Comparative study of annuities contractual design

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INTRODUCTION

Long prospectus
The .

ANNUITIES

Fixed Annuity
Fixed annuity provides **no exposure** to market performance. The insurance company usually provides a guaranteed interest rate for each account over a predetermined period of time.

Fixed Indexed Annuity (FIA)
Fixed indexed annuity provides **limited exposure** to **positive** index returns and **no exposure** to **downward** market performance.
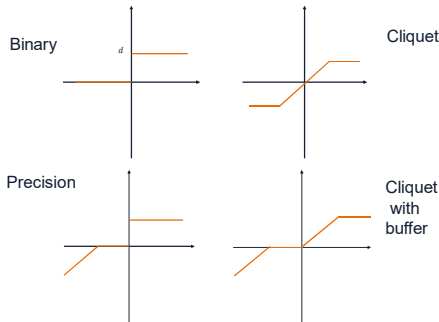
Variable Annuity (VA)
Variable annuity enables **full exposure** to market performance. The insurance company usually provides a variety of guarantees for each account, including GMDB, GMMB, GMAB, GMWB and GMIB, to attract potential customers.

Options for guarantee base

Reset Option

Roll-up Option
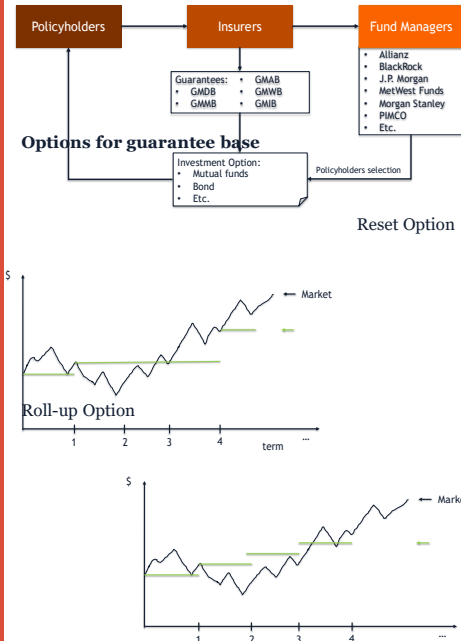
ANNUITIES

Indexed Variable Annuity (IVA)
Indexed variable annuity provides **some exposure** to **downside** market performance. Four kinds of features associated with IVA, including binary option, cliquet option, cliquet buffer option and precision option.

Binary — Cliquet
Precision — Cliquet with buffer

METHOD

Step 1
- Find product prospectuses online:
Variable Annuity: Northwestern Mutual, MetLife
Indexed Annuity: Allianz
- Identify their features and parameters:
Variable Annuity: GMDB, GMIB, roll-up, step-up
Indexed Annuity: binary, cliquet, cliquet buffer, precision

Step 2
- Summarize features into a spreadsheet.

Step 3
- Formulate and model different features of products in R/Python using parameter values extracted from prospectus.
- Simulate stock prices based on log normal model and investigate density of account value.
- Download historical data of S&P 500 from 1990s and visualize the evolution of account value.

Step 4
- We treat each feature like an investment, therefore it could be weighted.
- Simulate weights(π_i) of features and generate expected return and volatility of different portfolio combinations.
- Given a particular return, find portfolios with lowest risk. $\min_{\pi} (\pi^T \Sigma \pi)$, where $\sum_{i=1}^{4} \pi_i = 1$.
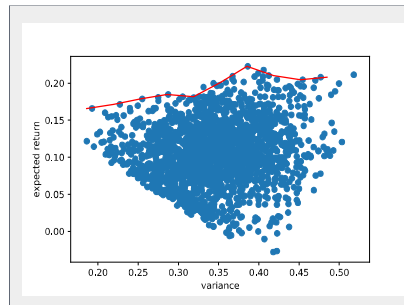
RESULTS
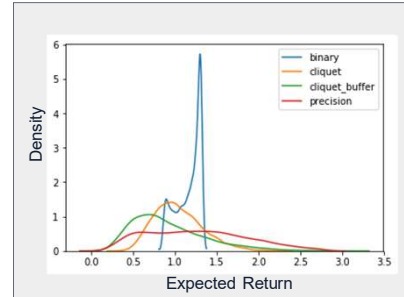
Density distributions of account terminal value for four features in Allianz IVA product.
Simulate 10000 paths using Brownian motion.
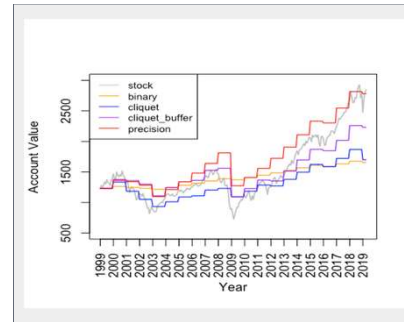
Evolution of account value for four features in Allianz IVA product.
Data from S&P 500 1999 to 2019.

Expected return v.s. variance for different combinations of four features in Allianz IVA product.

RESULTS

Optimal combinations of features on efficient frontier

Expected Return	Binary	Cliquet	Cliquet buffer	Precision
16.58%	0.8273	0.0202	0.0486	0.1036
17.12%	0.7211	0.0580	0.0413	0.1795
17.86%	0.6423	0.0895	0.0234	0.2446
18.47%	0.5695	0.0878	0.0297	0.3128
18.18%	0.4048	0.2092	0.0099	0.3760
20.09%	0.4216	0.0728	0.0386	0.4669
20.27%	0.4062	0.0332	0.0053	0.5552
20.46%	0.1510	0.1541	0.0595	0.6353

CONCLUSIONS

According to the graphs and our analysis, the precision feature of IVA provides with the greatest potential and variance for returns and the binary feature of IVA allows for a stable and healthy return. We recommend the customers to invest in a combination of these two options. Customers can always choose different features according to their risk tolerance.

We hope our analysis can help ordinary customers better understand different features associated with annuity products available in the market and find their best choice.

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