Nature of solutions. What to do YOU think? (Polls)

These examples will help you answer the polls.

The following systems of equations have all already been reduced to Row-echelon form.

For each of them, determine if they have no solutions, exactly one solution (unique solution) or infinitely many solutions

In the case of infinitely many solutions, how can you tell how many parameters are needed to describe the solutions?

a) $\left(\begin{array}{ccc|c} 1 & 4 & 7 & 4 \\ 0 & 3 & -1 & 5 \\ 0 & 0 & 8 & 2 \end{array}\right)$ b) $\left(\begin{array}{ccc|c} 5 & 0 & 0 & 14 \\ 0 & 2 & 1 & 6 \\ 0 & 0 & 8 & 7 \end{array}\right)$

c) $\left(\begin{array}{ccc|c} 0 & 5 & 1 & 4 \\ 0 & 0 & -1 & 6 \\ 0 & 0 & 0 & 5 \end{array}\right)$ d) $\left(\begin{array}{ccc|c} 1 & 2 & 0 & 4 \\ 0 & 0 & -1 & 6 \\ 0 & 0 & 0 & 0 \end{array}\right)$

e) $\left(\begin{array}{ccccc|c} 3 & 5 & 1 & 0 & 2 & 4 \\ 0 & 0 & -1 & 8 & 1 & 6 \end{array}\right)$ f) $\left(\begin{array}{cc|c} 1 & 1 & 8 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right)$

School of Mathematics and Statistics
Math1131-Algebra

Lec16: Solving systems of equations by Gaussian elimination

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Number of solutions of a system of linear equations.

Example 1. How many solutions do the systems of equations represented by the augmented matrices below have? Note that they are already in REF.

$$M_1 = \left(\begin{array}{ccc|c} \boxed{1} & 0 & 2 & 1 \\ 0 & \boxed{1} & 6 & 0 \\ 0 & 0 & 0 & \boxed{4} \end{array}\right) \qquad M_2 = \left(\begin{array}{ccc|c} \boxed{1} & -5 & 5 & 7 \\ 0 & \boxed{2} & 1 & -9 \\ 0 & 0 & \boxed{3} & 1 \end{array}\right) \qquad M_3 = \left(\begin{array}{ccc|c} \boxed{1} & 2 & 5 & -7 \\ 0 & \boxed{3} & -12 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

Number of solutions of a system of linear equations.

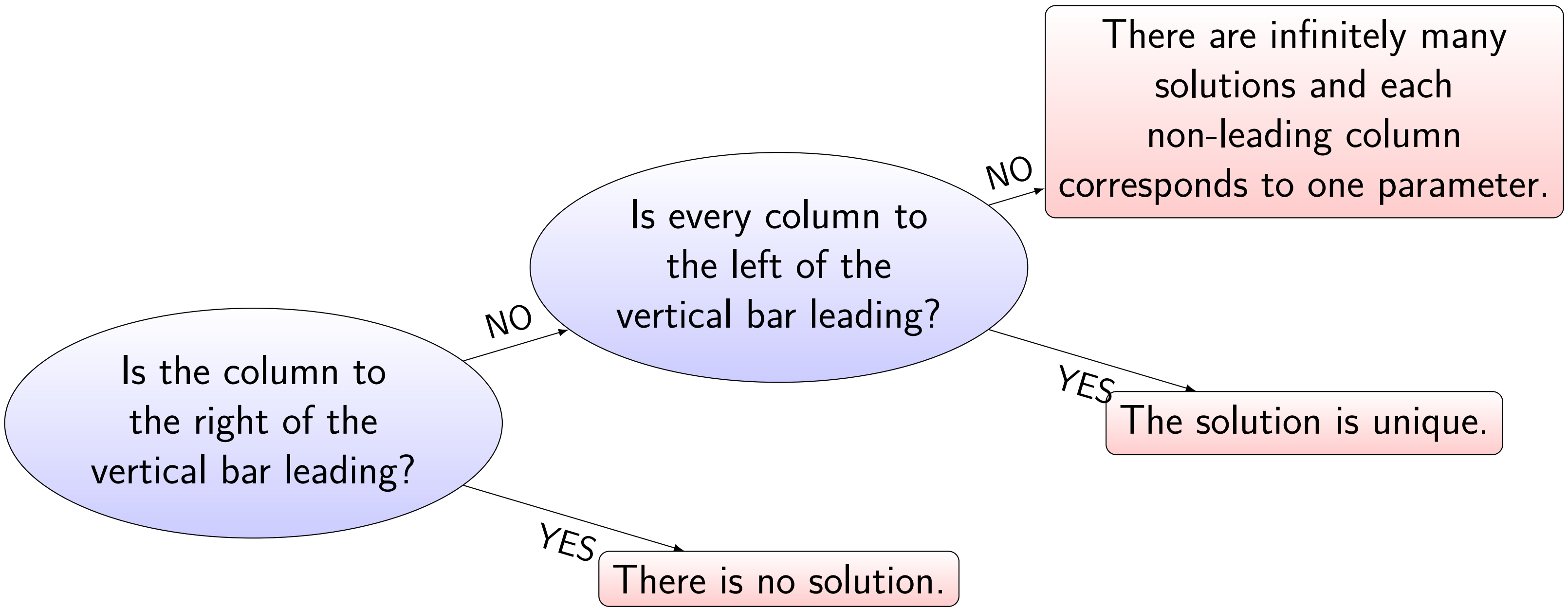
As we have seen, every time we solve a linear system of equations we either obtain, no solutions, exactly one solution (unique solution) or infinitely many solutions. You can tell how many solutions the system has by following these steps:

1. Rewrite your system in augmented matrix form $(A|\boldsymbol{b})$;
2. Row reduce, to obtain a row-echelon form $(U|\boldsymbol{y})$;
3. Circle or box the leading entries;
4. If y is a leading column, then the system has no solutions. If y is not a leading column then
   - If every column of U is leading then there is only one solution.
   - If U has non-leading columns, then the system has infinitely many solutions, and each the variable corresponding to the non-leading column will be treated as a parameter.

TIP! *N.B. Having (or not having) rows which are all zeros does not affect the number of solutions!!*

Number of solutions of a system of linear equations.

The same method described in a flow chart. After reduction to Row Echelon Form:



Example 1, again. How many solutions do the systems of equations represented by the augmented matrices below have? Note that they are already in REF.

$$M_1 = \left(\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & 6 & 0 \\ 0 & 0 & 0 & 4 \end{array}\right) \qquad M_2 = \left(\begin{array}{ccc|c} \boxed{1} & -5 & 5 & 7 \\ 0 & \boxed{2} & 1 & -9 \\ 0 & 0 & \boxed{3} & 1 \end{array}\right) \quad M_3 = \left(\begin{array}{ccc|c} \boxed{1} & 2 & 5 & -7 \\ 0 & \boxed{3} & -12 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

Gaussian elimination examples

Exercise 2. For the following system of linear equations, find conditions on the vector $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ such that the following system is consistent (ie has a solution)

$$\begin{aligned} x + 2y + 5z &= b_1 \\ 3x + 7y + 17z &= b_2 \\ x + 3y + 7z &= b_3. \end{aligned}$$

$$\left(\begin{array}{ccc|c} \boxed{1} & 2 & 5 & b_1 \\ 3 & 7 & 17 & b_2 \\ 1 & 3 & 7 & b_3 \end{array}\right) \begin{array}{l} \times -3 \\ \times 1 \\ \end{array}$$

$$\xrightarrow[R_3 \leftarrow R_3 - R_1]{R_2 \leftarrow R_2 - 3R_1} \left(\begin{array}{ccc|c} \boxed{1} & 2 & 5 & b_1 \\ 0 & \boxed{1} & 2 & -3b_1 + b_2 \\ 0 & 1 & 2 & -b_1 + b_3 \end{array}\right) \begin{array}{l} \\ \times -1 \\ \times 1 \end{array}$$

$$\xrightarrow{R_3 \leftarrow R_3 - R_2} \left(\begin{array}{ccc|c} \boxed{1} & 2 & 5 & b_1 \\ 0 & \boxed{1} & 2 & -3b_1 + b_2 \\ 0 & 0 & 0 & 2b_1 - b_2 + b_3 \end{array}\right)$$

The system is consistent iff $2b_1 - b_2 + b_3 = 0$

The vectors $\vec{b}$ for which there is a solution form a plane

Row-reduction with Maple

```
> # Load the LinearAlgebra package
  with(LinearAlgebra):

> # Enter the matrix column by column
  A := < <1,3,1> | <2,7,3> | <5,17,7> >:
  b := <b1, b2,b3>:

> # The augmented matrix is:
  Ab := <A|b>;
```

$$Ab := \begin{bmatrix} 1 & 2 & 5 & b1 \\ 3 & 7 & 17 & b2 \\ 1 & 3 & 7 & b3 \end{bmatrix}$$

```
> # R2 <- R2 - 3*R1
  RowOperation(Ab, [2, 1], -3);
```

$$\begin{bmatrix} 1 & 2 & 5 & b1 \\ 0 & 1 & 2 & b2 - 3\,b1 \\ 1 & 3 & 7 & b3 \end{bmatrix}$$

```
> # R3 <- R3 - R1
  RowOperation(%, [3, 1], -1);
```

$$\begin{bmatrix} 1 & 2 & 5 & b1 \\ 0 & 1 & 2 & b2 - 3\,b1 \\ 0 & 1 & 2 & b3 - b1 \end{bmatrix}$$

```
> # To get the Row Echelon form at once
  GaussianElimination(Ab);
```

$$\begin{bmatrix} 1 & 2 & 5 & b1 \\ 0 & 1 & 2 & b2 - 3\,b1 \\ 0 & 0 & 0 & b3 + 2\,b1 - b2 \end{bmatrix}$$

Gaussian elimination examples

Exercise 3.

Use the Maple output given below to determine for which values of λ (if any) the following system of linear equations will have : (1) no solutions; (2) a unique solution; (3) infinitely many solutions.

$$\begin{aligned} x + y + z &= 4 \\ x + \lambda y + 2z &= 5 \\ 2x + (\lambda+1)y + (\lambda^2-1)z &= \lambda + 7. \end{aligned}$$

```
> with(LinearAlgebra):

> A := < <1,1,2> | <1,lambda,lambda + 1> | <1,2,lambda^2 - 1> >:
  b := <4, 5, lambda + 7>:
  Ab := <A|b>;
```

$$Ab := \begin{bmatrix} 1 & 1 & 1 & 4 \\ 1 & \lambda & 2 & 5 \\ 2 & \lambda+1 & \lambda^2-1 & \lambda+7 \end{bmatrix}$$

```
> GaussianElimination(Ab);
```

$$\begin{bmatrix} 1 & 1 & 1 & 4 \\ 0 & \lambda-1 & 1 & 1 \\ 0 & 0 & \lambda^2-4 & \lambda-2 \end{bmatrix}$$

Gaussian elimination examples

Exercise 3.

Use the Maple output given below to determine for which values of λ (if any) the following system of linear equations will have : (1) no solutions; (2) a unique solution; (3) infinitely many solutions.

$$\begin{aligned} x + y + z &= 4 \\ x + \lambda y + 2z &= 5 \\ 2x + (\lambda+1)y + (\lambda^2-1)z &= \lambda + 7. \end{aligned}$$

```
> with(LinearAlgebra):
> A := < <1,1,2> | <1,lambda,lambda + 1> | <1,2,lambda^2 - 1> >:
  b := <4, 5, lambda + 7>:
  Ab := <A|b>;
```

$$Ab := \begin{bmatrix} 1 & 1 & 1 & 4 \\ 1 & \lambda & 2 & 5 \\ 2 & \lambda+1 & \lambda^2-1 & \lambda+7 \end{bmatrix}$$

```
> GaussianElimination(Ab);
```

$$\begin{bmatrix} 1 & 1 & 1 & 4 \\ 0 & \lambda-1 & 1 & 1 \\ 0 & 0 & \lambda^2-4 & \lambda-2 \end{bmatrix}$$

Case 1: $\lambda = 2$

$$\left(\begin{array}{ccc|c} \boxed{1} & 1 & 1 & 4 \\ 0 & \boxed{1} & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

infinitely many solutions

Case 2: $\lambda \neq 2$

$(\lambda - 2 \neq 0)$

$R_3 \leftarrow \frac{1}{\lambda-2} R_3$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & \lambda-1 & 1 & 1 \\ 0 & 0 & \lambda+2 & 1 \end{array}\right)$$

Gaussian elimination examples

Exercise 3, continued.

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & \lambda-1 & 1 & 1 \\ 0 & 0 & \lambda+2 & 1 \end{array}\right)$$

$$\left[\begin{array}{cccc} 1 & 1 & 1 & 4 \\ 0 & \lambda-1 & 1 & 1 \\ 0 & 0 & \lambda^2-4 & \lambda-2 \end{array}\right]$$

case 2α : $\lambda \neq 2$ and $\lambda = -2$

$$\left(\begin{array}{ccc|c} \boxed{1} & 1 & 1 & 4 \\ 0 & \boxed{-3} & 1 & 1 \\ 0 & 0 & 0 & \boxed{1} \end{array}\right)$$

no solution

case 2β : $\lambda \neq 2$ and $\lambda \neq -2$

case 2β i) $\lambda = 1$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 3 & 1 \end{array}\right) \xrightarrow[R_3 \leftarrow R_3 - 3R_2]{} \left(\begin{array}{ccc|c} \boxed{1} & 1 & 1 & 4 \\ 0 & 0 & \boxed{1} & 1 \\ 0 & 0 & 0 & \boxed{-2} \end{array}\right)$$

no solution

case 2β ii) $\lambda \neq -1$

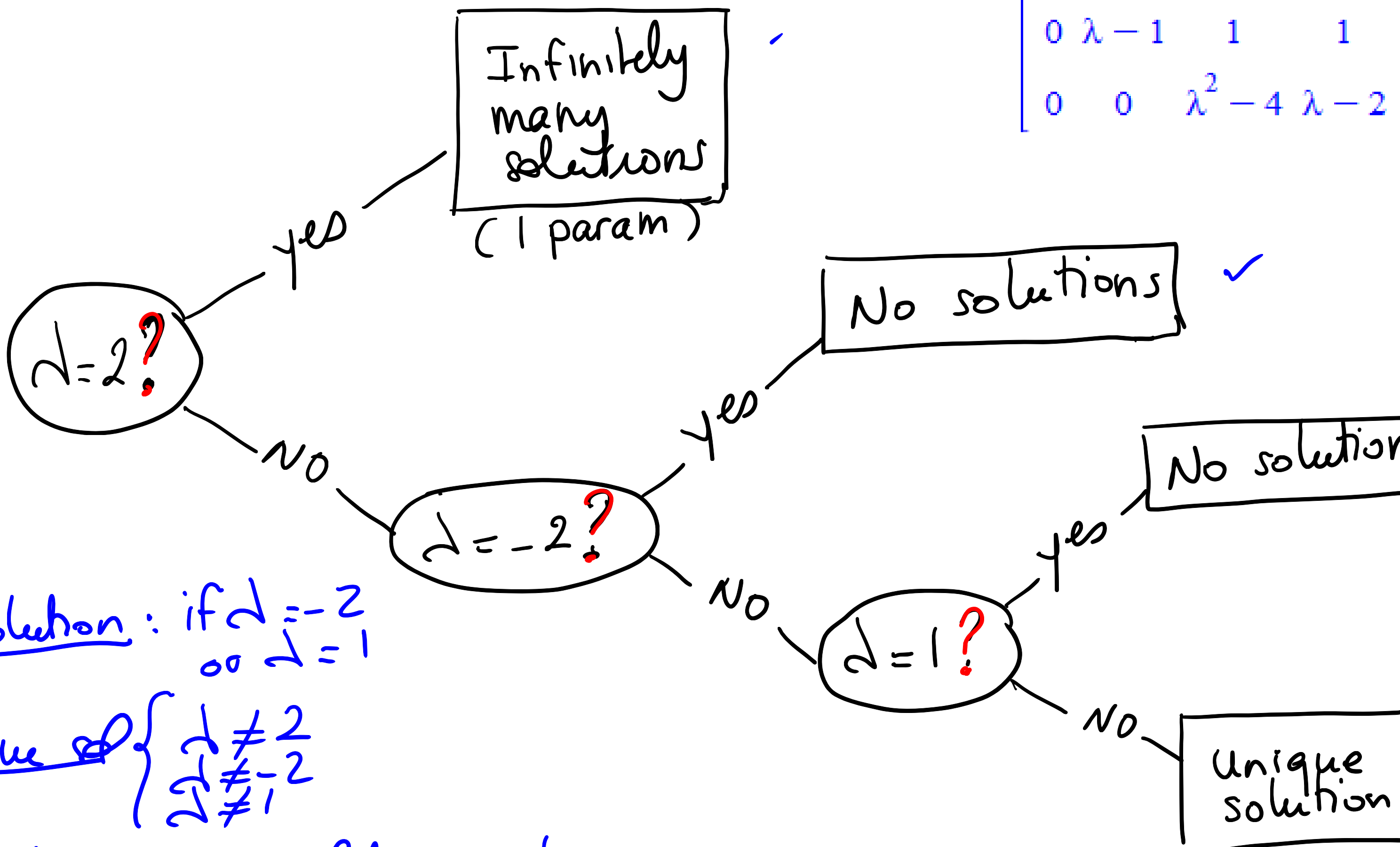
$$\left(\begin{array}{ccc|c} \boxed{1} & 1 & 1 & 4 \\ 0 & \boxed{\lambda-1} & 1 & 1 \\ 0 & 0 & \boxed{\lambda+2} & 1 \end{array}\right)$$

a unique solution

Gaussian elimination examples

Exercise 3, continued.

$$\left[\begin{array}{cccc} 1 & 1 & 1 & 4 \\ 0 & \lambda - 1 & 1 & 1 \\ 0 & 0 & \lambda^2 - 4 & \lambda - 2 \end{array}\right]$$



No solution : if $\lambda = -2$ or $\lambda = 1$

unique sol $\begin{cases} \lambda \neq 2 \\ \lambda \neq -2 \\ \lambda \neq 1 \end{cases}$

infinitely many solutions : $\lambda = 2$

Gaussian elimination examples

Exercise 4. Determine whether the line

$$\ell : \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + t \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad t \in \mathbb{R}$$

meets the plane

$$\Pi : \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 0 \\ -2 \end{pmatrix}, \quad \lambda, \mu \in \mathbb{R}$$

Solve this problem in two different ways.

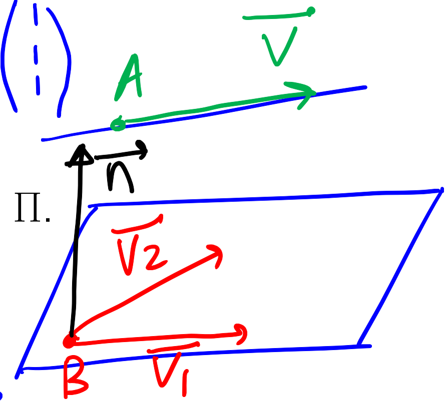
Method 1: Solve

$$\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + t \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 0 \\ -2 \end{pmatrix}$$

$$t \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

Gaussian elimination examples

$$\vec{n}\begin{pmatrix}-2\\-2\\-2\end{pmatrix}=-2\begin{pmatrix}1\\1\\1\end{pmatrix}$$



Exercise 4, continued. Determine whether the line ℓ meets the plane Π.

$$\begin{array}{ccc|c} \lambda & \mu & t & \\ \end{array}$$

$$\left(\begin{array}{ccc|c} \boxed{1} & -2 & 1 & 5 \\ -1 & 0 & 2 & 0 \\ 0 & 2 & 3 & 1 \end{array}\right)$$

$R_2 \leftarrow R_2 + R_1$

$$\left(\begin{array}{ccc|c} \boxed{1} & -2 & 1 & 5 \\ 0 & \boxed{-2} & 3 & 5 \\ 0 & 2 & 3 & 11 \end{array}\right)$$

$R_3 \leftarrow R_3 + R_2$

$$\begin{array}{c} \begin{array}{ccc} \lambda & \mu & t \end{array} \\ \left(\begin{array}{ccc|c} \boxed{1} & -2 & 1 & 5 \\ 0 & \boxed{-2} & 3 & 5 \\ 0 & 0 & \boxed{6} & 6 \end{array}\right) \end{array}$$

there is a unique solutions so there is a point of intersection

Let $\vec{n} = \vec{v_1} \times \vec{v_2}$

$$\begin{vmatrix} \vec{i} & -1 & 2 \\ \vec{j} & 1 & 0 \\ \vec{k} & 0 & -2 \end{vmatrix} = \begin{pmatrix} -2 \\ -(2) \\ -2 \end{pmatrix}$$

$\vec{n_1} = \begin{pmatrix}1\\1\\1\end{pmatrix}$ is normal to Π.

Let $\vec{v} = \begin{pmatrix}1\\2\\3\end{pmatrix}$ a vector giving the direction of ℓ

$\vec{n}\cdot\vec{v} = 1+2+3 \neq 0$

$\therefore$ ℓ is not parallel to Π so there is a point of intersection.

Gaussian elimination examples

Exercise 5. Consider the following system of linear equations.

$$\begin{aligned} x + 2y + z &= 8 \\ 4x + 3y - z &= 7 \\ 3x + y - 2z &= -1. \end{aligned}$$

1. Find the general solution.
2. Find the solution which has an x-value of 10.
3. Given x, y and z must all be non-negative, find the maximum value of y.

Gaussian elimination examples

Exercise 5. Consider the following system of linear equations.

$$\begin{aligned} x + 2y + z &= 8 \\ 4x + 3y - z &= 7 \\ 3x + y - 2z &= -1. \end{aligned}$$

1. Find the general solution.
2. Find the solution which has an x-value of 10.
3. Given x, y and z must all be non-negative, find the maximum value of y.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2 \\ 5 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

$\lambda \in \mathbb{R}$

1.
$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 8 \\ 4 & 3 & -1 & 7 \\ 3 & 1 & -2 & -1 \end{array}\right) \quad \begin{array}{l} R_2 \leftarrow R_2 - 4R_1 \\ R_3 \leftarrow R_3 - 3R_1 \end{array}$$

$$\sim \left(\begin{array}{ccc|c} 1 & 2 & 1 & 8 \\ 0 & -5 & -5 & -25 \\ 0 & -5 & -5 & -25 \end{array}\right) \quad \begin{array}{l} R_2 \leftarrow -\frac{1}{5} R_2 \\ R_3 \leftarrow R_3 - R_2 \\ \text{(using old } R_2) \end{array}$$

$$\sim \left(\begin{array}{ccc|c} \boxed{1} & 2 & 1 & 8 \\ 0 & \boxed{1} & 1 & 5 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

let $z = \lambda$, $\lambda \in \mathbb{R}$

Row 2 says

$$y + z = 5$$
$$\Leftrightarrow y = 5 - z = 5 - \lambda$$

Row 1 says

$$x + 2y + z = 8$$
$$\Leftrightarrow x = 8 - 2y - z = 8 - 2(5 - \lambda) - \lambda = -2 + \lambda$$

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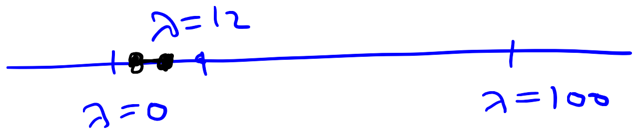
Gaussian elimination examples

Exercise 5, continued.

1. $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2 \\ 5 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

$\lambda \in \mathbb{R}$



2. Assume $x = 10$

$x = -2 + \lambda$

So we want

$10 = -2 + \lambda \iff \lambda = 12$

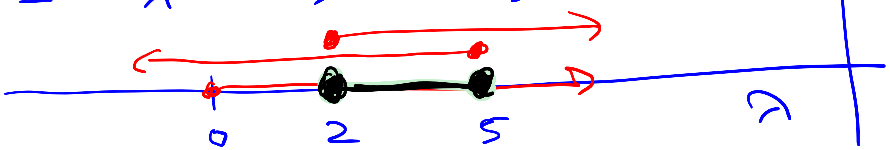
$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2 \\ 5 \\ 0 \end{pmatrix} + 12 \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 10 \\ -7 \\ 12 \end{pmatrix}$

3. x, y, z non-negative means

$x = -2 + \lambda \geq 0 \iff \lambda \geq 2$

$y = 5 - \lambda \geq 0 \iff \lambda \leq 5$

$z = \lambda \geq 0 \iff \lambda \geq 0$



So $2 \leq \lambda \leq 5$

y decrease as λ increase,

so y is between 0 and 3

$\boxed{\lambda = 2 : y = 3}$

$\lambda = 5 : y = 0$

The maximum value of y is 3.

Gaussian elimination examples

Exercise 6. A forest contains 3 different types of gums trees: the red gum, the blue gum and the swamp gum. Each red gum tree houses 1 koala and 3 bandicoots. Each blue gum houses 2 koalas and 1 bandicoot and each swamp gum houses 3 koalas and 2 bandicoots. Conservationists are planning to occupy and protect the forest from logging. They have assigned 1 person for each red gum, 2 people for each blue gum and 4 people for each swamp gum. There are a total 1700 koalas and 1200 bandicoots in the forest and 1900 conservationists.

Set up the system of equations needed to determine how many of each type of gum tree does the forest contains.

- let r be the number of red gums
- let b " " blue
- s " " swamp

Number of koalas: $1 \times r + 2b + 3s = 1700$

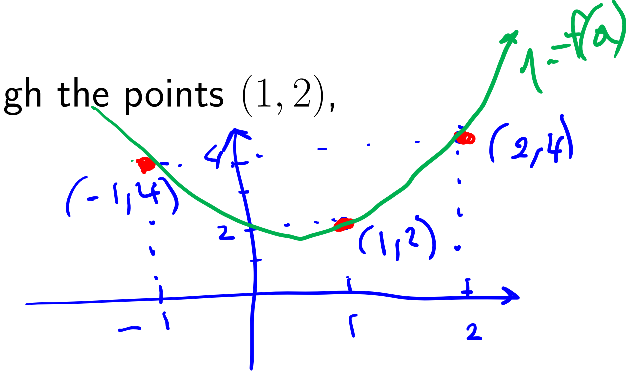
" bandicoots: $3r + b + 2s = 1200$

" conservatn: $r + 2b + 4s = 1900$

$\boxed{f(x) = x^2 - x + 2}$

Gaussian elimination examples

Exercise 7. Find the equation of the parabola that passes through the points $(1, 2)$, $(-1, 4)$, and $(2, 4)$.



Let $f(x) = ax^2 + bx + c$

We need to find a, b, c:

$(1,2) \in S$: $a\times 1^2 + b\times 1 + c = 2$

$(-1,4) \in S$: $a(-1)^2 + b(-1) + c = 4$

$(2,4) \in S$: $a(2)^2 + b\times 2 + c = 4$

$$\begin{cases} a + b + c = 2 \\ a - b + c = 4 \\ 4a + 2b + c = 4 \end{cases}$$

$$\begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \left(\begin{array}{ccc|c} \boxed{1} & 1 & 1 & 2 \\ 1 & -1 & 1 & 4 \\ 4 & 2 & 1 & 4 \end{array}\right) \end{array} \begin{array}{l} \times -4 \\ \\ \times 1 \end{array}$$

$$\left(\begin{array}{ccc|c} \boxed{1} & 1 & 1 & 2 \\ 0 & \boxed{-2} & 0 & 2 \\ 0 & -2 & -3 & -4 \end{array}\right) \begin{array}{l} R_2 \leftarrow R_2 - R_1 \\ R_3 \leftarrow R_3 - 4R_1 \end{array}$$

$$\left(\begin{array}{ccc|c} \boxed{1} & 1 & 1 & 2 \\ 0 & \boxed{-2} & 0 & 2 \\ 0 & 0 & \boxed{-3} & -6 \end{array}\right) \quad R_3 \leftarrow R_3 - R_2$$

$$\begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right) \end{array} \begin{array}{l} R_2 \leftarrow -\frac{1}{2} R_2 \\ R_3 \leftarrow -\frac{1}{3} R_3 \end{array}$$

$c = 2$, $b = -1$

$a + b + c = 2$ $\quad a - 1 + 2 = 2$ $\quad \boxed{a = 1}$

Checking our answers with Maple (Parabola)

```
> # Load the LinearAlgebra package
  with(LinearAlgebra):
```

```
> # Enter the matrix column by column
  A := < <1,1,4> | <1,-1,2> | <1,1,1> >:
  b := <2,4,4>:
```

```
> # The augmented matrix is:
  Ab := <A|b>;
```

$$Ab := \begin{bmatrix} 1 & 1 & 1 & 2 \\ 1 & -1 & 1 & 4 \\ 4 & 2 & 1 & 4 \end{bmatrix}$$

```
> # R2 <- R2 - R1
  RowOperation(Ab, [2, 1], -1);
```

$$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & -2 & 0 & 2 \\ 4 & 2 & 1 & 4 \end{bmatrix}$$

```
> # R3 <- R3 - 4*R1
  RowOperation(%, [3, 1], -4);
```

$$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & -2 & 0 & 2 \\ 0 & -2 & -3 & -4 \end{bmatrix}$$

```
> # R3 <- R3 - R2
  RowOperation(%, [3, 2], -1);
```

$$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & -2 & 0 & 2 \\ 0 & 0 & -3 & -6 \end{bmatrix}$$

```
> # Gaussian elimination  followed by back substitution
  REF := GaussianElimination(Ab);
  BackwardSubstitute(REF);
```

$$REF := \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & -2 & 0 & 2 \\ 0 & 0 & -3 & -6 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

```
> # Checking our answers
  f := x -> x^2 -x +2;
  f(1);f(-1);f(2);
```

$$f := x \mapsto x^2 - x + 2$$

$$2$$

$$4$$

$$4$$