Stat 5309 Semester Project

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An article in the AT&T Technical Journal (March/April 1986, Vol. 65, pp. 39-50) describes the application of two-level factorial designs to integrated circuit manufacturing. A basic processing step is to grow an epitaxial layer on polished silicon wafers. The wafers mounted on a susceptor are positioned inside a bell jar, and chemical vapors are introduced. The susceptor is rotated and heat is applied until the epitaxial layer is thick enough. An experiment was run using two factors: arsenic flow rate (A) and deposition time (B). Four replicates were run, and the epitaxial layer thickness was measured in um. The data are shown below:

Replicate Factor Levels

```
flow_rates <- c(0.55,0.59)
depo_times <- c(10,15)
epitaxial_data <- expand.grid(flow_rate=flow_rates,
                              depo_time=rep(depo_times,4))
epitaxial_data <- cbind(epitaxial_data,
                        thickness=c(14.037,13.880,14.821,14.888,
                                    16.165,13.860,14.757,14.921,
                                    13.972,14.032,14.843,14.415,
                                    13.907,13.914,14.878,14.932))
```

a

Estimate the factor effects.

```
epitaxial_model <- lm(formula = thickness~flow_rate*depo_time,
                      data=epitaxial_data)
epitaxial_model$coefficients %>% kable()
```

	x
(Intercept)	37.62656
flow_rate	-43.11875
depo_time	-1.48735
flow_rate:depo_time	2.81500

b

Conduct an analysis of variance. Which factors are important?

```
anova(epitaxial_model)
```

```
## Analysis of Variance Table
##
## Response: thickness
##                      Df Sum Sq Mean Sq F value  Pr(>F)
```

```
## flow_rate            1 0.4026 0.40259  1.2619 0.28327
## depo_time            1 1.3736 1.37358  4.3054 0.06016 .
## flow_rate:depo_time  1 0.3170 0.31697  0.9935 0.33856
## Residuals           12 3.8285 0.31904
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

deposition time is significant while flow rate and the interaction are not siginifcantly different from coefficient=0

```r
summary(epitaxial_model)
```

```
##
## Call:
## lm.default(formula = thickness ~ flow_rate * depo_time, data = epitaxial_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.61325 -0.14431 -0.00562  0.10187  1.64475
##
## Coefficients:
##                     Estimate Std. Error t value Pr(>|t|)
## (Intercept)           37.627     20.533   1.832   0.0918 .
## flow_rate            -43.119     36.001  -1.198   0.2542
## depo_time             -1.487      1.611  -0.923   0.3740
## flow_rate:depo_time    2.815      2.824   0.997   0.3386
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5648 on 12 degrees of freedom
## Multiple R-squared:  0.3535, Adjusted R-squared:  0.1918
## F-statistic: 2.187 on 3 and 12 DF,  p-value: 0.1425
```

c

Write down a regression equation that could be used to predict epitaxial layer thickness over the region of arsenic flow rate and deposition time used in this experiment.

$thickness = 37.627 - 43.119 flow rate - 1.148deposition time $

Build a RSM model (2nd order, 1st order with interaction). Choose one which works.

```r
thickness_rsm <- rsm( thickness ~ FO(depo_time,flow_rate) + TWI(depo_time,flow_rate) , data=epitaxial_da
summary(thickness_rsm)
```

```
##
## Call:
## rsm(formula = thickness ~ FO(depo_time, flow_rate) + TWI(depo_time,
##     flow_rate), data = epitaxial_data)
##
##                     Estimate Std. Error t value Pr(>|t|)
## (Intercept)          37.6266    20.5334  1.8325   0.0918 .
## depo_time            -1.4874     1.6108 -0.9234   0.3740
## flow_rate           -43.1188    36.0014 -1.1977   0.2542
## depo_time:flow_rate   2.8150     2.8242  0.9967   0.3386
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

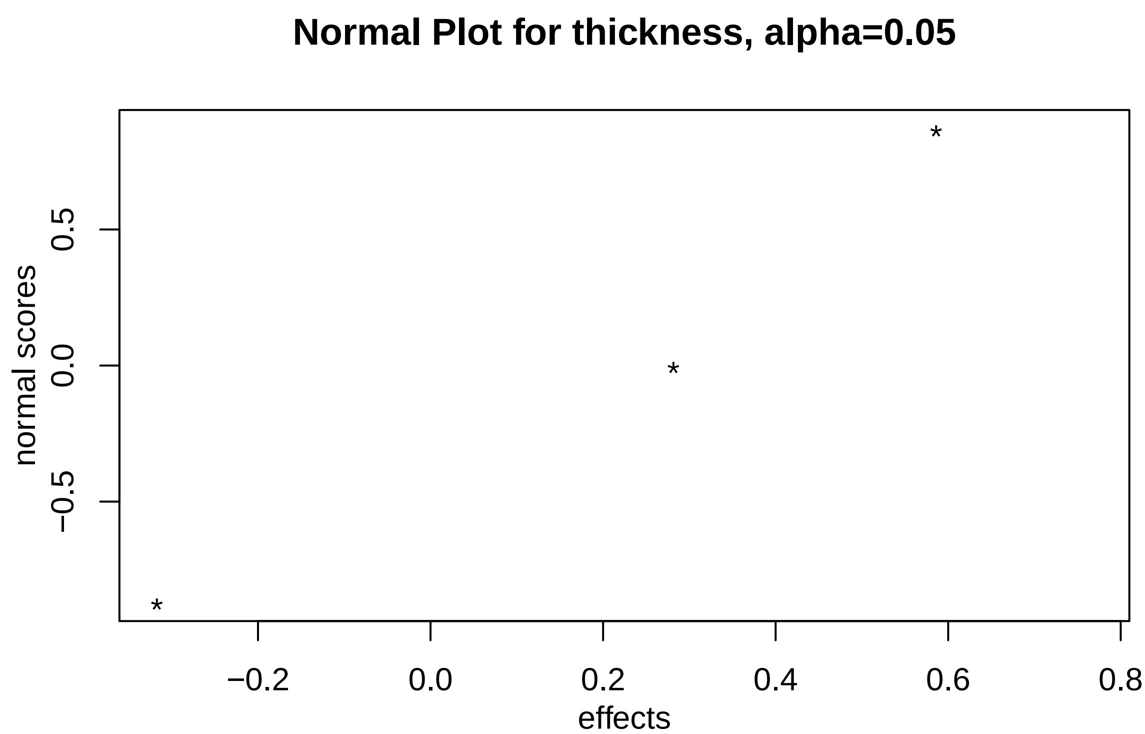
```
##
## Multiple R-squared:  0.3535, Adjusted R-squared:  0.1918
## F-statistic: 2.187 on 3 and 12 DF,  p-value: 0.1425
##
## Analysis of Variance Table
##
## Response: thickness
##                           Df Sum Sq Mean Sq F value Pr(>F)
## FO(depo_time, flow_rate)   2 1.7762       1  2.7836 0.1016
## TWI(depo_time, flow_rate)  1 0.3170       0  0.9935 0.3386
## Residuals                 12 3.8285       0
## Lack of fit                0 0.0000     Inf
## Pure error                12 3.8285       0
##
## Stationary point of response surface:
##  depo_time  flow_rate
## 15.3174956  0.5283659
##
## Eigenanalysis:
## eigen() decomposition
## $values
## [1]  1.4075 -1.4075
##
## $vectors
##                 [,1]       [,2]
## depo_time 0.7071068 -0.7071068
## flow_rate 0.7071068  0.7071068
```

Perform Daniel plot and Lenth plot. What is the model 's R-square.

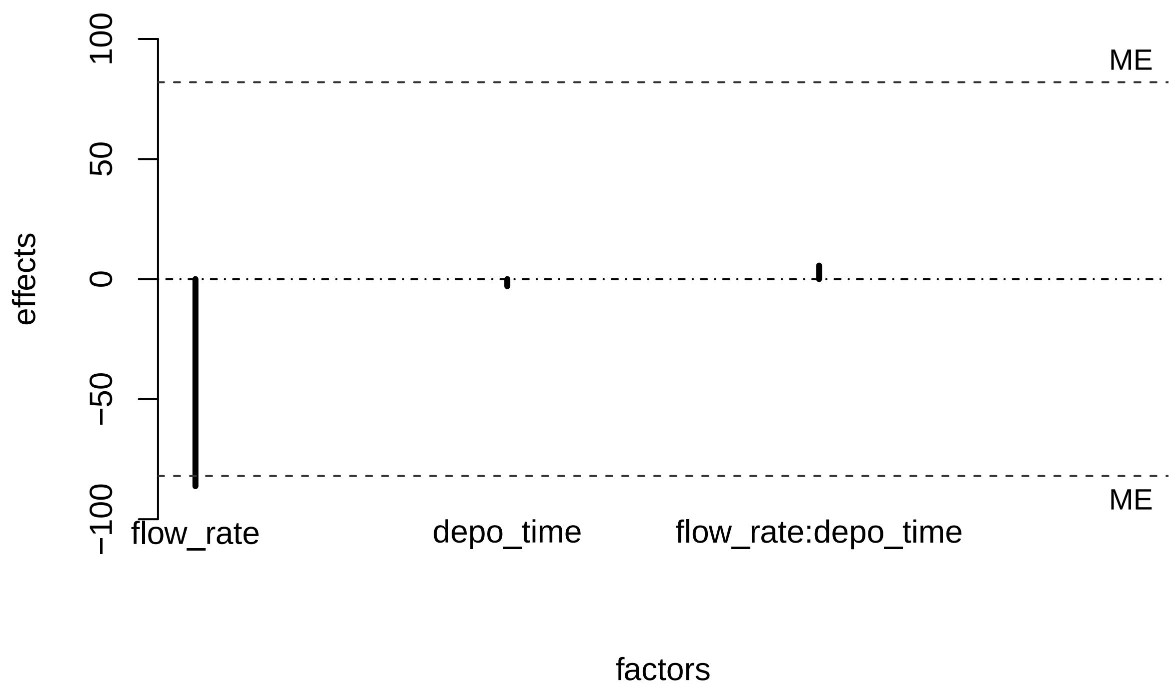
R-squared = 0.19

```r
DanielPlot(epitaxial_model)
```

```
## simulated critical values not available for all requests, used conservative ones
```




```
LenthPlot(epitaxial_model, alpha = 0.05, plt =TRUE, limits = TRUE)
```

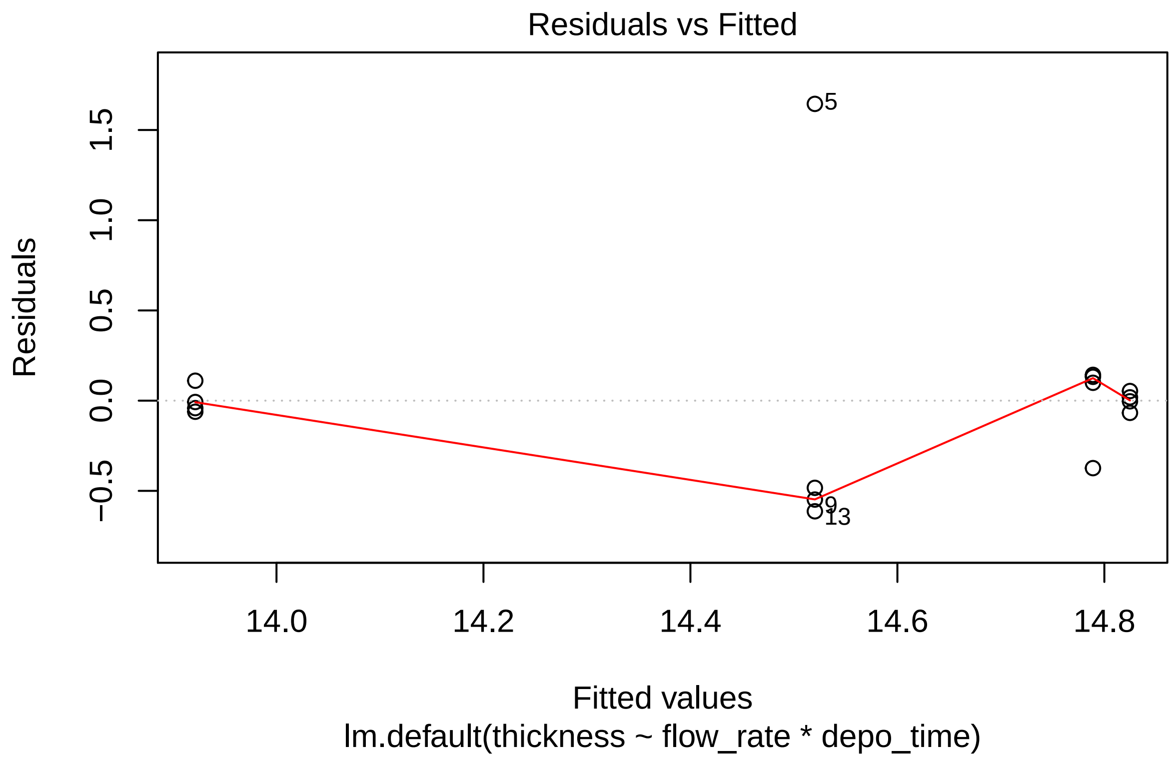


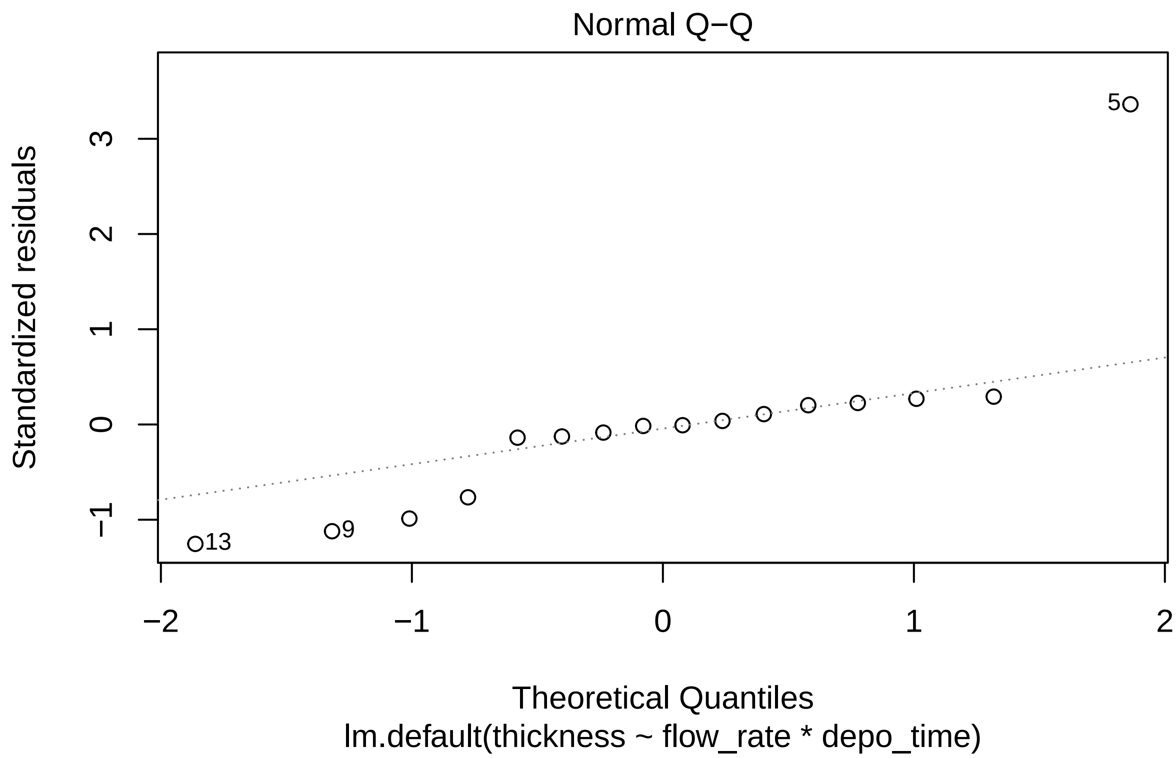

```
##      alpha        PSE         ME        SME
##   0.050000   6.453525  81.999810 242.293942
```

d

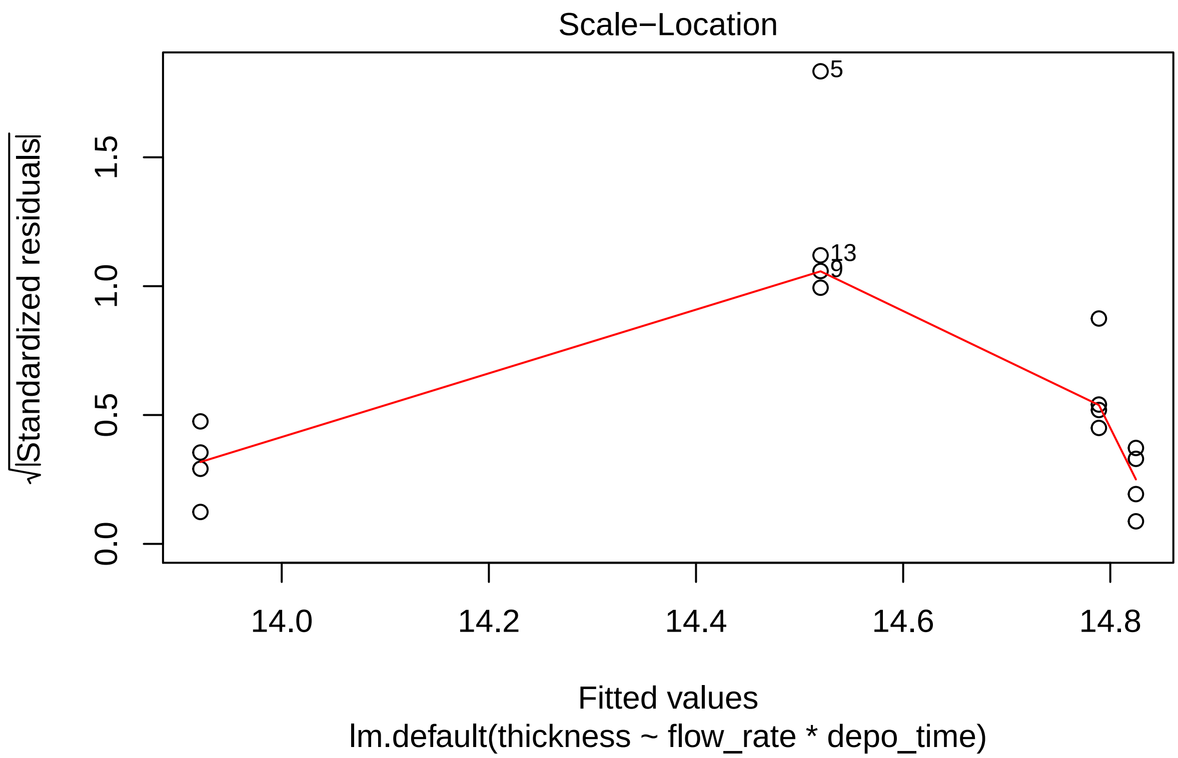
Analyze the residuals. Are there any residuals that should cause concern?

```r
plot(epitaxial_model)
```


Residuals vs Fitted
Residuals
1.5
1.0
0.5
0.0
−0.5
5
9
13
14.0
14.2
14.4
14.6
14.8
Fitted values
lm.default(thickness ~ flow_rate * depo_time)




```
## hat values (leverages) are all = 0.25
##  and there are no factor predictors; no plot no. 5
```

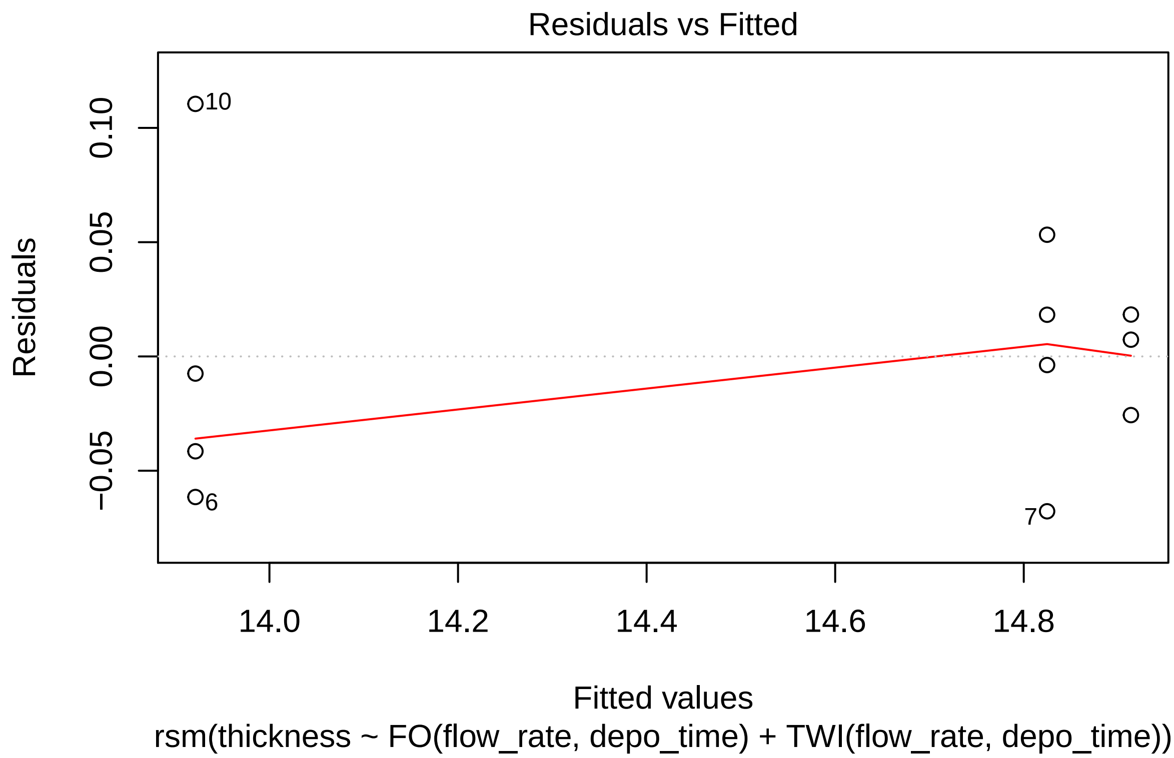

Scale−Location
√|Standardized residuals|
0.0
0.5
1.0
1.5
5
13
9
14.0
14.2
14.4
14.6
14.8
Fitted values
lm.default(thickness ~ flow_rate * depo_time)

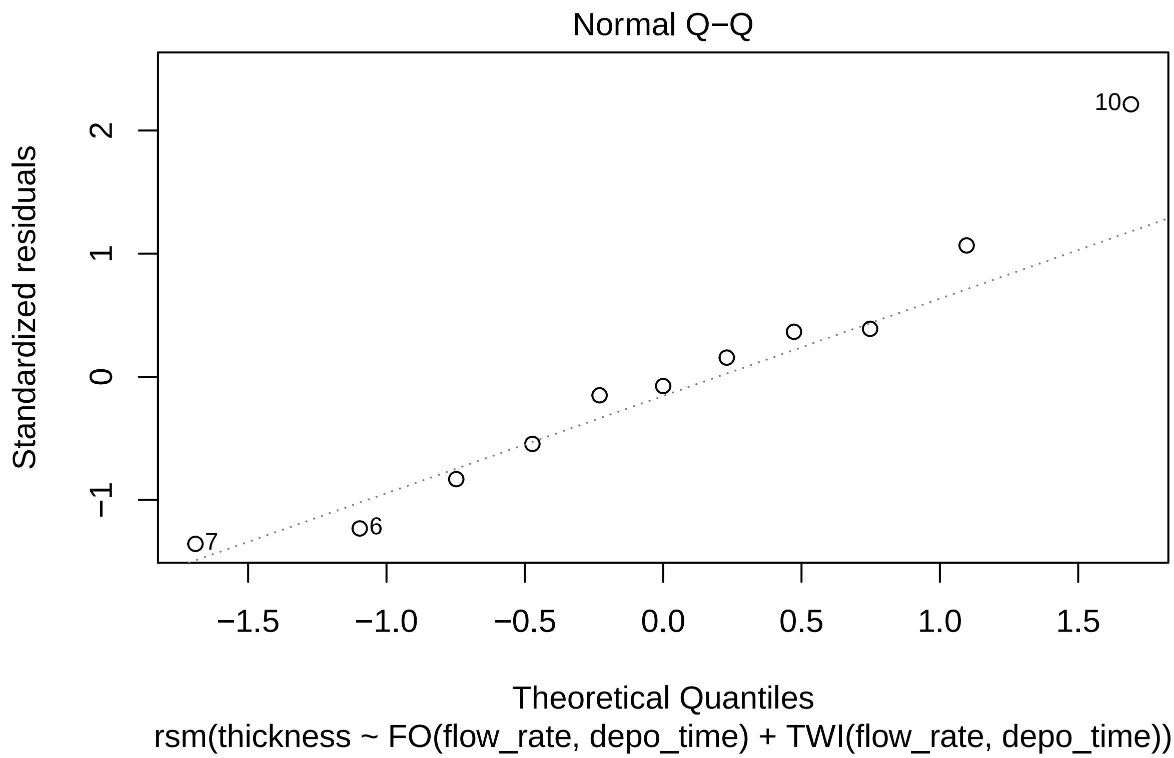
There is 1 very serious outlier.

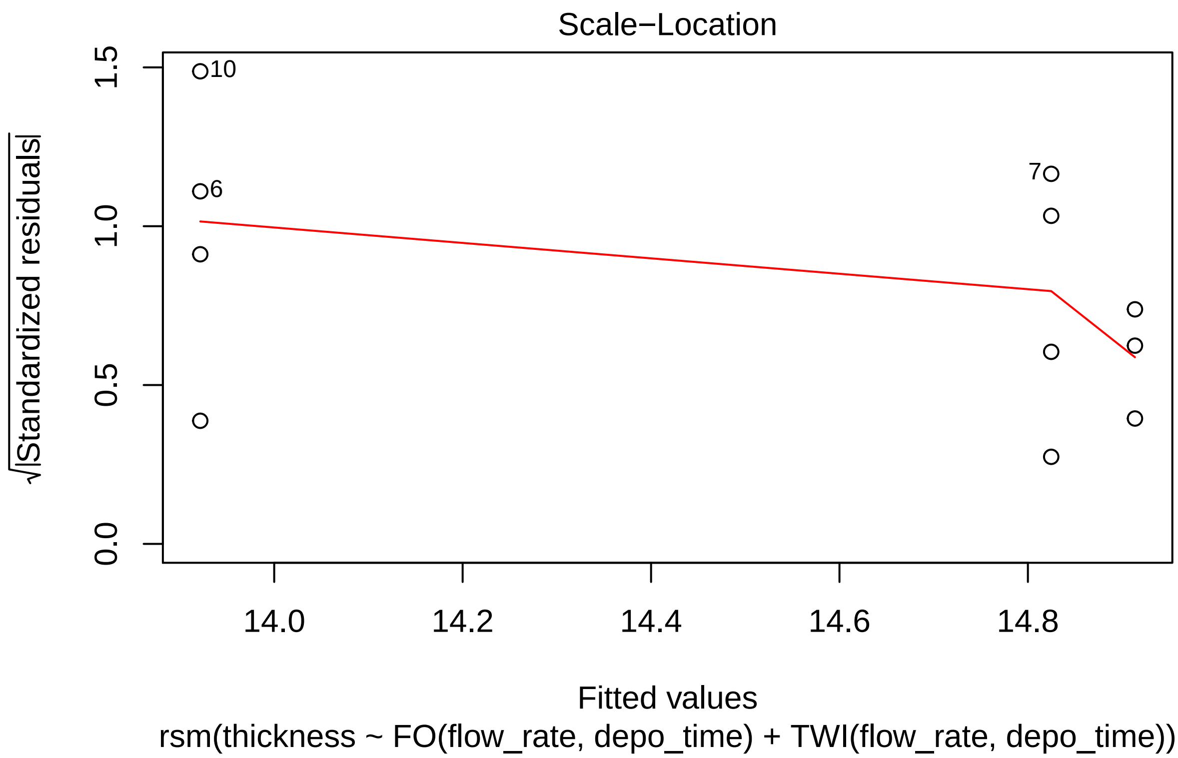
Find the cook distance. Take out the outlier(s). Rebuilt the rsm model on new data.

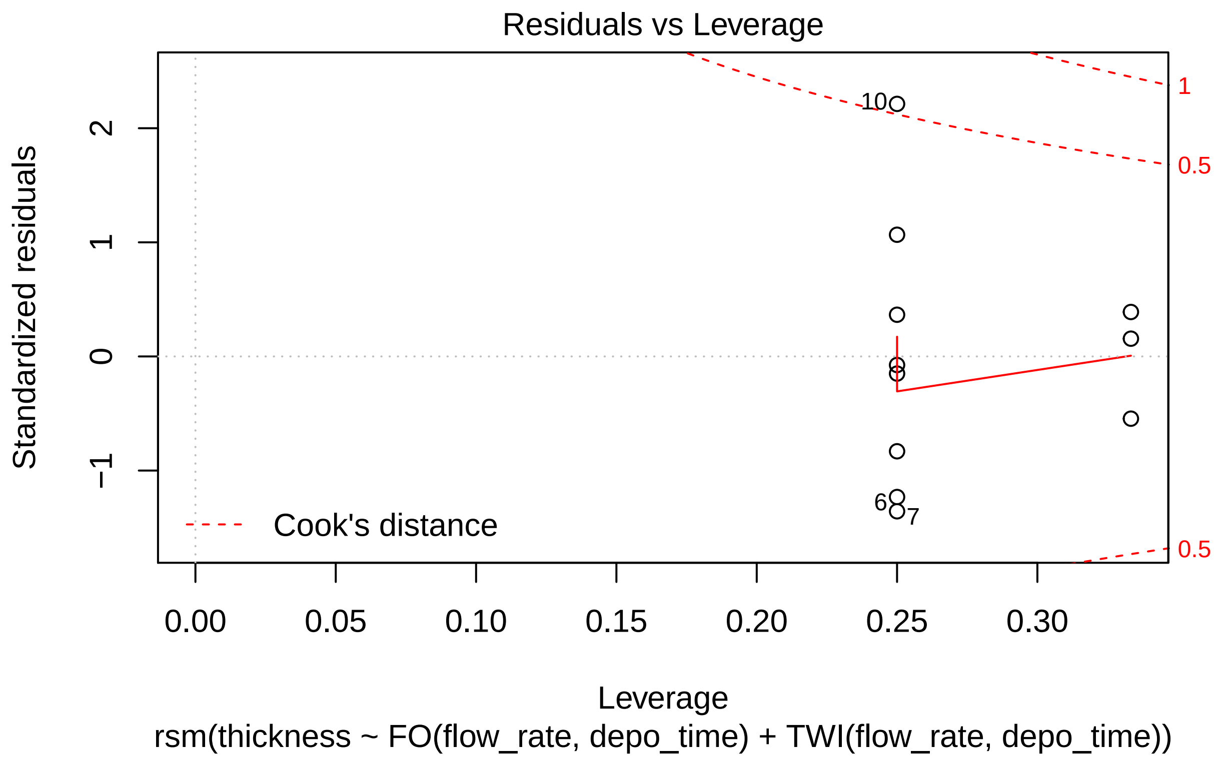
```
is_not_outlier <- cooks.distance(epitaxial_model)<0.025
epitaxial_outlier_removed <- rsm(formula = thickness~FO(flow_rate,depo_time)+TWI(flow_rate,depo_time),
                                 data=epitaxial_data[is_not_outlier,])
```

```
plot(epitaxial_outlier_removed)
```


Residuals vs Fitted
Residuals
0.10
0.05
0.00
−0.05
10
6
7
14.0
14.2
14.4
14.6
14.8
Fitted values
rsm(thickness ~ FO(flow_rate, depo_time) + TWI(flow_rate, depo_time))


Normal Q−Q
Standardized residuals
Theoretical Quantiles
rsm(thickness ~ FO(flow_rate, depo_time) + TWI(flow_rate, depo_time))
−1
0
1
2
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
10
6
7


Scale−Location
√|Standardized residuals|
0.0
0.5
1.0
1.5
10
6
7
14.0
14.2
14.4
14.6
14.8
Fitted values
rsm(thickness ~ FO(flow_rate, depo_time) + TWI(flow_rate, depo_time))



e

Discuss how you might deal with the potential outlier found in part (d).

f

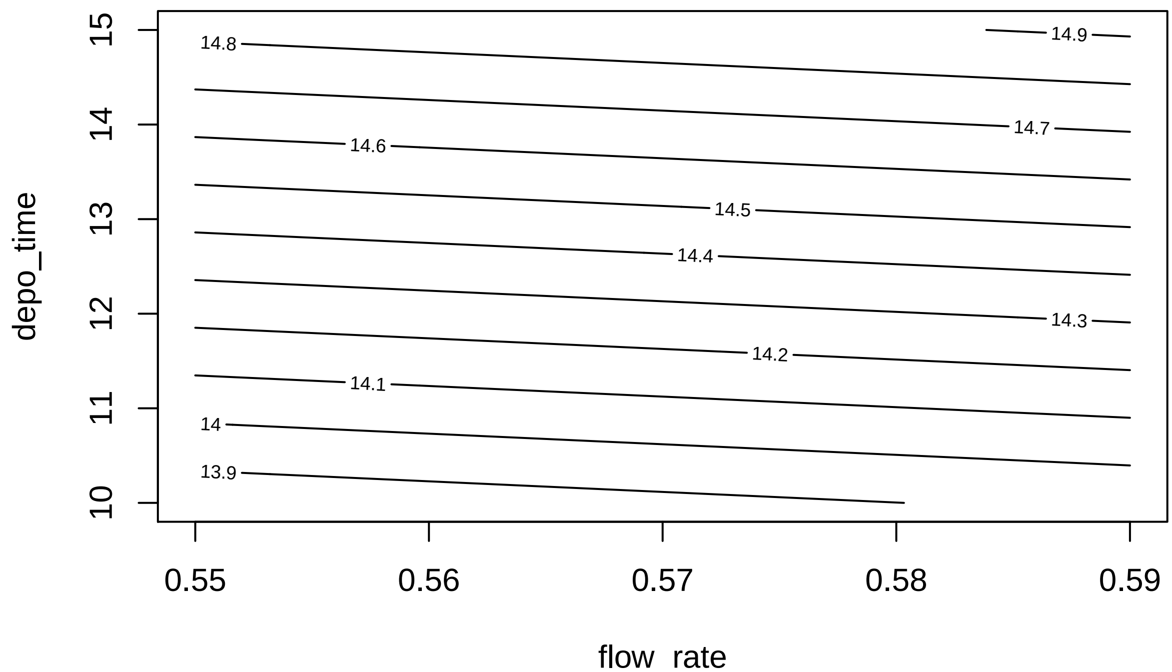
Perform a canonical analysis. Do a contour plot. Any optimal response?

```r
canonical(thickness_rsm)
```

```
## $xs
##  depo_time  flow_rate
## 15.3174956  0.5283659
##
## $eigen
## eigen() decomposition
## $values
## [1]  1.4075 -1.4075
##
## $vectors
##                 [,1]       [,2]
## depo_time 0.7071068 -0.7071068
## flow_rate 0.7071068  0.7071068
```

```r
contour(epitaxial_outlier_removed,~flow_rate+depo_time)
```

```
## Warning in predict.lm(lmobj, newdata = newdata): prediction from a rank-
## deficient fit may be misleading
```



2

A nickel-titanium alloy is used to make components for jet turbine aircraft engines. Cracking is a potentially serious problem in the final part, because it can lead to nonrecoverable failure. A test is run at the parts producer to determine the effect of four factors on cracks. The four factors are pouring temperature (A), titanium content (B), heat treatment method (C), and amount of grain refiner used (D). Two replicates of a 2^4 design are run, and the length of crack in mm $\times 10^{-2}$? induced in a sample coupon subjected to a standard test is measured. The data are shown in the following table:

```r
low_high <- c('-','+')
cracking_data <- expand.grid(pouring_temperature=low_high,
                             titanium_content=low_high,
                             heat_treatment_method=low_high,
                             grain_refiners=rep(low_high,2))
cracking_data <- cbind(cracking_data,
                       crack_length=c(7.037,14.707,11.635,17.273,
                                      10.403,4.368,9.360,14.440,
                                      8.561,16.867,13.876,19.824,
                                      11.846,6.125,11.190,15.653,
                                      6.376,15.219,12.089,17.815,
```

```
                                          10.151,4.098,9.253,12.923,
                                          8.951,17.052,13.658,19.639,
                                          12.337,5.904,10.935,15.053))
cracking_data %>% kable()
```

pouring_temperature	titanium_content	heat_treatment_method	grain_refiners	crack_length
-	-	-	-	7.037
+	-	-	-	14.707
-	+	-	-	11.635
+	+	-	-	17.273
-	-	+	-	10.403
+	-	+	-	4.368
-	+	+	-	9.360
+	+	+	-	14.440
-	-	-	+	8.561
+	-	-	+	16.867
-	+	-	+	13.876
+	+	-	+	19.824
-	-	+	+	11.846
+	-	+	+	6.125
-	+	+	+	11.190
+	+	+	+	15.653
-	-	-	-	6.376
+	-	-	-	15.219
-	+	-	-	12.089
+	+	-	-	17.815
-	-	+	-	10.151
+	-	+	-	4.098
-	+	+	-	9.253
+	+	+	-	12.923
-	-	-	+	8.951
+	-	-	+	17.052
-	+	-	+	13.658
+	+	-	+	19.639
-	-	+	+	12.337
+	-	+	+	5.904
-	+	+	+	10.935
+	+	+	+	15.053

a

Estimate the factor effects, Which factor effects appear to be large? Change the factors name (Temp,Content,Method,Refiner,Length).

```
cracking_model <- aov(formula=crack_length~.,data=cracking_data)
summary(cracking_model)
```

```
##                       Df Sum Sq Mean Sq F value   Pr(>F)
## pouring_temperature    1  75.96   75.96   8.508 0.007038 **
## titanium_content       1 130.47  130.47  14.613 0.000706 ***
## heat_treatment_method  1  99.90   99.90  11.189 0.002427 **
## grain_refiners         1  28.74   28.74   3.219 0.084013 .
```

```
## Residuals             27 241.06    8.93
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

b

Conduct an analysis of variance. Do any of the factors affect cracking? Use $\alpha = 0.05$. Perform effects Daniel plot and Lenth plot.

```r
anova(cracking_model)
```

```
## Analysis of Variance Table
##
## Response: crack_length
##                        Df  Sum Sq Mean Sq F value    Pr(>F)
## pouring_temperature    1  75.959  75.959  8.5079 0.0070380 **
## titanium_content       1 130.468 130.468 14.6133 0.0007061 ***
## heat_treatment_method  1  99.899  99.899 11.1894 0.0024271 **
## grain_refiners         1  28.736  28.736  3.2186 0.0840129 .
## Residuals             27 241.057   8.928
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

c

Write down a regression model that can be used to predict crack length as a function of the significant main effects and interactions you have identified in part (b). Build a RSM model (2nd order, 1st order with interaction). Choose one which works.

d

Analyze the residuals from this experiment. Take out outliers, if any.

e

Is there an indication that any of the factors affect the variability in cracking?

f

What recommendations would you make regarding process operations? Use interaction and/or main effect plots to assist in drawing conclusions. Perform a canonical analysis on the model. Is there an optimal response? Perform contour plot of Temp and Content.

3

Consider the three-variable central composite design shown below. Analyze the data and draw conclusions, assuming that we wish to maximize conversion (y,) with activity (y:) between 55 and 60.

```r
conversion_data <- ccd(3, n0 = c(4,2), alpha = "rotatable", randomize = FALSE)
colnames(conversion_data) <- c('run','std.roder','time','temp','catalyst','Block')
conversion_data <- cbind(conversion_data,
                         conversion = c(74.00,51.00,88.00,70.00,
                                        71.00,90.00,66.00,97.00,
                                        81.00,75.00,76.00,83.00,
                                        76.00,79.00,85.00,97.00,
                                        35.00,81.00,80.00,91.00),
                         activity = c(53.20,62.90,53.40,62.60,
                                      57.30,67.90,59.80,67.80,
                                      59.20,60.40,59.10,60.60,
                                      59.10,65.90,60.00,60.70,
                                      57.40,63.20,60.80,38.90))
conversion_data %>% kable()
```

run	std.roder	time	temp	catalyst	Block	conversion	activity
1	1	-1.000000	-1.000000	-1.000000	1	74	53.2
2	2	1.000000	-1.000000	-1.000000	1	51	62.9
3	3	-1.000000	1.000000	-1.000000	1	88	53.4
4	4	1.000000	1.000000	-1.000000	1	70	62.6
5	5	-1.000000	-1.000000	1.000000	1	71	57.3
6	6	1.000000	-1.000000	1.000000	1	90	67.9
7	7	-1.000000	1.000000	1.000000	1	66	59.8
8	8	1.000000	1.000000	1.000000	1	97	67.8
9	9	0.000000	0.000000	0.000000	1	81	59.2
10	10	0.000000	0.000000	0.000000	1	75	60.4
11	11	0.000000	0.000000	0.000000	1	76	59.1
12	12	0.000000	0.000000	0.000000	1	83	60.6
1	1	-1.681793	0.000000	0.000000	2	76	59.1
2	2	1.681793	0.000000	0.000000	2	79	65.9
3	3	0.000000	-1.681793	0.000000	2	85	60.0
4	4	0.000000	1.681793	0.000000	2	97	60.7
5	5	0.000000	0.000000	-1.681793	2	35	57.4
6	6	0.000000	0.000000	1.681793	2	81	63.2
7	7	0.000000	0.000000	0.000000	2	80	60.8
8	8	0.000000	0.000000	0.000000	2	91	38.9

a

Estimate the factor effects. Which factors appear to be large?

b

Perform an analysis of variance. Do any factor affects . Use

c

Build a RSM models (choose a model which works). Daniel plot/Lenth plot.

d

Perform a residual analysis. Take out any outlyers.

e

Perform a canonical analysis. Any optimal response. Do a contour plot of Time-Temperature, Time-Catalyst, Temp-Catalyst.

4

The following data were collected by a chemical engineer. The response y is filtration time, x_1 ; is temperature, and x_3 ; is pressure. Fit a second-order model.

```
fitration_data <- data.frame(x1=c(-1,-1,1,1,-1.414,1.414,0,0,0,0,0,0,0),
                             x2=c(-1,1,-1,1,0,0,-1.414,1.414,0,0,0,0,0),
                             y =c(54,45,32,47,50,53,47,51,41,39,44,42,40))
fitration_data %>% kable()
```

x1	x2	y
-1.000	-1.000	54
-1.000	1.000	45
1.000	-1.000	32
1.000	1.000	47
-1.414	0.000	50
1.414	0.000	53
0.000	-1.414	47
0.000	1.414	51
0.000	0.000	41
0.000	0.000	39
0.000	0.000	44
0.000	0.000	42
0.000	0.000	40

a

What operating conditions would you recommend if the objective is to minimize the filtration time?

b

What operating conditions would you recommend if the objective is to operate the process at a mean filtration rate very close to 46?