STAT 5309 – SPRING 2019

LAB 6

****Contents: Two-level factors. 2^K –Factorial design**

DUE: Sun, Mar 31

A. PRACTICE

##-----------------------2-level factors-------------------

------Example of 2^3 Full Factorial Design: 3 factors. Each with 2 levels.

The 2 levels are denoted by (-) [low level] and (+) [high]. The two levels need to be spread out in order to obtain a larger range in the response.

library(FrF2) **[SKIP]**

volt

```
    A   B   C   y
1  22 0.5 0.5 705
2  32 0.5 0.5 620
3  22   5 0.5 700
4  32   5 0.5 629
5  22 0.5   5 672
6  32 0.5   5 668
7  22   5   5 715
8  32   5   5 647
9  22 0.5 0.5 680
10 32 0.5 0.5 651
11 22   5 0.5 685
12 32   5 0.5 635
13 22 0.5   5 654
14 32 0.5   5 691
15 22   5   5 672
16 32   5   5 673
```

mod.1<- lm(y ~ A*B*C, data=volt, contrast=list(A=contr.FrF2, B=contr.FrF2, C=contr.FrF2))

summary(mod.1)

```
Call:
lm.default(formula = y ~ A * B * C, data = volt, contrasts = list
(A = contr.FrF2,
    B = contr.FrF2, C = contr.FrF2))

Residuals:
   Min     1Q Median     3Q    Max
-21.50 -11.75   0.00  11.75  21.50

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 668.5625     4.5178 147.985 4.86e-15 ***
A1          -16.8125     4.5178  -3.721  0.00586 **
B1            0.9375     4.5178   0.208  0.84079
C1            5.4375     4.5178   1.204  0.26315
```

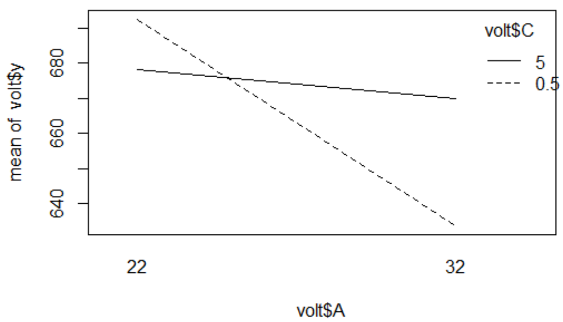
```
A1:B1         -6.6875     4.5178  -1.480  0.17707
A1:C1         12.5625     4.5178   2.781  0.02390 *
B1:C1          1.8125     4.5178   0.401  0.69878
A1:B1:C1      -5.8125     4.5178  -1.287  0.23422
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 18.07 on 8 degrees of freedom
Multiple R-squared:  0.772,    Adjusted R-squared:  0.5724
F-statistic: 3.869 on 7 and 8 DF,  p-value: 0.0385
```

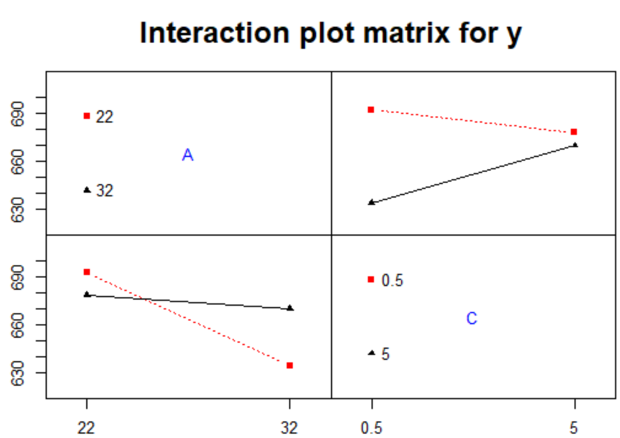
Residual standard error: 18.07 on 8 degrees of freedom
Multiple R-squared: 0.772,   Adjusted R-squared: 0.5724
F-statistic: 3.869 on 7 and 8 DF, p-value: 0.0385

##---------------Additive, Non-additive models; Interactions---------------

interaction.plot(volt$A, volt$C, volt$y)



IAPlot(mod_1, select=c(1,3))   **#package FrF2**



Note: One of the interaction plot at the lower left indicate INTERACTION

##----------------------2-level Full factorial design-------------------------
##----------------Example of 2^4 Full Factorial Design: 4 factors; Each has 2 levels.

data(chem)**[SKIP]**

```
    A  B  C  D  y
1  -1 -1 -1 -1 45
2   1 -1 -1 -1 41
3  -1  1 -1 -1 90
4   1  1 -1 -1 67
5  -1 -1  1 -1 50
6   1 -1  1 -1 39
7  -1  1  1 -1 95
8   1  1  1 -1 66
9  -1 -1 -1  1 47
10  1 -1 -1  1 43
11 -1  1 -1  1 95
12  1  1 -1  1 69
13 -1 -1  1  1 40
14  1 -1  1  1 51
15 -1  1  1  1 87
16  1  1  1  1 72
```

mod.2<- lm(y ~ A*B*C*D, data=chem) # **ADD Tukey1()**
summary(mod.1)

```
Call:
lm.default(formula = y ~ A * B * C * D, data = chem)

Residuals:
ALL 16 residuals are 0: no residual degrees of freedom!

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  62.3125         NA      NA       NA
A            -6.3125         NA      NA       NA
B            17.8125         NA      NA       NA
C             0.1875         NA      NA       NA
D             0.6875         NA      NA       NA
A:B          -5.3125         NA      NA       NA
A:C           0.8125         NA      NA       NA
B:C          -0.3125         NA      NA       NA
A:D           2.0625         NA      NA       NA
B:D          -0.0625         NA      NA       NA
C:D          -0.6875         NA      NA       NA
A:B:C        -0.1875         NA      NA       NA
A:B:D        -0.6875         NA      NA       NA
A:C:D         2.4375         NA      NA       NA
B:C:D        -0.4375         NA      NA       NA
A:B:C:D      -0.3125         NA      NA       NA
```

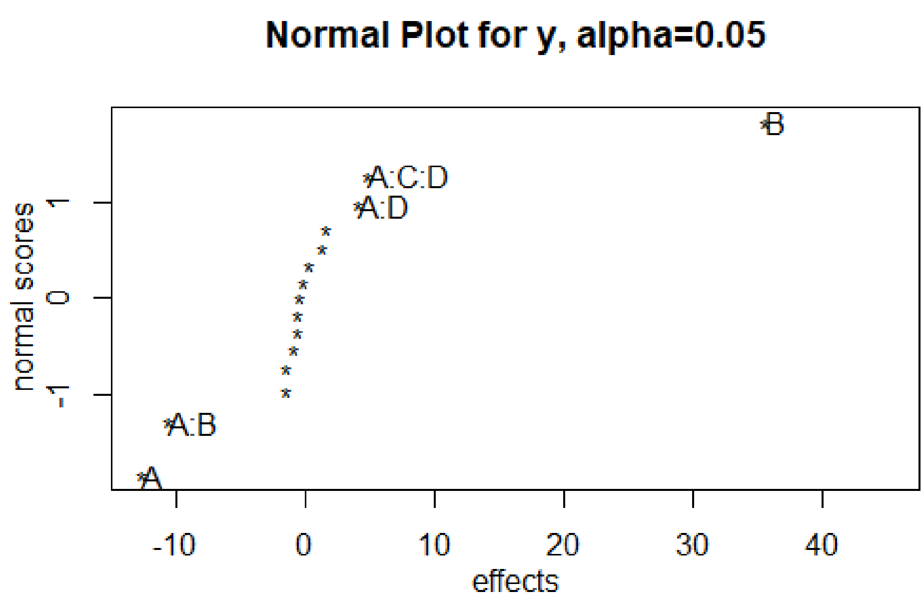
```
Residual standard error: NaN on 0 degrees of freedom
Multiple R-squared:      1,    Adjusted R-squared:    NaN
F-statistic:   NaN on 15 and 0 DF,  p-value: NA
```

- ➔ **Note: P-values can't be calculated by single replicate.**
- ➔ **Can use Tukey1() for single replicate**

##----------------------PLOTS -------------------

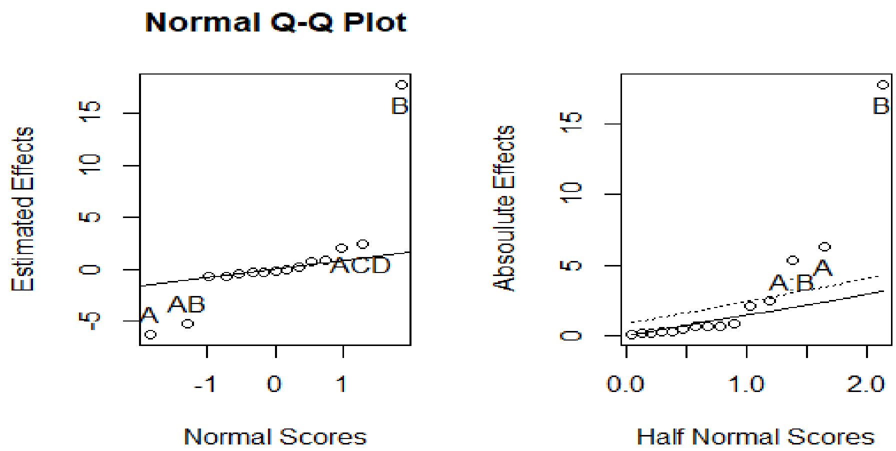
(1) Normal Probability/Half Normal Plots of Effects [Daniel Plot, Lenth Plot]: DanielPlot(), LenthPlot(), fullnormal()
(2) Main Effects Plot: MEPlot()
(3) Interaction Plot: IAPlot()

DanielPlot(mod.2) **#package FrF2**



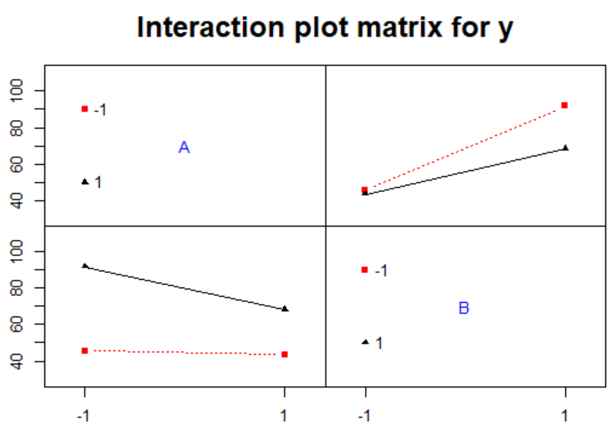
➔ **Note: If effects are off the straight line , then they are SIGNIFICANT**

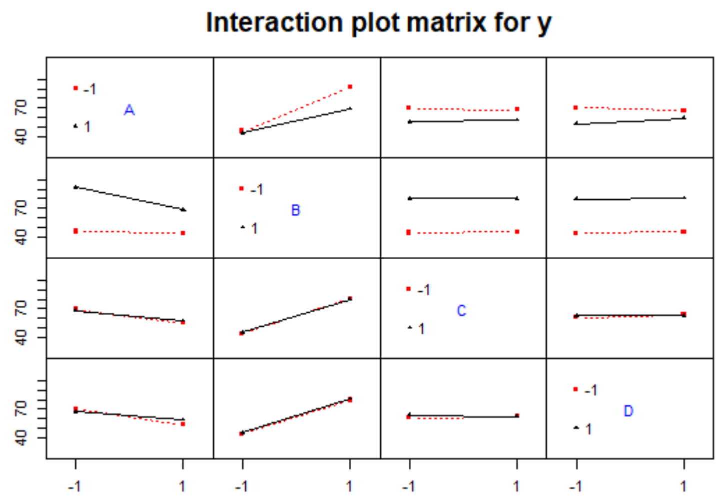
par(mfrow=c(1,2))
fullnormal(coef(mod.1)[-1]) **#package daewr**
LGB(coef(mod.1)[-1])



#Note: significant interaction AB, ACD

IAPlot(mod.1, select=c(1,2)) **# package FrF2**
IAPlot(mod.1, select=c(1,3,4))





Generate a 2^K $or\ 3^K - Design$

expand.grid();

```
D <- expand.grid(BW=c(3.25, 3.75, 4.25), WL =c(4,5,6))      #expand.grid()
D
```

```
    BW WL
1 3.25  4
2 3.75  4
3 4.25  4
4 3.25  5
5 3.75  5
6 4.25  5
7 3.25  6
8 3.75  6
9 4.25  6
```

##gen.factorial(); **Package "AlgDesign"** : Generate a full factoriral design

```
library(AlgDesign)
D.1 <- gen.factorial(levels=c(2,2,2),nVars=3, center=T, varNames=c("A", "B", "C"))
D.1                                         #generate a full factorial design
y <- c(1.5, 2.0,  2.1, 3.0, 1.2, 2.1, 2.5, 2.8)
data <- cbind(D.1,y)
data
```

```
   A  B  C   y
1 -1 -1 -1 1.5
2  1 -1 -1 2.0
3 -1  1 -1 2.1
4  1  1 -1 3.0
5 -1 -1  1 1.2
6  1 -1  1 2.1
7 -1  1  1 2.5
8  1  1  1 2.8
```

mod_3 <- lm(y ~ A*B*C, data=data)
summary(mod_3)

```
Call:
lm.default(formula = y ~ A * B * C, data = data)

Residuals:
ALL 8 residuals are 0: no residual degrees of freedom!

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  2.15e+00         NA      NA       NA
A            3.25e-01         NA      NA       NA
B            4.50e-01         NA      NA       NA
C            1.57e-16         NA      NA       NA
A:B         -2.50e-02         NA      NA       NA
A:C         -2.50e-02         NA      NA       NA
B:C          5.00e-02         NA      NA       NA
A:B:C       -1.25e-01         NA      NA       NA

Residual standard error: NaN on 0 degrees of freedom
Multiple R-squared:      1,	Adjusted R-squared:    NaN 
F-statistic:   NaN on 7 and 0 DF,  p-value: NA
```

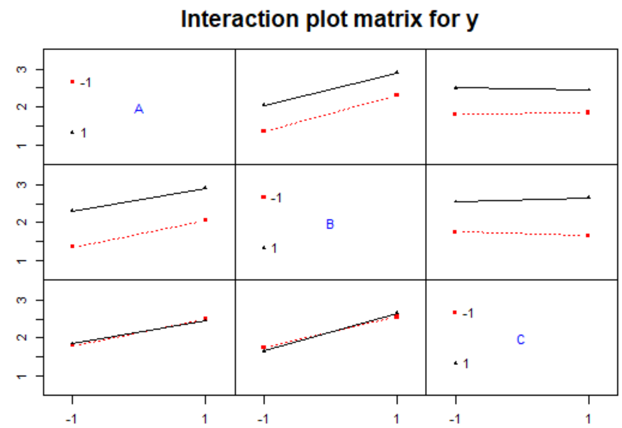
```
> anova(mod.3)
Analysis of Variance Table

Response: y
          Df Sum Sq Mean Sq F value Pr(>F)
A          1  0.845   0.845               
B          1  1.620   1.620               
C          1  0.000   0.000               
A:B        1  0.005   0.005               
A:C        1  0.005   0.005               
B:C        1  0.020   0.020               
A:B:C      1  0.125   0.125               
Residuals  0  0.000                       
```

➔ **Note: Single replicate causes P-values not calculated.**

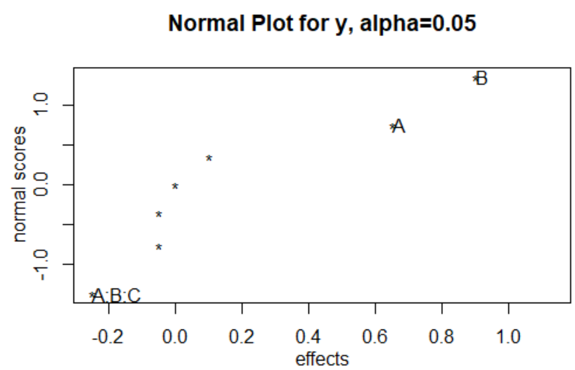
#--------Interaction plot---------------------------

IAPlot(mod_3, select=c(1,2, 3))



#-----------------Effects Half Normal Plot

DanielPlot(mod.3)



B. EXERCISE

1.

A router is used to cut locating notches on a printed circuit board. The vibration level at the surface of the board as it is cut is considered to be a major source of dimensional variation in the notches. Two factors are thought to influence vibration: bit size (A) and cutting speed (B). Two bit sizes ($\frac{1}{16}$ and $\frac{1}{8}$ inch) and two speeds (40 and 90 rpm) are selected, and four boards are cut at each set of conditions shown below. The response variable is vibration measured as the resultant vector of three accelerometers (x, y, and z) on each test circuit board.

			Replicate			
A	*B*	Treatment Combination	I	II	III	IV
−	−	(1)	18.2	18.9	12.9	14.4
+	−	*a*	27.2	24.0	22.4	22.5
−	+	*b*	15.9	14.5	15.1	14.2
+	+	*ab*	41.0	43.9	36.3	39.9

(a) Analyze the data from this experiment.
(b) Construct a normal probability plot of the residuals, and plot the residuals versus the predicted vibration level. Interpret these plots.
(c) Draw the *AB* interaction plot. Interpret this plot. What levels of bit size and speed would you recommend for routine operation?

2.

An engineer is interested in the effects of cutting speed (A), tool geometry (B), and cutting angle (C) on the life (in hours) of a machine tool. Two levels of each factor are chosen, and three replicates of a 2^3 factorial design are run. The results follow:

A	B	C	Treatment Combination	Replicate I	II	III
−	−	−	(1)	22	31	25
+	−	−	a	32	43	29
−	+	−	b	35	34	50
+	+	−	ab	55	47	46
−	−	+	c	44	45	38
+	−	+	ac	40	37	36
−	+	+	bc	60	50	54
+	+	+	abc	39	41	47

(a) Estimate the factor effects. Which effects appear to be large?
(b) Use the analysis of variance to confirm your conclusions for part (a).
(c) Write down a regression model for predicting tool life (in hours) based on the results of this experiment.
(d) Analyze the residuals. Are there any obvious problems?
(e) Based on an analysis of main effect and interaction plots, what levels of A, B, and C would you recommend using?

3.

An experiment was performed to improve the yield of a chemical process. Four factors were selected, and two replicates of a completely randomized experiment were run. The results are shown in the following table:

Treatment Combination	Replicate I	II	Treatment Combination	Replicate I	II
(1)	90	93	d	98	95
a	74	78	ad	72	76
b	81	85	bd	87	83
ab	83	80	abd	85	86
c	77	78	cd	99	90
ac	81	80	acd	79	75
bc	88	82	bcd	87	84
abc	73	70	$abcd$	80	80

(a) Estimate the factor effects.

(b) Prepare an analysis of variance table, and determine which factors are important in explaining yield.

(c) Write down a regression model for predicting yield, assuming that all four factors were varied over the range from -1 to $+1$ (in coded units).

(d) Plot the residuals versus the predicted yield and on a normal probability scale. Does the residual analysis appear satisfactory?

(e) Two three-factor interactions, *ABC* and *ABD*, apparently have large effects. Draw a cube plot in the factors *A*, *B*, and *C* with the average yields shown at each corner. Repeat using the factors *A*, *B*, and *D*. Do these two plots aid in data interpretation? Where would you recommend that the process be run with respect to the four variables?