The consistent histories approach to the Stern-Gerlach experiment

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The Stern-Gerlach (SG) Experiment

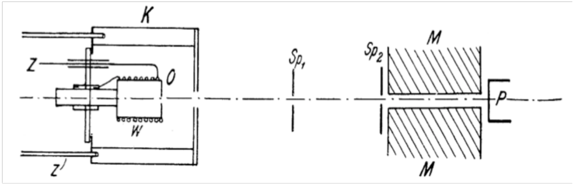


Figure: Otto Stern's original schematic (1922)

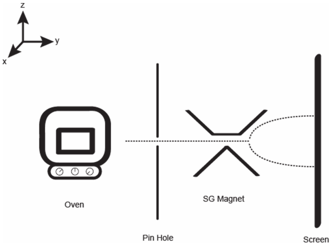
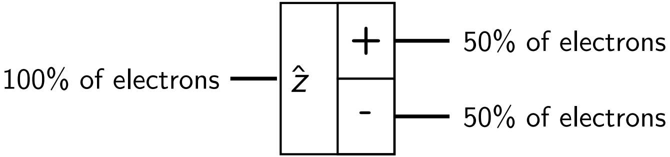
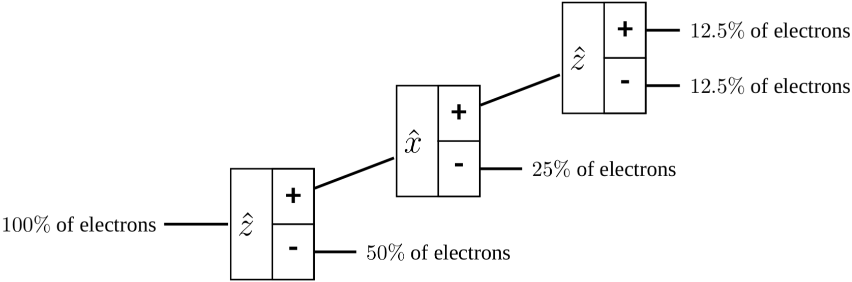


Figure: Adapted from a modern analysis of the experiment by Rodríguez et al. (2016)

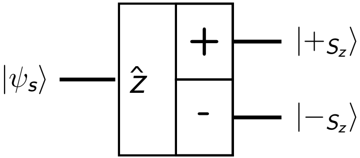
Results of the SG Experiment

- Spin angular momentum is quantized
- Complementary physical variables





Standard approach



- Foundational assumptions are made about measurement
- Eigenvalues for operator variables are “measurement results”
- Born rule assigns probabilities to the frequency of “measurement results”
- State collapse prescribes special non-deterministic dynamics upon “measurement”
- Dependent on ill-defined “classical observer”

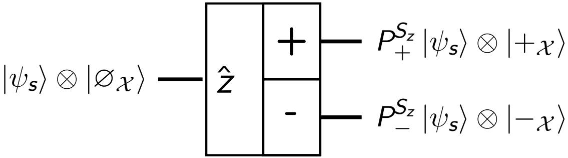
Motivating the removal of state collapse

- Ambiguity of "classical observer" makes quantum mechanics vulnerable to pseudoscience
- Quantum mechanics is limited to predictions involving classical observers
- State collapse is asymmetric in time

How can we describe the Stern-Gerlach experiment without postulating state collapse?

Unitary measurement

- Measurement can be described by the Schrödinger equation
- We begin by formalizing the spin-position interaction



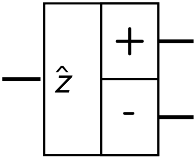
$P_\pm^{S_z} = |\pm\rangle\langle\pm|$ is the projection operator for the $|\pm\rangle_{S_z}$ state.

$$U(t_1, t_0)\,|\psi_s\rangle = P_+^{S_z}\,|\psi_s\rangle \otimes |+_{\mathcal{X}}\rangle \;+\; P_-^{S_z}\,|\psi_s\rangle \otimes |-_{\mathcal{X}}\rangle$$

Interaction with the environment

- Newton's third law requires that the electron exerts a force on the magnet
- The "environment" includes all systems other than the electron (including the magnet)

$$|\psi_s\rangle \otimes |\varnothing_{\mathcal{X}}\rangle \otimes |\varnothing_\epsilon\rangle$$



$$P_+^{S_z}|\psi_s\rangle \otimes |+_{\mathcal{X}}\rangle \otimes |+_\epsilon\rangle$$

$$P_-^{S_z}|\psi_s\rangle \otimes |-_{\mathcal{X}}\rangle \otimes |-_\epsilon\rangle$$

$$U(t_1, t_0) = \left(P_+^{S_z} \otimes \mathcal{S}^{\mathcal{X}}_{\varnothing,+} \otimes \mathcal{S}^{\epsilon}_{\varnothing,+}\right) + \left(P_-^{S_z} \otimes \mathcal{S}^{\mathcal{X}}_{\varnothing,-} \otimes \mathcal{S}^{\epsilon}_{\varnothing,-}\right)$$

Environment as a record keeper

- The causal effects of the force on the magnet encode *which-state* information about the electron
- The environment is responsible for "measuring" the electron instead of an undefined classical observer
- The environment records "facts of the universe"

Consistent histories

- Probabilistic predictions can no longer be made in the context of "measurement results"
- The consistent histories approach makes predictions about exhaustive and mutually exclusive sets of event sequences
- Events are represented by projectors $P_n^A = |n_a\rangle \langle n_a|$

Class operators

- Sequences of events (histories) are represented by *class operators*
- Class operators consist of projection operators to select events, and unitary operators to evolve the state
- The history for measuring a general initial state as spin-up in the z and then x direction is

$$\begin{aligned} C_h^\dagger &= U^\dagger(t_2, t_0) P_+^{S_x} U(t_2, t_1) P_+^{S_z} U(t_1, t_0) P_\varnothing \\ &= P_+^{S_x}(t_2) P_+^{S_z}(t_1) P_\varnothing(t_0) \end{aligned}$$

Extending the Born Rule

- The standard Born Rule assigns a probability distribution to a set of "state collapse" measurement outcomes

$$\mathcal{P}(h_n) = \langle\psi| P_n^A |\psi\rangle$$

- The event projector is replaced by the class operator

$$\mathcal{P}(h_n) = \langle\psi| C_h^\dagger |\psi\rangle$$

Reproducing the standard predictions

Using consistent histories, the predictions of the standard Born Rule are reproduced for the Stern-Gerlach Experiment

$$\begin{aligned}\mathcal{P}(\pm) &= \langle\psi| P_{\pm}^{S_z}(t_1) P_{\varnothing}(t_0) |\psi\rangle \\ &= \langle\psi_s| P_{\pm}^{S_z} |\psi_s\rangle \langle\pm_{\mathcal{X}}|\pm_{\mathcal{X}}\rangle \langle\pm_{\epsilon}|\pm_{\epsilon}\rangle \\ &= \langle\psi_s| P_{\pm}^{S_z} |\psi_s\rangle\end{aligned}$$

Open research questions

- Problem of outcomes

$$U(t_1, t_0)\,|\psi_s\rangle = P_+^{S_z}\,|\psi_s\rangle \otimes |+\chi\rangle \;+\; P_-^{S_z}\,|\psi_s\rangle \otimes |-\chi\rangle$$

- Many-worlds interpretation