

Statistical Inference - Project - Simulation Exercise

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Overview

In the simulation below, I will investigate the exponential distribution in R and compare it to the Central Limit Theorem. We will compare the simulated results with the theoretical results for the following 3 questions: 1. Show the sample mean and compare it to the theoretical mean of the distribution. 2. Show how variable the sample is (via variance) and compare it to the theoretical variance of the distribution. 3. Show that the distribution is approximately normal.

Simulation

The exponential distribution can be simulated in R using `rexp(n,lambda)` where `lambda` is the rate parameter. In this experiment we will be set `lambda = 0.2` for all the simulations.

```
set.seed(12345)

numOfSim <- 1000
lambda <- 0.2
n <- 40

mns = NULL
for (i in 1:numOfSim)
  mns = c(mns, mean(rexp(n, lambda)))
```

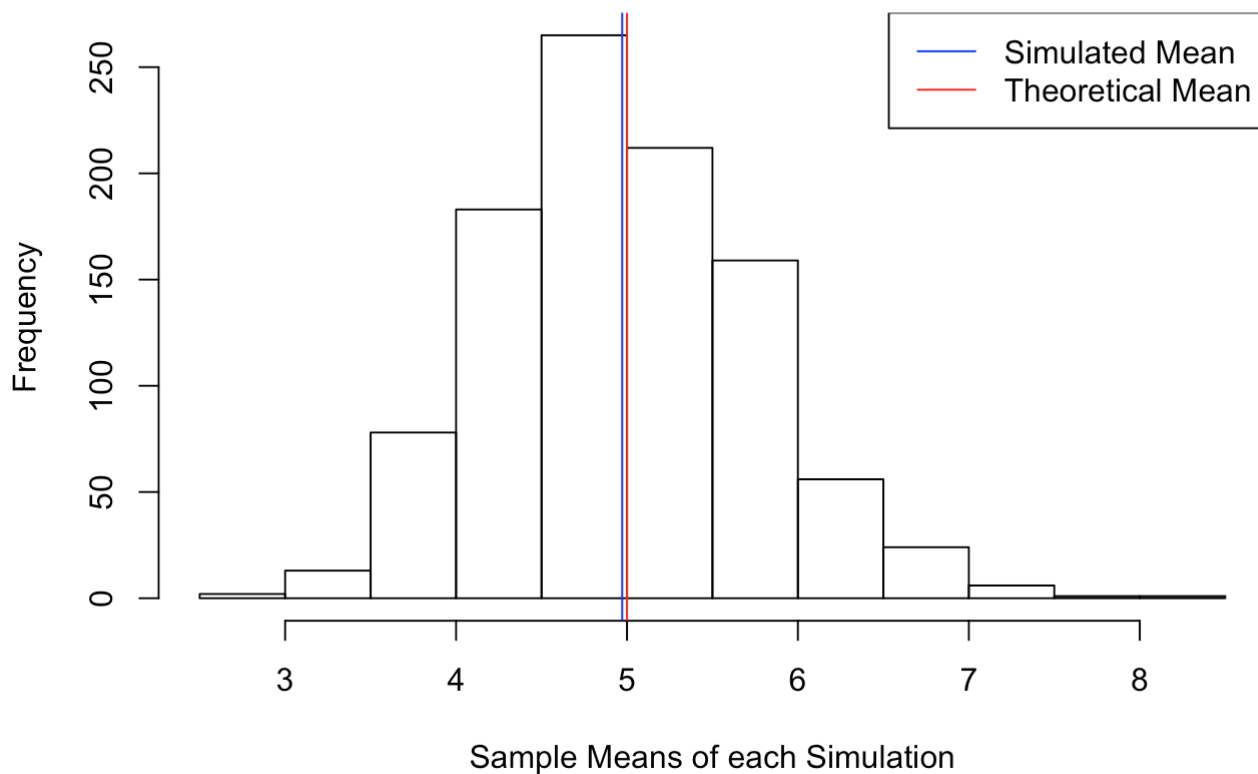
Sample Mean versus Theoretical Mean

The theoretical mean is $1/\lambda$. We will calculate and compare the theoretical and simulated mean.

```
theoMean <- 1/lambda
simMean <- mean(mns)

hist(mns,
     main="Exponential Distribution - \nSample mean vs Theoretical mean",
     xlab="Sample Means of each Simulation",
     ylab="Frequency"
)
abline(v=simMean, col='blue', lwd=1)
abline(v=theoMean, col='red', lwd=1)
legend('topright', c("Simulated Mean", "Theoretical Mean"), col=c("blue", "red"),
     lty=c(1,1))
```

Exponential Distirbution - Sample mean vs Theoretical mean



```
theoMean
```

```
## [1] 5
```

```
simMean
```

```
## [1] 4.971972
```

Conclusion: The simulated mean is very similar to the theoretical mean.

Sample Variance versus Theoretical Variance

The theoretical variance is $(\text{Std})^2/n$. The theoretical standard deviation is $1/\lambda$. We will calculate and compare the theoretical and simulated variances.

```
theoVar <- (1/lambda)^2/n  
simVar <- var(mns)
```

```
theoVar
```

```
## [1] 0.625
```

```
simVar
```

```
## [1] 0.5954369
```

Conclusion: The simulated variance is very similar to the theoretical variance.

Show simulated data is approximately normal

In order to investigate if the simulated data is approximately normal, we have to generate sample normal data with the same mean and standard distribution as the simulated exponential data. We can do this by using the R function `dnorm`.

```
theoSd <- 1/lambda/sqrt(n)
theoSd
```

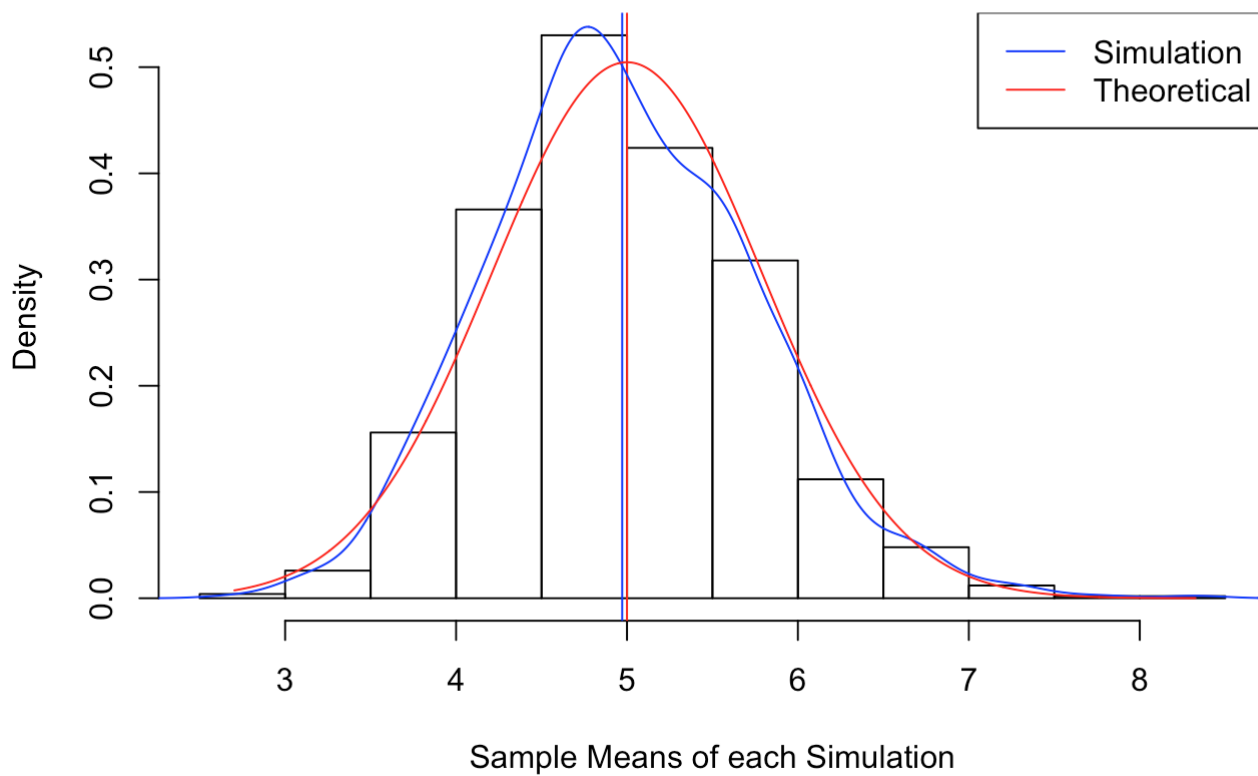
```
## [1] 0.7905694
```

```
normPoints <- seq(min(mns), max(mns), length=numOfSim)
normData <- dnorm(normPoints, mean=theoMean, sd=theoSd)

hist(mns, prob=T,
     main="Comparing Sample Exponential Distribution\n with Theoretical Normal Distribution",
     xlab="Sample Means of each Simulation",
     ylab="Density")
abline(v=simMean, col='blue', lwd=1)
abline(v=theoMean, col='red', lwd=1)

lines(density(mns), col="blue", lwd=1)
lines(normPoints, normData, pch=22, col="red", lwd=1)
legend('topright', c("Simulation", "Theoretical"), col=c("blue", "red"),
      lty=c(1,1))
```

Comparing Sample Exponential Distribution with Theoretical Normal Distribution



Conclusion: The simulated curve is very similar to the theoretical standard normal curve. Given this information, I believe the simulated data is approximately normal.