CHAPTER 6—STRUCTURAL ANALYSIS

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6.1—Scope

6.1.1 This chapter shall apply to methods of analysis, modeling of members and structural systems, and calculation of load effects.

6.2—General

6.2.1 Members and structural systems shall be permitted to be modeled in accordance with 6.3.

6.2.2 All members and structural systems shall be analyzed to determine the maximum load effects including the arrangements of live load in accordance with 6.4.

6.2.3 Methods of analysis permitted by this chapter shall be (a) through (e):

(a) The simplified method for analysis of continuous beams and one-way slabs for gravity loads in 6.5
(b) Linear elastic first-order analysis in 6.6
(c) Linear elastic second-order analysis in 6.7
(d) Inelastic analysis in 6.8
(e) Finite element analysis in 6.9

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R6.1—Scope

R6.1.1 The provisions of this chapter apply to analyses used to determine load effects for design.

Section 6.2 provides general requirements that are applicable for all analysis procedures.

Section 6.2.4 directs the licensed design professional to specific analysis provisions that are not contained in this chapter. Sections 6.2.4.1 and 6.2.4.2 identify analysis provisions that are specific to two-way slabs and walls.

Section 6.3 addresses modeling assumptions used in establishing the analysis model.

Section 6.4 prescribes the arrangements of live loads that are to be considered in the analysis.

Section 6.5 provides a simplified method of analysis for nonprestressed continuous beams and one-way slabs that can be used in place of a more rigorous analysis when the stipulated conditions are satisfied.

Section 6.6 includes provisions for a comprehensive linear elastic first-order analysis. The effects of cracked sections and creep are included in the analysis through the use of effective stiffnesses.

Section 6.7 includes provisions for linear elastic second-order analysis. Inclusion of the effects of cracking and creep is required.

Section 6.8 includes provisions for inelastic analysis.

Section 6.9 includes provisions for the use of the finite element method.

R6.2—General

R6.2.3 A first-order analysis satisfies the equations of equilibrium using the original undeformed geometry of the structure. When only first-order results are considered, slenderness effects are not accounted for. Because these effects can be important, 6.6 provides procedures to calculate both individual member slenderness ($P\delta$) effects and sidesway ($P\Delta$) effects for the overall structure using the first-order results.

A second-order analysis satisfies the equations of equilibrium using the deformed geometry of the structure. If the second-order analysis uses nodes along compression members, the analysis accounts for slenderness effects due to lateral deformations along individual members, as well as sidesway of the overall structure. If the second-order analysis uses nodes at the member intersections only, the analysis captures the sidesway effects for the overall structure but neglects individual member slenderness effects. In this case, the moment magnifier method (6.6.4) is used to determine individual member slenderness effects.

An inelastic analysis i) represents the nonlinear stress-strain response of the materials composing the structure; ii) satisfies compatibility of deformations; and iii) satisfies equilibrium in the undeformed configuration for first-order analysis or in the deformed configuration for second-order analysis.

Finite element analysis was introduced in the 2014 Code to explicitly recognize a widely used analysis method.

6.2.4 Additional analysis methods that are permitted include 6.2.4.1 through 6.2.4.4.

6.2.4.1 Two-way slabs shall be permitted to be analyzed for gravity loads in accordance with (a) or (b):

(a) Direct design method for nonprestressed slabs
(b) Equivalent frame method for nonprestressed and prestressed slabs

R6.2.4.1 Code editions from 1971 to 2014 contained provisions for use of the direct design method and the equivalent frame method. These methods are well-established and are covered in available texts. These provisions for gravity load analysis of two-way slabs have been removed from the Code because they are considered to be only two of several analysis methods currently used for the design of two-way slabs. The direct design method and the equivalent frame method of the 2014 Code, however, may still be used for the analysis of two-way slabs for gravity loads.

6.2.4.2 Slender walls shall be permitted to be analyzed in accordance with 11.8 for out-of-plane effects.

6.2.4.3 Diaphragms shall be permitted to be analyzed in accordance with 12.4.2.

6.2.4.4 A member or region shall be permitted to be analyzed and designed using the strut-and-tie method in accordance with Chapter 23.

6.2.5 *Slenderness effects*

R6.2.5 *Slenderness effects*

Second-order effects in many structures are negligible. In these cases, it is unnecessary to consider slenderness effects, and compression members, such as columns, walls, or braces, can be designed based on forces determined from first-order analyses. Slenderness effects can be neglected in both braced and unbraced systems, depending on the slenderness ratio ($k\ell_u/r$) of the member.

The sign convention for M_1/M_2 has been updated so that M_1/M_2 is negative if bent in single curvature and positive if bent in double curvature. This reflects a sign convention change from the 2011 Code.

The primary design aid to estimate the effective length factor k is the Jackson and Moreland Alignment Charts (Fig. R6.2.5.1), which provide a graphical determination of k for a column of constant cross section in a multi-bay frame (ACI SP-17(09); Column Research Council 1966).

Equations (6.2.5.1b) and (6.2.5.1c) are based on Eq. (6.6.4.5.1) assuming that a 5 percent increase in moments due to slenderness is acceptable (MacGregor et al. 1970).

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6.2.5.1 Slenderness effects shall be permitted to be neglected if (a) or (b) is satisfied:

(a) For columns not braced against sidesway

$$\frac{k\ell_u}{r} \leq 22 \qquad (6.2.5.1a)$$

(b) For columns braced against sidesway

$$\frac{k\ell_u}{r} \leq 34 + 12(M_1/M_2) \qquad (6.2.5.1b)$$

and

$$\frac{k\ell_u}{r} \leq 40 \qquad (6.2.5.1c)$$

where M_1/M_2 is negative if the column is bent in single curvature, and positive for double curvature.

If bracing elements resisting lateral movement of a story have a total stiffness of at least 12 times the gross lateral stiffness of the columns in the direction considered, it shall be permitted to consider columns within the story to be braced against sidesway.

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As a first approximation, k may be taken equal to 1.0 in Eq. (6.2.5.1b) and (6.2.5.1c).

The stiffness of the lateral bracing is considered based on the principal directions of the framing system. Bracing elements in typical building structures consist of structural walls or lateral braces. Torsional response of the lateral-force-resisting system due to eccentricity of the structural system can increase second-order effects and should be considered.

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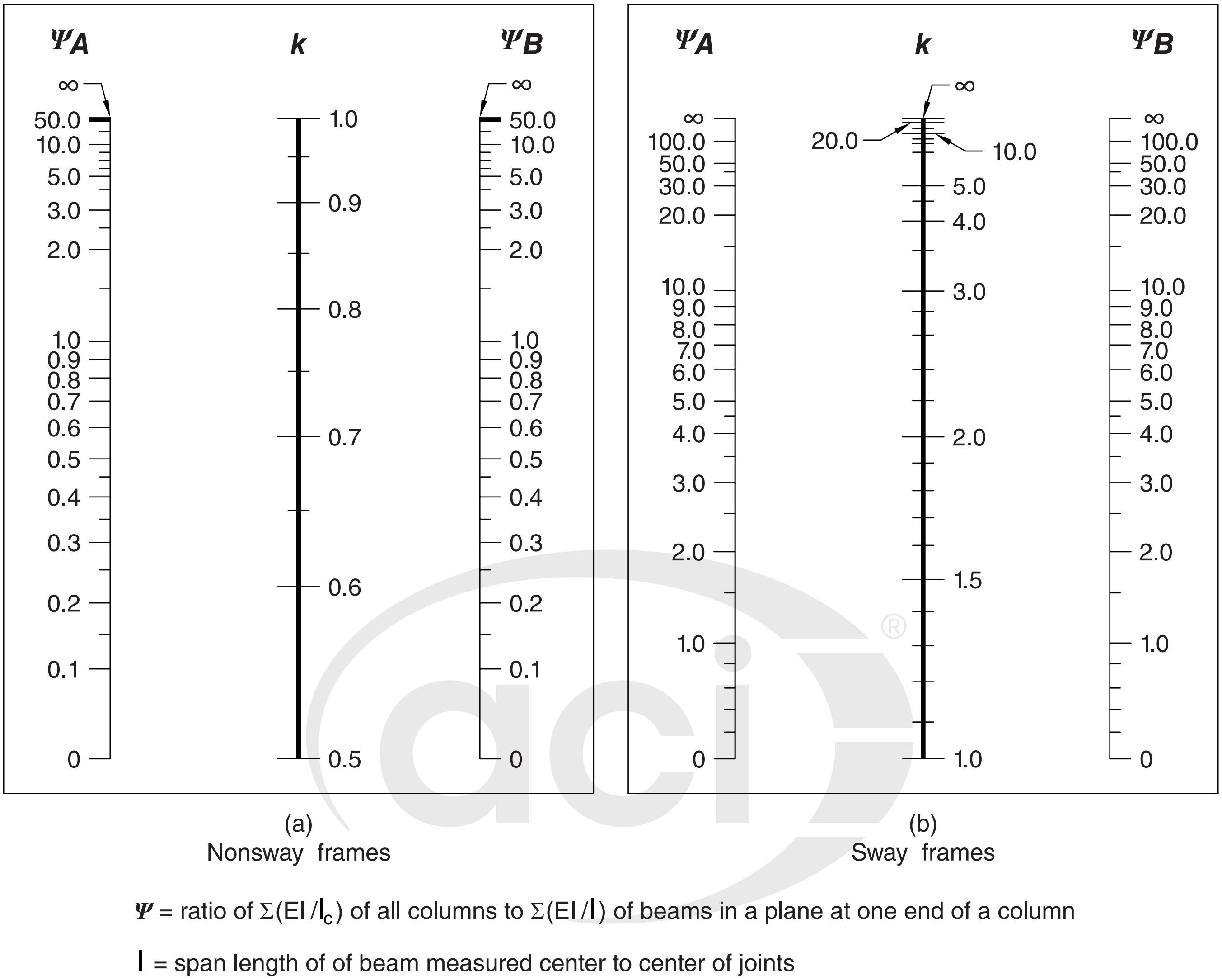

Fig. R6.2.5.1—Effective length factor k.

6.2.5.2 The radius of gyration, r, shall be permitted to be calculated by (a), (b), or (c):

(a) $r = \sqrt{\frac{I_g}{A_g}}$ (6.2.5.2)

(b) 0.30 times the dimension in the direction stability is being considered for rectangular columns

(c) 0.25 times the diameter of circular columns

6.2.5.3 Unless slenderness effects are neglected as permitted by 6.2.5.1, the design of columns, restraining beams, and other supporting members shall be based on the factored forces and moments considering second-order effects in accordance with 6.6.4, 6.7, or 6.8. M_u of end moments including second-order effects shall not exceed $1.4M_u$ due to first-order effects.

R6.2.5.3 Design considering second-order effects may be based on the moment magnifier approach (MacGregor et al. 1970; MacGregor 1993; Ford et al. 1981), an elastic second-order analysis, or a nonlinear second-order analysis. Figure R6.2.5.3 is intended to assist designers with application of the slenderness provisions of the Code.

End moments in compression members, such as columns, walls, or braces, should be considered in the design of adjacent flexural members. In nonsway frames, the effects of magnifying the end moments need not be considered in the

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design of adjacent beams. In sway frames, the magnified end moments should be considered in designing the adjoining flexural members.

Several methods have been developed to evaluate slenderness effects in compression members subject to biaxial bending. A review of some of these methods is presented in Furlong et al. (2004).

If the weight of a structure is high in proportion to its lateral stiffness, excessive $P\Delta$ effects, where secondary moments are more than 25 percent of the primary moments, may result. The $P\Delta$ effects will eventually introduce singularities into the solution to the equations of equilibrium, indicating physical structural instability (Wilson 1997). Analytical research (MacGregor and Hage 1977) on reinforced concrete frames showed that the probability of stability failure increases rapidly when the stability index Q, defined in 6.6.4.4.1, exceeds 0.2, which is equivalent to a secondary-to-primary moment ratio of 1.25. According to ASCE/SEI 7, the maximum value of the stability coefficient θ, which is close to the ACI stability coefficient Q, is 0.25. The value 0.25 is equivalent to a secondary-to-primary moment ratio of 1.33. Hence, the upper limit of 1.4 on the secondary-to-primary moment ratio was chosen.

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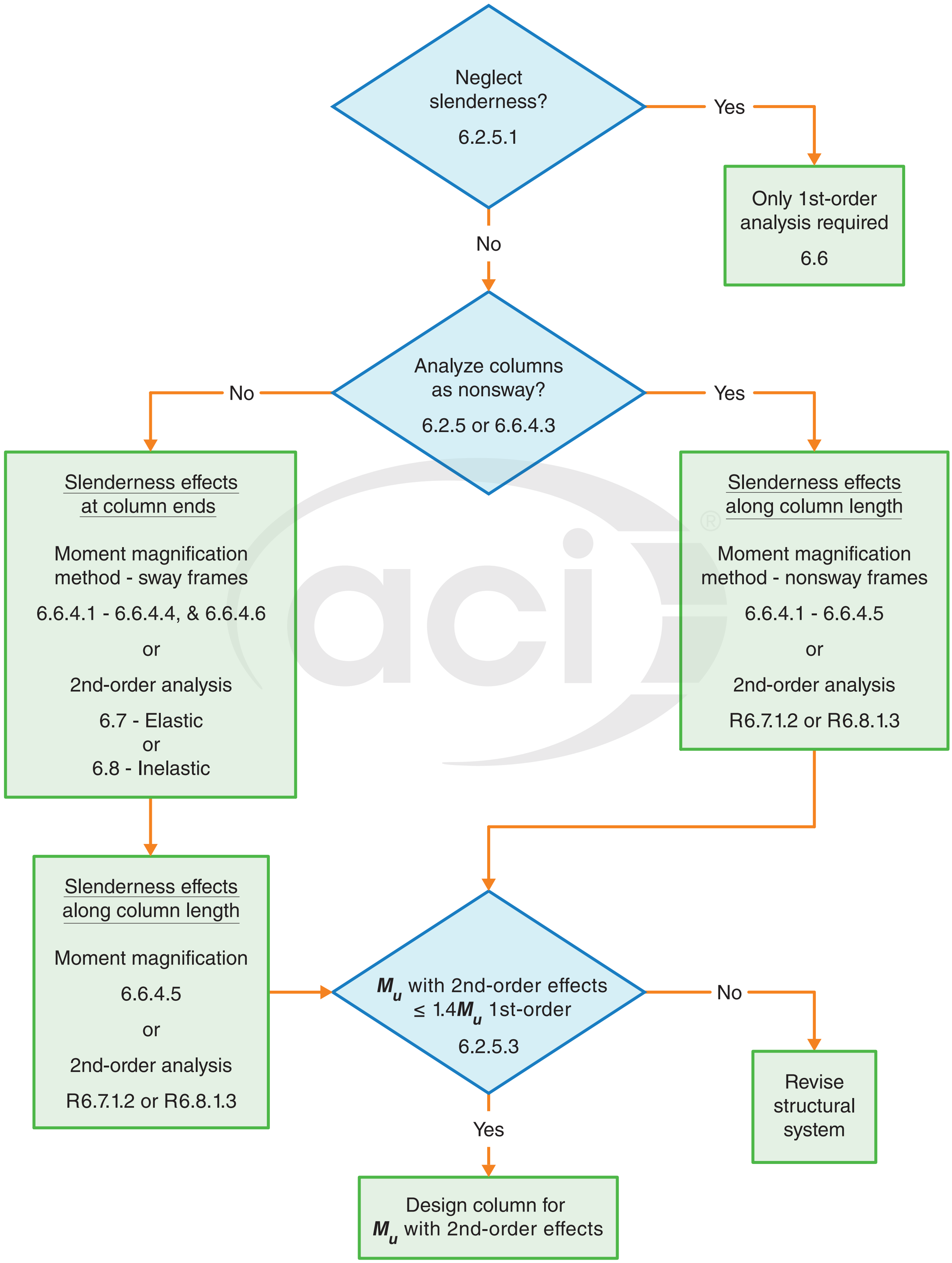

Fig. R6.2.5.3—Flowchart for determining column slenderness effects.

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6.3—Modeling assumptions

6.3.1 *General*

6.3.1.1 Member stiffnesses for analysis of structural systems shall be selected based on a reasonable set of assumptions. The assumptions shall be consistent throughout each analysis.

6.3.1.2 To calculate moments and shears caused by gravity loads in columns, beams, and slabs, it shall be permitted to use a model limited to the members in the level being considered and the columns above and below that level. It shall be permitted to assume far ends of columns built integrally with the structure to be fixed.

6.3.1.3 The analysis model shall consider the effects of variation of member cross-sectional properties, such as that due to haunches.

6.3.2 *T-beam geometry*

6.3.2.1 For nonprestressed T-beams supporting monolithic or composite concrete slabs, the effective flange width b_f shall include the beam web width b_w plus an effective

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R6.3—Modeling assumptions

R6.3.1 *General*

R6.3.1.1 Separate analyses with different stiffness assumptions may be performed for different objectives such as to check serviceability or strength criteria or to bound the demands on elements where stiffness assumptions are critical.

Ideally, the member stiffnesses E_cI and GJ should reflect the degree of cracking and inelastic action that has occurred along each member before yielding. However, the complexities involved in selecting different stiffnesses for all members of a frame would make frame analyses inefficient in the design process. Simpler assumptions are required to define member stiffnesses.

Relative stiffnesses of members are sufficient to estimate the distribution of load effects for many circumstances, such as, moments and shears in continuous beams. For analyses where deflections, second-order effects, or dynamic response are considered, an estimate of effective stiffness of members is required.

Guidance for the selection of effective stiffness is given in 6.6.3.

Two conditions determine whether it is necessary to consider torsional stiffness in the analysis of a given structure: 1) the relative magnitude of the torsional and flexural stiffnesses; and 2) whether torsion is required for equilibrium of the structure (equilibrium torsion) or is due to members twisting to maintain deformation compatibility (compatibility torsion). In the case of equilibrium torsion, torsional stiffness should be included in the analysis. It is, for example, necessary to consider the torsional stiffnesses of edge beams. In the case of compatibility torsion, torsional stiffness usually is not included in the analysis. This is because the cracked torsional stiffness of a beam is a small fraction of the flexural stiffness of the members framing into it. Torsion should be considered in design as required in Chapter 9.

R6.3.1.3 Stiffness and fixed-end moment coefficients for haunched members may be obtained from the Portland Cement Association (1972).

R6.3.2 *T-beam geometry*

R6.3.2.1 In ACI 318-11, the width of the slab effective as a T-beam flange was limited to one-fourth the span. The Code now allows one-eighth of the span on each side of the

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overhanging flange width in accordance with Table 6.3.2.1, where h is the slab thickness and s_w is the clear distance to the adjacent web.

Table 6.3.2.1—Dimensional limits for effective overhanging flange width for T-beams

Flange location	Effective overhanging flange width, beyond face of web	
Each side of web	Least of:	$8h$
		$s_w/2$
		$\ell_n/8$
One side of web	Least of:	$6h$
		$s_w/2$
		$\ell_n/12$

6.3.2.2 Isolated nonprestressed T-beams in which the flange is used to provide additional compression area shall have a flange thickness greater than or equal to $0.5b_w$ and an effective flange width less than or equal to $4b_w$.

6.3.2.3 For prestressed T-beams, it shall be permitted to use the geometry provided by 6.3.2.1 and 6.3.2.2.

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beam web. This was done to simplify Table 6.3.2.1 and has negligible impact on designs.

R6.3.2.3 The empirical provisions of 6.3.2.1 and 6.3.2.2 were developed for nonprestressed T-beams. The flange widths in 6.3.2.1 and 6.3.2.2 should be used unless experience has proven that variations are safe and satisfactory. Although many standard prestressed products in use today do not satisfy the effective flange width requirements of 6.3.2.1 and 6.3.2.2, they demonstrate satisfactory performance. Therefore, determination of an effective flange width for prestressed T-beams is left to the experience and judgment of the licensed design professional. It is not always considered conservative in elastic analysis and design considerations to use the maximum flange width as permitted in 6.3.2.1.

6.4—Arrangement of live load

6.4.1 For the design of floors or roofs to resist gravity loads, it shall be permitted to assume that live load is applied only to the level under consideration.

6.4.2 For one-way slabs and beams, it shall be permitted to assume (a) and (b):

(a) Maximum positive M_u near midspan occurs with factored L on the span and on alternate spans
(b) Maximum negative M_u at a support occurs with factored L on adjacent spans only

6.4.3 For two-way slab systems, factored moments shall be calculated in accordance with 6.4.3.1, 6.4.3.2, or 6.4.3.3, and shall be at least the moments resulting from factored L applied simultaneously to all panels.

6.4.3.1 If the arrangement of L is known, the slab system shall be analyzed for that arrangement.

R6.4—Arrangement of live load

R6.4.2 The most demanding sets of design forces should be established by investigating the effects of live load placed in various critical patterns.

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6.4.3.2 If L is variable and does not exceed $0.75D$, or the nature of L is such that all panels will be loaded simultaneously, it shall be permitted to assume that maximum M_u at all sections occurs with factored L applied simultaneously to all panels.

6.4.3.3 For loading conditions other than those defined in 6.4.3.1 or 6.4.3.2, it shall be permitted to assume (a) and (b):

(a) Maximum positive M_u near midspan of panel occurs with 75% of factored L on the panel and alternate panels
(b) Maximum negative M_u at a support occurs with 75% of factored L on adjacent panels only

6.4.4 For embedments creating voids parallel to the slab plane with cross-section dimension larger than one-third the slab thickness, or spaced closer than three void diameters or widths on center, the arrangement of live load on only a portion of the slab shall be considered.

6.5—Simplified method of analysis for nonprestressed continuous beams and one-way slabs

6.5.1 It shall be permitted to calculate M_u and V_u due to gravity loads in accordance with this section for continuous beams and one-way slabs satisfying (a) through (e):

(a) Members are prismatic
(b) Loads are uniformly distributed
(c) $L \leq 3D$
(d) There are at least two spans
(e) The longer of two adjacent spans does not exceed the shorter by more than 20%

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R6.4.3.3 The use of only 75% of the full factored live load for maximum moment loading patterns is based on the fact that maximum negative and maximum positive live load moments cannot occur simultaneously and that redistribution of maximum moments is thus possible before failure occurs. This procedure, in effect, permits some local overstress under the full factored live load if it is distributed in the prescribed manner, but still ensures that the design strength of the slab system after redistribution of moment is not less than that required to resist the full factored dead and live loads on all panels.

R6.4.4 If large or closely-spaced embedments are created within slabs, maximum bending and shear stresses may occur near the void. In these cases, the designer should consider the effect of applying the live load to only a portion of the slab (example Fig. R6.4.4) to determine if localized bending and shear behavior controls the design. Effects of concentrated live loads should also be considered.

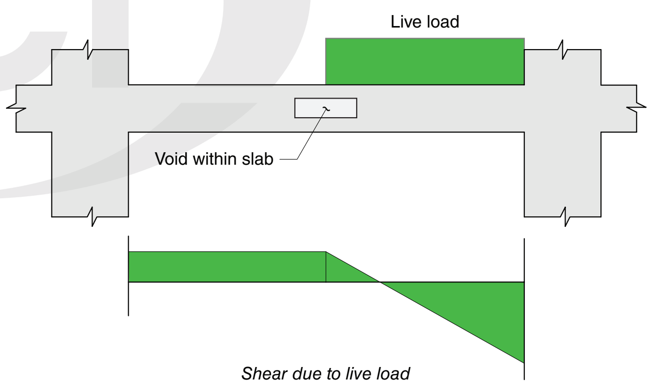

Fig. R6.4.4—Example of applying live load to a portion of the slab to calculate localized shear at the internal void

R6.5—Simplified method of analysis for nonprestressed continuous beams and one-way slabs

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6.5.2 M_u due to gravity loads shall be calculated in accordance with Table 6.5.2.

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R6.5.2 The approximate moments and shears give reasonable values for the stated conditions if the continuous beams and one-way slabs are part of a frame or continuous construction. Because the load patterns that produce critical values for moments in columns of frames differ from those for maximum negative moments in beams, column moments should be evaluated separately.

Table 6.5.2—Approximate moments for nonprestressed continuous beams and one-way slabs

Moment	Location	Condition	M_u
Positive	End span	Discontinuous end integral with support	$w_u\ell_n^2/14$
		Discontinuous end unrestrained	$w_u\ell_n^2/11$
	Interior spans	All	$w_u\ell_n^2/16$
Negative[1]	Interior face of exterior support	Member built integrally with supporting spandrel beam	$w_u\ell_n^2/24$
		Member built integrally with supporting column	$w_u\ell_n^2/16$
	Exterior face of first interior support	Two spans	$w_u\ell_n^2/9$
		More than two spans	$w_u\ell_n^2/10$
	Face of other supports	All	$w_u\ell_n^2/11$
	Face of all supports satisfying (a) or (b)	(a) slabs with spans not exceeding 10 ft (b) beams where ratio of sum of column stiffnesses to beam stiffness exceeds 8 at each end of span	$w_u\ell_n^2/12$

[1]To calculate negative moments, ℓ_n shall be the average of the adjacent clear span lengths.

6.5.3 Moments calculated in accordance with 6.5.2 shall not be redistributed.

6.5.4 V_u due to gravity loads shall be calculated in accordance with Table 6.5.4.

Table 6.5.4—Approximate shears for nonprestressed continuous beams and one-way slabs

Location	V_u
Exterior face of first interior support	$1.15w_u\ell_n/2$
Face of all other supports	$w_u\ell_n/2$

6.5.5 Floor or roof level moments shall be resisted by distributing the moment between columns immediately above and below the given floor in proportion to the relative column stiffnesses considering conditions of restraint.

R6.5.5 This section is provided to make certain that moments are included in column design. The moment refers to the difference between the end moments of the members framing into the column and exerted at the column centerline.

6.6—Linear elastic first-order analysis

6.6.1 *General*

6.6.1.1 Slenderness effects shall be considered in accordance with 6.6.4, unless they are allowed to be neglected by 6.2.5.1.

R6.6—Linear elastic first-order analysis

R6.6.1 *General*

R6.6.1.1 When using linear elastic first-order analysis, slenderness effects are calculated using the moment magnifier approach (MacGregor et al. 1970; MacGregor 1993; Ford et al. 1981).

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6.6.1.2 Redistribution of moments calculated by an elastic first-order analysis shall be permitted in accordance with 6.6.5.

6.6.2 *Modeling of members and structural systems*

6.6.2.1 Floor or roof level moments shall be resisted by distributing the moment between columns immediately above and below the given floor in proportion to the relative column stiffnesses and considering conditions of restraint.

6.6.2.2 For frames or continuous construction, consideration shall be given to the effect of floor and roof load patterns on transfer of moment to exterior and interior columns, and of eccentric loading due to other causes.

6.6.2.3 It shall be permitted to simplify the analysis model by the assumptions of (a), (b), or both:

(a) Solid slabs or one-way joist systems built integrally with supports, with clear spans not more than 10 ft, shall be permitted to be analyzed as continuous members on knife-edge supports with spans equal to the clear spans of the member and width of support beams otherwise neglected.
(b) For frames or continuous construction, it shall be permitted to assume the intersecting member regions are rigid.

6.6.3 *Section properties*

6.6.3.1 *Factored load analysis*

6.6.3.1.1 Moment of inertia and cross-sectional area of members shall be calculated in accordance with Tables 6.6.3.1.1(a) or 6.6.3.1.1(b), unless a more rigorous analysis is used. If sustained lateral loads are present, I for columns and walls shall be divided by $(1 + \beta_{ds})$, where β_{ds} is the ratio of maximum factored sustained shear within a story to the maximum factored shear in that story associated with the same load combination.

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R6.6.2 *Modeling of members and structural systems*

R6.6.2.1 This section is provided to make certain that moments are included in column design if members have been proportioned using 6.5.1 and 6.5.2. The moment refers to the difference between the end moments of the members framing into the column and exerted at the column centerline.

R6.6.2.3 A common feature of modern frame analysis software is the assumption of rigid connections. Section 6.6.2.3(b) is intended to apply to intersecting elements in frames, such as beam-column joints.

R6.6.3 *Section properties*

R6.6.3.1 *Factored load analysis*

R6.6.3.1.1 The values of I and A have been chosen from the results of frame tests and analyses, and include an allowance for the variability of the calculated deflections. The moments of inertia are taken from MacGregor and Hage (1977), which are multiplied by a stiffness reduction factor $\phi_K = 0.875$ (refer to R6.6.4.5.2). For example, the moment of inertia for columns is $0.875(0.80I_g) = 0.70I_g$.

The moment of inertia of T-beams should be based on the effective flange width defined in 6.3.2.1 or 6.3.2.2. It is generally sufficiently accurate to take I_g of a T-beam as $2I_g$ for the web, $2(b_wh^3/12)$.

If the factored moments and shears from an analysis based on the moment of inertia of a wall, taken equal to $0.70I_g$, indicate that the wall will crack in flexure, based on the modulus of rupture, the analysis should be repeated with $I = 0.35I_g$ in those stories where cracking is predicted using factored loads.

The values of the moments of inertia were derived for nonprestressed members. For prestressed members, the moments of inertia may differ depending on the amount, location, and type of reinforcement, and the degree of

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cracking prior to reaching ultimate load. The stiffness values for prestressed concrete members should include an allowance for the variability of the stiffnesses.

Shear area A_{shear} is used to calculate shear deformations. A_{shear} depends on the shape of the gross cross section, the amount of concrete degradation the member has sustained, and is generally less than the gross cross-sectional area. For a rectangular cross section, $A_{shear} \leq 5/6A_g$.

The equations in Table 6.6.3.1.1(b) provide more refined values of I considering axial load, eccentricity, reinforcement ratio, and concrete compressive strength as presented in Khuntia and Ghosh (2004a,b). The stiffnesses provided in these references are applicable for all levels of loading, including service and ultimate, and consider a stiffness reduction factor ϕ_K comparable to that for the moment of inertias included in Table 6.6.3.1.1(a). For use at load levels other than ultimate, P_u and M_u should be replaced with their appropriate values at the desired load level. For factored lateral load deflection analysis if inelastic behavior is expected, stiffnesses should be calculated according to 6.6.3.1.2.

Table 6.6.3.1.1(a)—Moments of inertia and cross-sectional areas permitted for elastic analysis at factored load level

Member and condition		Moment of inertia	Cross-sectional area for axial deformations	Cross-sectional area for shear deformations
Columns		$0.70I_g$	$1.0A_g$	$1.0A_{shear}$
Walls	Uncracked	$0.70I_g$		
	Cracked	$0.35I_g$		
Beams		$0.35I_g$		
Flat plates and flat slabs		$0.25I_g$		

Table 6.6.3.1.1(b)—Alternative moments of inertia for elastic analysis at factored load

Member	Alternative value of I for elastic analysis		
	Minimum	I	Maximum
Columns and walls	$0.35I_g$	$\left(0.80+25\frac{A_{st}}{A_g}\right)\left(1-\frac{M_u}{P_u h}-0.5\frac{P_u}{P_o}\right)I_g$	$0.875I_g$
Beams, flat plates, and flat slabs	$0.25I_g$	$(0.10+25\rho)\left(1.2-0.2\frac{b_w}{d}\right)I_g$	$0.5I_g$

Notes: For continuous flexural members, I shall be permitted to be taken as the average of values obtained for the critical positive and negative moment sections. P_u and M_u shall be calculated from the load combination under consideration, or the combination of P_u and M_u that produces the least value of I.

6.6.3.1.2 For calculation of deflections under factored lateral loads, if inelastic response is expected, member effective stiffness shall be calculated in accordance with (a), (b), or (c) except for two-way slab systems without beams designated as part of the seismic-force-resisting system, where I for slab members shall be defined by a model that is in substantial agreement with results of comprehensive tests and analysis. For all cases, it shall be permitted to calculate cross-sectional area as indicated in Table 6.6.3.1.1(a).

(a) Section properties defined in Table 6.6.3.1.1(a)
(b) $I = 0.5I_g$ for all members
(c) I by a more detailed analysis, considering the effective stiffness of all members under the loading conditions.

R6.6.3.1.2 The lateral deflection of a structure under factored lateral loads can be substantially different from that calculated using linear analysis, in part because of the inelastic response of the members and the decrease in effective stiffness. Selection of the appropriate effective stiffness for reinforced concrete frame members has dual purposes: 1) to provide realistic estimates of lateral deflection; and 2) to determine deflection-imposed actions on the gravity system of the structure. A detailed nonlinear analysis of the structure would adequately capture these two effects. A simple way to estimate an equivalent nonlinear lateral deflection using linear analysis is to reduce the modeled stiffness of the concrete members in the structure. The type of lateral

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load analysis affects the selection of appropriate effective stiffness values. For analyses with wind loading, where it is desirable to prevent nonlinear action in the structure, effective stiffnesses representative of pre-yield behavior may be appropriate. For earthquake-induced loading, the level of nonlinear deformation depends on the intended structural performance and earthquake recurrence interval.

Varying degrees of confidence can be obtained from a simple linear analysis based on the computational rigor used to define the effective stiffness of each member. This stiffness can be based on the secant stiffness to a point at or beyond yield or, if yielding is not expected, to a point before yield occurs.

Options (a) and (b) use values that approximate the stiffness for reinforced concrete building systems loaded to near or beyond the yield level, and have been shown to produce reasonable correlation with both experimental and detailed analytical results (Moehle 1992; Lepage 1998). For earthquake-induced loading, the use of option (a) or (b) may require a deflection amplification factor to account for inelastic deformations. In general, for effective section properties, E_c may be calculated or specified in accordance with 19.2.2, the shear modulus maybe taken as $0.4E_c$, and cross-sectional areas may be taken as given in Table 6.6.3.1.1(a).

Analysis of buildings with two-way slab systems without beams requires that the model represents the transfer of lateral loads between vertical members. The model should result in prediction of stiffness in substantial agreement with results of comprehensive tests and analysis. Several acceptable models have been proposed to accomplish this objective (Dovich and Wight 2005; Hwang and Moehle 2000; Vanderbilt and Corley 1983).

6.6.3.2 *Service load analysis*

6.6.3.2.1 Immediate and time-dependent deflections due to gravity loads shall be calculated in accordance with 24.2.

6.6.3.2.2 It shall be permitted to calculate immediate lateral deflections using a moment of inertia of 1.4 times I defined in 6.6.3.1.2, or using a more detailed analysis, but the value shall not exceed I_g.

R6.6.3.2 *Service load analysis*

R6.6.3.2.2 Analyses of deflections, vibrations, and building periods are needed at various service (unfactored) load levels (Grossman 1987, 1990) to determine the performance of the structure in service. The moments of inertia of the structural members in the service load analyses should be representative of the degree of cracking at the various service load levels investigated. Unless a more accurate estimate of the degree of cracking at service load level is available, it is satisfactory to use 1.0/0.70 = 1.4 times the moments of inertia provided in 6.6.3.1, not to exceed I_g, for service load analyses. Serviceability considerations for vibrations are discussed in R24.1.

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6.6.4 *Slenderness effects, moment magnification method*

6.6.4.1 Unless 6.2.5.1 is satisfied, columns and stories in structures shall be designated as being nonsway or sway. Analysis of columns in nonsway frames or stories shall be in accordance with 6.6.4.5. Analysis of columns in sway frames or stories shall be in accordance with 6.6.4.6.

6.6.4.2 The cross-sectional dimensions of each member used in an analysis shall be within 10% of the specified member dimensions in construction documents or the analysis shall be repeated. If the stiffnesses of Table 6.6.3.1.1(b) are used in an analysis, the assumed member reinforcement ratio shall also be within 10% of the specified member reinforcement in construction documents.

6.6.4.3 It shall be permitted to analyze columns and stories in structures as nonsway frames if (a) or (b) is satisfied:

(a) The increase in column end moments due to second-order effects does not exceed 5% of the first-order end moments

(b) Q in accordance with 6.6.4.4.1 does not exceed 0.05

COMMENTARY

R6.6.4 *Slenderness effects, moment magnification method*

R6.6.4.1 This section describes an approximate design procedure that uses the moment magnifier concept to account for slenderness effects. Moments calculated using a first-order frame analysis are multiplied by a moment magnifier that is a function of the factored axial load P_u and the critical buckling load P_c for the column. For the sway case, the moment magnifier is a function of the sum of P_u of the story and the sum of P_c of the sway-resisting columns in the story considered. Nonsway and sway frames are treated separately. A first-order frame analysis is an elastic analysis that excludes the internal force effects resulting from deflections.

The moment magnifier design method requires the designer to distinguish between nonsway frames, which are designed according to 6.6.4.5, and sway frames, which are designed according to 6.6.4.6. Frequently this can be done by comparing the total lateral stiffness of the columns in a story to that of the bracing elements. A compression member, such as a column, wall, or brace, may be assumed nonsway if it is located in a story in which the bracing elements (structural walls, shear trusses, or other types of lateral bracing) have such substantial lateral stiffness to resist the lateral deflections of the story that any resulting lateral deflection is not large enough to affect the column strength substantially. If not readily apparent without calculations, 6.6.4.3 provides two possible ways of determining if sway can be neglected.

R6.6.4.3 In 6.6.4.3(a), a story in a frame is classified as nonsway if the increase in the lateral load moments resulting from $P\Delta$ effects does not exceed 5% of the first-order moments (MacGregor and Hage 1977). Section 6.6.4.3(b) provides an alternative method of determining if a frame is classified as nonsway based on the stability index for a story, Q. In calculating Q, $\sum P_u$ should correspond to the lateral loading case for which $\sum P_u$ is greatest. A frame may contain both nonsway and sway stories.

If the lateral load deflections of the frame are calculated using service loads and the service load moments of inertia given in 6.6.3.2.2, it is permissible to calculate Q in Eq. (6.6.4.4.1) using 1.2 times the sum of the service gravity loads, the service load story shear, and 1.4 times the first-order service load story deflections.

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6.6.4.4 *Stability properties*

6.6.4.4.1 The stability index for a story, Q, shall be calculated by:

$$Q = \frac{\sum P_u \Delta_o}{V_{us}\ell_c} \tag{6.6.4.4.1}$$

where $\sum P_u$ and V_{us} are the total factored vertical load and horizontal story shear, respectively, in the story being evaluated, and Δ_o is the first-order relative lateral deflection between the top and the bottom of that story due to V_{us}.

6.6.4.4.2 The critical buckling load P_c shall be calculated by:

$$P_c = \frac{\pi^2 (EI)_{eff}}{(k\ell_u)^2} \tag{6.6.4.4.2}$$

6.6.4.4.3 The effective length factor k shall be calculated using E_c in accordance with 19.2.2 and I in accordance with 6.6.3.1.1. For nonsway members, k shall be permitted to be taken as 1.0, and for sway members, k shall be at least 1.0.

6.6.4.4.4 For columns, $(EI)_{eff}$ shall be calculated in accordance with (a), (b), or (c):

(a) $$(EI)_{eff} = \frac{0.4E_cI_g}{1+\beta_{dns}} \tag{6.6.4.4.4a}$$

(b) $$(EI)_{eff} = \frac{(0.2E_cI_g + E_sI_{se})}{1+\beta_{dns}} \tag{6.6.4.4.4b}$$

(c) $$(EI)_{eff} = \frac{E_cI}{1+\beta_{dns}} \tag{6.6.4.4.4c}$$

where β_{dns} shall be the ratio of maximum factored sustained axial load to maximum factored axial load associated with the same load combination and I in Eq. (6.6.4.4.4c) is calculated according to Table 6.6.3.1.1(b) for columns and walls.

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R6.6.4.4 *Stability properties*

R6.6.4.4.2 In calculating the critical axial buckling load, the primary concern is the choice of a stiffness $(EI)_{eff}$ that reasonably approximates the variations in stiffness due to cracking, creep, and nonlinearity of the concrete stress-strain curve. Section 6.6.4.4.4 may be used to calculate $(EI)_{eff}$.

R6.6.4.4.3 The effective length factor for a compression member, such as a column, wall, or brace, considering braced behavior, ranges from 0.5 to 1.0. It is recommended that a k value of 1.0 be used. If lower values are used, the calculation of k should be based on analysis of the frame using I values given in 6.6.3.1.1. The Jackson and Moreland Alignment Charts (Fig. R6.2.5.1) can be used to estimate appropriate values of k (ACI SP-17(09); Column Research Council 1966).

R6.6.4.4.4 The numerators of Eq. (6.6.4.4.4a) to (6.6.4.4.4c) represent the short-term column stiffness. Equation (6.6.4.4.4b) was derived for small eccentricity ratios and high levels of axial load. Equation (6.6.4.4.4a) is a simplified approximation to Eq. (6.6.4.4.4b) and is less accurate (Mirza 1990). For improved accuracy, $(EI)_{eff}$ can be approximated using Eq. (6.6.4.4.4c).

Creep due to sustained loads will increase the lateral deflections of a column and, hence, the moment magnification. Creep effects are approximated in design by reducing the stiffness $(EI)_{eff}$ used to calculate P_c and, hence, δ, by dividing the short-term EI provided by the numerator of Eq. (6.6.4.4.4a) through (6.6.4.4.4c) by $(1 + \beta_{dns})$. For simplification, it can be assumed that $\beta_{dns} = 0.6$. In this case, Eq. (6.6.4.4.4a) becomes $(EI)_{eff} = 0.25E_cI_g$.

In reinforced concrete columns subject to sustained loads, creep transfers some of the load from the concrete to the longitudinal reinforcement, increasing the reinforcement stresses. In the case of lightly reinforced columns, this load transfer may cause the compression reinforcement to yield prematurely, resulting in a loss in the effective EI. Accordingly, both the concrete and longitudinal reinforcement terms in Eq. (6.6.4.4.4b) are reduced to account for creep.

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6.6.4.5 *Moment magnification method: Nonsway frames*

6.6.4.5.1 The factored moment used for design of columns and walls, M_c, shall be the first-order factored moment M_2 amplified for the effects of member curvature.

$$M_c = \delta M_2 \qquad (6.6.4.5.1)$$

6.6.4.5.2 Magnification factor δ shall be calculated by:

$$\delta = \frac{C_m}{1-\dfrac{P_u}{0.75P_c}} \geq 1.0 \qquad (6.6.4.5.2)$$

6.6.4.5.3 C_m shall be in accordance with (a) or (b):

(a) For columns without transverse loads applied between supports

$$C_m = 0.6 - 0.4\frac{M_1}{M_2} \qquad (6.6.4.5.3a)$$

where M_1/M_2 is negative if the column is bent in single curvature, and positive if bent in double curvature. M_1 corresponds to the end moment with the lesser absolute value.

(b) For columns with transverse loads applied between supports.

$$C_m = 1.0 \qquad (6.6.4.5.3b)$$

6.6.4.5.4 M_2 in Eq. (6.6.4.5.1) shall be at least $M_{2,min}$ calculated according to Eq. (6.6.4.5.4) about each axis separately.

$$M_{2,min} = P_u(0.6 + 0.03h) \qquad (6.6.4.5.4)$$

If $M_{2,min}$ exceeds M_2, C_m shall be taken equal to 1.0 or calculated based on the ratio of the calculated end moments M_1/M_2, using Eq. (6.6.4.5.3a).

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R6.6.4.5 *Moment magnification method: Nonsway frames*

R6.6.4.5.2 The 0.75 factor in Eq. (6.6.4.5.2) is the stiffness reduction factor ϕ_K, which is based on the probability of understrength of a single isolated slender column. Studies reported in Mirza et al. (1987) indicate that the stiffness reduction factor ϕ_K and the cross-sectional strength reduction ϕ factors do not have the same values. These studies suggest the stiffness reduction factor ϕ_K for an isolated column should be 0.75 for both tied and spiral columns. In the case of a multistory frame, the column and frame deflections depend on the average concrete strength, which is higher than the strength of the concrete in the critical single understrength column. For this reason, the value of ϕ_K implicit in I values in 6.6.3.1.1 is 0.875.

R6.6.4.5.3 The factor C_m is a correction factor relating the actual moment diagram to an equivalent uniform moment diagram. The derivation of the moment magnifier assumes that the maximum moment is at or near midheight of the column. If the maximum moment occurs at one end of the column, design should be based on an equivalent uniform moment C_mM_2 that leads to the same maximum moment at or near midheight of the column when magnified (MacGregor et al. 1970).

The sign convention for M_1/M_2 has been updated to follow the right hand rule convention; hence, M_1/M_2 is negative if bent in single curvature and positive if bent in double curvature. This reflects a sign convention change from the 2011 Code.

In the case of columns that are subjected to transverse loading between supports, it is possible that the maximum moment will occur at a section away from the end of the member. If this occurs, the value of the largest calculated moment occurring anywhere along the member should be used for the value of M_2 in Eq. (6.6.4.5.1). C_m is to be taken as 1.0 for this case.

R6.6.4.5.4 In the Code, slenderness is accounted for by magnifying the column end moments. If the factored column moments are small or zero, the design of slender columns should be based on the minimum eccentricity provided in Eq. (6.6.4.5.4). It is not intended that the minimum eccentricity be applied about both axes simultaneously.

The factored column end moments from the structural analysis are used in Eq. (6.6.4.5.3a) in determining the ratio M_1/M_2 for the column when the design is based on the minimum eccentricity. This eliminates what would

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6.6.4.6 *Moment magnification method: Sway frames*

6.6.4.6.1 Moments M_1 and M_2 at the ends of an individual column shall be calculated by (a) and (b).

(a) $M_1 = M_{1ns} + \delta_s M_{1s}$ (6.6.4.6.1a)

(b) $M_2 = M_{2ns} + \delta_s M_{2s}$ (6.6.4.6.1b)

6.6.4.6.2 The moment magnifier δ_s shall be calculated by (a), (b), or (c). If δ_s exceeds 1.5, only (b) or (c) shall be permitted:

(a) $\delta_s = \dfrac{1}{1-Q} \geq 1$ (6.6.4.6.2a)

(b) $\delta_s = \dfrac{1}{1-\dfrac{\sum P_u}{0.75\sum P_c}} \geq 1$ (6.6.4.6.2b)

(c) Second-order elastic analysis

where $\sum P_u$ is the summation of all the factored vertical loads in a story and $\sum P_c$ is the summation for all sway-resisting columns in a story. P_c is calculated using Eq. (6.6.4.4.2) with k determined for sway members from 6.6.4.4.3 and $(EI)_{eff}$ from 6.6.4.4.4 with β_{ds} substituted for β_{dns}.

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otherwise be a discontinuity between columns with calculated eccentricities less than the minimum eccentricity and columns with calculated eccentricities equal to or greater than the minimum eccentricity.

R6.6.4.6 *Moment magnification method: Sway frames*

R6.6.4.6.1 The analysis described in this section deals only with plane frames subjected to loads causing deflections in that plane. If the lateral load deflections involve significant torsional displacement, the moment magnification in the columns farthest from the center of twist may be underestimated by the moment magnifier procedure. In such cases, a three-dimensional second-order analysis should be used.

R6.6.4.6.2 Three different methods are allowed for calculating the moment magnifier. These approaches include the Q method, the sum of P concept, and second-order elastic analysis.

(a) Q method:
The iterative $P\Delta$ analysis for second-order moments can be represented by an infinite series. The solution of this series is given by Eq. (6.6.4.6.2a) (MacGregor and Hage 1977). Lai and MacGregor (1983) show that Eq. (6.6.4.6.2a) closely predicts the second-order moments in a sway frame until δ_s exceeds 1.5.
The $P\Delta$ moment diagrams for deflected columns are curved, with Δ related to the deflected shape of the columns. Equation (6.6.4.6.2a) and most commercially available second-order frame analyses have been derived assuming that the $P\Delta$ moments result from equal and opposite forces of $P\Delta/\ell_c$ applied at the bottom and top of the story. These forces give a straight-line $P\Delta$ moment diagram. The curved $P\Delta$ moment diagrams lead to lateral displacements on the order of 15% larger than those from the straight-line $P\Delta$ moment diagrams. This effect can be included in Eq. (6.6.4.6.2a) by writing the denominator as $(1 - 1.15Q)$ rather than $(1 - Q)$. The 1.15 factor has been omitted from Eq. (6.6.4.6.2a) for simplicity.
If deflections have been calculated using service loads, Q in Eq. (6.6.4.6.2a) should be calculated in the manner explained in R6.6.4.3.
The Q factor analysis is based on deflections calculated using the I values from 6.6.3.1.1, which include the equivalent of a stiffness reduction factor ϕ_K. These I values lead to a 20 to 25% over-estimation of the lateral deflections that corresponds to a stiffness reduction factor ϕ_K between 0.80 and 0.85 on the $P\Delta$ moments. As a result, no additional ϕ factor is needed. Once the moments are established using Eq. (6.6.4.6.2a), selection of the cross sections of the columns involves the strength reduction factors ϕ from 21.2.2.
(b) Sum of P concept:

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To check the effects of story stability, δ_s is calculated as an averaged value for the entire story based on use of $\sum P_u/\sum P_c$. This reflects the interaction of all sway-resisting columns in the story on the $P\Delta$ effects because the lateral deflection of all columns in the story should be equal in the absence of torsional displacements about a vertical axis. In addition, it is possible that a particularly slender individual column in a sway frame could have substantial midheight deflections, even if adequately braced against lateral end deflections by other columns in the story. Such a column is checked using 6.6.4.6.4.

The 0.75 in the denominator of Eq. (6.6.4.6.2b) is a stiffness reduction factor ϕ, as explained in R6.6.4.5.2.

In the calculation of $(EI)_{eff}$, β_{ds} will normally be zero for a sway frame because the lateral loads are generally of short duration. Sway deflections due to short-term loads, such as wind or earthquake, are a function of the short-term stiffness of the columns following a period of sustained gravity load.

For this case, the definition of β_{ds} in 6.6.3.1.1 gives β_{ds} = 0. In the unusual case of a sway frame where the lateral loads are sustained, β_{ds} will not be zero. This might occur if a building on a sloping site is subjected to earth pressure on one side but not on the other.

6.6.4.6.3 Flexural members shall be designed for the total magnified end moments of the columns at the joint.

R6.6.4.6.3 The strength of a sway frame is governed by stability of the columns and the degree of end restraint provided by the beams in the frame. If plastic hinges form in the restraining beam, as the structure approaches a failure mechanism, its axial strength is drastically reduced. This section requires the restraining flexural members to have enough strength to resist the total magnified column end moments at the joint.

6.6.4.6.4 Second-order effects shall be considered along the length of columns in sway frames. It shall be permitted to account for these effects using 6.6.4.5, where C_m is calculated using M_1 and M_2 from 6.6.4.6.1.

R6.6.4.6.4 The maximum moment in a compression member, such as a column, wall, or brace, may occur between its ends. While second-order computer analysis programs may be used to evaluate magnification of the end moments, magnification between the ends may not be accounted for unless the member is subdivided along its length. The magnification may be evaluated using the procedure outlined in 6.6.4.5.

6.6.5 *Redistribution of moments in continuous flexural members*

R6.6.5 *Redistribution of moments in continuous flexural members*

Redistribution of moments is dependent on adequate ductility in plastic hinge regions. These plastic hinge regions develop at sections of maximum positive or negative moment and cause a shift in the elastic moment diagram. The usual result is a reduction in the values of maximum negative moments in the support regions and an increase in the values of positive moments between supports from those calculated by elastic analysis. However, because negative moments are typically determined for one loading arrange-

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ment and positive moments for another (6.4.3 provides an exception for certain loading conditions), economies in reinforcement can sometimes be realized by reducing maximum elastic positive moments and increasing negative moments, thus narrowing the envelope of maximum negative and positive moments at any section in the span (Bondy 2003). Plastic hinges permit utilization of the full capacity of more cross sections of a flexural member at ultimate loads.

The Code permissible redistribution is shown in Fig. R6.6.5. Using conservative values of limiting concrete strains and lengths of plastic hinges derived from extensive tests, flexural members with small rotation capacities were analyzed for redistribution of moments up to 20%, depending on the reinforcement ratio. As shown, the permissible redistribution percentages are conservative relative to the calculated percentages available for both f_y = 60 ksi and 80 ksi. Studies by Cohn (1965) and Mattock (1959) support this conclusion and indicate that cracking and deflection of beams designed for redistribution of moments are not significantly greater at service loads than for beams designed by the distribution of moments according to elastic theory. Also, these studies indicate that adequate rotational capacity for the redistribution of moments allowed by the Code is available if the members satisfy 6.6.5.1. The provisions for redistribution of moments apply equally to prestressed members (Mast 1992). The elastic deformations caused by a nonconcordant tendon change the amount of inelastic rotation required to obtain a given amount of redistribution of moments. Conversely, for a beam with a given inelastic rotational capacity, the amount by which the moment at the support may be varied is changed by an amount equal to the secondary moment at the support due to prestressing. Thus, the Code requires that secondary moments caused by reactions generated by prestressing forces be included in determining design moments. Redistribution of moments as permitted by 6.6.5 is not appropriate where approximate values of bending moments are used, such as provided by the simplified method of 6.5. Redistribution of moments is also not appropriate for two-way slab systems that are analyzed using the pattern loadings given in 6.4.3.3. These loadings use only 75% of the full factored live load, which is based on considerations of moment redistribution.

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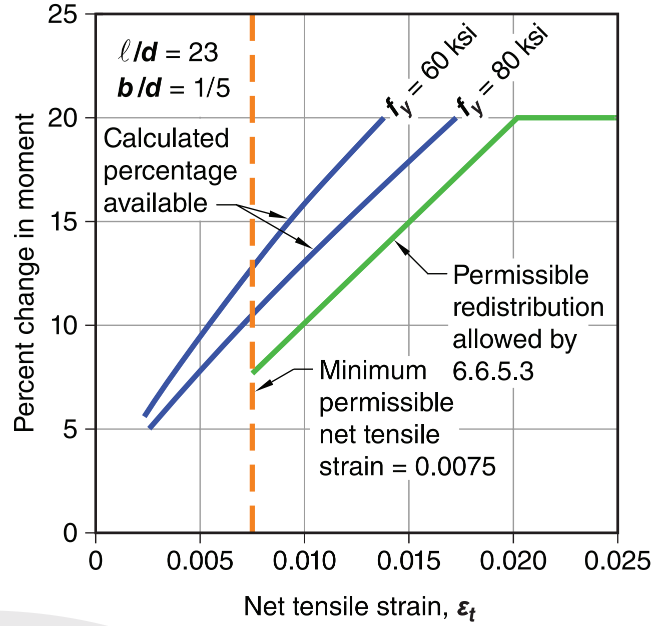

Fig. R6.6.5—Permissible redistribution of moments for minimum rotation capacity.

6.6.5.1 Except where approximate values for moments are used in accordance with 6.5, where moments have been calculated in accordance with 6.8, or where moments in two-way slabs are determined using pattern loading specified in 6.4.3.3, reduction of moments at sections of maximum negative or maximum positive moment calculated by elastic theory shall be permitted for any assumed loading arrangement if (a) and (b) are satisfied:

(a) Flexural members are continuous
(b) $\varepsilon_t \geq 0.0075$ at the section at which moment is reduced

6.6.5.2 For prestressed members, moments include those due to factored loads and those due to reactions induced by prestressing.

6.6.5.3 At the section where the moment is reduced, redistribution shall not exceed the lesser of $1000\varepsilon_t$ percent and 20%.

6.6.5.4 The reduced moment shall be used to calculate redistributed moments at all other sections within the spans such that static equilibrium is maintained after redistribution of moments for each loading arrangement.

6.6.5.5 Shears and support reactions shall be calculated in accordance with static equilibrium considering the redistributed moments for each loading arrangement

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6.7—Linear elastic second-order analysis

6.7.1 *General*

6.7.1.1 A linear elastic second-order analysis shall consider the influence of axial loads, presence of cracked regions along the length of the member, and effects of load duration. These considerations are satisfied using the cross-sectional properties defined in 6.7.2.

6.7.1.2 Slenderness effects along the length of a column shall be considered. It shall be permitted to calculate these effects using 6.6.4.5

6.7.1.3 The cross-sectional dimensions of each member used in an analysis to calculate slenderness effects shall be within 10% of the specified member dimensions in construction documents or the analysis shall be repeated.

6.7.1.4 Redistribution of moments calculated by an elastic second-order analysis shall be permitted in accordance with 6.6.5.

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R6.7—Linear elastic second-order analysis

R6.7.1 *General*

In linear elastic second-order analyses, the deformed geometry of the structure is included in the equations of equilibrium so that $P\Delta$ effects are determined. The structure is assumed to remain elastic, but the effects of cracking and creep are considered by using an effective stiffness *EI*. In contrast, linear elastic first-order analysis satisfies the equations of equilibrium using the original undeformed geometry of the structure and estimates $P\Delta$ effects by magnifying the column-end sway moments using Eq. (6.6.4.6.2a) or (6.6.4.6.2b).

R6.7.1.1 The stiffnesses *EI* used in an analysis for strength design should represent the stiffnesses of the members immediately prior to failure. This is particularly true for a second-order analysis that should predict the lateral deflections at loads approaching ultimate. The *EI* values should not be based solely on the moment-curvature relationship for the most highly loaded section along the length of each member. Instead, they should correspond to the moment-end rotation relationship for a complete member.

To allow for variability in the actual member properties in the analysis, the member properties used in analysis should be multiplied by a stiffness reduction factor ϕ_K less than 1. The cross-sectional properties defined in 6.7.2 already include this stiffness reduction factor. The stiffness reduction factor ϕ_K may be taken as 0.875. Note that the overall stiffness is further reduced considering that the modulus of elasticity of the concrete, E_c, is based on the specified concrete compressive strength, while the sway deflections are a function of the average concrete strength, which is typically higher.

R6.7.1.2 The maximum moment in a compression member may occur between its ends. In computer analysis programs, columns may be subdivided using nodes along their length to evaluate slenderness effects between the ends. If the column is not subdivided along its length, slenderness effects may be evaluated using the nonsway moment magnifier method specified in 6.6.4.5 with member-end moments from the second-order elastic analysis as input. Second-order analysis already accounts for the relative displacement of member ends.

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6.7.2 *Section properties*

6.7.2.1 *Factored load analysis*

6.7.2.1.1 It shall be permitted to use section properties calculated in accordance with 6.6.3.1.

6.7.2.2 *Service load analysis*

6.7.2.2.1 Immediate and time-dependent deflections due to gravity loads shall be calculated in accordance with 24.2.

6.7.2.2.2 Alternatively, it shall be permitted to calculate immediate deflections using a moment of inertia of 1.4 times I given in 6.6.3.1, or calculated using a more detailed analysis, but the value shall not exceed I_g.

6.8—Inelastic analysis

6.8.1 *General*

6.8.1.1 An inelastic analysis shall consider material nonlinearity. An inelastic first-order analysis shall satisfy equilibrium in the undeformed configuration. An inelastic second-order analysis shall satisfy equilibrium in the deformed configuration.

6.8.1.2 An inelastic analysis procedure shall have been shown to result in calculation of strength and deformations that are in substantial agreement with results of physical tests of reinforced concrete components, subassemblages, or structural systems exhibiting response mechanisms consistent with those expected in the structure being designed.

6.8.1.3 Unless slenderness effects are permitted to be neglected in accordance with 6.2.5.1, an inelastic analysis shall satisfy equilibrium in the deformed configuration. It shall be permitted to calculate slenderness effects along the length of a column using 6.6.4.5.

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R6.7.2 *Section properties*

R6.7.2.2 *Service load analysis*

R6.7.2.2.2 Refer to R6.6.3.2.2.

R6.8—Inelastic analysis

R6.8.1 *General*

R6.8.1.1 Material nonlinearity may be affected by multiple factors including duration of loads, shrinkage, and creep.

R6.8.1.2 Substantial agreement should be demonstrated at characteristic points on the reported response. The characteristic points selected should depend on the purpose of the analysis, the applied loads, and the response phenomena exhibited by the component, subassemblage, or structural system. For nonlinear analysis to support design under service-level loading, characteristic points should represent loads and deformations less than those corresponding to yielding of reinforcement. For nonlinear analysis to support design or assess response under design-level loading, characteristic points should represent loads and deformations less than those corresponding to yielding of reinforcement as well as points corresponding to yielding of reinforcement and onset of strength loss. Strength loss need not be represented if design loading does not extend the response into the strength-loss range. Typically, inelastic analysis to support design should employ specified material strengths and mean values of other material properties and component stiffnesses. Nonlinear response history analysis to verify the design of earthquake-resistant concrete structures should employ expected material strengths, expected material properties, and expected component stiffnesses, as specified in A.6.2.

R6.8.1.3 Refer to R6.7.1.2.

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6.8.1.4 The cross-sectional dimensions of each member used in an analysis to calculate slenderness effects shall be within 10% of the specified member dimensions in construction documents or the analysis shall be repeated.

6.8.1.5 Redistribution of moments calculated by an inelastic analysis shall not be permitted.

6.9—Acceptability of finite element analysis

6.9.1 Finite element analysis to determine load effects shall be permitted.

6.9.2 The finite element model shall be appropriate for its intended purpose.

6.9.3 For inelastic analysis, a separate analysis shall be performed for each factored load combination.

6.9.4 The licensed design professional shall confirm that the results are appropriate for the purposes of the analysis.

6.9.5 The cross-sectional dimensions of each member used in an analysis shall be within 10% of the specified member dimensions in construction documents or the analysis shall be repeated.

6.9.6 Redistribution of moments calculated by an inelastic analysis shall not be permitted.

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R6.8.1.5 Section 6.6.5 allows for redistribution of moments calculated using elastic analysis to account for inelastic response of the system. Moments calculated by inelastic analysis explicitly account for inelastic response; therefore, further redistribution of moments is not appropriate.

R6.9—Acceptability of finite element analysis

R6.9.2 The element types used should be capable of representing the response required. The model mesh size selected should be capable of determining the structural response in sufficient detail. The use of any set of reasonable assumptions for member stiffness is allowed.

R6.9.3 For an inelastic finite element analysis, the rules of linear superposition do not apply. To determine the ultimate member inelastic response, for example, it is not correct to analyze for service loads and subsequently combine the results linearly using load factors. A separate inelastic analysis should be performed for each factored load combination.

Notes