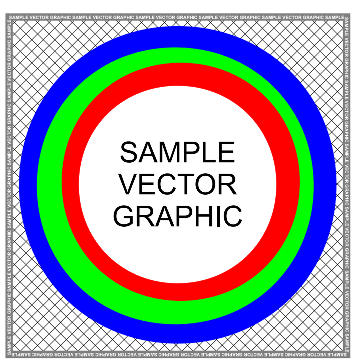
Insert Thesis Title Here
Use Manual Line Breaks If Necessary

Thesis by

Insert Author Name Here

In Partial Fulfillment of the Requirements
for the Degree of
Doctor of Philosophy
in
Electrical Engineering and Computer Science



University Institute of College
Springfield, New York, USA

2016
(Defended November 25, 2016)

DRAFT 2019-08-19 16:24

DRAFT 2019-08-19 16:24

Insert dedication here

Acknowledgments

Insert thesis acknowledgments here. Thesis acknowledgments typically include research advisers and mentors, thesis committee members, collaborators, and funding sources.

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

Abstract

Insert thesis abstract here. The thesis abstract provides a concise description of the main contributions in the thesis. Abstracting and indexing services usually include the thesis abstract in the catalog presented to users. A well-written abstract could improve the chances of your work being discovered and cited by others in the research community. The abstract should be self-contained (avoid citations and cross references) and should contain only plain text (avoid complicated mathematical expressions).

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a,

ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.

Quisque ullamcorper placerat ipsum. Cras nibh. Morbi vel justo vitae lacus tincidunt ultrices. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. In hac habitasse platea dictumst. Integer tempus convallis augue. Etiam facilisis. Nunc elementum fermentum wisi. Aenean placerat. Ut imperdiet, enim sed gravida sollicitudin, felis odio placerat quam, ac pulvinar elit purus eget enim. Nunc vitae tortor. Proin tempus nibh sit amet nisl. Vivamus quis tortor vitae risus porta vehicula.

Fusce mauris. Vestibulum luctus nibh at lectus. Sed bibendum, nulla a faucibus semper, leo velit ultricies tellus, ac venenatis arcu wisi vel nisl. Vestibulum diam. Aliquam pellentesque, augue quis sagittis posuere, turpis lacus congue quam, in hendrerit risus eros eget felis. Maecenas eget erat in sapien mattis porttitor. Vestibulum porttitor. Nulla facilisi. Sed a turpis eu lacus commodo facilisis. Morbi fringilla, wisi in dignissim interdum, justo lectus sagittis dui, et vehicula libero dui cursus dui. Mauris tempor ligula sed lacus. Duis cursus enim ut augue. Cras ac magna. Cras nulla. Nulla egestas. Curabitur a leo. Quisque egestas wisi eget nunc. Nam feugiat lacus vel est. Curabitur consectetuer.

Suspendisse vel felis. Ut lorem lorem, interdum eu, tincidunt sit amet, laoreet vitae, arcu. Aenean faucibus pede eu ante. Praesent enim elit, rutrum at, molestie non, nonummy vel, nisl. Ut lectus eros, malesuada sit amet, fermentum eu, sodales cursus, magna. Donec eu purus. Quisque vehicula, urna sed ultricies auctor, pede lorem egestas dui, et convallis elit erat sed nulla. Donec luctus. Curabitur et nunc. Aliquam dolor odio, commodo pretium, ultricies non, pharetra in, velit. Integer arcu est, nonummy in, fermentum faucibus, egestas vel, odio.

Contents

List of Figures

List of Tables

PART I

Background

1 Stern-Gerlach Experiments

1.1 Stern-Gerlach Backgorund

TODO

1.2 Stern-Gerlach Measurement

We begin by comparing standard and consistent descriptions of measuring a quantum states' spin along one axis using a Stern-Gerlach apparatus. In standard quantum mechanics, the act of measurement plays a special role in assigning probabilities to measurement outcomes through the probability postulate, and in determmining the evolution of the state through the projection postulate.

The probability postulate assigns probabilities to each measurement result by taking the inner product of $|\psi\rangle$ and the eigenstate $|n\rangle$ corresponding to measurement result n:

$$\mathcal{P}_n = |\langle n|\psi\rangle|^2 \tag{1.1}$$

The projection postulate describes an instantaneous evolution of the input state to the eigenstate corresponding to the measruement result. If input $|\psi\rangle$ is measured to have spin n (up or down), then the new state is

$$|\psi\rangle' = \frac{P_n |\psi\rangle}{\sqrt{\langle\psi| P_n |\psi\rangle}} \tag{1.2}$$

where $P_n = |n\rangle\langle n|$ is the projection operator for state $|n\rangle$. This operation projects $|\psi\rangle$ onto $|n\rangle$, then divides by the magnitude of that projection. The end result is that $|\psi\rangle$ becomes the normalized state $|n\rangle$ corresponding to measuring spin n, which is shown by the states exiting the each apparatus. This process is known as *state collapse* or *wavefunction collapse*.

TODO: show probability computations TODO: explain how measurement is not itself modeled as a physical process, and we are directed to use these postulates upon poorly defined "measurement"

In consistent quantum theory, measurement is modeled as a physical process. Each Stern-Gerlach apparatus has its own detector state space,

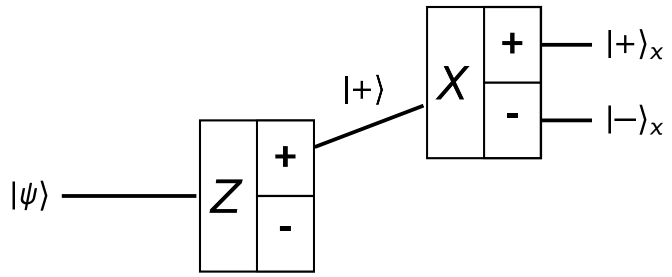


FIGURE 1.1 Demonstrating renormalizing upon measurment in standard quantum mechanics

containing orthonormal states representing each measurement result. A detector state could represent a needle pointing up, or a particle colliding with a screen in a distinguishable region. Let the state space of the z apparatus be represented by

$$\mathcal{H}_{Dz} = \{|D_+\rangle_z, |D_-\rangle_z\} \tag{1.3}$$

where

$$_z\langle D_+|D_+\rangle_z = 1 \tag{1.4}$$
$$_z\langle D_+|D_-\rangle_z = 0 \tag{1.5}$$

$\mathcal{H}_{Dx}$ is similarly defined. Each detector state space is a subset of a global detector state space $\mathcal{H}_D$, so that we can require orthogonality of states in seperate dector spaces:

$$\begin{aligned} &_z\langle D_n|D_m\rangle_x = 0 \\ &\qquad\qquad \forall\, |D_n\rangle_z \in \mathcal{H}_{Dz} \\ &\qquad\qquad \forall\, |D_m\rangle_x \in \mathcal{H}_{Dx} \end{aligned}$$

The act of measurement is described by correlating detector states with corresponding quantum states. The system then evolves by

$$\begin{aligned} &V: \\ &|\psi\rangle = \left(\sum_n P_n|\psi\rangle\right) \otimes \left(\sum_m P_m|D\rangle\right) \mapsto \sum_n P_n|\psi\rangle \otimes |D_n\rangle \end{aligned} \tag{1.6}$$

where P_n is the projection operator for the n^{th} spin eigenstate and P_m is the projection operator for the m^{th} detector state.

This evolution describes a change in the eigenstates of the system. Before,

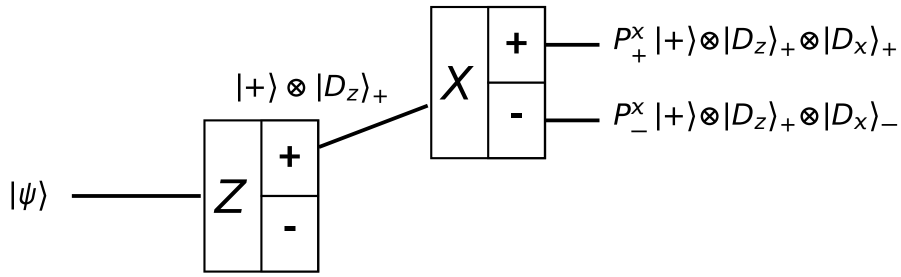


FIGURE 1.2 Demonstrating description of measurment as an abstract physical process in consistent quantum mechanics

$|\psi\rangle$ was a superposition of the tensor products of any spin eigenstate and any detector state. Afterwards, $|\psi\rangle$ is a superposition of the tensor products of a spin eigenstate and one specific detector state.

Measurment is now described by an abstract physical process. In this model, it is now feasible for "state collapse" to occur independent of physicists conducting clever experiments.

To compute probabilities of each outcome, we sum the magnitudes of each branch wavefunction that includes the corresponding detector state. This is accomplished by finding the trace of the projection operator of that detector state acting on the projection operator or the overall evolved state.

$$\mathcal{P}_n = Tr(P^D_n \cdot V \, |\psi\rangle \, \langle\psi| \, V^\dagger) \tag{1.7}$$

In this simple Stern-Gerlach example, there is only one branch wavefunction correlated with each detector state. So, computing probabilities is done by finding the magnitude of each branch wavefunction.

TODO: compute probabilities, show it is equal to std QM

1.3 Complementarity

We now compare standard and consistent quantum mechanics' treatment of the principle of complementarity. Arguably the most fundamental feature of quantum mechanics, the principle of complementarity states that a quantum system has pairs of physical observables which cannot be measured simultaneously. Components of spin on orthogonal axes are complemntary properties, so we examine measurements of succesive Stern-Gerlach experiments.

First, we compute the probabilities of each outcome using standard quantum mechanics. The first apparatus serves as a state preparation device with output $|+\rangle$. By the direction of the projection postulate, the

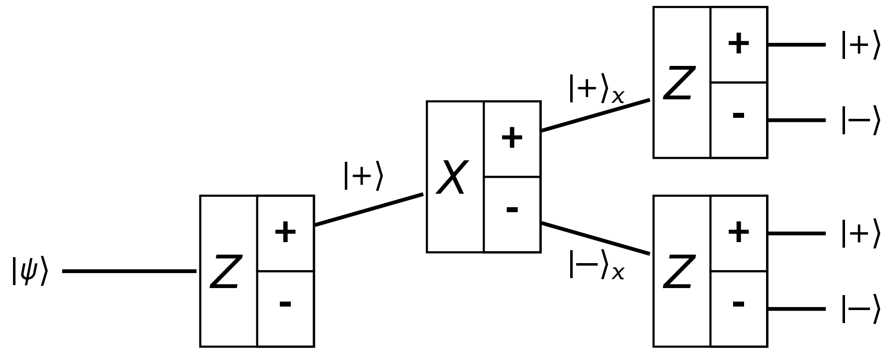


FIGURE 1.3 Demonstrating complementary measurments in standard quantum mechanics

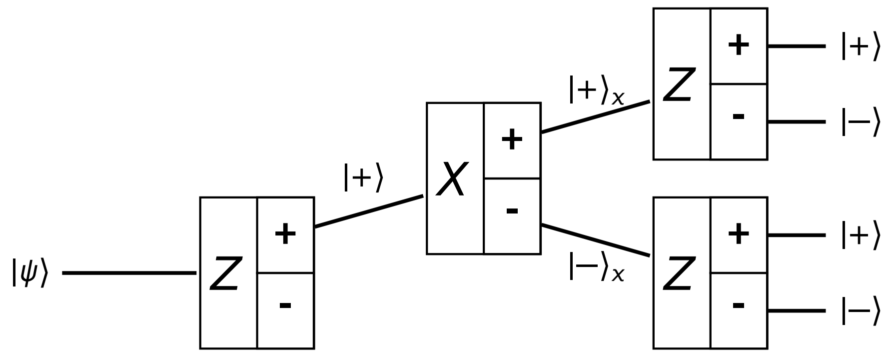


FIGURE 1.4 Demonstrating complementary measurments in consistent quantum mechanics

state is renormalized upon each measruement. After measuring a property complementary to what is known (such as spin along x, knowing spin along z), any information known about the input state is lost; the input state instantaneously changes to the state corresponding to the observed quantity. Consequently, there is an equal probability of observing the final state as $|+\rangle$ or $|-\rangle$ at either final apparatus, even though the state was initially prepared as $|+\rangle$, since

$$\begin{aligned}\mathcal{P}_n &= |\langle +|+\rangle_x|^2 \\ &= |\langle -|+\rangle_x|^2 \\ &= |\langle +|-\rangle_x|^2 \\ &= |\langle -|-\rangle_x|^2 \\ &= \frac{1}{4}\end{aligned}$$

This contradiction with classical intuition is a direct result of the projection postulate. TODO: describe measurement problem.

TODO: Demonstrate complementarity and single framework rule

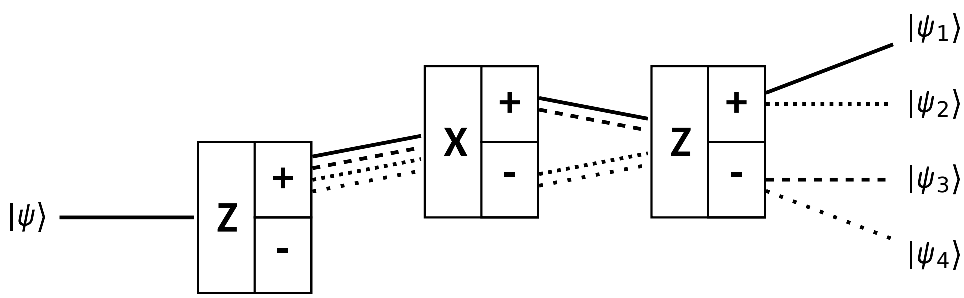


FIGURE 1.5 TODO: create section to discuss this example, and how it creates an inconsistent set of histories. Describe how the set can be made to be consistent

2 Insert Chapter Title Here

2.1 Introduction

Insert chapter introduction here. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

Related Work: Our work is related to [1–5]. Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu

Insert chapter footnote here. The chapter footnote could include citations to related publications by the author (“The material in this chapter was presented in part in ....”).

massa.

Quisque ullamcorper placerat ipsum. Cras nibh. Morbi vel justo vitae lacus tincidunt ultrices. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. In hac habitasse platea dictumst. Integer tempus convallis augue. Etiam facilisis. Nunc elementum fermentum wisi. Aenean placerat. Ut imperdiet, enim sed gravida sollicitudin, felis odio placerat quam, ac pulvinar elit purus eget enim. Nunc vitae tortor. Proin tempus nibh sit amet nisl. Vivamus quis tortor vitae risus porta vehicula.

Our Contribution: Fusce mauris. Vestibulum luctus nibh at lectus. Sed bibendum, nulla a faucibus semper, leo velit ultricies tellus, ac venenatis arcu wisi vel nisl. Vestibulum diam. Aliquam pellentesque, augue quis sagittis posuere, turpis lacus congue quam, in hendrerit risus eros eget felis. Maecenas eget erat in sapien mattis porttitor. Vestibulum porttitor. Nulla facilisi. Sed a turpis eu lacus commodo facilisis. Morbi fringilla, wisi in dignissim interdum, justo lectus sagittis dui, et vehicula libero dui cursus dui. Mauris tempor ligula sed lacus. Duis cursus enim ut augue. Cras ac magna. Cras nulla. Nulla egestas. Curabitur a leo. Quisque egestas wisi eget nunc. Nam feugiat lacus vel est. Curabitur consectetuer.

Suspendisse vel felis. Ut lorem lorem, interdum eu, tincidunt sit amet, laoreet vitae, arcu. Aenean faucibus pede eu ante. Praesent enim elit, rutrum at, molestie non, nonummy vel, nisl. Ut lectus eros, malesuada sit amet, fermentum eu, sodales cursus, magna. Donec eu purus. Quisque vehicula, urna sed ultricies auctor, pede lorem egestas dui, et convallis elit erat sed nulla. Donec luctus. Curabitur et nunc. Aliquam dolor odio, commodo pretium, ultricies non, pharetra in, velit. Integer arcu est, nonummy in, fermentum faucibus, egestas vel, odio.

Proofs of theorems are deferred to Section 2.5.

2.2 Some Examples

Sed commodo posuere pede. Mauris ut est. Ut quis purus. Sed ac odio. Sed vehicula hendrerit sem. Duis non odio. Morbi ut dui. Sed accumsan risus eget odio. In hac habitasse platea dictumst. Pellentesque non elit. Fusce sed justo eu urna porta tincidunt. Mauris felis odio, sollicitudin sed, volutpat a, ornare ac, erat. Morbi quis dolor. Donec pellentesque, erat ac sagittis semper, nunc dui lobortis purus, quis congue purus metus ultricies tellus. Proin et quam. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos hymenaeos. Praesent sapien turpis, fermentum vel, eleifend faucibus, vehicula eu, lacus.

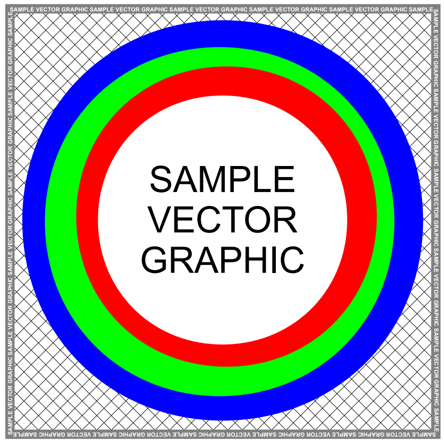


FIGURE 2.1 Insert the full caption here for this floating figure.

2.2.1 Examples of Figures and Tables

This is a reference to Figure 2.1. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Donec odio elit, dictum in, hendrerit sit amet, egestas sed, leo. Praesent feugiat sapien aliquet odio. Integer vitae justo. Aliquam vestibulum fringilla lorem. Sed neque lectus, consectetuer at, consectetuer sed, eleifend ac, lectus. Nulla facilisi. Pellentesque eget lectus. Proin eu metus. Sed porttitor. In hac habitasse platea dictumst. Suspendisse eu lectus. Ut mi mi, lacinia sit amet, placerat et, mollis vitae, dui. Sed ante tellus, tristique ut, iaculis eu, malesuada ac, dui. Mauris nibh leo, facilisis non, adipiscing quis, ultrices a, dui.

Here we say something about Figures 2.1 and 2.2. Note how the effect in Figure 2.2 is stronger that in Figure 2.1. Morbi luctus, wisi viverra faucibus pretium, nibh est placerat odio, nec commodo wisi enim eget quam. Quisque libero justo, consectetuer a, feugiat vitae, porttitor eu, libero. Suspendisse sed mauris vitae elit sollicitudin malesuada. Maecenas ultricies eros sit amet ante. Ut venenatis velit. Maecenas sed mi eget dui varius euismod. Phasellus aliquet volutpat odio. Vestibulum ante ipsum primis in faucibus orci luctus et ultrices posuere cubilia Curae; Pellentesque sit amet pede ac sem eleifend consectetuer. Nullam elementum, urna vel imperdiet sodales, elit ipsum pharetra ligula, ac pretium ante justo a nulla. Curabitur tristique arcu eu metus. Vestibulum lectus. Proin mauris. Proin eu nunc eu urna hendrerit faucibus. Aliquam auctor, pede consequat laoreet varius, eros tellus scelerisque quam, pellentesque hendrerit ipsum dolor sed augue. Nulla nec lacus.

We summarize our notation in Table 2.1. Suspendisse vitae elit. Aliquam arcu neque, ornare in, ullamcorper quis, commodo eu, libero. Fusce sagittis erat at erat tristique mollis. Maecenas sapien libero, molestie et, lobortis in,

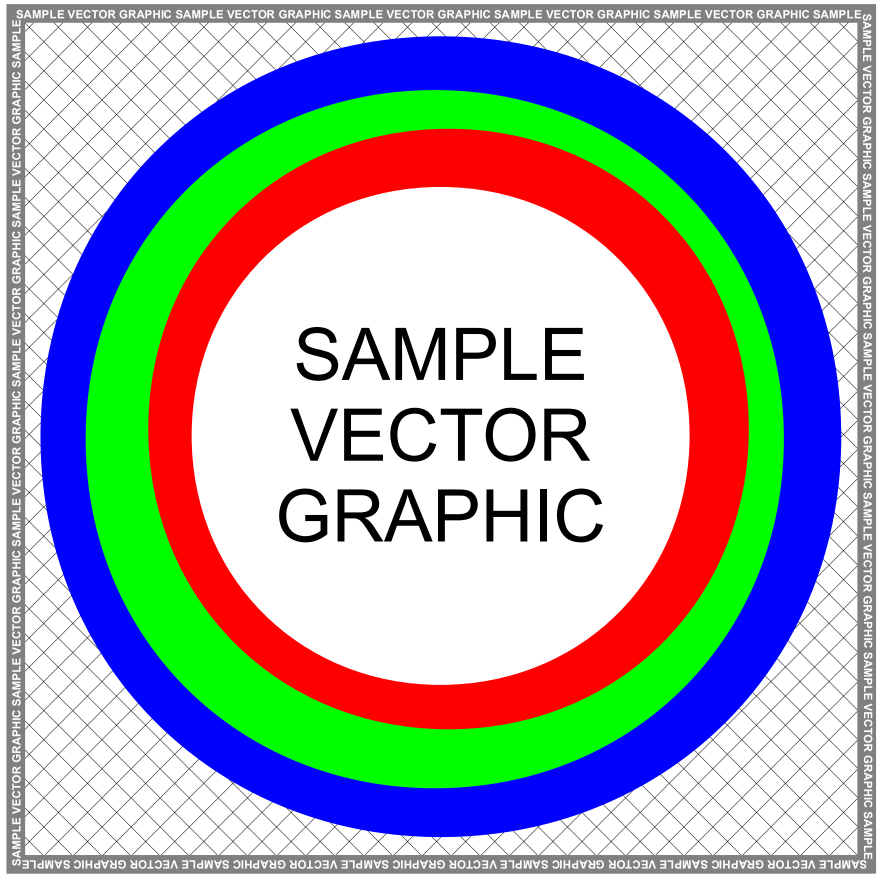


FIGURE 2.2 Insert the full caption here for this floating figure. The caption should provide sufficient context to interpret the figure. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris.

TABLE 2.1 Insert the full caption here for this floating table.

Symbol	Definition
α	insert definition of α here, $\alpha \geq 1$
β	insert definition of β here, $\beta \geq 2$
γ	insert definition of γ here, $\gamma \geq 3$
δ	insert definition of δ here, $\delta \geq 4$

TABLE 2.2 Insert the full caption here for this floating table. The caption should provide sufficient context to interpret the table. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris.

Variable	Initial Value	Value at $t = 100$
c	0.012	3.456
δ	0.312	1.416
γ	0.042	3.252
h	0.012	3.353
c	0.012	4.446
δ	0.015	3.556
γ	0.612	6.656
h	0.072	7.456
c	0.018	8.756
δ	0.912	9.456
γ	0.092	5.956
h	0.012	2.326

sodales eget, dui. Morbi ultrices rutrum lorem. Nam elementum ullamcorper leo. Morbi dui. Aliquam sagittis. Nunc placerat. Pellentesque tristique sodales est. Maecenas imperdiet lacinia velit. Cras non urna. Morbi eros pede, suscipit ac, varius vel, egestas non, eros. Praesent malesuada, diam id pretium elementum, eros sem dictum tortor, vel consectetuer odio sem sed wisi.

Table 2.2 summarizes our simulation results. Sed feugiat. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Ut pellentesque augue sed urna. Vestibulum diam eros, fringilla et, consectetuer eu, nonummy id, sapien. Nullam at lectus. In sagittis ultrices mauris. Curabitur malesuada erat sit amet massa. Fusce blandit. Aliquam erat volutpat. Aliquam euismod. Aenean vel lectus. Nunc imperdiet justo nec dolor.

2.2.2 Examples of Enumerated and Itemized Lists

Here are some citations [5–10]. The following is an enumerated list, or numbered list, with multiple levels:

1) First level item
2) First level item
    a) Second level item
    b) Second level item
        i) Third level item
            A) Fourth level item
            B) Fourth level item
        ii) Third level item
    c) Second level item
3) First level item

We draw your attention to items 1 and 3 in particular because they are very important in our study. The following is an itemized list, or unnumbered list, with multiple levels:

- First level item
- First level item
    - Second level item
    - Second level item
        - Third level item
            - Fourth level item
            - Fourth level item
        - Third level item
    - Second level item
- First level item

2.3 Some More Examples

According to [11], this behavior can be explained this way. Etiam euismod. Fusce facilisis lacinia dui. Suspendisse potenti. In mi erat, cursus id, nonummy sed, ullamcorper eget, sapien. Praesent pretium, magna in eleifend egestas, pede pede pretium lorem, quis consectetuer tortor sapien facilisis magna. Mauris quis magna varius nulla scelerisque imperdiet. Aliquam non quam.

Aliquam porttitor quam a lacus. Praesent vel arcu ut tortor cursus volutpat. In vitae pede quis diam bibendum placerat. Fusce elementum convallis neque. Sed dolor orci, scelerisque ac, dapibus nec, ultricies ut, mi. Duis nec dui quis leo sagittis commodo.

2.3.1 Examples of Mathematical Expressions, Definitions, and Theorems

We have the following unnumbered mathematical equation:

$$E = mc^2.$$

On the other hand, the following is a numbered mathematical inequality:

$$x \leq \frac{\sum_{i=1}^{n} y^2 \cdot \mathbb{1}[y > 1]}{\int_{-\infty}^{\infty} x^3 \, \mathrm{d}z \cdot \binom{\alpha}{\beta} \frac{\lfloor \frac{a}{b} \rfloor}{\lceil \frac{c}{d} \rceil}}. \tag{2.1}$$

Inequality (2.1) will be applied multiple times to prove our theorems, in a manner similar to [12, 13]. We now introduce the following definition:

DEFINITION 2.1 (Name of Term Being Defined) This is the definition of the term, along with relevant conditions, trivial cases, exceptions, etc.

We can rewrite the result of [13, Theorem 2.5] in the following convenient form for our problem:

PROPOSITION 2.2 *For all $a, b, c \in \mathbb{Z}^+$, we have*

$$a^2 + b^3 \leq c^4.$$

Based on our numerical observations, we make the following conjecture about the upper bound:

CONJECTURE 2.3 *If $x \geq 3$ and $0 < y < x^2$, then for all $n \in \mathbb{Z}^+$,*

$$\sum_{i=1}^{n} x_i = x_1 + x_2 + \cdots + x_n \leq T_{\text{all}}.$$

Here is a lemma that will be quite useful in deriving our results:

LEMMA 2.4 (Name of Lemma if any) *If $x, y, z \in \mathbb{Z}_0^+$, then $f(x + y + z) = 1$.*

Applying Lemma 2.4 to [11, Theorem 4.2] produces the following theorem:

THEOREM 2.5 (Name of Theorem if any) *If $x + y \geq z$, then*

$$\sum_{i=x}^{y} f(i) \leq z.$$

As a special case of Theorem 2.5, we have the following corollary:

COROLLARY 2.6 *If $x = 4$ and $y = z$, then $\sum_{i=x}^{y} f(i) = 5$.*

Aliquam lectus. Vivamus leo. Quisque ornare tellus ullamcorper nulla. Mauris porttitor pharetra tortor. Sed fringilla justo sed mauris. Mauris tellus. Sed non leo. Nullam elementum, magna in cursus sodales, augue est scelerisque sapien, venenatis congue nulla arcu et pede. Ut suscipit enim vel sapien. Donec congue. Maecenas urna mi, suscipit in, placerat ut, vestibulum ut, massa. Fusce ultrices nulla et nisl.

2.4 Conclusion and Future Work

Etiam ac leo a risus tristique nonummy. Donec dignissim tincidunt nulla. Vestibulum rhoncus molestie odio. Sed lobortis, justo et pretium lobortis, mauris turpis condimentum augue, nec ultricies nibh arcu pretium enim. Nunc purus neque, placerat id, imperdiet sed, pellentesque nec, nisl. Vestibulum imperdiet neque non sem accumsan laoreet. In hac habitasse platea dictumst. Etiam condimentum facilisis libero. Suspendisse in elit quis nisl aliquam dapibus. Pellentesque auctor sapien. Sed egestas sapien nec lectus. Pellentesque vel dui vel neque bibendum viverra. Aliquam porttitor nisl nec pede. Proin mattis libero vel turpis. Donec rutrum mauris et libero. Proin euismod porta felis. Nam lobortis, metus quis elementum commodo, nunc lectus elementum mauris, eget vulputate ligula tellus eu neque. Vivamus eu dolor.

Nulla in ipsum. Praesent eros nulla, congue vitae, euismod ut, commodo a, wisi. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Aenean nonummy magna non leo. Sed felis erat, ullamcorper in, dictum non, ultricies ut, lectus. Proin vel arcu a odio lobortis euismod. Vestibulum ante ipsum primis in faucibus orci luctus et ultrices posuere cubilia Curae; Proin ut est. Aliquam odio. Pellentesque massa turpis, cursus eu, euismod nec, tempor congue, nulla. Duis viverra gravida mauris. Cras tincidunt. Curabitur eros ligula, varius ut, pulvinar in, cursus faucibus, augue.

2.5 Proofs of Theorems

Remember to manually disable (and re-enable) updates to the table of contents (TOC), using

`\DisableTOCUpdates` and `\EnableTOCUpdates`,

if you want to omit subsections, tables, figures, etc., from the table of contents.

2.5.1 Proof of Lemma 2.4

Nulla mattis luctus nulla. Duis commodo velit at leo. Aliquam vulputate magna et leo. Nam vestibulum ullamcorper leo. Vestibulum condimentum rutrum mauris. Donec id mauris. Morbi molestie justo et pede. Vivamus eget turpis sed nisl cursus tempor. Curabitur mollis sapien condimentum nunc. In wisi nisl, malesuada at, dignissim sit amet, lobortis in, odio. Aenean consequat arcu a ante. Pellentesque porta elit sit amet orci. Etiam at turpis nec elit ultricies imperdiet. Nulla facilisi. In hac habitasse platea dictumst. Suspendisse viverra aliquam risus. Nullam pede justo, molestie nonummy, scelerisque eu, facilisis vel, arcu.

Curabitur tellus magna, porttitor a, commodo a, commodo in, tortor. Donec interdum. Praesent scelerisque. Maecenas posuere sodales odio. Vivamus metus lacus, varius quis, imperdiet quis, rhoncus a, turpis. Etiam ligula arcu, elementum a, venenatis quis, sollicitudin sed, metus. Donec nunc pede, tincidunt in, venenatis vitae, faucibus vel, nibh. Pellentesque wisi. Nullam malesuada. Morbi ut tellus ut pede tincidunt porta. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam congue neque id dolor.

■

2.5.2 Proof of Theorem 2.5

Donec et nisl at wisi luctus bibendum. Nam interdum tellus ac libero. Sed sem justo, laoreet vitae, fringilla at, adipiscing ut, nibh. Maecenas non sem quis tortor eleifend fermentum. Etiam id tortor ac mauris porta vulputate. Integer porta neque vitae massa. Maecenas tempus libero a libero posuere dictum. Vestibulum ante ipsum primis in faucibus orci luctus et ultrices posuere cubilia Curae; Aenean quis mauris sed elit commodo placerat. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos hymenaeos. Vivamus rhoncus tincidunt libero. Etiam elementum pretium justo. Vivamus est. Morbi a tellus eget pede tristique commodo. Nulla nisl. Vestibulum sed nisl eu sapien cursus rutrum.

The following lemma will be quite useful in deriving the theorem:

LEMMA 2.7 *If $a, b, c \in \mathbb{Z}$, then $g(a \cdot b \cdot c) \leq -1$.*

Proof of Lemma 2.7: Nulla non mauris vitae wisi posuere convallis. Sed eu nulla nec eros scelerisque pharetra. Nullam varius. Etiam dignissim elementum metus. Vestibulum faucibus, metus sit amet mattis rhoncus, sapien dui laoreet odio, nec ultricies nibh augue a enim. Fusce in ligula. Quisque at magna et nulla commodo consequat. Proin accumsan imperdiet sem. Nunc porta. Donec feugiat mi at justo. Phasellus facilisis ipsum quis ante. In ac elit eget ipsum pharetra faucibus. Maecenas viverra nulla in massa.

Nulla ac nisl. Nullam urna nulla, ullamcorper in, interdum sit amet, gravida ut, risus. Aenean ac enim. In luctus. Phasellus eu quam vitae turpis viverra pellentesque. Duis feugiat felis ut enim. Phasellus pharetra, sem id porttitor sodales, magna nunc aliquet nibh, nec blandit nisl mauris at pede. Suspendisse risus risus, lobortis eget, semper at, imperdiet sit amet, quam. Quisque scelerisque dapibus nibh. Nam enim. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Nunc ut metus. Ut metus justo, auctor at, ultrices eu, sagittis ut, purus. Aliquam aliquam.

■

Etiam pede massa, dapibus vitae, rhoncus in, placerat posuere, odio. Vestibulum luctus commodo lacus. Morbi lacus dui, tempor sed, euismod eget, condimentum at, tortor. Phasellus aliquet odio ac lacus tempor faucibus. Praesent sed sem. Praesent iaculis. Cras rhoncus tellus sed justo ullamcorper sagittis. Donec quis orci. Sed ut tortor quis tellus euismod tincidunt. Suspendisse congue nisl eu elit. Aliquam tortor diam, tempus id, tristique eget, sodales vel, nulla. Praesent tellus mi, condimentum sed, viverra at, consectetuer quis, lectus. In auctor vehicula orci. Sed pede sapien, euismod in, suscipit in, pharetra placerat, metus. Vivamus commodo dui non odio. Donec et felis.

Applying Lemma 2.7 yields the following:

$$\begin{multline} A + B + C + D + E + F + \alpha + \beta + \gamma + \delta + \Gamma \\ \leq \Omega + \Sigma + \omega + \sigma + \Theta + \theta + \epsilon + S + T + U + V + W + X + Y + Z. \tag{2.2} \end{multline}$$

Finally, the desired result is obtained by substituting $A = b$ into (2.2). ■

2.6 Acknowledgment

Insert chapter acknowledgment here. Etiam suscipit aliquam arcu. Aliquam sit amet est ac purus bibendum congue. Sed in eros. Morbi non orci. Pellentesque mattis lacinia elit. Fusce molestie velit in ligula. Nullam et orci vitae nibh vulputate auctor. Aliquam eget purus. Nulla auctor wisi sed ipsum. Morbi porttitor tellus ac enim. Fusce ornare. Proin ipsum enim, tincidunt in, ornare venenatis, molestie a, augue. Donec vel pede in lacus sagittis porta. Sed hendrerit ipsum quis nisl. Suspendisse quis massa ac nibh pretium cursus. Sed sodales. Nam eu neque quis pede dignissim ornare. Maecenas eu purus ac urna tincidunt congue.

3 Summary and Future Work

3.1 Summary

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.

Quisque ullamcorper placerat ipsum. Cras nibh. Morbi vel justo vitae lacus tincidunt ultrices. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. In hac habitasse platea dictumst. Integer tempus convallis augue. Etiam facilisis. Nunc elementum fermentum wisi. Aenean placerat. Ut imperdiet, enim sed gravida sollicitudin, felis odio placerat quam, ac pulvinar elit purus eget enim. Nunc vitae tortor. Proin tempus nibh sit amet nisl. Vivamus quis tortor vitae risus porta vehicula.

Fusce mauris. Vestibulum luctus nibh at lectus. Sed bibendum, nulla a faucibus semper, leo velit ultricies tellus, ac venenatis arcu wisi vel nisl. Vestibulum diam. Aliquam pellentesque, augue quis sagittis posuere, turpis lacus congue quam, in hendrerit risus eros eget felis. Maecenas eget erat in sapien mattis porttitor. Vestibulum porttitor. Nulla facilisi. Sed a turpis eu lacus commodo facilisis. Morbi fringilla, wisi in dignissim interdum, justo lectus sagittis dui, et vehicula libero dui cursus dui. Mauris tempor ligula sed lacus. Duis cursus enim ut augue. Cras ac magna. Cras nulla. Nulla egestas. Curabitur a leo. Quisque egestas wisi eget nunc. Nam feugiat lacus vel est. Curabitur consectetuer.

Suspendisse vel felis. Ut lorem lorem, interdum eu, tincidunt sit amet, laoreet vitae, arcu. Aenean faucibus pede eu ante. Praesent enim elit, rutrum at, molestie non, nonummy vel, nisl. Ut lectus eros, malesuada sit amet, fermentum eu, sodales cursus, magna. Donec eu purus. Quisque vehicula, urna sed ultricies auctor, pede lorem egestas dui, et convallis elit erat sed nulla. Donec luctus. Curabitur et nunc. Aliquam dolor odio, commodo pretium, ultricies non, pharetra in, velit. Integer arcu est, nonummy in, fermentum faucibus, egestas vel, odio.

3.2 Future Work

Sed commodo posuere pede. Mauris ut est. Ut quis purus. Sed ac odio. Sed vehicula hendrerit sem. Duis non odio. Morbi ut dui. Sed accumsan risus eget odio. In hac habitasse platea dictumst. Pellentesque non elit. Fusce sed justo eu urna porta tincidunt. Mauris felis odio, sollicitudin sed, volutpat a, ornare ac, erat. Morbi quis dolor. Donec pellentesque, erat ac sagittis semper, nunc dui lobortis purus, quis congue purus metus ultricies tellus. Proin et quam. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos hymenaeos. Praesent sapien turpis, fermentum vel, eleifend faucibus, vehicula eu, lacus.

Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Donec odio elit, dictum in, hendrerit sit amet, egestas sed, leo. Praesent feugiat sapien aliquet odio. Integer vitae justo. Aliquam vestibulum fringilla lorem. Sed neque lectus, consectetuer at, consectetuer sed, eleifend ac, lectus. Nulla facilisi. Pellentesque eget lectus. Proin eu metus. Sed porttitor. In hac habitasse platea dictumst. Suspendisse eu lectus. Ut mi mi, lacinia sit amet, placerat et, mollis vitae, dui. Sed ante tellus, tristique ut, iaculis eu, malesuada ac, dui. Mauris nibh leo, facilisis non, adipiscing quis, ultrices a, dui.

Morbi luctus, wisi viverra faucibus pretium, nibh est placerat odio, nec commodo wisi enim eget quam. Quisque libero justo, consectetuer a, feugiat vitae, porttitor eu, libero. Suspendisse sed mauris vitae elit sollicitudin malesuada. Maecenas ultricies eros sit amet ante. Ut venenatis velit. Maecenas sed mi eget dui varius euismod. Phasellus aliquet volutpat odio. Vestibulum ante ipsum primis in faucibus orci luctus et ultrices posuere cubilia Curae; Pellentesque sit amet pede ac sem eleifend consectetuer. Nullam elementum, urna vel imperdiet sodales, elit ipsum pharetra ligula, ac pretium ante justo a nulla. Curabitur tristique arcu eu metus. Vestibulum lectus. Proin mauris. Proin eu nunc eu urna hendrerit faucibus. Aliquam auctor, pede consequat laoreet varius, eros tellus scelerisque quam, pellentesque hendrerit ipsum dolor sed augue. Nulla nec lacus.

Suspendisse vitae elit. Aliquam arcu neque, ornare in, ullamcorper quis, commodo eu, libero. Fusce sagittis erat at erat tristique mollis. Maecenas sapien libero, molestie et, lobortis in, sodales eget, dui. Morbi ultrices rutrum lorem. Nam elementum ullamcorper leo. Morbi dui. Aliquam sagittis. Nunc placerat. Pellentesque tristique sodales est. Maecenas imperdiet lacinia velit. Cras non urna. Morbi eros pede, suscipit ac, varius vel, egestas non, eros. Praesent malesuada, diam id pretium elementum, eros sem dictum tortor, vel consectetuer odio sem sed wisi.

Sed feugiat. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Ut pellentesque augue sed urna. Vestibulum diam eros, fringilla et, consectetuer eu, nonummy id, sapien. Nullam at lectus. In sagittis ultrices mauris. Curabitur malesuada erat sit amet massa. Fusce blandit. Aliquam erat volutpat. Aliquam euismod. Aenean vel lectus. Nunc imperdiet justo nec dolor.

Etiam euismod. Fusce facilisis lacinia dui. Suspendisse potenti. In mi erat, cursus id, nonummy sed, ullamcorper eget, sapien. Praesent pretium, magna in eleifend egestas, pede pede pretium lorem, quis consectetuer tortor sapien facilisis magna. Mauris quis magna varius nulla scelerisque imperdiet. Aliquam non quam. Aliquam porttitor quam a lacus. Praesent vel arcu ut tortor cursus volutpat. In vitae pede quis diam bibendum placerat. Fusce elementum convallis neque. Sed dolor orci, scelerisque ac, dapibus nec, ultricies ut, mi. Duis nec dui quis leo sagittis commodo.

Bibliography

[1] A. Bo, C. Deng, E. Fang, G. Huang, J. Koh, L. Meng, and N. Ong, "This is the title of the conference paper about TFT and MySQL," in *Proc. IEEE International Conference on Many Topics (ICMT)*, Las Vegas, Nevada, USA, Jan. 1993, pp. 290–294.

[2] J. Doe, A. B. Charles, and X. Y. Zane, "This is the title of the article," *This is the Journal Name*, vol. 11, pp. 13–15, Mar. 2006.

[3] A. Bo, C. Deng, E. Fang, G. Huang, J. Koh, L. Meng, and N. Ong, "This is the title of the conference paper about TFT and MySQL," in *Proc. IEEE International Conference on Many Topics (ICMT)*, Las Vegas, Nevada, USA, Oct. 1994, pp. 190–194.

[4] J. Doe, A. B. Charles, and X. Y. Zane, "This is the title of the article," *This is the Journal Name*, vol. 12, pp. 103–110, Jun. 2007.

[5] A. Bo, C. Deng, E. Fang, G. Huang, J. Koh, L. Meng, and N. Ong, "This is the title of the conference paper about TFT and MySQL," in *Proc. IEEE International Conference on Many Topics (ICMT)*, Las Vegas, Nevada, USA, Feb. 1995, pp. 90–94.

[6] J. Doe, A. B. Charles, and X. Y. Zane, "This is the title of the article," *This is the Journal Name*, vol. 13, pp. 123–130, Apr. 2008.

[7] A. Bo, C. Deng, E. Fang, G. Huang, J. Koh, L. Meng, and N. Ong, "This is the title of the conference paper about TFT and MySQL," in *Proc. IEEE International Conference on Many Topics (ICMT)*, Las Vegas, Nevada, USA, Aug. 1996, pp. 1290–1294.

[8] J. Doe, A. B. Charles, and X. Y. Zane, "This is the title of the article," *This is the Journal Name*, vol. 14, pp. 143–150, Sep. 2009.

[9] A. Bo, C. Deng, E. Fang, G. Huang, J. Koh, L. Meng, and N. Ong, "This is the title of the conference paper about TFT and MySQL," in *Proc. IEEE International Conference on Many Topics (ICMT)*, Las Vegas, Nevada, USA, Nov. 1997, pp. 2290–2294.

[10] J. Doe, A. B. Charles, and X. Y. Zane, “This is the title of the article,” *This is the Journal Name*, vol. 12, pp. 1123–1130, Dec. 2010.

[11] B. D. Cullity, *Introduction to Magnetic Materials*. Reading, MA: Addison-Wesley, 1972.

[12] S. Zhang, C. Zhu, J. K. O. Sin, and P. K. T. Mok, “A novel ultrathin elevated channel low-temperature poly-Si TFT,” *Journal Name*, vol. 20, pp. 569–571, Nov. 1999.

[13] R. K. Gupta and S. D. Senturia, “Pull-in time dynamics as a measure of pull-in time dynamics as a measure of absolute pressure,” in *Proc. IEEE International Workshop on Microelectromechanical Systems (MEMS)*, Nagoya, Japan, Jan. 1997, pp. 290–294.