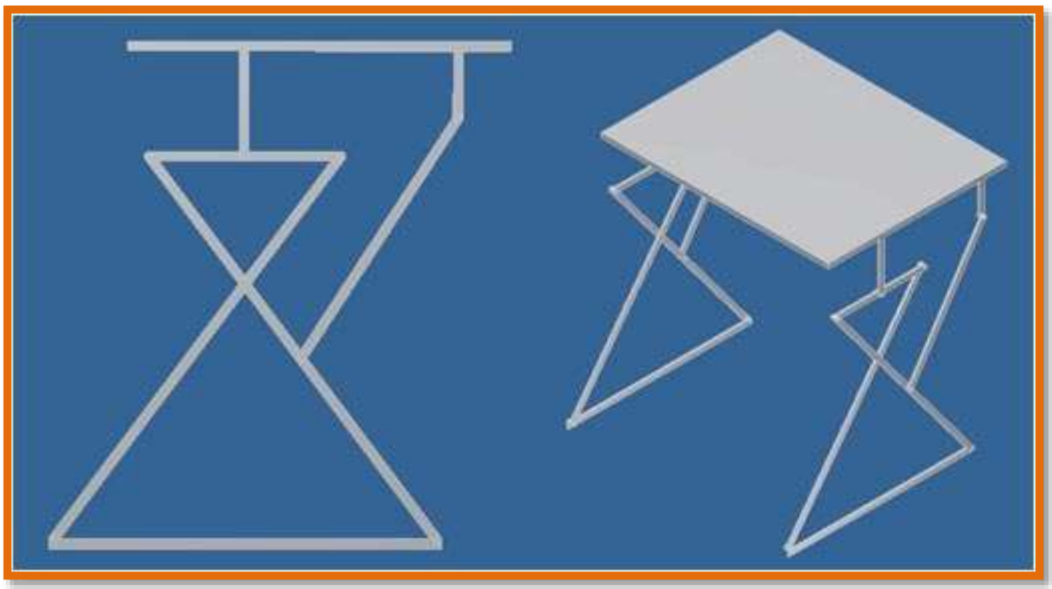
# SYNTHESIS OF A HORIZONTAL-PLATFORM MECHANISM



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Abstract

The objective of this project is to design and simulate a horizontal-platform mechanism using CAD and Matlab and then analyzed the mechanism position, velocity, and motor's torque. The platform must fulfill certain criteria during its horizontal motion to satisfy and pass the prerequisite. This mechanism, once finished, has a quick return and a well-defined horizontal motion. This makes this mechanism a good mechanism for performing slow linear motion and quick return to repeat the action. During the final steps we realized the Chebyshev is the easiest application to make this mechanism into a linear motion mechanism. The finished product has a similar coupler curve as the Chebyshev if the point is chosen on link 6.

# List of Symbols

| Symbols | Description |
|---|---|
| $\overrightarrow{R_1}$ | Vector for link 1 (notation same for other vectors) |
| $S$ | Arbitrary close loop vector |
| $\theta$ | Angle of link from horizontal position |
| T | Torque (Nm) |
| F | Force (N) |
| L | Length (m or mm) |
| V | Velocity (most of the time in mm/s) |
| a | Acceleration ($m/s^2$) |
| $I_c$ | Instant center (as display in figure 1 also consider as points in matlab code) |
| w | Angular velocity (rad/sec) |
| P | Power (W) |

# Introduction

The objective of this project is to design and simulate a horizontal-platform mechanism using CAD and Matlab. The platform needs to meet specific conditions to be considered useful for the task it needs to perform. Our objective is to design a mechanism that could meet the following conditions:

- transverse 850mm horizontally – During this time these criteria must be met:
  - less than 4mm vertical translation
  - tabletop angle less than 1° from horizontal
- table cannot flip over
- initial position fit within a 850mm by 1050mm perimeter
- motor to drive a link on the mechanism without interference from other links

To create this mechanism a person must be knowledgeable in mechanical mechanism. Using the knowledge for mechanical mechanism we decided to pick the motor between links 2 and 3 (another similar alternative is links 3 and 4). From this point on a detail analysis was required to create this mechanism.

# Analysis and Synthesis Details

To start off, design a simple but easily adjustable CAD model turns out to be the best way to visualize the mechanism. Once the CAD model was build the next step was choosing the placement of the motor. By analyzing each link the motor can only be place between link 2 and 3 or link 3 and 4 without flipping the table over (other possible locations such as $I_{36}$ were not analyzed). We cannot choose the link 1 and 2 since that would flip the table over and same goes for many other joints. After choosing the motor placement between link 2 and 4 we have to move on to the Matlab coding.

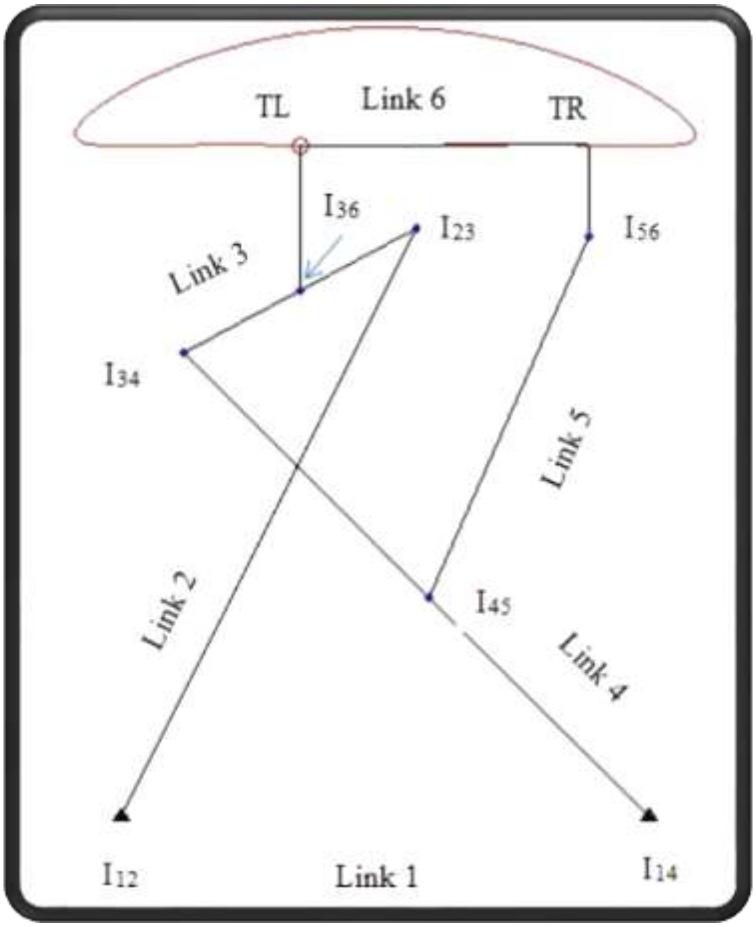
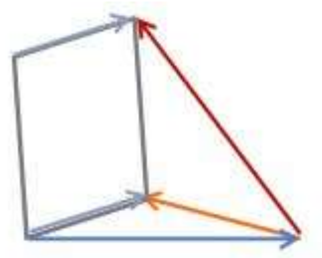



**Figure 1: Horizontal motion mechanism. (Variables used in Matlab were based on this figure)**

The first step is to write loop closure rule for the 4-bar link Chebyshev. The thing we need to take into account is the location of the motor as this will determine which $\theta$ will be the independent variable. Since my motor is place at $I_{23}$ the loop closure equation is the following: ($R_1$ = link 1, $R_2$ = link 2 and so on as label in figure 1)



$\overrightarrow{R_1} + \overrightarrow{S_1} = \overrightarrow{R_3}$ (1) From this equation we break it into real and imaginary. The equation we obtain from this were

$\theta_S = tan^{-1}(\frac{R_3 \sin(\theta_3) - R_1 sin(\theta_1)}{R_3 \cos(\theta_3) - R_1 cos(\theta_1)})$ (2) note: the $\theta_1 = 0$

$S = [(R_3 cos(\theta_3) - R_1)^2 + (R_3 sin(\theta_3))^2]^{0.5}$ (3)

From here we used the cosine law to obtain the following equations:

$\theta_2 = \theta_S + pi + arccos((S^2+R_2^2-R_4^2)/(2*S*R_2))$ (4)

$\theta_4 = \theta_S - arccos((S^2-R_2^2+R_4^2)/(2*S*R_4))$ (5)

Now since we have $\theta_1$, $\theta_2$, $\theta_3$, $\theta_4$, and $\theta_S$ for the first 4 links (1,2,3, and 4) and $S_1$ link we can transverse to any location on these 4 links. This means we could create vectors leading to any point on the 4 links. The next step was to create the necessary equation to extend these points to link 5 and $6_2$ (note: link $6_2$ is the vector $R_{6\_2}=I_{56}-I_{36}$ and not the link of the table top as shown in Figure 1). To do this we need to create another loop closure. This simple triangle loop closure contains these points $I_{36}$, $I_{45}$, and $I_{56}$.

(6) $\overrightarrow{R_6} + \overrightarrow{S_2} = \overrightarrow{R_5}$ From equation 6 we again break it into component and look for $\theta_5$ and $\theta_6$ using the cosine law. In this case, it turns out that $S_2$ is easily found in Matlab since we already defined $I_{36}$ and $I_{56}$. All we had to do was take the difference between the two points and take the arctan to find the magnitude and angle of $S_2$.

```
%% S2 vector (From I45 to I36)
S2x = I36x - I45x;
S2y = I36y - I45y;
S2 = (S2x.^2+S2y.^2).^.5;
thetaS2 = atan2(S2y, S2x);
```

Since we now have $S_2$ and its angle we can find the angle of link 5 and link $6_2$:

$\theta_5 = \theta_{S2} - \arccos[(S_2^2 + R_5^2 - R_6^2)/(2S_2R_5)]$
$\theta_6 = \theta_{S2} - pi + \arccos[(S_2^2 - R_5^2 + R_6^2)./( 2S_2R_6)]$

Once this was finished, the core base of the matlab code was done and we can transverse to any point on these links. Link 6, the table top link was easily added with straight line vertical vectors to $I_{35}$ and $I_{56}$. From this point on joining any two points with matlab code we can create lines.

During this point we could only look for all the lengths using matlab plot function and verify if all of the values fall within the range. This took hours of changing until the value finally got close enough. But using the Chebyshev linkage relationship I was able to quickly find the straight line generator. From this point on it was mainly adjusting R1, R5, R6, TL, TR, and I45 lengths or locations.

| $R_1$(mm) | $R_2$(mm) | $R_3$(mm) | $R_4$(mm) | $R_5$(mm) | $R_6$(mm) |
|---|---|---|---|---|---|
| 800 | 1000 | 400 | 1000 | 600 | 447.5 |
| $T_{LeftL}$(mm) | | $T_{RightL}$(mm) | | $I_{45Locatiion}$(mm) | |
| 220 | | 139.9 | | 472 | |

**Table 1: The lengths obtained for the horizontal motion platform.**

```
r1= 800 % The values I tested out here were 400, 600, 800, 830.
r2 = 2.5/2*r1 % Changes based on R1
r3 = r1/2 % Changes based on R1
r4 = r3+(r2^2-r1^2)^.5 % Changes based on R1
r5 = 600 % Here I just change it around until it works
r6 = 447.5 % Here are the values I tested 447.5, 445, 480_3, etc.
TLeftL = 220 % It's easier to find once we get the I36 and I56 to have constant
height difference
TRightL = 139.9 % This was similar to TLeftL case: 136, 170, etc.
I45L = 472 % Tested values: 480, 495 (position of I45 on link 4 from I14)
```

Once we found the corresponding values for the links we would have to do velocity and torque plot. To find the velocity we just took the derivative of the position vector to the center of the table. For the torque we took the derivative of the velocity vector at the center of the table. This gives us acceleration which we can use to find the forces. These forces are in turn used in a torque balance to find the amount of torque the motor need to provide to drive the mechanism.

# Results and Discussions

By this time we have achieve the desired outcome for the mechanism. The CAD images of the mechanism are shown below along with its dimension. It seems like this mechanism makes a close to perfect horizontal translation for a good 50 sec. Since this 6 bar linkage is build base on the Chebyshev linkage it still sustain a horizontal translation and could be used for similar application like the Chebyshev. James Watson did research on this for the steam engine as well. In this case the quick return for this mechanism seems really useful as well. Once the quick return is done it starts its straight line motion. An engine design could have this type of mechanism since it has both a quick return and linear motion.

To create this straight line mechanism the links 1 and 3 must be parallel if we follow the figure in the assignment handout (in my Figure 1 it is links 2 and 5). Also following the Chebyshev's ratio for length will also give us a straight line mechanism right away. Then from that point on the length and position of R1, R5, R6, TL, TR, and I45 will determine the straight line and angle of link 6 (refer to Figure 1 for links information). Each time a change was made the link 2 and link 5 lengths should be change to make them parallel again for straight line to appear. The most crucial dimension that gives the straight line is from the Chebyshev linkage. Without that the straight line would be difficult to find. The next crucial thing is making the links 2 and 5 parallel for straight line motion for link 6.

Below are the position, velocity, and acceleration for the center of the table. We should take notice of the quick return with the short time it the graphs and then the long linear motion. This action could be easily picked out from the vertical distance vs. time graph below.

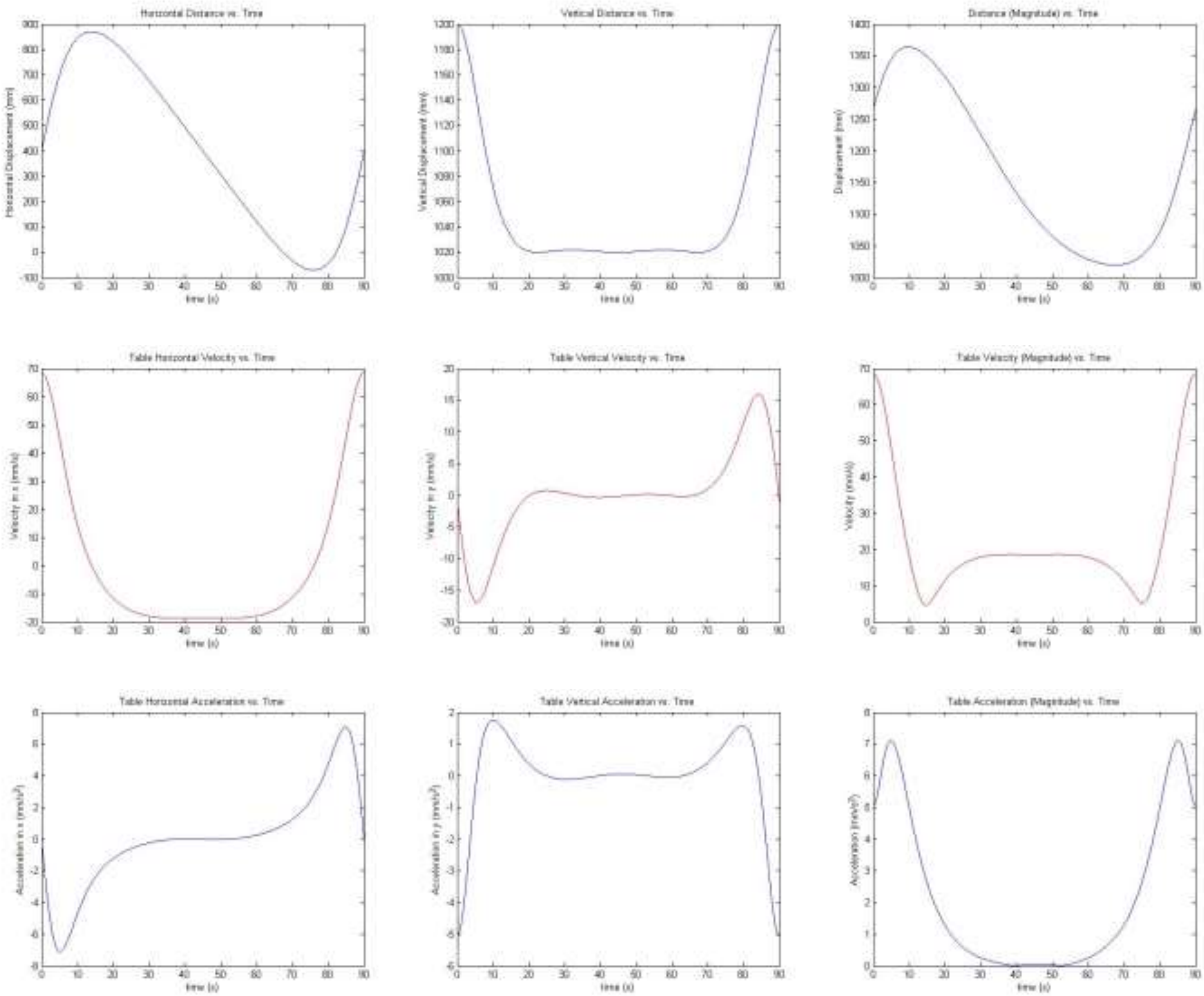


Figure 2: Position, Velocity, and Acceleration of the center of the table

From the position we differentiate to obtain velocity and again differentiate to obtain acceleration. As the graphs above easily show the displacement, velocity, and acceleration corresponds to each other when looking at the graphs slopes. We also perform an instant center analysis for the velocity. The point we choose to use for the velocity is at $I_{36}$. It should be noted that $I_{36}$ is sufficient for the velocity at that very instant for the center of the table. Since $V_{\text{center of table}}$ ($V_c$) and $V_{I\_36}$ share the same link they have the same velocity. The velocity was calculated from: $V_{I\_36} = V_c = w_3*(I_{13}I_{36})$. The instant center figure is in the Appendix. Torque was also calculated using the same figure IC diagram in the appendix. So the two equations for velocity and torque calculation were:

- $V_{I\_36} = V_c = w_3*(I_{13}I_{36})$
- $P_{out} = P_{in} \rightarrow P_{out} = F_{in}V_{in} \rightarrow T_{in}w_3 = F_{in}V_{in}$.

These are the value at this very instant that was used in my calculation from matlab.

time = 51.9900 (s) theta3 = 3.6296 (Deg) velocity = 18.6297 (mm/s)

Fx = 4.0853e-005 ($Nm/s^2$) Fy = -98.0999 ($Nm/s^2$) Torque = 21.5859 (Nm)

| | Velocity (mm/s) | Torque (Nm) |
|---|---|---|
| Matlab Coding (more exact) | 18.63 | 21.6 |
| IC Method (length are scaled) | 17.71 | 20.9 |
| % difference | 5.1% | 3.3% |

**Table 2: Comparison of % difference between matlab code and IC calculation**

The IC method used was performed on paper and lengths were scaled so errors were expected. Surprisingly the values were closely related so there was not much error in the two techniques. Though matlab coding was probably the longer approach but more efficient as it provide a whole range of value for all time.

The diagram below show the torque profile as it travels through one cycle. The figure to the left shows the link 6 angle profile. Again we should notice the peak in torque at the beginning meaning it is lifting the force during this portion. Then during the linear motion or during the center section between 30 and 80 sec it’s the horizontal travel so less torque is required.

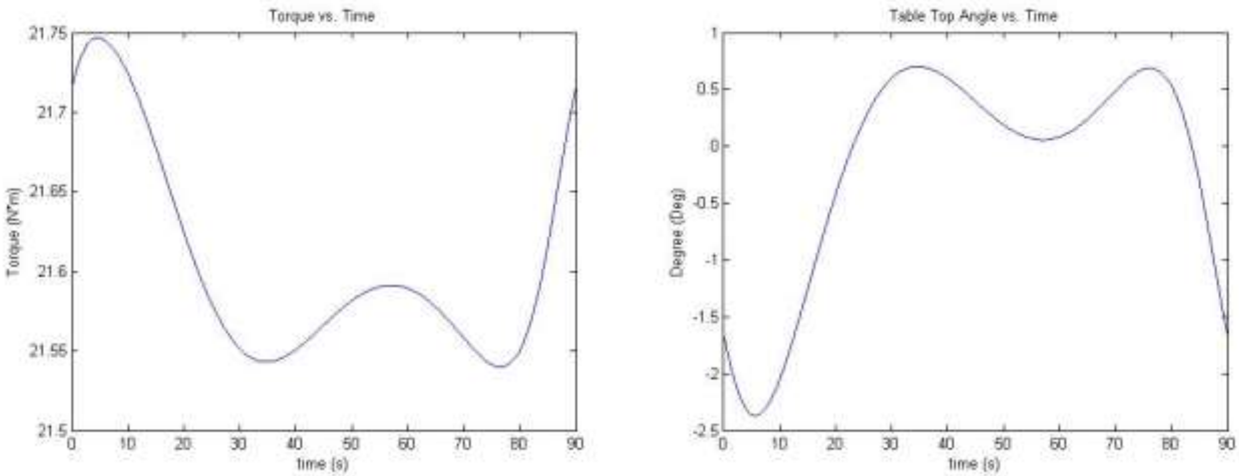



**Figure 3: Torque vs. time for one full cycle (right) and Angle vs. time (left)**

It should be noted that the torque remain between 21-22 Nm and never really change much. This is due to the constant mass that is applied. Our torque should change though since there is slight acceleration and this caused inertia and therefore have a F=ma term. The forces we used to calculate for torque were:

```
Fx = m*ax;              % N*m/s^2
Fy = m*ay+m*g;          % N*m/s^2
L1 = (I56x-I36x)/1000;     % finding length for torque (meters)
L2 = (I56y-I36y)/1000;     % finding length for torque (meters)
Torque = -(L1).*Fy./2-(L2).*Fx./2;  % Torque balance about I56
```

The accelerations were found by differentiating velocity.

In my design the motor is place at $I_{23}$ though other possible places are possible as described earlier. This location is ideal since it does not flip the table over and let the motor run continuously without any interference. We were also able to show that it remained within the 4mm height.

# Conclusion

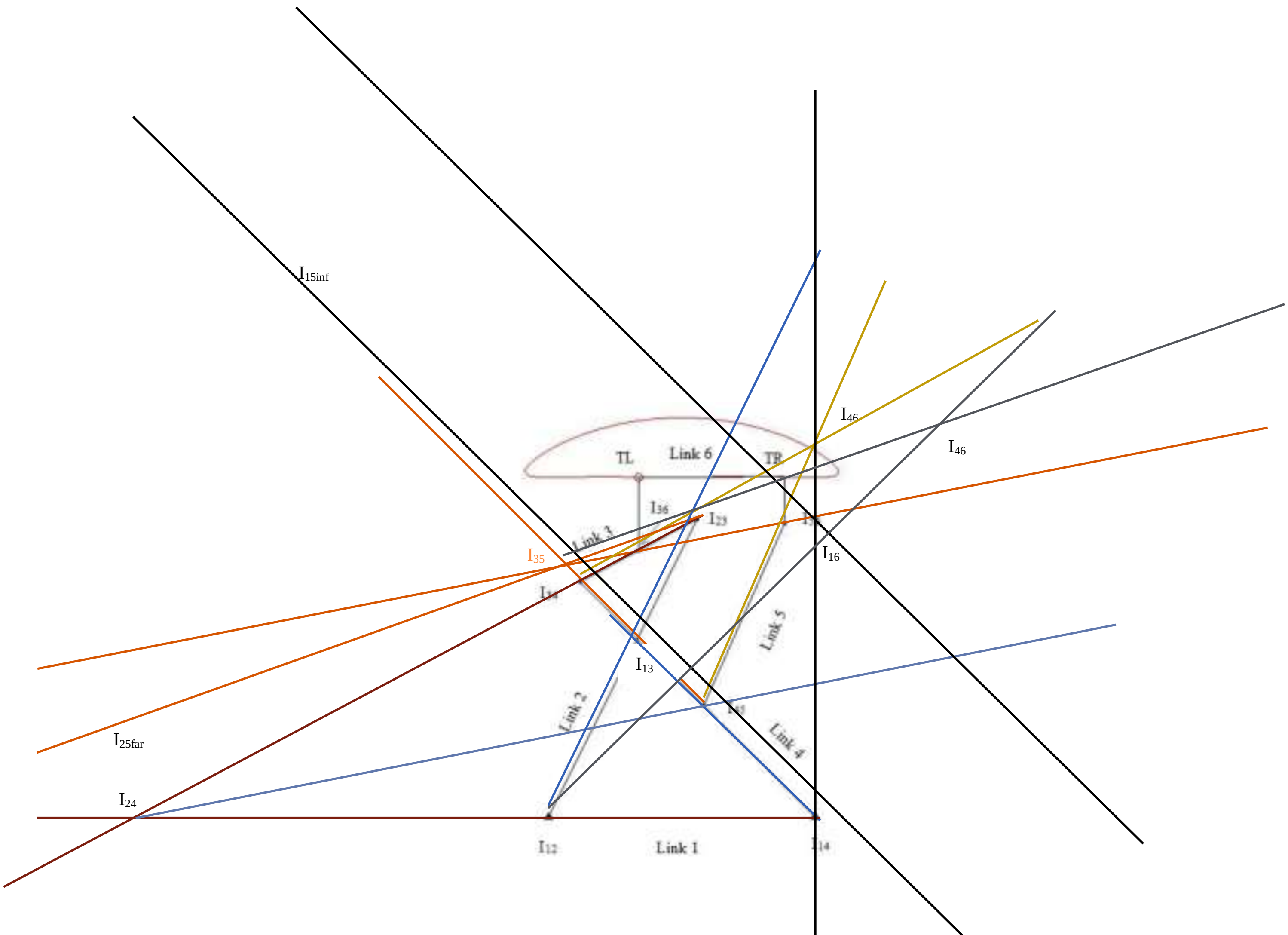
In conclusion this project was a great success we were able to meet all the requirements. The most difficult part was just the starting point and choosing the length of the links. Once that step is finished the work becomes significantly easier to do. We were able to work accomplish the basic measurement and verify the matlab values with the IC values. They were all smaller than 6% difference which was nice.

In future project it is probably better if there were some clues that could have help us approach the linear line motion as well since we were just guessing the values until it works. Beside that the project was a great success.

# Appendix

Instant Center Diagrams and other diagrams

## Instant Center Diagram




Scale 41mm = 800mm

Equation for Velocity and Torque were:

- $V_{I\_36} = V_c = w_3*(I_{13}I_{36})$
- $P_{out} = P_{in} \rightarrow P_{out} = F_{in}V_{in} \rightarrow T_{in}w_3 = F_{in}V_{in}$

The solution are listed in an earlier table.

## Other Diagrams

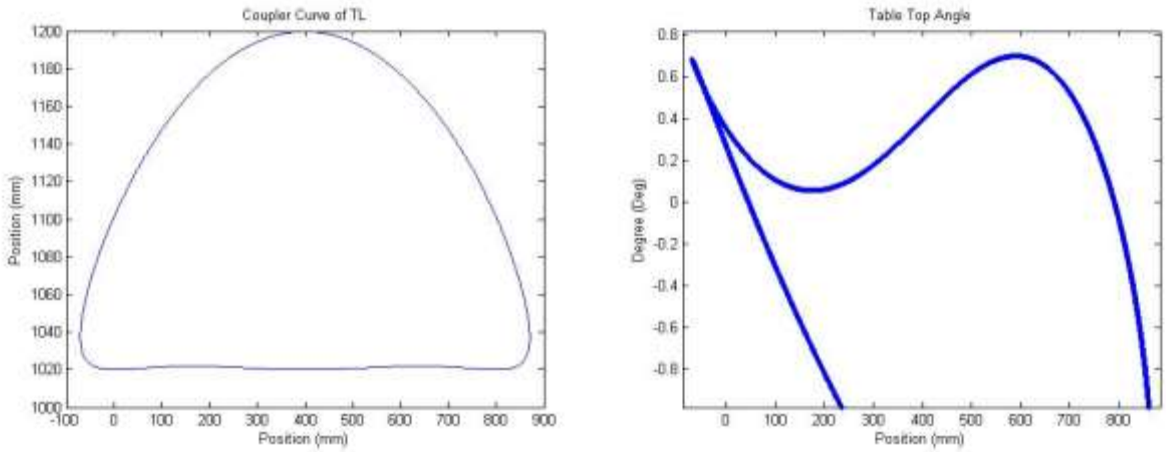



**Figure 4: (Left) Coupler curve of TL lets us know that there is a horizontal translation. (Right) Tells us that the angle is within the 1 deg. limit during the horizontal translation**

This figure above to the right is used to verify that the motion of the table does transverse at least 850mm and fall below the 1050mm height limit. The figure to the left verify angle (less than 1 deg) during this travel distance.

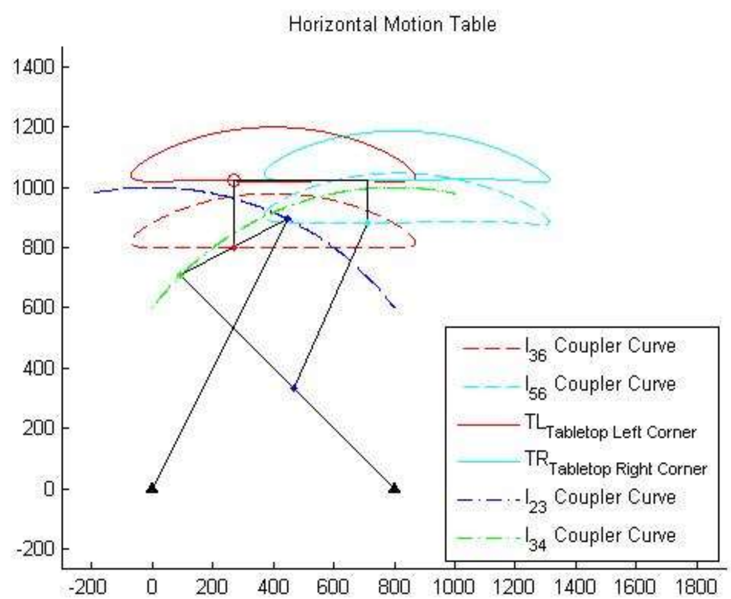



**Figure 5: Easily show that the mechanism fit within the 850mm by 1050mm specification**

# AutoCAD Drawings

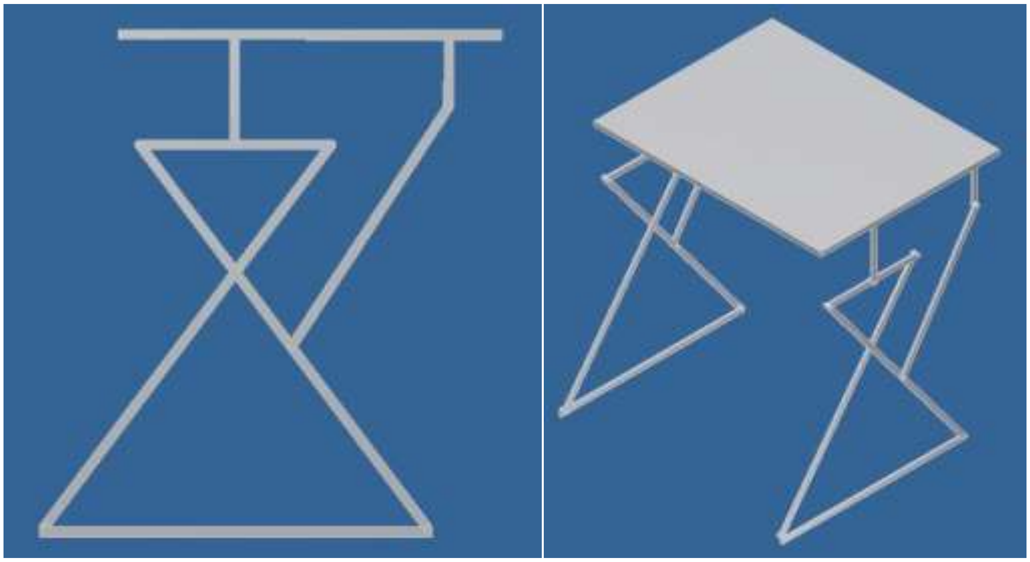


**Figure 6: (Left) Side view of mechanism (Right) Table design using this mechanism top corner view**