Bayes' Theorem

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conditional probability *Elementary Statistics.*

$P\ B\ A$

B A

$P\ B\ A$

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

B A

A

B

Bayes' theorem *Bayes' rule*

sequential

prior

probability *posterior probability*

Definitions

prior probability

posterior probability

Example 1

Solution

P

P

–

Bayes' Theorem

$A,$ B

$$P(A|B) = \frac{P(A) \cdot P(B|A)}{[P(A) \cdot P(B|A)] + [P(\overline{A}) \cdot P(B|\overline{A})]}$$

Example 2

Solution

$\overline{\mathrm{M}}$
$\overline{\mathrm{C}}$

P

P

P $\overline{\mathrm{M}}$

P

P $\overline{\mathrm{M}}$

Intuitive Bayes Theorem

Assume some convenient value for the total of all items involved, then construct a table of rows and columns with the individual cell frequencies based on the known probabilities.

Finding the number of males who smoke cigars:

– ×

not –

Finding the number of females who smoke cigars:

– ×

not –

		$\overline{C}$	**Total**
			51,000
$\overline{M}$			**49,000**
Total	**5678**	**94,322**	**100,000**

P

Bayes' Theorem Generalized

A

$\overline{A}$

disjoint

exhaustive

Example 3

Solution

$\overline{D}$

P

P
P
P

P
P
P

$$P\ A\ D \qquad \frac{P(A)\cdot P(D|A)}{[P(A)\cdot P(D|A)]+[P(B)\cdot P(D|B)]+[P(C)\cdot P(D|C)]}$$

$$\frac{0.80\cdot 0.04}{[0.80\cdot 0.04]+[0.15\cdot 0.06]+[0.05\cdot 0.09]}$$

Intuitive Baye's Theorem: *P*

		$\overline{\text{D}}$	**Total**
			8,000
			1,500
			500
Total	**455**	**9545**	**10,000**

Exercises

Pregnancy Test Results. *In Exercises 1 and 2, refer to the results summarized in the table below.*

1. a.

b.

2. a.

b.

3. ***Survey Results***

a.
b.

4. ***Emergency Locator Transmitters***

a.

b.

5. ***Emergency Locator Transmitters***

a.

b.

6. ***Emergency Locator Transmitters***

not

7. ***Pleas and Sentences***

a.

b.

8. ***Pleas and Sentences***

a.

b.

9. ***HIV***

10. ***HIV***

11. ***Extending Bayes' Theorem***

P

12. ***Extensions of Bayes' Theorem***

$\overline{\text{A}}$

$\overline{\text{A}}$ P

P

Answers to Odd–Numbered Exercises

$$P\ A\ Z \quad \frac{P(A)\cdot P(Z|A)}{[P(A)\cdot P(Z|A)]+[P(B)\cdot P(Z|B)]+[P(C)\cdot P(Z|C)]+[P(D)\cdot P(Z|D)]}$$