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A New Classification of Variables in Design of Experiments

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Abstract: The standard system factor classification of process input variables as controllable or uncontrollable does not reflect the observed structure of some processes. Specifically, some factors that are classified as controllable are actually only semi-controllable. In this paper the classic variable structure is extended to controllable, semi-controllable, and uncontrollable. This extension is proposed in an effort to deal more accurately with real world problems.

Keywords: Process modeling, propagation of error.

1. Introduction

Historically the categorization of factors used in experimental design referred to treatments and blocks or plots based on the agriculture origins of designed experimentation (Box, [1]). In the 1980's the terminology began to shift toward control factors and noise factors based on the nomenclature popularized by Taguchi (Mori, [7]). By the 1990's this had evolved into controllable and uncontrollable variables (Montgomery, [6]). This categorization can logically be refined to reflect six types of factors -- controllable (experimental and fixed), uncontrollable (in-process, experimental and fixed), and blocking factors. Controllable factors are, as the name implies, controllable or selectable (e.g., conveyor belt speed). Uncontrollable factors on the other hand are factors that can not be controlled in the production environment (e.g., outside humidity) though some may be controllable during experimentation. Uncontrollable factors are frequently referred to as covariates, noise or random factors in the literature.

Unfortunately, this classification of input variables does not reflect the observed structure of some processes. Specifically, some factors that are typically classified as controllable are actually only semi-controllable. For example, in a wave solder process the solder pot temperature is usually classified as a controllable variable. However, only the control panel setting is controllable, the actual observed temperature of the molten solder is really semi-controllable. That is, the temperature may be set at 260 °C, but at the critical PCB-solder interface, where it actually counts, it might be 255 ± 10 °C. So while it is usually assumed that temperature is a controllable variable in reality it is actually a variable that is time dependent (i.e., changing the set point does not instantaneously change the temperature at the PCB-solder interface since the temperature of a large mass of material must be changed). The automatic temperature control system samples the solder pot temperature and the control algorithm then turns the power to the heater on and off in an attempt to achieve a “stable” melt temperature. This action by the automatic controller induces cyclic temperature variation at the PCB-solder interface. Further, the amount of variation induced may be different at different set points.

The historical classification of variables in experimental design fails to incorporate semi-controllable variables and this can have significant implications in both the design and analysis of experiments -- particularly experiments that target robust process development.

2. The Theory

Based on the above argument it seems that there are, excluding blocking variables, three types of process variables (see Figure 1). These variables will be designated as x, w, and z, and are classified as follows:

1. The x_i are controllable variables (experimental or fixed) that are set, but are not or can not be measured in-process so their variance is assumed to be zero. Or, if they are measured in-process, their variance is so small as to have little effect on process performance. The mean of these controllable variables in-process should ideally be the same as their input set points and analysis of empirical data can be used to establish the nature of the functional relationship.

2. The w_i are semi-controllable variables (experimental or fixed) that are set at the control panel and measured in-process. They are viewed as x's at the input level. However, the set value and the observed values in-process may differ over time with the observed values changing both in terms of their mean and/or their variance.

3. The z_i are uncontrollable variables (i.e., noise variables) that are measured in-process. There are two types – ones that cannot be controlled in either the experimental or production environment (denoted z_u), and those that can be controlled in the experimental environment but not in the production environment (denoted z). The effect of the first type ends up in the error term whereas the second type can become a variable in the process model. In what follows all the z_i are controllable during experimentation but not during production.

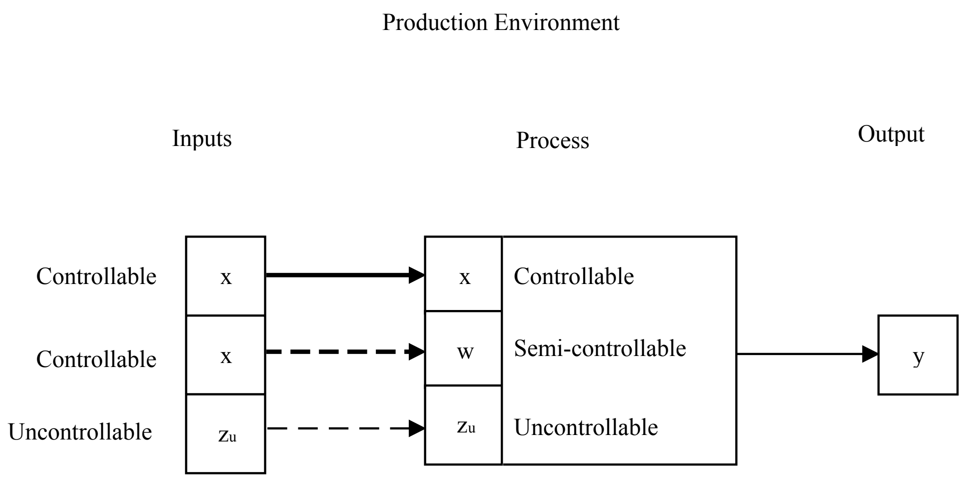

Figure 1. The process variable model for the production environment.

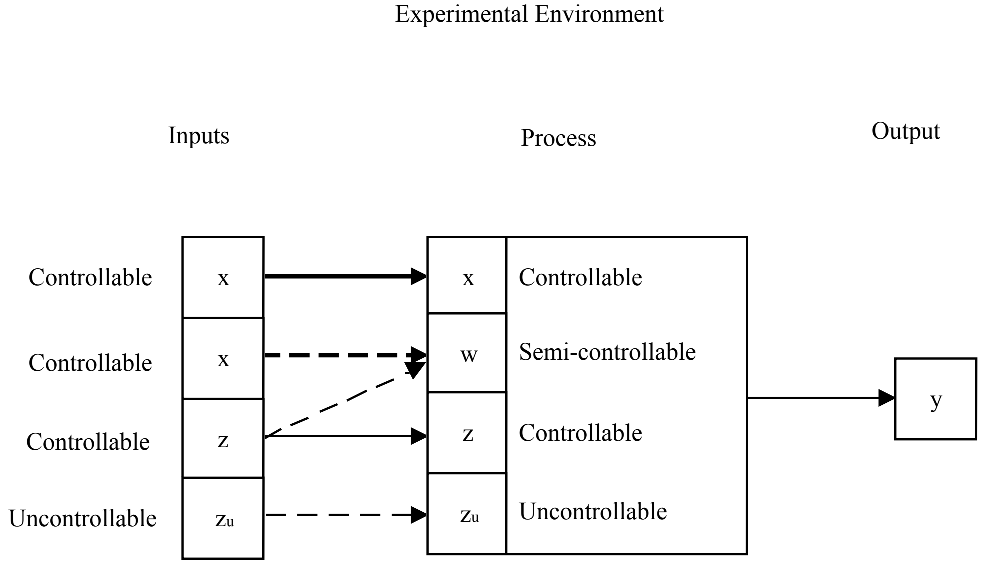

Figure 2. The process variable model for the experimental environment.

The practitioner should note that the classification of variables is a function of whether they are observed at the input level (i.e., at the controller), during experimentation, or during production. These distinctions are illustrated in Figure 1, and Figure 2.

Assuming an adequate model for the performance of a critical output quality characteristic y could be estimated through designed experiments, where the model for y is based on coded values of the input variables, the model could be represented by:

$$y = f(x, w, z) + e_m, \tag{0}$$

where f is a function in x, w, z, and where e_m is the model's residual error $e_m \sim N(0, \sigma_e)$. The output response y is a random variable and the input variables x, w, and z are assumed to be normal and independently distributed random variables with the following statistical properties in the experiment:

1. $E(x_i) = \mu_x$, $V(x_i) \approx 0$
2. $E(w_i) = \mu_w$, $V(w_i) > 0$
3. $E(z_i) = 0$, $V(z_i) = 1$, for controllable coded noise variables (i.e., $z_i \sim N(0, 1)$) (Koksoy, [5])

Let the mean value of y be the real valued function E(y). Further, since $E(z_i) = 0$, then all terms in f(x, w, z) that contain z drop out, as does e_m since $E(e_m) = 0$. Therefore, E(y) is given by:

$$E(y) = f(x, w, 0) + E(e_m) = g(x, w). \tag{1}$$

If f is a differentiable function, then the variance of y can be approximated by the real valued function h(x, w, z), at the mean of x, w, and z. Further, applying the theory of propagation of errors (Deming [3] and Box [2]), the first order Taylor series linear approximation for the transmitted variance of y is:

$$V(y) = \sum_{i=1}^{n_1}\left(\frac{\partial y}{\partial x_i}\bigg|_{\mu_1,\mu_2,\ldots,\mu_{n_1}}\right)^2 V(x_i) + \sum_{i=1}^{n_2}\left(\frac{\partial y}{\partial w_i}\bigg|_{\mu_1,\mu_2,\ldots,\mu_{n_2}}\right)^2 V(w_i) + \sum_{i=1}^{n_3}\left(\frac{\partial y}{\partial z_i}\bigg|_{\mu_1,\mu_2,\ldots,\mu_{n_3}}\right)^2 V(z_i) + V(e_m)$$

where all the terms that contain $V(x_i)$ drop out since $V(x_i) = 0$, $V(z_i) = 1$, and $V(e_m) = \sigma^2$. Therefore, V(y) is given by:

$$V(y) = \sum_{i=1}^{n_2}\left(\frac{\partial y}{\partial w_i}\bigg|_{\mu_1,\mu_2,\ldots,\mu_{n_2}}\right)^2 V(w_i) + \sum_{i=1}^{n_3}\left(\frac{\partial y}{\partial z_i}\bigg|_{\mu_1,\mu_2,\ldots,\mu_{n_3}}\right)^2 + \sigma^2 . \tag{2}$$

The model's residual error variance $V(e_m)$ at the mean can be estimated from the sample variance of the response function at the center point. This value is computed from the replicated runs performed at the center point of the experimental region. The residual term represents model uncertainty, which derives from the selection of the model and the input variables. If the practitioner has a measure of $V(x_i)$ for all x_i in the process and all have $V(x_i) \approx 0$, then the variance model of Equation 2 is reasonable. However, suppose that it is not possible to measure $V(x_i)$ for some x_i in the process, then the practitioner could allocate some of the model's residual error to this variable. This would essential result in an x variable being treated like a w variable.

What is the relationship between the semi-controllable variable x_i and its associated process level variable w_i? Intuitively it might seem reasonable that $w_i = mx_i + b$, where m and b are constants. This would be true if the relationship between the variables was linear. However, if the relationship is nonlinear, then a more complex situation arises and in such cases it becomes critically important to have in-process measurement of the w variable. Can such a situation arise? Consider a chemical reaction where the temperature is set at the controller but inside the process the reaction generates heat as a function of the reaction rate, then the set temperature and the observed temperature will differ. The reaction rate increases exponentially as a function of the input temperature (e.g., Arrhenius's equation), then heat added to the process will increase nonlinearly (i.e., the relationship between x and w will be nonlinear). Because of this, it is necessary to measure both the set value for w variables and the observed value so that the relationship between the two can be determined.

where $n'_2 < n_2$ and it is clear that V(y) has been changed, perhaps significantly. The robust optimization of the system requires that V(y) be considered but the input variable settings that minimize Equation 2 and the settings that minimize Equation 3 may be significantly different. This argument establishes the need to carefully delineate the type of variables in the experimental design and to recognize the existence and importance of semi-controllable variables.

Another, incorrect assumption might be to consider w variables as z type since they are noisy variables. The problem with this assumption is that E(z) is not controllable in the production environment whereas E(w) is controllable in production. This is a fundamental difference, since w's can be used to adjust E(y) whereas z's cannot (see Equation 1). Further, though both w's and z's affect V(y), (see Equation 2) the magnitude of V(y) can be changed by adjusting the w's, because they change V(w), whereas changing z's in the experiment would not change V(y) because V(z) is constant (i.e., V(z) = 1, see Equation 2) and in production the z's are uncontrollable.

Is the transmitted variance function, V(y), a good estimate of the observed variance of y, $V_o(y)$, (or put another way is $V(y) \approx V_o(y)$)? It may or may not be. For example, if some variable, say x_k is not included in the model for y due to low significance, then it will not appear in the transmitted variance function V(y). If there are replicated experimental runs, then the observed variance function $V_o(y)$ can be estimated and compared with V(y). Occasionally some variables that were left out of the mean model due to lack of significance may turn out to have a significant effect on the variance and hence should be included in the observed variance model (see Flaig, [4] for an example).

3. Example

Consider the following wave soldering experiment. At the control panel the solder pot temperature is set to the desired value (Tc = 260 °C) and at the actual PCB-solder interface (where product performance is determined) it is monitored by an external thermocouple or infrared sensor (Ti). The temperature of this system at the interface can be modeled using the following variables:

Tc = Temperature at the control panel set point

Ti = Temperature at the PCB-solder interface

Ti = f(Tc, t, e_c) where t is the time lag due to the solder pots thermal mass and e_c is the error due to the automatic temperature control system.

If the system is working properly and a run of a single PCB part number is being processed, the time lag variable can be removed from the process model assuming that production is not started until the system comes up to temperature and is not manually adjusted after that time. Further, it is assumed that the automatic temperature controller causes Ti to be a uniformly distributed random variable with mean T_0 and standard deviation s_0. Good process design would suggest that it would be desirable to have Ti = T_0 (an unbiased measurement system), and s_0 as small as possible (i.e., s_0 should be semi-adjustable by programming the controller).

The experiment designed to characterize the wave solder process had eight input variables and three response variables:

1. w_1: Solder Pot Temperature Ti (measured with an external thermocouple)
2. x_2: Conveyor Speed (machine set)

3. x_3: Solder Pot Temperature Tc (machine set)
4. x_4: Pre-Heater (machine set)
5. x_5: Fluxer Speed (machine set)
6. x_6: Air Knife (machine set)
7. x_7: Air Pressure (machine set)
8. x_8: Wave Height (machine set)
9. y_1: Insufficient Solder
10. y_2: Excess Solder
11. y_3: Solder Bridge

Table 1. Process performance before DOE.

Process Results (Defects/Board)	Insufficient Solder	Excess Solder	Solder Bridge
Average	0.57	0.33	0.25
Sigma	0.054	0.041	0.036

The general form of the insufficient solder response function is:

$$y_1 = f(x, w, z) + e_{m.}$$

From the DOE the expected insufficient solder response function was estimated using JMP software from SAS and standard least squares regression analysis. The resulting model was:

$$E(y_1) = -0.93 \ w_1 x_8 + 0.16 x_2 x_5 - 0.0029 x_3 x_6 + 243.05. \quad (4)$$

Applying the transmission of variance procedure described in Equation 2 to Equation 4 yields the estimated variance of the insufficient solder response function:

$$V(y_1) = [-0.93 \ E(x_8)]^2 V(w_1) + \sigma^2 \, . \quad (5)$$

It should be noted that Equation 5 would be a constant, if w_1 were to be treated as though it were a controllable variable (i.e., $V(w_1) = 0$). Under this assumption process optimization would consist of determining the feasible values of the input variables in Equation 4 that minimize the expected insufficient solder defects per board. Evaluating this assumption the insufficient solder defect rate per 100 boards rate drops from 57 at the nominal operating conditions to 33 assuming w_1 is considered controllable. This is certainly a significant improvement, but utilizing all the information about variability in the system the best result achieved was 8 defects per 100 boards. This is a 75% improvement from 33 defects per 100 boards, and illustrates the important role that semi-controllable variables play in system performance optimization.

Table 2. Process performance after DOE.

Process Results (Defects per Board)	Insufficient Solder	Excess Solder	Solder Bridge
Average	0.08	0	0.12
Sigma	0.017	0	0.021

Given Equations 4 and 5 the problem now becomes one of dual response optimization. Several good approaches to solving this problem are referenced in the literature (Koksoy, [5]). The approach we used in 1991 was to reduce $V(w_1)$ by fine tuning the solder pot automatic temperature controller algorithm. If the process model is adequate, then this should reduce the system variation with respect to insufficient solder. In addition, mean and variance response models were also developed for the other critical quality characteristics (Excess Solder and Solder Bridge) and multi-response optimization methods, based on the GRG algorithm, were used to adjust the key input variables. The results of this improvement program are displayed in Table 2.

4. Results

There were 40,000 keyboard PCB's produced using the new DOE based process and temperature controller settings. The verification results are displayed in Table 3.

Table 3. Process performance improvement summary.

	Before Defects/Board	After Defects/Board	Percentage Improvement
Insufficient Solder	0.57	0.08	86%
Excess Solder	0.33	0.00	100%
Solder Bridge	0.25	0.12	52%
Total Defects	11,500	2,000	83%

5. Conclusions

The addition of semi-controllable input variables into the general process model structure allows the practitioner to more accurately account for the effect of all types of variables on process performance and to develop more accurate models for estimating the mean response and the response variance. If more accurate process models result from this approach, then this should lead to better process optimization.

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