MAE 294 Final Exam
Due: Monday, March 17

Exam paper submission by noon to my office (38-137B, E4)
Program demonstration 3-6 pm in CAD lab (38-138, E4)
You may either use your own computer or a computer in CAD lab for demonstration.
Presentation order is according to last name, and your presentation must be ready when your name is called.

This is a take home exam. No discussion allowed among classmates. Identical (or clearly copied) portion of the returns will be discarded for both (or all) parties.

(25) **Problem 1**

Write a computer program to find the "geodesic path" between two separated locations on a surface.

Use surface $\overline{p}(u,v)=\begin{bmatrix} x(u,v) \\ y(u,v) \\ z(u,v) \end{bmatrix}=\begin{bmatrix} u \\ v \\ \sin u+\cos v+5 \end{bmatrix},\ u,v\in[-\pi,\pi]$

The starting location is $(-\frac{\pi}{2},\frac{3\pi}{4})$ and the end location is $(\frac{3\pi}{4},-\frac{\pi}{2})$.

Select your "geodesic path" by evaluating κ_g for 101 points along the curve.
Your program needs to perform an optimization search to a path that minimizes the following κ_g index: $\kappa_g index=\frac{1}{10}\sqrt{\sum_{i=0}^{100}\kappa_{gi}^2}$

Plot the surface by using the four boundary curves and plot the geodesic path by using your 100 points. Show your κ_g index value. Also show κ_g values for all 100 points and indicate their maximum and mean absolute values.

Hint: You need to design $u(\tau),v(\tau)$. I think you need 4 variables for your path to interpolate the two end points. Therefore in your $u(\tau),v(\tau)$, you need at least 5 variables in order to perform an optimization. This means that at least one of your $u(\tau),v(\tau)$ needs to be quadratic.

(50) **Problem 2**

Write computer programs the following three tasks. Provide two examples for each program. One example has its maximum curvature ≥ 50 and the other has its maximum torsion ≥ 30.

Program 1 Inputs: Hermite geometric coefficient matrix
Outputs: A short program description, the input data, and the results. The results include both the numerical values and their associated plots of the pc curves. Show the end positions and the end unit tangent vectors (including component values and direction arrowheads) of the curves. Show also the locations and magnitudes of the maximum curvature and torsion.

Program 2 Inputs: Bezier control points for n = 4 (Value n doesn't need to be a variable.)
Outputs: A short program description, the input data, and the results. The results include both the numerical values and their associated plots of the Bezier curves and the control polygons (including control points). Show also the locations and magnitudes of the maximum curvature and torsion.

Program 3 Inputs: Number n (Value n must be a variable.), NURBS control points, weights, type (closed or open), for k = 4 (Value k doesn't need to be a variable.)
Outputs: A short program description, the input data, and the results. The results include the numerical values and their associated plots of the NURBS curves and the control polygons (including control points). Provide two examples for closed curve and two for open curve with different ns. Show also the locations and magnitudes of the maximum curvature and torsion.

Note: No hand drawings are allowed in your plots. Send all your computer code to me via email by the deadline.

(25) **Problem 3**

Write a computer program to fit the five points $\mathbf{p}_0$=(1, 3, -2), $\mathbf{p}_1$=(2, 5, 4), $\mathbf{p}_2$=(-3, 4, 8), $\mathbf{p}_3$=(3, 8, 14), and $\mathbf{p}_4$=(7, -5, -2) by 4-segment composite parametric cubic curves with C^2 continuity and plot the resulted curves together with the given data points for parts a and b:

a. Normalized parameter u (u ε [0, 1]) for all segments with boundary conditions $\overset{o}{p}_0 = \overset{o}{p}_4 = 0$,

b. Non-normalized parameter t (time in seconds), and $t_0 = 0$, $t_1 = 3$, $t_2 = 8$, $t_3 = 12$, $t_4 = 20$, with initial conditions $\overset{\tau}{p}_0 = \overset{\tau}{p}_4 = 0$.