School of Mathematics and Statistics
Math1131-Algebra

Lec06: Orthogonality and Projections

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Learning outcomes for this lecture

Perpendicular, orthogonal and orthonormal vectors – Projections

At the the end of this lecture,

- ☐ you should know what it means for vectors to be **orthogonal** or **perpendicular** and how it can be checked using the dot product.
- ☐ you should be able to use it to solve problems in **Geometry** involving **right angle**s (showing that some lines are perpendicular ... etc.)
- ☐ you should know what it means for vectors to be **orthogonal** or **orthonormal**; you should know how to use the scalar product to get the coefficients of vector written as a linear combination of orthonormal vectors.
- ☐ you should be able to find the **projection** of a vector onto another one and have an intuitive idea of what the projection is: It should correspond to a picture in your head.
- ☐ you should be able to use projections to find the **shortest distance between a point and a line**.
- ☐ you should have learned that drawing is really helpful!

TIP!

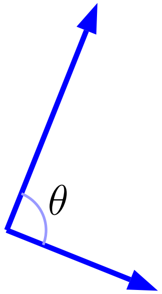
You can use this list as a check list to get ready for our next class: After studying the lecture notes, come back to this list, and for each item, check that you have indeed mastered it. Then tick the corresponding box ... or go back to the notes.

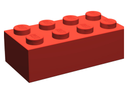
Orthogonal (and Perpendicular) vectors

Recall that for $\overrightarrow{u}$ and $\overrightarrow{v}$ in $\mathbb{R}^n$,

$$\overrightarrow{u} \cdot \overrightarrow{v} = |\overrightarrow{u}||\overrightarrow{v}|\cos\theta$$

where θ is the angle between $\overrightarrow{u}$ and $\overrightarrow{v}$, with $\theta \in [0, \pi]$.





Orthogonal and perpendicular vectors

- Two vectors $\overrightarrow{u}$ and $\overrightarrow{v}$ are *orthogonal* if and only if $\overrightarrow{u} \cdot \overrightarrow{v} = 0$.
  We denote this by $\overrightarrow{u} \perp \overrightarrow{v}$.

- For ***non-zero*** vectors $\overrightarrow{u}$ and $\overrightarrow{v}$, $\overrightarrow{u} \cdot \overrightarrow{v} = 0$ if and only if $\theta = \frac{\pi}{2}$.
  We say such vectors are *perpendicular*.

Perpendicular lines

Exercise 1. Prove that the lines

$$\ell_1 : \begin{pmatrix} x \\ y \end{pmatrix} = \underbrace{\begin{pmatrix} -3 \\ 1 \end{pmatrix}}_{A_1} + \lambda \underbrace{\begin{pmatrix} 4 \\ -2 \end{pmatrix}}_{\vec{v_1}}, \ \lambda \in \mathbb{R} \qquad \text{and} \qquad \ell_2 : \begin{pmatrix} x \\ y \end{pmatrix} = \underbrace{\begin{pmatrix} 2 \\ 1 \end{pmatrix}}_{A_2} + \mu \underbrace{\begin{pmatrix} -1 \\ -2 \end{pmatrix}}_{\vec{v_2}}, \ \mu \in \mathbb{R}$$

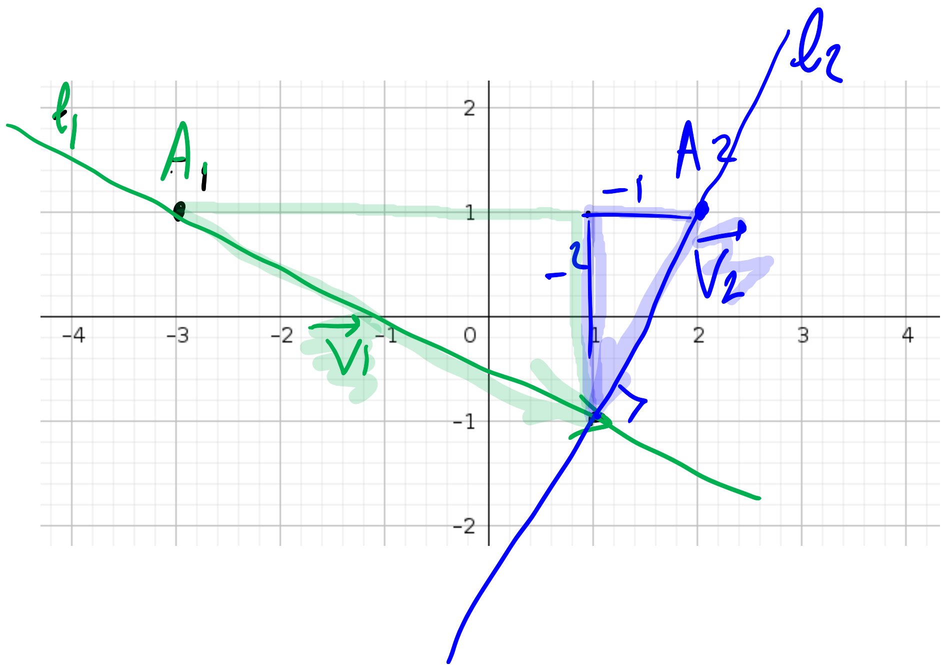
are perpendicular and represent them in the coordinate system below.

ℓ_1 and ℓ_2 are perpendicular if and only if $\vec{v_1} \cdot \vec{v_2} = 0$

$$\vec{v_1} \cdot \vec{v_2} = \begin{pmatrix} 4 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -2 \end{pmatrix}$$

$$= -4 + 4 = 0$$

$\therefore \ \ell_1 \perp \ell_2$



Orthogonal and orthonormal sets

Orthogonal and orthonormal sets of vectors

- A set of vectors in $\mathbb{R}^n$ is said to be an *orthogonal* set of vectors if all vectors are mutually (pairwise) orthogonal.
- A set of vectors in $\mathbb{R}^n$ is said to be an *orthonormal* set of vectors if it is an orthogonal set *and* all vectors are **unit** vectors.

The same definitions with symbols

- The vectors $\overrightarrow{\boldsymbol{v_1}}, \overrightarrow{\boldsymbol{v_2}}, \ldots, \overrightarrow{\boldsymbol{v_p}}$ are *orthogonal* $\overset{def}{\Longleftrightarrow}$ $\overrightarrow{\boldsymbol{v_i}} \cdot \overrightarrow{\boldsymbol{v_j}} = 0 \quad$ for $i \neq j$.
- The vectors $\overrightarrow{\boldsymbol{v_1}}, \overrightarrow{\boldsymbol{v_2}}, \ldots, \overrightarrow{\boldsymbol{v_p}}$ are *orthonormal*

$$\overset{def}{\Longleftrightarrow} \quad \overrightarrow{\boldsymbol{v_i}} \cdot \overrightarrow{\boldsymbol{v_j}} = \begin{cases} 1 & \text{for } i = j \quad \text{(unit vectors)} \\ 0 & \text{for } i \neq j \quad \text{(orthogonal)} \end{cases}$$

Orthogonal and orthonormal sets

Exercise 2.

Are each of the following orthogonal and/or orthonormal sets or neither?

a) $\left\{\overrightarrow{u_1} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \overrightarrow{u_2} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}\right\}$

$\overrightarrow{u_1} \cdot \overrightarrow{u_2} = 2-2 = 0$

$|\overrightarrow{u_1}| = \sqrt{2^2+1^2} = \sqrt{5} \neq 1$

$\therefore$ $\overrightarrow{u_1}$ and $\overrightarrow{u_2}$ are orthogonal
not orthonormal

b) $\left\{\overrightarrow{v_1} = \frac{1}{\sqrt{5}}\begin{pmatrix} 2 \\ -1 \end{pmatrix}, \overrightarrow{v_2} = \frac{1}{\sqrt{5}}\begin{pmatrix} 1 \\ 2 \end{pmatrix}\right\}$

$\overrightarrow{v_1} = \frac{1}{\sqrt{5}}\overrightarrow{u_1}$

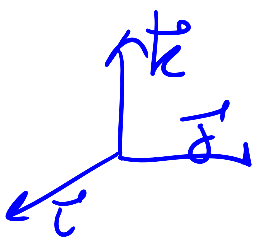
$\overrightarrow{v_1} \cdot \overrightarrow{v_2} = 0$ ✓

$|\overrightarrow{v_1}| = \frac{1}{\sqrt{5}}|\overrightarrow{v_1}| = \frac{1}{\sqrt{5}}\sqrt{5} = 1$ ✓

Similarly $|\overrightarrow{v_2}| = 1$ ✓

orthonormal

c) $\{\vec{i}, \vec{j}, \vec{k}\}$



orthonormal

d) $\{\overrightarrow{e_1}, \overrightarrow{e_2}, \ldots, \overrightarrow{e_n}\}$

$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ \vdots \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ \vdots \end{pmatrix}$

Why orthonormal sets of vectors are great

Exercise 3.

Write $\overrightarrow{v} = \frac{1}{\sqrt{5}}\begin{pmatrix}-4\\7\end{pmatrix}$ as a linear combination of $\overrightarrow{v_1} = \frac{1}{\sqrt{5}}\begin{pmatrix}2\\-1\end{pmatrix}$ and $\overrightarrow{v_2} = \frac{1}{\sqrt{5}}\begin{pmatrix}1\\2\end{pmatrix}$.

N.B. We have already shown that $\overrightarrow{v_1}$ and $\overrightarrow{v_2}$ are orthonormal.

$$\underset{\sim}{v} = \lambda_1 \underset{\sim}{v}_1 + \lambda_2 \underset{\sim}{v}_2$$

$$\Rightarrow \underset{\sim}{v}_1 \cdot \underset{\sim}{v} = \lambda_1 \underset{\sim}{v}_1 \cdot \underset{\sim}{v}_1 + \lambda_2 \underset{\sim}{v}_1 \cdot \underset{\sim}{v}_2$$

$$\Rightarrow \underset{\sim}{v}_1 \cdot \underset{\sim}{v} = \lambda_1 \times 1 + \lambda_2 \times 0$$

$$\Rightarrow \lambda_1 = \underset{\sim}{v}_1 \cdot \underset{\sim}{v} = \frac{1}{\sqrt{5}}\begin{pmatrix}2\\-1\end{pmatrix} \cdot \frac{1}{\sqrt{5}}\begin{pmatrix}-4\\7\end{pmatrix} = \frac{1}{5}(-8-7) = -3$$

similarly $\lambda_2 = \underset{\sim}{v}_2 \cdot \underset{\sim}{v} = \frac{1}{\sqrt{5}}\begin{pmatrix}1\\2\end{pmatrix} \cdot \frac{1}{\sqrt{5}}\begin{pmatrix}-4\\7\end{pmatrix} = \frac{1}{5}(-4+14) = 2$

So $\boxed{\underset{\sim}{v} = -3\,\underset{\sim}{v}_1 + 2\,\underset{\sim}{v}_2}$

? *How could we check our answer?*

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Checking our answers with Maple

```
> with(LinearAlgebra):
> # use : instead of ; if you do not want the echo

  v1 := 1/sqrt(5)*<2,-1>:
  v2 := 1/sqrt(5)*<1,2>:
  v := 1/sqrt(5)*<-4,7>;
```

$$v := \begin{bmatrix} -\frac{4\sqrt{5}}{5} \\ \frac{7\sqrt{5}}{5} \end{bmatrix}$$

```
> # Check orthonormality. I use ";" because I want the output displayed

  v1.v1; v1.v2; v1.v2;
```

$$1$$
$$0$$
$$0$$

```
> lambda1 := v1.v;
  lambda2 := v2.v;
```

$$\lambda 1 := -3$$
$$\lambda 2 := 2$$

Why orthonormal sets of vectors are great

Exercise 4. *[For fast students or for Independent practice]*

Consider the following set of vectors:

$$\left\{ \overrightarrow{\boldsymbol{w_1}} = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \overrightarrow{\boldsymbol{w_2}} = \frac{1}{\sqrt{42}}\begin{pmatrix} 4 \\ 1 \\ 5 \end{pmatrix}, \overrightarrow{\boldsymbol{w_3}} = \frac{1}{\sqrt{14}}\begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix} \right\}$$

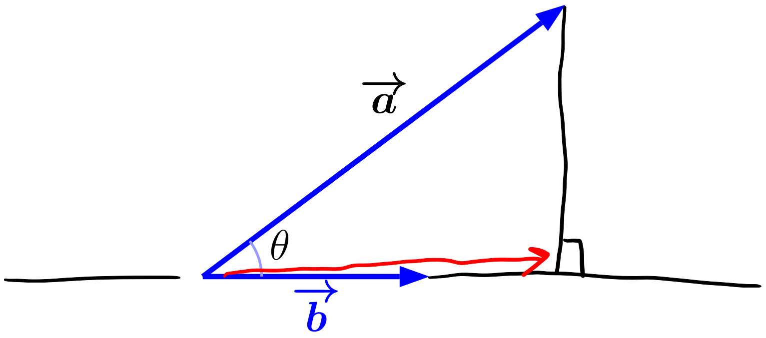
a) Is this an orthonormal set of vectors?

b) Let $\overrightarrow{\boldsymbol{v}} = \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix}$. Find $\lambda_1, \lambda_2, \lambda_3$ such that $\overrightarrow{\boldsymbol{v}} = \lambda_1 \overrightarrow{\boldsymbol{w_1}} + \lambda_2 \overrightarrow{\boldsymbol{w_2}} + \lambda_3 \overrightarrow{\boldsymbol{w_3}}$

Check your answer using Maple.

Projections

For $\overrightarrow{\boldsymbol{a}}, \overrightarrow{\boldsymbol{b}} \in \mathbb{R}^n$ with $\overrightarrow{\boldsymbol{b}} \neq \overrightarrow{\mathbf{0}}$, the **projection** of $\overrightarrow{\boldsymbol{a}}$ on $\overrightarrow{\boldsymbol{b}}$ is denoted $\textbf{proj}_{\overrightarrow{\boldsymbol{b}}}\overrightarrow{\boldsymbol{a}}$.



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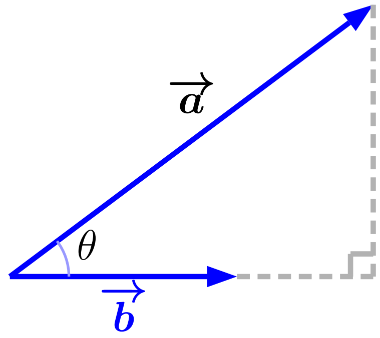

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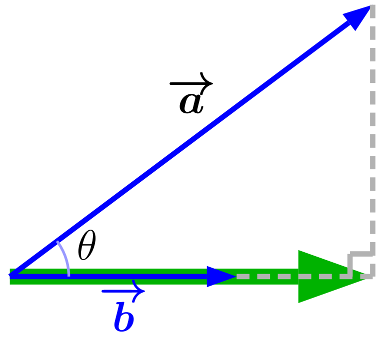

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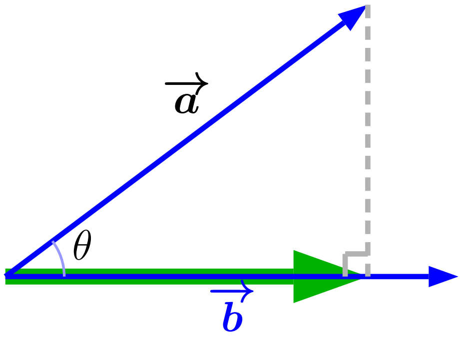

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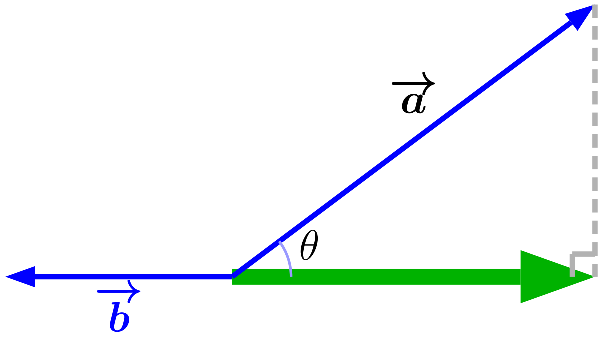

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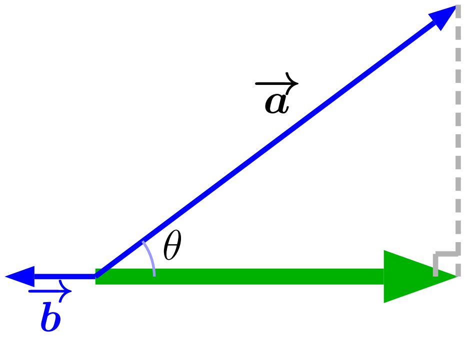

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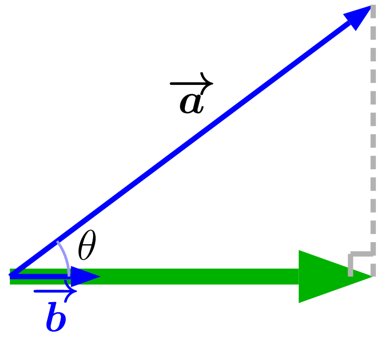

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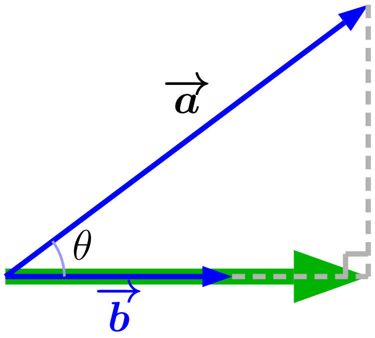
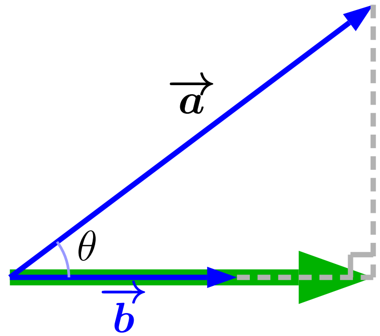


Projections

For $\vec{a}, \vec{b} \in \mathbb{R}^n$ with $\vec{b} \neq \vec{0}$, the **projection** of $\vec{a}$ on $\vec{b}$ is denoted $\mathbf{proj}_{\vec{b}}\,\vec{a}$.



Let's find a formula for the projection in the case when θ is acute:

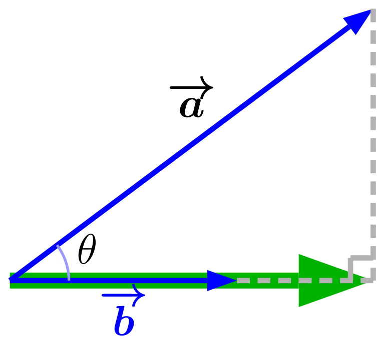
$$\mathbf{proj}_{\vec{b}}\,\vec{a} = \text{length of green arrow} \times \text{unit vector in direction of } \vec{b}$$

$$= |\vec{a}|\cos\theta\,\frac{1}{|\vec{b}|}\,\vec{b} = \underbrace{\boxed{|\vec{a}||\vec{b}|\cos\theta}}_{\vec{a}\cdot\vec{b}}\,\frac{1}{|\vec{b}|^2}\,\vec{b} = \underbrace{\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}}_{\text{real number}}\,\vec{b}$$

Projections

For $\overrightarrow{\boldsymbol{a}}, \overrightarrow{\boldsymbol{b}} \in \mathbb{R}^n$ with $\overrightarrow{\boldsymbol{b}} \neq \overrightarrow{\mathbf{0}}$, the **projection** of $\overrightarrow{\boldsymbol{a}}$ on $\overrightarrow{\boldsymbol{b}}$ is denoted $\mathbf{proj}_{\overrightarrow{b}}\, \overrightarrow{\boldsymbol{a}}$.

$$\mathbf{proj}_{\overrightarrow{b}}\, \overrightarrow{\boldsymbol{a}} = \frac{\overrightarrow{\boldsymbol{a}} \cdot \overrightarrow{\boldsymbol{b}}}{|\overrightarrow{\boldsymbol{b}}|^2}\, \overrightarrow{\boldsymbol{b}}$$



Let's find a formula for the projection in the case when θ is acute:

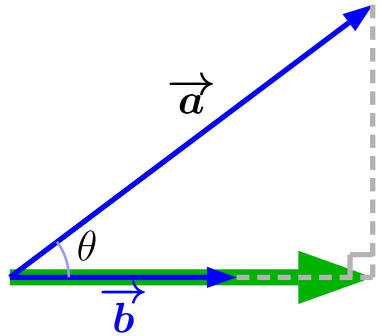
$$\begin{aligned} \mathbf{proj}_{\overrightarrow{b}}\, \overrightarrow{\boldsymbol{a}} &= \text{length of green arrow} \times \text{unit vector in direction of } \overrightarrow{\boldsymbol{b}} \\ &= |\overrightarrow{\boldsymbol{a}}| \cos\theta \, \frac{1}{|\overrightarrow{\boldsymbol{b}}|}\, \overrightarrow{\boldsymbol{b}} = |\overrightarrow{\boldsymbol{a}}||\overrightarrow{\boldsymbol{b}}| \cos\theta \, \frac{1}{|\overrightarrow{\boldsymbol{b}}|^2}\, \overrightarrow{\boldsymbol{b}} = \frac{\overrightarrow{\boldsymbol{a}} \cdot \overrightarrow{\boldsymbol{b}}}{|\overrightarrow{\boldsymbol{b}}|^2}\, \overrightarrow{\boldsymbol{b}} \end{aligned}$$

Projections

For $\overrightarrow{a}, \overrightarrow{b} \in \mathbb{R}^n$ with $\overrightarrow{b} \neq \overrightarrow{0}$, the **projection** of $\overrightarrow{a}$ on $\overrightarrow{b}$ is denoted $\mathbf{proj}_{\overrightarrow{b}}\,\overrightarrow{a}$.

$$\mathbf{proj}_{\overrightarrow{b}}\,\overrightarrow{a} = \frac{\overrightarrow{a} \cdot \overrightarrow{b}}{|\,\overrightarrow{b}\,|^2}\,\overrightarrow{b}$$

$$|\mathbf{proj}_{\overrightarrow{b}}\,\overrightarrow{a}| = \frac{|\,\overrightarrow{a} \cdot \overrightarrow{b}\,|}{|\,\overrightarrow{b}\,|}$$



Let's find a formula for the projection in the case when θ is acute:

$$\begin{aligned}
\mathbf{proj}_{\overrightarrow{b}}\,\overrightarrow{a} &= \text{length of green arrow} \times \text{unit vector in direction of } \overrightarrow{b} \\
&= |\,\overrightarrow{a}\,|\cos\theta\,\frac{1}{|\,\overrightarrow{b}\,|}\,\overrightarrow{b} = |\,\overrightarrow{a}\,||\,\overrightarrow{b}\,|\cos\theta\,\frac{1}{|\,\overrightarrow{b}\,|^2}\,\overrightarrow{b} = \frac{\overrightarrow{a} \cdot \overrightarrow{b}}{|\,\overrightarrow{b}\,|^2}\,\overrightarrow{b}
\end{aligned}$$

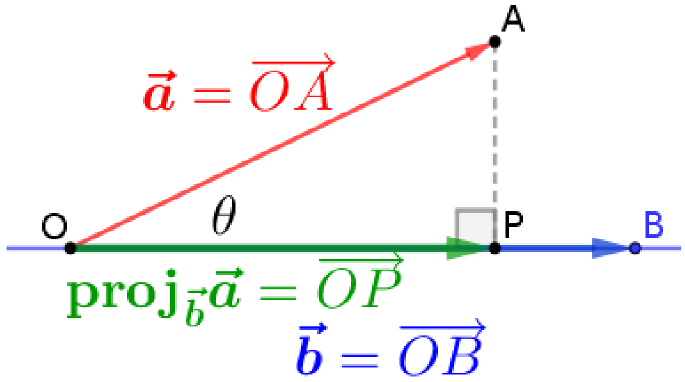
Can you prove this formula is also valid when θ is obtuse?

Projection, SUMMARY

Given two vectors $\vec{a}$ and $\vec{b}$, with $\vec{b} \neq \vec{0}$, drop a perpendicular AP to OB from A.

The vector $\overrightarrow{OP}$ is called the **projection** of $\vec{a}$ onto $\vec{b}$ and

$$\textbf{proj}_{\vec{b}}\,\vec{a} = \left(\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2}\right)\vec{b}.$$



Properties: The **projection** of $\vec{a}$ onto $\vec{b}$

- is parallel to $\vec{b}$;
- is unchanged if you replace $\vec{b}$ by a non-zero vector parallel to $\vec{b}$. (Can you prove it?)

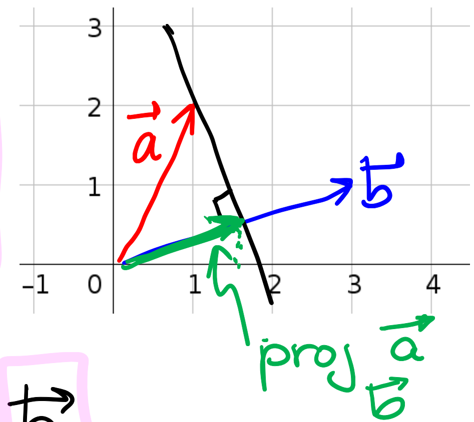
In that sense, you are projecting on the line span$(\vec{b})$, rather than onto any specific (non-zero) vector in this set.

Projections : Examples

Exercise 5. Find the projection of $\overrightarrow{a} = \begin{pmatrix}1\\2\end{pmatrix}$ onto $\overrightarrow{b} = \begin{pmatrix}3\\1\end{pmatrix}$.

$$\boxed{\text{proj}_{\vec{b}}\vec{a}} = \left(\frac{\vec{a}\cdot\vec{b}}{\vec{b}\cdot\vec{b}}\right)\vec{b}$$

$$= \frac{(3+2)}{(3^2+1^2)}\vec{b} = \frac{5}{10}\vec{b} = \frac{1}{2}\vec{b} = \boxed{\begin{pmatrix}3/2\\1/2\end{pmatrix}}$$



Check your work: $\textbf{proj}_{\vec{b}}\,\overrightarrow{a}$ should be parallel to vector $\boxed{\vec{b}}$..........

Exercise 6. Find the projection of $\begin{pmatrix}1\\2\\4\end{pmatrix}$ onto $\begin{pmatrix}-1\\-1\\-2\end{pmatrix}$.

$$\underset{\sim}{a} = \begin{pmatrix}1\\2\\4\end{pmatrix}, \quad \underset{\sim}{b} = \begin{pmatrix}-1\\-1\\-2\end{pmatrix}$$

$$\begin{pmatrix}1\\2\\4\end{pmatrix}\cdot\begin{pmatrix}-1\\-1\\-2\end{pmatrix} = -1-2-8 = -11$$

$$\boxed{\text{proj}_{\underset{\sim}{b}}\underset{\sim}{a}} = \frac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{\underset{\sim}{b}\cdot\underset{\sim}{b}}\underset{\sim}{b} = \frac{-11}{6}\begin{pmatrix}-1\\-1\\-2\end{pmatrix} \qquad \begin{pmatrix}-1\\-1\\-2\end{pmatrix}\cdot\begin{pmatrix}-1\\-1\\-2\end{pmatrix} = 1+1+4 = 6$$

$$= \boxed{\begin{pmatrix}11/6\\11/6\\11/3\end{pmatrix}}$$

Checking our answers with Maple

Exercise 6. Find the projection of $\begin{pmatrix}1\\2\\4\end{pmatrix}$ on $\begin{pmatrix}-1\\-1\\-2\end{pmatrix}$.

```
> with(LinearAlgebra):
> # Use : instead of ; if you do not want the echo.

  a := <1,2,4>:
  b := <-1,-1,-2>:
> proj_a_onto_b := (a.b)/(b.b)*b;
```

$$proj\_a\_onto\_b := \begin{bmatrix}\frac{11}{6}\\ \frac{11}{6}\\ \frac{11}{3}\end{bmatrix}$$

```
> # unchanged if we project on -b instead of b.

  b := <1,1,2>;
  proj_a_onto_b := (a.b)/(b.b)*b;
```

$$b := \begin{bmatrix}1\\1\\2\end{bmatrix}$$

$$proj\_a\_onto\_b := \begin{bmatrix}\frac{11}{6}\\ \frac{11}{6}\\ \frac{11}{3}\end{bmatrix}$$

Projections : Examples

Exercise 7. Find the shortest distance between the point $B(1,2,3)$ and the line

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -3 \\ -2 \end{pmatrix}, \qquad \lambda \in \mathbb{R}.$$

$$\overrightarrow{AB} = \begin{pmatrix} -4 \\ 2 \\ 2 \end{pmatrix}$$

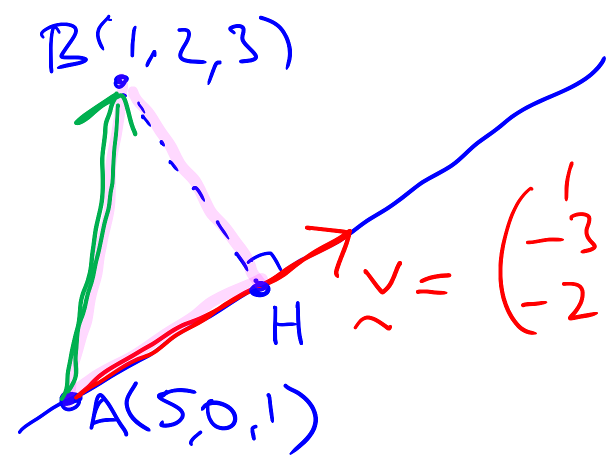
Also find the point on the line that is closest to B.

$$\overrightarrow{AH} = \text{proj}_{\underset{\sim}{v}} \overrightarrow{AB}$$

$$= \frac{\underset{\sim}{v} \cdot \overrightarrow{AB}}{\underset{\sim}{v} \cdot \underset{\sim}{v}} \underset{\sim}{v}$$

$$= \frac{-4-6-4}{1+9+4} \begin{pmatrix} 1 \\ -3 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix}$$



$$\overrightarrow{BH} = -\overrightarrow{AB} + \overrightarrow{AH} = \begin{pmatrix} 4 \\ -2 \\ -2 \end{pmatrix} + \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$$

dist of B from line

$$= |\overrightarrow{BH}| = \sqrt{3^2+1^2+0^2}$$

$$= \boxed{\sqrt{10}}$$

```
> with(LinearAlgebra):
> # The points A and B are defined by their position vectors.

  a := <5,0,1>:
  b := <1,2,3>:
  v := <1,-3,-2>:
> # This is vector AB

  u := b-a;
```

$$u := \begin{bmatrix} -4 \\ 2 \\ 2 \end{bmatrix}$$

```
> proj_u_onto_v := (u.v)/(v.v)*v;
```

$$proj\_u\_onto\_v := \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}$$

```
> # Position vector of H, the orthogonal projection of B on the line

  h := proj_u_onto_v + a;
```

$$h := \begin{bmatrix} 4 \\ 3 \\ 3 \end{bmatrix}$$

```
> # Norm of a vector = length = magnitude

  dist_BH := VectorCalculus[Norm](h-b);
```

$$dist\_BH := \sqrt{10}$$

or $\sqrt{\overrightarrow{BH} \cdot \overrightarrow{BH}}$, entered as sqrt((h-b).(h-b))