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Optimal Market Segmentation of Hotel Rooms—The Non-linear Case

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The optimal market segmentation pricing strategy for rooms of hotels is investigated to determine the optimal number of segments to be used and the accompanying number of rooms and price prevailing in each segment, under the assumption of an aggregate non-linear demand function. A single state-variable dynamic programming model is formulated to maximize profit, and an efficient reduction in the range of search for the solution of the model is outlined. A numerical example is provided for a 400 room hotel with an exponential demand curve.

Key words—market segmentation, yield management, hotel room management, segmentation pricing

1. INTRODUCTION

MANY HOTELS ARE faced with the reality that on any given day only a fraction of their rooms are higher than the available capacity, unbooked hotel rooms on any given day are perishable inventory. Thus, a management policy which

Revenue Management (PARM). Over 100 research articles have been published on the subject, dealing with issues of pricing, determining the size of a segment or of segments, booking polices under various static and dynamic conditions, starting with papers from the 1970s like, for example [14–17, 19–21], and later [9], and lately [2, 5, 25, 30]. A taxonomy and research overview is provided by Weatherford and Bodily [28], which summarizes the key questions that the researchers attempted to answer:

How many units should be made available initially at various prices (or, alternatively, for a given allocation scheme, what are the optimal pricing levels), and

How should this availability of units change over time, as the time of actual availability approaches; i.e. when should certain price levels be closed out (made unavailable) or opened up?

In all the above works, the number of segments (Weatherford and Bodily taxonomy 'E') was invariably given, implying an upper management directive. Thus, all these papers dealt only with *tactical* issues, how to attempt to optimize the performance under the constraint of a given number of segments, i.e. they treated only the problem of efficiency. No research has been performed on the *strategic* issue of *determining* first of all the *number of different segments*, i.e. the number of different price classes, and if possible to couple the determination of the number of different segments with the tactical issues of price and size of the segments. A strategic effectiveness objective—as opposed to a tactical efficiency objective—was never followed.

Neither has the field of marketing treated the problem differently. Since the pioneering work of Smith [26] in which he discussed the nature of market segmentation as compared to product differentiation, over 100 research articles (reviewed by Wind [29]) were published on market segmentation. Claycamp and Massy [7] and Massy and Weitz [23] provide the theory of market segmentation, while Frank *et al.* [11] describe market segmentation as a marketing strategy. Mahajan and Jain [22] deal with market segmentation as a form of research analysis directed at identification of, and allocation of resources among market segments. Bonoma and Shapiro [4] define market segmentation either as a process of aggregating individual customers into groups, or a process of disaggregating a total market into pieces. Moorthy [24], Gensch [12], and Blozan and Prabhaker [3] deal with the definition of market segments, Smith [27] associates market segmentation with adjustment of product and marketing effort to difference in demand, whereas Grover and Srinivasan [13] use brand choice probabilities as the basis of segmentation. Likewise, Dickson and Ginter [8] deal with the identification of market segments (and product differentiation opportunities) as the essence of market segmentation strategy.

No attempt has yet been made to determine the optimal number of segments. Although Bonoma and Shapiro [4, pp. 102–103] suggest a criterion of minimum combined direct cost and opportunity cost of segmentation for optimal degree of segmentation, they just remark that "this speculation deserves further research". Likewise, Chakravorti *et al.* [6, p. 65] state that in the electric utility market (which can be described as having a single aggregate demand function like a hotel—as contrasted with known separate demand functions in various potential segments), the number of segments, into which to divide the market, has to be based on judgmental decisions, since [6, p. 47] the optimum point may be difficult, if not impossible, to determine.

A single exception which attempted to determine the optimal number of market segments is [18], where the optimal number of passenger-cabin groups on a cruise-liner was determined, requiring price discrimination for non-differentiated cabins. There, a simplified and naive special case was considered of having a *linear* aggregate demand function, which allowed the derivation of optimal *equal*-sized market segments by simple calculus.

In the present work, the general and realistic case is considered in which in a given period the aggregate demand curve is characterized by a *non*-linear relationship between the price and the number of rooms demanded at that price, and it is treated in a deterministic manner. Obviously, it can be derived either from historical data, from experimentation, surveys, or from known relationships in similar environments.

The aim of this paper is to determine the optimal number of different market segments strategy that a hotel should follow. This strategy will be derived simultaneously with the optimal number of rooms to be assigned to each segment, and the price that should prevail in each one of them, under the deterministic demand curve assumption. The optimal policy is sought that will maximize the hotel's profit.

Since this is the first time that the problem of determining the optimal number of market segments has been addressed, the various tactical dynamic problems of booking and changing effective segment sizes as the booking date gets closer (problems caused by the stochastic nature of real-life demand), will not be considered here. Hopefully, it will take less than the 30 years it took to derive the existing solutions to the tactical problems to incorporate them optimally with simultaneous solution of the strategic problem. Nevertheless, already a two-stage optimization procedure can be pursued (though not necessarily generating an optimum optimorum), along the same general ideas as practised in corporate production planning, where first an aggregate plan is developed, and based on it a detailed production plan is generated. Thus, as the first stage, the optimal number of segments and the prevailing price in each segment can be determined using the deterministic demand curve approach outlined here. Then, as the second stage, using the derived number of segments and prevailing prices as inputs (instead of unfounded management directives), the tactical booking issues could be solved in a stochastic environment, according to the cited references and exactly in the same manner.

It should be stressed that since the optimal strategy depends on the demand curve, which in itself may change from day to day, and from season to season, it is quite possible that for the summer week-end days there will be one optimal strategy, a second strategy for summer mid-week days, a third strategy for winter week-end days, etc. For our investigation we assumed that the hotel has only a fixed number of identical rooms.

In Section 2 a dynamic programming model is constructed for the derivation of the optimal market segmentation room-pricing policy. The Appendix outlines a method for the reduction of the range of search in each stage of the dynamic programming model, while Section 3 provides a numerical example.

2. THE MODEL

In a hotel with a capacity of Q identical rooms, the aggregate demand curve which shows the price p per day per room to exactly satisfy a given number of rooms demanded per day, q, is given by the non-linear relationship (an example is illustrated in Fig. 1):

$$p = f(q) \text{ for } q > 0$$

Let us segment the market into n segments, where the number of hotel rooms assigned to each segment (at different prices per room in each segment) may not have to be necessarily equal.

Let the fixed cost per day of the hotel be F, and the variable cost per room per day be v. When several market segments are utilized, an additional fixed cost per day, K, occurs per each market segment (for keeping it separately from other segments and for advertising and administering in it separately).

We want to determine the optimal number of market segments, the optimal prices to be demanded per rooms in each market segment, and the resulting optimal number of rooms assigned to each market segment, in such a way that the profit per day will be maximized.

In essence, there is a demand curve (illustrated in Fig. 2), and price breaks are set at a number of points, resulting in revenue corresponding to the shaded area under the step

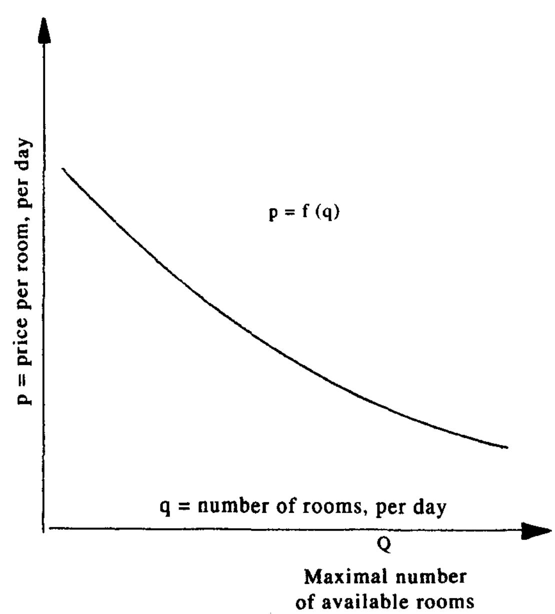

Fig. 1. Example demand curve.

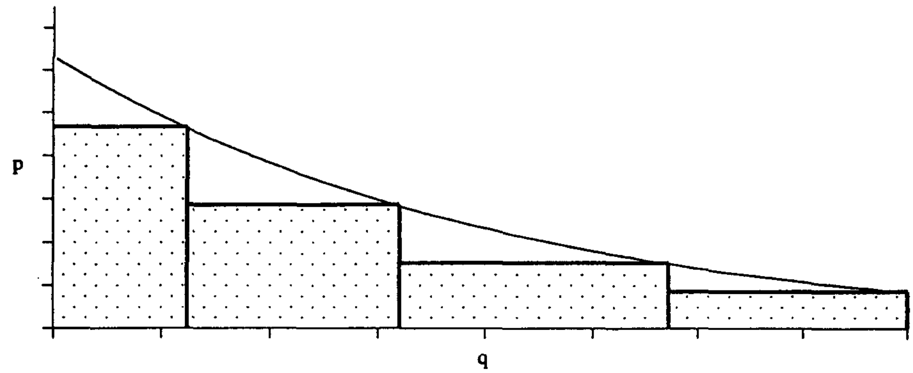

Fig. 2. Representation of the revenue obtained under a demand curve with several market segments.

function. The aim is to maximize the profit, which would imply making the shaded revenue area as large as possible, which would tend to make the steps very small—except that there is a fixed cost associated with creating each step, reducing profit, and so encouraging fewer steps. The objective is to find the number of steps, and their width (not necessarily of equal size), that will maximize the profit.

(1) Assume that only one market segment will be used, which we denote as $n = 1$, where n is the stage variable. The number of rooms assigned to this single market segment is q_1, which calls for a price per room of $p = f(q_1)$.

Let

$P_1(q_1)$ = the profit per day from a single market segment with q_1 rooms assigned to this segment at a price of $p = f(q_1)$.

$$P_1(q_1) = q_1 f(q_1) - [F + vq_1 + K] \quad \text{for } 1 \leq q_1 \leq Q \quad (1)$$

There is a value of q, q_o, which maximizes equation 1. It is obtained from

$$\frac{dP_1(q)}{dq} = \frac{d}{dq}[qf(q) - (F + vq + K)]$$
$$= f(q_o) + q_o f(q_o) - v = 0 \quad (2)$$

Since the solution of equation 2 will not necessarily provide an integer value, we will select as q_o the uprounded or downrounded integer that will maximize equation 1. Let

$\pi_1(q_1)$ = maximal profit per day from a single market segment with q_1 rooms assigned to this segment.

If $q_1 < q_o$ then the maximal profit is obtained when q_1 rooms are assigned, otherwise there is nothing to be gained from assigning more rooms than q_o. Thus

$$\pi_1(q_1) = \begin{cases} q_1 f(q_1) - [F + vq_1 + K] & \text{if } q_1 < q_o \\ q_o f(q_o) - [F + vq_o + K] & \text{if } q_1 \geq q_o \end{cases}$$
$$\text{for } 1 \leq q_1 \leq Q \quad (3)$$

(2) Assume now that 2 market segments will be utilized, $n = 2$, where the first market segment will be assigned q_1 rooms, and both segments altogether will be assigned q_2 rooms, i.e. the second segment will be assigned $(q_2 - q_1)$ rooms as illustrated in Fig. 3.

Let

$P_2(q_2)$ = Profit per day from 2 market segments, where both market segments are assigned q_2 rooms, and the first market segment is assigned q_1 rooms.

Then, the profit per day from the 2 market segments equals the profit from the first segment

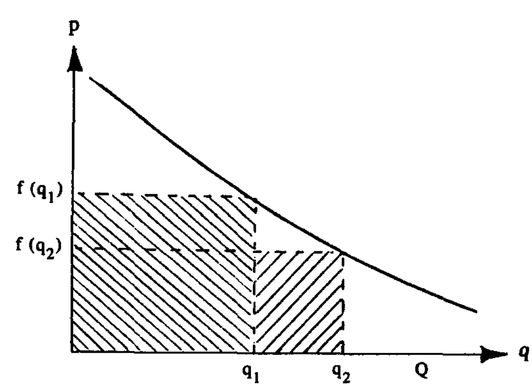

Fig. 3. Room prices with two segments.

with q_1 rooms, plus the profit generated from the second segment with $(q_2 - q_1)$ rooms. Hence,

$$P_2(q_2) = (q_2 - q_1)f(q_2) - v(q_2 - q_1) - K + P_1(q_1) \quad \text{for } q_1 \leq q_2 \leq Q \qquad (4)$$

Let

$\pi_2(q_2) =$ the maximal profit per day from 2 market segments where both market segments are assigned q_2 rooms, and the first market segment is assigned q_1 rooms.

It is possible to express the maximal profit from both market segments in a recursive manner, where it is the maximum profit obtained from the sum of the maximal profit from one segment (with q_1 rooms assigned to the first segment), and of the profit from the second segment (with $q_2 - q_1$ rooms assigned to the second segment), and where q_1 is the state variable:

$$\pi_2(q_2) \max_{\substack{q_1 \\ 1 \leq q_1 \leq q_2}} \{(q_2 - q_1)f(q_2) - v(q_2 - q_1) - K + \pi_1(q_1)\} \quad \text{for} \quad 2 \leq q_2 \leq Q \qquad (5)$$

When $\pi_2(q_2)$ is achieved for $q_1 = q_2$, it means that the maximal profit with hotel assignment of q_2 rooms is achieved with a single segment of $q_1 = q_2$ rooms.

(3) When n market segments are utilized, where the n market segments will be assigned a total of q_n rooms, i.e. where the nth market segment is assigned $(q_n - q_{n-1})$ rooms, let us define $\pi_n(q_n) =$ the maximal profit per day from all n market segments, and we obtain

$$\pi_n(q_n) = \max_{\substack{q_{n-1} \\ 1 \leq q_1 \leq q_2}} \{(q_n - q_{n-1})f(q_n) - v(q_n - q_{n-1}) - K + \pi_{n-1}(q_{n-1})\} \quad \text{for} \quad n \leq q_n \leq Q \text{ and } q_{n-1} \quad \text{for} \quad 2 \leq n \leq \text{Max } n \qquad (6)$$

Calculation should be continued until such n for which

$$\pi_{n+1}(q_{n+1} \leq Q) \leq \pi_n(q_n \leq Q) \geq \pi_{n-1}(q_{n-1} \leq Q) \qquad (7)$$

which assumes a concave function.

The highest n which satisfies equation 7 provides the optimal number of segments, the corresponding optimal values of q^*_1, q^*_2, ..., q^*_n, and the associated segment room prices $p^*_i = f(q^*_i)$ for $i = 1, ..., n$, as well as the value of the maximal profit of the hotel $\pi_n(Q)$.

The Appendix shows how the model may be solved in an efficient manner.

3. NUMERICAL EXAMPLE

For a hotel with a capacity of $Q = 400$ identical rooms, where the fixed cost per day is F \$5000/day, the variable cost per room per day is $v =$ \$6/(day, room), and the additional fixed cost for each segment is $K =$ \$350/(day, segment), let the demand function be

$$P = 126.99\exp(-0.004621q)$$

This is an exponential curve which passes throught the points ($p = 80$, $q = 100$) and ($p = 20$, $q = 400$).

Numerical solution of equation 3 by trial-and-error provides $q_o = 192$ rooms.

Solution of the dynamic programming model [equations 2, 5 and 6] provides for stages $n = 1, 2, ..., 8$ the results of Table 1. The table vividly illustrates the fact that, indeed, the maximal profit at stage n is monotonically increasing with n until the optimal stage of $n = 7$. It provides the optimal solution of:

38	rooms	in	Segment	1	with	a	price	of	\$106.54	per	room,
80–38 = 42	"	"	"	2	"	"	"	"	\$87.75	"	",
127–80 = 47	"	"	"	3	"	"	"	"	\$70.62	"	",
180–127 = 53	"	"	"	4	"	"	"	"	\$55.28	"	",
241–180 = 61	"	"	"	5	"	"	"	"	\$41.70	"	",
313–241 = 72	"	"	"	6	"	"	"	"	\$29.90	"	",
400–313 = 87	"	"	"	7	"	"	"	"	\$20.00	"	",

which provides a total profit of \$10,568.70 per day.

The calculations required 68 s computation time on an IBM 286 PC.

APPENDIX

Reduction in the range of search

In order to facilitate the solution of the dynamic programming model [equations 5 and 6] in an efficient manner, the range of search at each stage n of the model was reduced in the manner as follows:

(a) Range of search for $\pi_2(q_2)$

Let Q^*_2 be the optimal number of rooms to be assigned to 2 segments, when the first segment is assigned Q_1 rooms (the optimal number of rooms with a single segment), which maximizes the profit from both segments. It is illustrated in Fig. 4.

Q^*_2 is found from the condition that the combined profit with 2 segments, given that the first segment is with Q_1 rooms, has to be maximal:

$$\operatorname*{Max}_{Q^*_2}\{Q_1 f(Q_1) + (Q^*_2 - Q_1) f(Q^*_2) - [F + K + vQ^*_2]\} \quad (8)$$

When the first derivative in respect to Q^*_2 [of the term in the ornamental brackets of equation 8], is equated to zero, we obtain

$$f(Q^*_2) + f(Q^*_2)[Q^*_2 - Q_1] - v = 0 \quad (9)$$

from which Q^*_2 is found using numerical search.

Since solution of equation 9 will not necessarily provide an integer value, we will select (again, as with the case of q_o) as Q^*_2 the uprounded or downrounded integer that maximizes equation 8.

Let us further define

$$Q_1 = \mathrm{Min}(Q, q_Q),$$

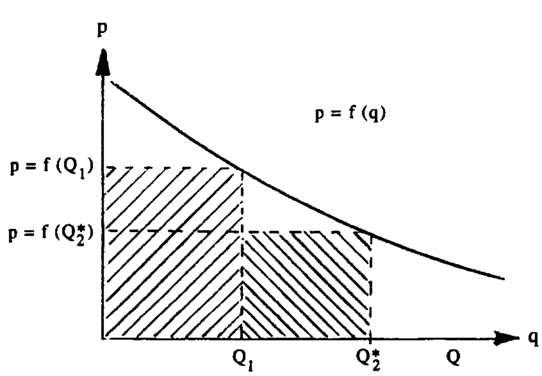

Fig. 4. Assignment of Q_1 rooms to First segment and Q^*_2

and

$$Q_2 = \mathrm{Min}(Q, Q^*_2).$$

Then, instead of searching (for each q_2, where $2 \le q_2 \le Q$) the value of q_1 that maximizes $\pi_2(q_2)$ in the range

$$1 \le q_1 \le q_2,$$

the search can be performed (for each q_2, where $2 \le q_2 \le Q_2$) for the value of q_1 that maximizes $\pi_2(q_2)$ in the range

$$1 \le q_1 \le q_2 \text{ if } q_2 \le Q_1$$

or

$$1 \le q_1 \le Q_1 \text{ if } q_2 > Q_1.$$

(b) Range of search for $\pi_n(q_n)$

For reasons similar to those outlined for the reduction of the range of search of $\pi_2(q_2)$, instead of searching (for each q_n, where $n \le q_n \le Q$) the value of q_{n-1} that maximizes $\pi_n(q_n)$ in the range

$$(n-1) \le q_{n-1} \le q_n,$$

the search can be performed (for each q_n, where $n \le q_n \le Q_n$) for the value of q_{n-1} that maximizes $\pi_n(q_n)$ in the range

$$(n-1) \le q_{n-1} \le q_n \text{ if } q_n \le Q_{n-1}$$

or

$$(n-1) \le q_{n-1} \le Q_{n-1} \text{ if } q_n > Q_{n-1}, \quad (10)$$

where

$$Q_n = \mathrm{Min}(Q, Q^*_n), \quad (11)$$

and Q^*_n is obtained from the solution (by numerical search) of

$$f(Q^*_n) + f'(Q^*_n)[Q^*_n - Q_{n-1}] - v = 0. \quad (12)$$

Equation 10 is valid for each stage $n > 2$, while for $n = 2$, as evident from equation 9, $Q_1 = \mathrm{Min}$ (Q, q_o).

Further reduction in the search (of the value of q_{n-1} that maximizes $\pi_n(q_n)$) can be obtained, when the search is started at the upper boundary [defined by equations 10–12] and the concavity assumption is utilized: the value of q_{n-1} should be reduced only as long as the resulting maximal

REFERENCES

1. American Hotel and Motel Association (1952) *The Hubert Formula for Evaluating Rate Structures of Hotel Rooms*. American Hotel Association, New York.
2. Belobaba PP (1989) Application of a probabilistic decision model to airline seat management control. *Opns Res*. **37**, 183–197.
3. Blozan W and Probhaker P (1984) Notes on aggregation criteria in market segmentation. *J. Mktg Res*. **XXI**, 332–335.
4. Bonoma TV and Shapiro BP (1983) *Segmenting the Industrial Market*. Lexington Books, Lexington, Mass.
5. Brumelle SL and McGill JI (1993) Airline seat allocation with multiple nested fare classes. *Opns Res*. **41**, 127–137.
6. Chakravarti D, Hendrix PE and Wilkie WL (1987) *Market Segmentation Research*, Vol. 1 and 2. Electric Power Research Institute, Palo Alto, Calif.
7. Claycamp HJ and Massy WF (1968) A theory of market segmentation. *J. Mktg Res*. **8**, 388–396.
8. Dickson PR and Ginter JL (1987) Market segmentation, product differentiation, and marketing strategy. *J. Mktg* **51**, No. 2, 1–10.
9. Dror M, Trudeau P and Ladany SP (1988) Network models for seat allocation on flights. *Transpn Res*. **22B**, 239–250.
10. Fay CR Jr and Rhoads RC (1976) *Managerial Accounting for the Hospitality Service Industry*, 2nd edition. Brown, Dubuque.
11. Frank RE, Massy WF and Wind Y (1972) *Market Segmentation*. Prentice–Hall, Englewood Cliffs, N.J.
12. Gensch DH (1978) Targeting the switchable industrial customer. *Mktg Sci*. **3**, 4.
13. Grover R and Srinivasan V (1987) A simultaneous approach to market segmentation and market structuring. *J. Mktg Res*. **XXIV**, 139–153.
14. Hersh M and Ladany S (1978) Optimal allocation of seats in flights with one intermediate stop. *Computers Opns Res*. **5**, 31–37.
15. Ladany S (1976) Dynamic operating rules for motel reservations. *Decis. Sci*. **7**, 829–840.
16. Ladany S (1977) A model for optimal hotel size. *Int. J. Syst. Sci*. **8**, 121–129.
17. Ladany S (1977) Bayesian dynamic operating rules for optimal hotel reservations. *Z. Opns Res*. **21**, B165–B176.
18. Ladany SP and Arbel A (1991) Optimal cruise-liner passenger cabin pricing policy. *Eur. J. Opns Res*. **55**, 136–147.
19. Ladany SP and Bedi DN (1975) Analysis of different boarding procedures. In *Management Science Applications to Leisure-Time Operations* (Edited by Ladany SP), pp. 197–213. North Holland, Amsterdam.
20. Ladany S and Bedi DN (1977) Dynamic booking rules for flights with an intermediate stop. *Omega* **5**, 721–730.
21. Ladany SP and Hersh M (1977) Non-stop versus one-stop flights. *Transpn Res*. **11**, 155–159.
22. Mahajan V and Jain AK (1984) An approach to normative segmentation. *J. Mktg Res*. **XV**, 338–345.
23. Massy WF and Weitz BA (1975) *A Normative Theory of Market Segmentation*. Graduate School of Business, Stanford University.
24. Moorthy SK (1978) Market segmentation, self-selection, and product line design. *Mktg Sci*. **3**, 288.
25. Pfeiffer PE (1989) The airline discount fare allocation problem. *Decis. Sci*. **20**, 149–157.
26. Smith WR (1956) Product differentiation and market segmentation as alternative marketing strategies. *J. Mktg* July, 3–8.
27. Smith WR (1978) Retrospective note on market segmentation. *J. Mktg Res*. **15**, 316.
28. Weatherford LR and Bodily SE (1992) A taxonomy and research overview of perishable-asset revenue management: yield management, overbooking and pricing. *Opns Res*. **40**, 831–844.
29. Wind Y (1978) Issues and advances in segmentation research. *J. Mkt Res*. **XV**, August.
30. Wollmer RD (1992) An airline management model for a single leg route when lower fare classes book first. *Opns Res*. **40**, 26–34.

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