EC516 HW3

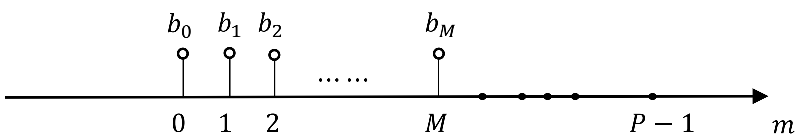
Due Thursday Oct 1st, 2015

HW3.1

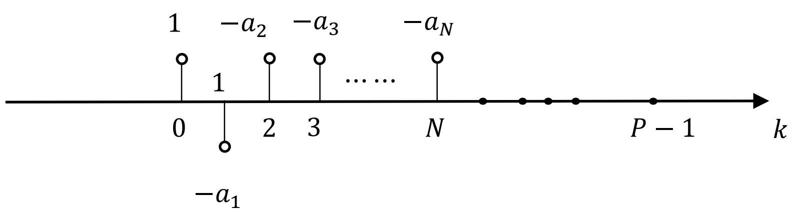
Let $P \geq Max(M+1, N+1)$ where P is an integer

then $H\left(e^{j\frac{2\pi}{P}n}\right) = \frac{\sum_{m=0}^{M} b_m e^{-j\frac{2\pi}{P}nm}}{1-\sum_{k=1}^{N} a_k e^{-j\frac{2\pi}{P}nk}}$ where $H\left(e^{j\frac{2\pi}{P}n}\right)$ stands for samples of $H(e^{j\omega})$ taken every $\frac{2\pi}{P}$.

Claim: $\sum_{m=0}^{M} b_m e^{-j\frac{2\pi}{P}nm}$ is the P-point DFT of



Claim: $1-\sum_{k=1}^{N} a_k e^{-j\frac{2\pi}{P}nk}$ is the P-point DFT of



Please verify the above two claims.

HW3.2

Consider a causal digital filter with system function

$$H(z) = \frac{\left(1-\frac{1}{2}z^{-1}\right)}{\left(1+\frac{1}{2}z^{-1}\right)\left(1-\frac{j}{2}z^{-1}\right)\left(1+\frac{j}{2}z^{-1}\right)}$$

(a) What is the difference equation for this filter?
(b) Plot the pole-zero plot for this filter.
(c) Use MATLAB to compute the values of $\left|H\left(e^{j\frac{2\pi k}{100}}\right)\right|$ for $k = 0, 1, \cdots, 99$.
(d) Determine the value of $\left|H\left(e^{-j\frac{4\pi}{100}}\right)\right|$ from the result in (c).

HW3.3

$$\sum_{k=-\infty}^{\infty} \delta[n-kM] = \frac{1}{M}\sum_{k=0}^{M-1} e^{j\frac{2\pi k}{M}n}$$

Show this using finite sum formula.

HW3.4

Let $x[n] = \left(\frac{1}{2}\right)^n \{u[n] - u[n-4]\}$

Let $g[n] = x[2n]$ and $h[n] = x[2n+1]$

(a) Sketch $x[n]$, $g[n]$ and $h[n]$ (you may use MATLAB)
(b) Determine the values of $X[k]_4$
(c) Determine the values of $G[k]_2$
(d) Determine the values of $H[k]_2$
(e) Show how the values of $G[k]_2$ and $H[k]_2$ may be merged to obtain $X[k]_4$ (Hint: use $X(e^{j\omega}) = G(e^{j2\omega}) + e^{-j\omega}H(e^{j2\omega})$).