Analysis of the San Francisco Crime Classification problem, from "kaggle.com": a practical project in big data and machine learning

Stephen R. Williams

Tuesday 3rd November, 2015

Abstract

This project is ongoing. Here I explain the problem and document my efforts in competing in the kaggle data science competition for San Francisco Crime Classification. The problem aspires towards the optimal prediction for the likelihood for each of 39 possible crime categories having occurred, given a training data set and measured against the test data set. The data features can be divided into two broad categories, firstly the precise time and date at which the crime was reported to police (temporal features), and secondly the location where it occurred (spatial features). Here I will steadily and systematically build up towards my chosen approach. This approach will combine a variation on a naive Bayes method with extensive data feature engineering. We will treat the temporal features as being independent of each other and to an extent of the spatial features. In addition we predict the time dependent behaviour by exploiting the cyclic nature of human behaviour. Python is the main software tool used throughout this project to implement the data science (making use of both pandas and Scikit-learn libraries).

The latest version of this document and all of the source files for the work it refers to are publicly available from my GitHub repository, https://github.com/wilstep/frisco-crime

1 Introduction (The Problem)

The competition may be found at https://www.kaggle.com/c/sf-crime The problem consists of crime reports to the San Francisco police over the period 1/01/2003 through to 13/05/2015 with alternating weeks divided between the *test*

data set and the training data set. The data set has the following data features (with some only in the training set (noted as: train.csv)),

- Dates timestamp of the crime incident
- Category category of the crime incident (only in train.csv). This is the target variable you are going to predict.
- Descript detailed description of the crime incident (only in train.csv)
- DayOfWeek the day of the week
- PdDistrict name of the Police Department District
- Resolution how the crime incident was resolved (only in train.csv)
- Address the approximate street address of the crime incident
- X Longitude
- Y Latitude,

and we are required to predict the *Category* for the test set. The *Category* provides which of the 39 possible crime categories has been recorded, e.g. Assault, Burglary, Fraud, etc. There is not enough information available to predict the particular crime *Category* with any reliability, and the aim is to predict the likelihood of each of the 39 possible category outcomes as best we can with the available information. This is forced upon us by the way our entries are scored, using a *multi-class logarithmic loss* formula

$$logloss = -\frac{1}{N} \sum_{i=1}^{N} \sum_{j=1}^{M} y_{ij} \ln(p_{ij}), \tag{1}$$

where N=884,262 is the number of crime incidents in the test set, M=39 is the number of possible crime category outcomes, y_{ij} is unity if the actual (not known to us) crime category for incident i happens to coincide with category j and zero otherwise, p_{ij} is the probability we have predicted for category j being the actual (not known to us) crime category reported for crime incident i, and $\ln(x)$ is the natural logarithm of x. It is important to note that we do not have the required information available to compute the value of Eq. (1) for our results from the test data. This information is kept secret from us by the kaggle administrators. However they do have the information and when an entry is submitted a score is reported back. A live leader board for these scores is maintained at https://www.kaggle.com/c/sf-crime/leaderboard

Given a suitable mathematical background, it is easy to see that this *logloss* quantity has the property that the absolute statistical errors in our predictions for each of the 39 categories, contribute in proportion to their relative error. This means that it is important to put effort into accurately predicting the probability of not just the most likely outcomes, but also the not so likely outcomes. One should realise that if we correctly identify an outcome as being very unlikely, then as a result it's absolute error must be correspondingly very small, and that will be good enough. Let us start by making a submission that makes use of this observation, and very little more.

2 Trial 1: An entropic approach

Given Eq. (1) let us consider the optimal way of scoring upon ignoring all information based on time and space. That is, if we submit the same 39 probabilities for all of the N=884,262 crime incidents, what is the optimal probability weighting? In the limit where $N \to \infty$ this score will be given by the entropy

$$S = -\frac{1}{39} \sum_{j=1}^{39} p_j \ln p_j, \tag{2}$$

where p_j is the global probability for the j_{th} crime category occurring, and in this limit the global probabilities will correspond to the optimal probability weightings. I will not go into the details of how and why this is the case, and just hope that you accept my assertion of it being so. With this in mind let us compute the global probabilities for each of the 39 crime categories, using the training data, then compute the entropy, and see what happens upon making a kaqqle submission. But first lets see what these global probabilities look like. There are some good graphing scripts that have been put up by the community on the kaggle site. I have slightly modified one of these to make a histogram of the various crime categories ordered by their prevalence. The *Python* script is here in my GitHub repository as "entropy\crime-graphs.py" and the relevant graph can be seen in Fig. (1). On average there are $884,262/39 \simeq 22,700$ crime reports per category (the frequency), and from the graph we see that the top crime category of LARCENY/THEFT is around 8 times more likely than the average, the twelfth most prevalent crime of ROBBERY is close to the average value, and by the time we are half way through the list of categories (say STOLEN PROPERTY) we are down to a frequency that is approximately 5 times less than the average. The top quarter to a third of most prominent categories contain the vast majority of all crime occurrences.

Given our script for making the histogram, it is straightforward to also compute the entropy and output a submission file using the already computed frequencies

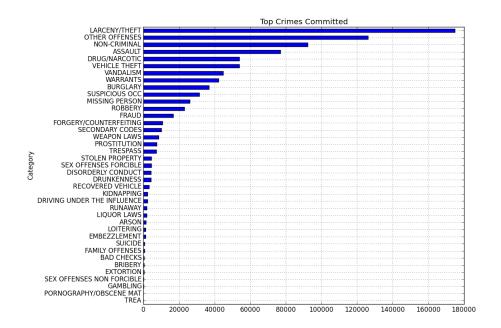


Figure 1: Histogram of crime categories, ranked from most to least prevalent. The x-axis gives the number of crime incidents over the full 12.4 year period.

to obtain the global probabilities. I start a new script, "entropy\global-prob.py", which does this after initially printing out the number of crime occurrences for each category (n.b. this script takes a little time to run). We see that the entropy has a value of S = 2.68033 and the score given by the kaggle entry is 2.68016, very similar indeed. This is a very unsophisticated entry; the competition started on the 2^{nd} June 2015 and when I submitted this on the 16^{th} of June it came in at position 52 out of 82 entries. At the time of writing 7^{th} July the best score is 2.27804. Given how rudimentary and crude this first entry is, it has done remarkably well.

The reader might well ponder the importance of what has been done here. Consider how I have put forward a couple of assertions, and have shown how one of these is consistent with the outcome we obtained for our first submission, i.e. the score was extremely close to the value given by the entropy. These assertions (which can be mathematically proved) have lead me to the conclusion that we need to accurately predict not just the most probable outcomes, but also the less likely outcomes. While the outcome for our submission was overall not great, upon accounting for how simple and crude our method was, it was surprisingly effective. This shows the importance of having an approach that attempts to accurately predict all the possible outcomes, and thus it is crucial that we continue to heed this aspect as we move forward.

3 The Temporal problem addressed through subdivision

To improve our method, we will make use of what time the crime occurred at. We will derive three different data features from the time stamp, making use of the the number of crimes that have occurred for each category, N_c , obtained from the training data. These derived data features are:

- For each category, initially divide the N_c data points into m_c sub-divided blocks, representing equally spaced time segments. We make the integer m_c as large as we can such that $m_c \leq 16$, and that $(N_c + N_{min})/m_c \geq N_{min}$. If $N_c < N_{min}$ we don't make any subdivisions. This is done to handle slow drift in the data. We will denote this data feature as d_{tdr} . We will end up setting $N_{min} = 1000$
- The day of the week, changing at 6am, is used provided that $N_c \ge 7000$. The 6am boundary is chosen because criminal activity is at a low ebb at this time. We will denote this data feature as d_{tdy} .
- The hour of the day, provided $N_c \ge 24,000$. We will denote this data feature as d_{thr} .

To provide some insight for the motivation behind these choices we will first consider some highly relevant graphs. We know that humans tend to organise their lives around the clock and the calendar. For this reason various crimes may be more prevalent at certain times or days of the week, and also depend upon the season. For example wallets may be more likely to be stolen from the beach during the day time in the summer, etc. Initially I attempted to analyse this periodicity through a Fourier analysis, but that proved ineffective. However I leave the Fourier predictions in the graphs I present here, as it does no harm. We will focus on the typical time scales on which humans organise their behaviour and home in on the time scales that are important. Fortunately this is rather straightforward using the training data.

The first graph we consider shows a histogram of the sum of all the crime categories over the full 12.4 years (645 weeks) spanned by the data. The Python script "Fourier\time-1.py", which makes use of "Fourier\fourier.py", produces this histogram, Fig. (2). We can see how the crime rate fluctuates and drifts. Given the long time period that this data spans, the drift is to be expected. When we consider specific crime categories we can expect these behaviours to become more pronounced, as a result of the law of large numbers. We now focus on a much shorter time scale for the exact same data, Fig (3), see "Fourier\time-2.py". Here we see how the crime rate is strongly dependent on the time of the day and to a

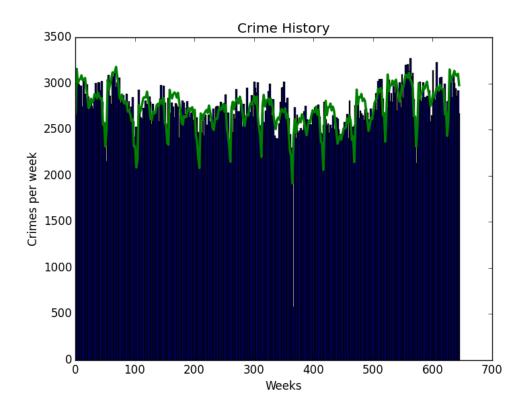


Figure 2: Histogram of crimes per week (all crime categories combined) and Fourier series (solid green line) estimate of the same, over the entire time period of 12.4 years spanned by the training data.

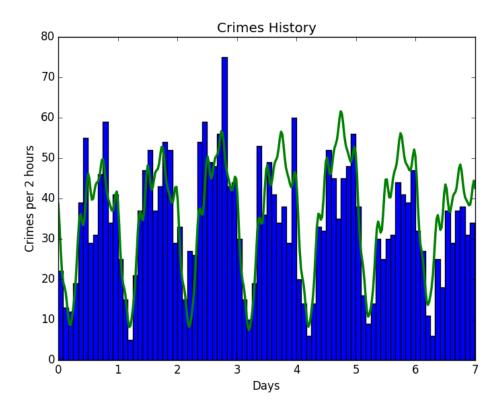


Figure 3: Histogram of crimes per 2 hour period over the first week of training data and Fourier series (solid green line) estimate of the same, over the first week spanned by the training data. The data for *Days* starts at 00:00 hours Monday

lesser extent the day of the week. The crime rate slows right down in the early morning and is less active on a Sunday. Let us now take a look at some graphs of specific crime categories. In Fig. (4) we plot a histogram for the category of Vehicle Theft over the full 12.4 years spanned by the data. The script to compute this is given in "Fourier\time-3.py". Here we see that the drift is far more dramatic than was the case for the sum of all the categories in Fig. (2). Importantly there is an artefact in this data seen as a sudden drop in the rate of thefts around the 150 week mark. The reason for this is not of primary concern to us, just so long as our approach makes use of this information. Referring back to the first point in our list of derived data features at the start of this section, we see that the first derived data feature is just what we need to handle this type of artefact.

We will now construct another two graphs along similar lines to Fig. (3), but for specific crime categories. We will look at two categories of particular interest, firstly *Larceny/Theft* because it is the most prevalent as can be seen in Fig. (1) and

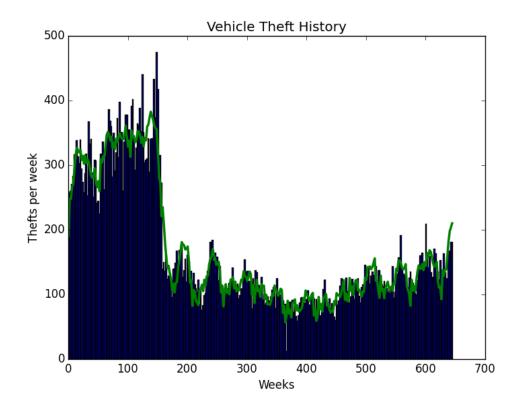


Figure 4: Histogram of vehicle thefts per week and Fourier series (solid green line) estimate of the same, over the entire time period of 12.4 years spanned by the training data.

secondly *Drunkenness*. One would expect *Drunkenness* may be interesting from a temporal perspective because people tend to get in trouble for it on the weekend and at night. So this is a crime that may stand out as happening at atypical times. The graph for *Larceny/Theft* crime category is shown in Fig. (5) and was made using the script 'Fourier\time-4.py". We have summed up the training data (we only have every second week) over a duration of 100 weeks here. We can see that much like was the case for the sum of all the categories shown in Fig. (3) the rate for this crime category tends to be pretty low early in the morning, around 06:00.

We now move onto the Drunkenness category. The graph for the *Drunkenness* crime category is shown in Fig. (6) and was made using the script "Fourier\time-5.py". Here we see some interesting behaviour that is quite different to what we saw in the sum of all the categories shown in Fig. (3). There are notably more cases of *Drunkenness* on Friday and Saturday night, and the data is skewed towards late in the evening.

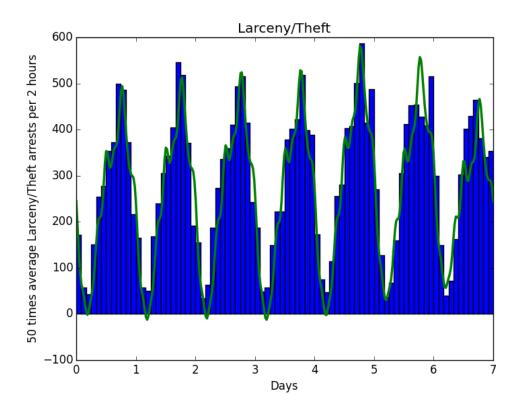


Figure 5: Histogram of Larceny/Theft per two hours, accumulated over the first 100 weeks of the test data and Fourier series (solid green line) estimate of same. The data for Days starts at 00:00 hours Monday

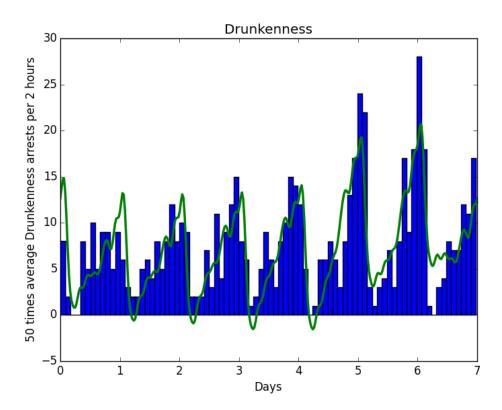


Figure 6: Histogram of Drunkness per two hours, accumulated over the first 100 weeks of the test data and Fourier series (solid green line) estimate of same. The data for Days starts at 00:00 hours Monday

So let's now see what we can do to implement this data, using the derived data features that we have listed at the top of this section. We implement this in the script, "time-sub-div\sdiv.py". Upon assuming each of our features to be independent we can express the probability in terms of our temporal features as

$$P(f_{cr}|\mathbf{d_t}) \propto P(f_{cr}) \frac{P(f_{cr}|d_{tdr})P(f_{cr}|d_{tdy})P(f_{cr}|d_{thr})}{P^3(f_{cr})},$$
 (3)

where $\mathbf{d_t} = [d_{tdr}, d_{tdu}, d_{thr}]$ is a vector representing all the temporal features, f_{cr} is the crime category and P(f|x) is the conditional probability of observing f given we have the value x. This is very similar to what is done in a Bernoulli naive Bayes classifier, except we compute the posterior directly rather than obtaining it from the prior. This is convenient here as it will make it easier to experiment with feature engineering as we progress further. So we implement this simple approach in the Python script "time-sub-div\sub.py", which has several parameters that may be altered. First we implement this using only the drift variable which has the parameter N_{min} as given in the list at the beginning of this section. I settled on a value of $N_{min} = 1000$ that returned a score of 2.66062 which is an improvement on our earlier score of 2.68016. Next the day of the week, d_{tdy} , was additionally included with the parameter $D_{min} = 2000$. This parameter represents the minimum number of crimes a given category must have recorded against it, if this data feature is to be used. This reduced the score further to 2.65794. Finally we additionally include the d_{thr} feature, with the parameter H_{min} which represents the minimum category score for the hour, just as was done for the day. This reduced the score further to 2.62751 which is our best improvement so far. Thus far we have done enough to get about half way up the leaderboard (things become more competitive as time progresses). As it turns out the temporal features just aren't as strongly correlated with the crime categories as the spatial features are. However as we move further towards the top of the leaderboard, small improvements become more and more important. We will next move onto the spatial features and drop the temporal features for now. In the long run it will become important to combine both types of features.

4 Using the spatial information

There is quite a bit of information provided in the data features about the location where the crime occurred. Let us start by noting the police department (PD) that handled the crime. There are 10 separate PDs given, each of which has recorded a comparable number of the crimes over the 12.4 year duration. So let us start by making a simple submission based on the which PD the crime was handled by. To do this we simply enumerate the probability for each crime given the PD using

the training data, and then use this with the PD provided for each crime in the test data. This is done by our script "position\sub-1.py" and resulted in a score of 2.61645 which is our best so far. So we see that the spatial features appears to have much stronger correlation than the temporal features. This is good because there is still much more we can do with the spatial features.

Let us now consider the address where the crime occurred. One of the interesting aspects about the address is that there are a small subset of addresses that have a disproportionately large number of crimes listed against them. Our script "position\worst-address.py" provides some basic statistics that bring this out very simply. There are 878,049 crimes provided in the training data and on average there are 37.8 crimes per address. Of these there are 83 addresses that have 750 crimes or more each against them. These 83 addresses account for 16.8% of all crimes but only consist of 0.36% of all addresses. This is good news for us, because this is something we can exploit. The precise address is very detailed information but a small subset of them are associated with a remarkably large portion of crime. So we can get good statistics on this small subset of addresses, and given the address is such specific information we should get very strong variability between the addresses. We will incorporate this into our script, by first determining if our crime happened at one of these addresses. If it did we will use the probabilities computed for the specific address, and if it didn't we will use the probabilities computed for the PD as we did previously. This is done in the script "position\sub-2.py" and this script now has a parameter we can vary called Amin which is the minimum number of crimes an address must have if we are going to use it. We set Amin = 1000 and obtain the score 2.585. This has improved our position on the leader board considerably, to approximately two thirds the way to the top. But we still have quite a large amount of work to do and the strong exception that our position can be improved substantially.

4.1 Being more even handed

So far when we divided our data between addresses and PDs, we treated all crime category possibilities the same. But there is a different amount of training data points for each crime category, and so this is not an even handed approach. We also need to make use of the coordinates, X & Y, which we will do by dividing the area spanned by the data up into a rectangular $N \times N$ grid. Additionally, thus far we have resorted to the use of kaggle submissions to assess how good our analysis is working, but we could divide the training data up still using some of it as training data and the remainder as test data. To do this we make use of the Scikit-learn module $train_test_split$. In the end, after much searching using $train_test_split$, the following was settled upon:

- Four levels of spatial division where formed, the address, a fine grid of 80 × 80 divisions, a coarser grid of 18 × 18 divisions and finally the PD that handled the crime.
- Each of these four different levels was trained using all of the training data
- The test data was first given a probability for each of the assumed crime categories using the address, if the address had 6 or more crimes of the given category recorded against it from the training data this probability was retained. Otherwise the fine grid was appealed to with 7 or more crimes for retention. This was then continued on for the coarser grid and finally the PD was appealed to if all else failed.
- The probabilities for each of the 39 crime categories for the given crime instance was then renormalised, a crude but small correction.
- Some of the data has invalid coordinate values. This data is identified, and if there isn't enough crime incidents given by the address the PD is used.

The Python script that was used to do this can be found at "evenh\sub.py" which uses "evenh\grid.py". This improved our score to 2.446, an improvement of 0.139 on our previous best, and moved us up the leaderboard to position 42 of 364. This puts us 88.5% of the way to the top, which is pretty good.

5 Logistic Regression

The form of logistic regression provided in the Scikit-learn library is multinomial logistic regression. Multinomial logistic regression uses the soft-max function to minimise a modified version of the logarithmic loss function, when an arbitrary number of categorical outcomes need to be predicted. For the simpler case where the categorical outcomes are binary, regular logistic regression can be used which makes use of the logistic function, a special case of the soft-max function. The logarithmic loss function is modified by adding a sum of the logistic regression model's parameters squared. This is done to provide what is called regurlisation. The strength of the regurbisation may be adjusted, with strong regurbisation working against the data being over fitted. This provides us with opportunities to tune our logistic regression model. We can also tune the amount of regurbisation between different data features by multiplying all elements of a given data feature by some constant before feeding them into the logistic regression model for fitting. We will make use of this here, but won't document the details further, as it is too technical. Here we will use regular logistic regression, because it allows us to use munged data features which are particular to each categorical outcome. So

we will do a separate logistic regression for each categorical outcome to predict the probability of that outcome occurring or not occurring and we feed the log odds of our initial estimates for the probabilities of each categorical outcome into its separate logistic regression model. n.b. that the log odds is the inverse of the logistic function used in regular logistic regression and that multinomial logistic regression reduces to regular logistic regression when there are only two categorical outcomes, as we are doing here. These outcomes, p_{ri} , are then combined and renormalised to obtain the desired predictions

$$p_i = \frac{p_{ri}}{\sum_j^M p_{rj}}. (4)$$

Alternatively we could feed all of the data features into multinomial logistic regression, but this would slow things down. The way we have chosen gives us the opportunity to use more features, and we want to take advantage of this, mainly so we can fit a 5th order polynomial to the X & Y positions in the data by feeding the large number of data features this requires into the logistic regression. We won't go into much more detail of how we implemented the logistic regression using *Python* and *Scikit-learn*, other than to say we split the training data and did a systematic tuning of free parameters using the script "logistic\tune.py". This then allowed us to prepare a submission using "logistic\sub.py" which resulted in a score of 2.27476. This is a sizeable improvement of 0.171 on our previous submission given above, but a considerable time has elapsed between these submissions and the competition has hotted up considerably. None the less we have moved up to position 21 of 683, which is very competitive.

6 Moving forward

We have skipped over a great deal of detail about the logistic regression, so it needs to be pointed out that we have close to wrung as much as we can out of it. Moving forward there is one more thing I would like to try with logistic regression. That is to divide the data into blocks, such that each address is only in a single block. Then we can do logistic regression on each of these blocks separately. This will allow for correlations between the addresses and the times the crimes are committed at to be accounted for, that the current model ignores. Having done that, the smaller data blocks could also be modelled using the *support vector machine* that is available from the *Scikit-learn* library. Without doing this a *support vector machine* is not able to handle the large amount of data samples that the data sets for this problem have. Then we could try some ensemble averaging between the methods. We are not at the top spot, and the person who is is currently there is known to have got there legitimately, so obviously there is more that we can squeeze out of this yet.