LECTURE 13
Curve Sketching

A function f is said to be **odd** if $f(-x) = -f(x)$ over its domain.

A function f is said to be **even** if $f(-x) = f(x)$ over its domain.

To sketch an unknown graph of a function $y = f(x)$ we use a checklist to investigate its structure. We find:

✓ the y intercept by setting $x = 0$.
✓ the x intercept(s) by solving $f(x) = 0$ for x.
✓ whether or not the function is odd or even.
✓ (V.A.) the vertical asymptotes (usually a consequence of division by zero).
✓ (H.A.) the behaviour of the function for large $|x|$ by considering $\lim\limits_{x\longrightarrow\pm\infty} f(x)$.
(H.A. will determine horizontal and oblique asymptotes).
✓ the position and nature of any stationary points (this is crucial).
✓ !! If stuck plot points !!

Odd and Even Functions

A function f is said to be **odd** if $f(-x) = -f(x)$ over its domain.
Examples of odd functions are $y = x$, $y = x^3$,$y = x^5$, $y = \sin(x)$ and $y = \sin^{-1}(x)$.

Odd functions exhibit skew symmetry in their graphs.

Example 1:

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A function f is said to be **even** if $f(-x) = f(x)$ over its domain.

Examples of even functions are $y = 1$, $\quad y = x^2$, $\quad y = x^4$ and $y = \cos(x)$.

Even functions exhibit symmetry about the y axis in their graphs.

Example 2:

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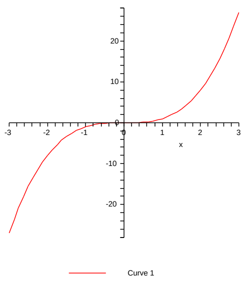
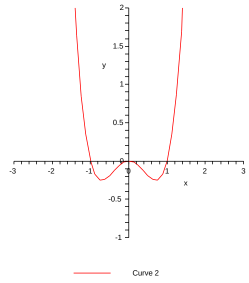
odd×odd=even,   odd×even=odd,   even×even=even.

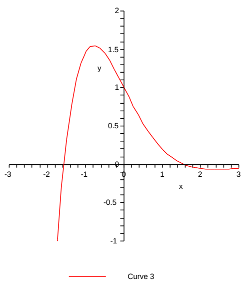
odd±odd=odd,   even±even=even,   even±odd=neither.

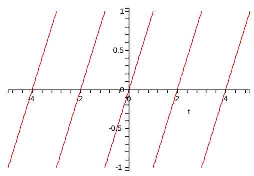
$$\int_{-a}^{a} odd = 0, \quad \int_{-a}^{a} even = 2\int_{0}^{a} even.$$

Example 3: Identify each of the following functions as odd, even or neither:

a) 

b) 

c) 

d) 

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Example 4: Identify each of the following functions as odd, even or neither:

i) $f(x) = \cos(3x)$

ii) $f(x) = x\cos(x)$

iii) $f(x) = \sin^2(x)$

iv) Prove your answer in iii) from the definition.

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Example 5: Prove that the derivative of an even function is an odd function.

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We now turn to the problem of sketching an unknown function. This can be quite tricky and our approach is to piece together a graph using a host of different clues as to the shape of the curve. Our checklist is to find:

✓ the y intercept by setting $x = 0$.

✓ the x intercept(s) by solving $f(x) = 0$ for x.

✓ whether or not the function is odd or even.

✓ (V.A.) the vertical asymptotes (usually a consequence of division by zero).

✓ (H.A.) the behaviour of the function for large $|x|$ by considering $\lim_{x \longrightarrow \pm\infty} f(x)$.
(H.A. will determine horizontal and oblique asymptotes).

✓ the position and nature of any stationary points (this is crucial).

✓ !! If stuck plot points !!

Let's have a look at a batch of examples.

Example 6: Sketch the graph of $y = f(x) = x^3 - 3x$.

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Example 7: Sketch the graph of $y = f(x) = \dfrac{2x-1}{x+1}$.

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Example 8: Consider the function $f(x) = \dfrac{x^2 - 5x + 29}{x - 4}$.

a) Sketch the graph of $y = f(x)$.

b) Hence find the domain and range of f.

c) By considering your sketch find all values of k for which the equation

$$x^2 - (5 + k)x + (29 + 4k) = 0$$

has exactly one solution.

d) **Homework:** Check your answer to c) by using the discriminant.

$\bigstar \quad (-1,-7) \quad \text{is a local max and} \quad (9,13) \quad \text{is a local min.} \qquad \bigstar$

$\bigstar \quad \text{Dom(f)} = \{x \in \mathbb{R} : x \neq 4\} \qquad \text{Range(f)} = [13,\infty) \cup (-\infty,-7] \quad \bigstar$

$\bigstar \quad k = -7, 13 \quad \bigstar$

Example 9: Sketch the graph of $y = f(x) = e^{-x}\sin(x)$.

We can just use common sense on this one!

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