Stat 5309 Lab 6

Tom Wilson

Mar 26, 2019

1

```
bit_sizes <- c("1/16","1/8")
speeds <- c("40","90")
treatments <- expand.grid(bit_size=rep(bit_sizes,4),speed=speeds)
treatments
```

```
##    bit_size speed
## 1      1/16    40
## 2       1/8    40
## 3      1/16    40
## 4       1/8    40
## 5      1/16    40
## 6       1/8    40
## 7      1/16    40
## 8       1/8    40
## 9      1/16    90
## 10      1/8    90
## 11     1/16    90
## 12      1/8    90
## 13     1/16    90
## 14      1/8    90
## 15     1/16    90
## 16      1/8    90
```

```
circuit_data <- data.frame(treatments,
                           vibration =c(18.2,27.2,
                                        18.9,24.0,
                                        12.9,22.4,
                                        14.4,22.5,
                                        15.9,41.0,
                                        14.5,43.9,
                                        15.1,36.3,
                                        14.2,39.9
                                        )
                           )
circuit_data %>% kable()
```

bit_size	speed	vibration
1/16	40	18.2
1/8	40	27.2
1/16	40	18.9
1/8	40	24.0
1/16	40	12.9
1/8	40	22.4
1/16	40	14.4

bit_size	speed	vibration
1/8	40	22.5
1/16	90	15.9
1/8	90	41.0
1/16	90	14.5
1/8	90	43.9
1/16	90	15.1
1/8	90	36.3
1/16	90	14.2
1/8	90	39.9

a

analyze the data from this experiment.

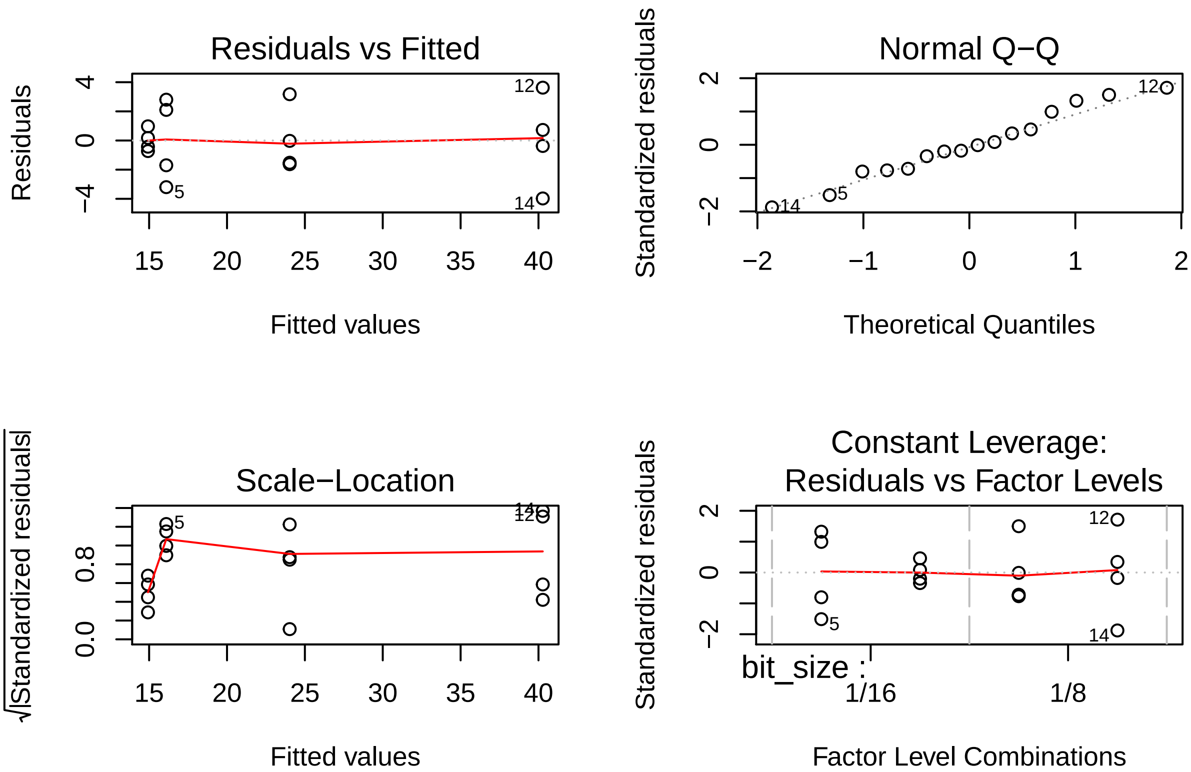
```r
vibration_model <- aov(formula = vibration~bit_size*speed, data=circuit_data)
summary(vibration_model)
```

```
##                Df Sum Sq Mean Sq F value   Pr(>F)    
## bit_size        1 1107.2  1107.2  185.25 1.17e-08 ***
## speed           1  227.3   227.3   38.02 4.83e-05 ***
## bit_size:speed  1  303.6   303.6   50.80 1.20e-05 ***
## Residuals      12   71.7     6.0                     
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

b

construct a normal probability plot of the residuals, and plot the residuals versus the predicted vibration level. Interpret these plots.

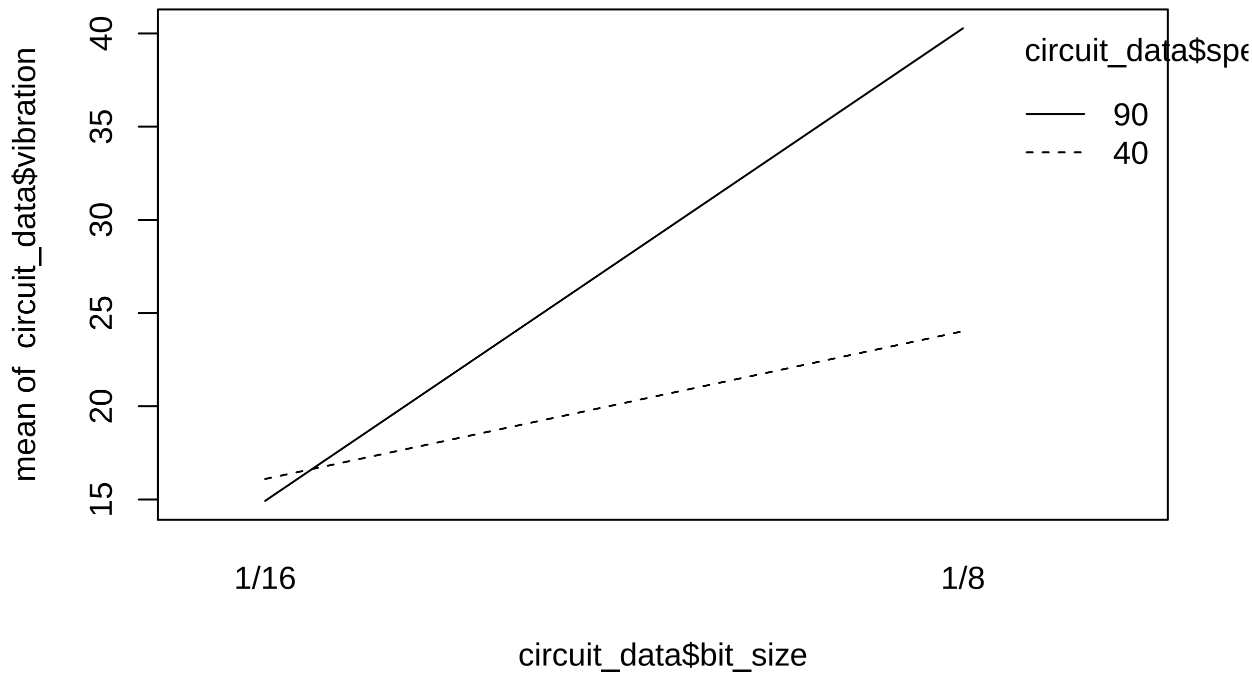
```r
par( mfrow = c(2,2) )
plot(vibration_model)
```



c

Draw the AB interaction plot. What levels of bit size and speed would you recommend for routine operation?

```
interaction.plot(x.factor = circuit_data$bit_size,
                 trace.factor = circuit_data$speed,
                 response = circuit_data$vibration)
```



2

```
cutting_speeds <- c("-","+")
tool_geometries <- c("-","+")
cutting_angles <- c("-","+")
machine_trts <- expand.grid(cutting_speed=rep(cutting_speeds,3),
                            tool_geometry=tool_geometries,
                            cutting_angle=cutting_angles)
machine_trts
```

```
##    cutting_speed tool_geometry cutting_angle
## 1              -             -             -
## 2              +             -             -
## 3              -             -             -
## 4              +             -             -
## 5              -             -             -
## 6              +             -             -
## 7              -             +             -
## 8              +             +             -
## 9              -             +             -
## 10             +             +             -
## 11             -             +             -
## 12             +             +             -
## 13             -             -             +
```

```
## 14              +             -             +
## 15              -             -             +
## 16              +             -             +
## 17              -             -             +
## 18              +             -             +
## 19              -             +             +
## 20              +             +             +
## 21              -             +             +
## 22              +             +             +
## 23              -             +             +
## 24              +             +             +
```

```
machine_data <- data.frame(machine_trts,
                           lifetime = c(22,32,
                                        31,42,
                                        25,29,
                                        35,55,
                                        34,47,
                                        50,46,
                                        44,40,
                                        45,37,
                                        38,36,
                                        60,39,
                                        50,41,
                                        54,47
                                        )
                           )
machine_data %>% kable()
```

cutting_speed	tool_geometry	cutting_angle	lifetime
-	-	-	22
+	-	-	32
-	-	-	31
+	-	-	42
-	-	-	25
+	-	-	29
-	+	-	35
+	+	-	55
-	+	-	34
+	+	-	47
-	+	-	50
+	+	-	46
-	-	+	44
+	-	+	40
-	-	+	45
+	-	+	37
-	-	+	38
+	-	+	36
-	+	+	60
+	+	+	39
-	+	+	50
+	+	+	41
-	+	+	54
+	+	+	47

a

Estimate the factor effects. Which effect appears to be large?

```
lifetime_model <- aov(lifetime~cutting_speed*tool_geometry*cutting_angle,data=machine_data)
lifetime_model$coefficients
```

```
##                                  (Intercept)
##                                    26.000000
##                               cutting_speed+
##                                     8.333333
##                               tool_geometry+
##                                    13.666667
##                               cutting_angle+
##                                    16.333333
##                cutting_speed+:tool_geometry+
##                                     1.333333
##                cutting_speed+:cutting_angle+
##                                   -13.000000
##                tool_geometry+:cutting_angle+
##                                    -1.333333
## cutting_speed+:tool_geometry+:cutting_angle+
##                                    -9.000000
```

b

Use the analysis of variance to confirm your conclusions for part a.

```
anova(lifetime_model)
```

```
## Analysis of Variance Table
##
## Response: lifetime
##                                          Df Sum Sq Mean Sq F value
## cutting_speed                             1   0.38    0.38  0.0129
## tool_geometry                             1 782.04  782.04 26.8129
## cutting_angle                             1 287.04  287.04  9.8414
## cutting_speed:tool_geometry               1  15.04   15.04  0.5157
## cutting_speed:cutting_angle               1 459.38  459.38 15.7500
## tool_geometry:cutting_angle               1  51.04   51.04  1.7500
## cutting_speed:tool_geometry:cutting_angle  1  30.37   30.37  1.0414
## Residuals                                16 466.67   29.17
##                                             Pr(>F)
## cutting_speed                             0.911132
## tool_geometry                            9.154e-05 ***
## cutting_angle                             0.006365 **
## cutting_speed:tool_geometry               0.483031
## cutting_speed:cutting_angle               0.001102 **
## tool_geometry:cutting_angle               0.204473
## cutting_speed:tool_geometry:cutting_angle  0.322673
## Residuals
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

c

Write down a regression model for predicting tool life (in hours) based on the results of this experiment.

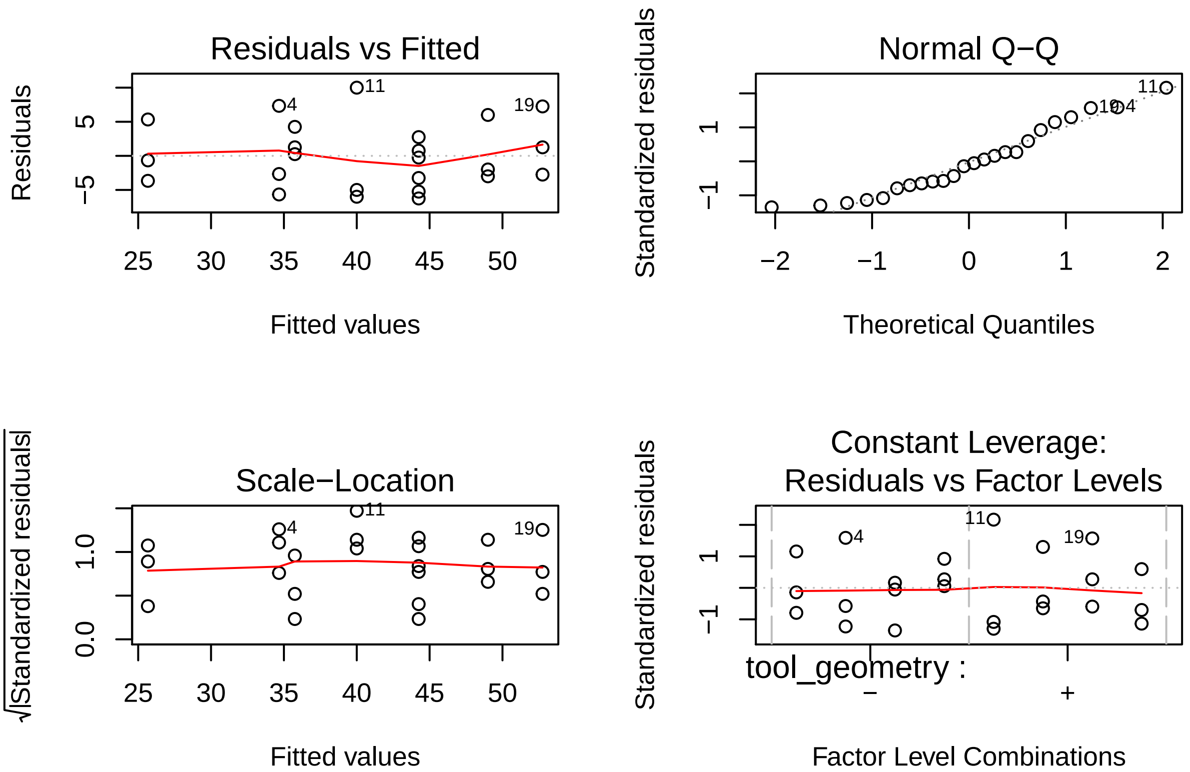
```
reduced_lifetime_model <- lm(lifetime~tool_geometry*cutting_angle+cutting_speed:cutting_angle,data=machi
summary(reduced_lifetime_model)
```

```
##
## Call:
## lm(formula = lifetime ~ tool_geometry * cutting_angle + cutting_speed:cutting_angle,
##     data = machine_data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -6.2500 -3.3542 -0.4583  3.1250 10.0000
##
## Coefficients:
##                               Estimate Std. Error t value Pr(>|t|)
## (Intercept)                     25.667      2.667   9.624  1.6e-08 ***
## tool_geometry+                  14.333      3.079   4.655 0.000197 ***
## cutting_angle+                  18.583      3.772   4.927 0.000109 ***
## tool_geometry+:cutting_angle+   -5.833      4.355  -1.339 0.197091
## cutting_angle-:cutting_speed+    9.000      3.079   2.923 0.009090 **
## cutting_angle+:cutting_speed+   -8.500      3.079  -2.760 0.012888 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 5.334 on 18 degrees of freedom
## Multiple R-squared:  0.7552, Adjusted R-squared:  0.6872
## F-statistic: 11.11 on 5 and 18 DF,  p-value: 5.361e-05
```

d

Analyze the residuals. Are there any obvious problems?

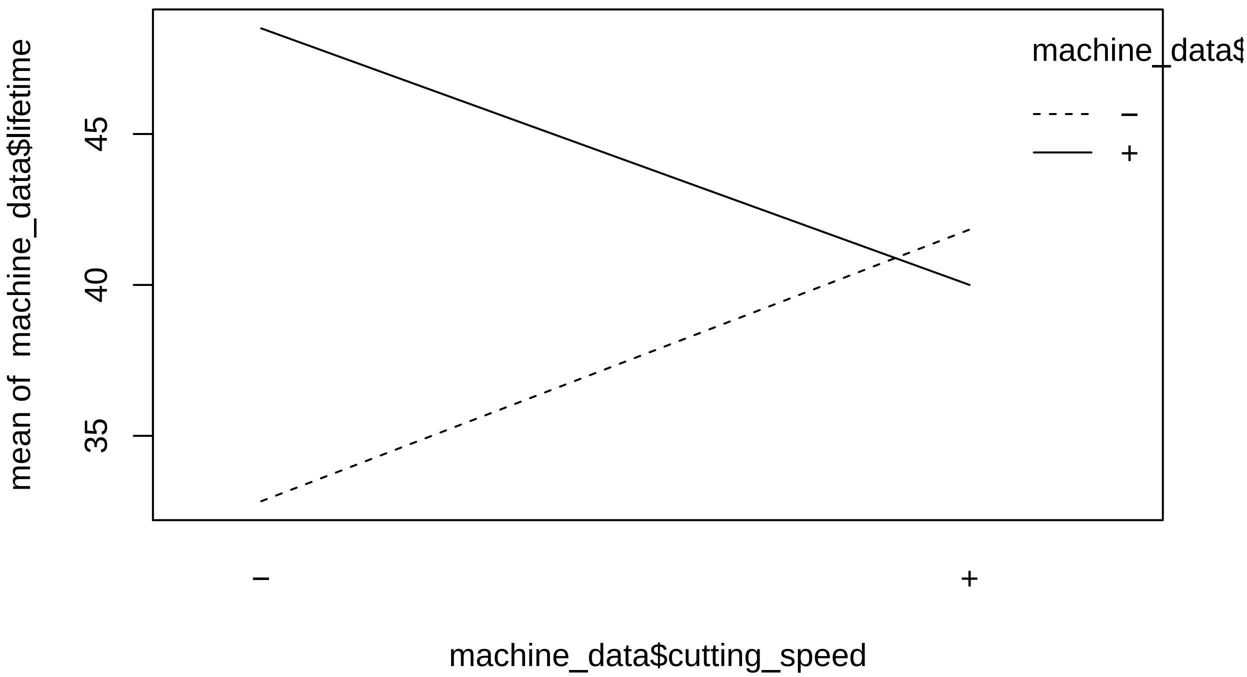
```
par( mfrow = c(2,2) )
plot(reduced_lifetime_model)
```



e

based on an analysis of main effect and interaction plots, what levels of A, B, and C would you recommend using?

```
interaction.plot(x.factor = machine_data$cutting_speed,
                 trace.factor = machine_data$cutting_angle,
                 response = machine_data$lifetime)
```



low cutting speed and high cutting angle is associated with the longest lifetime.

3

An experiment was performed to improve the yield of a chemical process. Four factors were selectd, and two replicates of a completely randomized experiment were run. The results are shown in the following table.

```
lowhigh <- c("-","+")
treatments <- expand.grid(factor1 = rep(lowhigh,2),
                          factor2 = lowhigh,
                          factor3 = lowhigh,
                          factor4 = lowhigh
                          )
chem_data <- cbind(treatments,
                   yield = c(90,74,93,78,
                             81,83,85,80,
                             77,81,78,80,
                             88,73,82,70,
                             98,72,98,76,
                             87,85,83,86,
                             99,79,90,75,
                             87,80,84,80
                             )
                   )
chem_data %>% kable()
```

factor1	factor2	factor3	factor4	yield
-	-	-	-	90
+	-	-	-	74
-	-	-	-	93
+	-	-	-	78
-	+	-	-	81
+	+	-	-	83
-	+	-	-	85
+	+	-	-	80
-	-	+	-	77
+	-	+	-	81
-	-	+	-	78
+	-	+	-	80
-	+	+	-	88
+	+	+	-	73
-	+	+	-	82
+	+	+	-	70
-	-	-	+	98
+	-	-	+	72
-	-	-	+	98
+	-	-	+	76
-	+	-	+	87
+	+	-	+	85
-	+	-	+	83
+	+	-	+	86
-	-	+	+	99
+	-	+	+	79
-	-	+	+	90
+	-	+	+	75
-	+	+	+	87
+	+	+	+	80
-	+	+	+	84
+	+	+	+	80

a

Estimate the factor effects.

```
chem_model <- aov(formula = yield ~ factor1*factor2*factor3*factor4,
                  data=chem_data)

chem_model$coefficients %>% kable()
```

	x
(Intercept)	91.5
factor1+	-15.5
factor2+	-8.5
factor3+	-14.0
factor4+	6.5
factor1+:factor2+	14.0
factor1+:factor3+	18.5
factor2+:factor3+	16.0

	x
factor1+:factor4+	-8.5
factor2+:factor4+	-4.5
factor3+:factor4+	10.5
factor1+:factor2+:factor3+	-30.5
factor1+:factor2+:factor4+	10.5
factor1+:factor3+:factor4+	-12.0
factor2+:factor3+:factor4+	-12.0
factor1+:factor2+:factor3+:factor4+	18.0

b

Prepare an analysis of variance table and determine which factors are important in explaining yield.

```
summary(chem_model)
```

```
##                                   Df Sum Sq Mean Sq F value   Pr(>F)    
## factor1                            1  684.5   684.5  92.814  4.6e-08 ***
## factor2                            1   18.0    18.0   2.441 0.137785    
## factor3                            1   66.1    66.1   8.966 0.008580 ** 
## factor4                            1  136.1   136.1  18.458 0.000555 ***
## factor1:factor2                    1  144.5   144.5  19.593 0.000423 ***
## factor1:factor3                    1    6.1     6.1   0.831 0.375652    
## factor2:factor3                    1    1.1     1.1   0.153 0.701269    
## factor1:factor4                    1   45.1    45.1   6.119 0.024964 *  
## factor2:factor4                    1    1.1     1.1   0.153 0.701269    
## factor3:factor4                    1   18.0    18.0   2.441 0.137785    
## factor1:factor2:factor3            1  231.1   231.1  31.339  4.0e-05 ***
## factor1:factor2:factor4            1  190.1   190.1  25.780 0.000112 ***
## factor1:factor3:factor4            1    4.5     4.5   0.610 0.446129    
## factor2:factor3:factor4            1    4.5     4.5   0.610 0.446129    
## factor1:factor2:factor3:factor4  1   40.5    40.5   5.492 0.032355 *  
## Residuals                         16  118.0     7.4                     
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

c

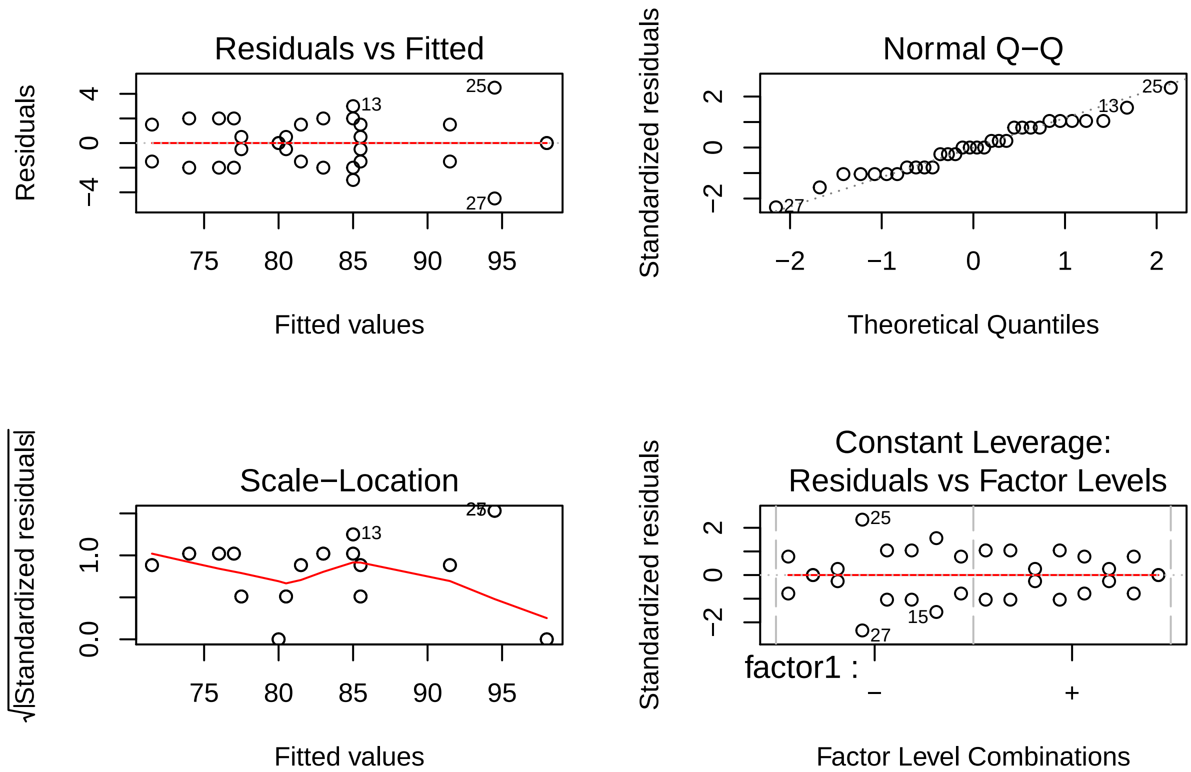
Write down a regression model for predicting yield, assuming that all four factors were varied over the range from -1 to +1.

$$yield = 91.5 - 15.5f1 - 8.5f2 - 14.0f3 + 6.5f4 + 14.0f1f2 - 30.5f1f2f3 + 10.5f1f2f4$$

d

Plot the residuals versus the predicted yield and on a normal probability scale. Does the residual analysis appear satisfactory?

```
par( mfrow = c(2,2) )
plot(chem_model)
```



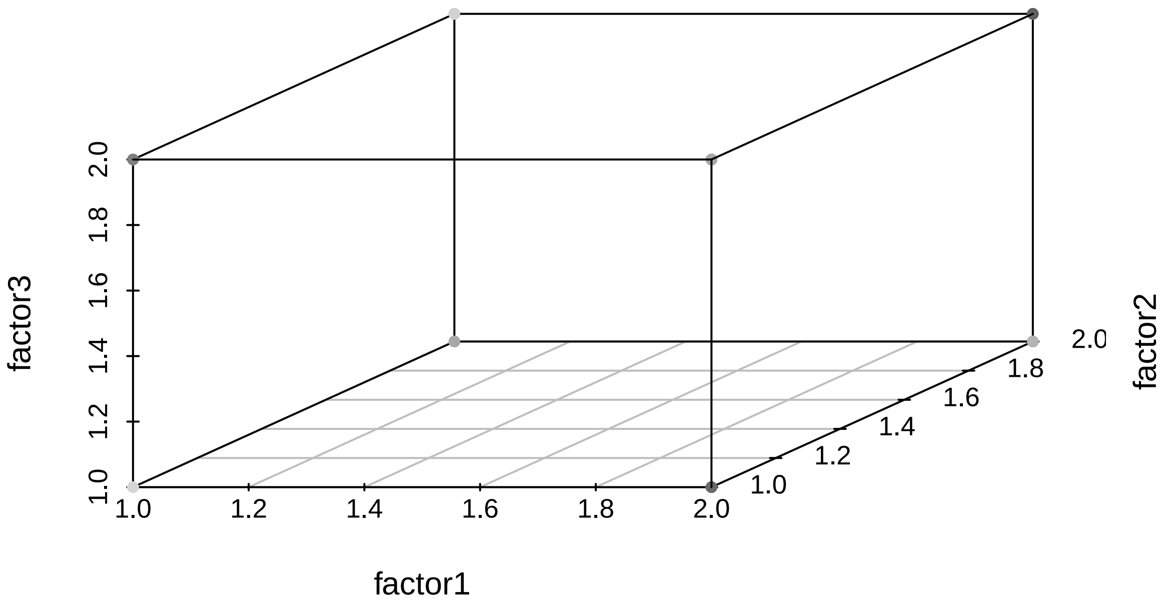
e

Two three-factor interactions, ABC and ABD, apparently have large effects. Draw a cube plot in th factors A, B, and C with the average yields shown at each corner. Repeat using the factors A, B, and D. Do these two plots aid in a data interpretation? Where would you recommend that the process be run with respect to the four variables?

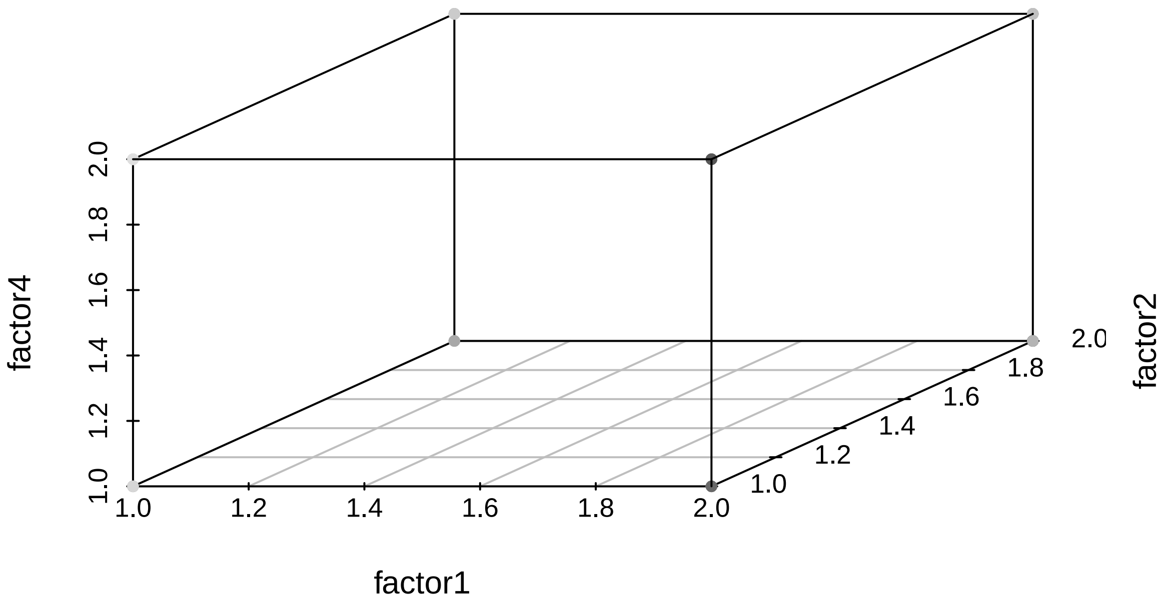
```
colorpalette <- gray.colors(length(chem_data$yield))
colorpalette
```

```
##  [1] "#4D4D4D" "#575757" "#606060" "#696969" "#707070" "#777777" "#7E7E7E"
##  [8] "#848484" "#8A8A8A" "#8F8F8F" "#949494" "#999999" "#9E9E9E" "#A3A3A3"
## [15] "#A8A8A8" "#ACACAC" "#B0B0B0" "#B4B4B4" "#B8B8B8" "#BCBCBC" "#C0C0C0"
## [22] "#C4C4C4" "#C8C8C8" "#CBCBCB" "#CFCFCF" "#D2D2D2" "#D6D6D6" "#D9D9D9"
## [29] "#DCDCDC" "#DFDFDF" "#E2E2E2" "#E6E6E6"
```

```
colors <- colorpalette[rank(chem_data$yield)]
scatterplot3d(chem_data[1:3],pch=20,color = colors)
```




```
scatterplot3d(chem_data[c(1,2,4)],pch=20,color = colors)
```


factor4
1.0
1.2
1.4
1.6
1.8
2.0
factor1
2.0
factor2