School of Mathematics and Statistics

Math1131 Mathematics 1A

CALCULUS LECTURE 1

REVISION ON SETS AND INEQUALITIES

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MATH1131 CALCULUS
REVISION ON SETS AND INEQUALITIES

$$\mathbb{N} = \{0, 1, 2, 3, 4 \dots\}$$

$$\mathbb{Z} = \{\dots - 4, -3, -2, -1, 0, 1, 2, 3, 4 \dots\}$$

$$\mathbb{Q} = \{\frac{p}{q} \text{ where } p, q \in \mathbb{Z} \text{ with } q \neq 0\}$$

$\mathbb{R}$ is the set of all real numbers. (the real line)

(a, b) represents the open interval $a < x < b$

$[a, b]$ represents the closed interval $a \leq x \leq b$

Most of you have already seen some Calculus in high school and we will now build on that theory in a number of interesting ways. Note that our approach at university tends to be a little more formal and rigorous than that used in the schools.

We start with some notation which will streamline our future presentation.

Interval Notation

The backbone of all of your mathematical study has of course been the concept of a number. Starting with the counting numbers

$$\mathbb{N} = \{0, 1, 2, 3, 4 \dots\}$$

we progress to the integers

$$\mathbb{Z} = \{\dots - 4, -3, -2, -1, 0, 1, 2, 3, 4 \dots\}$$

the rationals

$$\mathbb{Q} = \{\frac{p}{q} \text{ where } p, q \in \mathbb{Z} \text{ with } q \neq 0\}$$

and finally the set of all real numbers $\mathbb{R}$ which includes such curious creatures as π and $\sqrt{2}$.

Example 1: Graph each of the following sets on the number line:

a) $\{x \in \mathbb{Z} : -3.5 < x \leq 3.5\}$

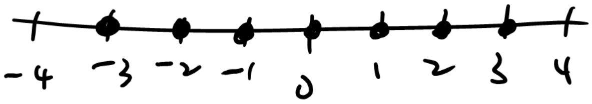
b) $\{x \in \mathbb{R} : -3.5 < x \leq 3.5\}$

$\in$ means "element of"

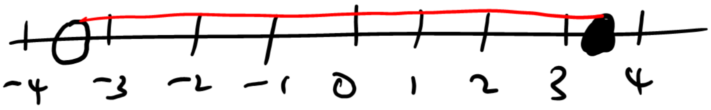
: OR | means "such that"

{ } means "set"

a)



b)



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It is important to be able to specify subsets of the real line without resorting to the use of variables. To do this we use what is called interval notation.

(a, b) represents the open interval $a < x < b$

and

$[a, b]$ represents the closed interval $a \leq x \leq b$.

Generally speaking "(" and ")" is an instruction to exclude the endpoint while "[" and "]" tells you to include the endpoint.

Example 2: Express each of the following sets in interval notation:

a) $\{x \in \mathbb{R} : 1 < x < 3\}$. $(1, 3)$

b) $\{x \in \mathbb{R} : 1 \leq x \leq 3\}$. $[1, 3]$

c) $\{x \in \mathbb{R} : 1 < x \leq 3\}$. $(1, 3]$

d) $\{x \in \mathbb{R} : 1 \leq x < 3\}$. $[1, 3)$

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Note that we often write $\{x \in \mathbb{R} : 1 \leq x < 3\}$ simply as $1 \leq x < 3$.

Example 3: Write out the following intervals using inequalities and sketch on the real numberline:

a) $(-2, 7]$.

b) $[3, \infty)$.

a) $\{x \in \mathbb{R} \mid -2 < x \leq 7\}$

$-2 < x \leq 7$

b) $\{x \in \mathbb{R} : x \geqslant 3\}$ OR $x \geqslant 3$

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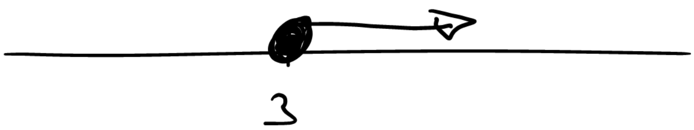
Note that ∞ never gets a "]" as it is not a real number and hence cannot be included in the interval.



Unions and Intersections

We now revise some very old theory on unions and intersections.

Given two sets A and B:

$A \cup B$ (read as A union B) corresponds to A **or** B. $\cup \equiv$ OR

$A \cap B$ (read as A intersection B) corresponds to A **and** B. $\cap \equiv$ and

A^c or $\overline{A}$ (read as A complement) corresponds to **not** A. $\overline{A} \equiv$ not A

Example 4: Display each of the following on a Venn diagram:

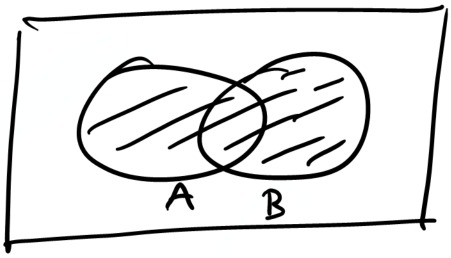
a) $A \cup B$.

b) $A \cap B$.

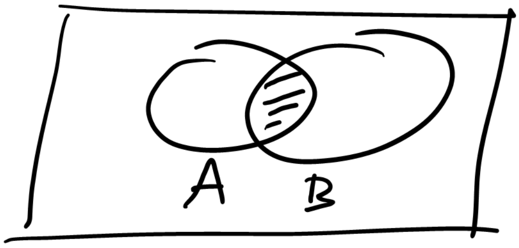
c) A^c.

d) $A \cap B^c$.

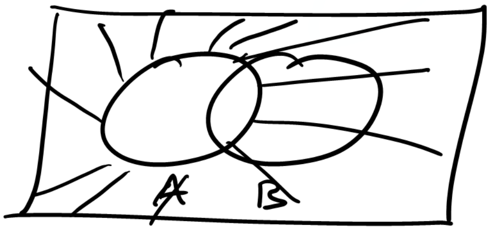
$A \cup B$



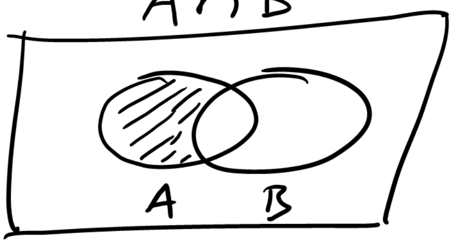
$A \cap B$



A^c



$A \cap B^c$



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Sketching Polynomials

It is a trivial task to sketch polynomials if they are presented to you in factored form. All you need to remember is that all odd powers $n = 3, 5, 7, \ldots$ will meet the x axis just like a cubic and all the even powers $n = 2, 4, 6, \ldots$ will bounce just like a quadratic. If ever in doubt a simple strategy is to just plot a few points.

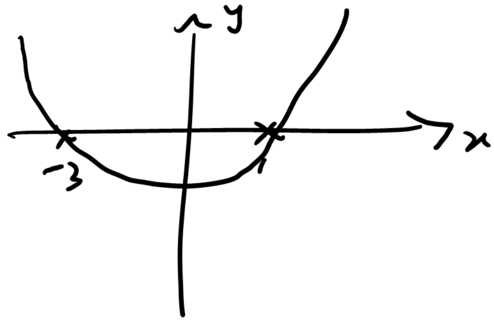
Example 5: Sketch each of the following polynomials:

a) $y = (x-1)(x+3)$.

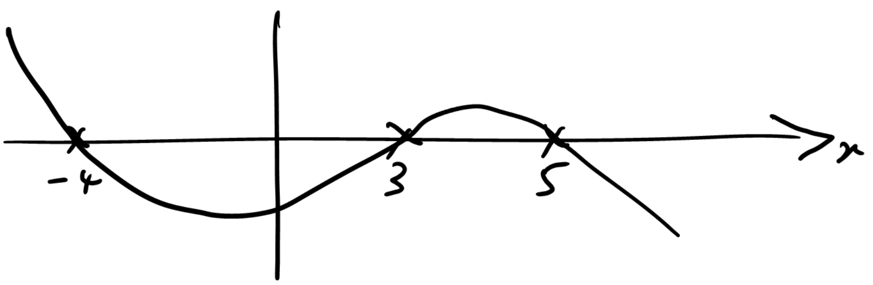
b) $y = (x-3)(5-x)(x+4)$.

c) $y = (x-4)(x+3)^{22}(5-x)^{12}(x-6)^{17}$.

a)



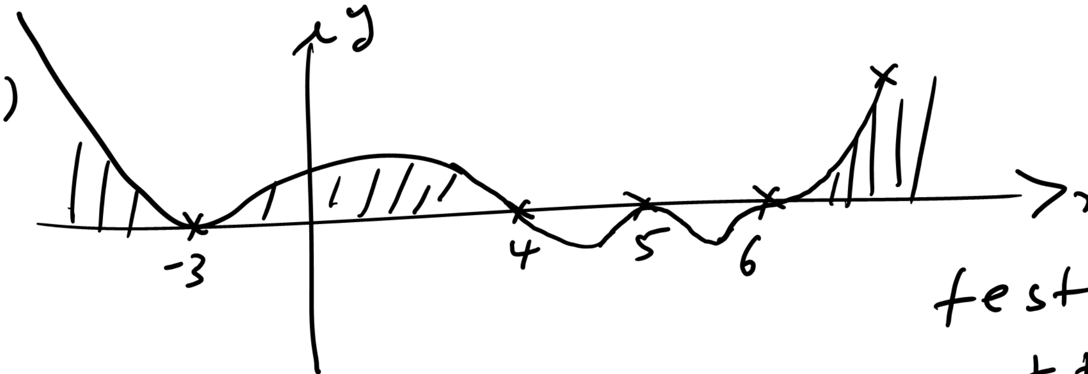
b)



test $x=10$:

$+ - + = -$

c)



test $x=10$:

$++++ = +$

Example 6: Use the sketch in c) above to solve the inequality

$$(x-4)(x+3)^{22}(5-x)^{12}(x-6)^{17} \geq 0$$

$x \leq 4$ OR $x \geq 6$ OR $x = 5$

$(-\infty, 4] \cup [6, \infty) \cup \{5\}$

★ $(-\infty, 4] \cup [6, \infty) \cup \{5\}$ ★

Inequalities

We adopt a range of different techniques when solving inequalities. Each of the following is slightly different.

Example 7: Solve each of the following inequalities. Sketch your solution on the number line and express your solution in interval notation:

a) $-3x + 1 < 16$.

b) $x^2 \leq 5x - 6$.

c) $\dfrac{4}{x-1} \leq x + 2$.

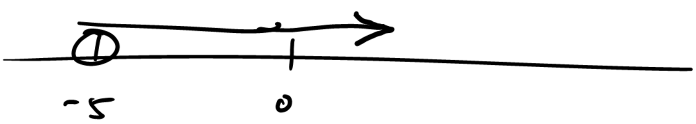
d) $|5x - 4| > 16$.

e) $|x - 1| > 5 - 2x$.

a) $-3x+1<16 \Rightarrow -3x < 15$

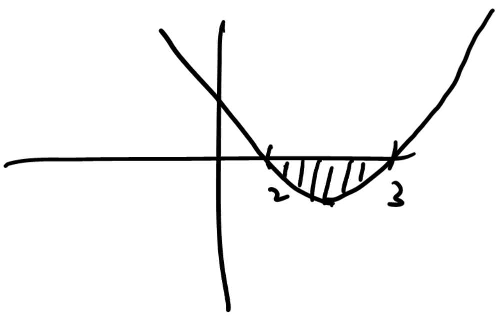
$\Rightarrow x > -5$

$(-5, \infty)$



b) $x^2 \leq 5x - 6 \leq 0 \Rightarrow x^2 - 5x + 6 \leq 0$

$\Rightarrow (x-2)(x-3) \leq 0$



$\therefore \quad 2 \leq x \leq 3$

$[2, 3]$

c) $\dfrac{4}{x-1} \leq x+2 \Rightarrow \cancel{4 \leq (x+2)(x-1)}$

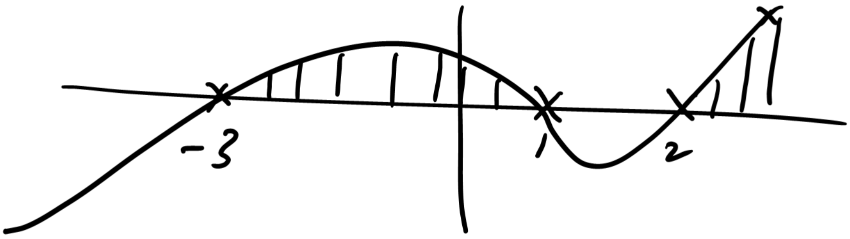
$\times(x-1)^2: \quad \dfrac{4}{(x-1)}(x-1)^2 \leq (x+2)(x-1)^2$

$4(x-1) \leq (x+2)(x-1)^2$

$0 \leq (x+2)(\underline{x-1})^2 - 4(\underline{x-1})$

$$0 \leq (x-1)\{(x+2)(x-1)-4\}$$
$$0 \leq (x-1)\{x^2 - x + 2x - 2 - 4\}$$
$$0 \leq (x-1)\{x^2 + x - 6\}$$
$$0 \leq (x-1)(x-2)(x+3)$$

Test $x = 10$
$+++ = +$



$\therefore -3 \leq x \leq 1$ OR $x \geq 2$

!!! But $x \neq 1$!!!

$\therefore -3 \leq x < 1$ OR $x \geq 2$

$[-3, 1) \cup [2, \infty)$ #

d)

$$|5x - 4| > 16$$

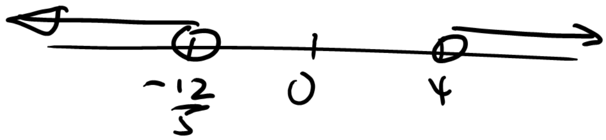
$+$ $-$

$5x - 4 > 16$ OR $-(5x-4) > 16$

$5x > 20$ $\quad$ $5x - 4 < -16$

$x > 4$ $\quad$ $5x < -12$

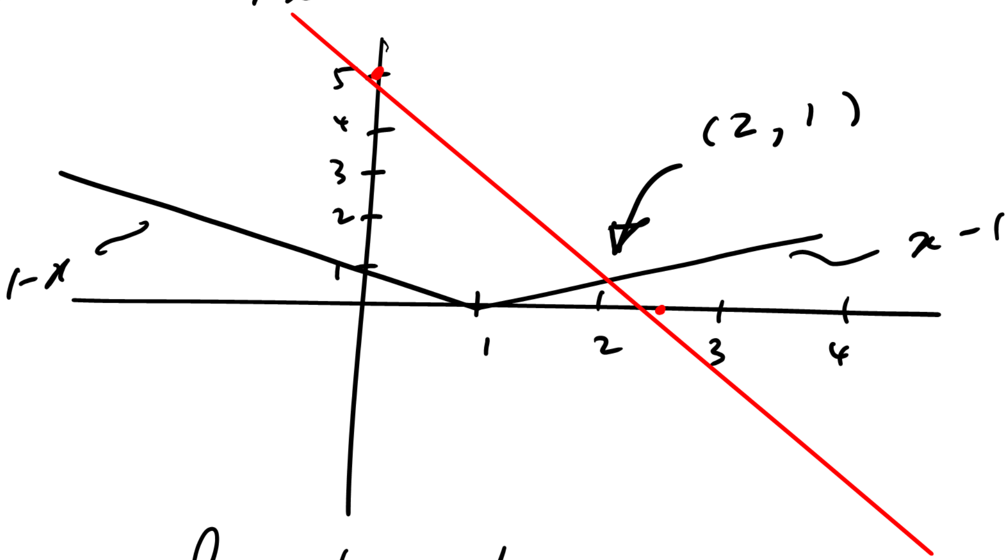
$x < -\frac{12}{5}$



$(4, \infty) \cup (-\infty, -\frac{12}{5})$

★ $a)$ $x > -5$ $b)$ $[2, 3]$ $c)$ $[-3, 1) \cup [2, \infty)$ $d)$ $(-\infty, -\frac{12}{5}) \cup (4, \infty)$ $e)$ $x > 2$ ★

e) $|x-1| > 5-2x$.



Point of intersection:

$$x-1 = 5-2x$$
$$3x = 6$$
$$x = 2$$

By considering sketch:

$$x > 2$$
$$(2, \infty)$$