Time: Wednesday 9am-10am

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Q1

let $z = -\frac{(1-i\sqrt{3})^{11}}{(-i)^{47}}$, $z \in \mathbb{C}$

$\therefore \quad z = -\frac{\left(2e^{\frac{-\pi}{3}i}\right)^{11}}{\left(e^{\frac{-\pi}{2}i}\right)^{47}}$

$= -\frac{2048e^{\frac{-11\pi}{3}i}}{e^{\frac{-47}{2}i}}$

$= -\frac{2048e^{\frac{\pi}{3}i}}{e^{\frac{\pi}{2}i}}$

$= -2048e^{\left(\frac{\pi}{3}-\frac{\pi}{2}\right)i}$

$= -2048e^{\frac{-\pi}{6}i}$

$\therefore$ in polar form, $z = -2048e^{\frac{-\pi}{6}i}$

Q2

For set $D := \{ z \in \mathbb{C} \mid Im(z^2) = 0 \}$:

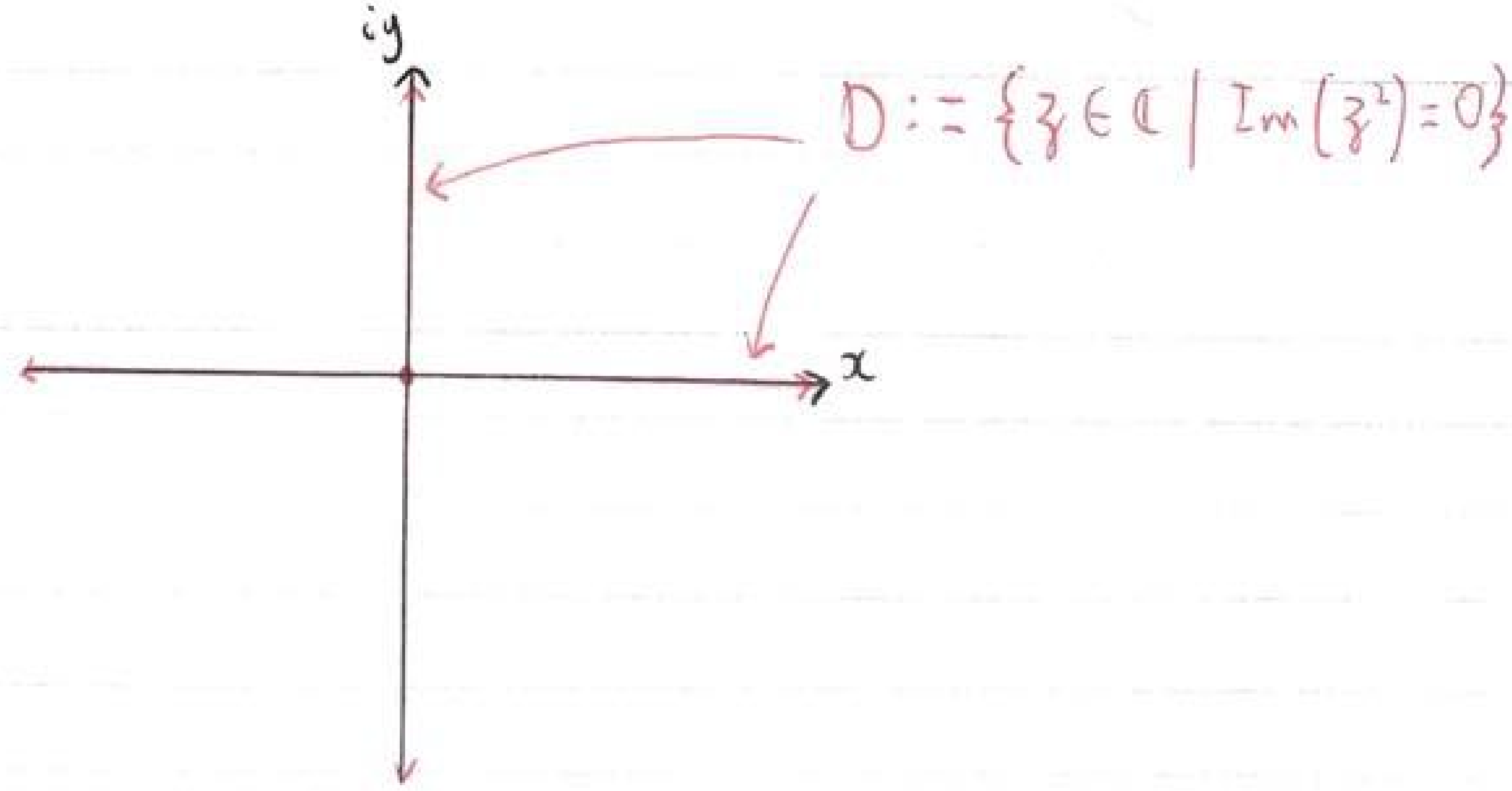
let $z = a + ib$, $z \in D$; $a, b \in \mathbb{R}$:

$$\therefore z^2 = (a+ib)^2$$
$$= a^2 - b^2 + 2abi$$

$$Im(z^2) = 2ab$$
$$\therefore 2ab = 0$$
$$ab = 0, \quad a = 0 \text{ or } b = 0$$

$\therefore$ The set D describes all points that lie on the real number line $(x=0)$, and all points that are imaginary $(iy=0)$

Sketch: (on the Argand Plane)



Q3

To find all solutions $z \in \mathbb{C}$ to the equation:

$$z^2 + 2z = -3$$

We first rearrange the equation:

$$z^2 + 2z + 3 = 0$$

Then we apply the quadratic formula:

$$z = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(3)}}{2} \qquad \left(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},\ ax^2 + bx + c = 0 \right)$$

$$= \frac{-2 \pm \sqrt{-8}}{2}$$

$$= \frac{-2 \pm 2\sqrt{2}\,i}{2} \qquad \left(\text{let } i = \sqrt{-1} \right)$$

$$= -1 \pm \sqrt{2}\,i$$

$\therefore$ The 2 solutions are:

$$z = -1 + \sqrt{2}\,i$$

or

$$z = -1 - \sqrt{2}\,i$$

Q 4

let z be all solution to $z^7 = \omega$, where $z \in \mathbb{C}$, and $\omega = 1 - i\sqrt{3}$:

We then find all 7 roots of ω as follows:

$$z = \left(1 - i\sqrt{3}\right)^{\frac{1}{7}}$$
$$= \left(2e^{\left(\frac{-\pi}{3} + 2k\pi\right)i}\right)^{\frac{1}{7}}, \quad \text{where } k \in \{0, \pm 1, \pm 2, \pm 3\},$$
$$= \left(2e^{\frac{6k-1}{3}\pi i}\right)^{\frac{1}{7}}$$
$$= \sqrt[7]{2}\, e^{\frac{6k-1}{21}\pi i}$$

$$\therefore z = \sqrt[7]{2}\, e^{\frac{-\pi}{21}i}, \sqrt[7]{2}\, e^{\frac{5\pi}{21}i}, \sqrt[7]{2}\, e^{\frac{-7\pi}{21}i}, \sqrt[7]{2}\, e^{\frac{11\pi}{21}i}, \sqrt[7]{2}\, e^{\frac{-13\pi}{21}i}, \sqrt[7]{2}\, e^{\frac{17\pi}{21}i}, \sqrt[7]{2}\, e^{\frac{-19\pi}{21}i}$$

The 7 solutions are: $\sqrt[7]{2}\, e^{\frac{-\pi}{21}i}, \sqrt[7]{2}\, e^{\frac{5\pi}{21}i}, \sqrt[7]{2}\, e^{\frac{-\pi}{3}i}, \sqrt[7]{2}\, e^{\frac{11\pi}{21}i}, \sqrt[7]{2}\, e^{\frac{-13\pi}{21}i}, \sqrt[7]{2}\, e^{\frac{17\pi}{21}i}, \sqrt[7]{2}\, e^{\frac{-19\pi}{21}i}$

Q5

let $z \in \mathbb{C}$ be all solutions to $z^4 + 8z^2 - 9 = 0$.

We rearrange the above as follows:

$$(z^2)^2 + 8(z^2) - 9 = 0$$

By inspection, this is a quadratic with 2 real roots, and can be written as:

$$(z+9)(z-1) = 0$$

$$\therefore z = -9 \quad \text{or} \quad z - 1 = 0,$$
$$z = 1$$

However, as this is also a polynomial of order 4, there are 4 Complex roots.

If $z = -3i$ is one such root, then by the Conjugate pair theorem, $z = 3i$ must also be a root.

The 4 roots are thus as follows:

$$z = -9, \; z = 1, \; z = -3i, \; z = 3i.$$