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$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right)^2} \leq \phi\left(\frac{V_c}{b_w d} + 8\sqrt{f_c'}\right) \quad (22.7.7.1a)$$

(b) For hollow sections

$$\left(\frac{V_u}{b_w d}\right) + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right) \leq \phi\left(\frac{V_c}{b_w d} + 8\sqrt{f_c'}\right) \quad (22.7.7.1b)$$

22.7.7.1.1 For prestressed members, the value of d used in 22.7.7.1 need not be taken less than $0.8h$.

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hand side are the shear stresses due to shear and torsion. The sum of these stresses may not exceed the stress causing shear cracking plus $8\sqrt{f_c'}$, similar to the limiting strength given in 22.5.1.2 for shear without torsion. The limit is expressed in terms of V_c to allow its use for nonprestressed or prestressed concrete. It was originally derived on the basis of crack control. It is not necessary to check against crushing of the web because crushing occurs at higher shear stresses.

In a hollow section, the shear stresses due to shear and torsion both occur in the walls of the box as shown in Fig. R22.7.7.1(a) and hence are directly additive at Point A as given in Eq. (22.7.7.1b). In a solid section, the shear stresses due to torsion act in the tubular outside section while the shear stresses due to V_u are spread across the width of the section, as shown in Fig. R22.7.7.1(b). For this reason, stresses are combined in Eq. (22.7.7.1a) using the square root of the sum of the squares rather than by direct addition.

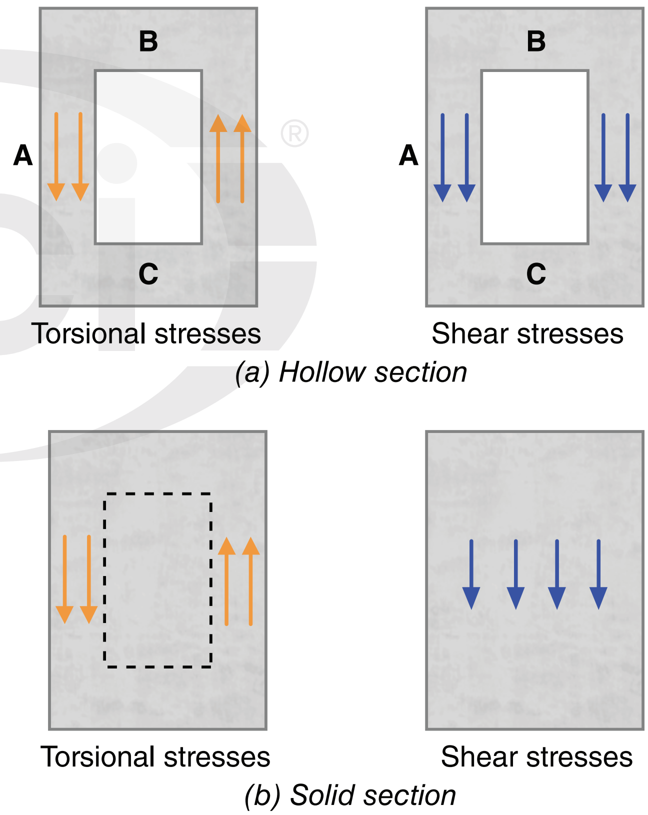

Fig. R22.7.7.1—Addition of torsional and shear stresses.

R22.7.7.1.1 Although the value of d may vary along the span of a prestressed beam, studies (MacGregor and Hanson 1969) have shown that, for prestressed concrete members, d need not be taken less than $0.8h$. The beams considered had some straight prestressed reinforcement or reinforcing bars at the bottom of the section and had stirrups that enclosed the longitudinal reinforcement.

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22.7.7.1.2 For hollow sections where the wall thickness varies around the perimeter, Eq. (22.7.7.1b) shall be evaluated at the location where the term

$$\left(\frac{V_u}{b_w d}\right) + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right)$$

is a maximum.

R22.7.7.1.2 Generally, the maximum torsional stress will be on the wall where the torsional and shearing stresses are additive (Point A in Fig. R22.7.7.1(a)). If the top or bottom flanges are thinner than the vertical webs, it may be necessary to evaluate Eq. (22.7.7.1b) at Points B and C in Fig. R22.7.7.1(a). At these points, the stresses due to the shear are usually negligible

22.7.7.2 For hollow sections where the wall thickness is less than A_{oh}/p_h, the term $(T_u p_h/1.7A_{oh}^2)$ in Eq. (22.7.7.1b) shall be taken as $(T_u/1.7A_{oh}t)$, where t is the thickness of the wall of the hollow section at the location where the stresses are being checked.

22.8—Bearing

22.8.1 *General*

22.8.1.1 Section 22.8 shall apply to the calculation of bearing strength of concrete members.

22.8.1.2 Bearing strength provisions in 22.8 shall not apply to post-tensioned anchorage zones.

22.8.2 *Required strength*

22.8.2.1 Factored compressive force transferred through bearing shall be calculated in accordance with the factored load combinations defined in Chapter 5 and analysis procedures defined in Chapter 6.

22.8.3 *Design strength*

22.8.3.1 Design bearing strength shall satisfy:

$$\phi B_n \geq B_u \tag{22.8.3.1}$$

for each applicable factored load combination.

22.8.3.2 Nominal bearing strength B_n shall be calculated in accordance with Table 22.8.3.2, where A_1 is the loaded area, and A_2 is the area of the lower base of the largest frustum of a pyramid, cone, or tapered wedge contained wholly within the support and having its upper base equal to the loaded area. The sides of the pyramid, cone, or tapered wedge shall be sloped 1 vertical to 2 horizontal.

R22.8—Bearing

R22.8.1 *General*

R22.8.1.2 Because post-tensioned anchorage zones are usually designed in accordance with 25.9, the bearing strength provisions in 22.8 are not applicable.

R22.8.3 *Design strength*

R22.8.3.2 The permissible bearing stress of $0.85f_c'$ is based on tests reported in Hawkins (1968). Where the supporting area is wider than the loaded area on all sides, the surrounding concrete confines the bearing area, resulting in an increase in bearing strength. No minimum depth is given for the support, which will most likely be controlled by the punching shear requirements of 22.6.

A_1 is the loaded area but not greater than the bearing plate or bearing cross-sectional area.

Where the top of the support is sloped or stepped, advantage may still be taken of the condition that the supporting member is larger than the loaded area, provided the supporting member does not slope at too great an angle. Figure R22.8.3.2 illustrates the application of the frustum to find A_2 for a support under vertical load transfer.

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Table 22.8.3.2—Nominal bearing strength

Geometry of bearing area	B_n		
Supporting surface is wider on all sides than the loaded area	Lesser of (a) and (b)	$\sqrt{A_2/A_1}(0.85f_c'A_1)$	(a)
		$2(0.85f_c'A_1)$	(b)
Other cases	$0.85f_c'A_1$		(c)

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Adequate bearing strength needs to be provided for cases where the compression force transfer is in a direction other than normal to the bearing surface. For such cases, this section applies to the normal component and the tangential component needs to be transferred by other methods, such as anchor bolts or shear lugs.

The frustum should not be confused with the path by which a load spreads out as it progresses downward through the support. Such a load path would have steeper sides. However, the frustum described has somewhat flat side slopes to ensure that there is concrete immediately surrounding the zone of high stress at the bearing.

Where tensile forces occur in the plane of bearing, it may be desirable to reduce the allowable bearing stress, provide confinement reinforcement, or both. Guidelines are provided in the *PCI Design Handbook* for precast and prestressed concrete (PCI MNL 120).

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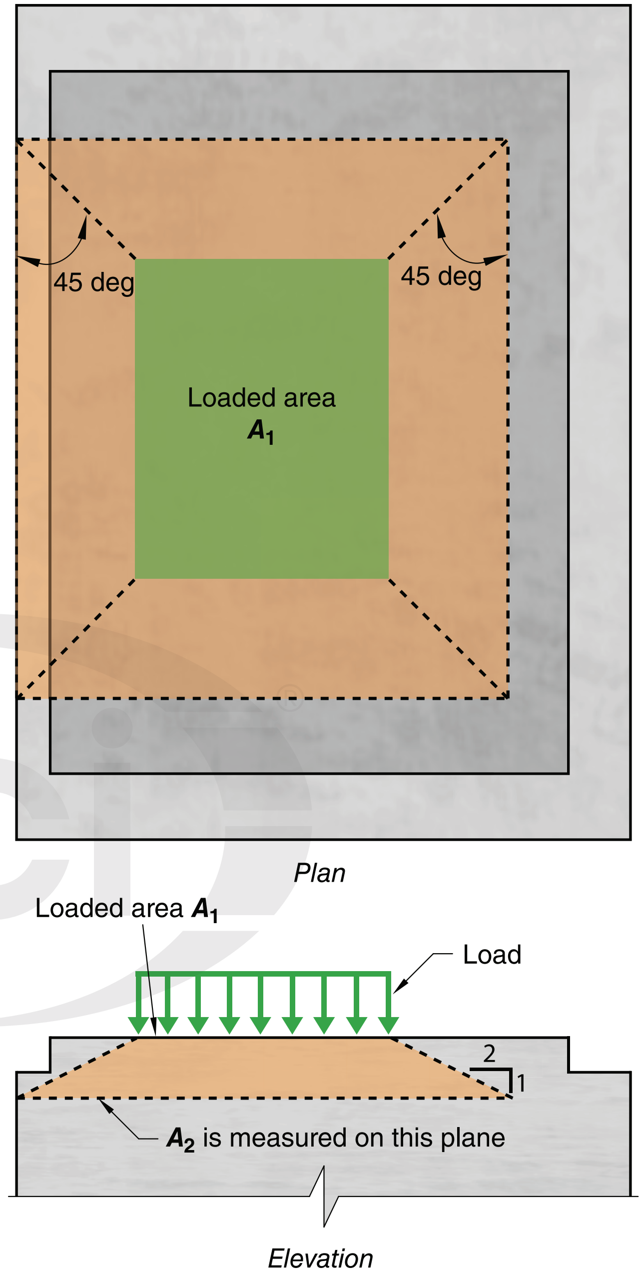

Fig. R22.8.3.2—Application of frustum to find A_2 *in stepped or sloped supports.*

22.9—Shear friction

22.9.1 *General*

22.9.1.1 This section shall apply where it is appropriate to consider shear transfer across any given plane, such as an existing or potential crack, an interface between dissimilar materials, or an interface between two concretes cast at different times.

R22.9—Shear friction

R22.9.1 *General*

R22.9.1.1 The purpose of this section is to provide a design method to address possible failure by shear sliding on a plane. Such conditions include a plane formed by a crack in monolithic concrete, an interface between concrete and steel, and an interface between concretes cast at different times (Birkeland and Birkeland 1966; Mattock and Hawkins 1972).

Although uncracked concrete is relatively strong in direct shear, there is always the possibility that a crack will form in an unfavorable location. The shear-friction concept

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assumes that such a crack will form, and that reinforcement is provided across the crack to resist relative displacement along it. When shear acts along a crack, one crack face slips relative to the other. If the crack faces are rough and irregular, this slip is accompanied by separation of the crack faces. At nominal strength, the separation is sufficient to stress, in tension, the reinforcement crossing the crack to its specified yield strength. The reinforcement in tension provides a clamping force $A_{vf}f_y$ across the crack faces. The applied shear is then resisted by friction between the crack faces, by resistance to the shearing off of protrusions on the crack faces, and by dowel action of the reinforcement crossing the crack. Successful application of this section depends on proper selection of the location of an assumed crack (PCI MNL 120; Birkeland and Birkeland 1966).

The requirements of 22.9 were developed based on monotonic testing and may be unconservative for interfaces that are part of the seismic-force-resisting system and experience strength degradation due to force and displacement reversals. Palieraki et al. (2022) provides design guidance for interfaces subject to cyclic loading that could cause sliding.

22.9.1.2 The required area of shear-friction reinforcement across the assumed shear plane, A_{vf}, shall be calculated in accordance with 22.9.4. Alternatively, it shall be permitted to use shear transfer design methods that result in prediction of strength in substantial agreement with results of comprehensive tests.

R22.9.1.2 The relationship between shear-transfer strength and the reinforcement crossing the shear plane can be expressed in various ways. Equations (22.9.4.2) and (22.9.4.3) are based on the shear-friction model and provide a conservative estimate of the shear-transfer strength.

Other relationships that provide a more accurate estimate of shear-transfer strength can be used under the requirements of this section. Examples of such procedures can be found in the *PCI Design Handbook* (PCI MNL 120), Mattock et al. (1976b), and Mattock (1974).

22.9.1.3 The value of f_y used to calculate V_n for shear friction shall not exceed the limit in 20.2.2.4.

22.9.1.4 Surface preparation of the shear plane assumed for design shall be specified in the construction documents.

R22.9.1.4 For concrete cast against hardened concrete or structural steel, 26.5.6.1 requires the licensed design professional to specify the surface preparation in the construction documents.

22.9.2 *Required strength*

22.9.2.1 Factored forces across the assumed shear plane shall be calculated in accordance with the factored load combinations defined in Chapter 5 and analysis procedures defined in Chapter 6.

22.9.3 *Design strength*

22.9.3.1 Design shear strength across the assumed shear plane shall satisfy:

$$\phi V_n \geq V_u \tag{22.9.3.1}$$

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for each applicable factored load combination.

22.9.4 *Nominal shear strength*

22.9.4.1 Value of V_n across the assumed shear plane shall be calculated in accordance with 22.9.4.2 or 22.9.4.3. V_n shall not exceed the value calculated in accordance with 22.9.4.4.

22.9.4.2 If shear-friction reinforcement is perpendicular to the shear plane, nominal shear strength across the assumed shear plane shall be calculated by:

$$V_n = \mu\left(A_{vf}f_y + N_u\right) \tag{22.9.4.2}$$

where μ is the coefficient of friction in accordance with Table 22.9.4.2, and N_u is the minimum factored compressive force acting concurrently with V_u. It shall be permitted to take N_u equal to zero even if compression across the interface is present.

Table 22.9.4.2—Coefficients of friction

Contact surface condition	Coefficient of friction μ[1]	
Concrete placed monolithically	1.4λ	(a)
Concrete placed against hardened concrete that is clean, free of laitance, and intentionally roughened to a trough-to-peak amplitude of approximately 1/4 in.[2]	1.0λ	(b)
Concrete placed against hardened concrete that is clean, free of laitance, and not intentionally roughened	0.6	(c)
Concrete placed against as-rolled structural steel that is clean, free of paint, and with shear transferred across the contact surface by headed studs or by welded deformed bars or wires.	0.7λ	(d)

[1]λ = 1.0 for normalweight concrete. For lightweight concrete, λ is calculated as given in 19.2.4, but shall not exceed 0.85.
[2]Refer to 26.5.6.2(e) for compliance requirements for intentional roughening.

22.9.4.3 If shear-friction reinforcement is inclined to the shear plane and the shear force induces tension in the shear-friction reinforcement, nominal shear strength across the assumed shear plane shall be calculated by:

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R22.9.4 *Nominal shear strength*

R22.9.4.2 The required area of shear-friction reinforcement, A_{vf}, is calculated using:

$$A_{vf} = \frac{V_u - \phi\mu N_u}{\phi f_y \mu} \tag{R22.9.4.2}$$

Only compressive normal force is considered in Eq. (22.9.4.2); 22.9.4.5 requires reinforcement across the interface to resist net factored tension. Normal force *U* is factored in accordance with the load combinations of Chapter 5. All applicable load combinations should be considered to determine the most critical design condition, recognizing that non-permanent loads should only be considered if they add to V_u and load combinations that include a 0.9 factor on dead load should be considered.

The upper limit on shear strength that can be achieved using Eq. (22.9.4.2) is given in 22.9.4.4.

In the shear-friction method of calculation, it is assumed that all the shear resistance is due to the friction between the crack faces. It is therefore necessary to use artificially high values of the coefficient of friction in the shear-friction equations so that the calculated shear strength will be in reasonable agreement with test results.

For concrete cast against hardened concrete not roughened in accordance with 22.9.4.2, shear resistance is primarily due to dowel action of the reinforcement. Test results (Mattock 1977) indicate that the reduced value of μ = 0.6 specified for this case is appropriate.

Beginning with the 2025 Code, the λ factor was removed from (c) of Table 22.9.4.2 based on research by Krc et al. (2016), which examined the use of λ for different concrete interface conditions. For contact surfaces not intentionally roughened, the lower strength of lightweight aggregate does not reduce shear transfer strength because the interfacial crack does not propagate through the aggregate.

For concrete placed against as-rolled structural steel, the shear-transfer reinforcement may be either reinforcing bars or headed studs. The design of shear connectors for composite action of concrete slabs and steel beams is not covered by these provisions. ANSI/AISC 360 contains design provisions for these systems.

R22.9.4.3 Inclined shear-friction reinforcement is illustrated in Fig. R22.9.4.3a (Mattock 1974), where α is the acute angle between the bar and the shear plane. Equation (22.9.4.3) applies only when the shear force component parallel to the reinforcement produces tension in the rein-

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$$V_n = A_{vf}f_y(\mu\sin\alpha + \cos\alpha) + \mu N_u \qquad (22.9.4.3)$$

where α is the angle between shear-friction reinforcement and assumed shear plane, and μ and N_u are as defined in 22.9.4.2.

22.9.4.4 The value of V_n across the assumed shear plane shall not exceed the limits in Table 22.9.4.4. Where concretes of different strengths are cast against each other, the lesser value of f_c' shall be used in Table 22.9.4.4.

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forcement and the force component parallel to the shear plane resists part of the shear, as shown in Fig. R22.9.4.3a.

If the shear-friction reinforcement is inclined such that the shear force component parallel to the reinforcement produces compression in the reinforcement, as shown in Fig. R22.9.4.3b, then shear friction does not apply ($V_n = 0$).

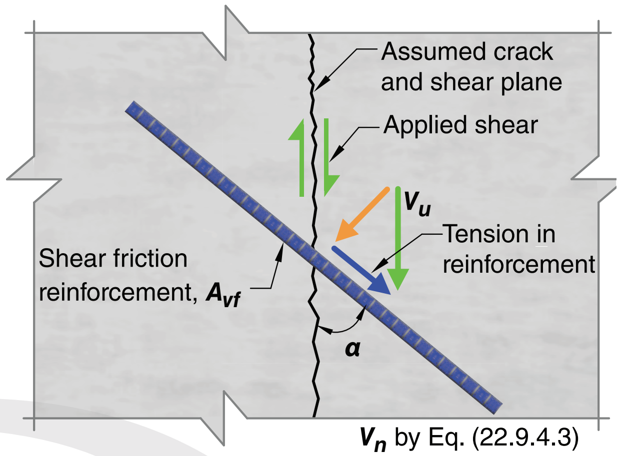

Fig. R22.9.4.3a—Tension in shear friction reinforcement.

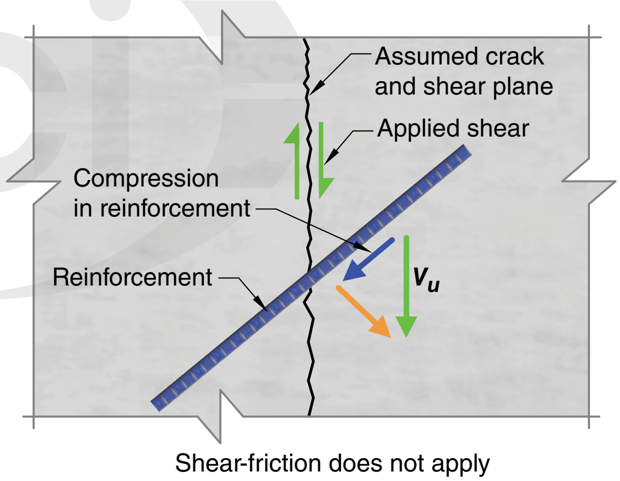

Fig. R22.9.4.3b—Compression in reinforcement.

R22.9.4.4 Upper limits on shear friction strength are necessary, as Eq. (22.9.4.2) and (22.9.4.3) may become unconservative for some cases (Kahn and Mitchell 2002; Mattock 2001).

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Table 22.9.4.4—Maximum V_n across the assumed shear plane

Condition	Maximum V_n		
Normalweight concrete placed monolithically or placed against hardened concrete that is clean, free of laitance, and intentionally roughened to a trough-to-peak amplitude of approximately 1/4 in.[1]	Least of (a), (b), and (c)	$0.2f_c'A_c$	(a)
		$(480 + 0.08f_c')A_c$	(b)
		$1600A_c$	(c)
Other cases	Lesser of (d) and (e)	$0.2f_c'A_c$	(d)
		$800A_c$	(e)

[1]Refer to 26.5.6.2(e) for compliance requirements for intentional roughening.

22.9.4.5 Area of reinforcement required to resist a net factored tension across an assumed shear plane shall be added to the area of reinforcement required for shear friction crossing the assumed shear plane.

22.9.5 *Detailing for shear-friction reinforcement*

22.9.5.1 Reinforcement crossing the shear plane to satisfy 22.9.4 shall develop f_y in tension on both sides of the shear plane.

22.9.5.2 It shall be permitted to consider inverted U-bars extending into composite topping slabs sufficient to develop f_y in tension at the interface if specified details satisfy (a) through (e):

(a) U-bar size does not exceed No. 7.
(b) Cross-leg of each U-bar extends at least $5d_b$ above the interface, where d_b is the diameter of the U-bar leg.
(c) Upper corners of each U-bar enclose a bar or strand.

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R22.9.4.5 Tension across the shear plane may be caused by restraint of deformations due to temperature change, creep, and shrinkage.

Where moment acts on a shear plane, the flexural compression and tension forces are in equilibrium and do not change the resultant compression $A_{vf}f_y$ acting across the shear plane or the shear-friction resistance. It is therefore not necessary to provide additional reinforcement to resist the flexural tension stresses, unless the required flexural tension reinforcement exceeds the amount of shear-transfer reinforcement provided in the flexural tension zone (Mattock et al. 1975).

R22.9.5 *Detailing for shear-friction reinforcement*

R22.9.5.1 If no moment acts across the shear plane, reinforcement should be uniformly distributed along the shear plane to minimize crack widths. If a moment acts across the shear plane, the shear-transfer reinforcement should be placed primarily in the flexural tension zone.

Anchorage may be developed by bond, by a mechanical device, or by threaded dowels and screw inserts. Space limitations often require the use of mechanical anchorage devices. For anchorage of headed studs in concrete, refer to *PCI Design Handbook* for precast and prestressed concrete (PCI MNL 120).

The shear-friction reinforcement anchorage should engage the primary reinforcement; otherwise, a potential crack may pass between the shear-friction reinforcement and the body of the concrete. This requirement applies particularly to welded headed studs used with steel inserts.

R22.9.5.2 Detailing requirements of this provision are based on tests by Mattock (1987) and Waweru et al. (2018) and are illustrated in Fig. R22.9.5.2. Although inverted U-bars are typically used, this provision is intended to also apply to rectangular ties. The top leg of the U-bar should extend as close to the top surface as cover requirements permit.

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(d) Spacing between U-bar legs in both directions is at least $8d_b$.

(e) Below the interface, vertical legs of the U-bars develop f_y in tension in accordance with 25.4.2.

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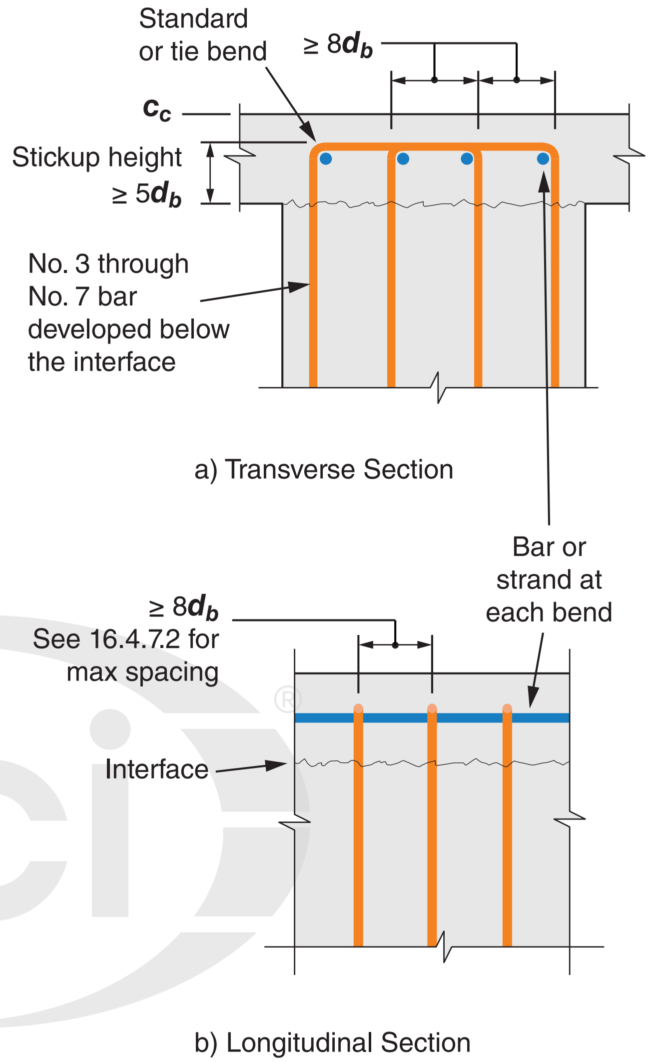

Fig. R22.9.5.2—Details for development of U-bars across a shear-friction interface.

Notes