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Table 22.6.6.1—Two-way shear strength v_c for members with shear reinforcement

Type of shear reinforcement	Critical sections	v_c		
Stirrups	All	$2\lambda_s\lambda\sqrt{f_c'}$		(a)
Headed shear stud reinforcement	According to 22.6.4.1	Least of (b), (c), and (d):	$3\lambda_s\lambda\sqrt{f_c'}$	(b)
			$\left(2+\frac{4}{\beta}\right)\lambda_s\lambda\sqrt{f_c'}$	(c)
			$\left(2+\frac{\alpha_s d}{b_o}\right)\lambda_s\lambda\sqrt{f_c'}$	(d)
	According to 22.6.4.2		$2\lambda_s\lambda\sqrt{f_c'}$	(e)

Notes: (i) λ_s is the size effect factor given in 22.5.5.1.3. (ii) β is the ratio of long to short sides of the column, concentrated load, or reaction area. (iii) α_s is given in 22.6.5.3.

22.6.6.2 It shall be permitted to take λ_s as 1.0 if (a) or (b) is satisfied:

(a) Stirrups are designed and detailed in accordance with 8.7.6 and $A_v/s \geq 2\sqrt{f_c'}b_o/f_{yt}$.
(b) Smooth headed shear stud reinforcement with stud shaft length not exceeding 10 in. is designed and detailed in accordance with 8.7.7 and $A_v/s \geq 2\sqrt{f_c'}b_o/f_{yt}$.

22.6.6.3 For members with shear reinforcement, effective depth shall be selected such that two-way shear stress v_u calculated at critical sections does not exceed the values in Table 22.6.6.3.

COMMENTARY

R22.6.6.2 The size effect in slabs with $d > 10$ in. can be mitigated if a minimum amount of shear reinforcement is provided. The ability of ordinary (smooth) headed shear stud reinforcement to effectively mitigate the size effect on the two-way shear strength of slabs may be compromised if studs longer than 10 in. are used. Until experimental evidence becomes available, it is not permitted to use λ_s equal to 1.0 for slabs with $d > 10$ in. without headed shear stud reinforcement with stud shaft length not exceeding 10 in. Stacking or "piggybacking" of headed shear studs, as shown in Fig. R22.6.6.2, introduces an intermediate head that contributes to further anchor the stacked stud.

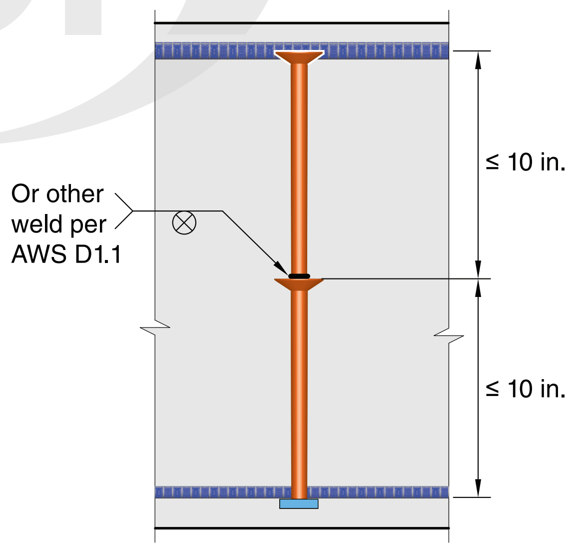

Fig. R22.6.6.2—Stacking (piggybacking) of headed shear stud reinforcement.

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Table 22.6.6.3—Maximum two-way shear stress v_u for members with shear reinforcement

Type of shear reinforcement	Maximum v_u at critical sections defined in 22.6.4.1	
Stirrups	$\phi 6\sqrt{f_c'}$	(a)
Headed shear stud reinforcement	$\phi 8\sqrt{f_c'}$	(b)

22.6.7 *Two-way shear strength provided by single- or multiple-leg stirrups*

22.6.7.1 Single- or multiple-leg stirrups fabricated from bars or wires shall be permitted to be used as shear reinforcement in slabs and footings satisfying (a) and (b):

(a) d is at least 6 in.
(b) d is at least $16d_b$, where d_b is the diameter of the stirrups

22.6.7.2 For members with stirrups, v_s shall be calculated by:

$$v_s = \frac{A_v f_{yt}}{b_o s} \tag{22.6.7.2}$$

where A_v is the sum of the area of all legs of reinforcement on one peripheral line that is geometrically similar to the perimeter of the column section, and s is the spacing of the peripheral lines of shear reinforcement in the direction perpendicular to the column face.

22.6.8 *Two-way shear strength provided by headed shear stud reinforcement*

22.6.8.1 Headed shear stud reinforcement shall be permitted to be used as shear reinforcement in slabs and footings if the placement and geometry of the headed shear stud reinforcement satisfies 8.7.7.

22.6.8.2 For members with headed shear stud reinforcement, v_s shall be calculated by:

$$v_s = \frac{A_v f_{yt}}{b_o s} \tag{22.6.8.2}$$

where A_v is the sum of the area of all shear studs on one peripheral line that is geometrically similar to the perimeter of the column section, and s is the spacing of the peripheral

COMMENTARY

R22.6.7 *Two-way shear strength provided by single- or multiple-leg stirrups*

R22.6.7.2 Because shear stresses are used for two-way shear in this chapter, shear strength provided by transverse reinforcement is averaged over the cross-sectional area of the critical section.

R22.6.8 *Two-way shear strength provided by headed shear stud reinforcement*

Tests (ACI PRC-421.1) show that headed shear stud reinforcement mechanically anchored as close as practicable to the top and bottom of slabs is effective in resisting punching shear. The critical section beyond the shear reinforcement is generally assumed to have a polygonal shape (refer to Fig. R22.6.4.2a, R22.6.4.2b, and R22.6.4.2c). Equations for calculating shear stresses on such sections are given in ACI PRC-421.1.

R22.6.8.2 Because shear stresses are used for two-way shear in this chapter, shear strength provided by transverse reinforcement is averaged over the cross-sectional area of the critical section.

CODE

lines of headed shear stud reinforcement in the direction perpendicular to the column face.

22.6.8.3 If headed shear stud reinforcement is provided, A_v/s shall satisfy:

$$\frac{A_v}{s} \geq 2\sqrt{f_c'}\frac{b_o}{f_{yt}} \qquad (22.6.8.3)$$

22.7—Torsional strength

COMMENTARY

R22.7—Torsional strength

The design for torsion in this section is based on a thin-walled tube space truss analogy. A beam subjected to torsion is idealized as a thin-walled tube with the core concrete cross section in a solid beam neglected as shown in Fig. R22.7(a). Once a reinforced concrete beam has cracked in torsion, its torsional strength is provided primarily by closed stirrups and longitudinal bars located near the surface of the member. In the thin-walled tube analogy, the strength is assumed to be provided by the outer skin of the cross section roughly centered on the closed stirrups. Both hollow and solid sections are idealized as thin-walled tubes both before and after cracking.

In a closed thin-walled tube, the product of the shear stress τ and the wall thickness t at any point in the perimeter is known as the shear flow, $q = \tau t$. The shear flow q due to torsion acts as shown in Fig. R22.7(a) and is constant at all points around the perimeter of the tube. The path along which it acts extends around the tube at midthickness of the walls of the tube. At any point along the perimeter of the tube, the shear stress due to torsion is $\tau = T/(2A_o t)$, where A_o is the gross area enclosed by the shear flow path, shown shaded in Fig. R22.7(b), and t is the thickness of the wall at the point where τ is being calculated. For a hollow member with continuous walls, A_o includes the area of the hole.

The concrete contribution to torsional strength is ignored, and in cases of combined shear and torsion, the concrete contribution to shear strength does not need to be reduced. The design procedure is derived and compared with test results in MacGregor and Ghoneim (1995) and Hsu (1997). Detailed information on the thin-walled tube space truss analogy is provided in ACI PRC-445.1.

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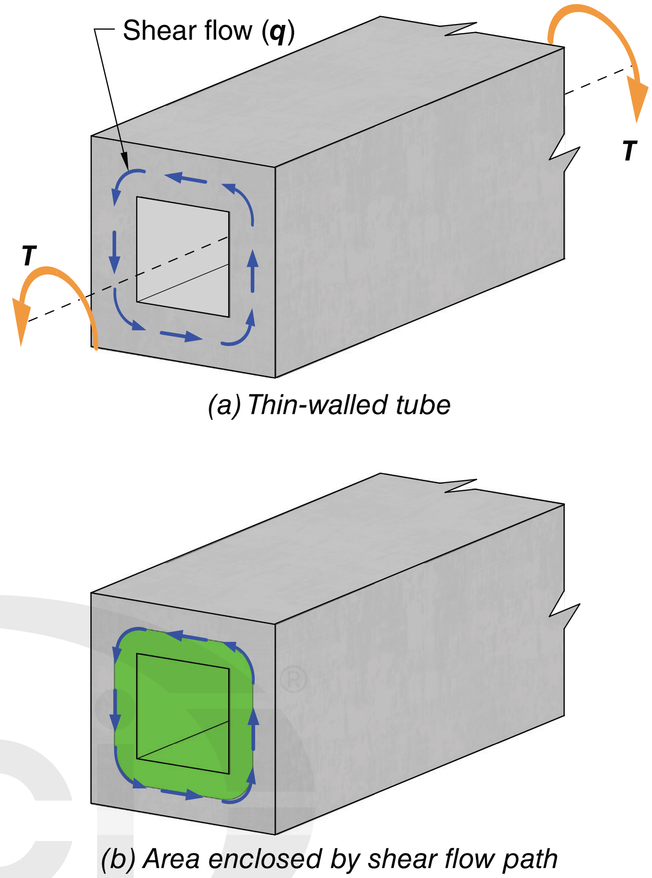

Fig. R22.7—(a) Thin-walled tube; and (b) area enclosed by shear flow path.

22.7.1 *General*

22.7.1.1 This section shall apply to members if $T_u \geq \phi T_{th}$, where ϕ is given in Chapter 21 and threshold torsion T_{th} is given in 22.7.4. If $T_u < \phi T_{th}$, it shall be permitted to neglect torsional effects.

22.7.1.2 Nominal torsional strength shall be calculated in accordance with 22.7.6.

22.7.1.3 For calculation of T_{th} and T_{cr}, λ shall be in accordance with 19.2.4.

R22.7.1 *General*

R22.7.1.1 Torsional moments that do not exceed the threshold torsion T_{th} will not cause a structurally significant reduction in either flexural or shear strength and can be ignored.

22.7.2 *Limiting material strengths*

22.7.2.1 The value of $\sqrt{f_c'}$ used to calculate T_{th} and T_{cr} shall not exceed 100 psi.

22.7.2.2 The values of f_y and f_{yt} for longitudinal and transverse torsional reinforcement shall not exceed the limits in 20.2.2.4.

R22.7.2 *Limiting material strengths*

R22.7.2.1 Because of a lack of test data and practical experience with concretes having compressive strengths greater than 10,000 psi, the Code imposes a maximum value of 100 psi on $\sqrt{f_c'}$ for use in the calculation of torsional strength.

R22.7.2.2 The upper limit of 60,000 psi on the value of f_y and f_{yt} used in design is intended to control diagonal crack width.

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22.7.3 *Factored design torsion*

COMMENTARY

R22.7.3 *Factored design torsion*

In designing for torsion in reinforced concrete structures, two conditions may be identified (Collins and Lampert 1973; Hsu and Burton 1974):

(a) The torsional moment cannot be reduced by redistribution of internal forces (22.7.3.1). This type of torsion is referred to as equilibrium torsion because the torsional moment is required for the structure to be in equilibrium. For this condition, illustrated in Fig. R22.7.3(a), torsional reinforcement must be provided to resist the total design torsional moments.

(b) The torsional moment can be reduced by redistribution of internal forces after cracking (22.7.3.2) if the torsion results from the member twisting to maintain compatibility of deformations. This type of torsion is referred to as compatibility torsion.

For this condition, illustrated in Fig. R22.7.3(b), the torsional stiffness before cracking corresponds to that of the uncracked section according to St. Venant's theory. At torsional cracking, however, a large twist occurs under an essentially constant torsional moment, resulting in a large redistribution of forces in the structure (Collins and Lampert 1973; Hsu and Burton 1974). The cracking torsional moment under combined shear, moment, and torsion corresponds to a principal tensile stress somewhat less than the $4\lambda\sqrt{f_c'}$ used in R22.7.5.

If the torsional moment exceeds the cracking torsional moment (22.7.3.2), a maximum factored torsional moment equal to the cracking torsional moment may be assumed to occur at critical sections. The maximum factored torsional moment has been established to limit the width of torsional cracks.

Provision 22.7.3.2 applies to typical and regular framing conditions. With layouts that impose significant torsional rotations within a limited length of the member, such as a large torsional moment located close to a stiff column, or a column that rotates in the reverse directions because of other loading, a more detailed analysis is advisable.

If the factored torsional moment from an elastic analysis based on uncracked section properties is between ϕT_{th} and ϕT_{cr}, torsional reinforcement should be designed to resist the calculated torsional moments.

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22.7.3.1 If $T_u \geq \phi T_{cr}$ and T_u is required to maintain equilibrium, the member shall be designed to resist T_u.

22.7.3.2 In a statically indeterminate structure where $T_u \geq \phi T_{cr}$ and a reduction of T_u can occur due to redistribution of internal forces after torsional cracking, it shall be permitted to reduce T_u to ϕT_{cr}, where the cracking torsion T_{cr} is calculated in accordance with 22.7.5.

22.7.3.3 If T_u is redistributed in accordance with 22.7.3.2, the factored moments and shears used for design of the adjoining members shall be in equilibrium with the reduced torsion.

COMMENTARY

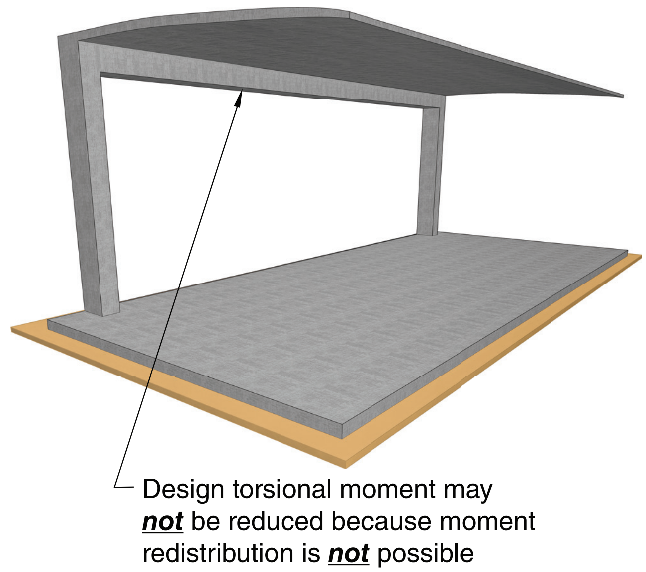

Fig. R22.7.3a—Equilibrium torsion, the design torsional moment may not be reduced (22.7.3.1).

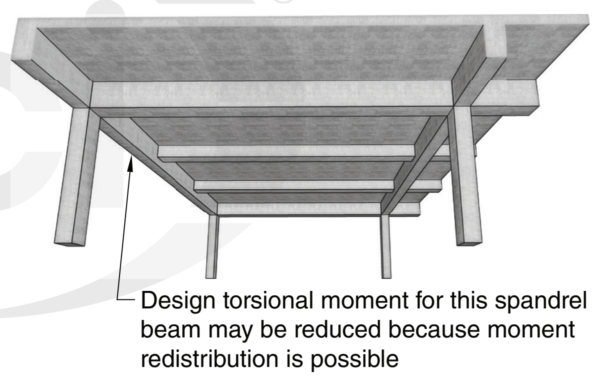

Fig. R22.7.3b—Compatibility torsion, the design torsional moment may be reduced (22.7.3.2).

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22.7.4 *Threshold torsion*

22.7.4.1 Threshold torsion T_{th} shall be calculated in accordance with Table 22.7.4.1(a) for solid cross sections and Table 22.7.4.1(b) for hollow cross sections, where N_u is positive for compression and negative for tension.

Table 22.7.4.1(a)—Threshold torsion for solid cross sections

Type of member	T_{th}	
Nonprestressed member	$\lambda\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)$	(a)
Prestressed member	$\lambda\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)\sqrt{1+\frac{f_{pc}}{4\lambda\sqrt{f_c'}}}$	(b)
Nonprestressed member subjected to axial force	$\lambda\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)\sqrt{1+\frac{N_u}{4A_g\lambda\sqrt{f_c'}}}$	(c)

COMMENTARY

R22.7.4 *Threshold torsion*

The threshold torsion is defined as one-fourth the cracking torsional moment T_{cr}. For sections of solid members, the interaction between the cracking torsional moment and the inclined cracking shear is approximately circular or elliptical. For such a relationship, a threshold torsional moment of T_{th}, as used in 22.7.4.1, corresponds to a reduction of less than 5% in the inclined cracking shear, which is considered negligible.

For torsion, a hollow section is defined as having one or more longitudinal voids, such as a single-cell or multiple-cell box girder. Small longitudinal voids, such as ungrouted post-tensioning ducts that result in $A_g/A_{cp} \geq 0.95$, can be ignored when calculating T_{th}. The interaction between torsional cracking and shear cracking for hollow sections is assumed to vary from the elliptical relationship for members with small voids, to a straight-line relationship for thin-walled sections with large voids. For a straight-line interaction, a torsional moment of T_{th} would cause a reduction in the inclined cracking shear of approximately 25%, which was considered to be significant. Therefore, the expressions for solid sections are modified by the factor $(A_g/A_{cp})^2$ to develop the expressions for hollow sections. Tests of solid and hollow beams (Hsu 1968) indicate that the cracking torsional moment of a hollow section is approximately (A_g/A_{cp}) times the cracking torsional moment of a solid section with the same outside dimensions. An additional multiplier of (A_g/A_{cp}) reflects the transition from the circular interaction between the inclined cracking loads in shear and torsion for solid members, to the approximately linear interaction for thin-walled hollow sections.

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Table 22.7.4.1(b)—Threshold torsion for hollow cross sections

Type of member	T_{th}	
Nonprestressed member	$\lambda\sqrt{f_c'}\left(\frac{A_g^2}{p_{cp}}\right)$	(a)
Prestressed member	$\lambda\sqrt{f_c'}\left(\frac{A_g^2}{p_{cp}}\right)\sqrt{1+\frac{f_{pc}}{4\lambda\sqrt{f_c'}}}$	(b)
Nonprestressed member subjected to axial force	$\lambda\sqrt{f_c'}\left(\frac{A_g^2}{p_{cp}}\right)\sqrt{1+\frac{N_u}{4A_g\lambda\sqrt{f_c'}}}$	(c)

22.7.5 *Cracking torsion*

22.7.5.1 Cracking torsion T_{cr} shall be calculated in accordance with Table 22.7.5.1 for solid and hollow cross sections, where N_u is positive for compression and negative for tension.

Table 22.7.5.1—Cracking torsion

Type of member	T_{cr}	
Nonprestressed member	$4\lambda\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)$	(a)
Prestressed member	$4\lambda\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)\sqrt{1+\frac{f_{pc}}{4\lambda\sqrt{f_c'}}}$	(b)
Nonprestressed member subjected to axial force	$4\lambda\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)\sqrt{1+\frac{N_u}{4A_g\lambda\sqrt{f_c'}}}$	(c)

22.7.6 *Nominal torsional strength*

22.7.6.1 For nonprestressed and prestressed members, T_n shall be the lesser of (a) and (b):

COMMENTARY

R22.7.5 *Cracking torsion*

R22.7.5.1 The cracking torsional moment under pure torsion, T_{cr}, is derived by replacing the actual section with an equivalent thin-walled tube with a wall thickness $t$prior to cracking of $0.75A_{cp}/p_{cp}$ and an area enclosed by the wall centerline A_o equal to $2A_{cp}/3$. Cracking is assumed to occur when the principal tensile stress reaches $4\lambda\sqrt{f_c'}$. The stress at cracking, $4\lambda\sqrt{f_c'}$, has purposely been taken as a lower-bound value. In a nonprestressed beam loaded with torsion alone, the principal tensile stress is equated to the torsional shear stress, $\tau = T/(2A_ot)$. Thus, cracking occurs when τ reaches $4\lambda\sqrt{f_c'}$, giving the cracking torsional moment T_{cr} as defined by expression (a) in Table 22.7.5.1.

For prestressed members, the torsional cracking load is increased by the prestress given by expression (b) in Table 22.7.5.1. A Mohr's Circle analysis based on average stresses indicates the torsional moment required to cause a principal tensile stress equal to $4\lambda\sqrt{f_c'}$ is $\sqrt{1+f_{pc}/\left(4\lambda\sqrt{f_c'}\right)}$times the corresponding torsional cracking moment in a nonprestressed beam. A similar modification is made in expression (c) in Table 22.7.5.1 for members subjected to axial force and torsion.

If the factored torsional moment exceeds ϕT_{cr} in a statically indeterminate structure, a maximum factored torsional moment equal to ϕT_{cr} may be assumed to occur at critical sections. This limit has been established to control the width of the torsional cracks. The replacement of A_{cp} with A_g, as in the calculation of T_{th} for hollow sections in 22.7.4.1, is not applied here. Thus, the torsional moment after redistribution is larger and, hence, more conservative.

R22.7.6 *Nominal torsional strength*

In the calculation of T_n, all the torsion is assumed to be resisted by stirrups and longitudinal reinforcement, neglecting any concrete contribution to torsional strength. At the same time, the nominal shear strength provided by concrete, V_c, is assumed to be unchanged by the presence of torsion.

R22.7.6.1 Equation (22.7.6.1a) is based on the space truss analogy shown in Fig. R22.7.6.1a with compression diago-

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(a) $T_n = \dfrac{2A_oA_tf_{yt}}{s}\cot\theta$ (22.7.6.1a)

(b) $T_n = \dfrac{2A_oA_\ell f_y}{p_h}\tan\theta$ (22.7.6.1b)

where A_o shall be determined by analysis; θ shall not be taken less than 30 degrees nor greater than 60 degrees; A_t is the area of one leg of a closed stirrup resisting torsion; A_ℓ is the area of longitudinal torsional reinforcement; and p_h is the perimeter of the centerline of the outermost closed stirrup.

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nals at an angle θ, assuming the concrete resists no tension and the reinforcement yields. After torsional cracking develops, the torsional strength is provided mainly by closed stirrups, longitudinal reinforcement, and compression diagonals. The concrete outside these stirrups is relatively ineffective. For this reason A_o, the gross area enclosed by the shear flow path around the perimeter of the tube, is defined after cracking in terms of A_{oh}, the area enclosed by the centerline of the outermost closed transverse torsional reinforcement.

The shear flow q in the walls of the tube, discussed in R22.7, can be resolved into the shear forces V_1 to V_4 acting in the individual sides of the tube or space truss, as shown in Fig. R22.7.6.1a.

As shown in Figure R22.7.6.1b, on a given wall of the tube, the shear flow V_i is resisted by a diagonal compression component, $D_i = V_i/\sin\theta$, in the concrete. An axial tension force, $N_i = V_i(\cot\theta)$, is required in the longitudinal reinforcement to complete the resolution of V_i.

Because the shear flow due to torsion is constant at all points around the perimeter of the tube, the resultants of D_i and N_i act through the midheight of side i. As a result, half of N_i can be assumed to be resisted by each of the top and bottom chords as shown. Longitudinal reinforcement with a strength $A_\ell f_y$ is required to resist the sum of the N_i forces, $\sum N_i$, acting in all of the walls of the tube.

In the derivation of Eq. (22.7.6.1b), axial tension forces are summed along the sides of the area A_o. These sides form a perimeter length p_o approximately equal to the length of the line joining the centers of the bars in the corners of the tube. For ease in calculation, this has been replaced with the perimeter of the closed stirrups, p_h.

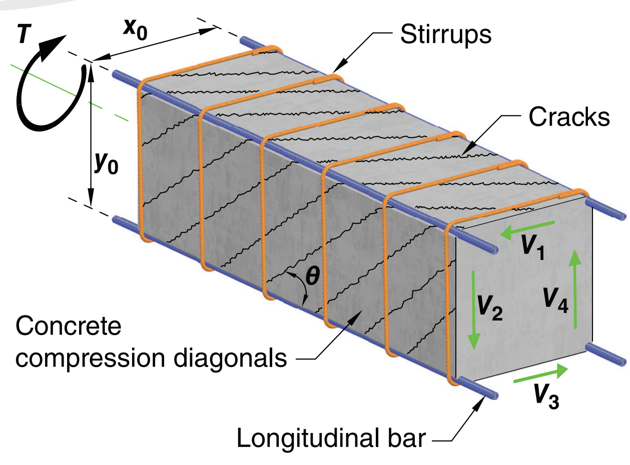

Fig. R22.7.6.1a—Space truss analogy.

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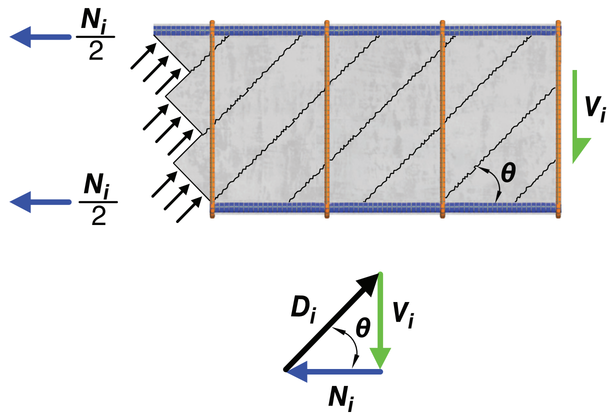

Fig. R22.7.6.1b—Resolution of shear force V_i *into diagonal compression force* D_i *and axial tension force* N_i *in one wall of tube.*

22.7.6.1.1 In Eq. (22.7.6.1a) and (22.7.6.1b), it shall be permitted to take A_o equal to $0.85A_{oh}$.

R22.7.6.1.1 The area A_{oh} is shown in Fig. R22.7.6.1.1 for various cross sections. In I-, T-, L-shaped, or circular sections, A_{oh} is taken as that area enclosed by the outermost transverse reinforcement.

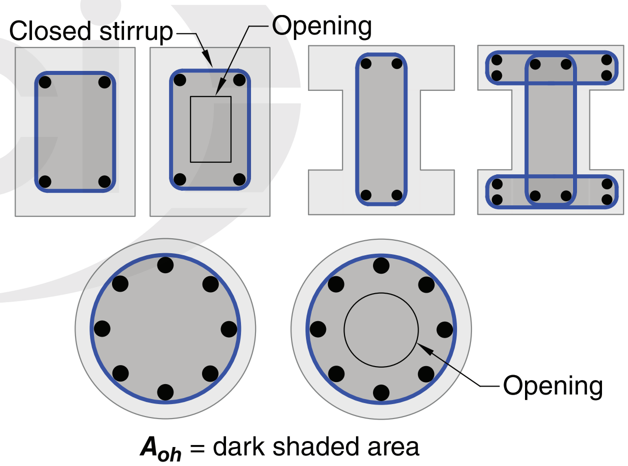

Fig. R22.7.6.1.1—Definition of A_{oh}.

22.7.6.1.2 In Eq. (22.7.6.1a) and (22.7.6.1b), it shall be permitted to take θ equal to (a) or (b):

(a) 45 degrees for nonprestressed members or members with $A_{ps}f_{se} < 0.4(A_{ps}f_{pu} + A_sf_y)$

(b) 37.5 degrees for prestressed members with $A_{ps}f_{se} \geq 0.4(A_{ps}f_{pu} + A_sf_y)$

R22.7.6.1.2 The angle θ can be obtained by analysis (Hsu 1990) or may be taken equal to the values given in 22.7.6.1.2(a) or (b). The same value of θ is required to be used in both Eq. (22.7.6.1a) and (22.7.6.1b). With smaller values of θ, the amount of stirrups required by Eq. (22.7.6.1a) decreases. At the same time, the amount of longitudinal reinforcement required by Eq. (22.7.6.1b) increases.

22.7.7 *Cross-sectional limits*

R22.7.7 *Cross-sectional limits*

22.7.7.1 Cross-sectional dimensions shall be selected such that (a) or (b) is satisfied:

(a) For solid sections

R22.7.7.1 The size of a cross section is limited for two reasons: first, to reduce excessive cracking, and second, to minimize the potential for crushing of the surface concrete due to inclined compressive stresses due to shear and torsion. In Eq. (22.7.7.1a) and (22.7.7.1b), the two terms on the left-