STAT 5310 Take-Home Test #2

Tom Wilson

November 7, 2018

1. (5 points)

Enter the variables X1, X2, and X3 into a multiple regression model predicting Y. Display the regression output.

```
##
## Call:
## lm(formula = Y ~ X1 + X2 + X3, data = data1)
##
## Residuals:
##          1          2          3          4          5
## -2.493e-13  2.924e-13  3.733e-14 -3.893e-14 -4.144e-14
##
## Coefficients:
##               Estimate Std. Error    t value Pr(>|t|)
## (Intercept) -1.000e+03  2.728e-10 -3.665e+12 1.74e-13 ***
## X1           1.000e+00  2.728e-13  3.666e+12 1.74e-13 ***
## X2           1.000e+00  2.727e-13  3.667e+12 1.74e-13 ***
## X3           1.330e-14  2.156e-13  6.200e-02    0.961
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.902e-13 on 1 degrees of freedom
## Multiple R-squared:      1,  Adjusted R-squared:      1
## F-statistic: 4.992e+25 on 3 and 1 DF,  p-value: 1.04e-13
```

2. (15 points)

Create a table of R2adjusted, AIC, AICC, and BIC for the best subset of each size. Identify the optimal model or models from the approach based on all possible subsets.

best metrics for each size

k	max(adj.r.squared)	min(AIC)	min(AICc)	min(BIC)
1	0.8764847	15.88064	17.21397	14.70895
2	1.0000000	-273.89202	-267.89202	-275.45426
3	1.0000000	-279.69529	-255.69529	-281.64810

best models for each metric

model	k	adj.r.squared	AIC	AICc	BIC
Y ~ X2+X1+X1	2	1	-273.8920	-267.8920	-275.4543
Y ~ X3+X1+X2	3	1	-279.6953	-255.6953	-281.6481
Y ~ X2+X1+X1	2	1	-273.8920	-267.8920	-275.4543
Y ~ X3+X1+X2	3	1	-279.6953	-255.6953	-281.6481

3. (10 points)

Use the Forward Elimination method to determine the regression equation when starting with the same predictor variables listed in 1.

```
## Start:  AIC=9.59
## Y ~ 1
##
##        Df Sum of Sq     RSS     AIC
## + X3    1   20.6879  2.1121 -0.3087
## + X1    1    8.6112 14.1888  9.2151
## + X2    1    8.5064 14.2936  9.2519
## <none>              22.8000  9.5866
##
## Step:  AIC=-0.31
## Y ~ X3
##
##        Df Sum of Sq    RSS      AIC
## <none>              2.1121 -0.30875
## + X2    1  0.066328 2.0458  1.53172
## + X1    1  0.064522 2.0476  1.53613
##
## Call:
## lm(formula = Y ~ X3, data = data1)
##
## Residuals:
##        1        2        3        4        5
##  0.03434  0.13124 -0.43912 -0.82850  1.10203
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   0.7975     1.3452   0.593   0.5950
## X3            0.6947     0.1282   5.421   0.0123 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.8391 on 3 degrees of freedom
## Multiple R-squared:  0.9074, Adjusted R-squared:  0.8765
## F-statistic: 29.38 on 1 and 3 DF,  p-value: 0.01232
```

Forward selection recommends Y~X3 as the model which minimizes AIC.

4. (10 points)

Repeat Part 3, except use the Backward Elimination method.
Is the solution different from the one you got using the Forward method?

```
## Start:  AIC=-285.77
## Y ~ X1 + X2 + X3
```

```
## Warning: attempting model selection on an essentially perfect fit is
## nonsense
```

```
##        Df Sum of Sq    RSS      AIC
## - X3    1    0.0000 0.0000 -287.749
## <none>              0.0000 -285.768
## - X1    1    2.0458 2.0458    1.532
## - X2    1    2.0476 2.0476    1.536
##
## Step:  AIC=-287.75
## Y ~ X1 + X2
```

```
## Warning: attempting model selection on an essentially perfect fit is
## nonsense
```

```
##        Df Sum of Sq    RSS      AIC
## <none>               0.000 -287.749
## - X2    1    14.189 14.189    9.215
## - X1    1    14.294 14.294    9.252
```

```
##
## Call:
## lm(formula = Y ~ X1 + X2, data = data1)
##
## Residuals:
##          1          2          3          4          5
## -2.545e-13  2.854e-13  5.287e-14 -2.928e-14 -5.448e-14
##
## Coefficients:
##               Estimate Std. Error    t value Pr(>|t|)
## (Intercept) -1.000e+03  7.388e-11 -1.354e+13   <2e-16 ***
## X1           1.000e+00  7.312e-14  1.368e+13   <2e-16 ***
## X2           1.000e+00  7.339e-14  1.363e+13   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.764e-13 on 2 degrees of freedom
## Multiple R-squared:      1,  Adjusted R-squared:      1
## F-statistic: 1.492e+26 on 2 and 2 DF,  p-value: < 2.2e-16
```

Backward elimination recommends Y~X1+X2 as the model which minimizes AIC.

5. (10 points)

Are different models chosen?
If so, carefully explain why different models are chosen.

Different models are found by forward and backward selection.

Forward selection finds Y~X3 is the best k=1 model (minimizes AIC) subsequently, adding X2 or X1 does not decrease AIC.

backward selection finds that removing Y3 from the full model, Y~X1+X2+X3 minimizes AIC and subsequently removing X1 or X2 does not decrease AIC.

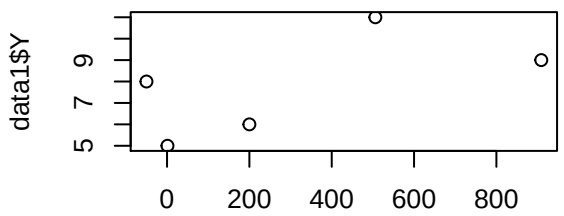
Decide on which model you would recommend.
At this point, describe and examine the assumptions of multiple linear regression for your recommended model.

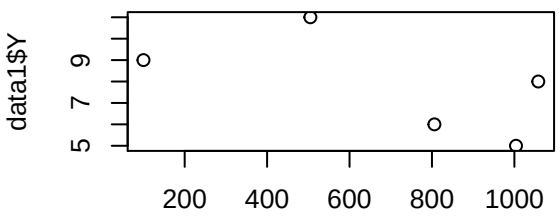
If any assumptions are violated – discuss what steps would/should be performed.

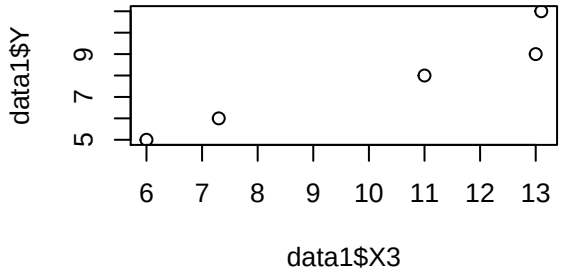
Checking Assumptions

Linear relationship

There is an assumption that Y is related to each x by the simple linear regression. We can confirm this with scatterplots.

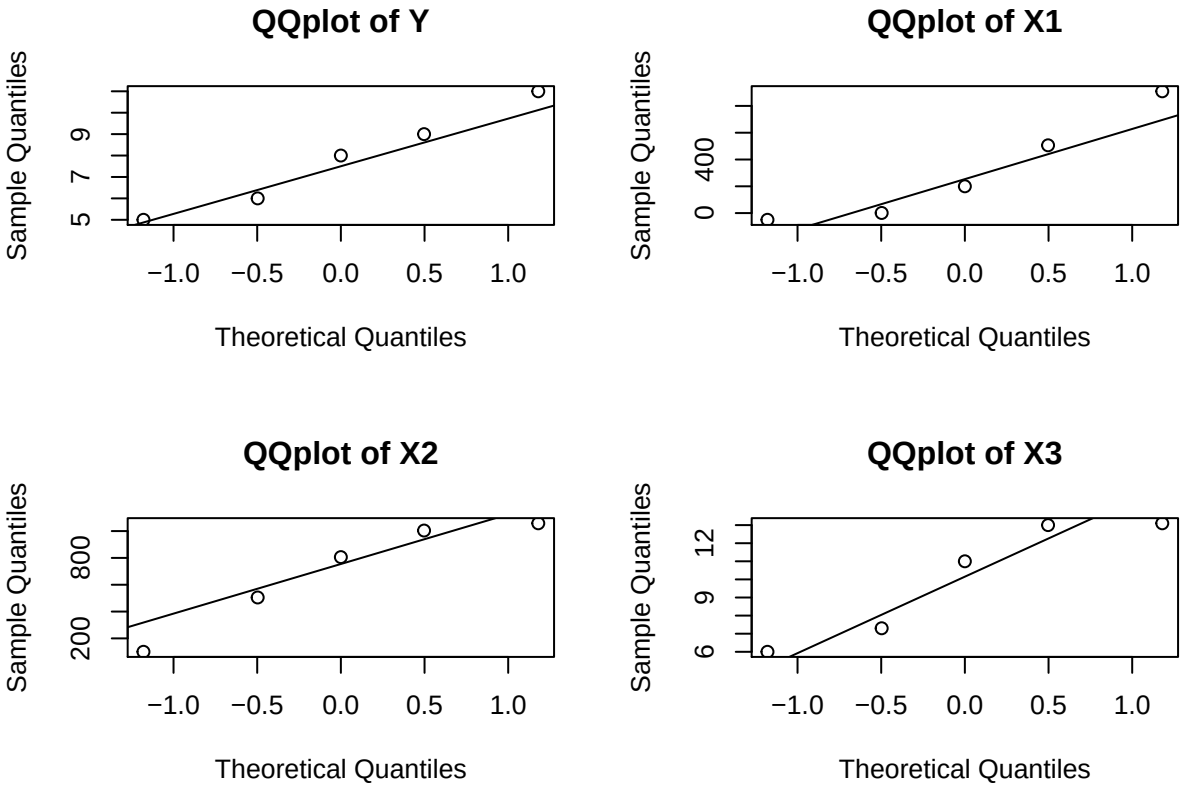






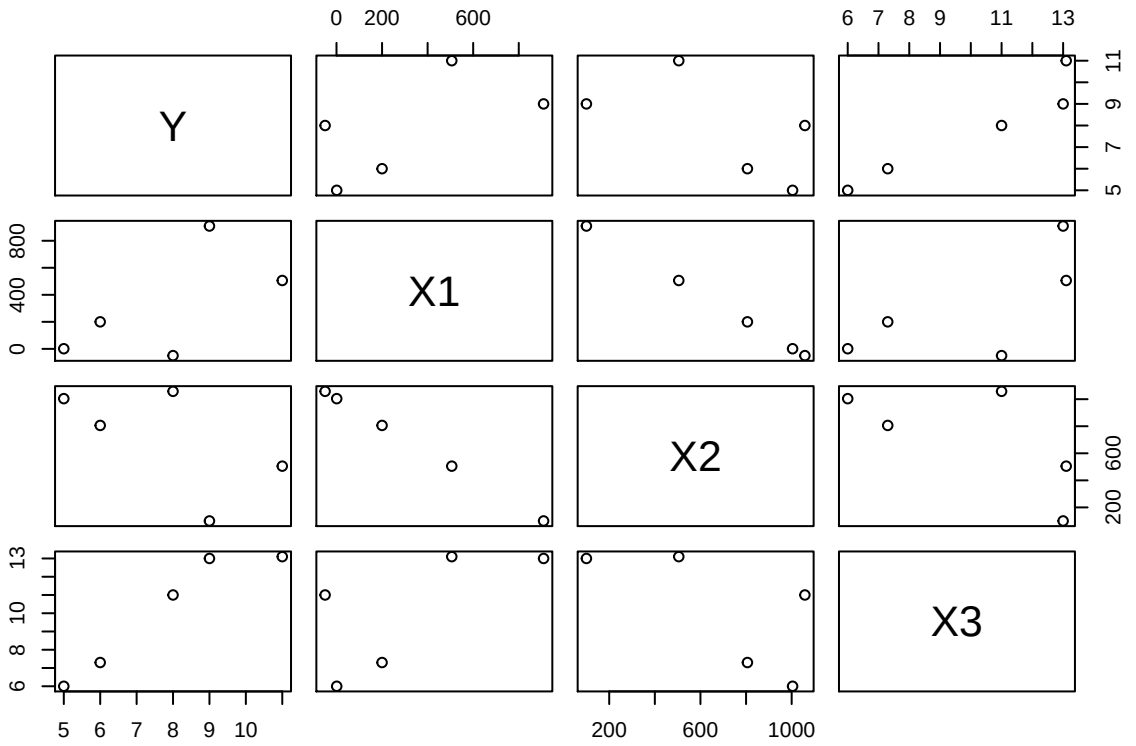
Visually, there is a plausible linear relationship between Y and each X.

Multivariate normality



visually, each X and Y appeark to be normally distributed.

little to no multicollinearity

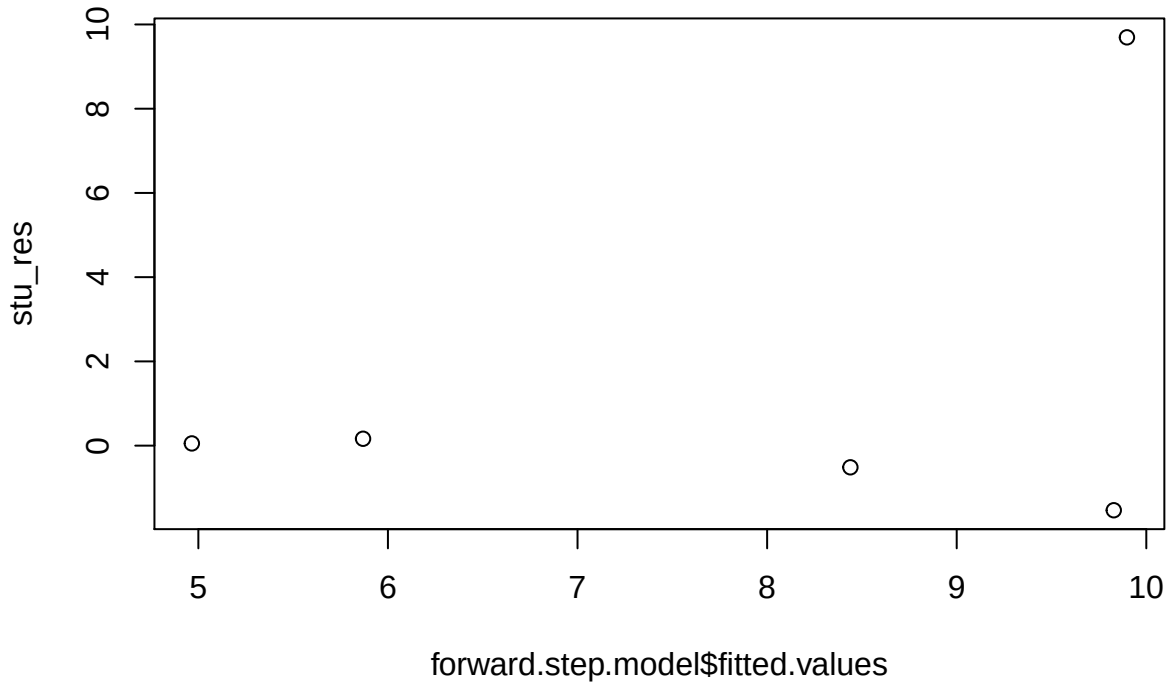


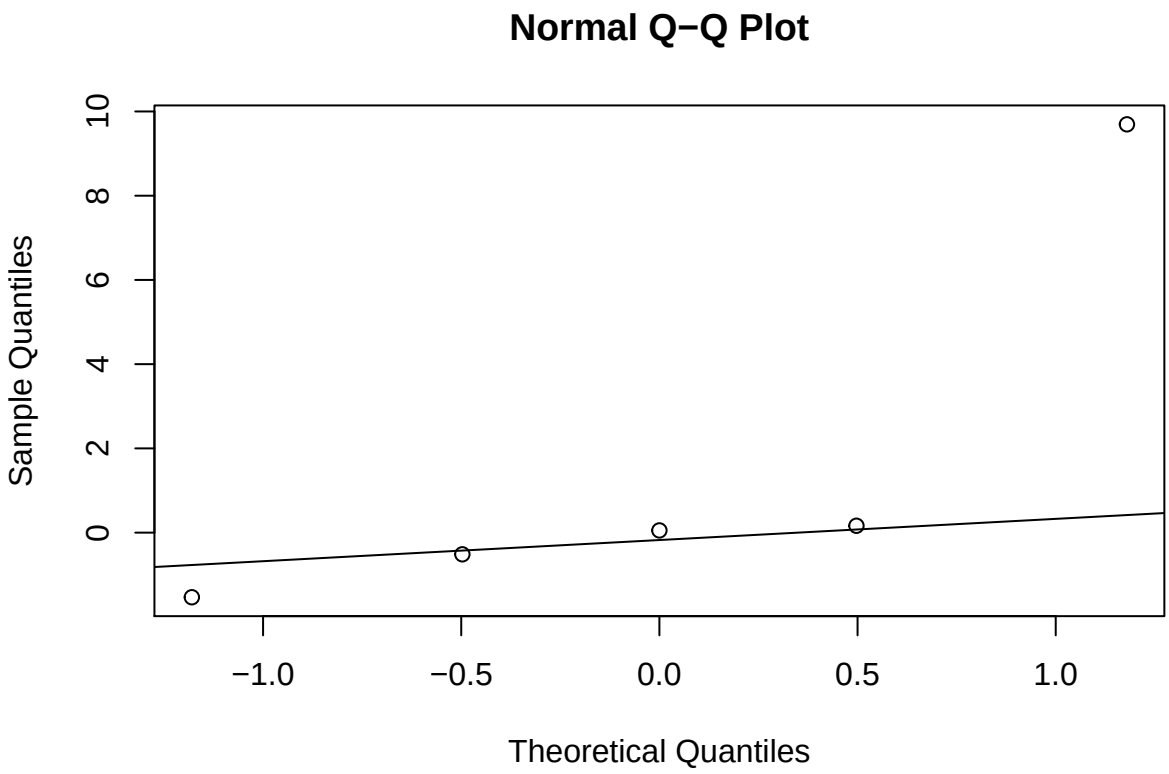

```
##             Y         X1         X2         X3
## Y   1.0000000  0.6145611 -0.6108095  0.9525563
## X1  0.6145611  1.0000000 -0.9999887  0.6858141
## X2 -0.6108095 -0.9999887  1.0000000 -0.6826107
## X3  0.9525563  0.6858141 -0.6826107  1.0000000
```

X1 and X2 are highly correlated. This violates the assumption of no multicollinearity. If we consider models which include X1 or X2, we cannot infer information about one without considering the other.

Homoscedasticity

This assumption means that the variance is equal. Moreover, the residuals are independent of each other, and are identically normally distributed with a mean of 0 and variance of σ^2. We can confirm this by inspecting the residuals.


stu_res
0
2
4
6
8
10
5
6
7
8
9
10
forward.step.model$fitted.values



Visually, 1/5 data points are outliers with large residuals.

At this point, I would recommend the model

$$Y = \beta_0 + \beta_1 X3 + \epsilon$$

It is the simplest model which satisfies most assumptions while showing adequate predictive performance.

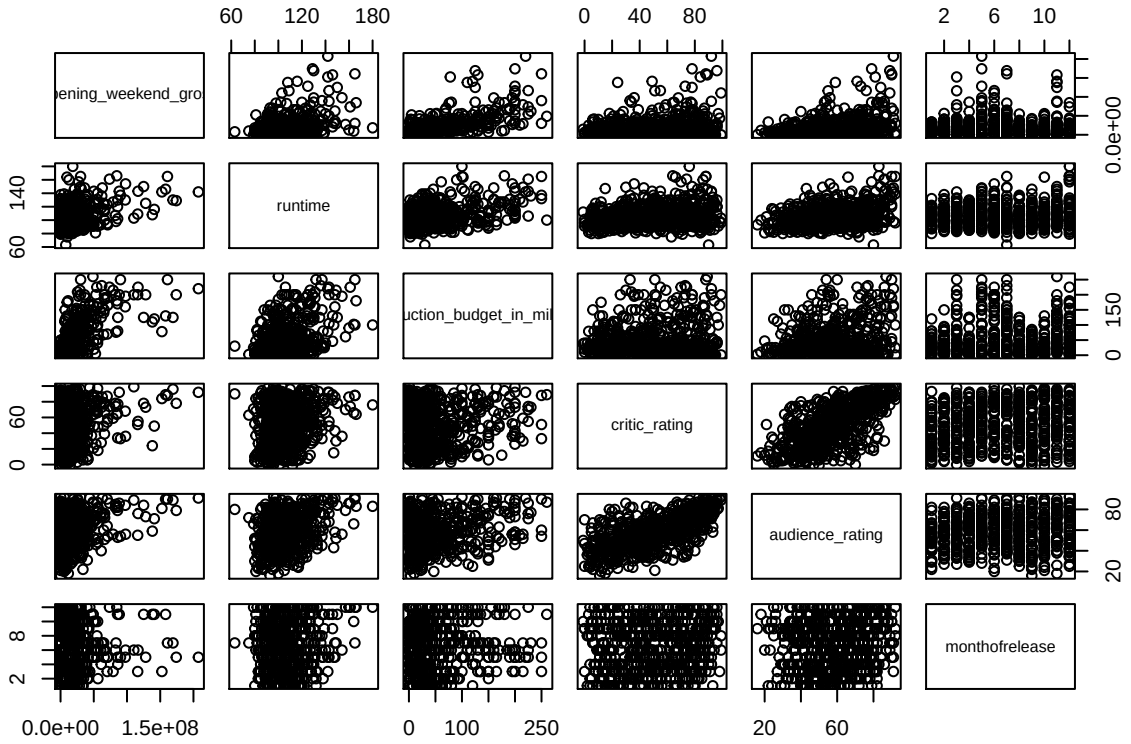
6. (50 points)

Using the sheet/page "2010 to 2013 Wide Release Movies" from the CreditCard dataset of Lab/Homework #3.

read in data and subset

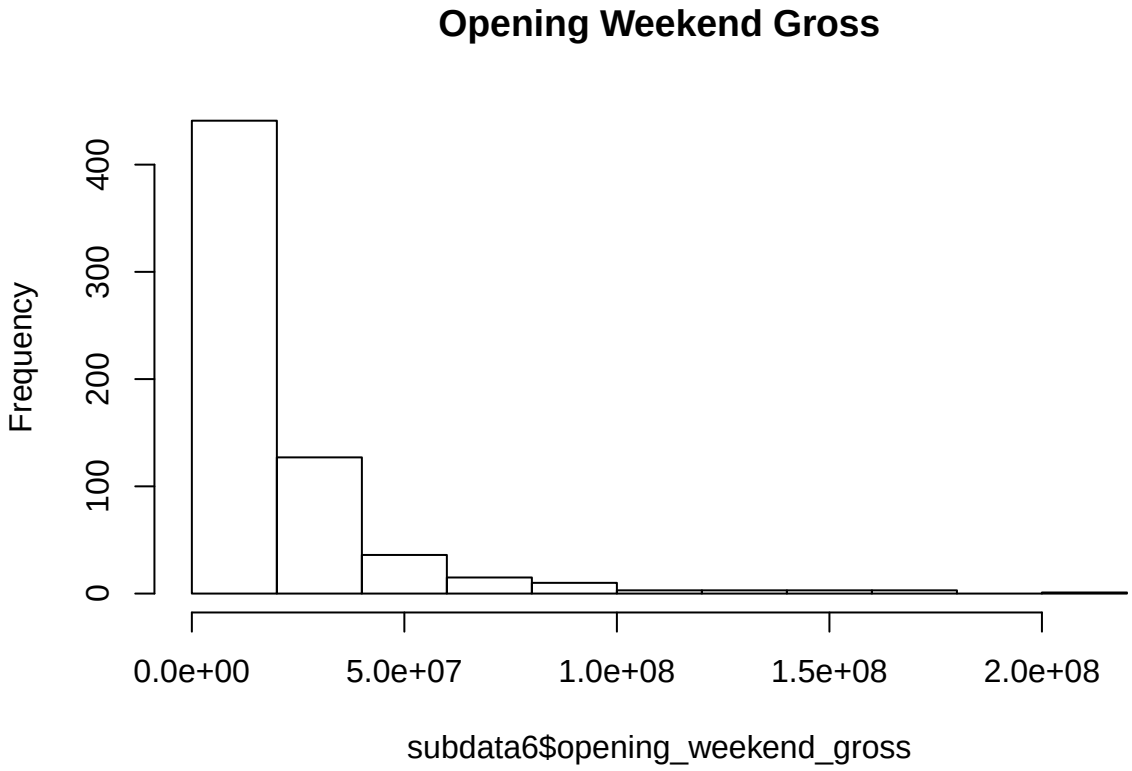
Recommend a model to predict the y variable – "Opening Weekend Gross" with the possible predictors (no interactions) – Runtime, Production Budget, Critic Rating, Audience Rating, and/or Month of Release.

Explain how you determined your model and why you recommended it over other models.

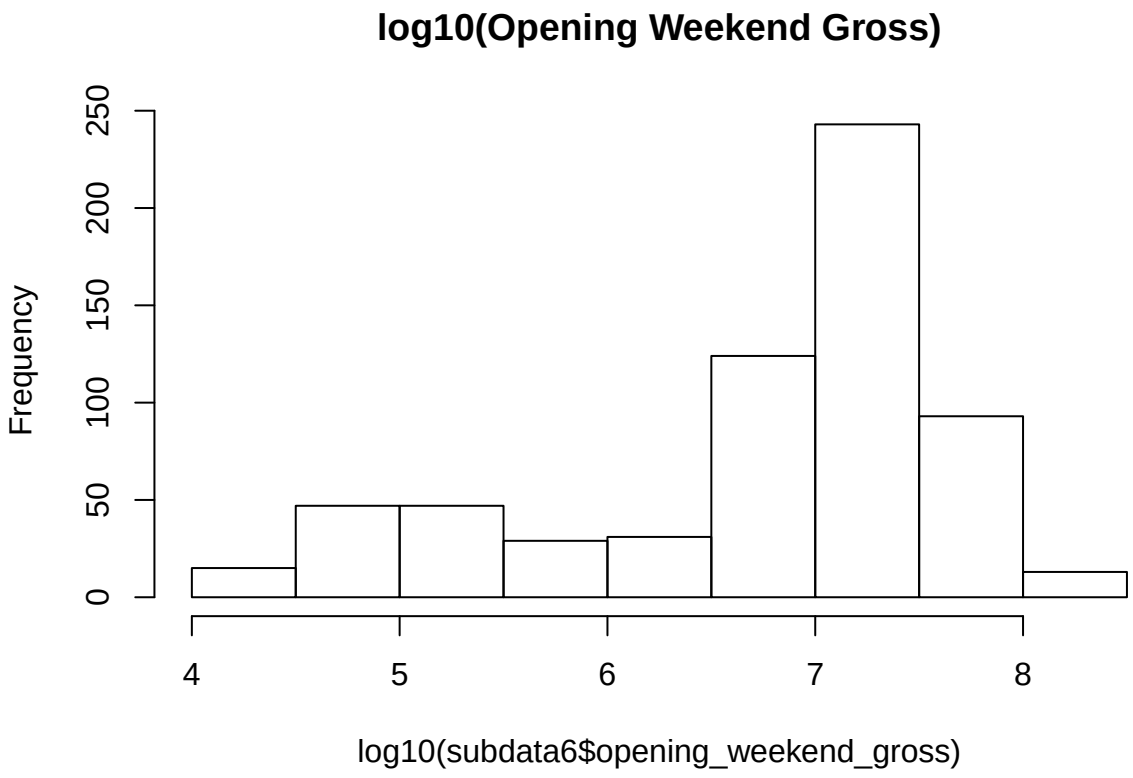


based on the scatterplot matrix, there is not much linearity between opening weekend gross and the specified predictors. Furthermore, each predictor has a different scale. it is advantageous to scale each feature so that our model does not overweight larger values.

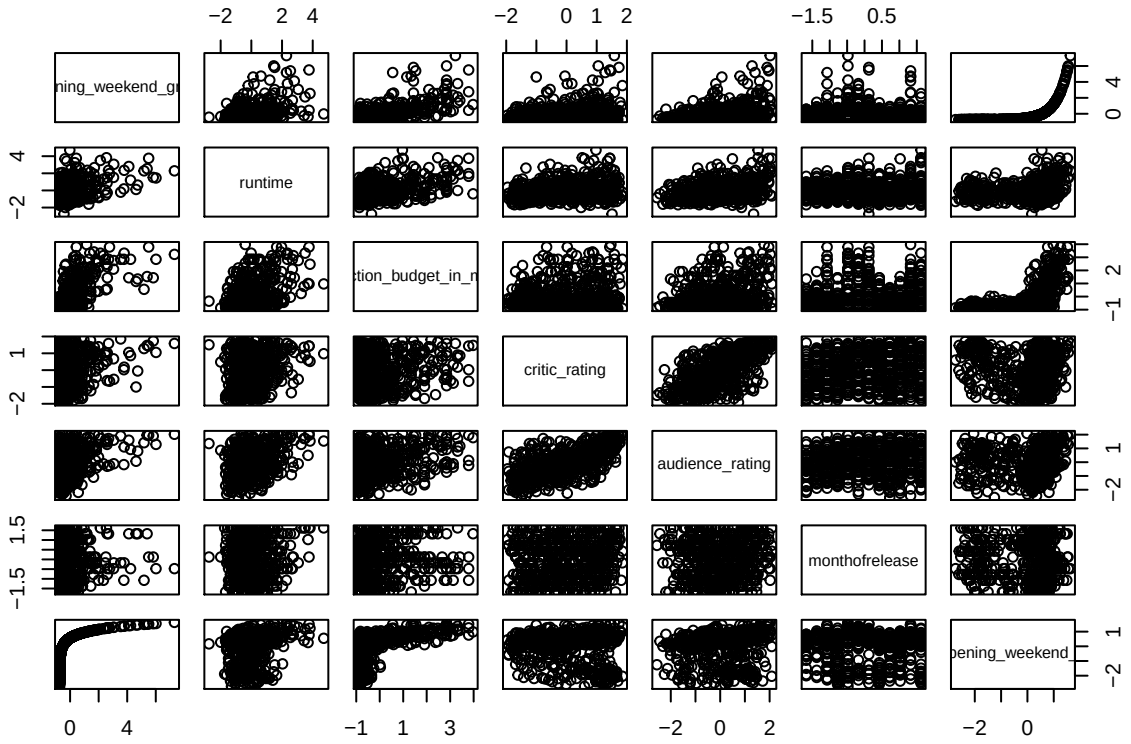
There is strong correlation between audience and critic ratings.



based on the histogram, opening weekend gross is highly skewed. It may be beneficial to predict a transformation of the data instead of the raw values.



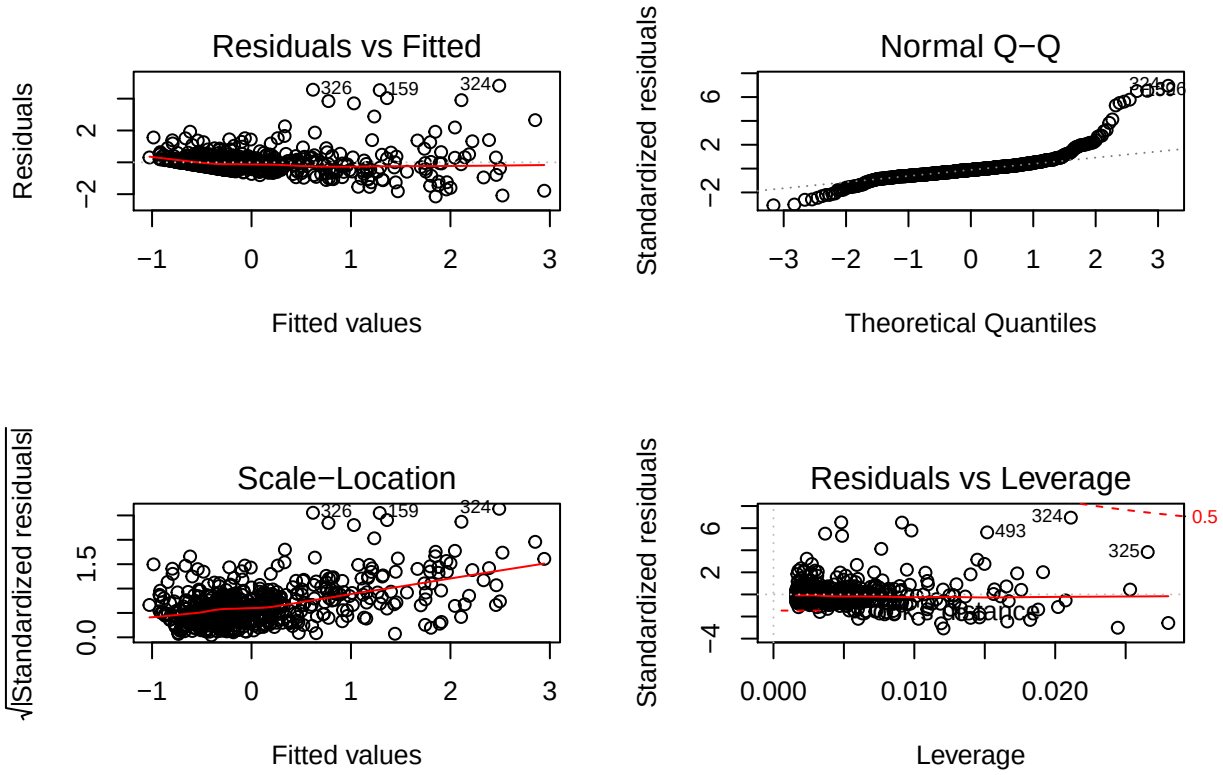
As shown, the distribution of the log tranformed data is much more normally distributed though still a bit skewed.



Since there are just a few predictors and no interactions considered, let's consider an exhaustive search of the model space similar to Question #2.

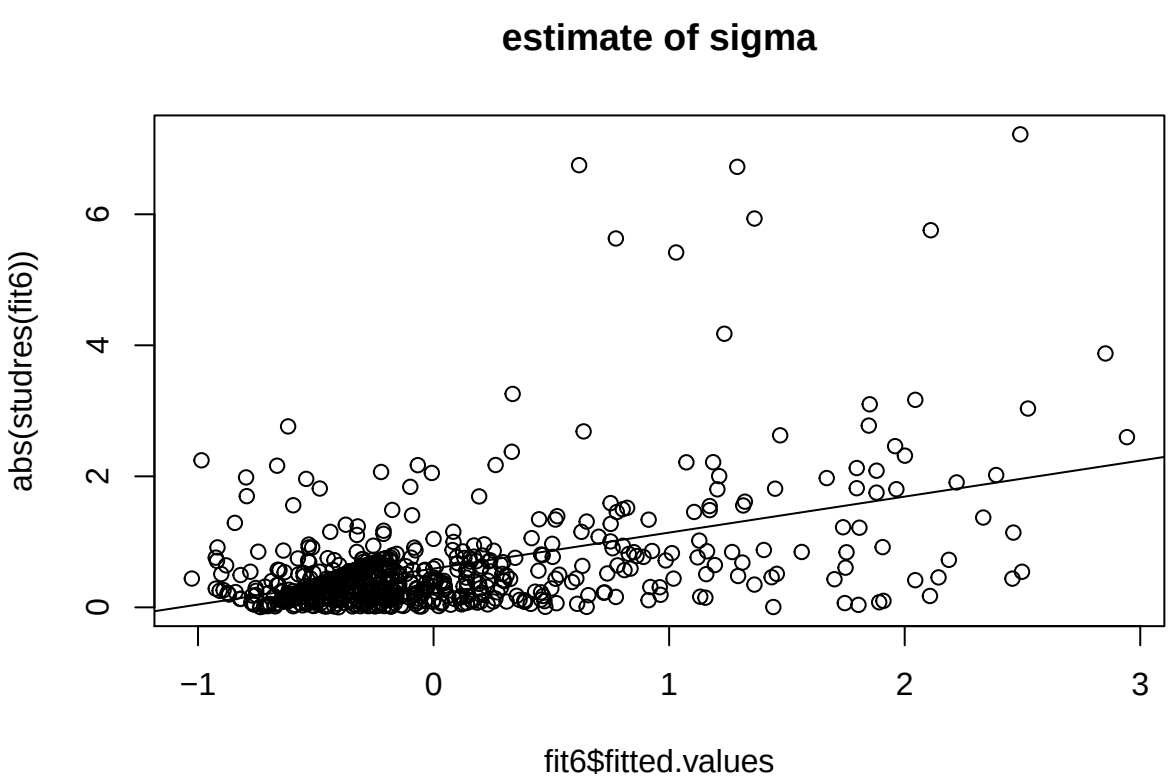
From here, consider the model which minimizes both AIC and AICc,

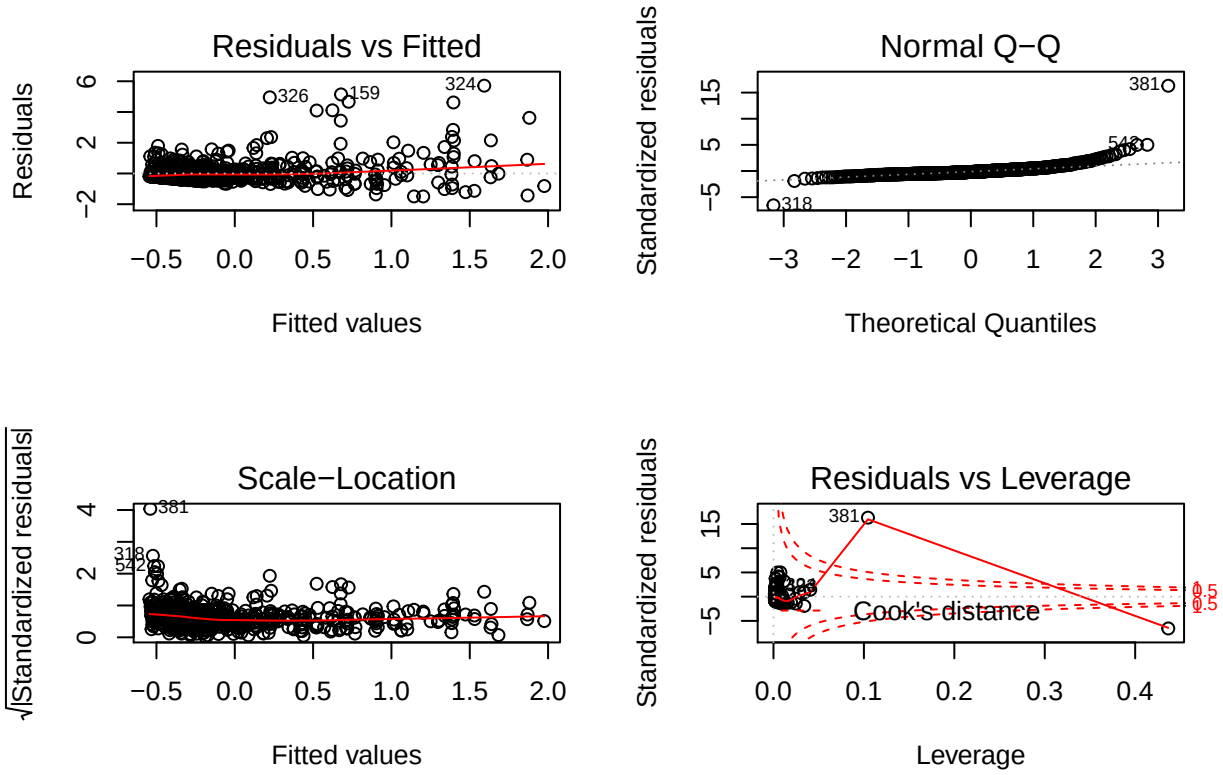
$$opening\ weekend\ gross = \beta_0 + \beta_1 production\ budget + \beta_2 audience\ rating + \epsilon$$



Based on analysis of the residual, this model has a several outliers, there is decreasing variance at large predicitons.

One way to address this issue is to assign a weight based on an estimate of σ

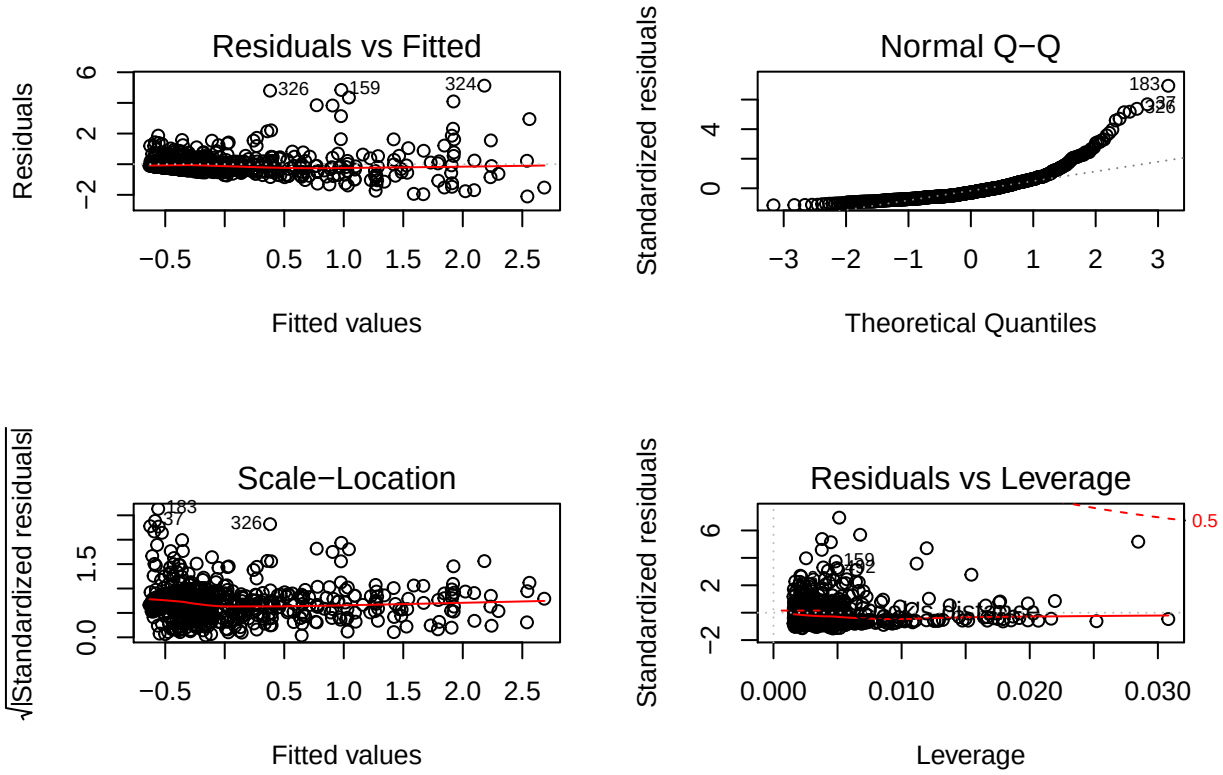

estimate of sigma
abs(studres(fit6))
0
2
4
6
−1
0
1
2
3
fit6$fitted.values




```
##
## Call:
## lm(formula = opening_weekend_gross ~ production_budget_in_millions +
##     audience_rating, data = subdata6, weights = (1/(rfit6$fitted.values^2)))
##
## Weighted Residuals:
##     Min      1Q  Median      3Q     Max
## -7.0265 -0.6582 -0.2600  0.3581 22.0119
##
## Coefficients:
##                               Estimate Std. Error t value Pr(>|t|)
## (Intercept)                  -0.067641   0.037115  -1.823   0.0688 .
## production_budget_in_millions  0.511129   0.050391  10.143   <2e-16 ***
## audience_rating                0.006238   0.015682   0.398   0.6909
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.427 on 639 degrees of freedom
## Multiple R-squared:  0.1566, Adjusted R-squared:  0.154
## F-statistic: 59.33 on 2 and 639 DF,  p-value: < 2.2e-16
```

Based on the residuals of the weighted fit, there are two large outliers the rest of the data has acceptable homoscedasticity.

From the summary of the weighted model, we see that the coefficient of audience rating is not significantly different from zero. Let's consider a model with audience rating removed.



From this residual, the simpler model performs much worse when the predictions are large.

In summary, I propose a weigted lease squares fit of the form

$$opening\ weekend\ gross = \beta_0 + \beta_1 production\ budget + \beta_2 audience\ rating + \epsilon$$

Which minimizes information criteria based on a exhaustive search while maintaining predictive performance.