School of Mathematics and Statistics

Math1131-Algebra

Lec03: Lines

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Jonathan Kress
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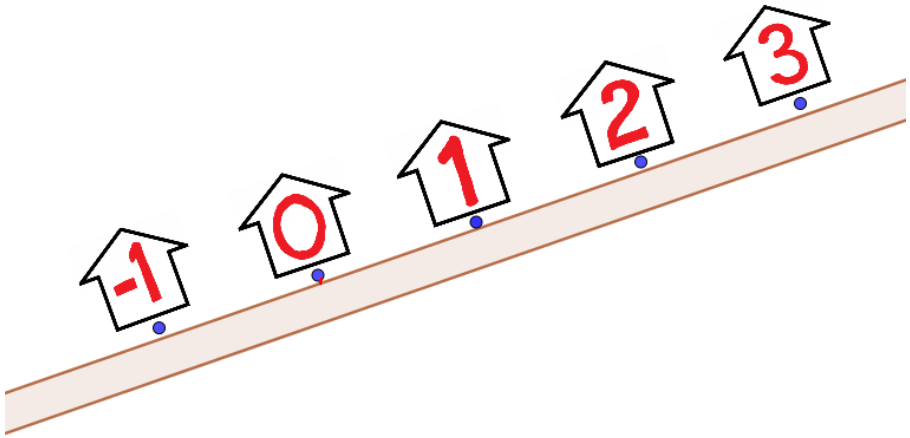
Red-Centre, Room 3090

2019 Term 1

Lines

How do you find a given house on a street?

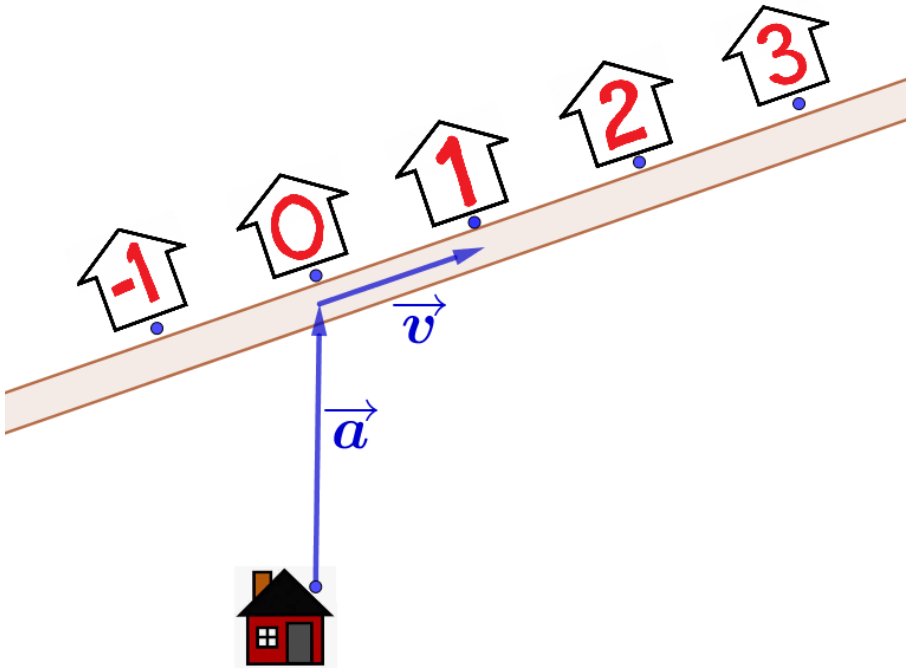
Go to somewhere on the street and then travel in the direction of the street until you reach the correct house.



Lines

How do you find house number 1 on a street?

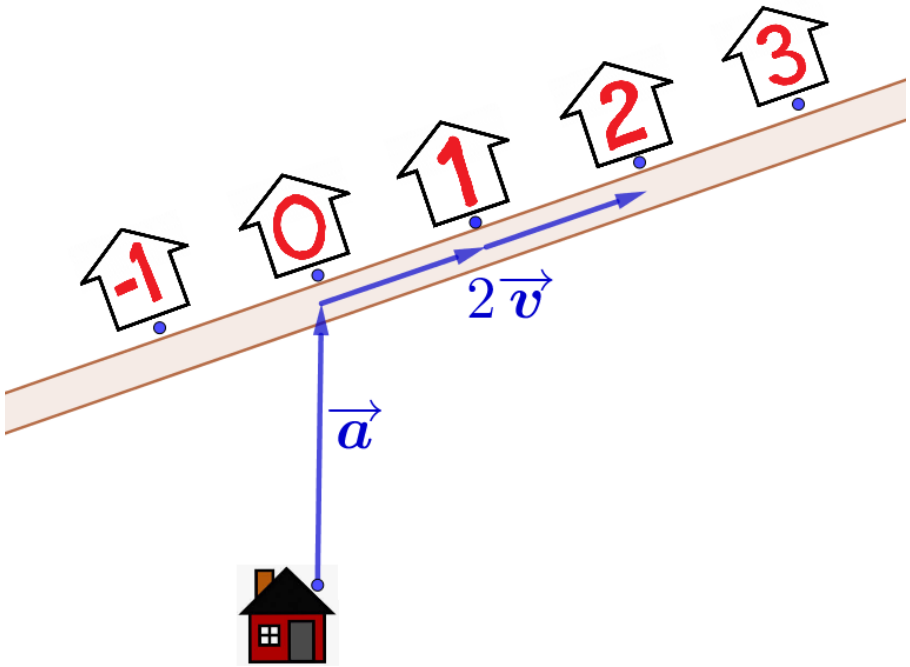
Go to somewhere on the street and then travel in the direction of the street until you reach house number 1.



Lines

How do you find house number 2 on a street?

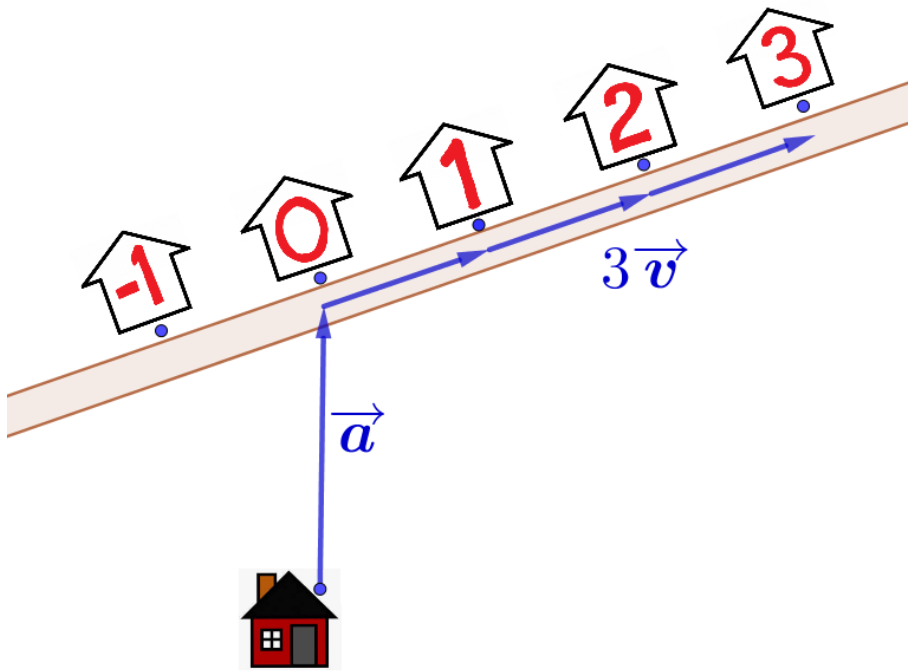
Go to somewhere on the street and then travel in the direction of the street until you reach house number 2.



Lines

How do you find house number 3 on a street?

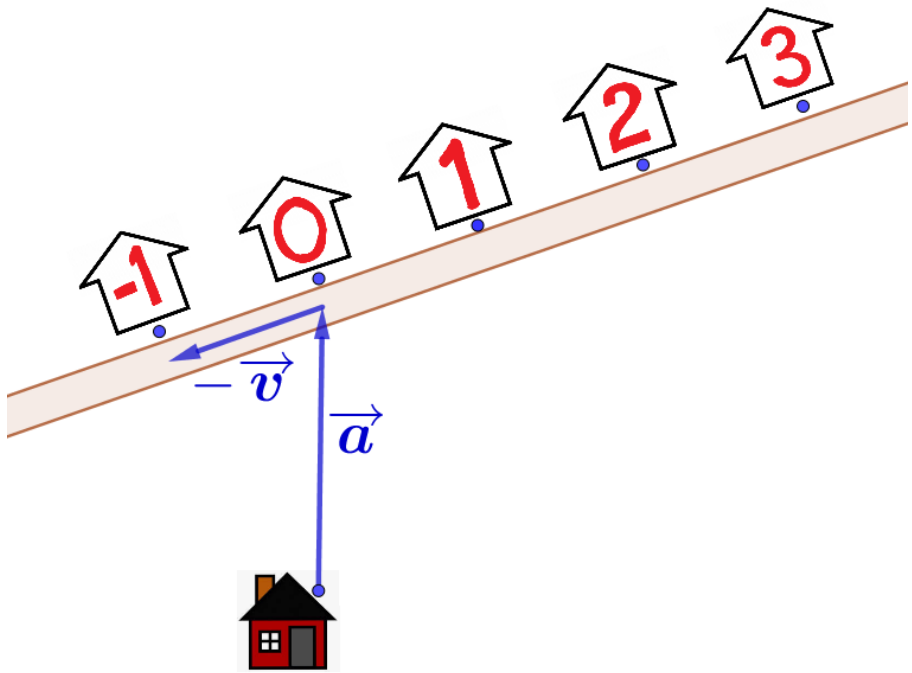
Go to somewhere on the street and then travel in the direction of the street until you reach house number 3.



Lines

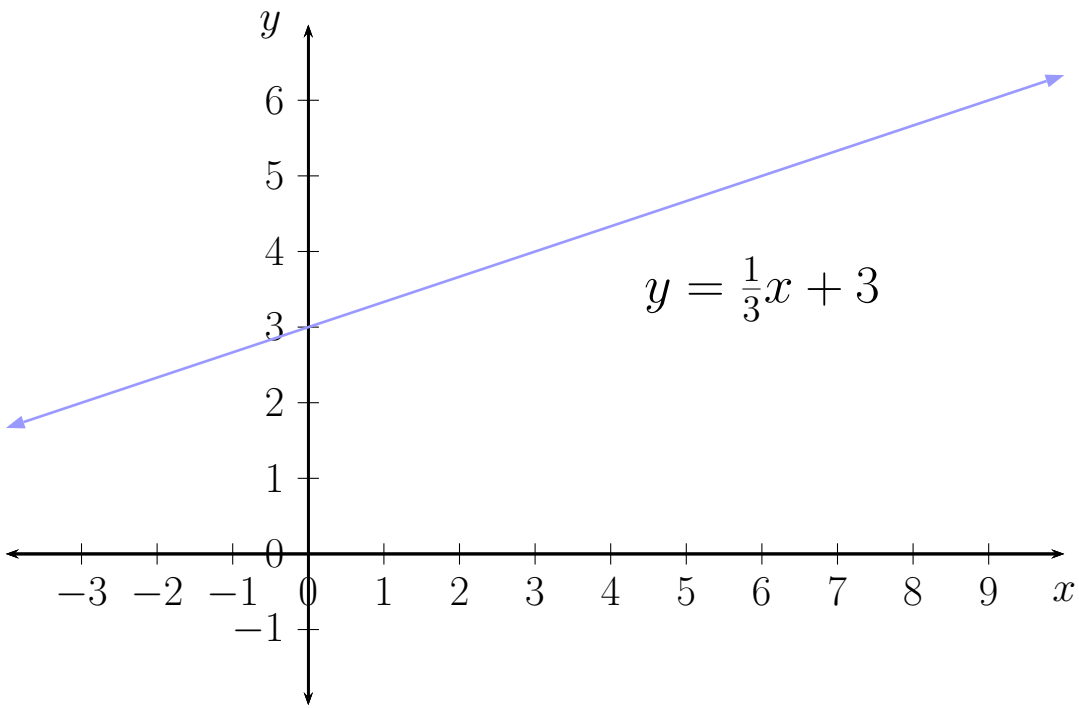
How do you find house number -1 on a street?

Go to somewhere on the street and then travel in the direction of the street until you reach house number -1.



Lines in two dimensions

We can describe a line in 2D using a Cartesian equation.



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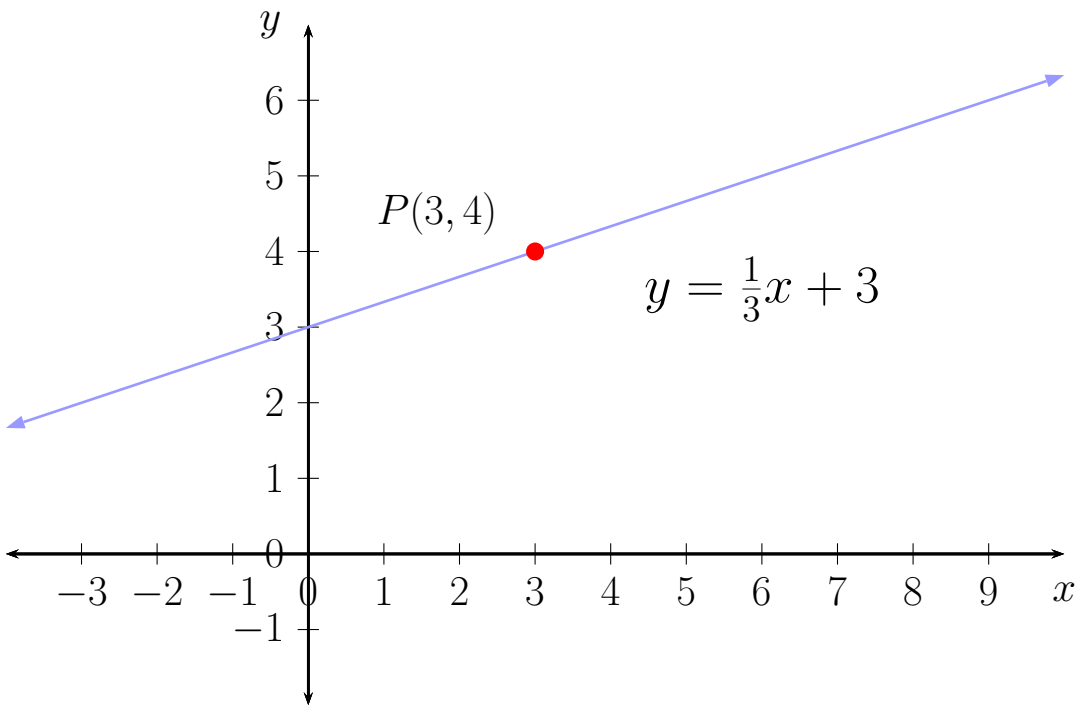

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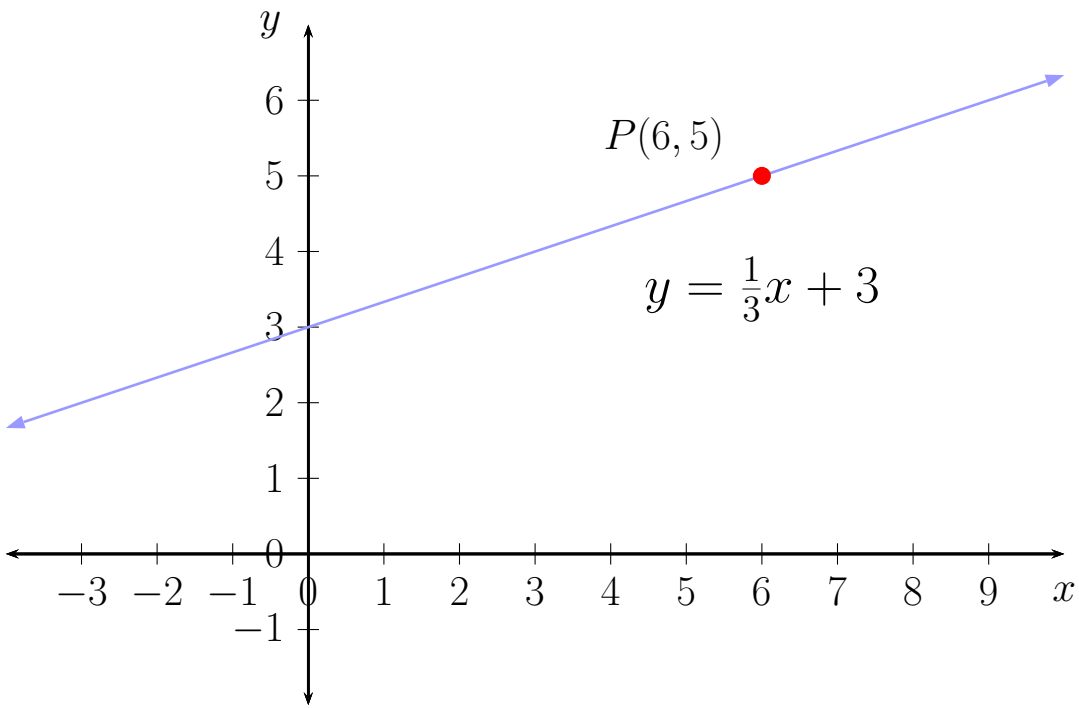

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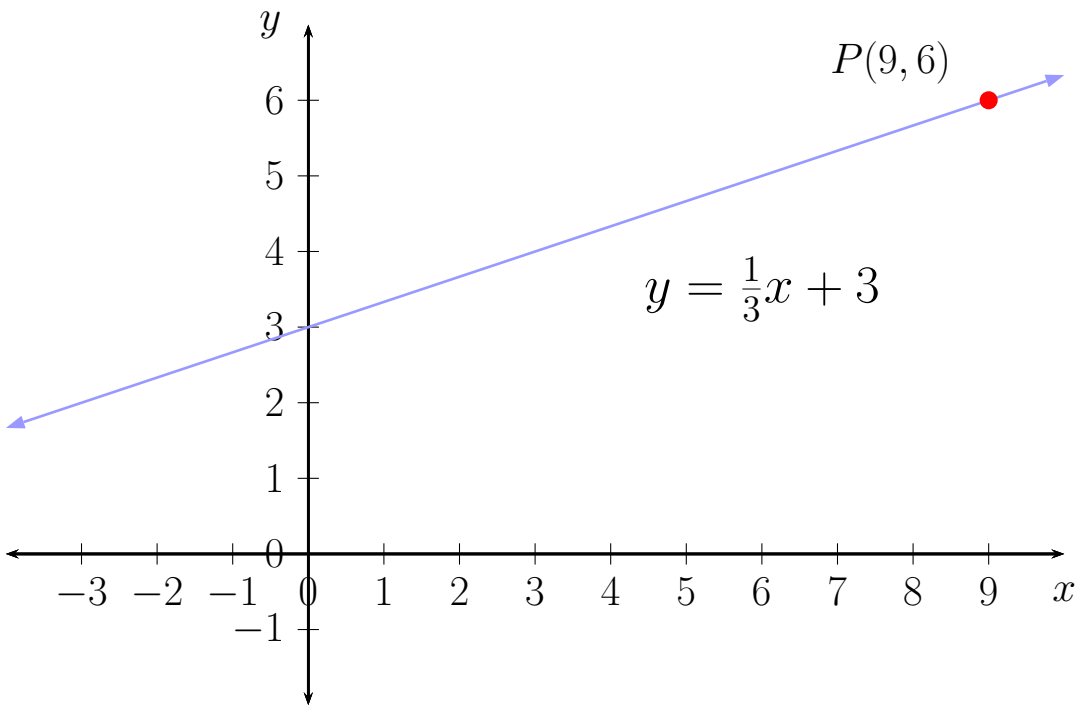

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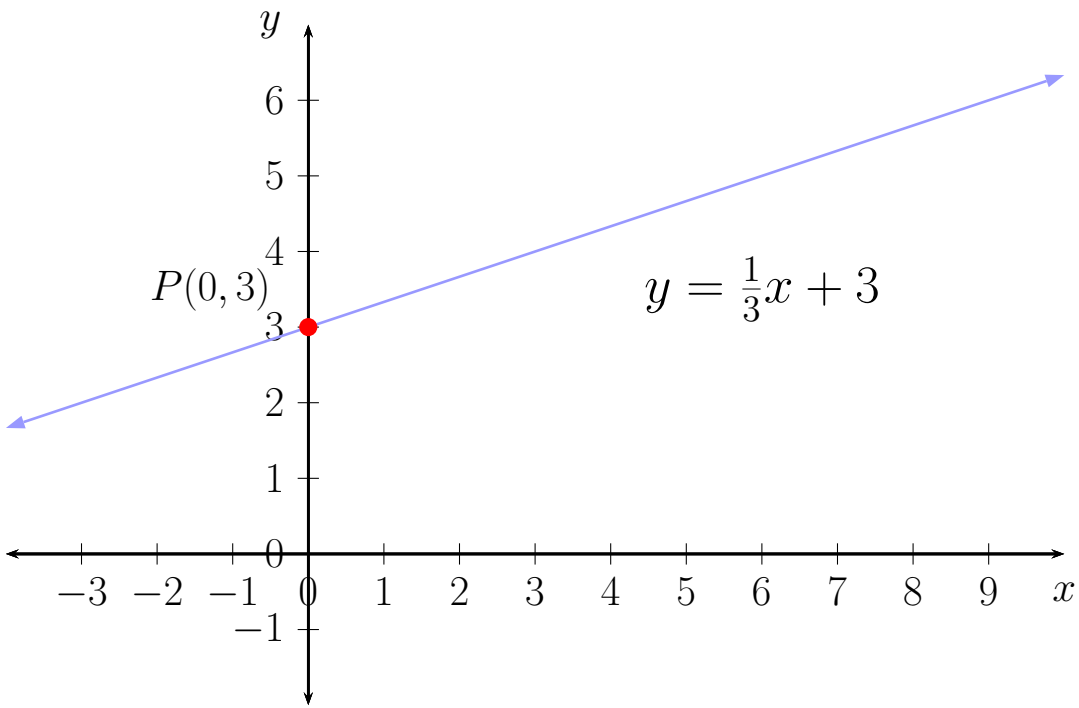

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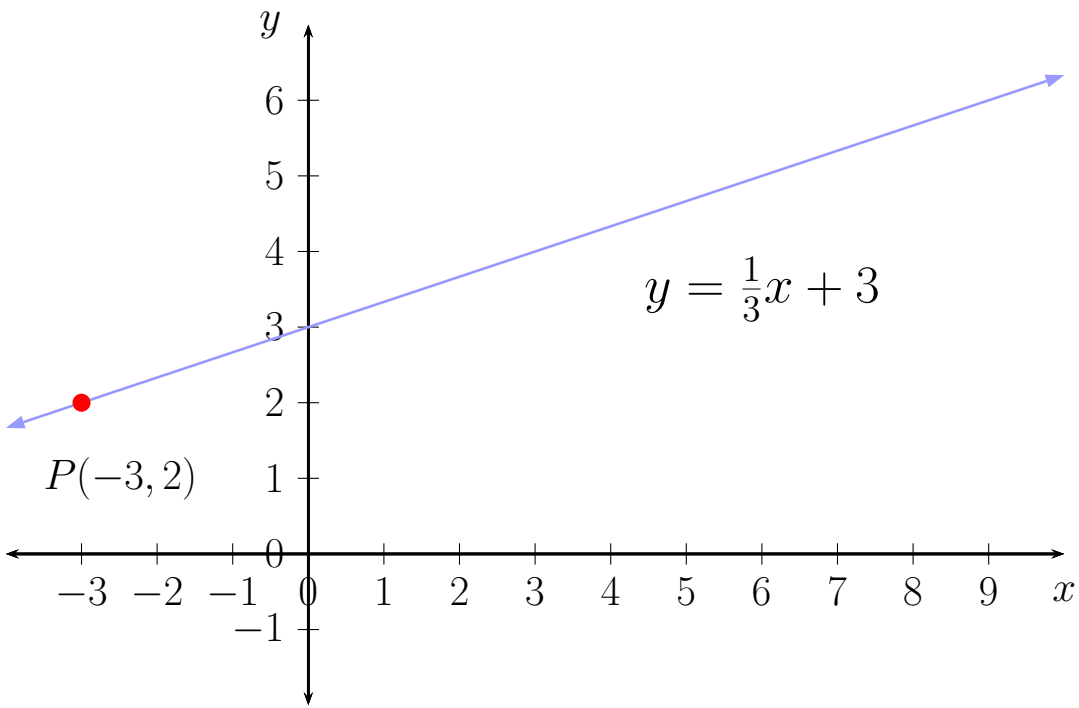

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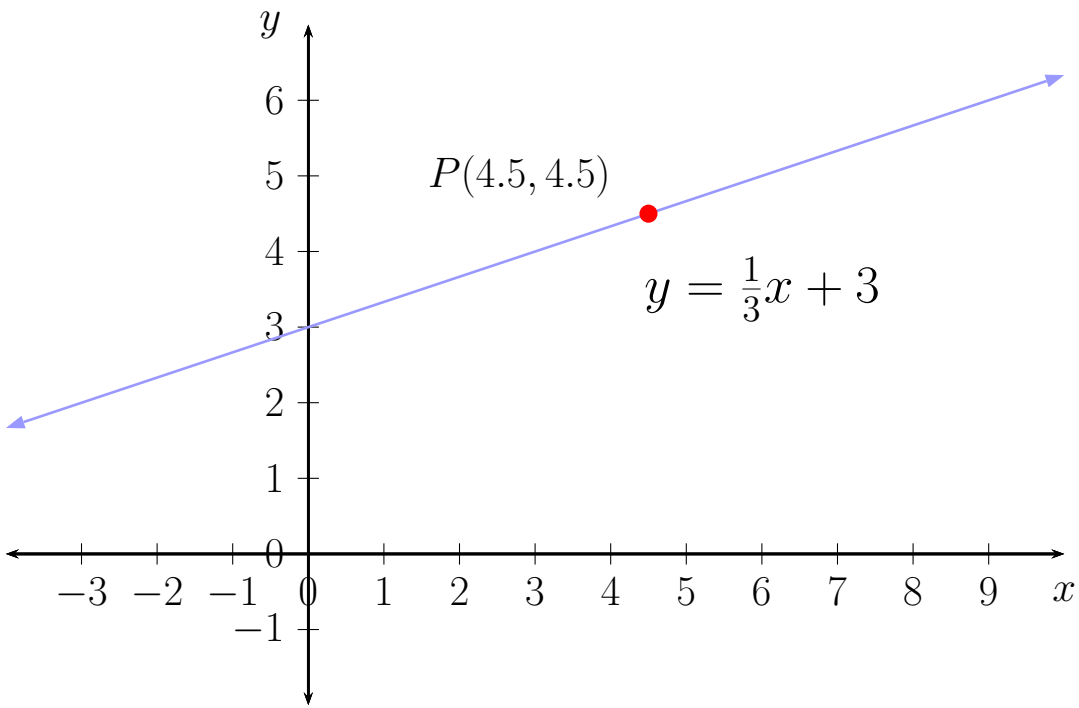

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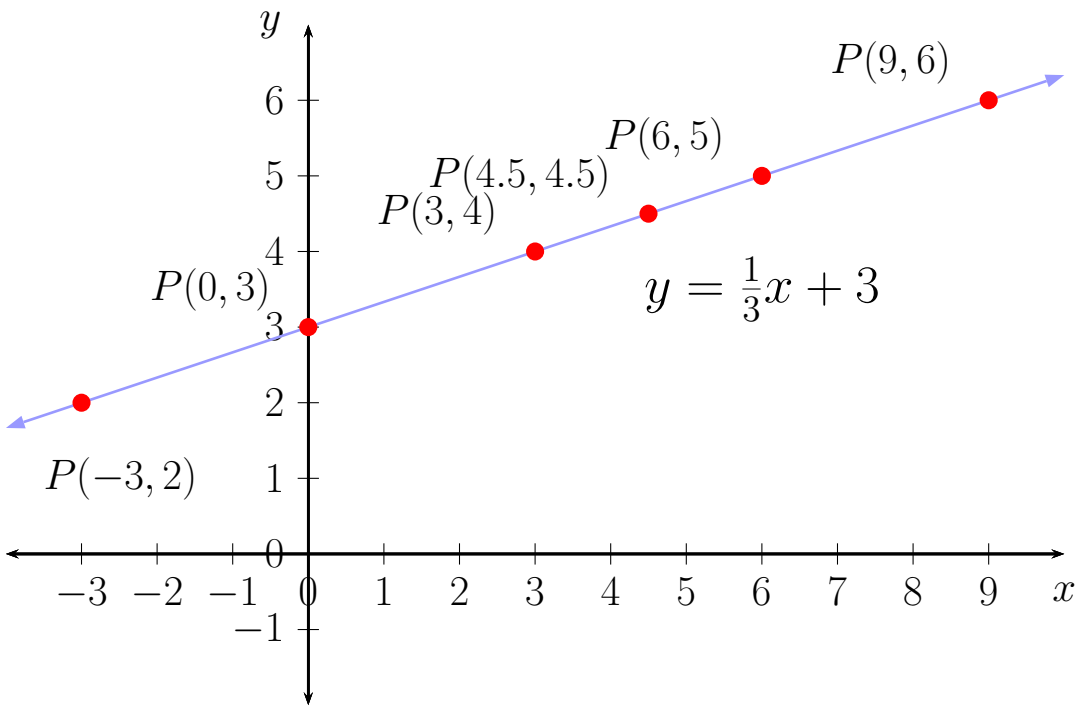
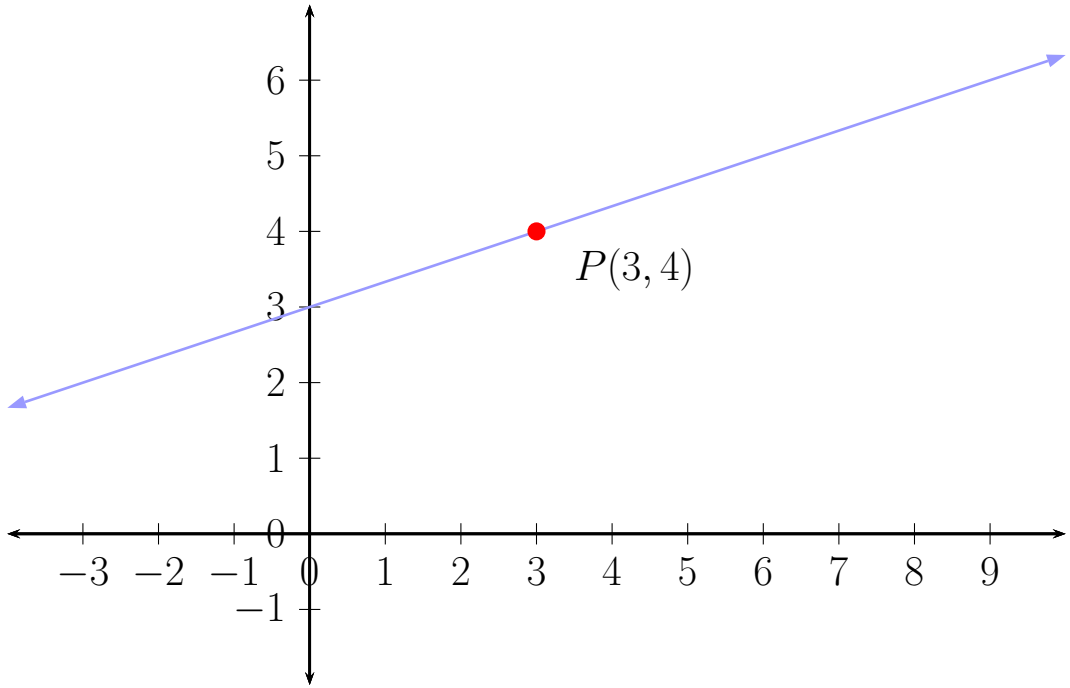


Lines in two dimensions

Now let's describe this line using vectors.



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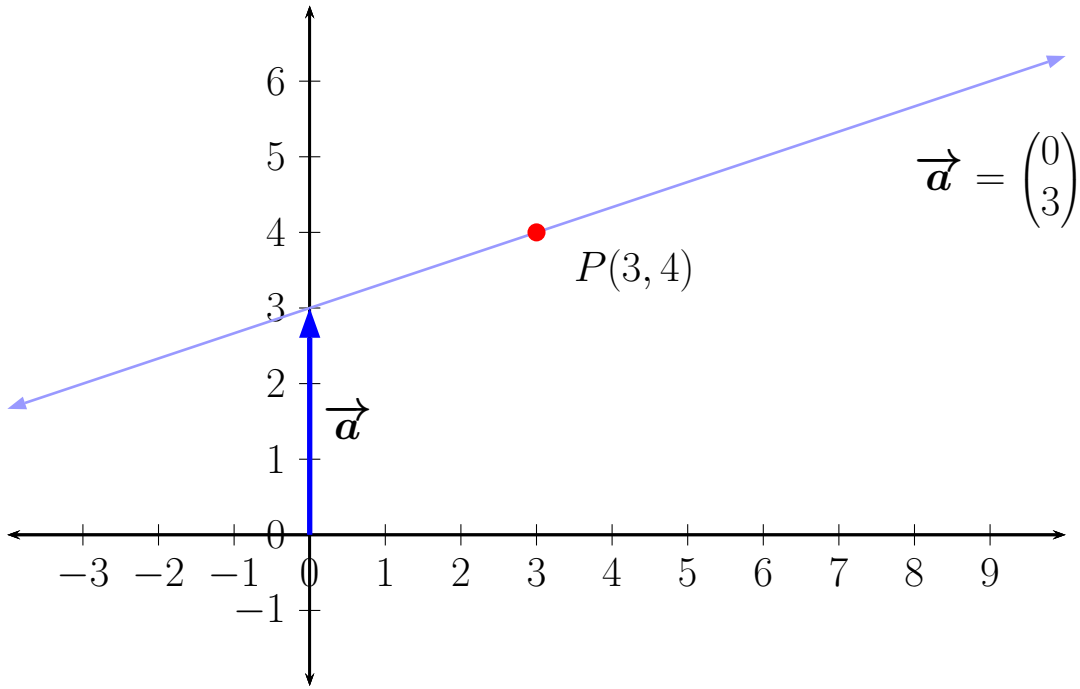

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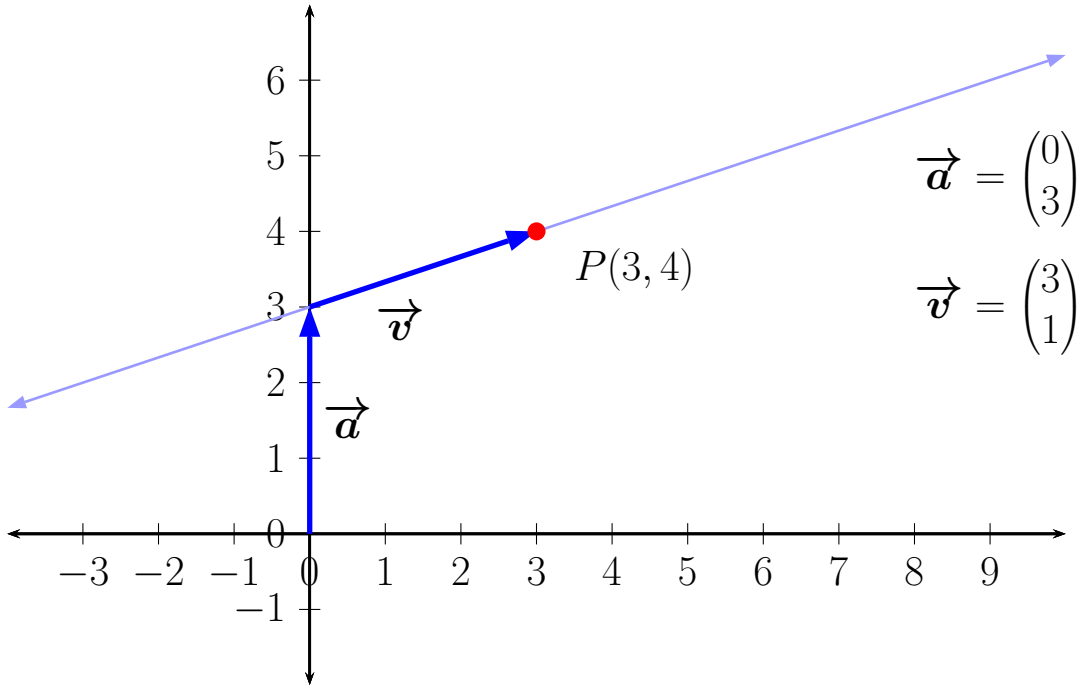

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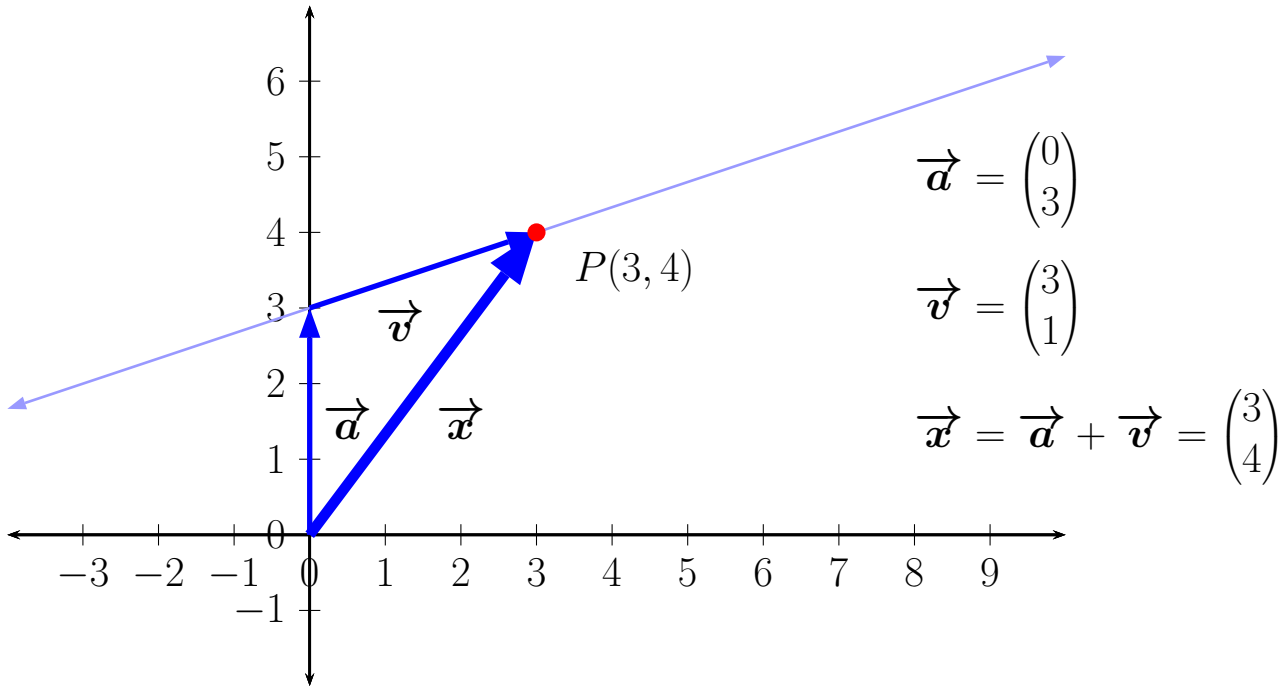

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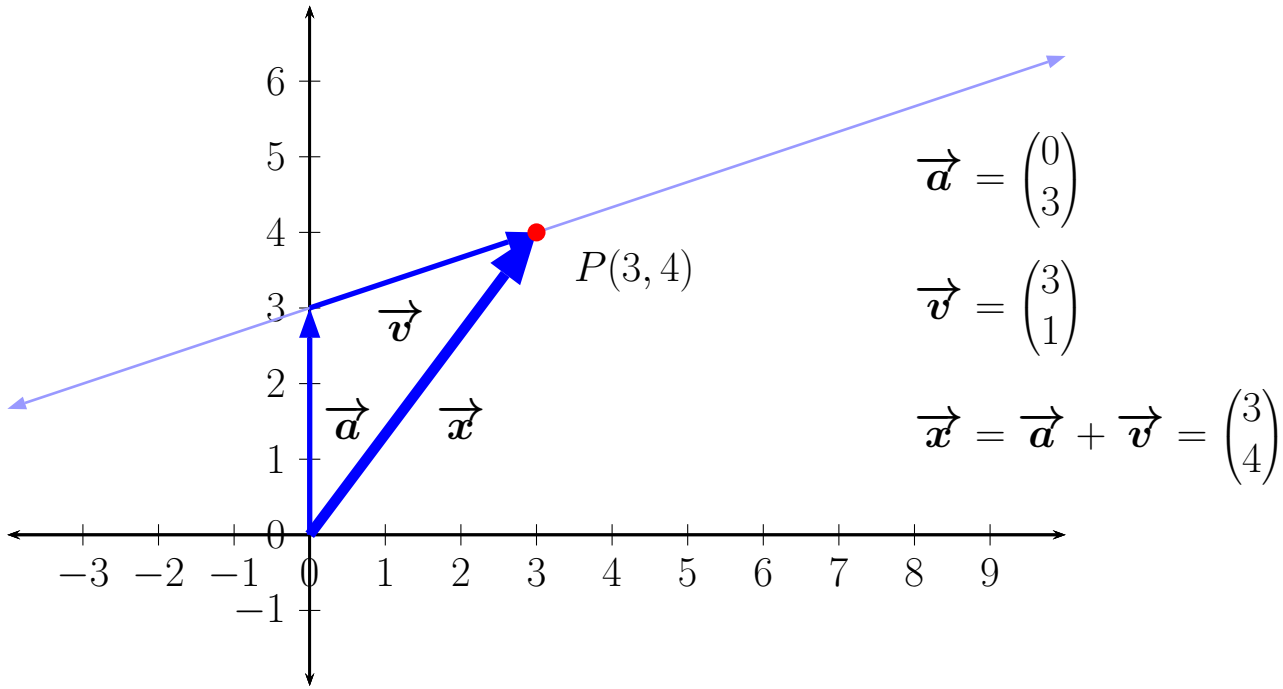

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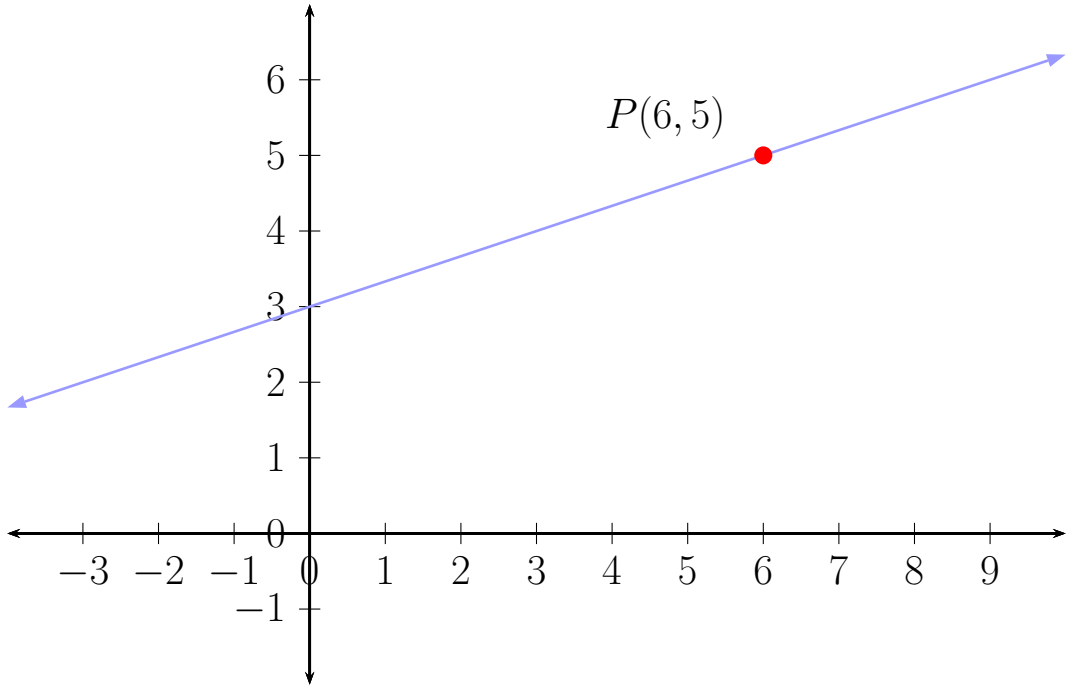

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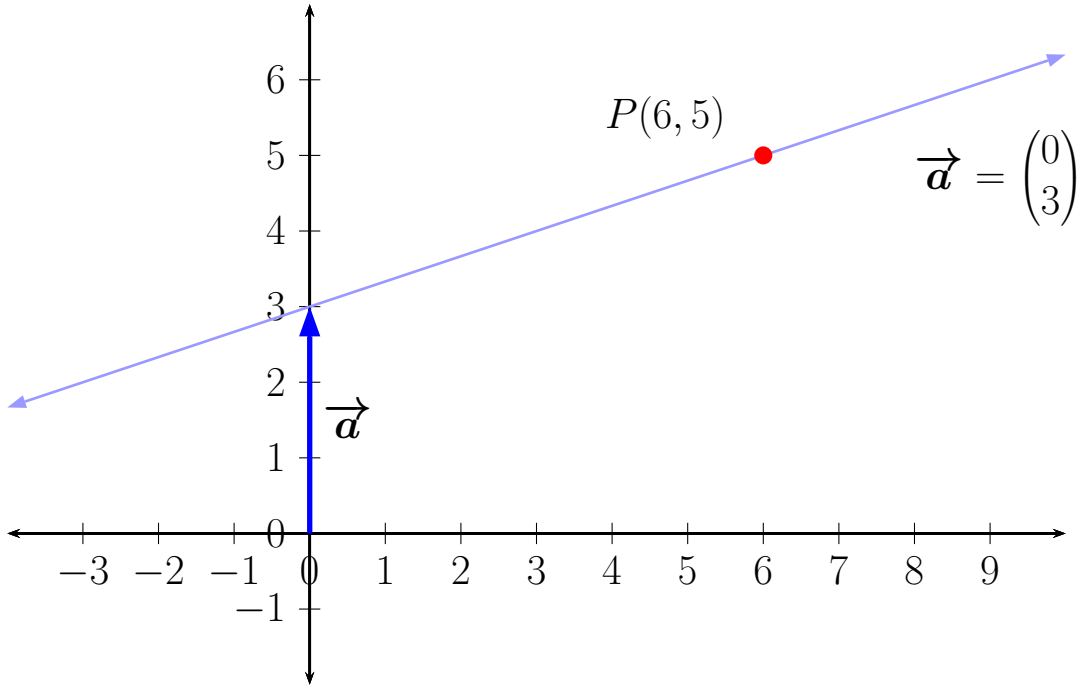

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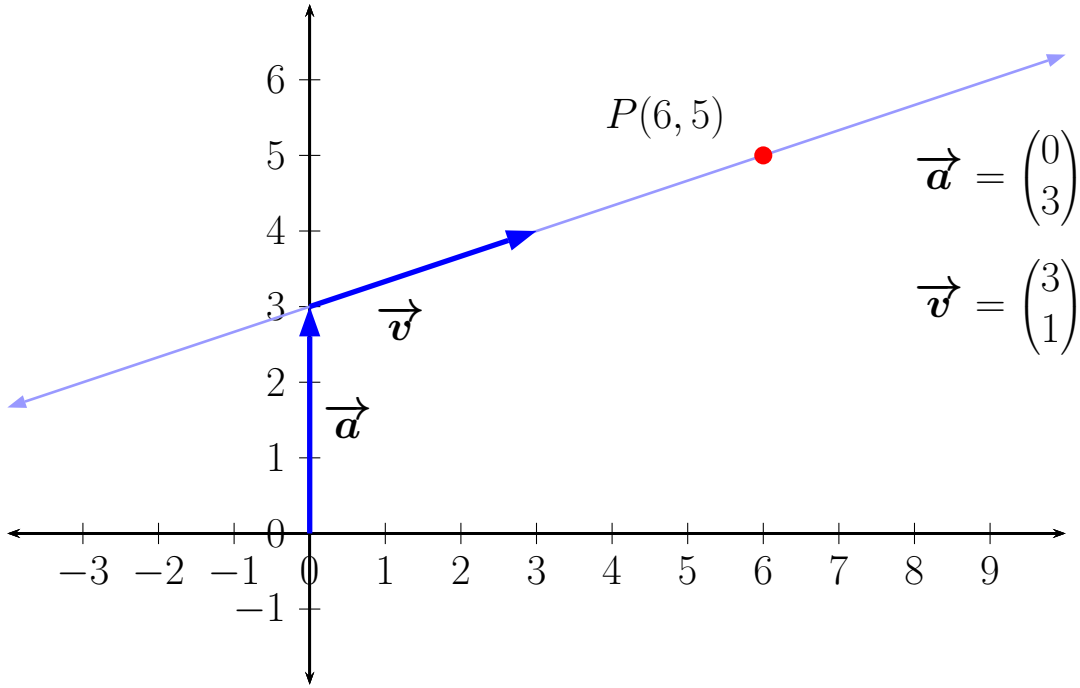

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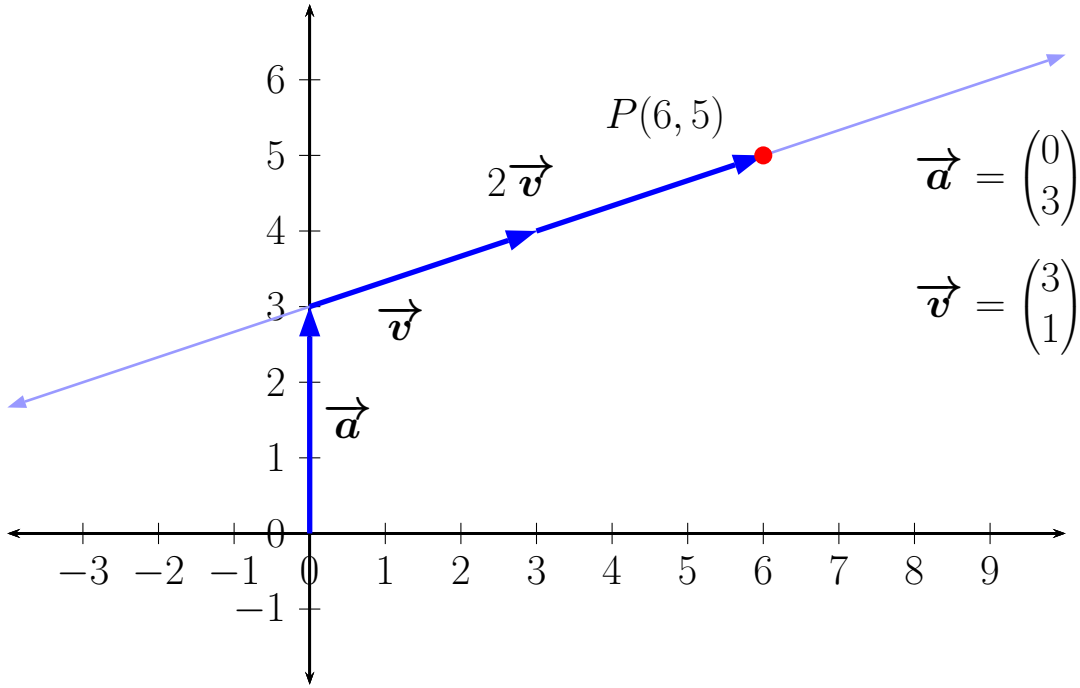

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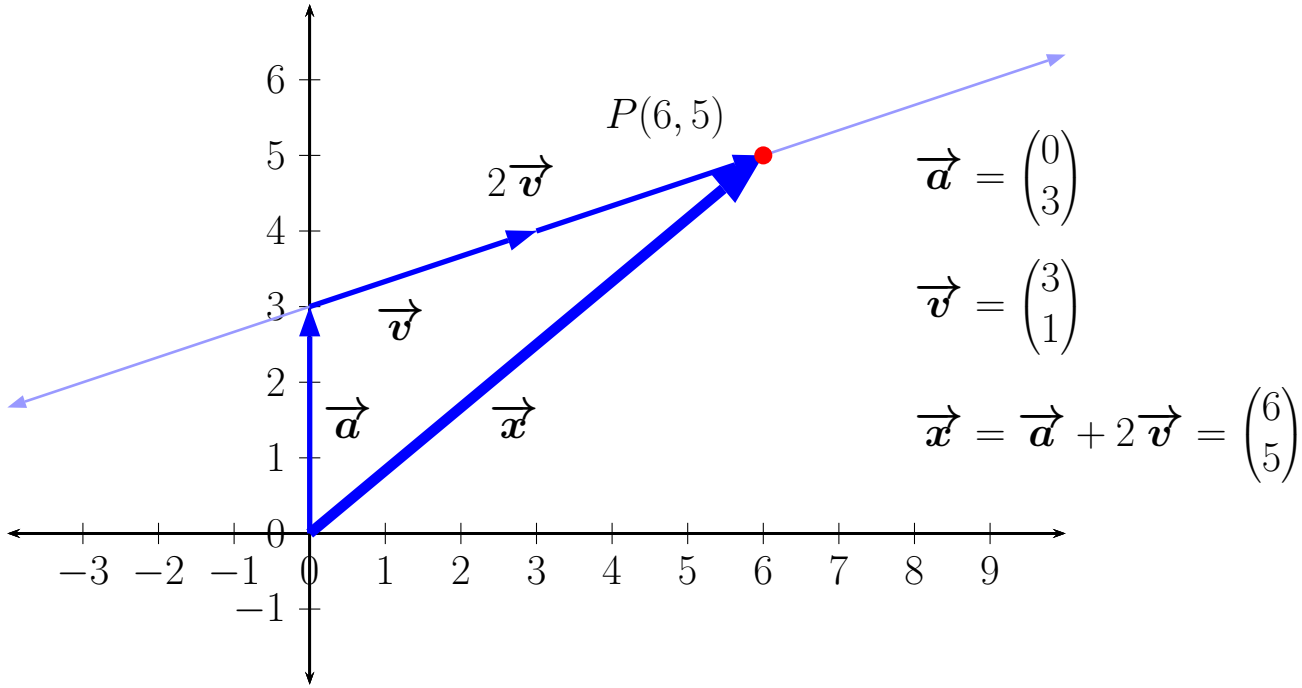

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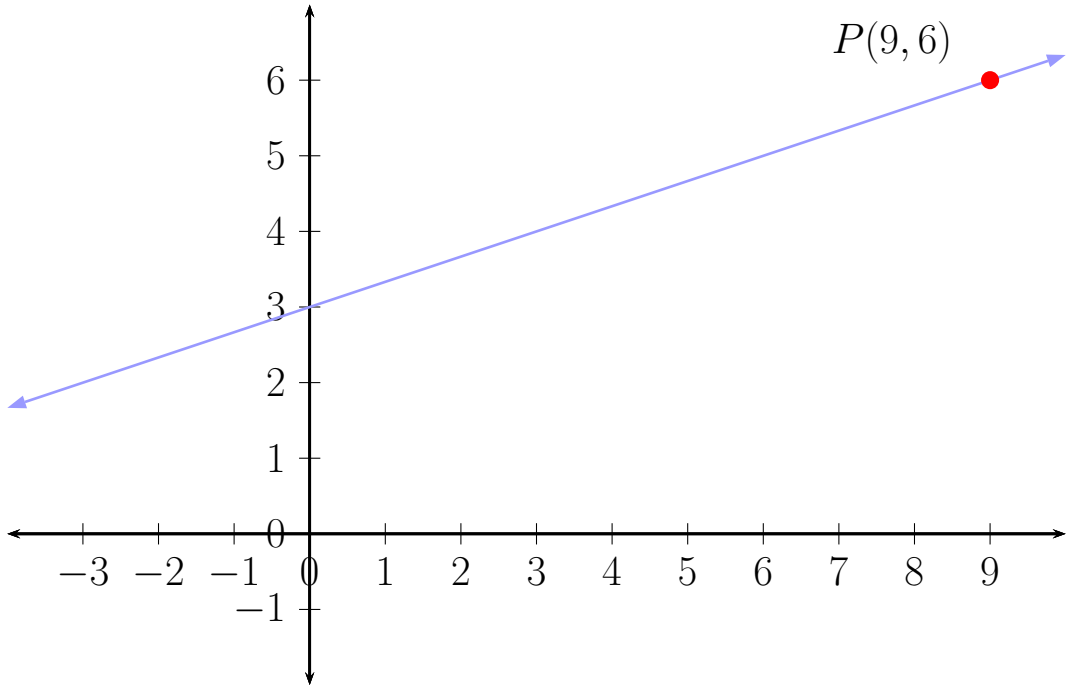

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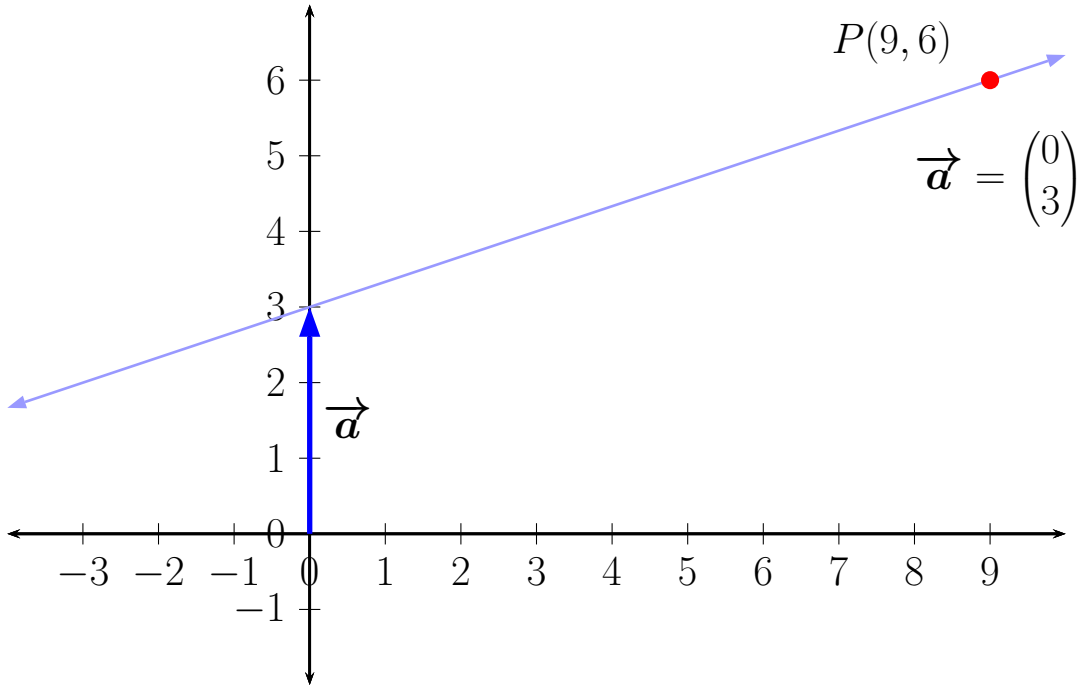

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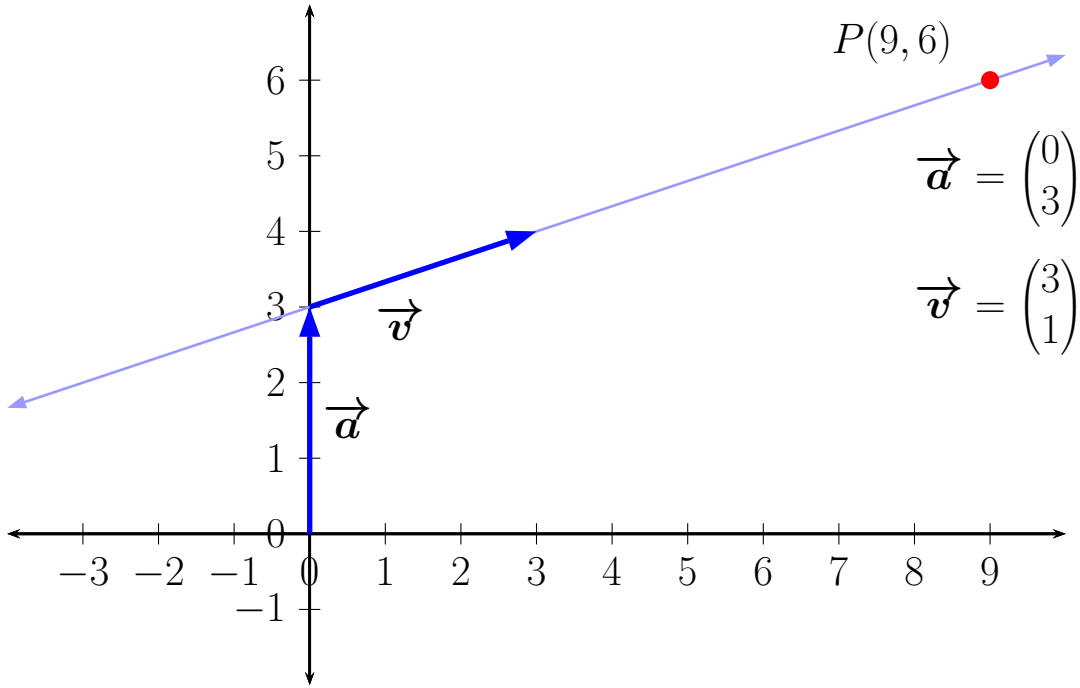

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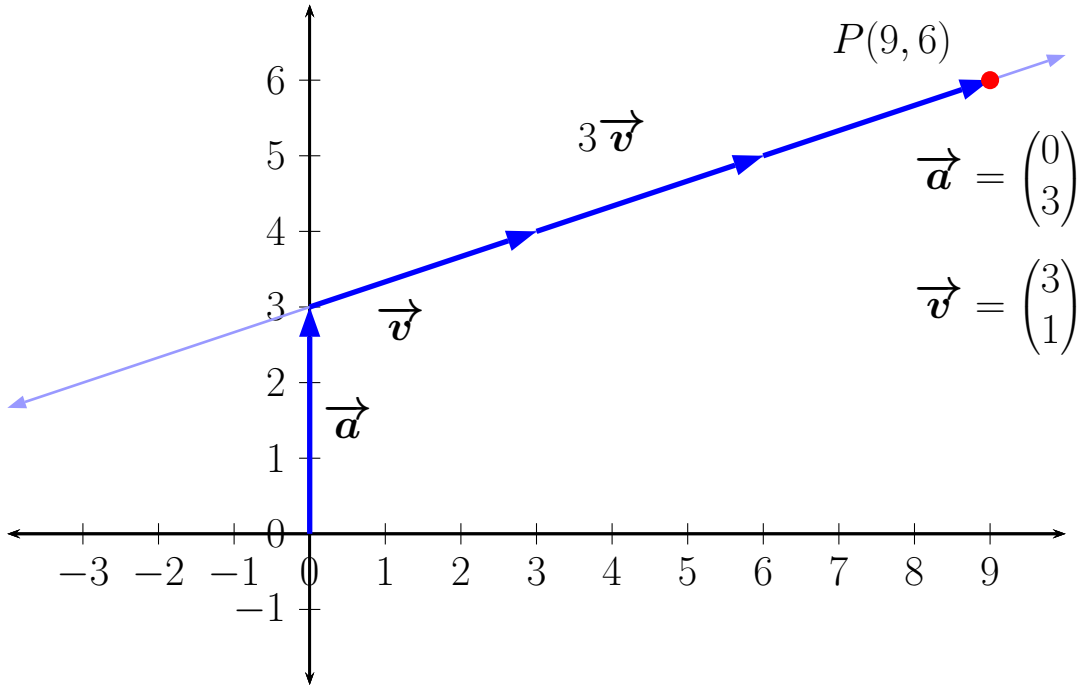

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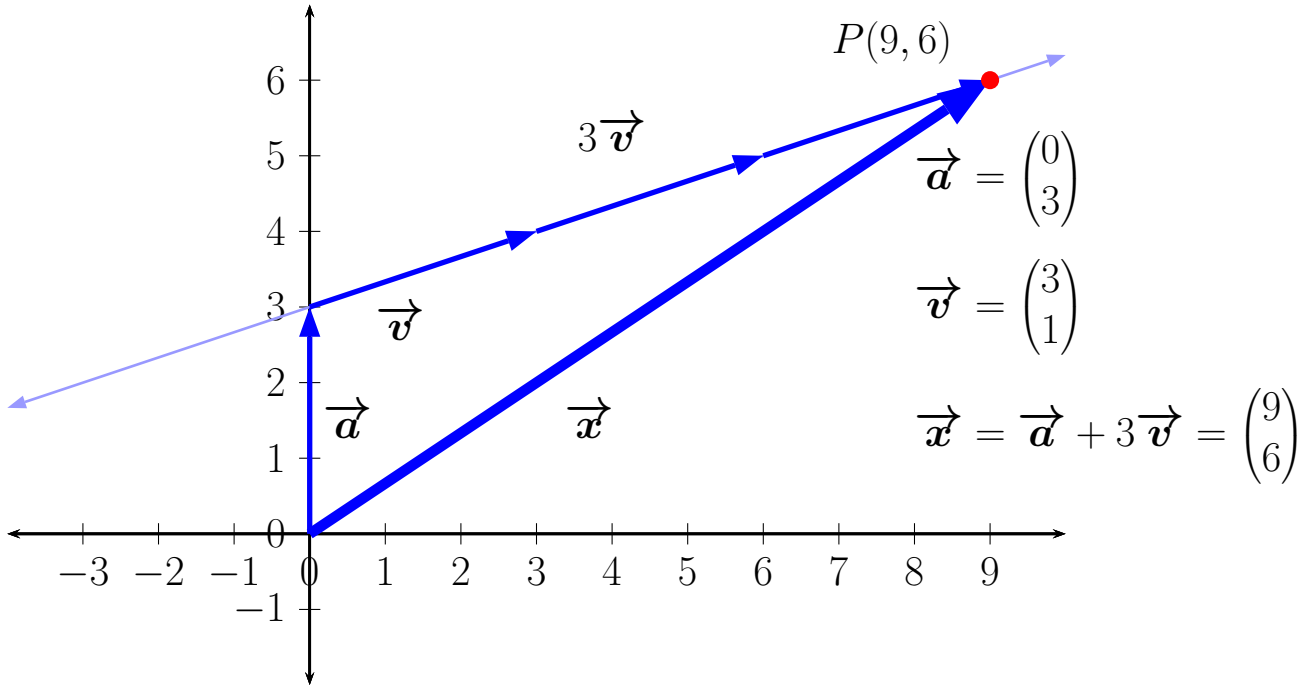

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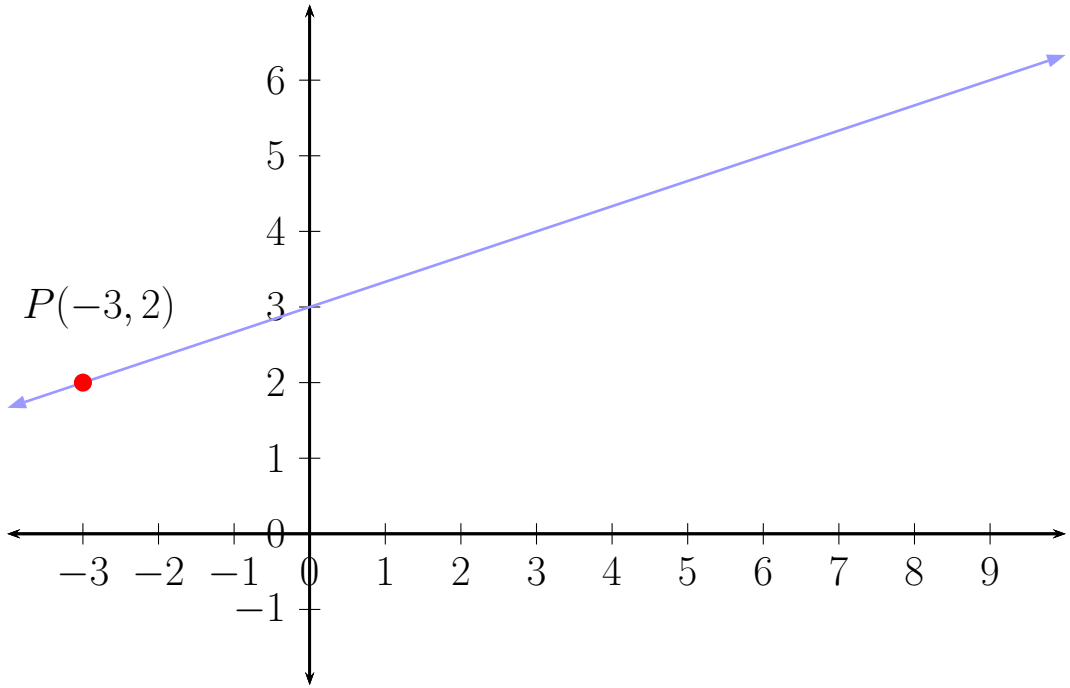

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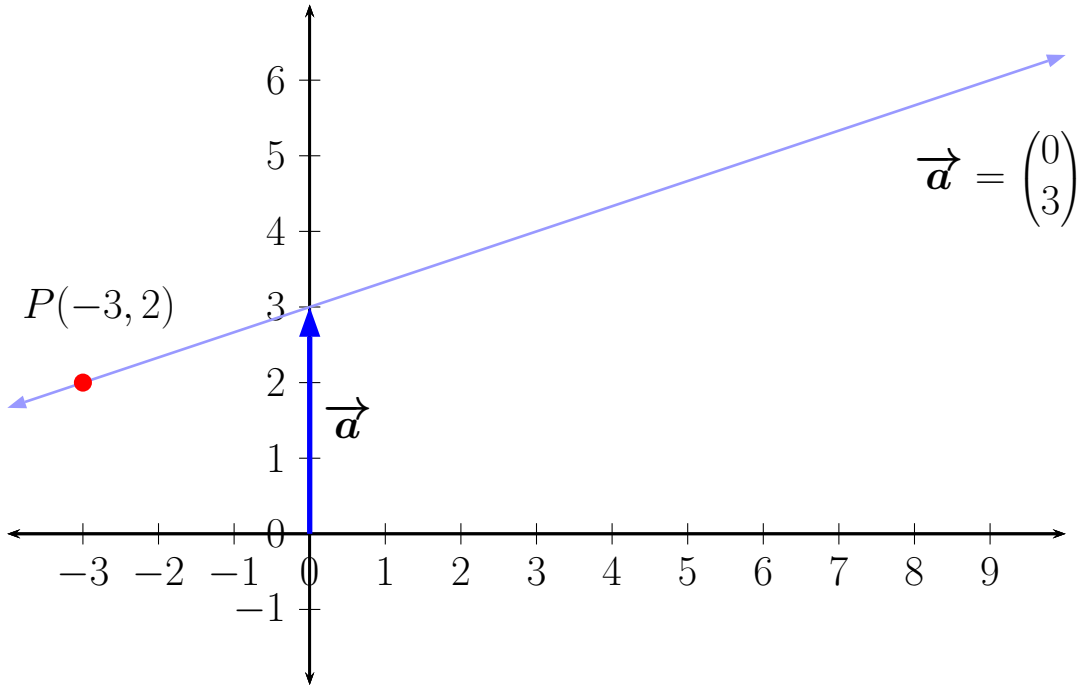

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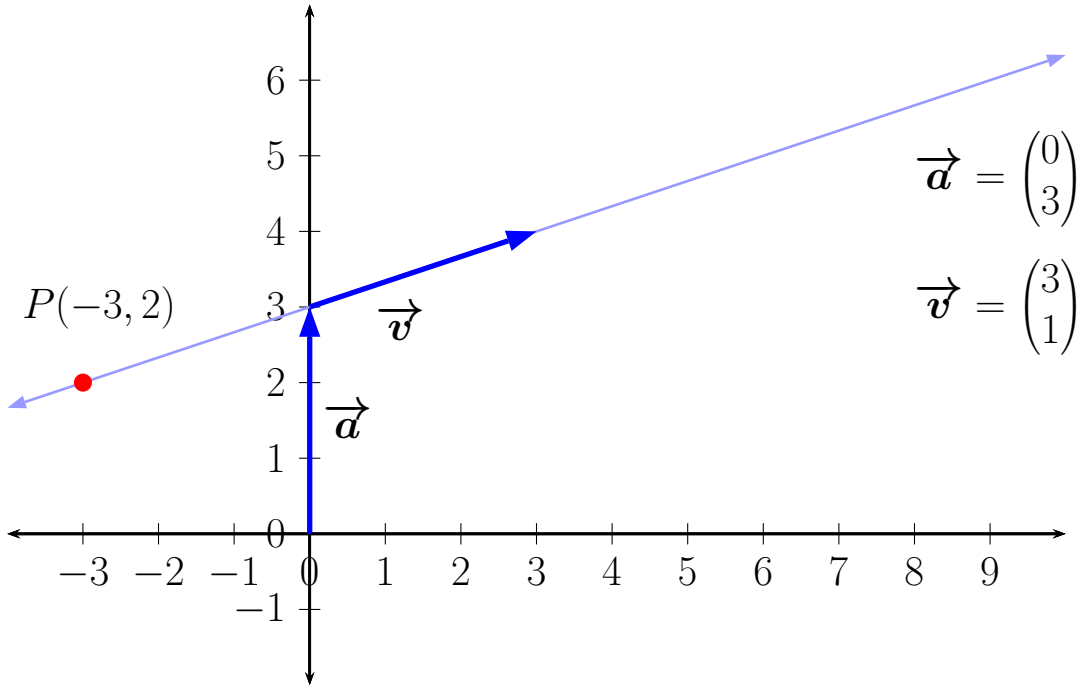

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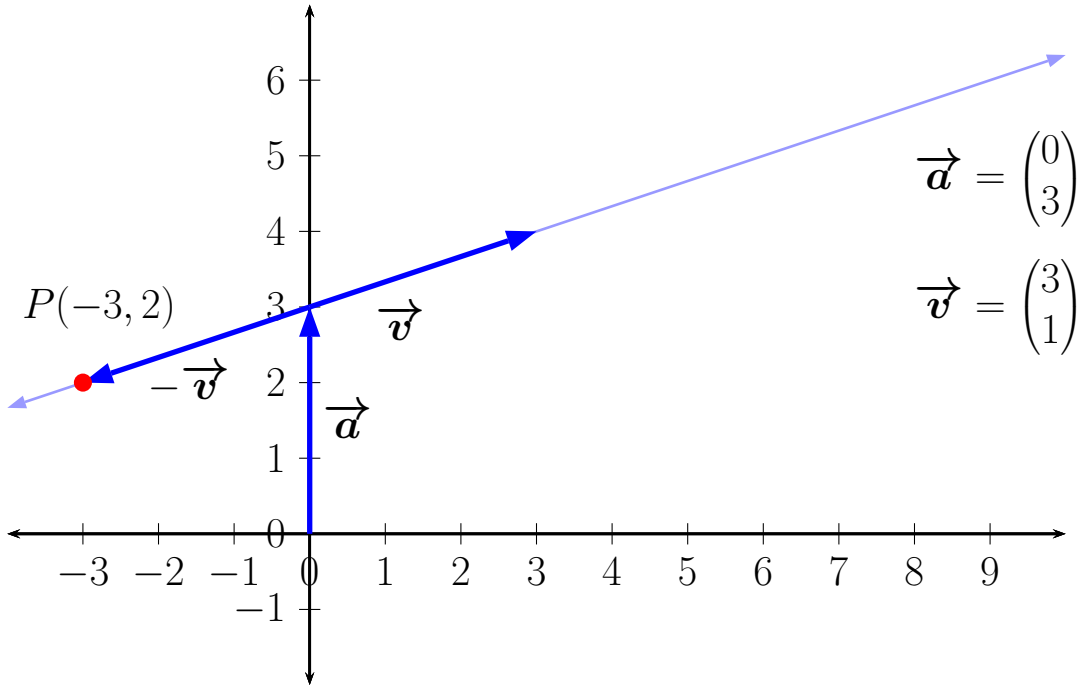

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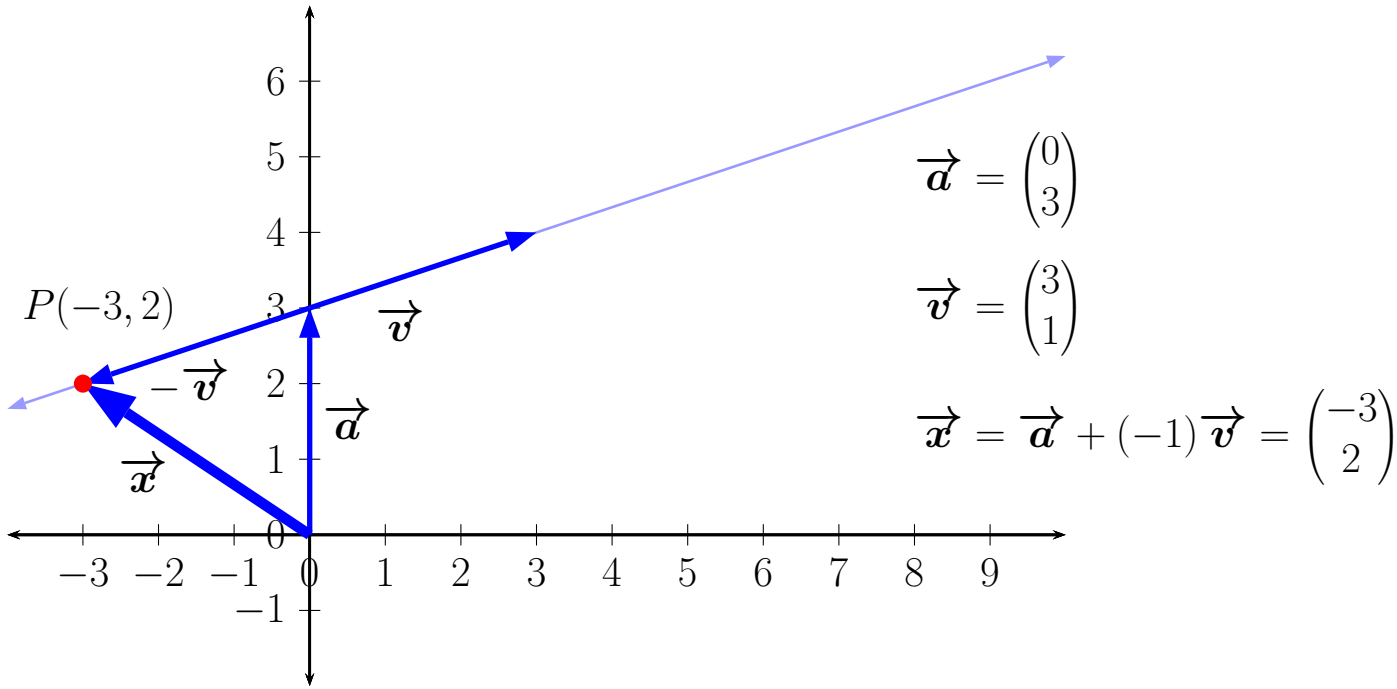

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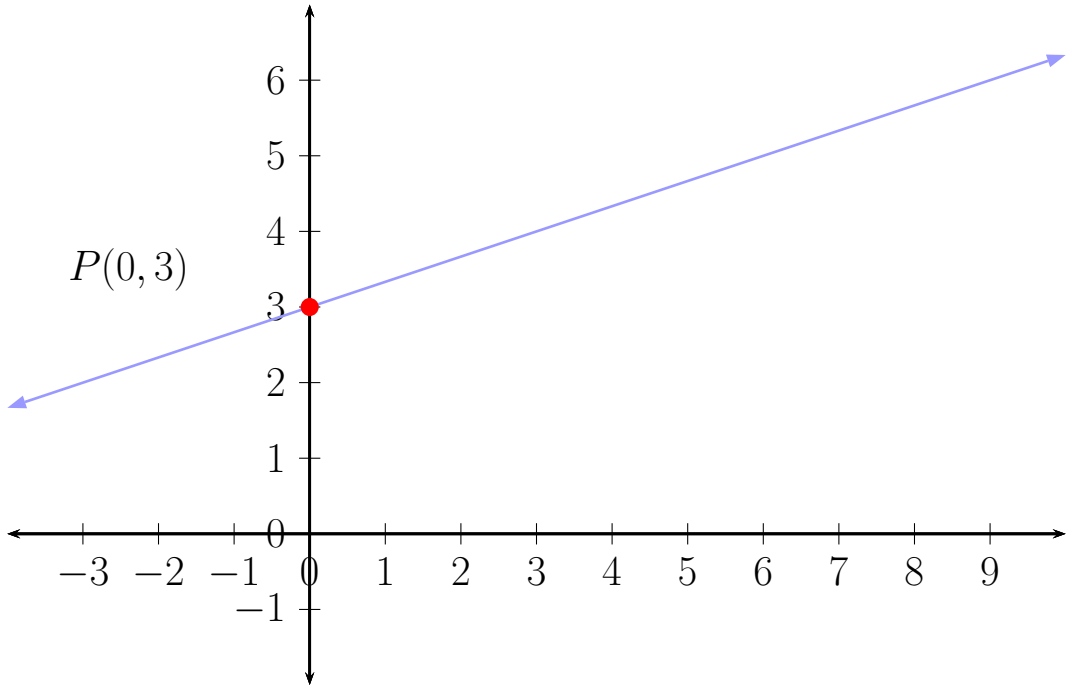

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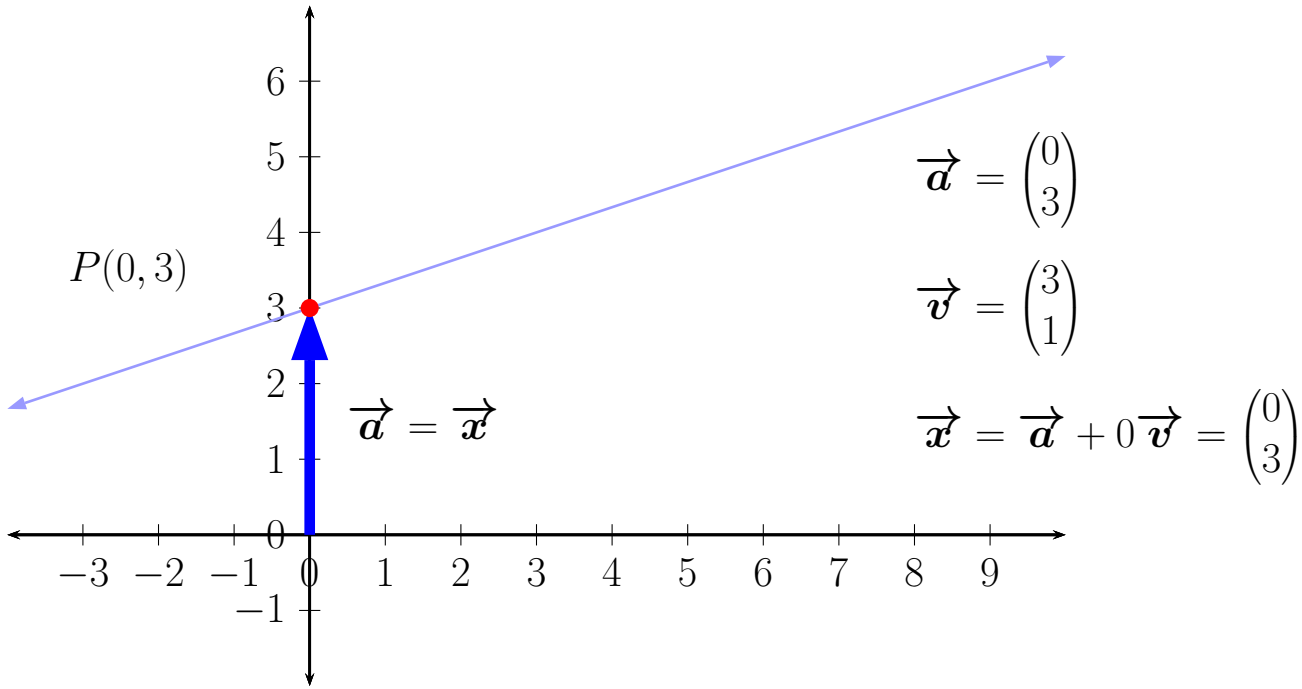

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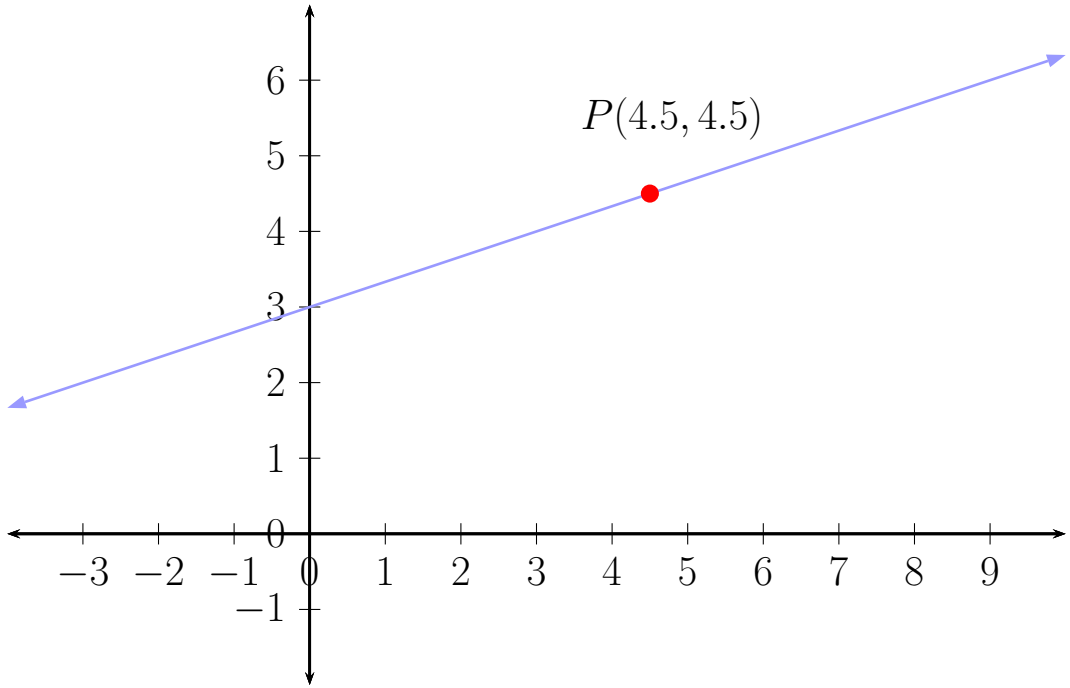

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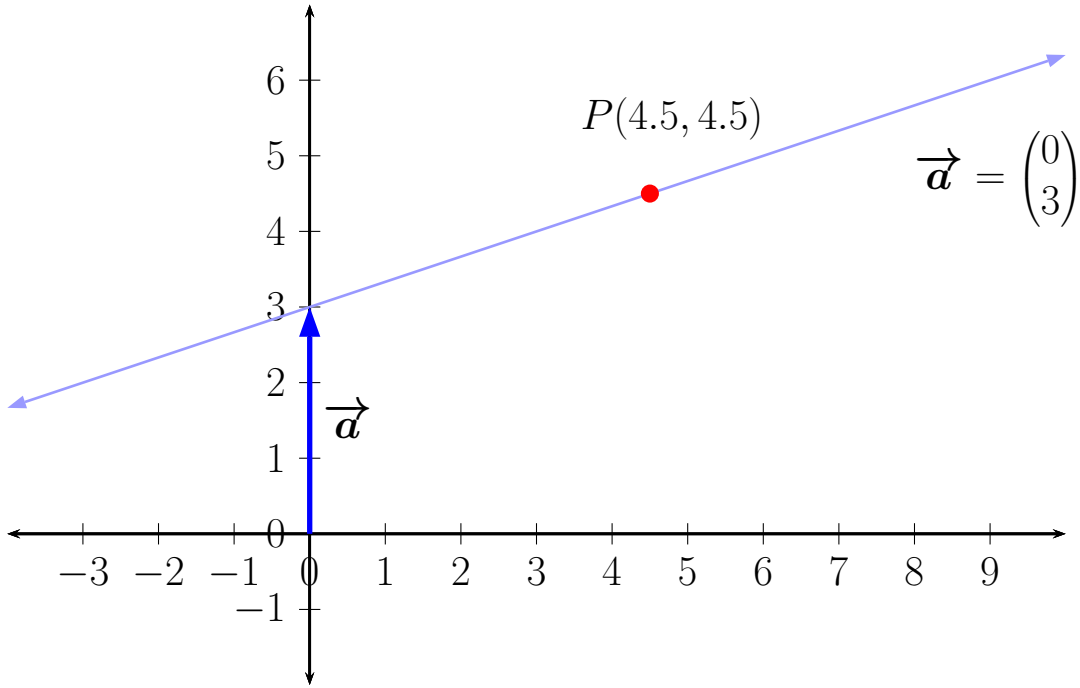

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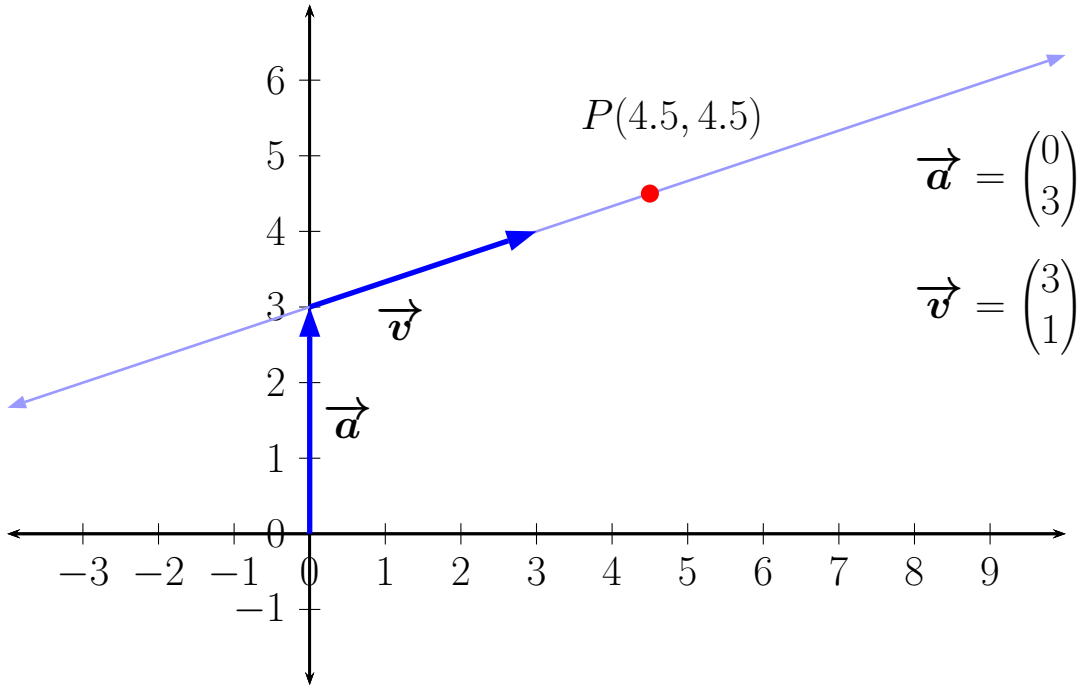

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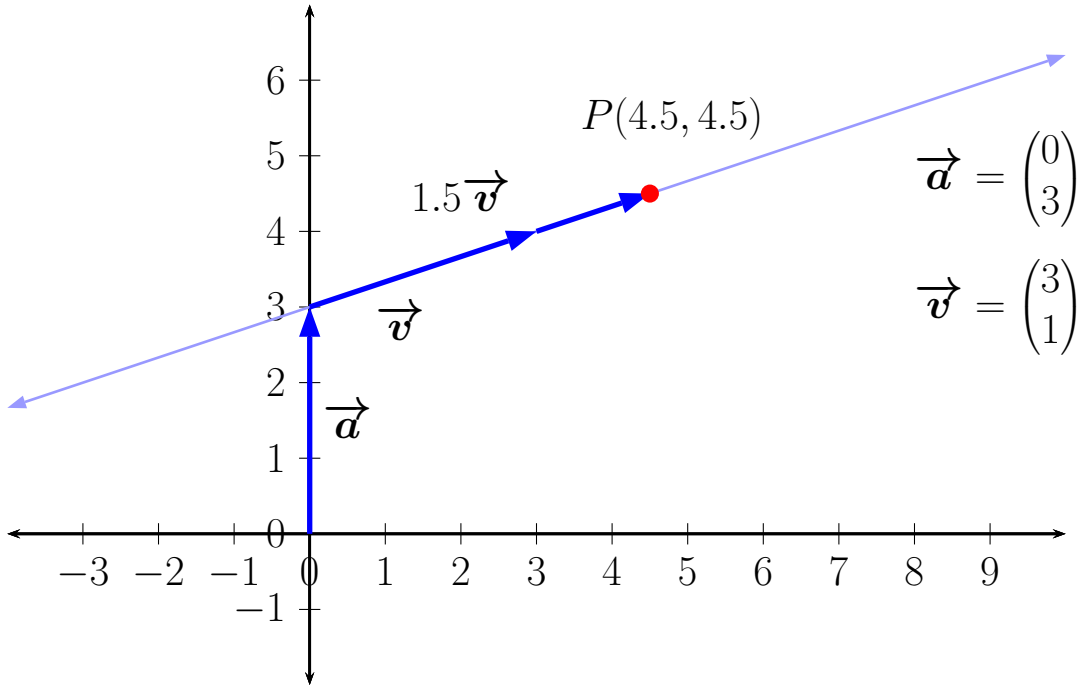

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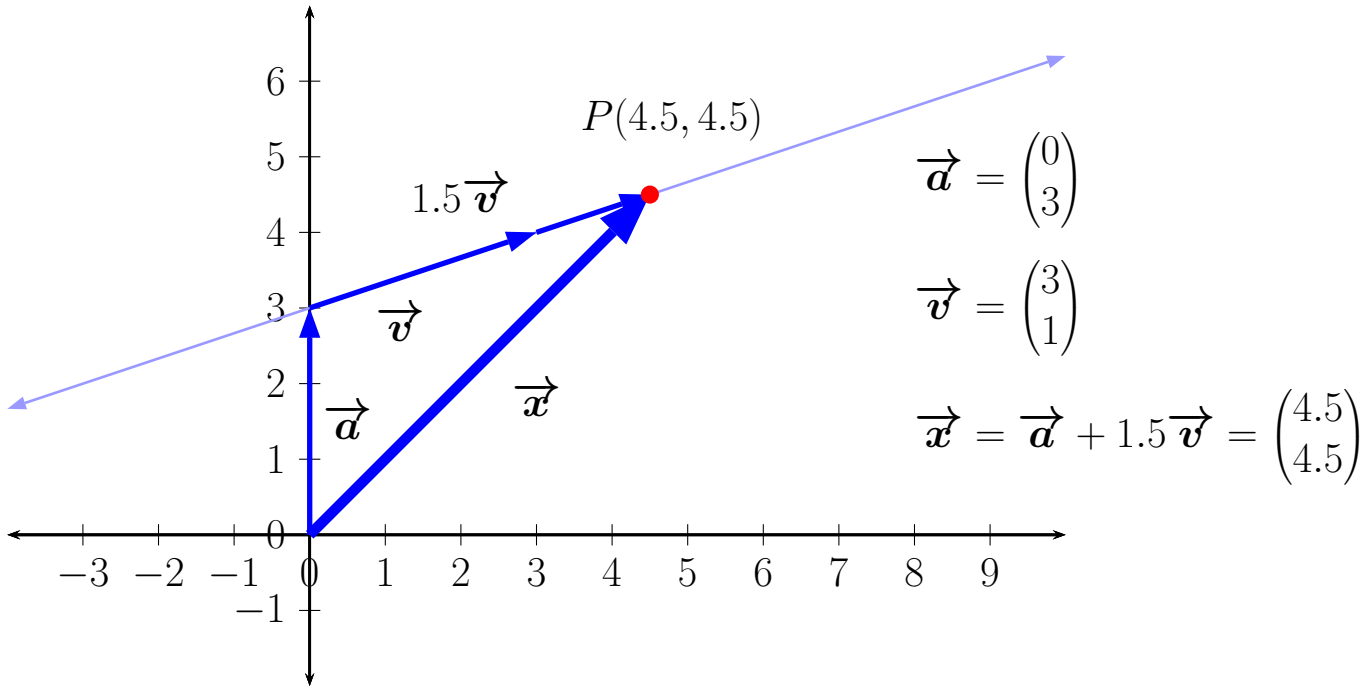

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Lines in two dimensions

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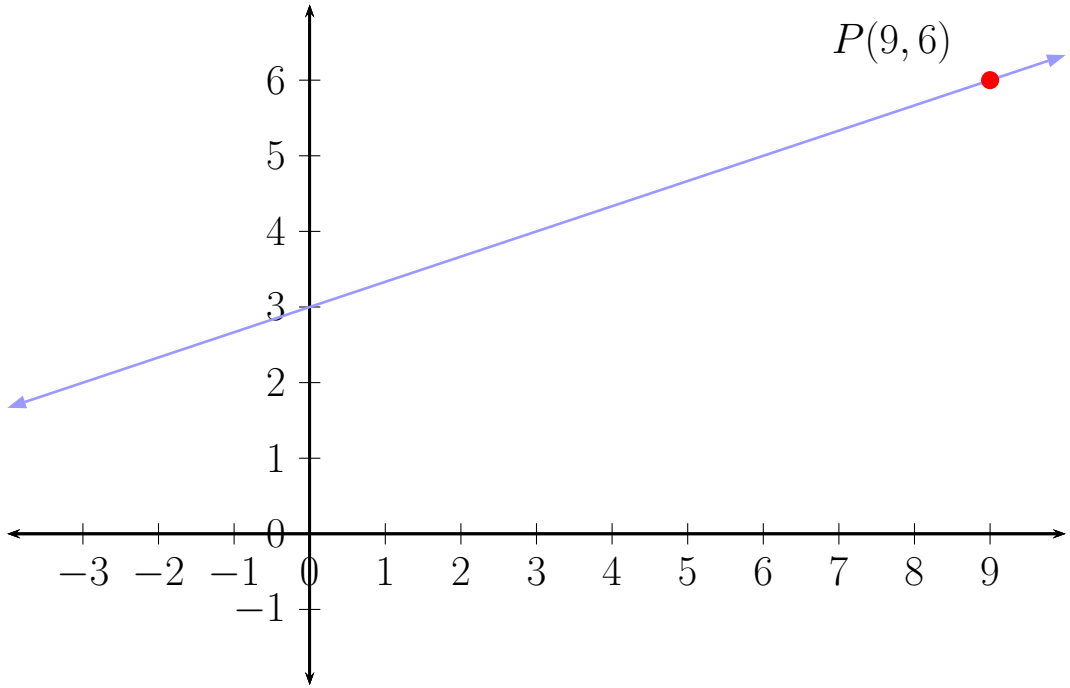

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Lines in two dimensions

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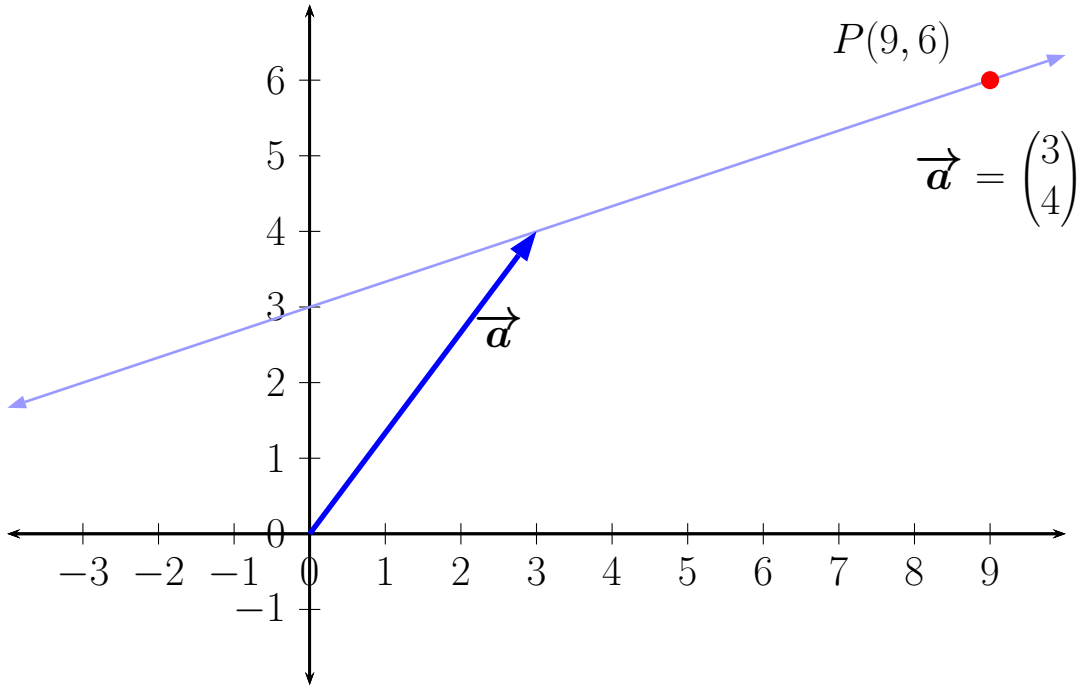

First we pick a vector that takes us to a point on the line.

Then we pick a vector in the direction of the line.

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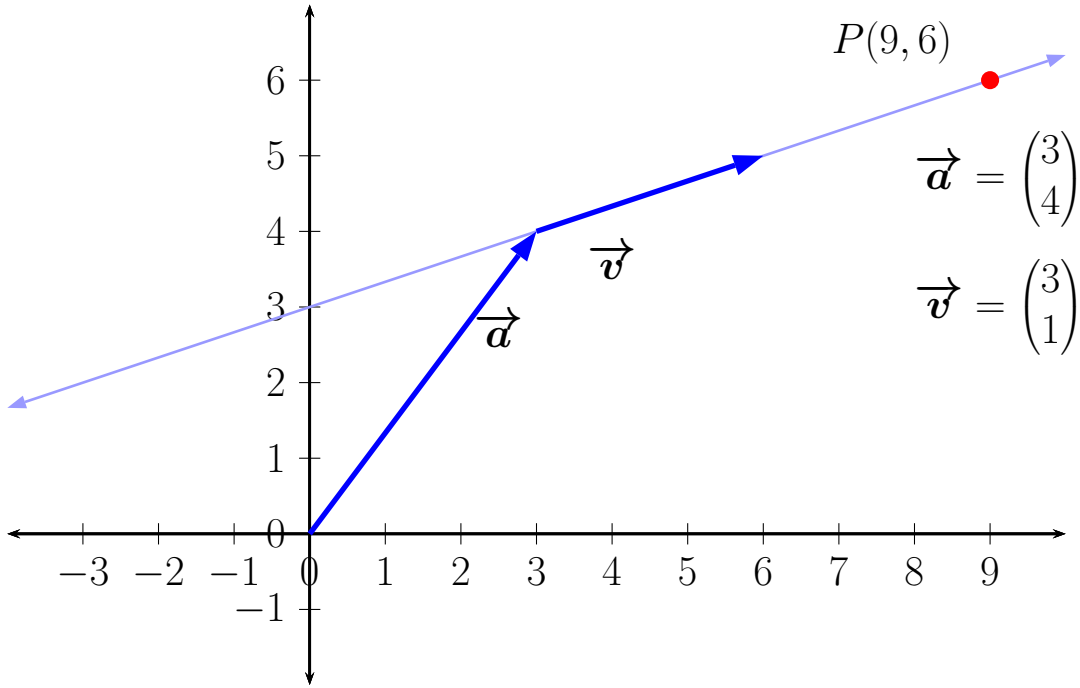

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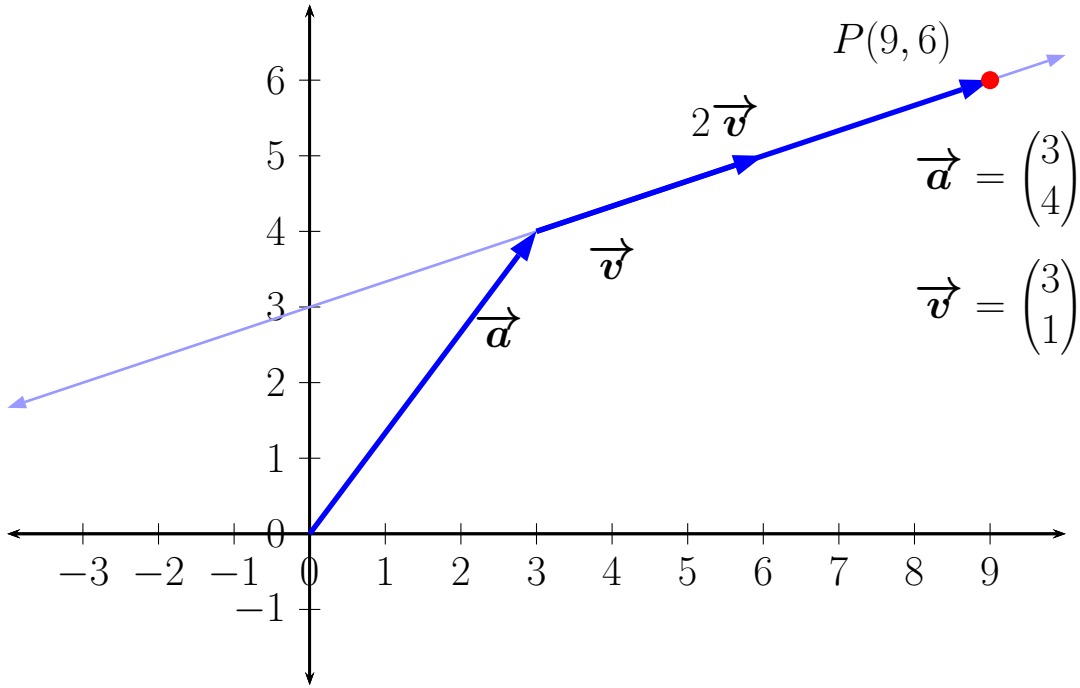

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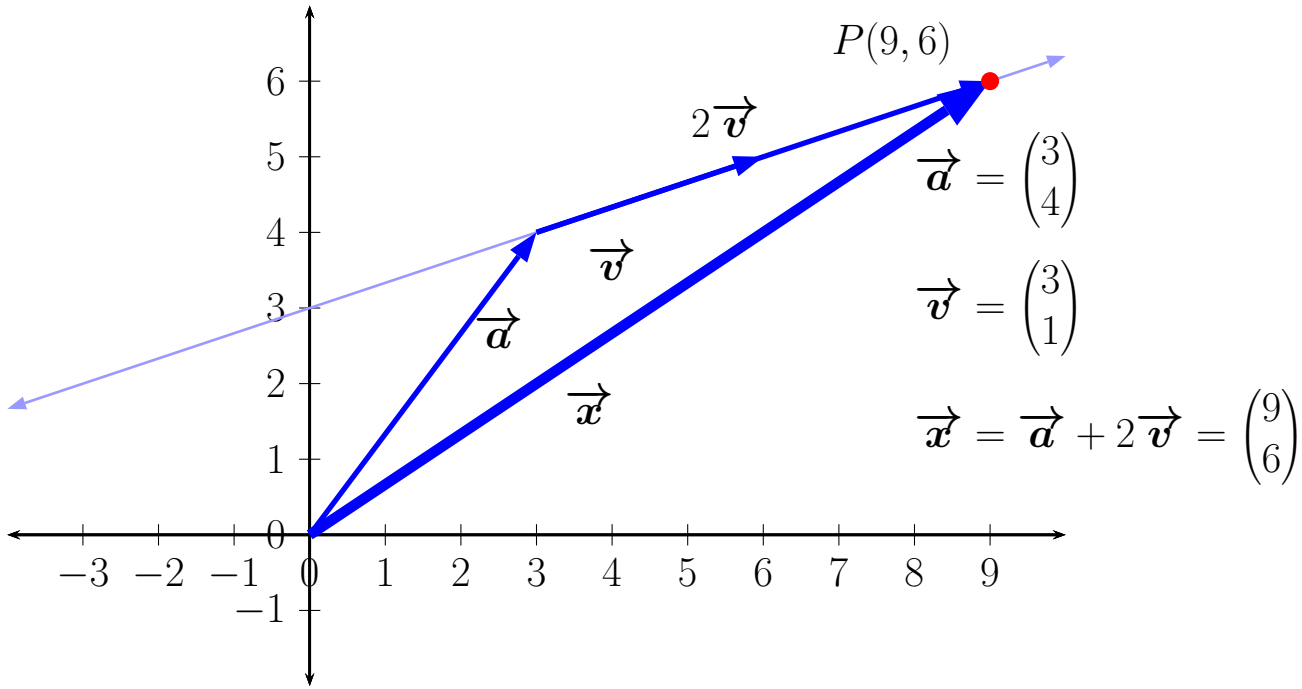

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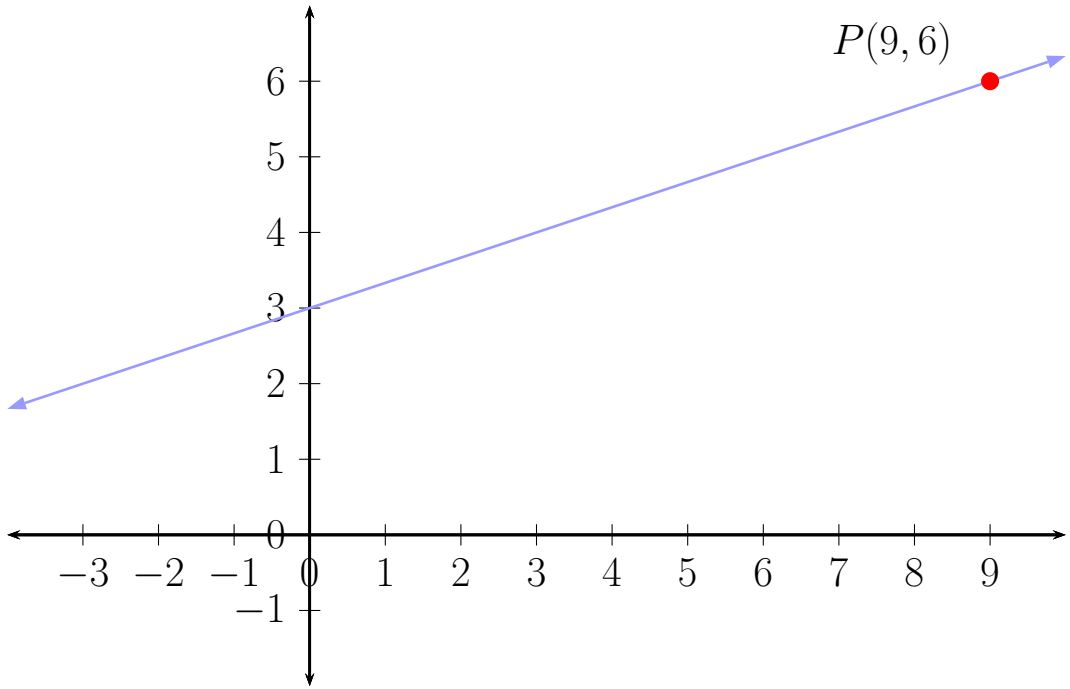

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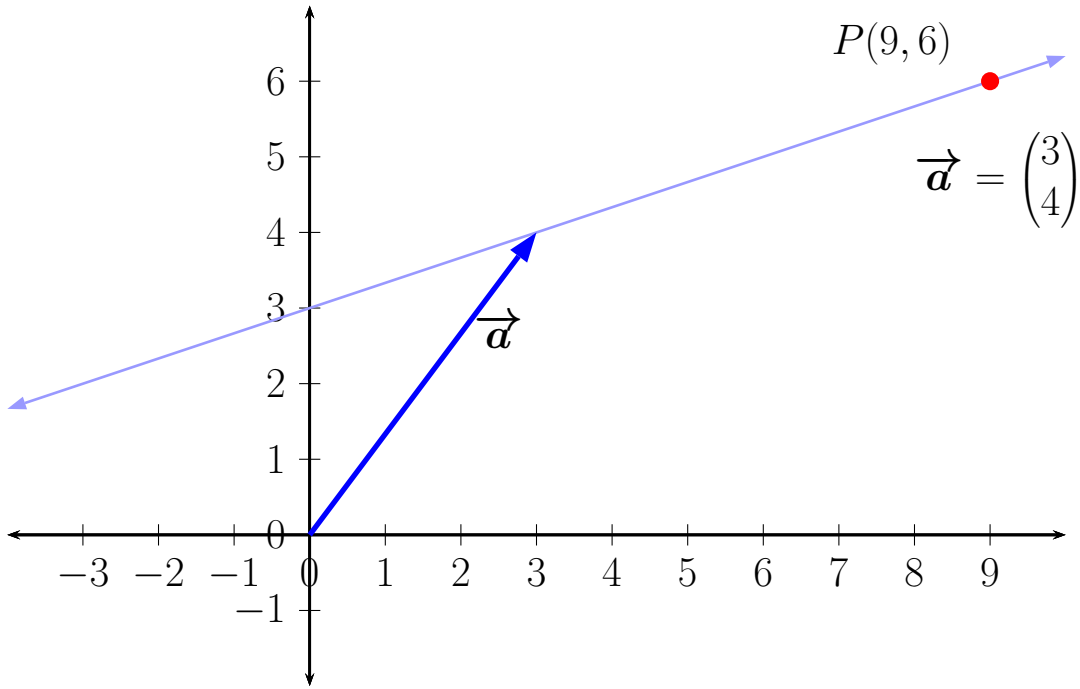

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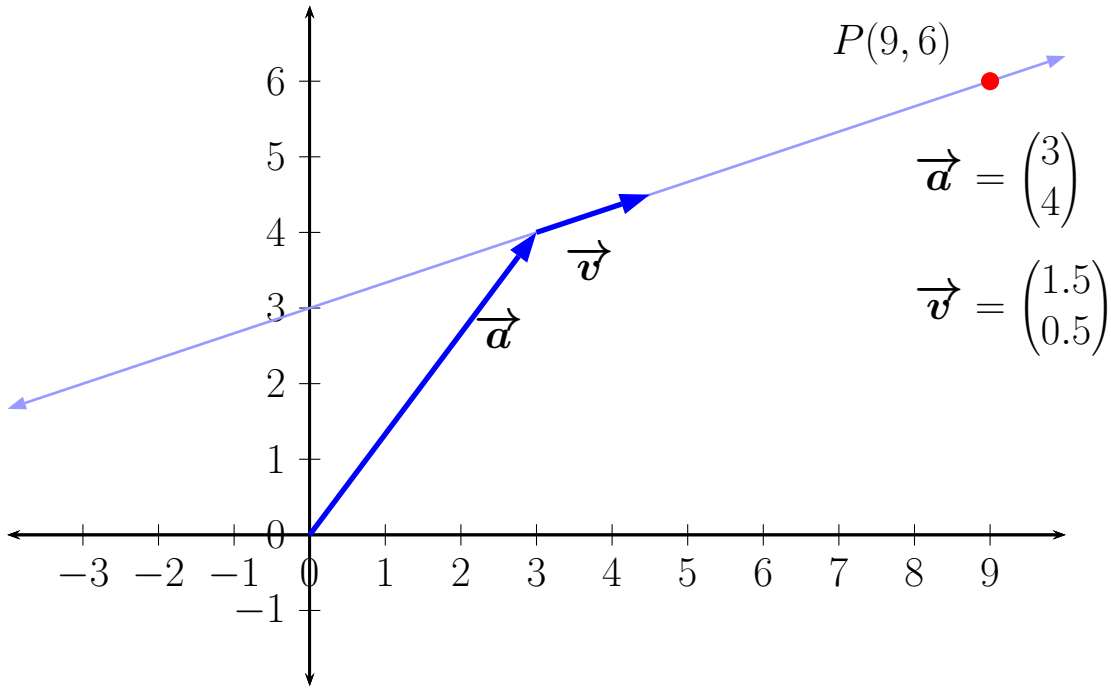

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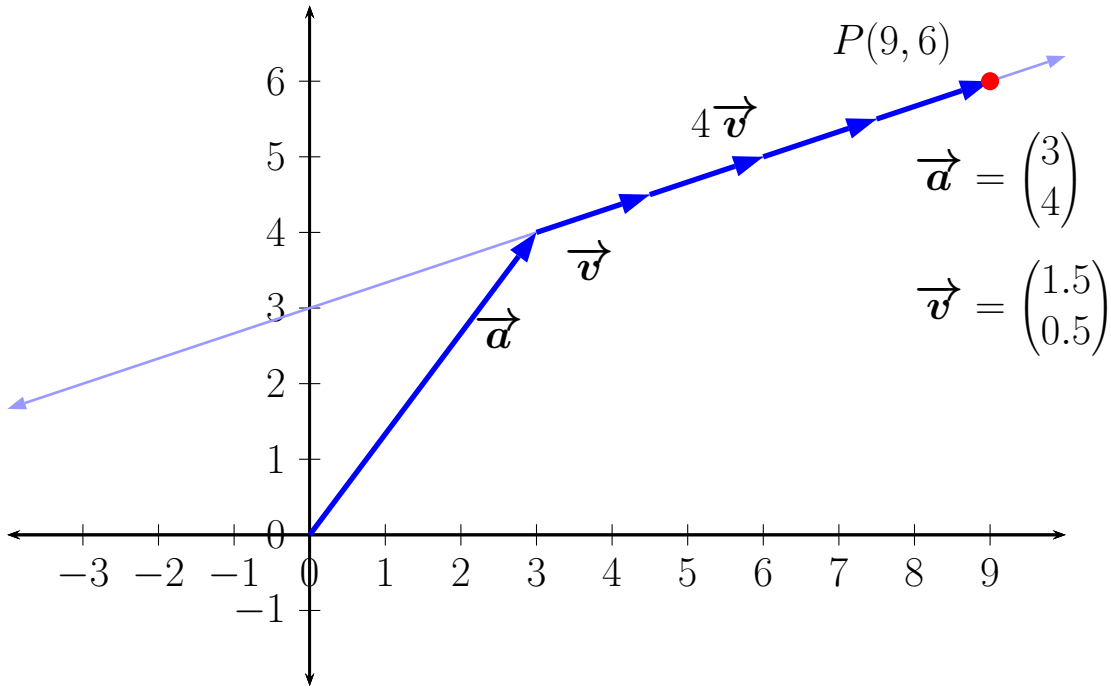

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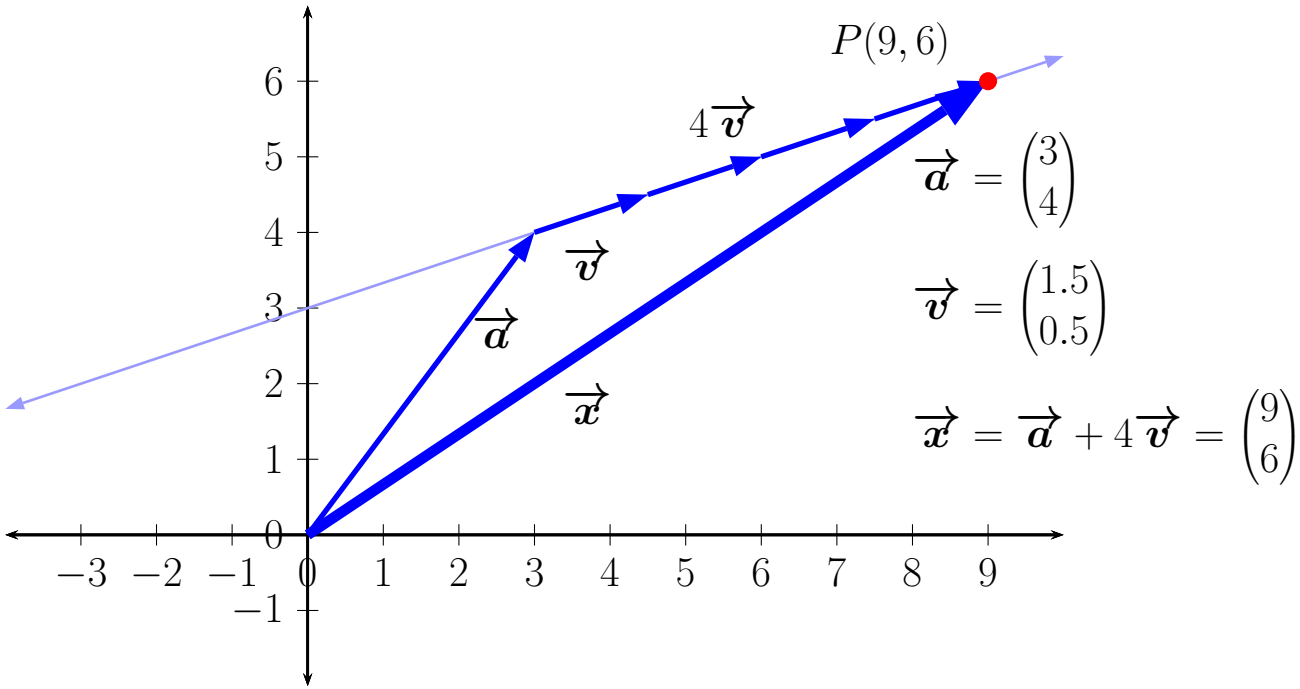

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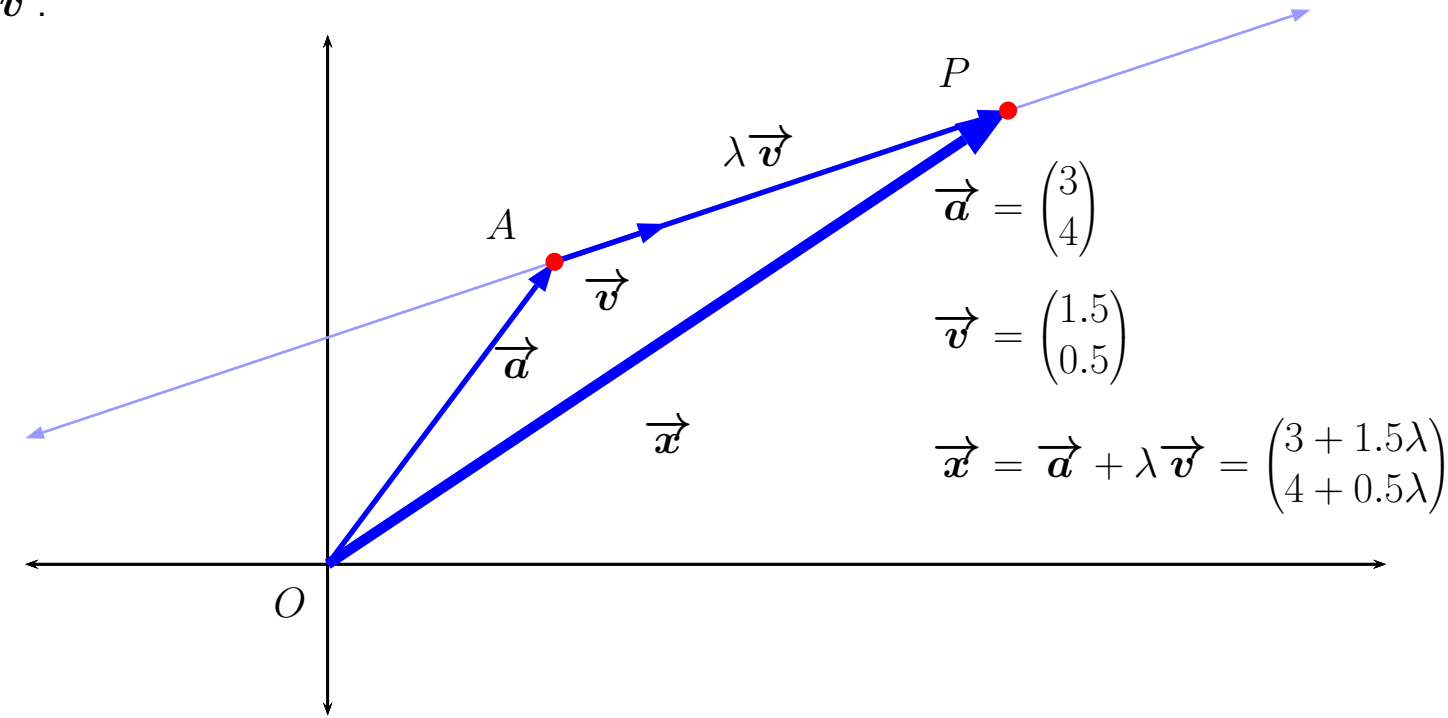
Vector parametric form of a line

We can describe points on a line with a point A on the line and a vector in the direction of the line $\vec{v}$.



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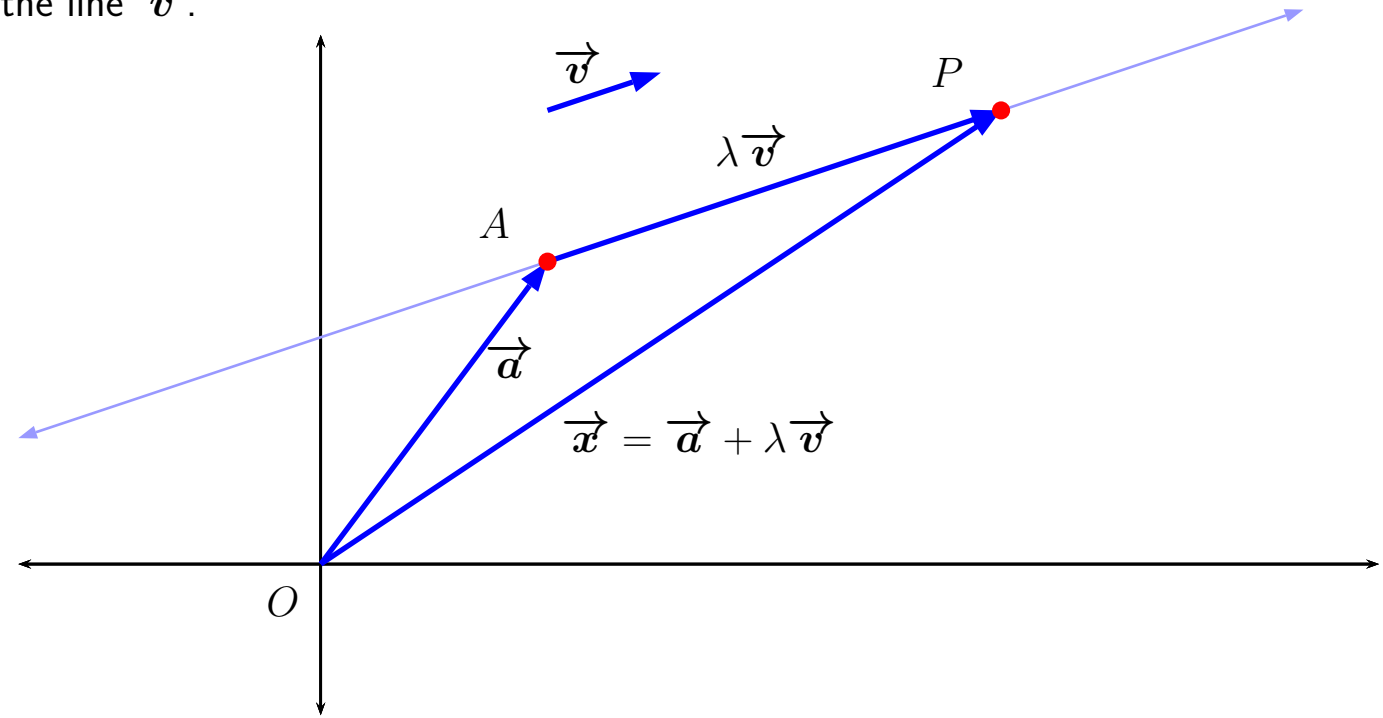

Remember that the location of a vector isn't fixed.

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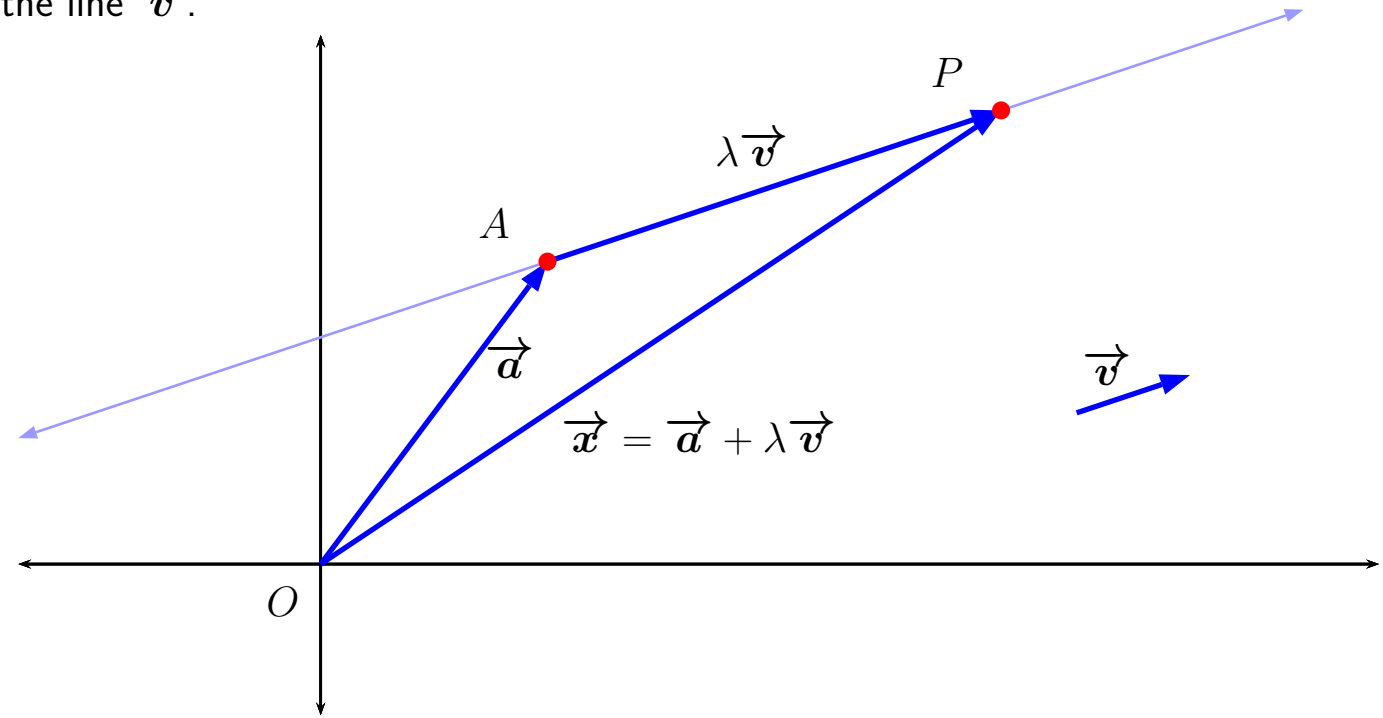

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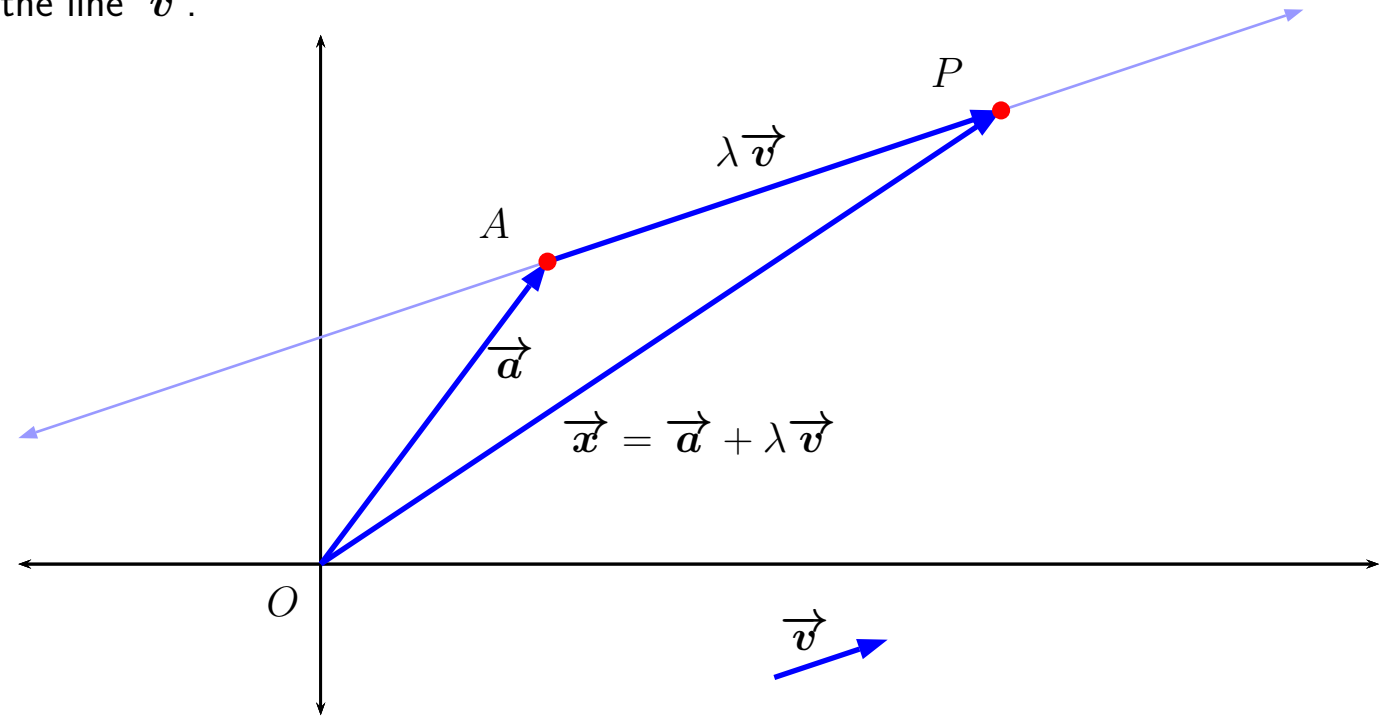

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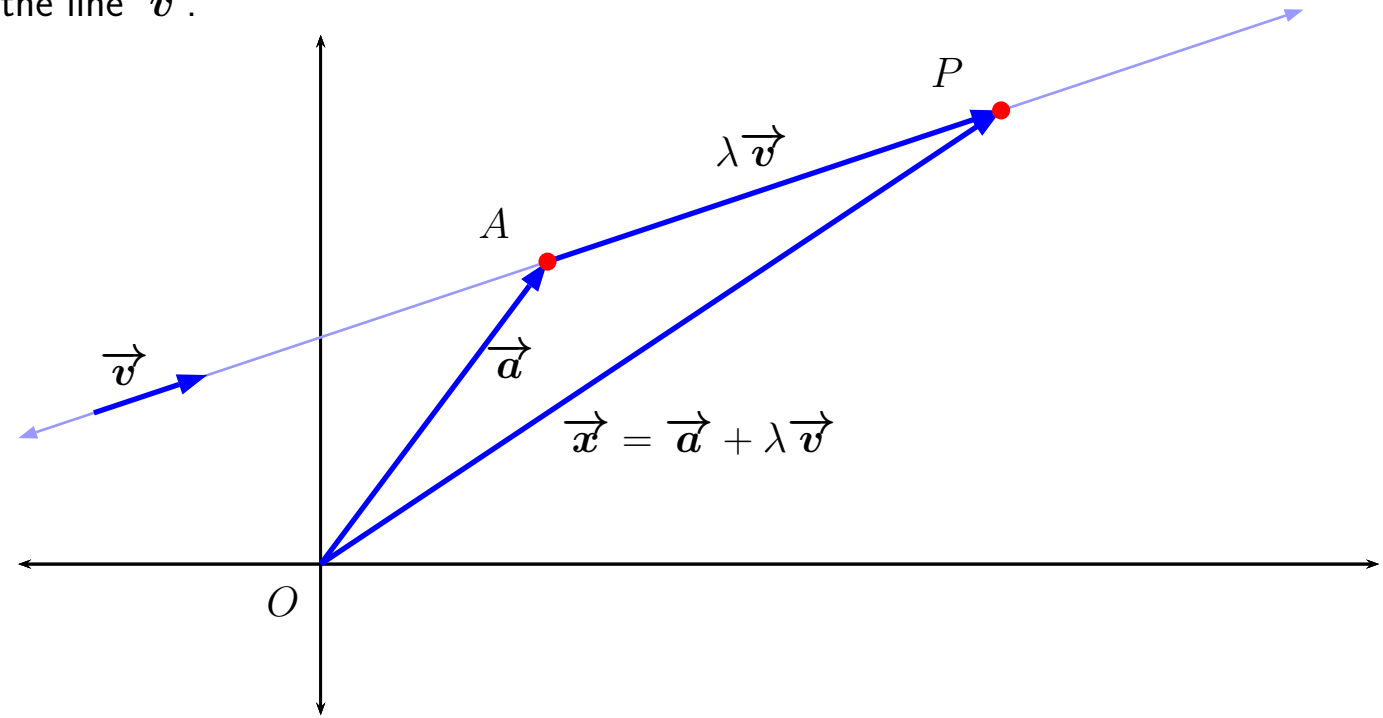

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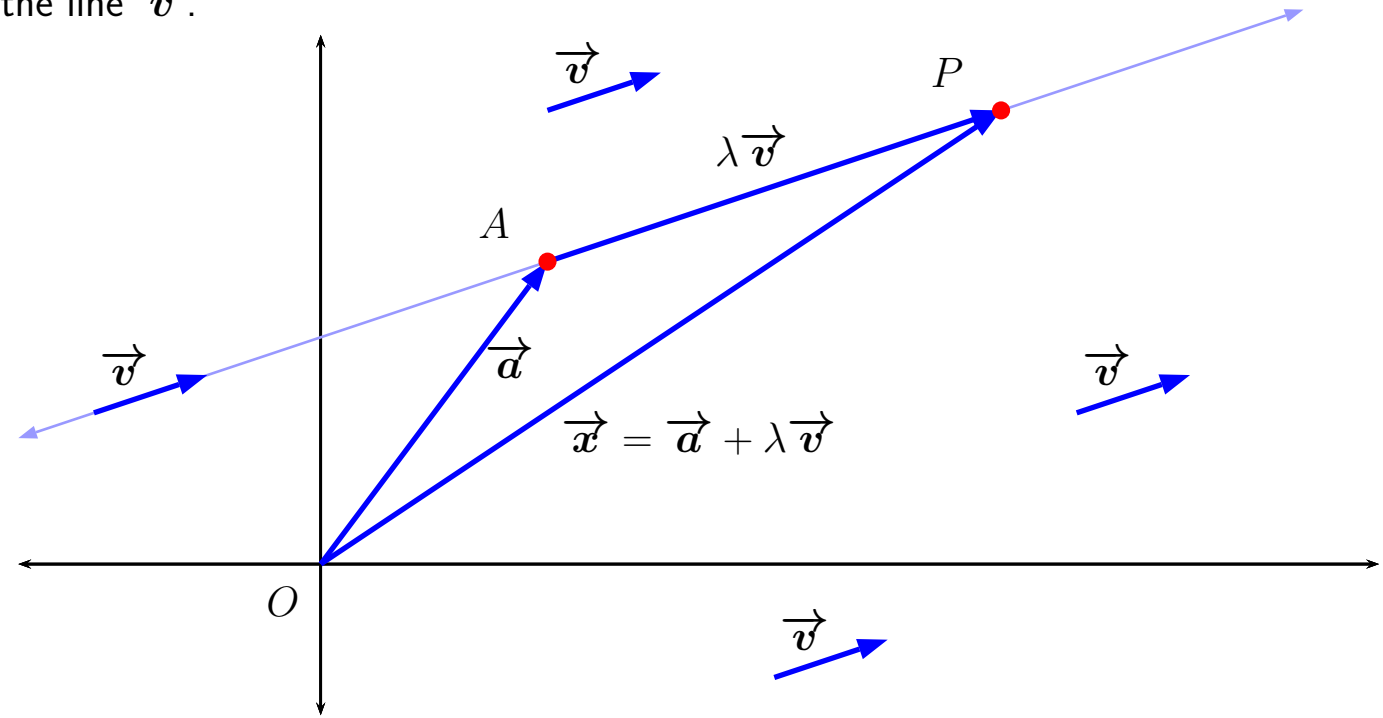

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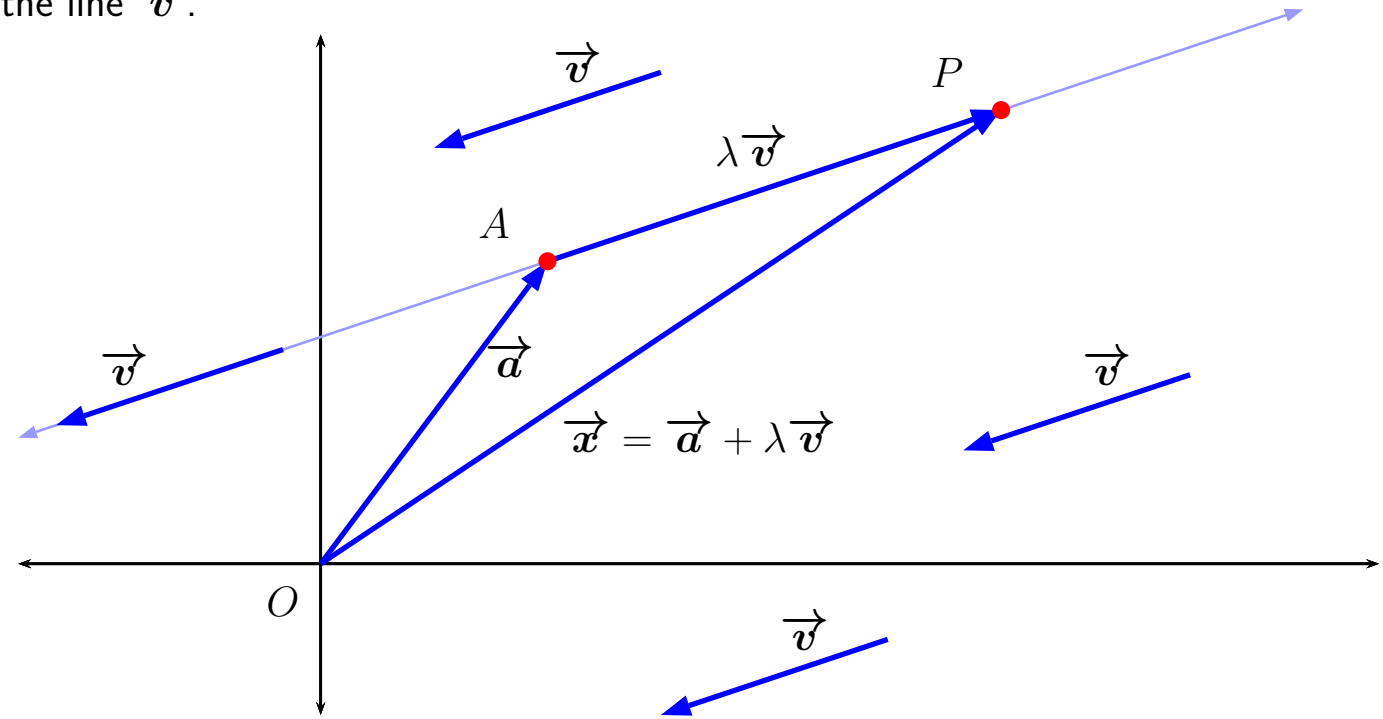

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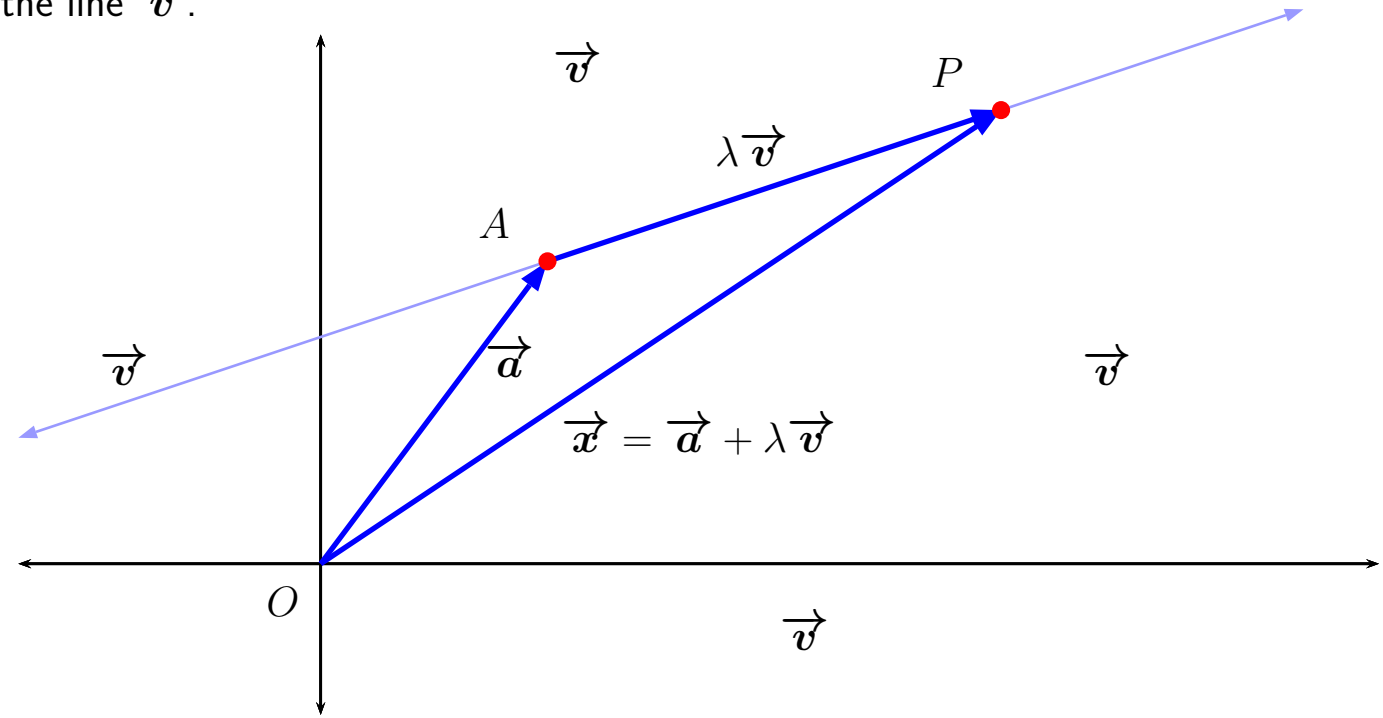

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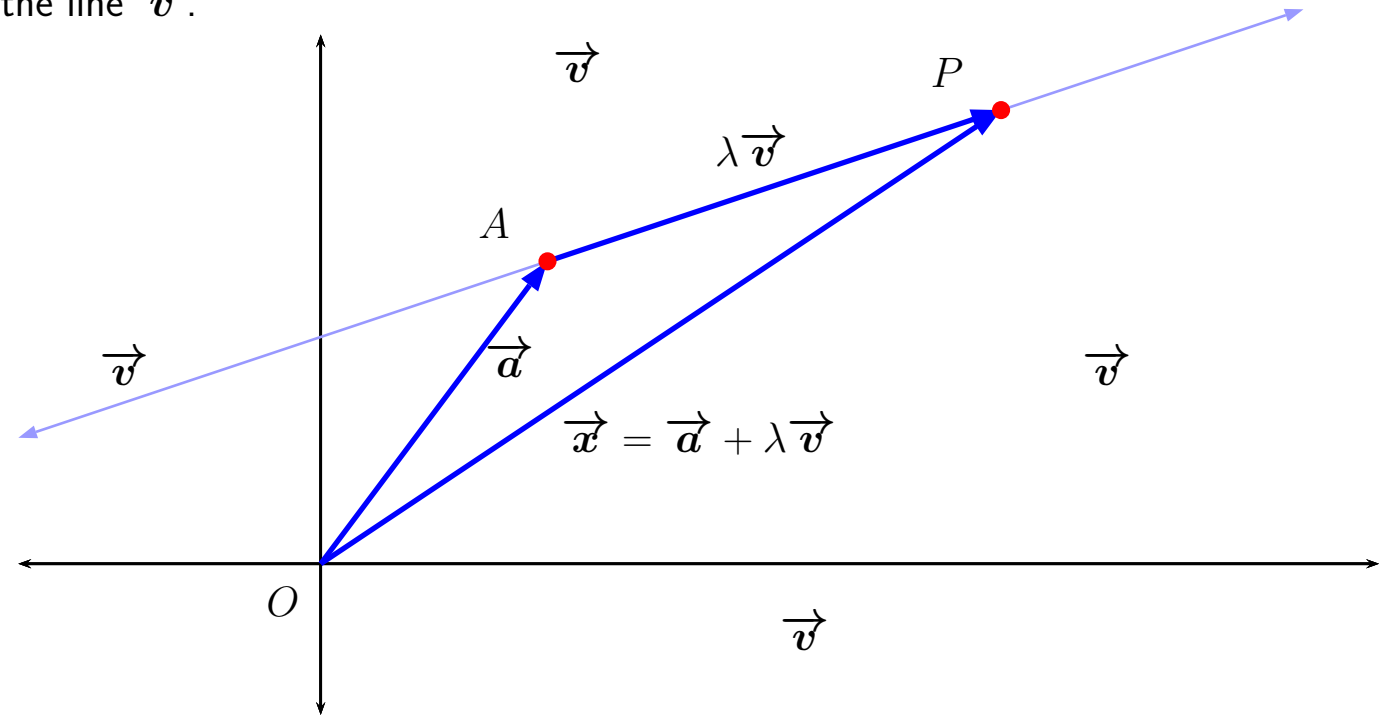

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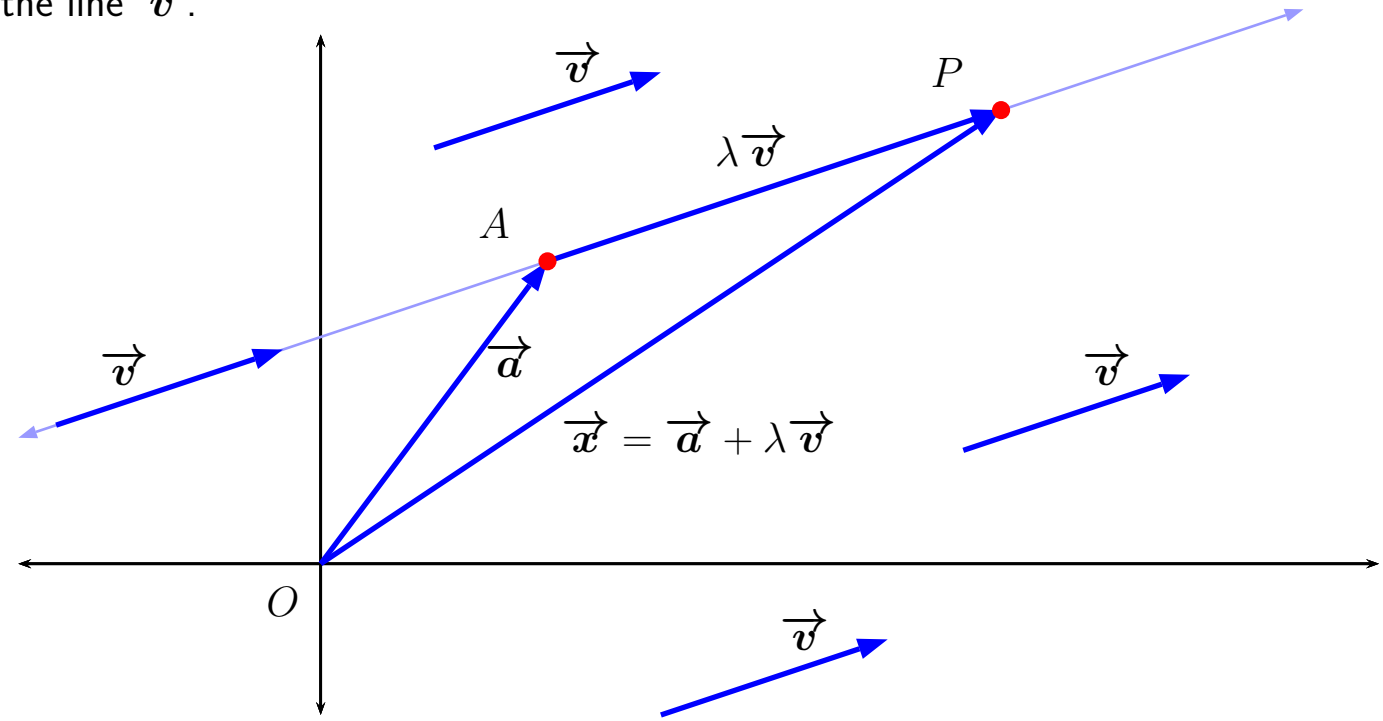


Any vector in the direction of the line will do.

Vector parametric form of a line

Here's a GeoGebra app showing the vector equation of a line:

https://www.geogebra.org/m/O9FqJOkn

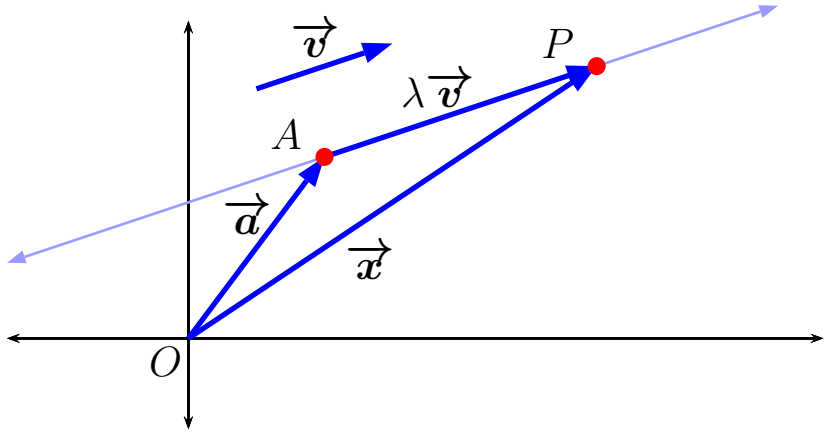
SUMMARY: Vector parametric form of a line

A parametric equation for the points P with position vectors $\vec{\boldsymbol{x}} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$ on a line in $\mathbb{R}^n$ passing through a point A with position vector $\vec{\boldsymbol{a}} \in \mathbb{R}^n$ and parallel to the nonzero vector $\vec{\boldsymbol{v}} \in \mathbb{R}^n$ is:

$$\vec{\boldsymbol{x}} = \vec{\boldsymbol{a}} + \lambda \vec{\boldsymbol{v}}$$

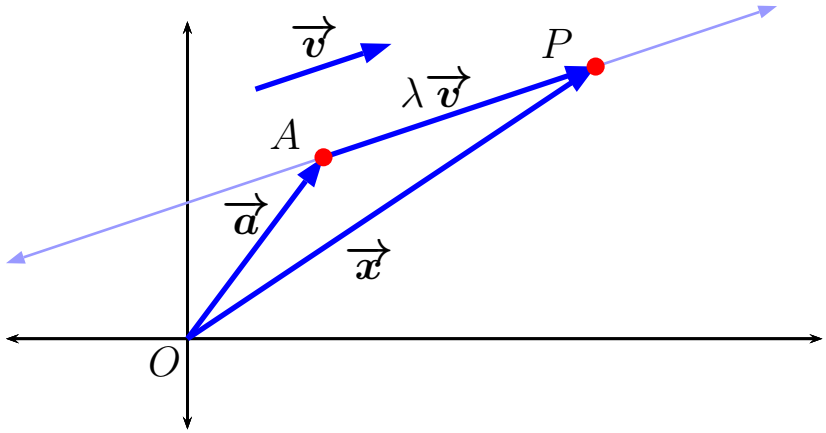
i.e. $\overrightarrow{\boldsymbol{OP}} = \overrightarrow{\boldsymbol{OA}} + \lambda \vec{\boldsymbol{v}}$

where $\lambda \in \mathbb{R}$.



The scalar λ is called a parameter.
Each point on the line has its own value of λ.

SUMMARY: Vector parametric form of a line



A point P is on the line passing through the point A and parallel to the nonzero vector $\vec{v}$

if and only if $\overrightarrow{AP}$ is parallel to $\vec{v}$

i.e. *if and only if* there exist a scalar λ such that $\overrightarrow{AP} = \lambda\vec{v}$

i.e. *if and only if* there exist a scalar λ such that $\overrightarrow{OP} = \overrightarrow{OA} + \lambda\vec{v}$.

Example: Points on a line

Example 1. Let

$$\vec{x} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}, \qquad \lambda \in \mathbb{R}$$

be a line in $\mathbb{R}^3$.

$\vec{OP} = \vec{x} = \vec{OA} + \lambda \vec{v}$, $\vec{v}$, $\vec{a}$, A is a point on the line

1. Give two points that lie on the line.
2. Does $B(9, -2, -5)$ lie on the line?
3. Does $C(-3, 4, 0)$ lie on the line?

$$\begin{cases} 9 = 1 + 2\lambda \\ -2 = 2 - \lambda \\ -5 = -1 - \lambda \end{cases}$$

① $A(1, 2, -1)$ $\quad \lambda = 0$

$D(3, 1, -2)$ $\quad \lambda = 1$

$$\Leftrightarrow \begin{cases} \lambda = \frac{9-1}{2} = 4 \\ \lambda = 2 + 2 = 4 \\ \lambda = -1 + 5 = 4 \end{cases}$$

There exists a solution ($\lambda = 4$) so B is on the line

② Is there a λ such that

$\vec{OB} = \vec{OA} + \lambda \vec{v}$

$$\begin{pmatrix} 9 \\ -2 \\ -5 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$$

$\lambda = 4$

check $\begin{pmatrix} 1 + 8 \\ 2 - 1 \times 4 \\ -1 + 4 \times 1 \end{pmatrix} = \begin{pmatrix} 9 \\ -2 \\ -5 \end{pmatrix}$ ✓

Example: Points on a line

Example 1. Let

$$\overrightarrow{\boldsymbol{x}} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}, \qquad \lambda \in \mathbb{R}$$

be a line in $\mathbb{R}^3$.

1. Give two points that lie on the line.
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Parallel lines

How to tell if two lines are parallel from their parametric forms?

Two lines

$$\begin{aligned} \overrightarrow{\boldsymbol{x}} &= \overrightarrow{\boldsymbol{a_1}} + \lambda \overrightarrow{\boldsymbol{v_1}} \\ \overrightarrow{\boldsymbol{x}} &= \overrightarrow{\boldsymbol{a_2}} + \lambda \overrightarrow{\boldsymbol{v_2}} \end{aligned}$$

are parallel if the $\overrightarrow{\boldsymbol{v_1}}$ and $\overrightarrow{\boldsymbol{v_2}}$ are parallel.

Example 2. Let

$$\ell : \quad \overrightarrow{\boldsymbol{x}} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}, \quad \lambda \in \mathbb{R}$$

be a line in $\mathbb{R}^3$. Write down a parametric vector equation of the line ℓ' through $(1, 2, 3)$ that is parallel to the line ℓ above.

Example

Example 3. Find an equation of the line ℓ in $\mathbb{R}^4$ which passes through $A(2, -3, -1, 2)$ and $B(-1, 2, 2, 7)$.

Cartesian equation of a line in $\mathbb{R}^2$

Example 4. Find a Cartesian equation of the line ℓ given parametrically as

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$