# CHAPTER 23—STRUT-AND-TIE METHOD

## CODE

### 23.1—Scope

## COMMENTARY

### R23.1—Scope

A discontinuity in stress distribution occurs at a change in the geometry of a structural element or at a concentrated load or reaction. St. Venant's principle indicates that the stresses due to axial force and bending approach a linear distribution at a distance approximately equal to the overall depth of the member, $h$, away from the discontinuity. For this reason, discontinuity regions are assumed to extend a distance $h$ from the section where the load or change in geometry occurs.

Shaded regions in Fig. R23.1(a) and (b) show typical D-regions (Schlaich et al. 1987). The plane sections assumption of 22.2.1.2 is not applicable in such regions. In general, any portion of a member outside a D-region is a B-region where the plane sections assumptions of flexural theory can be applied. The strut-and-tie design method, as described in this chapter, is based on the assumption that D-regions can be analyzed and designed using hypothetical pin-jointed trusses consisting of struts and ties connected at nodes.

The idealized truss specified in 23.2.1, which forms the basis of the strut-and-tie method, is not intended to apply to structural systems configured as actual trusses because secondary effects, such as moments, are not included in the model.

## CODE

## COMMENTARY

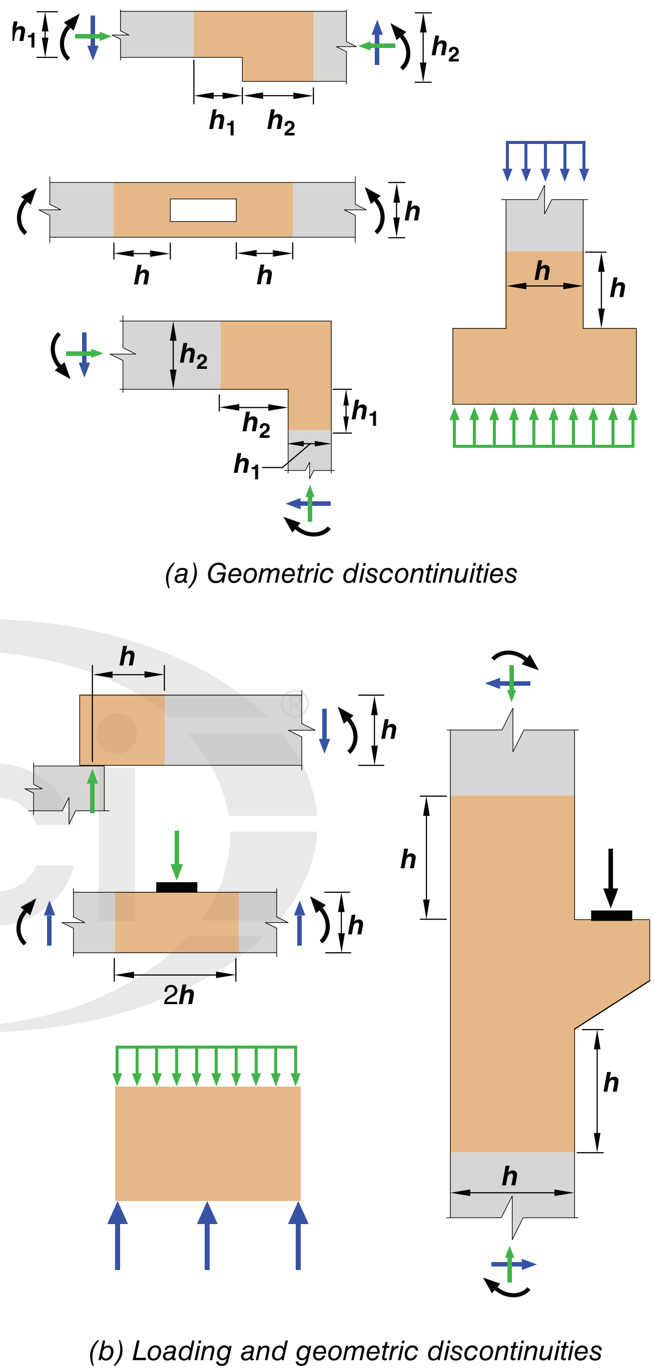


*Fig. R23.1—D-regions and discontinuities.*

**23.1.1** This chapter shall apply to the design of structural concrete members, or regions of members, where load or geometric discontinuities cause a nonlinear distribution of longitudinal strains within the cross section.

**23.1.2** Any structural concrete member, or discontinuity region in a member, shall be permitted to be designed by modeling the member or region as an idealized truss in accordance with this chapter.

## CODE

### 23.2—General

**23.2.1** Strut-and-tie models shall consist of struts and ties connected at nodes to form an idealized truss in two or three dimensions.

## COMMENTARY

### R23.2—General

**R23.2.1** For the idealized truss, struts are the compression members, ties are the tension members, and nodes are the joints. Uniformly distributed loads are usually idealized as a series of concentrated loads applied at nodes. Similarly, distributed reinforcement is usually modeled as discrete ties representing groups of individual bars or wires. Additional information is provided in "Strut-and-Tie Method Guidelines for ACI 318-19—Guide" (ACI PRC-445.2-21). Design examples for the strut-and-tie method are given in ACI SP-208 (Reineck 2002) and ACI SP-273 (Reineck and Novak 2010). The process of designing by the strut-and-tie method to support imposed forces acting on and within a D-region is referred to as the strut-and-tie method, and it includes the following four steps:

(1) Define and isolate each D-region.
(2) Calculate resultant forces on each D-region boundary.
(3) Select the model and calculate the forces in the struts and ties to transfer resultant forces across the D-region. The axes of the struts and ties are chosen to approximately coincide with the axes of the compression and tension fields, respectively.
(4) Design the struts, ties, and nodal zones so that they have sufficient strength. Widths of struts and nodal zones are determined considering the effective concrete strengths defined in 23.4.3 and 23.9.2. Reinforcement is provided for the ties considering the steel strengths defined in 23.7.2. Reinforcement should be anchored in or beyond the nodal zones.

Components of a strut-and-tie model of a single-span deep beam loaded with two concentrated loads are identified in Fig. R23.2.1. Cross-sectional dimensions of a strut or tie are designated as thickness and width, and both directions are perpendicular to the axis of the strut or tie. Thickness is perpendicular to the plane, and width is in the plane of the strut-and-tie model. A tie consists of nonprestressed or prestressed reinforcement plus a portion of the surrounding concrete that is concentric with the axis of the tie.

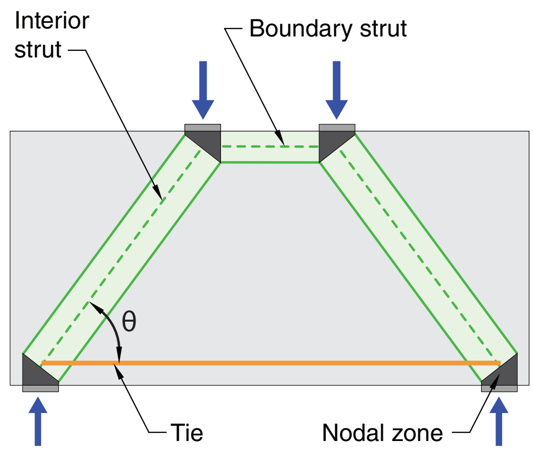


*Fig. R23.2.1—Description of strut-and-tie model.*

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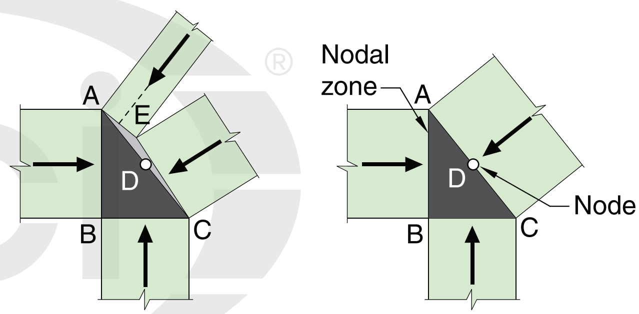
**23.2.2** Geometry of the idealized truss shall be consistent with the dimensions of the struts, ties, nodal zones, bearing areas, and supports.

## COMMENTARY

**R23.2.2** The struts, ties, and nodal zones making up the strut-and-tie model all have finite widths, typically in the plane of the model, and thicknesses, typically the out-of-plane dimension of the structure, which should be taken into account in selecting dimensions of the truss. Figures R23.2.2(a) and (b) show a node and the corresponding nodal zone. The vertical and horizontal forces equilibrate the forces in the inclined strut.

If more than three forces act on a nodal zone in a two-dimensional strut-and-tie model, as shown in Fig. R23.2.2(a), it is suggested to resolve some of the forces to form three forces that intersect at a single point. The strut forces acting on Faces A-E and C-E in Fig. R23.2.2(a) can be replaced with one force acting on Face A-C as shown in Fig. R23.2.2(b). This force passes through the node at D.

If the width of the support, or a loaded region, in the direction perpendicular to the member is less than the width of the member, transverse reinforcement may be required to restrain splitting in the plane of the node. This can be modeled using a transverse strut-and-tie model.




*(a) Struts A-E and C-E may be replaced by A-C*

*(b) Three struts acting on a nodal zone*

*Fig. R23.2.2—Resolution of forces on a nodal zone.*

**23.2.3** Strut-and-tie models shall be capable of transferring all factored loads to supports or adjacent B-regions.

**R23.2.3** Analysis results from the strut-and-tie method represent lower-bound strength limit states. Section 23.5.1 requires distributed reinforcement in D-regions designed by this chapter unless struts are laterally restrained. Distributed reinforcement in D-regions will improve serviceability performance. In addition, crack widths in a tie can be controlled using 24.3.2, assuming the tie is encased in a prism of concrete corresponding to the area of the tie from R23.8.1.

**23.2.4** The internal forces in strut-and-tie models shall be in equilibrium with the applied loads and reactions.

**23.2.5** Ties shall be permitted to cross struts and other ties.

**23.2.6** Struts shall intersect or overlap only at nodes.

**R23.2.6** A hydrostatic nodal zone, by definition, has equal stresses on the loaded faces; these faces are perpendicular to the axes of the struts and ties that act on the node. This type of node is considered a hydrostatic nodal zone because in-plane stresses are the same in all directions. Strictly speaking, this terminology is incorrect because the in-plane stresses are not equal to the out-of-plane stresses.

## CODE

## COMMENTARY

Figure R23.2.6a(i) shows a C-C-C nodal zone. If stresses on the face of the nodal zone are the same in all three struts, the ratios of lengths of sides of the nodal zone, $w_{n1}$:$w_{n2}$:$w_{n3}$, are in the same proportions as the three forces, $C_1$:$C_2$:$C_3$.

A C-C-T nodal zone can be represented as a hydrostatic nodal zone if the tie is assumed to extend through the node and is anchored by a plate on the far side of the node, as shown in Fig. R23.2.6a(ii), provided that the size of the plate results in bearing stresses that are equal to the stresses in the struts. The bearing plate on the left side of Fig. R23.2.6a(ii) is used to represent an actual tie anchorage. The tie force can be anchored by a plate or through embedment of straight bars (Fig. R23.2.6a(iii)), headed bars, or hooked bars. For non-hydrostatic nodes, the face with the highest stress will control the dimensions of the node.

The lightly shaded area in Fig. R23.2.6b is an extended nodal zone. An extended nodal zone is that portion of a member bounded by the intersection of the effective strut width $w_s$ and the effective tie width $w_t$.

For equilibrium, at least three forces should act on each node in a strut-and-tie model, as shown in Fig. R23.2.6c. Nodes are classified according to the signs of these forces. A C-C-C node resists three compressive forces, a C-C-T node resists two compressive forces and one tensile force, and a C-T-T node resists one compressive force and two tensile forces.

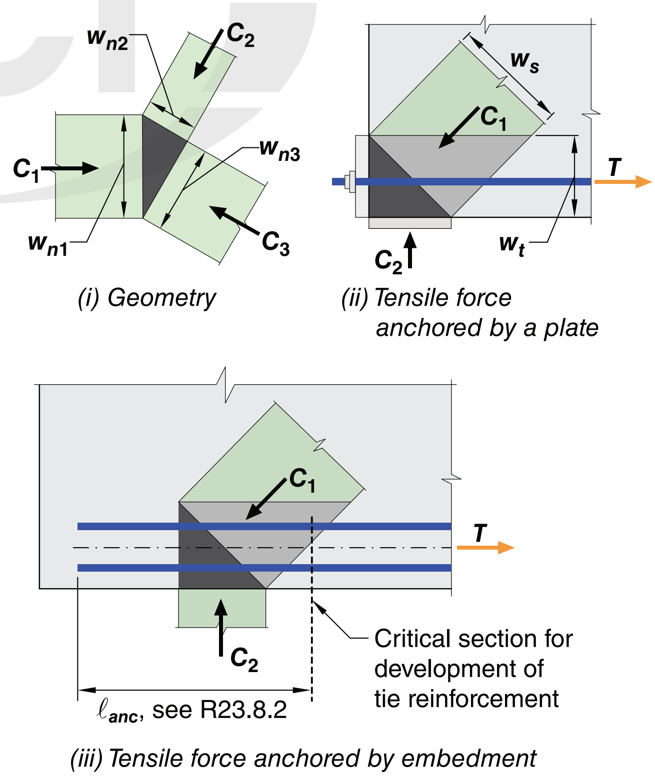



*Fig. R23.2.6a—Hydrostatic nodes.*

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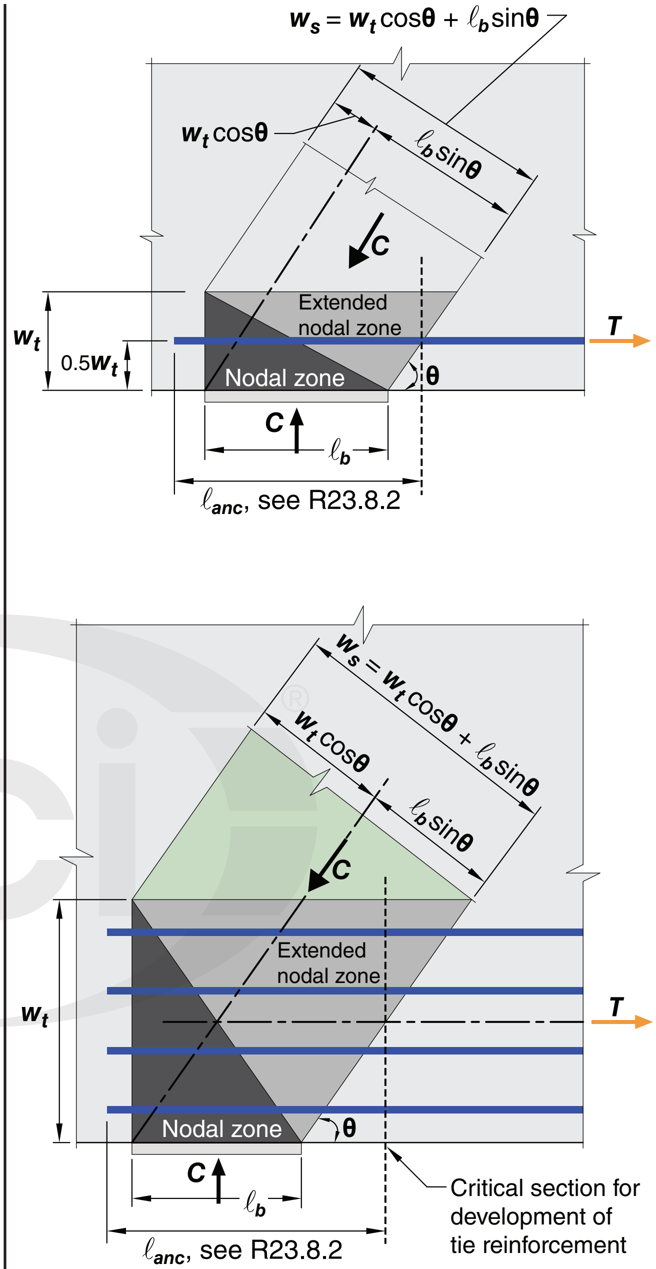
## COMMENTARY




*(ii) Distributed reinforcement*

*Fig. R23.2.6b—Extended nodal zone showing the effect of the distribution of the force.*

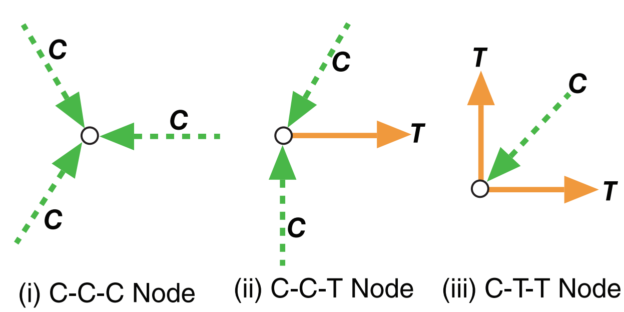



*Fig. R23.2.6c—Classification of nodes.*

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**23.2.7** The angle between the axes of any strut and any tie entering a single node shall be at least 25 degrees.

## COMMENTARY

**R23.2.7** The angle between the axes of a strut and a tie acting on a node should be large enough to mitigate cracking and to avoid incompatibilities due to shortening of the strut and lengthening of the tie occurring in approximately the same direction. This limitation on the angle prevents modeling shear spans in slender beams using struts inclined at less than 25 degrees from the longitudinal reinforcement (Muttoni et al. 1997).

In some cases, strut-and-tie models can be adjusted to satisfy this requirement without excluding transverse reinforcement close to concentrated loads or reactions as illustrated in Fig. R23.2.7.

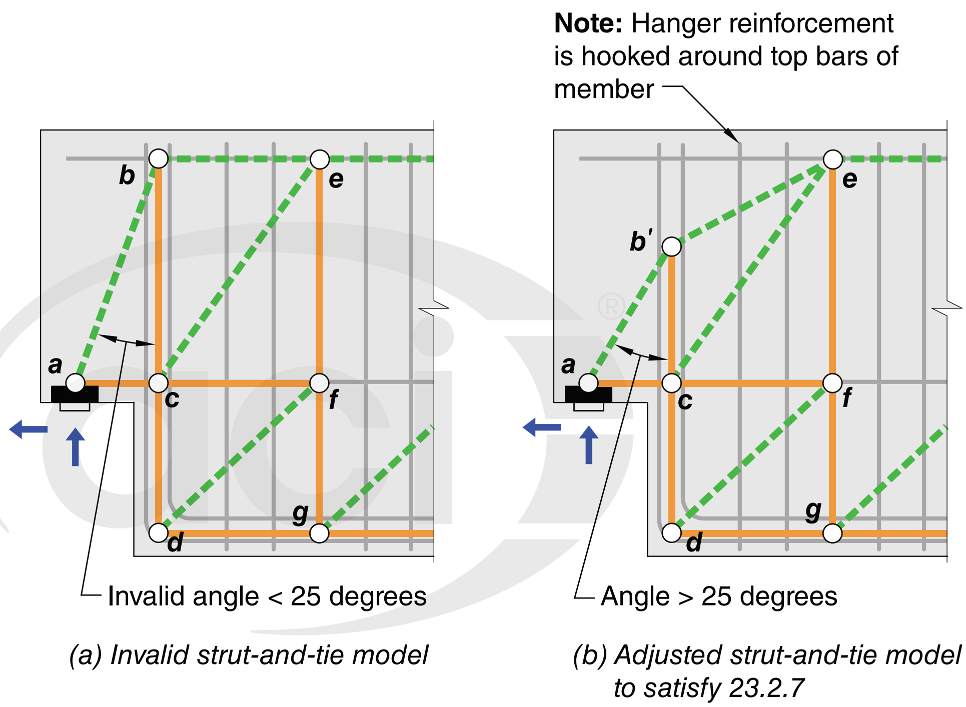


*Fig. R23.2.7—Strut and-tie model of dapped connection illustrating adjustment to comply with 23.2.7.*

**23.2.8** The effects of prestressing shall be included in the strut-and-tie model as external loads with load factors in accordance with 5.3.16. For pretensioned members, it shall be permitted to assume that the prestress force is applied at the end of the strand transfer length.

**R23.2.8** The flow of forces in the strut-and-tie model is unrealistic if prestressing effects are not considered as external loads. Including prestressing effects as external loads is required to identify regions where the effects of other external loads exceed the precompression force and vice versa. Prestressing effects are simulated by concentrated loads at the anchorages and transverse loads equivalent to the effects of tendon deviation or curvature. Provision 5.3.16 requires different load factors depending on the effects of prestressing on the strut-and-tie model. Applying the prestressing force at the end of the transfer length may require a deformed bar tie where the prestress force is being transferred.

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**23.2.9** Deep beams designed using the strut-and-tie method shall satisfy 9.9.2.1, 9.9.3.1, and 9.9.4.

**23.2.10** Walls designed using the strut-and-tie method shall satisfy 11.6, 11.7.2, and 11.7.3.

**23.2.11** Brackets and corbels with shear span-to-depth ratio $a_v/d < 2.0$ designed using the strut-and-tie method shall satisfy 16.5.2, 16.5.6, and Eq. (23.2.11).

$$A_{sc} \geq 0.04(f_c'/f_y)(b_w d) \qquad (23.2.11)$$

**23.2.12** The shear-friction requirements of 22.9 shall apply where it is appropriate to consider shear transfer across any given plane, such as an existing or potential crack, an interface between dissimilar materials, or an interface between two concretes cast at different times.

**23.2.13** Members designed using strut-and-tie models that are part of seismic-force-resisting system shall meet the additional requirements of 23.11, if applicable.

## 23.3—Design strength

**23.3.1** For each applicable factored load combination, design strength of each strut, tie, and nodal zone in a strut-and-tie model shall satisfy $\phi S_n \geq U$, including (a) through (c):

(a) Struts: $\phi F_{ns} \geq F_{us}$
(b) Ties: $\phi F_{nt} \geq F_{ut}$
(c) Nodal zones: $\phi F_{nn} \geq F_{un}$

**23.3.2** $\phi$ shall be in accordance with 21.2.

## 23.4—Strength of struts

**23.4.1** The nominal compressive strength of a strut, $F_{ns}$, shall be calculated by (a) or (b):

(a) Strut without longitudinal reinforcement

$$F_{ns} = f_{ce} A_{cs} \qquad (23.4.1a)$$

(b) Strut with longitudinal reinforcement

$$F_{ns} = f_{ce} A_{cs} + A_s' f_s' \qquad (23.4.1b)$$

where $F_{ns}$ shall be evaluated at each end of the strut and taken as the lesser value; $A_{cs}$ is the cross-sectional area at the end of the strut under consideration; $f_{ce}$ is given in 23.4.3; $A_s'$ is the area of compression reinforcement along the length of the strut; and $f_s'$ is the stress in the compression reinforcement at the nominal axial strength of the strut. It shall be permitted to take $f_s'$ equal to $f_y$ for Grade 40 or 60 reinforcement.

COMMENTARY

**R23.2.12** A construction joint between a corbel and face of a column is an example of an interface where shear-friction requirements of 22.9 apply.

## R23.3—Design strength

**R23.3.1** Factored loads are applied to the strut-and-tie model, and forces in all struts, ties, and nodal zones are calculated. If several load combinations exist, each should be investigated separately. For a given strut, tie, or nodal zone, $F_{us}$, $F_{ut}$, or $F_{un}$ is the largest force in that element for all load combinations considered.

## R23.4—Strength of struts

**R23.4.1** The width of strut, $w_s$, used to calculate $A_{cs}$ is the dimension perpendicular to the axis of the strut at the ends of the strut. This strut width is illustrated in Fig. R23.2.6a(i) and Fig. R23.2.6b. If two-dimensional strut-and-tie models are appropriate, such as for deep beams, the thickness of the struts may be taken as the width of the member except at bearing supports where the thickness of the strut must equal the least thickness of the member or supporting element.

The contribution of reinforcement to the strength of the strut is given by the last term in Eq. (23.4.1b). The stress $f_s'$ in the reinforcement in a strut at nominal strength can be obtained from the strain in the strut when the strut crushes. Detailing requirements in 23.6 must be met including confinement reinforcement to prevent buckling of the strut reinforcement.

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**23.4.2** Effective compressive strength of concrete in a strut, $f_{ce}$, shall be calculated in accordance with 23.4.3.

**23.4.3** Effective compressive strength of concrete in a strut, $f_{ce}$, shall be calculated by:

$$f_{ce} = 0.85\beta_c\beta_s f_c' \tag{23.4.3}$$

where $\beta_s$ is in accordance with Table 23.4.3(a) and $\beta_c$ is in accordance with Table 23.4.3(b).

**Table 23.4.3(a)—Strut coefficient $\beta_s$**

| Strut location | Strut type | Reinforcement | Member dimensions | $\beta_s$ | |
|---|---|---|---|---|---|
| Tension members or tension zones of members | Any | Any | Any | 0.4 | (a) |
| Beam-column joints | Interior | Satisfying applicable strength and detailing provisions of Chapters 15 and 18 | | 0.75 | (b) |
| All other cases | Boundary | Any | Any | 1.0 | (c) |
| | Interior | Satisfying (a) or (b) of Table 23.5.1 | Any | 0.75 | (d) |
| | | Not satisfying (a) or (b) of Table 23.5.1 | Satisfying 23.4.4 | 0.75 | (e) |
| | | | Not satisfying 23.4.4 | 0.4 | (f) |

**Table 23.4.3(b)—Strut and node confinement modification factor $\beta_c$**

| Location | $\beta_c$ | | |
|---|---|---|---|
| • End of a strut connected to a node that includes a bearing surface<br>• Node that includes a bearing surface | Lesser of | $\sqrt{A_2/A_1}$, where $A_1$ is defined by the bearing surface | (a) |
| | | 2.0 | (b) |
| Other cases | 1.0 | | (c) |

## COMMENTARY

**R23.4.2** In design, struts are usually idealized as prismatic compression members. If the area of a strut differs at its two ends, due either to different nodal zone strengths at the two ends or to different bearing lengths, the strut is idealized as a uniformly tapered compression member.

**R23.4.3** The strength coefficient $0.85f_c'$ in Eq. (23.4.3) represents the effective concrete strength in compression.

The value of $\beta_s$ in (a) of Table 23.4.3(a) applies, for example, to a transverse model of a ledger beam used to proportion hanger and ledge reinforcement, where longitudinal tension in the flange reduces the strength of the transverse struts. The low value of $\beta_s$ reflects that these struts need to transfer compression in a zone where tensile stresses act perpendicular to the plane of the strut-and-tie model.

The value of $\beta_s$ in (b) of Table 23.4.3(a) reflects the requirements for reinforcement or confinement for beam-column joints in Chapters 15 and 18.

The value of $\beta_s$ in (c) of Table 23.4.3(a) applies to a boundary strut and results in a stress state that is comparable to the rectangular stress block in the compression zone of a beam or column. Boundary struts are not subject to transverse tension and therefore have a higher effective strength, $f_{ce}$, than interior struts (Fig. R23.2.1).

The value of $\beta_s$ in (d) of Table 23.4.3(a) reflects the beneficial effect of distributed reinforcement.

The value of $\beta_s$ in (e) of Table 23.4.3(a) applies to interior struts in regions with sufficient diagonal tension strength to satisfy Eq. (23.4.4).

The value of $\beta_s$ in (f) of Table 23.4.3(a) is reduced to preclude diagonal tension failure in regions without transverse reinforcement that do not meet or are not evaluated under 23.4.4. Evaluation of test results from the ACI shear database for members without transverse reinforcement indicates that diagonal tension failures are precluded if struts are proportioned based on $\beta_s$ of 0.4 (Reineck and Todisco 2014). The ACI shear database includes test results for specimens with an average $d$ of 15 in. and not exceeding 38 in.; therefore, size effect would not be expected to significantly reduce the strength of members of this size. Because size effect may be significant for deeper members without transverse reinforcement, evaluation in accordance with Eq. (23.4.4) is considered appropriate.

The influence of concrete confinement on the effective compressive strength of a strut or node is taken into account by $\beta_c$. The bearing surface can be a bearing plate or the area from a well-defined compressive load from another member, such as a column. It is the same confining effect as used for bearing areas in 22.8.3. The increase in compressive strength associated with confinement provided by surrounding concrete for a strut-and-tie model is described by Tuchscherer et al. (2010) and Breen et al. (1994).

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**23.4.4** If use of $\beta_s$ of 0.75 is based on line (e) of Table 23.4.3(a), member dimensions shall be selected to satisfy Eq. (23.4.4), where $\lambda_s$ is defined in 23.4.4.1.

$$V_u \leq \phi 5\tan(\theta)\lambda\lambda_s\sqrt{f_c'}\,b_w d \qquad (23.4.4)$$

**23.4.4.1** The size effect modification factor, $\lambda_s$, shall be determined by (a) or (b), as applicable:

(a) If distributed reinforcement is provided in accordance with 23.5, $\lambda_s$ shall be taken as 1.0.
(b) If distributed reinforcement is not provided in accordance with 23.5, $\lambda_s$ shall be taken in accordance with Eq. (23.4.4.1).

$$\lambda_s = \sqrt{\frac{2}{1+\frac{d}{10}}} \leq 1 \qquad (23.4.4.1)$$

## COMMENTARY

**R23.4.4** Equation (23.4.4) is intended to preclude diagonal tension failure. In discontinuity regions, diagonal tension strength increases as the strut angle increases. For very steeply inclined struts, $V_u$ can exceed $\phi 10\lambda\lambda_s\sqrt{f_c'}b_w d$.

## 23.5—Minimum distributed reinforcement

## R23.5—Minimum distributed reinforcement

The strut-and-tie method is derived from the lower-bound theorem of plasticity; therefore, a member designed using this method requires sufficient reinforcement to promote redistribution of internal forces in the cracked state (Marti 1985). In addition to allowing force redistribution, distributed reinforcement controls cracking at service loads and promotes ductile behavior (Smith and Vantsiotis 1982; Rogowsky and MacGregor 1986; Tan et al. 1977).

Interior struts are typically oriented parallel to compression fields and are therefore oriented perpendicular to diagonal tension fields. Tensile stresses across the strut may also develop where compressive stress at the node spreads out along the length of a strut. Minimum distributed reinforcement helps control cracking from these tensile stresses.

The distributed reinforcement ratio required by 23.5.1 is the total on both faces plus any interior layers placed in wide members. Figure R23.5.1 illustrates unidirectional distributed reinforcement crossing interior struts at angle $\alpha$.

Although minimum distributed reinforcement is not required where interior struts are laterally restrained, distributed reinforcement may be beneficial in large discontinuity regions. A continuous corbel supporting a slab is an example of a discontinuity region that includes struts that are laterally restrained in accordance with 23.5.3(a). Pile caps and beam ledges supporting concentrated loads are examples of discontinuity regions that include struts that are laterally restrained in accordance with 23.5.3(b). Side faces of the strut in 23.5.3(b) are the faces parallel to the plane of the model. For pile caps evaluated using three-dimensional strut-and-tie models, the plane of the model in 23.5.3 is defined by the strut in question and the pile to which it connects.

**23.5.1** In D-regions designed using the strut-and-tie method, minimum distributed reinforcement shall be in accordance with Table 23.5.1.

**R23.5.1** More restrictive requirements than 23.5.1 and 23.5.2 may be applicable as defined in the respective member chapters. Refer to 23.2.9 to 23.2.13.

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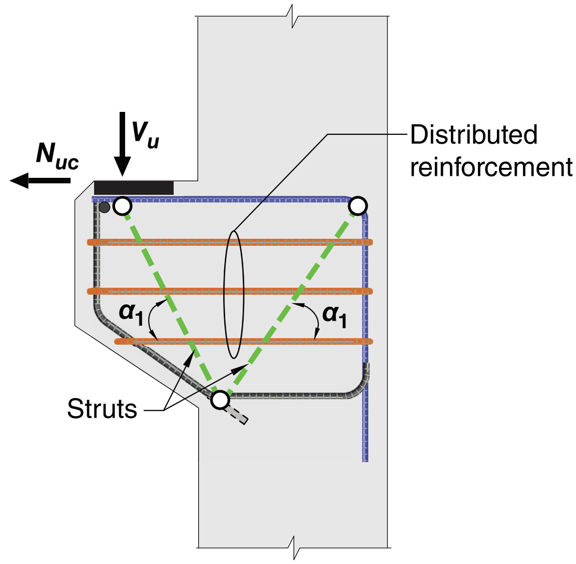
**Table 23.5.1—Minimum distributed reinforcement**

| Lateral restraint of strut | Reinforcement configuration | Minimum distributed reinforcement ratio | |
|---|---|---|---|
| Not restrained | Orthogonal grid | 0.0025 in each direction | (a) |
| | Reinforcement in one direction crossing strut at angle $\alpha$ | $\dfrac{0.0025}{\sin^2\alpha_1}$ | (b) |
| Restrained in accordance with 23.5.3 | Distributed reinforcement not required | | (c) |

## COMMENTARY




*Distributed reinforcement crossing interior struts. Note that $\alpha_1$ is different for the two struts above; the minimum distributed reinforcement ratio is controlled by the smaller angle $\alpha_1$.*

*Fig. R23.5.1—Distributed reinforcement crossing interior struts.*

**23.5.2** Distributed reinforcement required by 23.5.1 shall be determined based on the entire cross-sectional area of the member and shall satisfy (a) through (f) :

(a) Reinforcement shall be developed beyond the extent of the strut in accordance with 25.4.
(b) Reinforcement satisfying the "Not restrained" strut conditions in row (a) or (b) of Table 23.5.1 shall have spacing $s_{\ell d}$ and $s_{td}$, measured in the plane defined by the strut-and-tie model, not exceeding 12 in.
(c) Angle $\alpha_1$ shall not be less than 40 degrees.
(d) For members with a cross-sectional dimension greater than 10 in. measured perpendicular to the plane defined by the strut-and-tie model, distributed reinforcement required by 23.5.1 shall be placed in at least two planes with one near each side face.
(e) Where at least two planes of reinforcement are required in accordance with 23.5.2(d), the spacing between adjacent reinforcement planes $s_{wd}$ shall not exceed 24 in.
(f) Where concrete extends beyond the node by more than 12 in., reinforcement shall be provided to connect the extension to the concrete within the strut-and-tie model.

**R23.5.2** Because the primary purpose of the distributed reinforcement is to control cracking, the quantity of distributed reinforcement is evaluated based on the entire cross section, even for cases where the node geometry or strut dimension is narrower than the overall member thickness. Spacing limits are consistent with the maximum spacing of shear reinforcement legs in 9.7.6.2.2. Distributed reinforcement in a planar member satisfying row (a) of Table 23.5.1 is similar to the configuration illustrated in R9.9.4.3.

Concrete extending beyond the volume defined by the strut-and-tie model will be subjected to three dimensional stresses arising from deformation compatibility, including the influence of axial and transverse strain distributions that differ from the uniform axial strain conditions in idealized struts. Reinforcement to control cracking and provide integrity for the concrete in these regions should be provided in multiple directions. Figure R23.5.2 illustrates the requirements in accordance with 23.5.2(d) through (f) for a wide member designed using a strut-and-tie model width narrower than the member width.

To the extent practicable, multiple planes of reinforcement, whether parallel with the member or strut depth or width, should be placed symmetrically with respect to the center plane of the member.

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## COMMENTARY

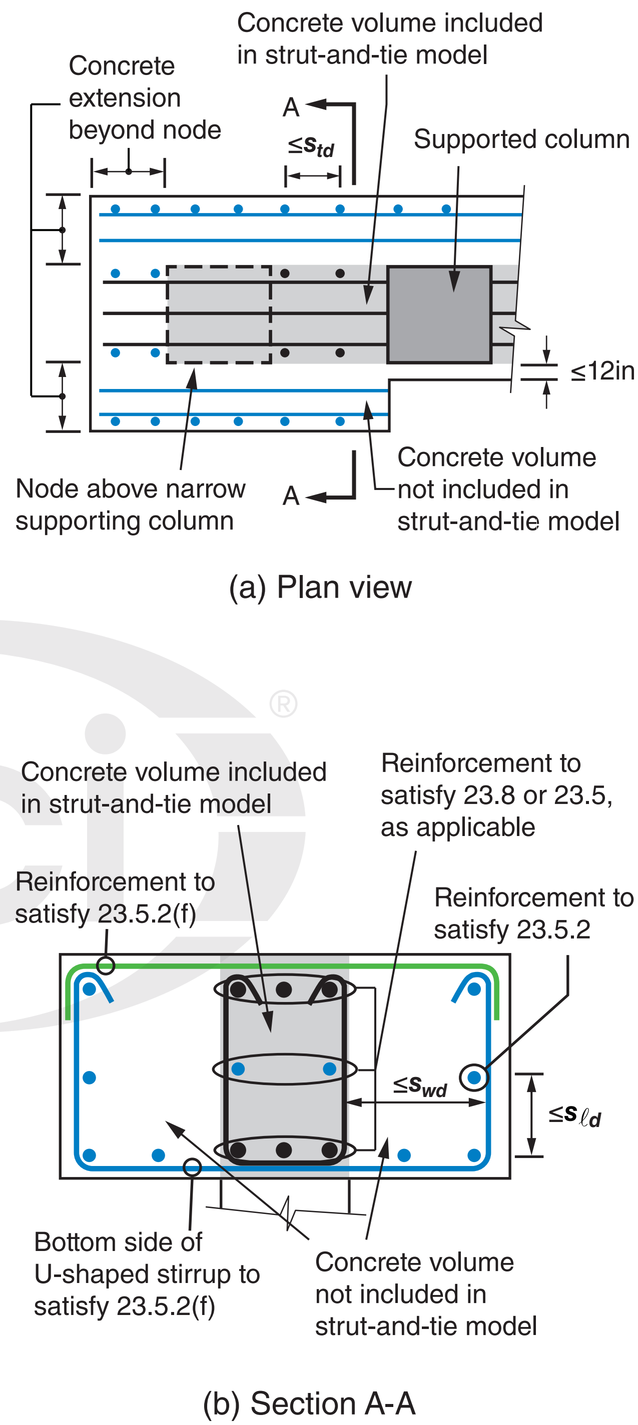


*Fig. R23.5.2—Reinforcement in a wide member designed with a narrow strut-and-tie model*

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**23.5.3** Struts are considered laterally restrained if they are restrained perpendicular to the plane of the strut-and-tie model in accordance with (a), (b), or (c):

(a) The discontinuity region is continuous perpendicular to the plane of the strut-and-tie model.
(b) The concrete restraining the strut extends beyond each side face of the strut a distance not less than half the width of the strut.
(c) The strut is in a joint that is restrained in accordance with 15.5.2.5.

## 23.6—Strut reinforcement detailing

**23.6.1** Compression reinforcement in struts shall be parallel to the axis of the strut and enclosed along the length of the strut by closed ties in accordance with 23.6.3 or by spirals in accordance with 23.6.4.

**23.6.2** Compression reinforcement in struts shall develop $f_s'$ in compression at the face of the nodal zone, where $f_s'$ is calculated in accordance with 23.4.1.

**23.6.3** Closed ties enclosing compression reinforcement in struts shall satisfy 25.7.2 and this section.

**23.6.3.1** Spacing of closed ties, $s$, along the length of the strut shall not exceed the smallest of (a) through (c):

(a) Smallest dimension of cross section of strut
(b) $48d_b$ of bar or wire used for closed tie reinforcement
(c) $16d_b$ of compression reinforcement

**23.6.3.2** The first closed tie shall be located not more than $0.5s$ from the face of the nodal zone at each end of a strut.

**23.6.3.3** Closed ties shall be arranged such that every corner and alternate longitudinal bar shall have lateral support provided by crossties or the corner of a tie with an included angle of not more than 135 degrees and no longitudinal bar shall be farther than 6 in. clear on each side along the tie from such a laterally supported bar.

**23.6.4** Spirals enclosing compression reinforcement in struts shall satisfy 25.7.3.

## 23.7—Strength of ties

**23.7.1** Tie reinforcement shall be nonprestressed or prestressed.

**23.7.2** The nominal tensile strength of a tie, $F_{nt}$, shall be calculated by:

COMMENTARY

## R23.6—Strut reinforcement detailing

**R23.6.1** Refer to R23.4.1.

**R23.6.3.3** Refer to R25.7.2.3.

## R23.7—Strength of ties

**R23.7.2** The tie strength in 23.7.2 is based on including any effects of prestressing as external loads in accordance

## CODE

$$F_{nt} = A_{ts}f_y + A_{tp}\Delta f_p \quad (23.7.2)$$

where $A_{tp}$ is zero for nonprestressed members.

**23.7.2.1** In Eq. (23.7.2), it shall be permitted to take $\Delta f_p$ equal to 60,000 psi for bonded prestressed reinforcement and 10,000 psi for unbonded prestressed reinforcement. Higher values of $\Delta f_p$ shall be permitted if justified by analysis, but $\Delta f_p$ shall not be taken greater than $(f_{py} - f_{se})$.

### 23.8—Tie reinforcement detailing

**23.8.1** The centroidal axis of the tie reinforcement shall coincide with the axis of the tie assumed in the strut-and-tie model.

**23.8.2** If located within the tie, distributed reinforcement in 23.5 may also be considered as tie reinforcement.

**23.8.3** The transverse spacing of tie reinforcement across the thickness of a node shall not exceed the lesser of (a) or (b):

(a) 24 in.
(b) The maximum reinforcement spacing defined in the respective member chapters.

**23.8.4** Tie reinforcement shall be anchored by mechanical devices, post-tensioning anchorage devices, standard hooks, or straight bar development in accordance with 23.8.5,

## COMMENTARY

with 23.2.8. The total strength of a prestressed tie is $A_{tp}(f_{se} + \Delta f_p)$.

### R23.8—Tie reinforcement detailing

**R23.8.1** The effective tie height assumed in design, $w_t$, can vary between the following limits, depending on the distribution of the tie reinforcement:

(a) If bars in the tie are in one layer and spaced transversely through the thickness of the member, the effective tie height $w_t$ can be taken as the diameter of the bars in the tie plus twice the cover to the surface of the bars, as shown in Fig. R23.2.6b(i).
(b) A practical upper limit of the tie height can be taken as the corresponding dimension of a hydrostatic nodal zone, calculated as $w_{t,max} = F_{nt}/(f_{ce}b_s)$, where $f_{ce}$ is calculated for the nodal zone in accordance with 23.9.2.

If the tie height exceeds the value from (a), the tie reinforcement should be distributed approximately uniformly over the tie height, as shown in Fig. R23.2.6b(ii), and over the member thickness. Spacing of tie reinforcement in the thickness direction should satisfy 23.8.3.

**R23.8.3** If geometric or other constraints prevent the inclusion of transverse tie reinforcement spaced in accordance with 23.8.3 across the full member width, multiple strut-and-tie models each occupying a portion of the overall member width may be used. In this case, each model should be aligned with the tie reinforcement locations. The individual strut-and-tie models should also be connected together with an adequate load path to allow load sharing and deformation compatibility. It is good practice to include not less than the minimum distributed reinforcement in 23.5 in the regions between the individual models. Refer to ACI PRC-445.2.

More restrictive requirements than 23.8.3 may be applicable as defined in the respective member chapters. Refer to 23.2.9 to 23.2.13.

**R23.8.4** Anchorage of ties often requires special attention in nodal zones of corbels or in nodal zones adjacent to exterior supports of deep beams. The reinforcement in a tie

## CODE

except for ties extending from curved-bar nodes designed in accordance with 23.10.

**23.8.5** Tie force shall be developed in each direction at the point where the centroid of the reinforcement in the tie leaves the extended nodal zone.

## COMMENTARY

should be anchored before it exits the extended nodal zone at the point defined by the intersection of the centroid of the bars in the tie and the extensions of the outlines of either the strut or bearing area. This length is $\ell_{anc}$. In Fig. R23.2.6b, this occurs where the outline of the extended nodal zone is crossed by the centroid of the reinforcement in the tie. Some of the anchorage may be achieved by extending the reinforcement through the nodal zone, as shown in Fig. R23.2.6a(iii) and R23.2.6b, and developing it beyond the nodal zone. If the tie is anchored using 90-degree hooks, the hooks should be confined within reinforcement to avoid cracking along the outside of the hooks in the support region.

Figure R23.8.4 shows two ties anchored at a nodal zone. Development is required where the centroid of the tie crosses the outline of the extended nodal zone.

The development length of the tie reinforcement can be reduced through hooks, headed bars, mechanical devices, additional confinement, or by splicing it with layers of smaller bars.

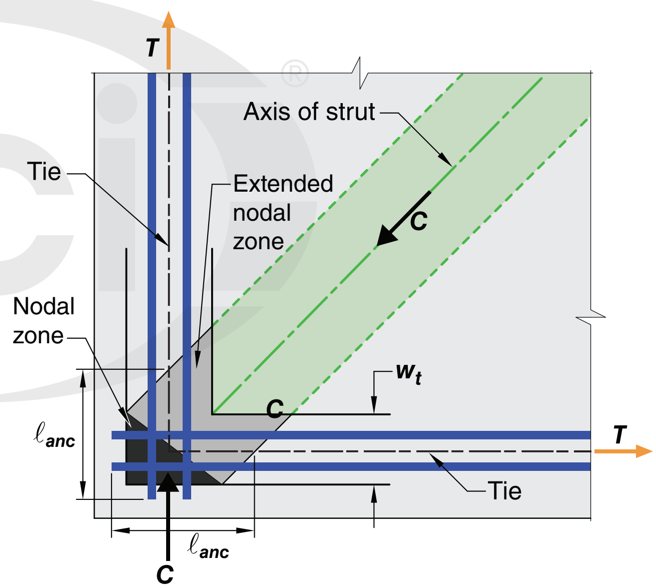


*Fig. R23.8.4—Extended nodal zone anchoring two ties.*



### 23.9—Strength of nodal zones

**23.9.1** The nominal compressive strength of a nodal zone, $F_{nn}$, shall be calculated by:

$$F_{nn} = f_{ce}A_{nz} \tag{23.9.1}$$

where $f_{ce}$ is defined in 23.9.2 or 23.9.3 and $A_{nz}$ is given in 23.9.4 or 23.9.5.

**23.9.2** The effective compressive strength of concrete at a face of a nodal zone, $f_{ce}$, shall be calculated by:

### R23.9—Strength of nodal zones

**R23.9.2** Nodes in two-dimensional models can be classified as shown in Fig. R23.2.6c. The effective compressive

## CODE

$$f_{ce} = 0.85\beta_c\beta_n f_c' \qquad (23.9.2)$$

where $\beta_n$ shall be in accordance with Table 23.9.2 and $\beta_c$ is in accordance with Table 23.4.3(b).

**Table 23.9.2—Nodal zone coefficient $\beta_n$**

| Configuration of nodal zone | $\beta_n$ | |
|---|---|---|
| Nodal zone bounded by struts, bearing areas, or both | 1.0 | (a) |
| Nodal zone anchoring one tie | 0.80 | (b) |
| Nodal zone anchoring two or more ties | 0.60 | (c) |

**23.9.3** If confining reinforcement is provided within the nodal zone and its effect is documented by tests and analyses, it shall be permitted to use an increased value of $f_{ce}$ when calculating $F_{nn}$.

**23.9.4** The area of each face of a nodal zone, $A_{nz}$, shall be taken as the smaller of (a) and (b):

(a) Area of the face of the nodal zone perpendicular to the line of action of $F_{us}$

(b) Area of a section through the nodal zone perpendicular to the line of action of the resultant force on the section

**23.9.5** In a three-dimensional strut-and-tie model, the area of each face of a nodal zone shall be at least that given in 23.9.4, and the shape of each face of the nodal zone shall be similar to the shape of the projection of the end of the strut onto the corresponding face of the nodal zone.

### 23.10—Curved-bar nodes

**23.10.1** Curved-bar nodes shall be designed and detailed in accordance with this section.

**23.10.2** If specified clear cover to the side face normal to plane of bend is $1.5d_b$ or greater, the bend radius $r_b$ shall be in accordance with (a) or (b), but shall not be less than half the minimum bend diameter specified in 25.3.

(a) Curved bar nodes with bends less than 180 degrees:

$$r_b \geq \frac{2A_{ts}f_y}{b_s f_c'} \qquad (23.10.2a)$$

(b) Ties anchored by 180-degree bends:

## COMMENTARY

strength of the nodal zone is given by Eq. (23.9.2) where the value for $\beta_n$ is given in Table 23.9.2.

Lower $\beta_n$ values reflect the increasing degree of disruption of the nodal zones due to incompatibility of tensile strains in the ties and compressive strains in the struts. The stress on any face of the nodal zone or on any section through the nodal zone should not exceed the value given by Eq. (23.9.2).

As described in R23.4.3, $\beta_c$ accounts for the effect of concrete confinement on the effective compressive strength of a node containing a bearing surface. $\beta_c$ is the same for the node as for the node-strut interface.

**R23.9.4** If stresses in all struts meeting at a node are equal, a hydrostatic nodal zone can be used. Faces of such a nodal zone are perpendicular to the axes of the struts, and widths of faces of the nodal zone are proportional to forces in the struts.

Stresses on nodal faces that are perpendicular to the axes of struts and ties are principal stresses, and 23.9.4(a) is used. If, as shown in Fig. R23.2.6b(ii), the face of a nodal zone is not perpendicular to the axis of the strut, there will be both shear stresses and normal stresses on the face of the nodal zone. Typically, these stresses are replaced by the normal (principal compressive) stress acting on the cross-sectional area, $A_{nz}$, of the strut, taken perpendicular to the axis of the strut as given in 23.9.4(a).

### R23.10—Curved-bar nodes

**R23.10.1** A curved-bar node is formed by the bend region of a continuous reinforcing bar (or bars) where two ties extending from the bend region are intersected by a strut or the resultant of two or more struts (Fig. R23.10.5), or where a single tie is anchored by a 180-degree bend (Fig. R23.10.2).

**R23.10.2** Equations (23.10.2a) and (23.10.2b) are intended to avoid $f_{ce}$ exceeding the limit for C-T-T nodes given by 23.9.2 (Klein 2008). In Eq. (23.10.2a), $b_s$ is the width of the strut transverse to the plane of the strut-and-tie model. Equation (23.10.2a) applies whether tie forces at the node are equal or different; where tie forces are different, the larger tie force is considered in determining $A_{ts}$.

Ties anchored by 180-degree bends can be used at C-C-T or C-T-T nodes. Parallel straight legs of the bar(s) that extend into the member form a single tie, where $A_{ts}$ is taken as the

## CODE

$$r_b \geq \frac{A_{ts}f_y}{w_t f_c'} \qquad (23.10.2b)$$

## COMMENTARY

total area of reinforcement extending from both ends of the bend(s). Equation (23.10.2b) is intended to ensure that $f_{ce}$ does not exceed the limit for C-T-T nodes given by 23.9.2. Width $w_t$ is the effective tie width as illustrated in Fig. R23.10.2.

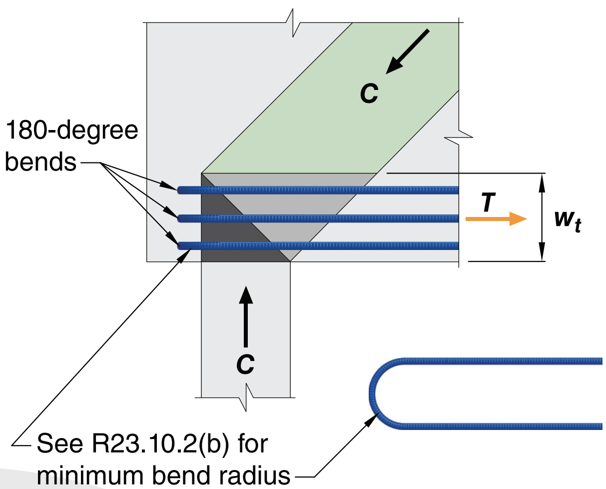


*Fig. R23.10.2—C-C-T node using ties anchored by 180-degree bends.*

**23.10.3** If specified clear cover to the side face normal to plane of bend is less than $1.5d_b$, $r_b$ required by 23.10.2 shall be multiplied by the ratio $1.5d_b/c_c$, where $c_c$ is the specified clear cover to the side face.

**R23.10.3** Larger bar bend radii at curved-bar nodes are required to reduce the likelihood of side splitting where concrete cover perpendicular to the plane of the bend is limited. The multiplier on $d_b/c_c$ was changed from 2 to 1.5 for the 2025 Code based on testing of knee joints with a range of clear cover (Wang 2020).

**23.10.4** If curved-bar nodes are formed by more than one layer of reinforcement, $A_{ts}$ shall be taken as the total area of tie reinforcement, and $r_b$ shall be taken as the bend radius of the innermost layer.

**R23.10.4** Figure R23.10.4 illustrates the use of a curved-bar node with two layers of reinforcing bars. In such cases, the total area of tie reinforcement contributes to the compressive stress on the face of the nodal zone (Face ab in Fig. R23.10.4).

23 Strut-and-Tie

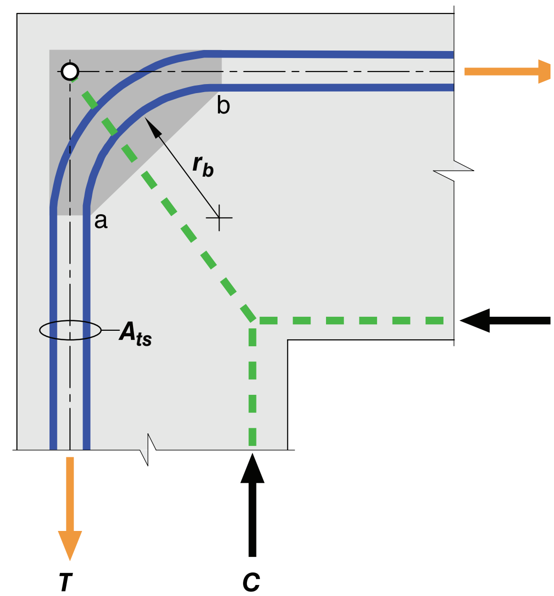


*Fig. R23.10.4—Curved-bar node with two layers of reinforcement (nodal zone is shaded).*

**23.10.5** At frame corners, the joint and reinforcement shall be proportioned such that the center of bar curvature is located within the joint.

**R23.10.5** The radius of the bend should be consistent with the geometry of the truss used for the strut-and-tie model. Figure R23.10.5 illustrates the region in which the center of curvature must be located for a typical frame corner.

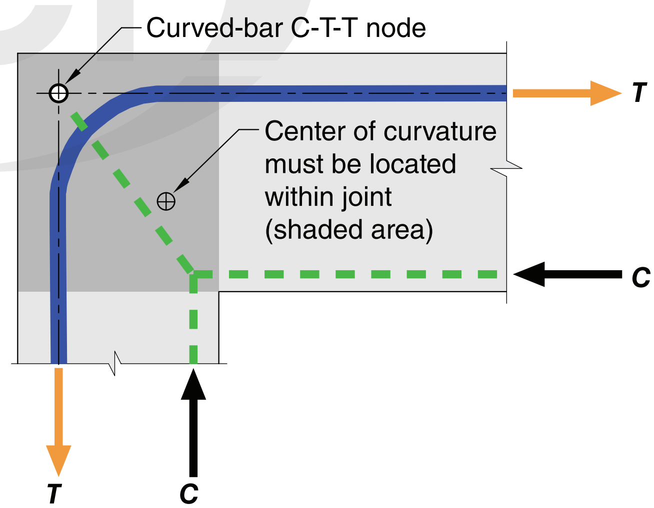


*Fig. R23.10.5—Permissible zone for the center of curvature of a curved-bar node at a frame corner.*

## 23.11—Earthquake-resistant design using the strut-and-tie method

**23.11.1** Regions of a seismic-force-resisting system assigned to Seismic Design Category (SDC) D, E, or F and designed with the strut-and-tie method shall be in accordance with (a) and (b):

## R23.11—Earthquake-resistant design using the strut-and-tie method

**R23.11.1** Strut-and-tie elements of a seismic-force-resisting system may experience strength degradation due to force and displacement reversals. Strut-and-tie elements do not require seismic detailing when the design force is amplified by $\Omega_o$. It is preferable that the strength of the seismic-force-resisting system not be limited by the strength of the discontinuity region designed by the strut-and-tie method.

## CODE

(a) Chapter 18
(b) 23.11.2 through 23.11.5 unless design earthquake-induced force, $E$, in the strut-and-tie element is multiplied by an overstrength factor, $\Omega_o$, not less than 2.5 unless a smaller value of $\Omega_o$ can be justified by a detailed analysis.

## COMMENTARY

For diaphragm design that includes amplified seismic forces, an additional amplification factor is not required.

Load combinations are provided in 5.3.1, and Eq. (5.3.1e) and (5.3.1g) are used for seismic design. The effects of $E$ may cause reversal of forces in strut and tie elements. In such cases, different strut-and-tie models are developed for each loading direction.

### 23.11.2 *Strut strength*

### R23.11.2 *Strut strength*

**23.11.2.1** Effective compressive strength determined in accordance with 23.4 shall be multiplied by 0.8.

**R23.11.2.1** A reduction factor is applied to account for cracking that is likely to develop in struts located in a region subjected to force reversals.

### 23.11.3 *Strut detailing*

### R23.11.3 *Strut detailing*

**23.11.3.1** Struts shall have reinforcement satisfying the detailing requirements of 23.11.3.2 or 23.11.3.3.

**R23.11.3.1** Two confinement options for struts are permitted. For 23.11.3.2, each strut contains longitudinal and transverse reinforcement as required for columns of special moment frames. For 23.11.3.3, the entire cross section of the region or member containing the struts are confined instead of the individual struts.

Expressions (a) and (b) in Table 23.11.3.2(a) are the same as those in Table 18.7.5.4 for columns of special moment frames with the exception of $A_{cs}$ substituted for $A_g$.

**23.11.3.2** Struts shall be reinforced with a minimum of four longitudinal bars with a bar in each corner of the transverse reinforcement. Transverse reinforcement shall be placed perpendicular to the direction of the strut and satisfy (a) through (d):

(a) Detailed in accordance with 18.7.5.2(a) through (e)
(b) $A_{sh}/sb_c$ determined in accordance with Table 23.11.3.2(a)
(c) Spacing satisfying 18.7.5.3(d) and not exceeding the values specified in Table 23.11.3.2(b)
(d) Continued through the nodal zone

**Table 23.11.3.2(a)—Transverse reinforcement for struts[1][2]**

| Transverse reinforcement | Applicable expressions | | |
|---|---|---|---|
| $A_{sh}/sb_c$ for rectilinear hoops | Greater of | $0.3\left(\frac{A_{cs}}{A_{ch}}-1\right)\frac{f_c'}{f_{yt}}$ | (a) |
| | | $0.09\frac{f_c'}{f_{yt}}$ | (b) |

[1] $A_{ch}$ is measured to the outside edges of the transverse reinforcement for the strut.
[2] It shall be permitted to configure hoops using two pieces of reinforcement as specified in 18.6.4.3.

## CODE

**Table 23.11.3.2(b)—Transverse reinforcement spacing limitation**

| Reinforcement | Maximum transverse bar spacing | |
|---|---|---|
| Grade 60 | Lesser of | $6d_b$ |
| | | 6 in. |
| Grade 80 | Lesser of | $5d_b$ |
| | | 6 in. |
| Grade 100 | Lesser of | $4d_b$ |
| | | 6 in. |

**23.11.3.3** Transverse reinforcement shall be provided in each orthogonal direction and through the thickness of the entire member cross section or for the region of the member containing struts and shall satisfy (a) through (d).

(a) Detailed in accordance with 18.7.5.2(a) through (e)
(b) $A_{sh}/sb_c$ determined in accordance with Table 23.11.3.3.
(c) Spacing measured along the longitudinal axis of the member not exceeding the values specified in Table 23.11.3.2(b).
(d) Spacing of crossties or legs of hoops both vertically and horizontally in the plane of the member cross section shall not exceed 8 in. Each crosstie and each hoop leg shall engage a longitudinal bar of equal or greater diameter.

**Table 23.11.3.3—Transverse reinforcement for the entire member cross section**

| Transverse reinforcement | Applicable expressions | | |
|---|---|---|---|
| $A_{sh}/sb_c$ for rectilinear hoops | Greater of | $0.3\left(\frac{A_g}{A_{ch}}-1\right)\frac{f_c'}{f_{yt}}$ | (a) |
| | | $0.09\frac{f_c'}{f_{yt}}$ | (b) |

### 23.11.4 *Strength of ties*

**23.11.4.1** Tie reinforcement shall develop $1.25f_y$ in tension in accordance with 25.4.

### 23.11.5 *Strength of nodes*

**23.11.5.1** The nominal compressive strength of a nodal zone calculated in accordance with 23.9 shall be multiplied by 0.8.

## COMMENTARY

### R23.11.4 *Strength of ties*

**R23.11.4.1** Because the actual yield strength of tie reinforcement may exceed the specified yield strength and strain hardening of the reinforcement is likely to occur, development lengths for tie reinforcement are determined considering a stress of $1.25f_y$.

### R23.11.5 *Strength of nodes*

**R23.11.5.1** A reduction of the nominal compressive strength at nodes is provided to account for tie yielding and the effect of reversed cyclic loading (Mansour and Hsu 2005; To et al. 2009; Ruggiero et al. 2016).