Stat 5309 Semester Project

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An article in the AT&T Technical Journal (March/April 1986, Vol. 65, pp. 39-50) describes the application of two-level factorial designs to integrated circuit manufacturing. A basic processing step is to grow an epitaxial layer on polished silicon wafers. The wafers mounted on a susceptor are positioned inside a bell jar, and chemical vapors are introduced. The susceptor is rotated and heat is applied until the epitaxial layer is thick enough. An experiment was run using two factors: arsenic flow rate (A) and deposition time (B). Four replicates were run, and the epitaxial layer thickness was measured in um. The data are shown below:

Table 1: Predictors of Filament usage

flow_rate	depo_time	thickness
0.55	10	14.037
0.59	10	13.880
0.55	15	14.821
0.59	15	14.888
0.55	10	16.165
0.59	10	13.860
0.55	15	14.757
0.59	15	14.921
0.55	10	13.972
0.59	10	14.032
0.55	15	14.843
0.59	15	14.415
0.55	10	13.907
0.59	10	13.914
0.55	15	14.878
0.59	15	14.932

a

Estimate the factor effects. Based on a simple first order analysis of variance, the model effects are summarized below.

```
##
## Call:
## lm.default(formula = thickness ~ flow_rate * depo_time, data = epitaxial_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.61325 -0.14431 -0.00562  0.10187  1.64475
##
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)         37.627     20.533   1.832   0.0918 .
```

```
## flow_rate             -43.119      36.001  -1.198    0.2542
## depo_time              -1.487       1.611  -0.923    0.3740
## flow_rate:depo_time     2.815       2.824   0.997    0.3386
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5648 on 12 degrees of freedom
## Multiple R-squared:  0.3535, Adjusted R-squared:  0.1918
## F-statistic: 2.187 on 3 and 12 DF,  p-value: 0.1425
```

	effect
(Intercept)	37.62656
flow_rate	-43.11875
depo_time	-1.48735
flow_rate:depo_time	2.81500

b

Conduct an analysis of variance. Which factors are important?

```
##                     Df Sum Sq Mean Sq F value Pr(>F)
## flow_rate            1  0.403  0.4026   1.262 0.2833
## depo_time            1  1.374  1.3736   4.305 0.0602 .
## flow_rate:depo_time  1  0.317  0.3170   0.994 0.3386
## Residuals           12  3.828  0.3190
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Important factors should have both significant and large effects. deposition time is significant at a level of 90% and the effect is large. flow rate effect is large, but insignificant.

c

Write down a regression equation that could be used to predict epitaxial layer thickness over the region of arsenic flow rate and deposition time used in this experiment.

$$thickness = 37.627 - 43.119 flow\ rate - 1.148 deposition\ time$$

Build a RSM model (2nd order, 1st order with interaction). Choose one which works.

```
## Warning in rsm(thickness ~ SO(depo_time, flow_rate), data = epitaxial_data): Some coefficients are al
##   Returning an 'lm' object.
```

Perform Daniel plot and Lenth plot. What is the model 's R-square.

both response surface models have an R-squared of 0.19. Quadratic terms were not able to be calculated from this experiment. a Center point is needed to test for inflection.

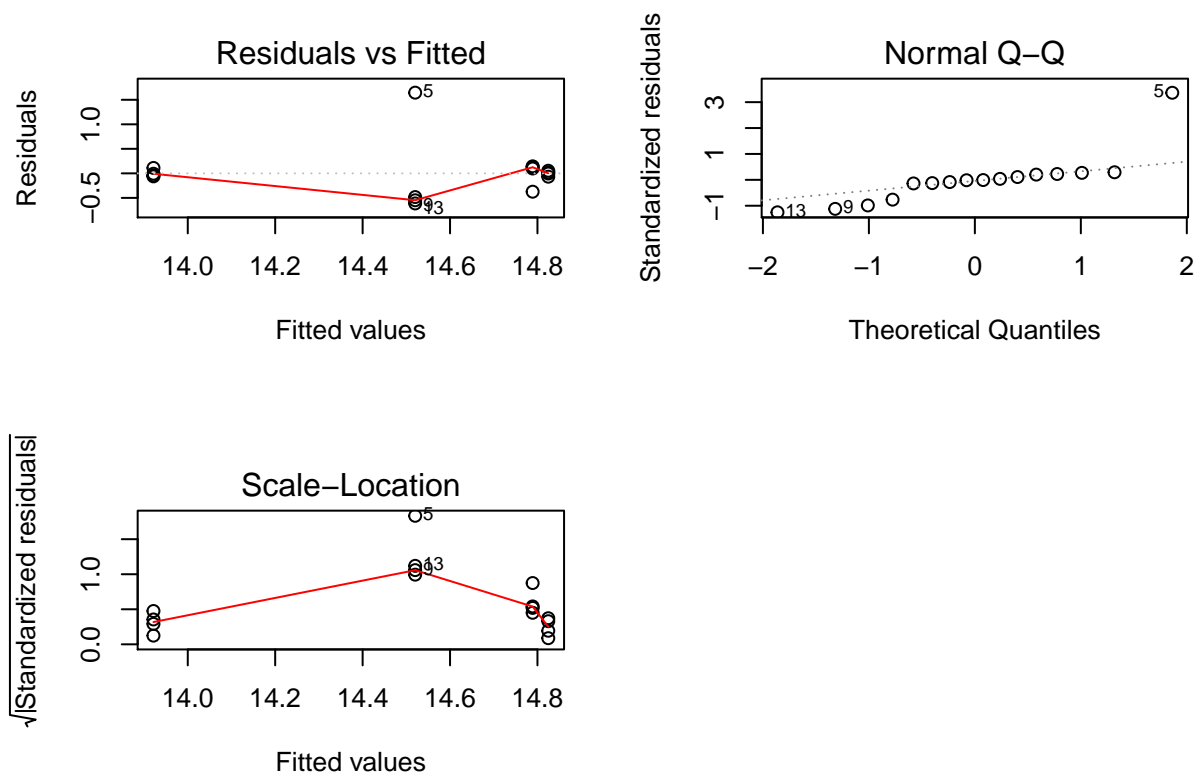**Normal Plot for thickness, alpha=0.05**
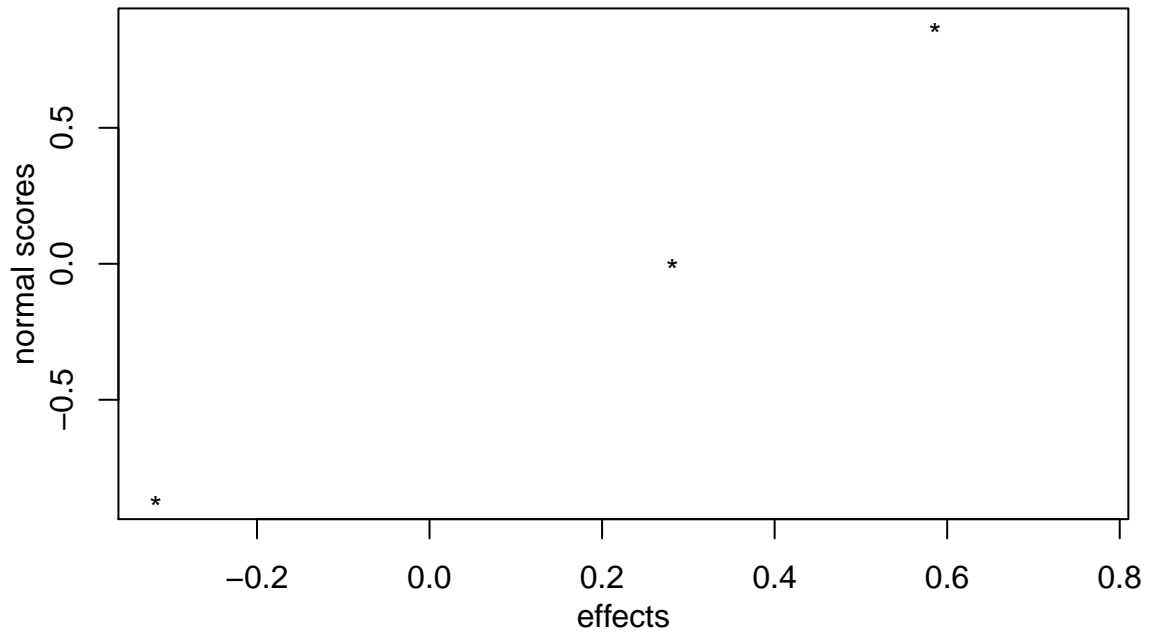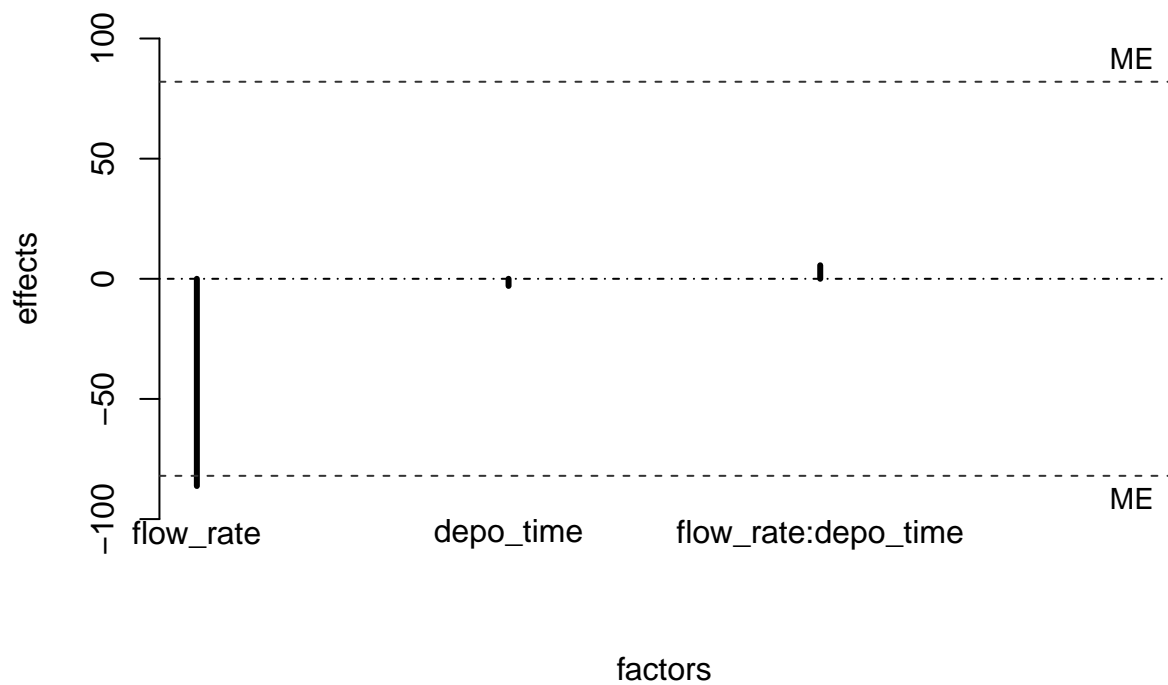
```
##      alpha        PSE        ME        SME
##   0.050000   6.453525  81.999810 242.293942
```

Flow rate has a large effect while depo time has a small effect.

d

Analyze the residuals. Are there any residuals that should cause concern?

```
## hat values (leverages) are all = 0.25
##  and there are no factor predictors; no plot no. 5
```

Residuals vs Fitted

Normal Q–Q

Scale–Location

There is 1 very serious large outlier and a few less serious small outliers.

Find the cook distance. Take out the outlier(s). Rebuild the rsm model on new data.
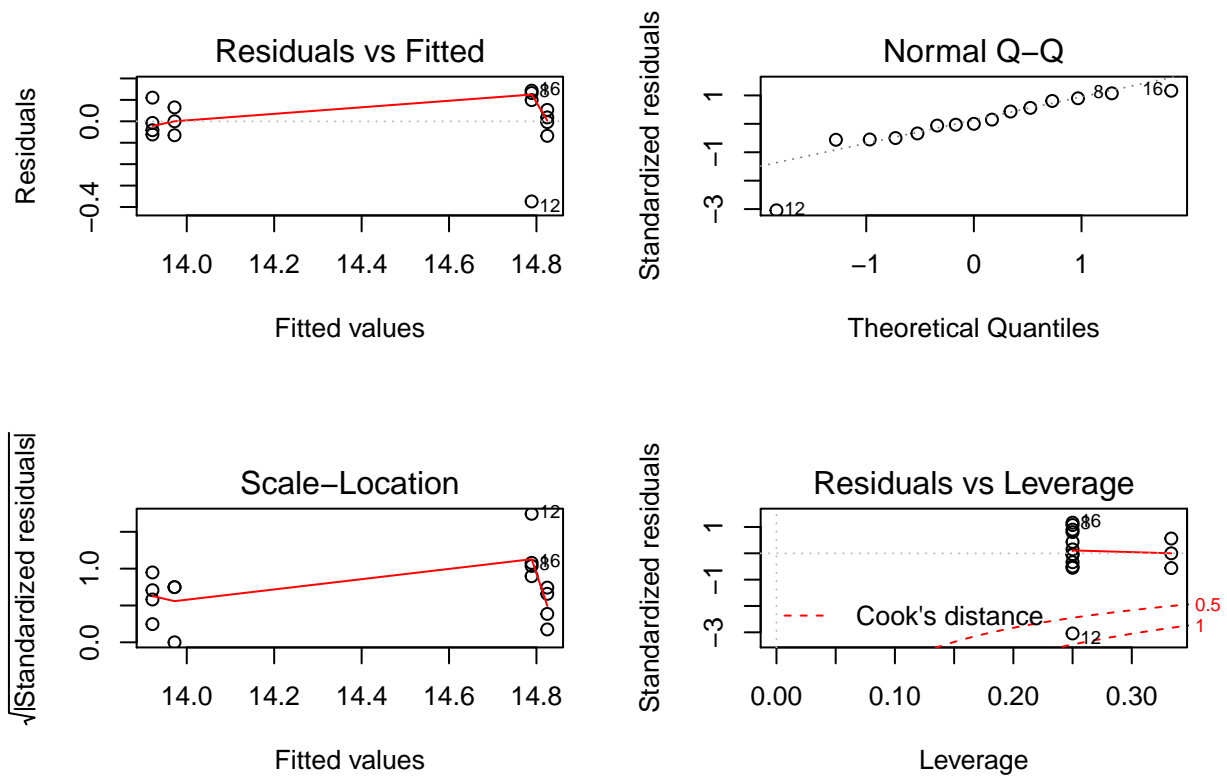
e

Discuss how you might deal with the potential outlier found in part (d).

- Analyse the data with and without outliers removed.
- create an indication of outliers and analyze its effect on thickness.
- Cross validate

f

Perform a canonical analysis. Do a contour plot. Any optimal response?

```
## $xs
##  depo_time  flow_rate
## 15.3174956  0.5283659
##
## $eigen
## eigen() decomposition
## $values
## [1]  1.4075 -1.4075
##
## $vectors
##                 [,1]       [,2]
## depo_time 0.7071068 -0.7071068
## flow_rate 0.7071068  0.7071068
```
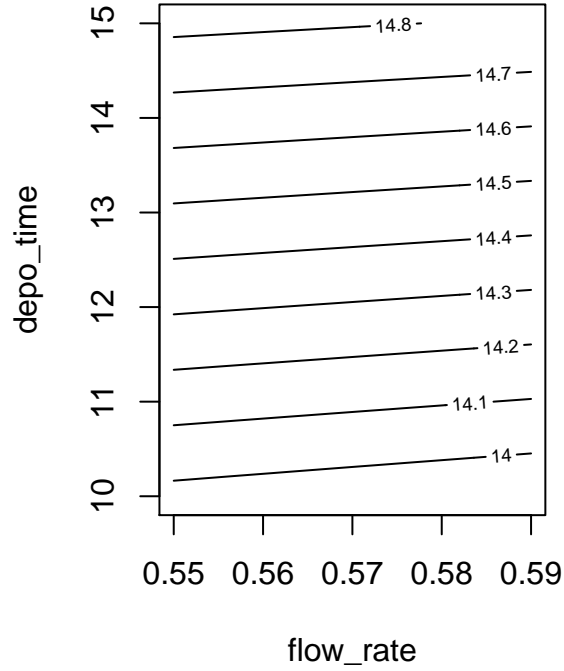
There is a maximum thickness at depo time = 15.3174956 flow rate = 0.5283659 contours are planar, so the design space is not suffcient to find a global extremum. I would suggest the next experiment characterize along the gradient of this plane.

2

A nickel-titanium alloy is used to make components for jet turbine aircraft engines. Cracking is a potentially serious problem in the final part, because it can lead to nonrecoverable failure. A test is run at the parts producer to determine the effect of four factors on cracks. The four factors are pouring temperature (A), titanium content (B), heat treatment method (C), and amount of grain refiner used (D). Two replicates of a 2^4 design are run, and the length of crack in mm $\times 10^{-2}$? induced in a sample coupon subjected to a standard test is measured. The data are shown in the following table:

temp	content	method	refiner	length
-1	-1	-1	-1	7.037
1	-1	-1	-1	14.707
-1	1	-1	-1	11.635
1	1	-1	-1	17.273
-1	-1	1	-1	10.403
1	-1	1	-1	4.368
-1	1	1	-1	9.360
1	1	1	-1	14.440
-1	-1	-1	1	8.561
1	-1	-1	1	16.867
-1	1	-1	1	13.876

temp	content	method	refiner	length
1	1	-1	1	19.824
-1	-1	1	1	11.846
1	-1	1	1	6.125
-1	1	1	1	11.190
1	1	1	1	15.653
-1	-1	-1	-1	6.376
1	-1	-1	-1	15.219
-1	1	-1	-1	12.089
1	1	-1	-1	17.815
-1	-1	1	-1	10.151
1	-1	1	-1	4.098
-1	1	1	-1	9.253
1	1	1	-1	12.923
-1	-1	-1	1	8.951
1	-1	-1	1	17.052
-1	1	-1	1	13.658
1	1	-1	1	19.639
-1	-1	1	1	12.337
1	-1	1	1	5.904
-1	1	1	1	10.935
1	1	1	1	15.053

a

Estimate the factor effects, Which factor effects appear to be large? Change the factors name (Temp,Content,Method,Refiner,Length).

	effect
(Intercept)	12.0193125
temp	1.5406875
content	2.0191875
method	-1.7668750
refiner	0.9476250
temp:content	0.9983125
temp:method	-1.9726250
content:method	0.0792500
temp:refiner	0.0070000
content:refiner	-0.0076250
method:refiner	-0.0696875
temp:content:method	1.6000000
temp:content:refiner	0.0177500
temp:method:refiner	-0.0216875
content:method:refiner	-0.0134375
temp:content:method:refiner	-0.0241875

```
##              Df Sum Sq Mean Sq F value   Pr(>F)
## temp          1  75.96   75.96 524.861 1.17e-13 ***
## content       1 130.47  130.47 901.505 1.69e-15 ***
## method        1  99.90   99.90 690.282 1.38e-14 ***
## refiner       1  28.74   28.74 198.558 1.94e-10 ***
```

```
## temp:content               1  31.89   31.89 220.368 8.93e-11 ***
## temp:method                1 124.52  124.52 860.407 2.44e-15 ***
## content:method             1   0.20    0.20   1.389    0.256
## temp:refiner               1   0.00    0.00   0.011    0.918
## content:refiner            1   0.00    0.00   0.013    0.911
## method:refiner             1   0.16    0.16   1.074    0.315
## temp:content:method        1  81.92   81.92 566.050 6.48e-14 ***
## temp:content:refiner       1   0.01    0.01   0.070    0.795
## temp:method:refiner        1   0.02    0.02   0.104    0.751
## content:method:refiner     1   0.01    0.01   0.040    0.844
## temp:content:method:refiner 1  0.02    0.02   0.129    0.724
## Residuals                 16   2.32    0.14
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There are significant interactions between pouring temperature, titanium content, and heat treatment method. temp, content, method, refiner, temp:content, temp:method are also significant.
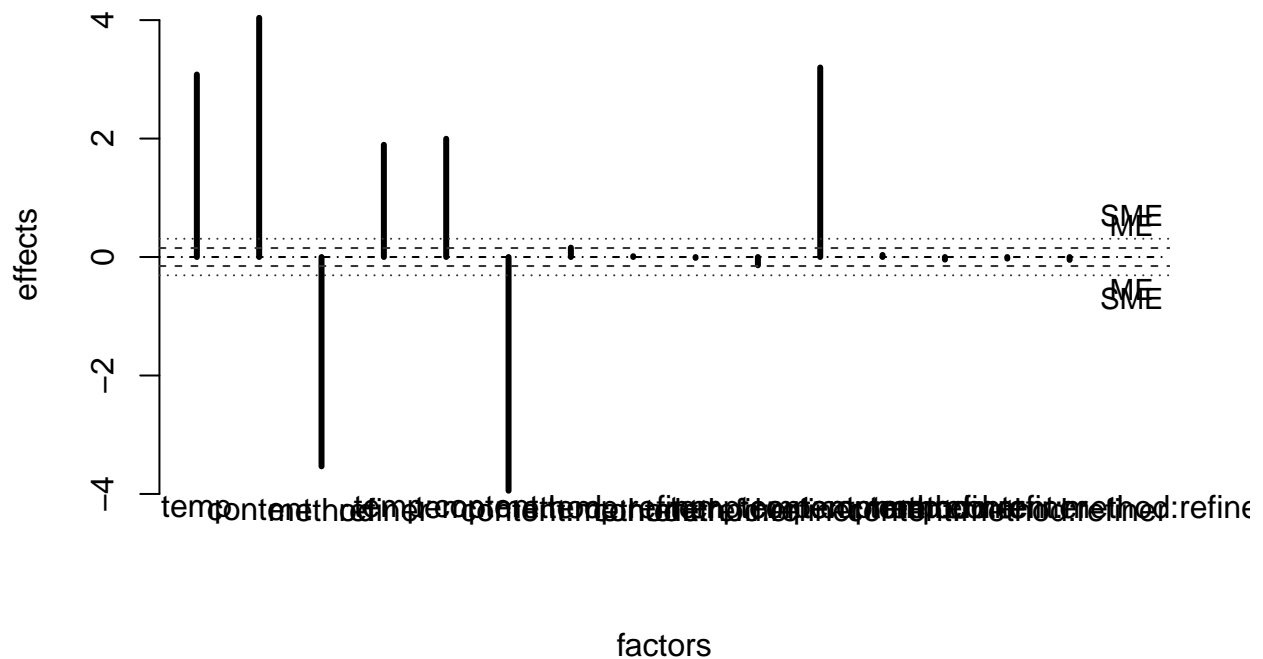

```
##      alpha        PSE         ME        SME
## 0.05000000 0.05915625 0.15206598 0.30871584
```

These 6 coefficients are large: content+:method+ -6.126 temp+:method+ -14.3005 temp+:content+:method+ 12.9935 temp+ 8.2565 content+ 5.1555 method+ 3.5705

b

Conduct an analysis of variance. Do any of the factors affect cracking? Use $\alpha = 0.05$. Perform effects Daniel plot and Lenth plot.

```
##                              Df Sum Sq Mean Sq F value   Pr(>F)
## temp                          1  75.96   75.96 524.861 1.17e-13 ***
## content                       1 130.47  130.47 901.505 1.69e-15 ***
## method                        1  99.90   99.90 690.282 1.38e-14 ***
## refiner                       1  28.74   28.74 198.558 1.94e-10 ***
## temp:content                  1  31.89   31.89 220.368 8.93e-11 ***
## temp:method                   1 124.52  124.52 860.407 2.44e-15 ***
## content:method                1   0.20    0.20   1.389    0.256
## temp:refiner                  1   0.00    0.00   0.011    0.918
## content:refiner               1   0.00    0.00   0.013    0.911
## method:refiner                1   0.16    0.16   1.074    0.315
## temp:content:method           1  81.92   81.92 566.050 6.48e-14 ***
## temp:content:refiner          1   0.01    0.01   0.070    0.795
## temp:method:refiner           1   0.02    0.02   0.104    0.751
## content:method:refiner        1   0.01    0.01   0.040    0.844
## temp:content:method:refiner   1   0.02    0.02   0.129    0.724
## Residuals                    16   2.32    0.14
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Normal Plot for length, alpha=0.05
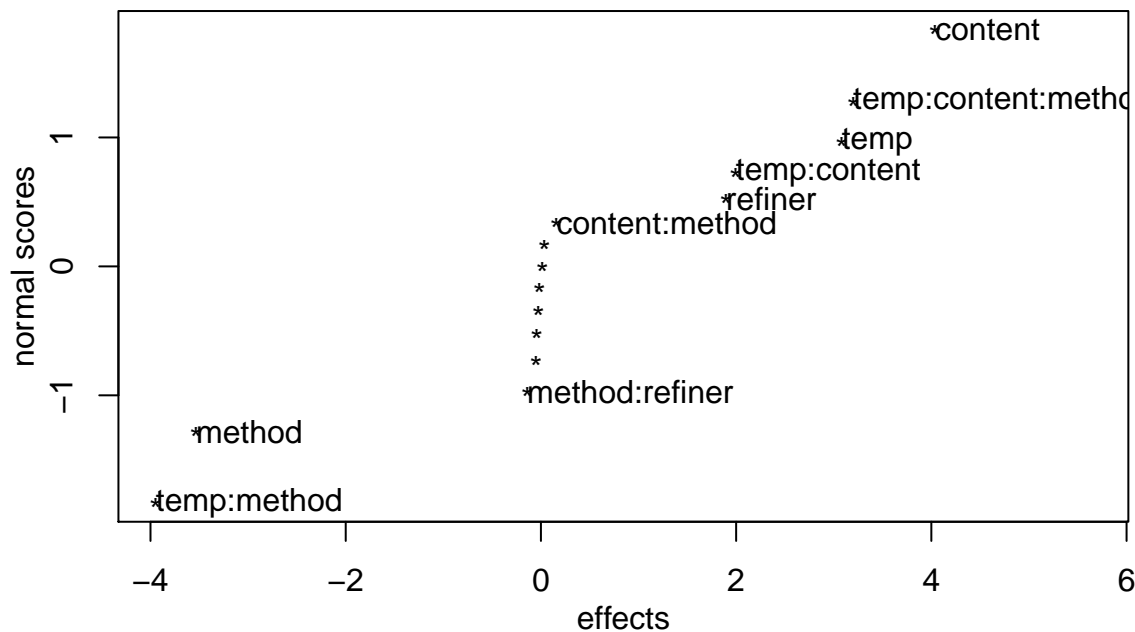
content, method, and the interaction between temp:method, and temp:content:method are the most important effects.

10

c

Write down a regression model that can be used to predict crack length as a function of the significant main effects and interactions you have identified in part (b). Build a RSM model (2nd order, 1st order with interaction). Choose one which works.

$$crack\ length = 8.2565\ pouring\ temperature + 5.1555\ titanium\ content+$$

$$3.5705\ heat\ treatment\ method + 2.0495\ grain\ refiners-$$

$$2.5745\ pouring\ temperature \cdot titanium\ content - 14.3005\ pouring\ temperature \cdot heat\ treatment\ method-$$

$$6.1260\ titanium\ content\dot{\cdot}heat\ treatment\ method + 12.9935\ pouring\ temperature \cdot titanium\ content \cdot heat\ treatment\ method$$

cannot create rsm without numeric values for high and low settings of each factor.

```
##
## Call:
## rsm(formula = length ~ FO(temp, content, method, refiner) + TWI(temp,
##     content, method, refiner), data = cracking_data)
##
##                 Estimate Std. Error t value  Pr(>|t|)
## (Intercept)    12.019312   0.354153 33.9382 < 2.2e-16 ***
## temp            1.540687   0.354153  4.3503 0.0002811 ***
## content         2.019188   0.354153  5.7015 1.168e-05 ***
## method         -1.766875   0.354153 -4.9890 6.151e-05 ***
## refiner         0.947625   0.354153  2.6757 0.0141495 *
## temp:content    0.998313   0.354153  2.8189 0.0102847 *
## temp:method    -1.972625   0.354153 -5.5700 1.581e-05 ***
## temp:refiner    0.007000   0.354153  0.0198 0.9844171
## content:method  0.079250   0.354153  0.2238 0.8250963
## content:refiner -0.007625   0.354153 -0.0215 0.9830259
## method:refiner  -0.069687   0.354153 -0.1968 0.8458977
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Multiple R-squared:  0.8537, Adjusted R-squared:  0.784
## F-statistic: 12.25 on 10 and 21 DF,  p-value: 1.136e-06
##
## Analysis of Variance Table
##
## Response: length
##                                Df Sum Sq Mean Sq  F value     Pr(>F)
## FO(temp, content, method, refiner)   4 335.06  83.765  20.8705 4.529e-07
## TWI(temp, content, method, refiner)  6 156.77  26.129   6.5101 0.0005388
## Residuals                           21  84.29   4.014
## Lack of fit                          5  81.97  16.394 113.2786 6.612e-12
## Pure error                          16   2.32   0.145
##
## Stationary point of response surface:
##      temp    content    method   refiner
## -2.407638 20.052847 11.162240 65.602587
##
## Eigenanalysis:
## eigen() decomposition
## $values
```

```
## [1]  1.090337000  0.040756823 -0.009049772 -1.122044052
##
## $vectors
##                  [,1]        [,2]        [,3]        [,4]
## temp      0.71172876  0.03059364 -0.01518856  0.70162348
## content   0.30273133  0.80989339  0.37507180 -0.33428667
## method   -0.63350999  0.40711470  0.19289525  0.62905814
## refiner   0.02147111 -0.42118208  0.90657701  0.01621023
```

d

Analyze the residuals from this experiment. Take out outliers, if any.

```
## hat values (leverages) are all = 0.5
##  and there are no factor predictors; no plot no. 5
```
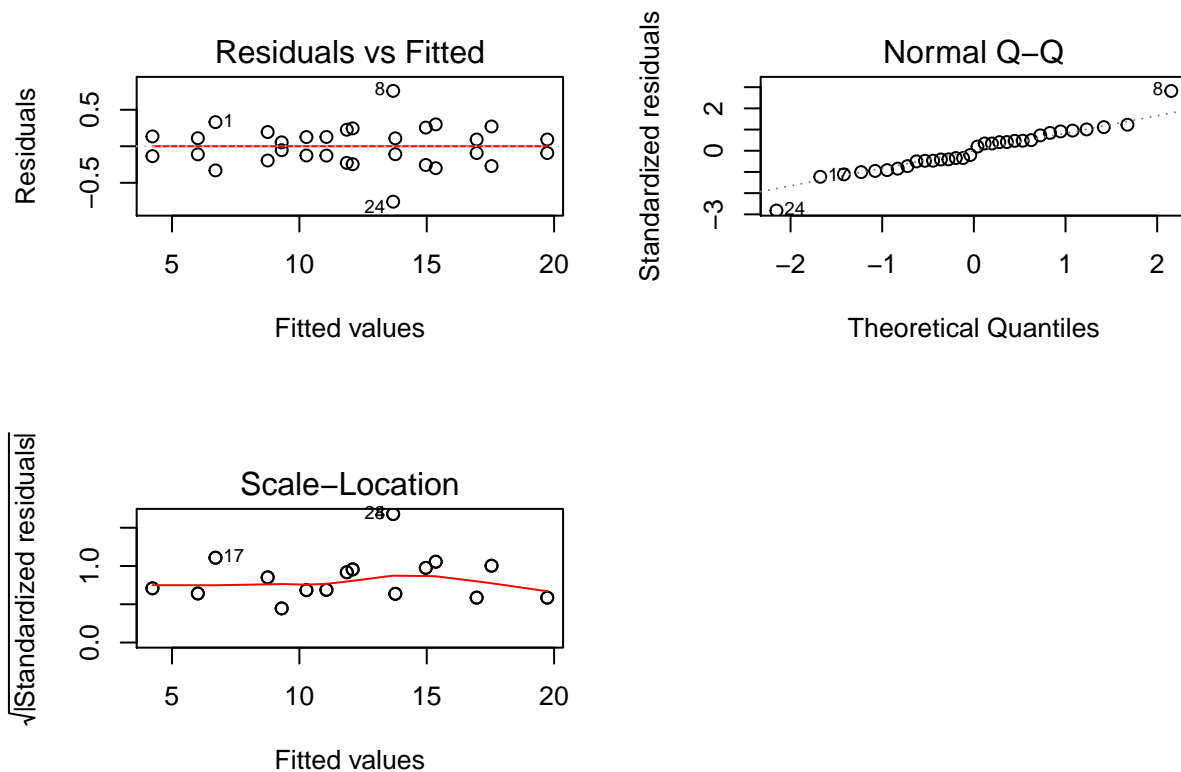

The residuals are homoscedastic. No serious outliers.

e

Is there an indication that any of the factors affect the variability in cracking?

No

f

What recommendations would you make regarding process operations? Use interaction and/or main effect plots to assist in drawing conclusions. Perform a canonical analysis on the model. Is there an optimal response? Perform contour plot of Temp and Content.

Recommend to focus on temp, content, and method. Leverage the interaction between them.

3

Consider the three-variable central composite design shown below. Analyze the data and draw conclusions, assuming that we wish to maximize conversion (y,) with activity (y:) between 55 and 60.

run	std.roder	time	temp	catalyst	Block	conversion	activity
1	1	-1.000000	-1.000000	-1.000000	1	74	53.2
2	2	1.000000	-1.000000	-1.000000	1	51	62.9
3	3	-1.000000	1.000000	-1.000000	1	88	53.4
4	4	1.000000	1.000000	-1.000000	1	70	62.6
5	5	-1.000000	-1.000000	1.000000	1	71	57.3
6	6	1.000000	-1.000000	1.000000	1	90	67.9
7	7	-1.000000	1.000000	1.000000	1	66	59.8
8	8	1.000000	1.000000	1.000000	1	97	67.8
9	9	0.000000	0.000000	0.000000	1	81	59.2
10	10	0.000000	0.000000	0.000000	1	75	60.4
11	11	0.000000	0.000000	0.000000	1	76	59.1
12	12	0.000000	0.000000	0.000000	1	83	60.6
1	1	-1.681793	0.000000	0.000000	2	76	59.1
2	2	1.681793	0.000000	0.000000	2	79	65.9
3	3	0.000000	-1.681793	0.000000	2	85	60.0
4	4	0.000000	1.681793	0.000000	2	97	60.7
5	5	0.000000	0.000000	-1.681793	2	35	57.4
6	6	0.000000	0.000000	1.681793	2	81	63.2
7	7	0.000000	0.000000	0.000000	2	80	60.8
8	8	0.000000	0.000000	0.000000	2	91	38.9

a

Estimate the factor effects. Which factors appear to be large?

b

Perform an analysis of variance. Do any factor affects . Use

c

Build a RSM models (choose a model which works). Daniel plot/Lenth plot.

13

d

Perform a residual analysis. Take out any outlyers.

e

Perform a canonical analysis. Any optimal response. Do a contour plot of Time-Temperature, Time-Catalyst, Temp-Catalyst.

4

The following data were collected by a chemical engineer. The response y is filtration time, x_1 ; is temperature, and x_2 ; is pressure. Fit a second-order model.

x1	x2	y
-1.000	-1.000	54
-1.000	1.000	45
1.000	-1.000	32
1.000	1.000	47
-1.414	0.000	50
1.414	0.000	53
0.000	-1.414	47
0.000	1.414	51
0.000	0.000	41
0.000	0.000	39
0.000	0.000	44
0.000	0.000	42
0.000	0.000	40

a

What operating conditions would you recommend if the objective is to minimize the filtration time?

```
## $xs
##          x1          x2
## -0.2428700  0.3283412
##
## $eigen
## eigen() decomposition
## $values
## [1]  3 -3
##
## $vectors
##         [,1]       [,2]
## x1 0.7071068 -0.7071068
## x2 0.7071068  0.7071068
```

There is a minimum at x1=-0.243 and x2=0.328

b

What operating conditions would you recommend if the objective is to operate the process at a mean filtration rate very close to 46?