (-1) \\ &= 1. \end{aligned}$$

So $\quad i^5 = i^4 \times i = i,$

More examples

Example

Simplify
$$1, i, i^2, i^3, i^4, i^5, i^6, i^7, i^8, i^9, \dots$$

Since $i^2 = -1$, we know
$$\begin{aligned} i^3 &= i^2 \times i \\ &= (-1) \times i \\ &= -i. \end{aligned}$$

Similarly,
$$\begin{aligned} i^4 &= i^2 \times i^2 \\ &= (-1) \times (-1) \\ &= 1. \end{aligned}$$

So $\quad i^5 = i^4 \times i = i, \quad i^6 = i^4 \times i^2 = -1,$

More examples

Example

Simplify
$$1, i, i^2, i^3, i^4, i^5, i^6, i^7, i^8, i^9, \ldots$$

Since $i^2 = -1$, we know
$$\begin{aligned} i^3 &= i^2 \times i \\ &= (-1) \times i \\ &= -i. \end{aligned}$$

Similarly,
$$\begin{aligned} i^4 &= i^2 \times i^2 \\ &= (-1) \times (-1) \\ &= 1. \end{aligned}$$

So $\quad i^5 = i^4 \times i = i, \quad i^6 = i^4 \times i^2 = -1, \quad i^7 = i^4 \times i^3 = -i, \quad$ etc.

More examples

Example

Simplify
$$1, i, i^2, i^3, i^4, i^5, i^6, i^7, i^8, i^9, \ldots$$

Since $i^2 = -1$, we know

$$\begin{aligned} i^3 &= i^2 \times i \\ &= (-1) \times i \\ &= -i. \end{aligned}$$

Similarly,

$$\begin{aligned} i^4 &= i^2 \times i^2 \\ &= (-1) \times (-1) \\ &= 1. \end{aligned}$$

So $\quad i^5 = i^4 \times i = i, \quad i^6 = i^4 \times i^2 = -1, \quad i^7 = i^4 \times i^3 = -i, \quad$ etc.

In general, we can write $i^{4n+k} = i^k$ for any integer n.

More examples

Example

Show that $z = 2 + i$ is a solution of the cubic equation

$$z^3 - 5z^2 + 9z - 5 = 0.$$

More examples

Example

Show that $z = 2 + i$ is a solution of the cubic equation

$$z^3 - 5z^2 + 9z - 5 = 0.$$

First find z^2:

Example

Show that $z = 2 + i$ is a solution of the cubic equation

$$z^3 - 5z^2 + 9z - 5 = 0.$$

First find z^2:

$$z^2 = (2 + i)(2 + i)$$

Example

Show that $z = 2 + i$ is a solution of the cubic equation

$$z^3 - 5z^2 + 9z - 5 = 0.$$

First find z^2:

$$z^2 = (2 + i)(2 + i) = 4 + 2i + 2i + i^2$$

More examples

Example

Show that $z = 2 + i$ is a solution of the cubic equation

$$z^3 - 5z^2 + 9z - 5 = 0.$$

First find z^2:

$$z^2 = (2+i)(2+i) = 4 + 2i + 2i + i^2 = 3 + 4i.$$

More examples

Example

Show that $z = 2 + i$ is a solution of the cubic equation

$$z^3 - 5z^2 + 9z - 5 = 0.$$

First find z^2:

$$z^2 = (2 + i)(2 + i) = 4 + 2i + 2i + i^2 = 3 + 4i.$$

Next find z^3:

More examples

Example

Show that $z = 2 + i$ is a solution of the cubic equation

$$z^3 - 5z^2 + 9z - 5 = 0.$$

First find z^2:

$$z^2 = (2+i)(2+i) = 4 + 2i + 2i + i^2 = 3 + 4i.$$

Next find z^3:

$$z^3 = z \times z^2$$

More examples

Example

Show that $z = 2 + i$ is a solution of the cubic equation

$$z^3 - 5z^2 + 9z - 5 = 0.$$

First find z^2:

$$z^2 = (2+i)(2+i) = 4 + 2i + 2i + i^2 = 3 + 4i.$$

Next find z^3:

$$z^3 = z \times z^2 = (2+i)(3+4i)$$

More examples

Example

Show that $z = 2 + i$ is a solution of the cubic equation

$$z^3 - 5z^2 + 9z - 5 = 0.$$

First find z^2:

$$z^2 = (2 + i)(2 + i) = 4 + 2i + 2i + i^2 = 3 + 4i.$$

Next find z^3:

$$z^3 = z \times z^2 = (2 + i)(3 + 4i) = 6 + 8i + 3i + 4i^2$$

More examples

Example

Show that $z = 2 + i$ is a solution of the cubic equation

$$z^3 - 5z^2 + 9z - 5 = 0.$$

First find z^2:

$$z^2 = (2 + i)(2 + i) = 4 + 2i + 2i + i^2 = 3 + 4i.$$

Next find z^3:

$$z^3 = z \times z^2 = (2 + i)(3 + 4i) = 6 + 8i + 3i + 4i^2 = 2 + 11i.$$

More examples

Example

Show that $z = 2 + i$ is a solution of the cubic equation

$$z^3 - 5z^2 + 9z - 5 = 0.$$

First find z^2:

$$z^2 = (2+i)(2+i) = 4 + 2i + 2i + i^2 = 3 + 4i.$$

Next find z^3:

$$z^3 = z \times z^2 = (2+i)(3+4i) = 6 + 8i + 3i + 4i^2 = 2 + 11i.$$

Substituting in:

More examples

Example

Show that $z = 2 + i$ is a solution of the cubic equation

$$z^3 - 5z^2 + 9z - 5 = 0.$$

First find z^2:

$$z^2 = (2+i)(2+i) = 4 + 2i + 2i + i^2 = 3 + 4i.$$

Next find z^3:

$$z^3 = z \times z^2 = (2+i)(3+4i) = 6 + 8i + 3i + 4i^2 = 2 + 11i.$$

Substituting in:

$$z^3 - 5z^2 + 9z - 5$$

More examples

Example

Show that $z = 2 + i$ is a solution of the cubic equation

$$z^3 - 5z^2 + 9z - 5 = 0.$$

First find z^2:

$$z^2 = (2+i)(2+i) = 4 + 2i + 2i + i^2 = 3 + 4i.$$

Next find z^3:

$$z^3 = z \times z^2 = (2+i)(3+4i) = 6 + 8i + 3i + 4i^2 = 2 + 11i.$$

Substituting in:

$$z^3 - 5z^2 + 9z - 5 \quad = \quad (2 + 11i) - 5(3 + 4i) + 9(2 + i) - 5$$

Example

Show that $z = 2 + i$ is a solution of the cubic equation

$$z^3 - 5z^2 + 9z - 5 = 0.$$

First find z^2:

$$z^2 = (2+i)(2+i) = 4 + 2i + 2i + i^2 = 3 + 4i.$$

Next find z^3:

$$z^3 = z \times z^2 = (2+i)(3+4i) = 6 + 8i + 3i + 4i^2 = 2 + 11i.$$

Substituting in:

$$\begin{aligned} z^3 - 5z^2 + 9z - 5 &= (2+11i) - 5(3+4i) + 9(2+i) - 5 \\ &= (2 - 15 + 18 - 5) + (11 - 20 + 9)i \end{aligned}$$

More examples

Example

Show that $z = 2 + i$ is a solution of the cubic equation

$$z^3 - 5z^2 + 9z - 5 = 0.$$

First find z^2:

$$z^2 = (2+i)(2+i) = 4 + 2i + 2i + i^2 = 3 + 4i.$$

Next find z^3:

$$z^3 = z \times z^2 = (2+i)(3+4i) = 6 + 8i + 3i + 4i^2 = 2 + 11i.$$

Substituting in:

$$\begin{aligned} z^3 - 5z^2 + 9z - 5 &= (2+11i) - 5(3+4i) + 9(2+i) - 5 \\ &= (2 - 15 + 18 - 5) + (11 - 20 + 9)i \\ &= 0. \end{aligned}$$