

# Stat 5309 Final Exam

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## Problem 1:

The effect of 4 types of graphite coater types on light box reading are to be studied. The readings might differ from day to day. Here assume we have a fixed effect model. Observations are taken for 3 days on the four types. The results are

```
days<-c('d1','d2','d3')
types<-c('t1','t2','t3','t4')
graphite_data <- expand.grid(day=days,type=types)
graphite_data <- cbind(graphite_data,light=c(4,4.8,4,
                                             4.8,5,4.8,
                                             5,5.2,5.6,
                                             4.6,4.6,5))
graphite_data %>% kable()
```

day	type	light
d1	t1	4.0
d2	t1	4.8
d3	t1	4.0
d1	t2	4.8
d2	t2	5.0
d3	t2	4.8
d1	t3	5.0
d2	t3	5.2
d3	t3	5.6
d1	t4	4.6
d2	t4	4.6
d3	t4	5.0

(a)

Note: Day is used as blocking factor

Write the model equation

State all the assumptions about residuals

(b)

Use definitions to estimate the model parameters ( 14 parameters)

$$\mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \tau_1, \tau_2, \tau_3, \tau_4, \sigma^2, \beta_1, \beta_2, \beta_3, \beta_4$$

where  $\tau_i, \beta_j$  are treatment effects and block effects.

(c)

Test the hypothesis that the effects ( ie, mean effect) of the 4 graphite coater types are the same.

Hypothesis: Ho\_\_\_\_\_ Ha\_\_\_\_\_

Test statistic\_\_\_\_\_

P-values \_\_\_\_\_

Conclusion\_\_\_\_\_

(d)

How would your analysis of variance be different if the experiment had not been blocked?

Write the ANOVA table with the value of the test statistic and p-value.

## Problem 2:

The response time in milliseconds was determined for 3 different types of circuits that could be used in an automatic valve shutoff mechanism. The results from a completely randomized experiment are shown in the following table:

```
circuit_types <- c('c1','c2','c3')
circuit_data <- data.frame(circuit_type=rep(circuit_types,5),
                           response_time=c(9,20,6,
                                           12,21,5,
                                           10,23,8,
                                           8,17,16,
                                           15,30,7))

circuit_data
```

##	circuit_type	response_time
## 1	c1	9
## 2	c2	20
## 3	c3	6
## 4	c1	12
## 5	c2	21
## 6	c3	5
## 7	c1	10
## 8	c2	23
## 9	c3	8
## 10	c1	8
## 11	c2	17
## 12	c3	16
## 13	c1	15
## 14	c2	30
## 15	c3	7

(a)

Write the model equation:\_\_\_\_\_ State all the assumptions about residuals: \_\_\_\_\_

(b)

Use definitions and R to find the estimates for the following parameters:

$$\mu_1, \mu_2, \mu_3, \mu, \tau_1, \tau_2, \tau_3, \sigma^2$$

$\tau_i$  are treatment effects

(c)

Perform a Hypothesis Testing that the hypothesis to be tested for response time being equal for different circuit types. Hypothesis.: Ho\_\_\_\_\_ Ha\_\_\_\_\_

Test statistics \_\_\_\_\_

P-value\_\_\_\_\_

Decision: \_\_\_\_\_ Use

(d)

Identify the pairs of treatment means which are different using Tukey's tests with Significant level.

(e)

Suppose we want to test for the significant difference of

$$\frac{\mu_1 + \mu_3}{2} = \mu_2$$

Perform the appropriate test for the contrast.

Write Contrast C: \_\_\_\_\_

Find the estimate :  $c =$ \_\_\_\_\_  $se(c) =$ \_\_\_\_\_

Hypothesis : Ho\_\_\_\_\_; Ha\_\_\_\_\_

Test statistics\_\_\_\_\_; P-value\_\_\_\_\_

Conclusion:\_\_\_\_\_

### Problem 3

A study is conducted to compare 4 menus in terms of numbers of calories. The 4 menus are: A- No calories B- Calories C-Rank-ordered Calories D-Color-ordered Calories

Suppose  $n=20$  per each menu. The sample means and estimated variance are

$$y_1 = 17.6 \ y_2 = 16.8 \ y_3 = 16 \ y_4 = 14.4 \ y_{..} = 16.2 \ s^2 = MSE = 196$$

(a)

Complete the following table to test the  $H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4$

Reject \_\_\_\_\_ Not reject \_\_\_\_\_

(b)

Difference between 2 means:  $y_1 - y_2$

Compute the standard error : \_\_\_\_\_

Compute the Fisher LSD \_\_\_\_\_

Write a 95%-CI for the difference, using Fisher LSD \_\_\_\_\_

(c)

Contrast C: Write the contrast C which compares Menu 1 vs Menus {1,2,3} combined. \_\_\_\_\_

Calculate an estimate of C :  $c =$  \_\_\_\_\_

Calculate the standard error of the contrast:  $se(c) =$  \_\_\_\_\_

Write a 95%-CI for C, using t-distribution : \_\_\_\_\_

## Problem 4

A router is used to cut locating notches on a printed circuit board. The vibration level at the surface of the board as it is cut is considered to be a major source of dimensional variation in the notches. Two factors are thought to influence vibration: bit size (A) and cutting speed (B). Two sizes (1/16 and 1/8 inch) two speeds (40 and 90 rpm ) are selected and four boards are cut at each set of conditions shown below. The response variable is vibration measured as the resultant vector of three accelerometers (x,y,z) on each test circuit board.

```
bit_sizes <- c('1/16th','1/8th')
speeds <- c('40','90')
vibration_data <- expand.grid(bit_size = bit_sizes,
                             speed = rep(speeds,4))
vibration_data <- cbind(vibration_data,
                       vibration=c(18.2,27.2,15.9,41.0,
                                   18.9,24.0,14.5,43.9,
                                   12.9,22.4,15.1,36.3,
                                   14.4,22.5,14.2,39.9
                                   ))
vibration_data %>% kable()
```

bit_size	speed	vibration
1/16th	40	18.2
1/8th	40	27.2
1/16th	90	15.9
1/8th	90	41.0
1/16th	40	18.9
1/8th	40	24.0
1/16th	90	14.5
1/8th	90	43.9
1/16th	40	12.9
1/8th	40	22.4
1/16th	90	15.1
1/8th	90	36.3
1/16th	40	14.4

bit_size	speed	vibration
1/8th	40	22.5
1/16th	90	14.2
1/8th	90	39.9

**(a)**

Calculate, from scratch, effects of A, B, AB[ Hint: Chapter 6]

**(b)**

Calculate the Sum Squares:  $SS(A)$ ,  $SS(B)$ ,  $SS(AB)$ .

**(c)**

Complete the ANOVA table: df, MS, F-stat, P-values of A, B, AB

**(d)**

Use R, find the model matrix X.

Use X to calculate the vector of coefficients , for the regression model.