CHAPTER 22—SECTIONAL STRENGTH

CODE

22.1—Scope

22.1.1 This chapter shall apply to calculating nominal strength at sections of members, including (a) through (g):

(a) Flexural strength
(b) Axial strength or combined flexural and axial strength
(c) One-way shear strength
(d) Two-way shear strength
(e) Torsional strength
(f) Bearing
(g) Shear friction

22.1.2 Sectional strength requirements of this chapter shall be satisfied unless the member or region of the member is designed in accordance with Chapter 23.

22.1.3 Design strength at a section shall be taken as the nominal strength multiplied by the applicable strength reduction factor ϕ given in Chapter 21.

22.2—Design assumptions for moment and axial strength

22.2.1 *Equilibrium and strain compatibility*

22.2.1.1 Equilibrium shall be satisfied at each section.

22.2.1.2 Strain in concrete and nonprestressed reinforcement shall be assumed proportional to the distance from neutral axis.

22.2.1.3 Strain in prestressed concrete and in bonded and unbonded prestressed reinforcement shall include the strain due to effective prestress.

22.2.1.4 Changes in strain for bonded prestressed reinforcement shall be assumed proportional to the distance from neutral axis.

COMMENTARY

R22.1—Scope

R22.1.1 The provisions in this chapter apply where the strength of the member is evaluated at critical sections.

R22.1.2 Chapter 23 provides methods for designing discontinuity regions where section-based methods do not apply.

R22.2—Design assumptions for moment and axial strength

R22.2.1 *Equilibrium and strain compatibility*

R22.2.1.1 The flexural and axial strength of a member calculated by the strength design method of the Code requires that two basic conditions be satisfied: 1) equilibrium; and 2) compatibility of strains. Equilibrium refers to the balancing of forces acting on the cross section at nominal strength. The relationship between the stress and strain for the concrete and the reinforcement at nominal strength is established within the design assumptions allowed by 22.2.

R22.2.1.2 It is reasonable to assume a linear distribution of strain over the depth of a reinforced concrete cross section for bending moment up to nominal strength, except in deep beams described in 9.9 and discontinuity regions addressed in Chapter 23. This assumption allows the determination of strain and corresponding stress in the reinforcement.

R22.2.1.4 The change in strain for bonded prestressed reinforcement is influenced by the change in strain at the section under consideration. For unbonded prestressed reinforcement, the change in strain is influenced by external load, reinforcement location, and boundary conditions along the length of the reinforcement. Current Code equations for calculating f_{ps} for unbonded tendons, as provided in 20.3.2.4, have been correlated with test results

CODE

22.2.2 *Design assumptions for concrete*

22.2.2.1 Maximum strain at the extreme concrete compression fiber shall be assumed equal to 0.003.

22.2.2.2 Tensile strength of concrete shall be neglected in flexural and axial strength calculations.

22.2.2.3 The relationship between concrete compressive stress and strain shall be represented by a rectangular, trapezoidal, parabolic, or other shape that results in prediction of strength in substantial agreement with results of comprehensive tests.

22.2.2.4 The equivalent rectangular concrete stress distribution in accordance with 22.2.2.4.1 through 22.2.2.4.3 satisfies 22.2.2.3.

22.2.2.4.1 Concrete stress of $0.85f_c'$ shall be assumed uniformly distributed over an equivalent compression zone bounded by edges of the cross section and a line parallel to the neutral axis located a distance a from the fiber of maximum compressive strain, as calculated by:

$$a = \beta_1 c \qquad (22.2.2.4.1)$$

22.2.2.4.2 Distance from the fiber of maximum compressive strain to the neutral axis, c, shall be measured perpendicular to the neutral axis.

22.2.2.4.3 Values of β_1 shall be in accordance with Table 22.2.2.4.3.

COMMENTARY

R22.2.2 *Design assumptions for concrete*

R22.2.2.1 The maximum concrete compressive strain at crushing of the concrete has been observed in tests of various kinds to vary from 0.003 to higher than 0.008 under special conditions. However, the strain at which strength of the member is developed is usually 0.003 to 0.004 for members of normal proportions, materials, and strength.

R22.2.2.2 The tensile strength of concrete in flexure (modulus of rupture) is a more variable property than the compressive strength and is approximately 10 to 15 percent of the compressive strength. Tensile strength of concrete in flexure is conservatively neglected in calculating the nominal flexural strength. The strength of concrete in tension, however, is important in evaluating cracking and deflections at service loads.

R22.2.2.3 At high strain levels, the stress-strain relationship for concrete is nonlinear (stress is not proportional to strain). As stated in 22.2.2.1, the maximum usable strain is set at 0.003 for design.

The actual distribution of concrete compressive stress within a cross section is complex and usually not known explicitly. The important properties of the concrete stress distribution can be approximated closely using any one of several different assumptions for the shape of the stress distribution.

R22.2.2.4 For design, the Code allows the use of an equivalent rectangular compressive stress distribution (stress block) to replace the more detailed approximation of the concrete stress distribution.

R22.2.2.4.1 The equivalent rectangular stress distribution does not represent the actual stress distribution in the compression zone at nominal strength, but does provide essentially the same nominal combined flexural and axial compressive strength as obtained in tests (Mattock et al. 1961).

R22.2.2.4.3 The values for β_1 were determined experimentally. The lower limit of β_1 is based on experimental data from beams constructed with concrete strengths greater than 8000 psi (Leslie et al. 1976; Karr et al. 1978).

CODE

Table 22.2.2.4.3—Values of β_1 for equivalent rectangular concrete stress distribution

f_c', psi	β_1	
$2500 \le f_c' \le 4000$	0.85	(a)
$4000 < f_c' < 8000$	$0.85 - \frac{0.05(f_c' - 4000)}{1000}$	(b)
$f_c' \ge 8000$	0.65	(c)

22.2.3 *Design assumptions for nonprestressed reinforcement*

22.2.3.1 Deformed reinforcement used to resist tensile or compressive forces shall conform to 20.2.1.

22.2.3.2 Stress-strain relationship and modulus of elasticity for deformed reinforcement shall be idealized in accordance with 20.2.2.1 and 20.2.2.2.

22.2.4 *Design assumptions for prestressed reinforcement*

22.2.4.1 For members with bonded prestressed reinforcement conforming to 20.3.1, stress at nominal flexural strength, f_{ps}, shall be calculated in accordance with 20.3.2.3.

22.2.4.2 For members with unbonded prestressed reinforcement conforming to 20.3.1, f_{ps} shall be calculated in accordance with 20.3.2.4.

22.2.4.3 If the embedded length of the prestressed strand is less than ℓ_d, the design stress of the prestressed strand shall not exceed the value given in 25.4.8.3, as modified by 25.4.8.1(b).

22.3—Flexural strength

22.3.1 *General*

22.3.1.1 Nominal flexural strength M_n shall be calculated in accordance with the assumptions of 22.2.

22.3.2 *Prestressed concrete members*

22.3.2.1 Deformed reinforcement conforming to 20.2.1, provided in conjunction with prestressed reinforcement, shall be permitted to be considered to contribute to the tensile force and be included in flexural strength calculations at a stress equal to f_y.

22.3.2.2 Other nonprestressed reinforcement shall be permitted to be considered to contribute to the flexural strength if a strain compatibility analysis is performed to calculate stresses in such reinforcement.

COMMENTARY

R22.3—Flexural strength

R22.3.2 *Prestressed concrete members*

R22.3.2.2 Bond length for nontensioned prestressing strand (Salmons and McCrate 1977; PCA 1980) should be sufficient to develop the stress consistent with strain compatibility analysis at the critical section.

CODE

22.3.3 *Composite concrete members*

22.3.3.1 Provisions of 22.3.3 apply to members constructed in separate placements but connected so that all elements resist loads as a unit.

22.3.3.2 For calculation of M_n for composite concrete slabs and beams, use of the entire composite section shall be permitted.

22.3.3.3 For calculation of M_n for composite concrete slabs and beams, no distinction shall be made between shored and unshored members.

22.3.3.4 For calculation of M_n for composite concrete members where the specified concrete compressive strength of different elements varies, properties of the individual elements shall be used in design. Alternatively, it shall be permitted to use the value of f_c' for the element that results in the most critical value of M_n.

22.4—Axial strength or combined flexural and axial strength

22.4.1 *General*

22.4.1.1 Nominal flexural and axial strength shall be calculated in accordance with the assumptions of 22.2.

22.4.2 *Maximum axial compressive strength*

22.4.2.1 Nominal axial compressive strength P_n shall not exceed $P_{n,max}$ in accordance with Table 22.4.2.1, where P_o is calculated by Eq. (22.4.2.2) for nonprestressed members and by Eq. (22.4.2.3) for prestressed members. The value of f_y shall be limited to a maximum of 80,000 psi.

Table 22.4.2.1—Maximum axial strength

Member	Transverse reinforcement	$P_{n,max}$	
Nonprestressed	Ties conforming to 22.4.2.4	$0.80P_o$	(a)
	Spirals conforming to 22.4.2.5	$0.85P_o$	(b)
Prestressed	Ties	$0.80P_o$	(c)
	Spirals	$0.85P_o$	(d)
Deep foundation member	Ties conforming to Ch. 13	$0.80P_o$	(e)

22.4.2.2 For nonprestressed members, P_o shall be calculated by:

COMMENTARY

R22.3.3 *Composite concrete members*

R22.3.3.1 The scope of Chapter 22 is intended to include composite concrete flexural members. Where separate placements of concrete are designed to act as a unit, the interface is designed for the forces that will be transferred across the interface. Composite structural steel-concrete beams are not covered in the Code. Design provisions for these types of composite members are covered in ANSI/AISC 360.

R22.4—Axial strength or combined flexural and axial strength

R22.4.2 *Maximum axial compressive strength*

R22.4.2.1 To account for accidental eccentricity, the design axial strength of a section in pure compression is limited to 80 to 85% of the nominal axial strength. These percentage values approximate the axial strengths at eccentricity-to-depth ratios of 0.10 and 0.05 for tied and spirally reinforced members conforming to 22.4.2.4 and 22.4.2.5, respectively. The same axial load limitation applies to both cast-in-place and precast compression members. The value of f_y is limited to 80,000 psi because the compression capacity of the concrete is likely to be reached before this stress is exceeded. The transverse reinforcement requirements for columns do not apply to deep foundation members. Chapter 13 provides the detailing requirements for these members.

CODE

$$P_o = 0.85f_c'(A_g - A_{st}) + f_yA_{st} \qquad (22.4.2.2)$$

where A_{st} is the total area of nonprestressed longitudinal reinforcement.

22.4.2.3 For prestressed members, P_o shall be calculated by:

$$P_o = 0.85f_c'(A_g - A_{st} - A_{pd}) + f_yA_{st} - (f_{se} - 0.003E_p)A_{pt} \qquad (22.4.2.3)$$

where A_{pt} is the total area of prestressing reinforcement, and A_{pd} is the total area occupied by duct, sheathing, and prestressing reinforcement; the value of f_{se} shall be at least $0.003E_p$. For grouted, post-tensioned tendons, it shall be permitted to assume A_{pd} equals A_{pt}.

22.4.2.4 Tie reinforcement for lateral support of longitudinal reinforcement in compression members shall satisfy 10.7.6.2 and 25.7.2.

22.4.2.5 Spiral reinforcement for lateral support of longitudinal reinforcement in compression members shall satisfy 10.7.6.3 and 25.7.3.

22.4.3 *Maximum axial tensile strength*

22.4.3.1 Nominal axial tensile strength of a nonprestressed or prestressed member, P_{nt}, shall not be taken greater than $P_{nt,max}$, calculated by:

$$P_{nt,max} = f_yA_{st} + (f_{se} + \Delta f_p)A_{pt} \qquad (22.4.3.1)$$

where $(f_{se} + \Delta f_p)$ shall not exceed f_{py}, and A_{pt} is zero for nonprestressed members.

22.5—One-way shear strength

22.5.1 *General*

22.5.1.1 Nominal one-way shear strength at a section, V_n, shall be calculated by:

$$V_n = V_c + V_s \qquad (22.5.1.1)$$

COMMENTARY

R22.4.2.3 The effects of prestressing on the axial strength of compression members are taken into account in Eq. (22.4.2.3). Equation (22.4.2.3) is similar to Eq. (22.4.2.2) for nonprestressed compression members. The effective area of concrete subjected to the limiting stress of $0.85f_c'$ is reduced by the term A_{pd} to account for the area of ducts, sheathing, and prestressing reinforcement. A third term is added to account for the reduction of column capacity due to the prestress force. At nominal strength, the stress in the prestressed reinforcement, f_{se}, is decreased by $0.003E_p$, where 0.003 is the assumed compressive strain at the axial capacity of the member.

R22.5—One-way shear strength

R22.5.1 *General*

R22.5.1.1 In a member without shear reinforcement, shear is assumed to be resisted by the concrete. In a member with shear reinforcement, a portion of the shear strength is assumed to be provided by the concrete and the remainder by the shear reinforcement.

The one-way shear equations for nonprestressed concrete were changed in the 2019 Code with the primary objectives of including effect of member depth, commonly referred to as the "size effect," and the effects of the longitudinal reinforcement ratio on shear strength.

The shear strength is based on an average shear stress over the effective cross section, b_wd.

Chapter 23 allows the use of the strut-and-tie method in the shear design of any structural concrete member, or discontinuity region in a member.

CODE

22.5.1.2 Cross-sectional dimensions shall be selected to satisfy Eq. (22.5.1.2).

$$V_u \leq \phi\left(V_c + 8\sqrt{f_c'}\, b_w d\right) \qquad (22.5.1.2)$$

22.5.1.3 For nonprestressed members, V_c shall be calculated in accordance with 22.5.5.

22.5.1.4 For prestressed members, V_c shall be calculated in accordance with 22.5.6 or 22.5.7.

22.5.1.5 For calculation of V_c, V_{ci}, and V_{cw}, λ shall be in accordance with 19.2.4.

22.5.1.6 V_s shall be calculated in accordance with 22.5.8.

22.5.1.7 Effect of any openings in members shall be considered in calculating V_n.

22.5.1.8 Effect of axial tension due to creep and shrinkage in members shall be considered in calculating V_c.

22.5.1.9 Effect of inclined flexural compression in variable depth members shall be permitted to be considered in calculating V_c.

22.5.1.10 The interaction of shear forces acting along orthogonal axes shall be permitted to be neglected if (a) or (b) is satisfied.

(a) $\dfrac{V_{u,x}}{\phi V_{n,x}} \leq 0.5$ (22.5.1.10a)

(b) $\dfrac{V_{u,y}}{\phi V_{n,y}} \leq 0.5$ (22.5.1.10b)

COMMENTARY

R22.5.1.2 The limit on cross-sectional dimensions in 22.5.1.2 is intended to control cracking under service load and to minimize the likelihood of diagonal compression failure.

R22.5.1.7 Openings in the web of a member can reduce its shear strength. The effects of openings are discussed in Section 4.7 of Joint ACI-ASCE Committee 426 (1973), Barney et al. (1977), and Schlaich et al. (1987). The strut-and-tie method as addressed in Chapter 23 can be used to design members with openings.

R22.5.1.8 Consideration of axial tension requires engineering judgment. Axial tension often occurs due to volume changes, but it may be low enough not to be detrimental to the performance of a structure with adequate expansion joints and satisfying minimum longitudinal reinforcement requirements. It may be desirable to design shear reinforcement to resist the total shear if there is uncertainty about the magnitude of axial tension.

R22.5.1.9 In a member of variable depth, the internal shear at any section is increased or decreased by the vertical component of the inclined flexural stresses.

R22.5.1.10 and **R.22.5.1.11** Reinforced concrete members, such as columns and beams, may be subjected to biaxial shear. For symmetrically reinforced circular sections, nominal one-way shear strength about any axis is the same. Therefore, when a circular section is subjected to shear along two centroidal axes, shear strength can be evaluated using the resultant shear. However, for rectangular and other cross sections, calculating nominal one-way shear strength along the axis of the resultant shear is not practical. Tests and analytical results for columns have indicated that for biaxial shear loading, the shear strength follows an elliptical interaction diagram that requires calculating nominal one-way shear strength along two orthogonal directions (Umehara and Jirsa 1984). Considering shear along each centroidal axis independently can be unconservative. Thus, linear interaction accounts for biaxial shear.

CODE

22.5.1.11 If $\frac{V_{u,x}}{\phi V_{n,x}} > 0.5$ and $\frac{V_{n,y}}{\phi V_{n,y}} > 0.5$ then Eq. (22.5.1.11) shall be satisfied.

$$\frac{V_{u,x}}{\phi V_{n,x}} + \frac{V_{u,y}}{\phi V_{n,y}} \leq 1.5 \qquad (22.5.1.11)$$

22.5.2 *Geometric assumptions*

22.5.2.1 For calculation of V_c and V_s, it shall be permitted to assume (a) through (d) :

(a) d equal to 0.8h for rectangular columns
(b) d equal to 0.8 times the diameter for circular sections
(c) b_w equal to the diameter for solid circular sections
(d) b_w equal to twice the wall thickness for hollow circular sections

22.5.2.2 For calculation of V_c and V_s in prestressed members, d shall be taken as the distance from the extreme compression fiber to the centroid of prestressed and any nonprestressed longitudinal reinforcement but need not be taken less than 0.8h.

22.5.3 *Limiting material strengths*

22.5.3.1 The value of $\sqrt{f_c'}$ used to calculate V_c, V_{ci}, and V_{cw} for one-way shear shall not exceed 100 psi, unless allowed in 22.5.3.2.

22.5.3.2 Values of $\sqrt{f_c'}$ greater than 100 psi shall be permitted in calculating V_c, V_{ci}, and V_{cw} for reinforced or prestressed concrete beams and concrete joist construction having minimum web reinforcement in accordance with 9.6.3.4 or 9.6.4.2.

22.5.3.3 The values of f_y and f_{yt} used to calculate V_s shall not exceed the limits in 20.2.2.4.

COMMENTARY

R22.5.2 *Geometric assumptions*

R22.5.2.1 The computation of d for rectangular columns and circular sections can be complicated considering variations in axial load as well as multiple layers of reinforcement. While the actual d can be used, simplified definitions of d are provided. Experimental results indicate that d computed as 80% of the overall column dimension is appropriate and results in good accuracy (Sezen et al. 2021).

Although the transverse reinforcement in a circular section may not consist of straight legs, tests indicate that Eq. (22.5.8.5.3) is conservative if d is taken as defined in 22.5.2.1 (Faradji and Diaz de Cossio 1965; Khalifa and Collins 1981).

R22.5.2.2 Although the value of d may vary along the span of a prestressed beam, studies (MacGregor and Hanson 1969) have shown that, for prestressed concrete members, d need not be taken less than 0.8h. The beams considered had some straight prestressed reinforcement or reinforcing bars at the bottom of the section and had stirrups that enclosed the longitudinal reinforcement.

R22.5.3 *Limiting material strengths*

R22.5.3.1 Because of a lack of test data and practical experience with concretes having compressive strengths greater than 10,000 psi, the Code imposes a maximum value of 100 psi on $\sqrt{f_c'}$ for use in the calculation of shear strength of concrete members. Exceptions to this limit are permitted in beams and joists if the transverse reinforcement satisfies the requirements in 22.5.3.2.

R22.5.3.2 Based on the beam test results in Mphonde and Frantz (1984), Elzanaty et al. (1986), Roller and Russell (1990), Johnson and Ramirez (1989), and Ozcebe et al. (1999), an increase in the minimum amount of transverse reinforcement is required for high-strength concrete. These tests indicate a reduction in reserve shear strength occurs as f_c' increases in beams reinforced with transverse reinforcement providing an effective shear stress of 50 psi. By providing minimum transverse reinforcement, which increases as f_c' increases, the reduction in shear strength is offset.

R22.5.3.3 The upper limit of 60,000 psi on the value of f_y and f_{yt} used in design is intended to control diagonal crack widths.

CODE

22.5.4 *Composite concrete members*

22.5.4.1 This section shall apply to members constructed in separate placements but connected so that all elements resist loads as a unit.

22.5.4.2 For calculation of V_n for composite concrete members, no distinction shall be made between shored and unshored members.

22.5.4.3 For calculation of V_n for composite concrete members where the specified concrete compressive strength, unit weight, or other properties of different elements vary, properties of the individual elements shall be used in design. Alternatively, it shall be permitted to use the properties of the element that results in the most critical value of V_n.

22.5.4.4 If an entire composite concrete member is assumed to resist vertical shear, it shall be permitted to calculate V_c assuming a monolithically cast member of the same cross-sectional shape.

22.5.4.5 If an entire composite concrete member is assumed to resist vertical shear, it shall be permitted to calculate V_s assuming a monolithically cast member of the same cross-sectional shape if shear reinforcement is fully anchored into the interconnected elements in accordance with 25.7.

22.5.5 V_c *for nonprestressed members*

22.5.5.1 For nonprestressed members, V_c shall be calculated in accordance with Table 22.5.5.1 and 22.5.5.1.1 through 22.5.5.1.3.

Table 22.5.5.1—V_c for nonprestressed members

Criteria	V_c		
$A_v \geq A_{v,min}$	Either of:	$\left[2\lambda\sqrt{f_c'} + \frac{N_u}{6A_g}\right]b_w d$	(a)
		$\left[8\lambda(\rho_w)^{1/3}\sqrt{f_c'} + \frac{N_u}{6A_g}\right]b_w d$	(b)
$A_v < A_{v,min}$	$\left[8\lambda_s\lambda(\rho_w)^{1/3}\sqrt{f_c'} + \frac{N_u}{6A_g}\right]b_w d$		(c)

Notes:
1. Axial load N_u is positive for compression and negative for tension.
2. V_c shall not be taken less than zero.

COMMENTARY

R22.5.4 *Composite concrete members*

R22.5.4.1 The scope of Chapter 22 includes composite concrete members. Composite structural steel-concrete beams are not covered in the Code. Design provisions for such composite members are covered in ANSI/AISC 360.

R22.5.5 V_c *for nonprestressed members*

R22.5.5.1 Test results for nonprestressed members without shear reinforcement indicate that measured shear strength, attributed to concrete, does not increase in direct proportion with member depth. This phenomenon is often referred to as "size effect." For example, if member depth doubles, shear at failure for the deeper beam may be less than twice the shear at failure for the shallower beam (Sneed and Ramirez 2010). Research (Angelakos et al. 2001; Lubell et al. 2004; Brown et al. 2006; Becker and Buettner 1985; Anderson 1978; Bažant et al. 2007) has shown that shear stress at failure is lower for beams and slabs with increased depth and a reduced area of longitudinal reinforcement. Changes were made in the ACI 318-19 code (Kuchma et al. 2019) to account for size effect and the effect of longitudinal reinforcement ratio on shear strength of members.

In Table 22.5.5.1, for $A_v > A_{v,min}$, either equation for V_c may be used. Equation (a) is provided as a simpler option.

When calculating V_c by Table 22.5.5.1, an axial tension force can cause V_c to have a negative value. In those cases, the Code specifies that V_c should be taken equal to zero.

The criteria column in Table 22.5.5.1 references $A_{v,min}$, which is defined in 7.6.3.3 for one-way slabs, 9.6.3.4 for beams, and 10.6.2.2 for columns and referenced throughout the Code.

CODE

22.5.5.1.1 V_c shall not be taken greater than $5\lambda\sqrt{f_c'}b_wd$. V_c need not be taken less than $\lambda\sqrt{f_c'}b_wd$ except in cases (a) or (b):

(a) elements subjected to net axial tension
(b) if 18.6.5.2 or 18.7.6.2.1 apply.

22.5.5.1.2 In Table 22.5.5.1, the value of $N_u/6A_g$ shall not be taken greater than $0.05f_c'$.

22.5.5.1.3 The size effect modification factor, λ_s, shall be determined by

$$\lambda_s = \frac{2}{1+\frac{d}{10}} \leq 1 \qquad (22.5.5.1.3)$$

22.5.5.1.4 For nonprestressed beams and one-way slabs constructed with steel fiber-reinforced concrete, conforming to 26.4.1.6.1(a), 26.4.2.2(h), and 26.12.8.1(a), V_c shall be the greater of Eq. (a) and 1.3 times Equation (b) of Table 22.5.5.1.

22.5.6 V_c *for prestressed members*

22.5.6.1 This section shall apply to the calculation of V_c for post-tensioned and pretensioned members in regions where the effective force in the prestressed reinforcement is fully transferred to the concrete. For regions of pretensioned members where the effective force in the prestressed reinforcement is not fully transferred to the concrete, 22.5.7 shall govern the calculation of V_c.

22.5.6.2 For prestressed members, V_c shall be permitted to be the lesser of V_{ci} calculated in accordance with 22.5.6.2.1 and V_{cw} calculated in accordance with 22.5.6.2.2 or 22.5.6.2.3.

COMMENTARY

R22.5.5.1.1 Because of the compounded effect of λ_s and ρ_w in Eq. 22.5.5.1c, V_c may tend to zero for large, lightly reinforced concrete members. The lower bound $\lambda\sqrt{f_c'}b_wd$ is intended to result in a concrete shear strength consistent with earlier, successful practice.

R22.5.5.1.3 The parameters within the size effect modification factor, λ_s, are consistent with fracture mechanics theory for reinforced concrete (Bažant et al. 2007; Frosch et al. 2017).

R22.5.5.1.4 The use of Eq. (a) and (b) in Table 22.5.5.1 for steel fiber-reinforced concrete and associated dosages in Chapter 26 are supported by experimental results (Dinh et al. 2010; Shoaib et al. 2014; Zarrinpour and Chao 2017). The presence of steel fibers is not considered in determining the value of $A_{v,min}$. Requirements for steel fibers are included in Chapter 26.

R22.5.6 V_c *for prestressed members*

R22.5.6.1 Editions of the Code prior to 2025 included an approximate method for the calculation of V_c for prestressed members. This method was removed because the method in Section 22.5.6.2 is a better predictor of shear strength.

R22.5.6.2 Two types of inclined cracking occur in concrete beams: web-shear cracking and flexure-shear cracking. These two types of inclined cracking are illustrated in Fig. R22.5.6.2.

Web-shear cracking begins from an interior point in a member when the principal tensile stresses exceed the tensile strength of the concrete. Flexure-shear cracking is initiated by flexural cracking. When flexural cracking occurs, the shear stresses in the concrete above the crack are increased. The flexure-shear crack develops when the combined shear and flexural-tensile stress exceeds the tensile strength of the concrete.

The nominal shear strength provided by the concrete, V_c, is assumed equal to the lesser of V_{ci} and V_{cw}. The derivations of Eq. (22.5.6.2.1a) and Eq. (22.5.6.2.2) are summarized in SP-10 (1965).

CODE

22.5.6.2.1 The flexure-shear strength V_{ci} shall be calculated by (a) but need not be taken less than (b) or (c):

(a) $$V_{ci} = 0.6\lambda\sqrt{f_c'}\,b_w d_p + V_d + \frac{V_i M_{cre}}{M_{max}} \quad (22.5.6.2.1a)$$

(b) For members with $A_{ps}f_{se} < 0.4(A_{ps}f_{pu} + A_s f_y)$,

$$V_{ci} = 1.7\lambda\sqrt{f_c'}\,b_w d \quad (22.5.6.2.1b)$$

(c) For members with $A_{ps}f_{se} \geq 0.4(A_{ps}f_{pu} + A_s f_y)$,

$$V_{ci} = 2\lambda\sqrt{f_c'}\,b_w d \quad (22.5.6.2.1c)$$

where d_p need not be taken less than $0.80h$, the values of M_{max} and V_i shall be calculated from the load combinations causing maximum factored moment to occur at section considered, and M_{cre} shall be calculated by:

$$M_{cre} = \left(\frac{I}{y_t}\right)\left(6\lambda\sqrt{f_c'} + f_{pe} - f_d\right) \quad (22.5.6.2.1d)$$

COMMENTARY

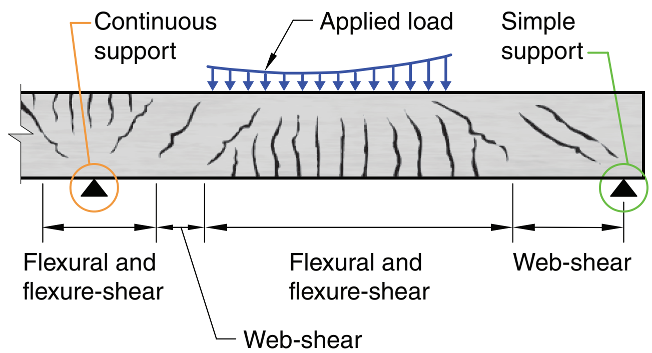

Fig. R22.5.6.3—Types of cracking in concrete beams.

R22.5.6.2.1 In deriving Eq. (22.5.6.2.1a), it was assumed that V_{ci} is the sum of the shear required to cause a flexural crack at the section in question given by:

$$V = \frac{V_i M_{cre}}{M_{max}} \quad (R22.5.6.2.1a)$$

plus an additional increment of shear required to change the flexural crack to a flexure-shear crack. The externally applied factored loads, from which V_i and M_{max} are determined, include superimposed dead load and live load. In calculating M_{cre} for substitution into Eq. (22.5.6.2.1a), I and y_t are the properties of the section resisting the externally applied loads.

For a composite concrete member, where part of the dead load is resisted by only a part of the section, appropriate section properties should be used to calculate f_d. The shear due to dead loads, V_d, and that due to other loads, V_i, are separated in this case. V_d is then the total shear force due to unfactored dead load acting on that part of the section resisting the dead loads acting prior to composite action plus the unfactored superimposed dead load acting on the composite member. The terms V_i and M_{max} may be taken as

$$V_i = V_u - V_d \quad (R22.5.6.2.1b)$$

$$M_{max} = M_u - M_d \quad (R22.5.6.2.1c)$$

where V_u and M_u are the factored shear and moment due to the total factored loads, and M_d is the moment due to unfactored dead load (the moment corresponding to f_d).

For noncomposite, uniformly loaded beams, the total cross section resists all the shear, and the live and dead load shear force diagrams are similar. In this case, Eq. (22.5.6.2.1a) and Eq. (22.5.6.2.1d) reduce to

$$V_{ci} = 0.6\lambda\sqrt{f_c'}\,b_w d + \frac{V_u M_{ct}}{M_u} \quad (R22.5.6.2.1d)$$

where

$$M_{ct} = (I/y_t)(6\lambda\sqrt{f_c'} + f_{pe}) \quad (R22.5.6.2.1e)$$

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22.5.6.2.2 The web-shear strength V_{cw} shall be calculated by:

$$V_{cw} = \left(3.5\lambda\sqrt{f_c'} + 0.3f_{pc}\right)b_w d_p + V_p \qquad (22.5.6.2.2)$$

where d_p need not be taken less than $0.80h$, and V_p is the vertical component of the effective prestress.

22.5.6.2.3 As an alternative to 22.5.6.2.2, it shall be permitted to calculate V_{cw} as the shear force corresponding to dead load plus live load that results in a principal tensile stress of $4\lambda\sqrt{f_c'}$ at location (a) or (b):

(a) Where the centroidal axis of the prestressed cross section is in the web, the principal tensile stress shall be calculated at the centroidal axis.
(b) Where the centroidal axis of the prestressed cross section is in the flange, the principal tensile stress shall be calculated at the intersection of the flange and the web.

22.5.6.2.4 In composite concrete members, the principal tensile stress shall be calculated at the location specified in 22.5.6.2.3 for the composite section, considering superposition of stresses calculated cross sections that resist the corresponding loads.

22.5.7 V_c *for pretensioned members in regions of reduced prestress force*

22.5.7.1 When calculating V_c, the transfer length of prestressed reinforcement, ℓ_{tr}, shall be assumed to be $50d_b$ for strand and $100d_b$ for wire.

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The cracking moment M_{ct} in the two preceding equations represents the total moment, including dead load, required to cause cracking at the extreme fiber in tension. This is not the same as M_{cre} in Eq. (22.5.6.2.1a) where the cracking moment is that due to all loads except the dead load. In Eq. (22.5.6.2.1a), the dead load shear is added as a separate term.

M_u is the factored moment on the beam at the section under consideration, and V_u is the factored shear force occurring simultaneously with M_u. Because the same section properties apply to both dead and live load stresses, there is no need to calculate dead load stresses and shears separately. M_{ct} reflects the total stress change from effective prestress to a tension of $6\lambda\sqrt{f_c'}$, assumed to cause flexural cracking.

R22.5.6.2.2 Equation (22.5.6.2.2) is based on the assumption that web-shear cracking occurs at a shear level causing a principal tensile stress of approximately $4\lambda\sqrt{f_c'}$ at the centroidal axis of the cross section. V_p is calculated from the effective prestress force without load factors.

R22.5.6.2.4 Generally, in unshored construction the principal tensile stresses due to dead load are caused before composite action and principal tensile stresses due to live load are caused after composite action is developed in a member. In shored construction the principal tensile stresses due to both the dead load and live load are caused after composite action is developed.

R22.5.7 V_c *for pretensioned members in regions of reduced prestress force*

R22.5.7.1 The effect of the reduced prestress near the ends of pretensioned beams on the shear strength should be taken into account. Provision 22.5.7.2 relates to the reduced shear strength at sections within the transfer length of prestressed reinforcement when bonding of prestressed reinforcement extends to the end of the member. Provision 22.5.7.3 relates to the reduced shear strength at sections within the length over which some of the prestressed reinforcement is not bonded to the concrete, or within the transfer length of

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22.5.7.2 If bonding of strands extends to the end of the member, (a) and (b) shall apply:

(a) The effective prestress force shall be assumed to vary linearly from zero at the end of the prestressed reinforcement to a maximum at a distance ℓ_{tr} from the end of the prestressed reinforcement.
(b) The reduced effective prestress force shall be used to calculate V_{cw} in 22.5.6.2.

22.5.7.3 If bonding of strands does not extend to the end of the member, (a) and (b) shall apply:

(a) The effective prestress force shall be assumed to vary linearly from zero at the point where bonding commences to a maximum at a distance ℓ_{tr} from that point.
(b) The reduced effective prestress force shall be used to calculate V_c in accordance with 22.5.6.2.

22.5.8 *One-way shear reinforcement*

22.5.8.1 At each section where $V_u > \phi V_c$, transverse reinforcement shall be provided such that Eq. (22.5.8.1) is satisfied.

$$V_s \geq \frac{V_u}{\phi} - V_c \qquad (22.5.8.1)$$

22.5.8.2 For one-way members reinforced with transverse reinforcement, V_s shall be calculated in accordance with 22.5.8.5.

22.5.8.3 For one-way members reinforced with bent-up longitudinal bars, V_s shall be calculated in accordance with 22.5.8.6.

22.5.8.4 If more than one type of shear reinforcement is provided to reinforce the same portion of a member, V_s shall be the sum of the V_s values for the various types of shear reinforcement.

22.5.8.5 *One-way shear strength provided by transverse reinforcement*

22.5.8.5.1 In nonprestressed and prestressed members, shear reinforcement satisfying (a), (b), or (c) shall be permitted:

(a) Stirrups, ties, or hoops perpendicular to longitudinal axis of member
(b) Welded wire reinforcement with wires located perpendicular to longitudinal axis of member

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the prestressed reinforcement for which bonding does not extend to the end of the beam.

R22.5.8 *One-way shear reinforcement*

R22.5.8.2 Provisions of 22.5.8.5 apply to all types of transverse reinforcement, including stirrups, ties, hoops, crossties, and spirals.

R22.5.8.5 *One-way shear strength provided by transverse reinforcement*

R22.5.8.5.1 Design of shear reinforcement is based on a modified truss analogy. In the truss analogy, the force in vertical ties is resisted by shear reinforcement. Shear reinforcement needs to be designed to resist only the shear exceeding that which causes inclined cracking, provided the diagonal members in the truss are assumed to be inclined at 45 degrees. The concrete is assumed to contribute to the shear capacity through resistance across the concrete compressive

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(c) Spiral reinforcement

22.5.8.5.2 Inclined stirrups making an angle of at least 45 degrees with the longitudinal axis of the member and crossing the plane of the potential shear crack shall be permitted to be used as shear reinforcement in nonprestressed members.

22.5.8.5.3 V_s for shear reinforcement in 22.5.8.5.1 shall be calculated by:

$$V_s = \frac{A_v f_{yt} d}{s} \qquad (22.5.8.5.3)$$

where s is the spiral pitch or the longitudinal spacing of the shear reinforcement, and A_v is given in 22.5.8.5.5 or 22.5.8.5.6.

22.5.8.5.4 V_s for shear reinforcement in 22.5.8.5.2 shall be calculated by:

$$V_s = \frac{A_v f_{yt}(\sin\alpha + \cos\alpha)d}{s} \qquad (22.5.8.5.4)$$

where α is the angle between the inclined stirrups and the longitudinal axis of the member, s is measured parallel to the longitudinal reinforcement, and A_v is given in 22.5.8.5.5.

22.5.8.5.5 For each rectangular tie, stirrup, hoop, or crosstie, A_v shall be the effective area of all bar legs or wires within spacing s.

22.5.8.5.6 For each circular tie or spiral, A_v shall be two times the area of the bar or wire within spacing s.

22.5.8.6 *One-way shear strength provided by bent-up longitudinal bars*

22.5.8.6.1 The center three-fourths of the inclined portion of bent-up longitudinal bars shall be permitted to be used as shear reinforcement in nonprestressed members if the angle α between the bent-up bars and the longitudinal axis of the member is at least 30 degrees.

22.5.8.6.2 If shear reinforcement consists of a single bar or a single group of parallel bars having an area A_v, all bent

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zone, aggregate interlock, and dowel action in an amount equivalent to that which caused inclined cracking.

Equations (22.5.8.5.3), (22.5.8.5.4), and (22.5.8.6.2a) are presented in terms of nominal shear strength provided by shear reinforcement, V_s. Where shear reinforcement perpendicular to the axis of the member is used, the required area of shear reinforcement, A_v, and its spacing, s, are calculated by

$$\frac{A_v}{s} = \frac{V_u - \phi V_c}{\phi f_{yt} d} \qquad (R22.5.8.5)$$

R22.5.8.5.2 Although inclined stirrups crossing the plane of the potential shear cracks are permitted, their use is not appropriate where the direction of net shear reverses due to changes in transient load.

R22.5.8.5.4 To be effective, it is critical that inclined stirrups cross potential shear cracks. If the inclined stirrups are generally oriented parallel to the potential shear cracks, the stirrups provide no shear strength.

R22.5.8.5.6 Although the transverse reinforcement in a circular section may not consist of straight legs, tests indicate that Eq. (22.5.8.5.3) is conservative if d is taken as defined in 22.5.2.2 (Faradji and Diaz de Cossio 1965; Khalifa and Collins 1981).

R22.5.8.6 *One-way shear strength provided by bent-up longitudinal bars*

R22.5.8.6.1 To be effective, it is critical that the inclined portion of the bent-up longitudinal bar cross potential shear cracks. If the inclined bars are generally oriented parallel to the potential shear cracks, the bars provide no shear strength.

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the same distance from the support, V_s shall be the lesser of (a) and (b):

(a) $V_s = A_v f_y \sin\alpha$ (22.5.8.6.2a)
(b) $V_s = 3\sqrt{f_c'} b_w d$ (22.5.8.6.2b)

where α is the angle between bent-up reinforcement and longitudinal axis of the member.

22.5.8.6.3 If shear reinforcement consists of a series of parallel bent-up bars or groups of parallel bent-up bars at different distances from the support, V_s shall be calculated by Eq. (22.5.8.5.4).

22.6—Two-way shear strength

22.6.1 *General*

22.6.1.1 Provisions 22.6.1 through 22.6.8 apply to the nominal two-way shear strength of members with and without shear reinforcement.

22.6.1.2 Nominal two-way shear strength for members without shear reinforcement shall be calculated by

$$v_n = v_c \quad (22.6.1.2)$$

22.6.1.3 Nominal two-way shear strength for members with shear reinforcement shall be calculated by

$$v_n = v_c + v_s \quad (22.6.1.3)$$

22.6.1.4 Two-way shear shall be resisted by a section with a depth d and an assumed critical perimeter b_o as defined in 22.6.4.

22.6.1.5 v_c for two-way shear shall be calculated in accordance with 22.6.5. For two-way shear in members with shear reinforcement, v_c shall not exceed the limits in 22.6.6.1.

22.6.1.6 For calculation of v_c, λ shall be in accordance with 19.2.4.

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R22.6—Two-way shear strength

Factored two-way shear stress due to shear and moment transfer is calculated in accordance with the requirements of 8.4.4. Section 22.6 provides requirements for determining nominal shear strength, either without shear reinforcement or with shear reinforcement in the form of stirrups or headed shear studs. Factored shear demand and strength are calculated in terms of stress, permitting superposition of effects from direct shear and moment transfer.

Design provisions for shearheads have been eliminated from the Code because this type of shear reinforcement is seldom used in current practice. Shearheads may be designed following the provisions of ACI CODE-318-14.

R22.6.1 *General*

R22.6.1.4 The critical section perimeter b_o is defined in 22.6.4.

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22.6.1.7 For two-way shear in members reinforced with single- or multiple-leg stirrups, v_s shall be calculated in accordance with 22.6.7.

22.6.1.8 For two-way shear in members reinforced with headed shear stud reinforcement, v_s shall be calculated in accordance with 22.6.8.

22.6.2 *Effective depth*

22.6.2.1 For calculation of v_c and v_s for two-way shear, d shall be the average of the effective depths in the two orthogonal directions.

22.6.2.2 For two-way shear in prestressed members, d need not be taken less than $0.8h$.

22.6.3 *Limiting material strengths*

22.6.3.1 The value of $\sqrt{f_c'}$ used to calculate v_c for two-way shear shall not exceed 100 psi.

22.6.3.2 The value of f_{yt} used to calculate v_s shall not exceed the limits in 20.2.2.4.

22.6.4 *Critical sections for two-way shear*

22.6.4.1 For two-way shear, critical sections shall be located so that the perimeter b_o is a minimum but need not be closer than $d/2$ to (a) and (b):

(a) Edges or corners of columns, concentrated loads, or reaction areas
(b) Changes in slab or footing thickness, such as edges of capitals, drop panels, or shear caps

22.6.4.1.1 For square or rectangular columns, concentrated loads, or reaction areas, critical sections for two-way shear in accordance with 22.6.4.1(a) and (b) shall be permitted to be defined assuming straight sides.

22.6.4.1.2 For a circular or regular polygon-shaped column, critical sections for two-way shear in accordance

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R22.6.3 *Limiting material strengths*

R22.6.3.1 There are limited test data on the two-way shear strength of high-strength concrete slabs. Until more experience is obtained for two-way shear in slabs constructed with concretes that have compressive strengths greater than 10,000 psi, it is prudent to limit $\sqrt{f_c'}$ to 100 psi for the calculation of shear strength.

R22.6.3.2 The upper limit of 60,000 psi on the value of f_{yt} used in design is intended to control cracking.

R22.6.4 *Critical sections for two-way shear*

R22.6.4.1 The critical section defined in 22.6.4.1(a) for two-way shear in slabs and footings follows the perimeter at the edge of the loaded area (Joint ACI-ASCE Committee 326 1962). Loaded area for two-way shear in slabs and footings includes columns, concentrated loads, and reaction areas. An idealized critical section located a distance $d/2$ from the periphery of the loaded area is considered.

For members of uniform thickness without shear reinforcement, it is sufficient to check shear using one section. For slabs with changes in thickness or with shear reinforcement, it is necessary to check shear at multiple sections as defined in 22.6.4.1(a) and (b) and 22.6.4.2.

For columns near an edge or corner, the critical perimeter may extend to the edge of the slab.

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with 22.6.4.1(a) and (b) shall be permitted to be defined assuming a square column of equivalent area.

22.6.4.2 For members reinforced for two-way shear with headed shear reinforcement or single- or multi-leg stirrups, a critical section with perimeter b_o located $d/2$ beyond the outermost peripheral line of shear reinforcement shall also be considered. The shape of this critical section shall be a polygon selected to minimize b_o.

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R22.6.4.2 For members reinforced for two-way shear with stirrup or headed stud shear reinforcement, it is required to check shear stress in concrete at a critical section located a distance $d/2$ beyond the point where shear reinforcement is discontinued. Calculated shear stress at this section must not exceed the limits given in expressions (a) and (e) in Table 22.6.6.1. The shape of this outermost critical section should correspond to the minimum value of b_o, as depicted in Fig. R22.6.4.2a, b, and c. Note that these figures depict slabs reinforced with stirrups. The shape of the outermost critical section is similar for slabs with headed shear reinforcement. The square or rectangular critical sections described in 22.6.4.1.1 will not result in the minimum value of b_o for the cases depicted in these figures. Additional critical section checks are required at a distance $d/2$ beyond any point where variations in shear reinforcement occur, such as changes in size, spacing, or configuration.

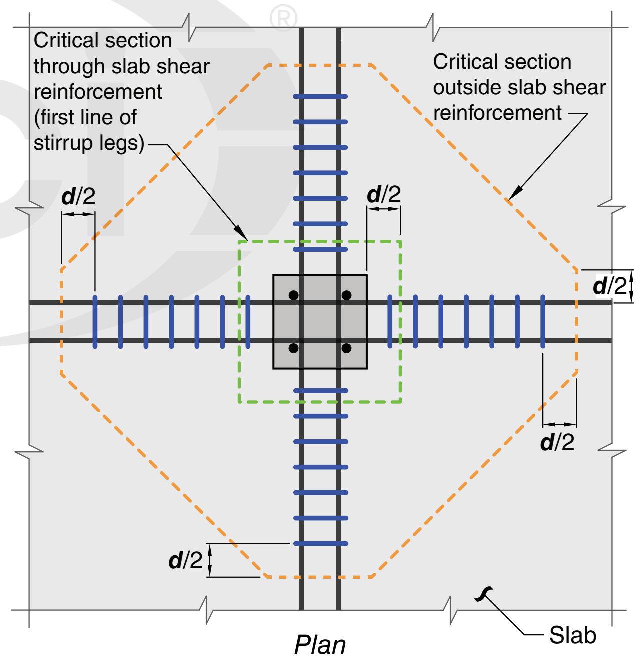

Fig. R22.6.4.2a—Critical sections for two-way shear in slab with shear reinforcement at interior column.

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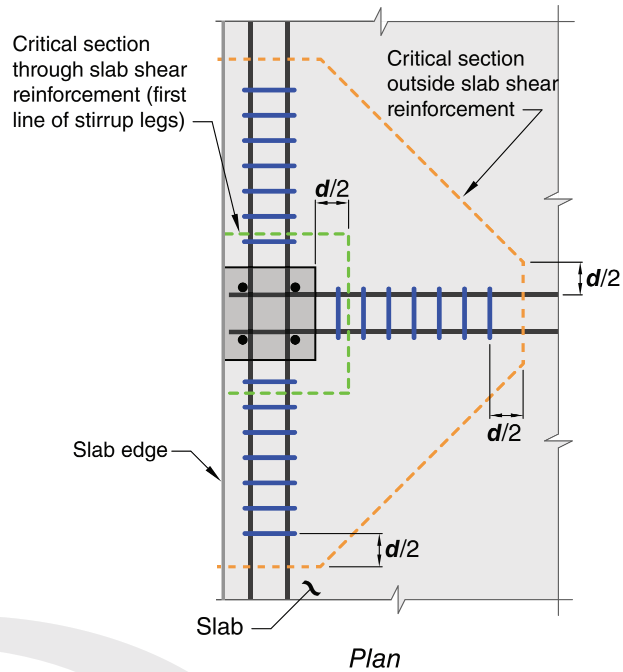

Fig. R22.6.4.2b—Critical sections for two-way shear in slab with shear reinforcement at edge column.

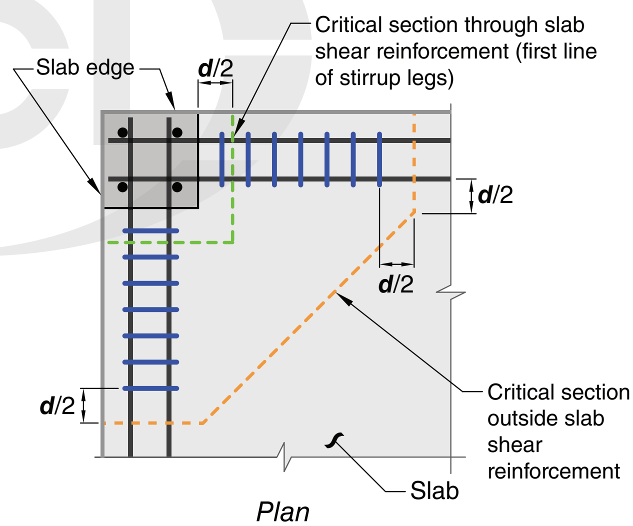

Fig. R22.6.4.2c—Critical sections for two-way shear in slab with shear reinforcement at corner column.

22.6.4.3 If an opening is located closer than $4h$ from the periphery of a column, concentrated load, or reaction area, the portion of b_o enclosed by straight lines projecting from the centroid of the column, concentrated load or reaction area and tangent to the boundaries of the opening shall be considered ineffective.

R22.6.4.3 Provisions for design of openings in slabs (and footings) were developed in Joint ACI-ASCE Committee 326 (1962). The locations of the effective portions of the critical section near typical openings and free edges are shown by the dashed lines in Fig. R22.6.4.3. Research (Joint ACI-ASCE Committee 426 1974) has confirmed that these provisions are conservative.

Research (Genikomsou and Polak 2017) has shown that when openings are located at distances greater than $4d$ from the periphery of a column, the punching shear strength is the same as that for a slab without openings.

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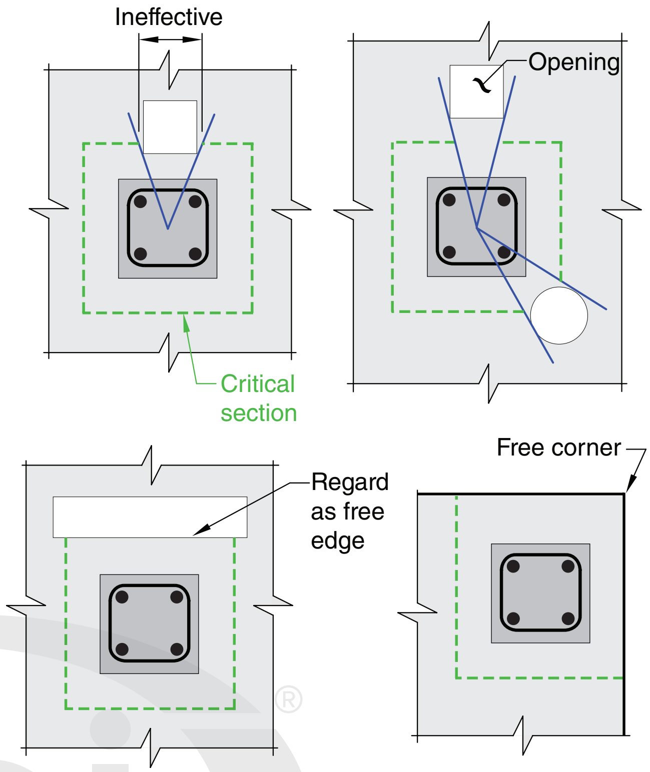

Note: Openings shown are located within 10***h*** of column.

Fig. R22.6.4.3—Effect of openings and free edges (effective perimeter shown with dashed lines).

22.6.5 *Two-way shear strength provided by concrete in members without shear reinforcement*

22.6.5.1 For nonprestressed members, v_c shall be calculated in accordance with 22.6.5.2. For prestressed members, v_c shall be calculated in accordance with (a) or (b):

(a) 22.6.5.2
(b) 22.6.5.5, if the conditions of 22.6.5.4 are satisfied

22.6.5.2 v_c shall be calculated in accordance with Table 22.6.5.2.

Table 22.6.5.2—Two-way shear strength v_c for members without shear reinforcement

	v_c	
Least of (a), (b), and (c):	$4\lambda_s\lambda\sqrt{f_c'}$	(a)
	$\left(2+\frac{4}{\beta}\right)\lambda_s\lambda\sqrt{f_c'}$	(b)
	$\left(2+\frac{\alpha_s d}{b_o}\right)\lambda_s\lambda\sqrt{f_c'}$	(c)

Notes: (i) λ_s is the size effect factor given in 22.5.5.1.3. (ii) β is the ratio of long to short sides of the column, concentrated load, or reaction area. (iii) α_s is given in 22.6.5.3.

R22.6.5 *Two-way shear strength provided by concrete in members without shear reinforcement*

R22.6.5.2 Experimental evidence indicates that the measured two-way concrete shear strength of members without shear reinforcement does not increase in direct proportion with member depth. This phenomenon is referred to as the "size effect." The modification factor λ_s accounts for the dependence of two-way shear strength of slabs on effective depth.

For nonprestressed two-way slabs without a minimum amount of shear reinforcement and with d > 10 in., the size effect specified in 22.5.5.1.3 reduces the shear strength of two-way slabs below $4\sqrt{f_c'}b_od$ (Hawkins and Ospina 2017; Dönmez and Bažant 2017).

For square columns, the stress corresponding to the nominal two-way shear strength provided by concrete in slabs subjected to bending in two directions is limited to $4\lambda_s\sqrt{f_c'}$. However, tests (Joint ACI-ASCE Committee 426 1974)

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have indicated that the value of $4\lambda_s\sqrt{f_c'}$ is unconservative when the ratio β of the lengths of the long and short sides of a rectangular column or loaded area is larger than 2.0. In such cases, the actual shear stress on the critical section at punching shear failure varies from a maximum of approximately $4\lambda_s\sqrt{f_c'}$ around the corners of the column or loaded area, down to $2\lambda_s\sqrt{f_c'}$ or less along the long sides between the two end sections. Other tests (Vanderbilt 1972) indicate that v_c decreases as the ratio b_o/d increases. Expressions (b) and (c) in Table 22.6.5.2 were developed to account for these two effects.

For shapes other than rectangular, β is taken to be the ratio of the longest overall dimension of the effective loaded area to the largest overall perpendicular dimension of the effective loaded area, as illustrated for an L-shaped reaction area in Fig. R22.6.5.2. The effective loaded area is that area totally enclosing the actual loaded area, for which the perimeter is a minimum.

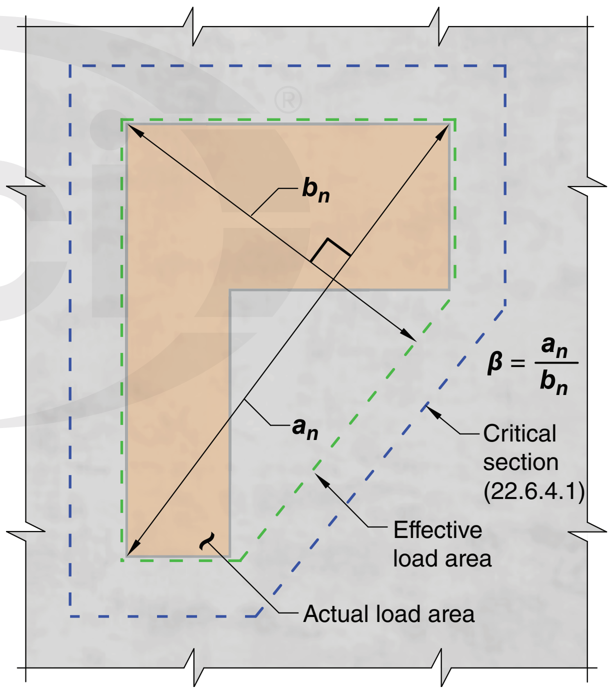

Fig. R22.6.5.2—Value of β for a nonrectangular loaded area.

22.6.5.3 The value of α_s is 40 for interior columns, 30 for edge columns, and 20 for corner columns.

R22.6.5.3 The terms "interior columns," "edge columns," and "corner columns" in this provision refer to critical sections with a continuous slab on four, three, and two sides, respectively.

22.6.5.4 For two-way shear in prestressed members, it shall be permitted to calculate v_c using 22.6.5.5, provided that (a) through (c) are satisfied:

(a) Bonded reinforcement is provided in accordance with 8.6.2.3 and 8.7.5.3

(b) No portion of the column cross section is closer to a discontinuous edge than four times the slab thickness h

R22.6.5.4 For prestressed members, modified forms of expressions (b) and (c) in Table 22.6.5.2 are specified. Research (ACI PRC-423.3) indicates that the shear strength of two-way prestressed slabs around interior columns is conservatively calculated by the expressions in 22.6.5.5, where v_c corresponds to a diagonal tension failure of the concrete initiating at the critical section defined in 22.6.4.1. The mode of failure differs from a punching shear failure around the perimeter of the loaded

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(c) Effective prestress f_{pc} in each direction is not less than 125 psi

22.6.5.5 For two-way shear in prestressed members conforming to 22.6.5.4, v_c shall be permitted to be the lesser of (a) and (b)

(a) $v_c = 3.5\lambda\sqrt{f_c'} + 0.3f_{pc} + \dfrac{V_p}{b_o d}$ (22.6.5.5a)

(b) $v_c = \left(1.5 + \dfrac{\alpha_s d}{b_o}\right)\lambda\sqrt{f_c'} + 0.3f_{pc} + \dfrac{V_p}{b_o d}$ (22.6.5.5b)

where α_s is given in 22.6.5.3; the value of f_{pc} is the average of f_{pc} in the two directions and shall not exceed 500 psi; V_p is the vertical component of all effective prestress forces crossing the critical section; and the value of $\sqrt{f_c'}$ shall not exceed 70 psi.

22.6.6 *Two-way shear strength provided by concrete in members with shear reinforcement*

22.6.6.1 For members where shear reinforcement is required to resist two-way shear, v_c at critical sections shall be calculated in accordance with Table 22.6.6.1.

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area of a nonprestressed slab calculated using expression (b) in Table 22.6.5.2. Consequently, the expressions in 22.6.5.5 differ from those for nonprestressed slabs. Values for $\sqrt{f_c'}$ and f_{pc} are restricted in design due to limited test data available beyond the specified limits. When calculating f_{pc}, loss of prestress due to restraint of the slab by structural walls and other structural elements should be taken into account.

R22.6.6 *Two-way shear strength provided by concrete in members with shear reinforcement*

Critical sections for members with shear reinforcement are defined in 22.6.4.1 for the sections adjacent to the column, concentrated load, or reaction area, and 22.6.4.2 for the section located just beyond the outermost peripheral line of stirrup or headed shear stud reinforcement. Values of maximum v_c for these critical sections are given in Table 22.6.6.1. Limiting values of v_u for the critical sections defined in 22.6.4.1 are given in Table 22.6.6.3.

The maximum v_c and limiting value of v_u at the innermost critical section (defined in 22.6.4.1) are higher where headed shear stud reinforcement is provided than the case where stirrups are provided (refer to R8.7.7). Maximum v_c values at the critical sections defined in 22.6.4.2 beyond the outermost peripheral line of shear reinforcement are independent of the type of shear reinforcement provided.

R22.6.6.1 For slabs with stirrups, the maximum value of v_c is taken as $2\lambda_s\lambda\sqrt{f_c'}$ because the stirrups resist all the shear beyond that at inclined cracking (which occurs at approximately half the capacity of a slab without shear reinforcement (that is, $0.5 \times 4\lambda_s\lambda\sqrt{f_c'} = 2\lambda_s\lambda\sqrt{f_c'}$) (Hawkins 1974). The higher value of v_c for slabs with headed shear stud reinforcement is based on research (Elgabry and Ghali 1987).

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Table 22.6.6.1—Two-way shear strength v_c for members with shear reinforcement

Type of shear reinforcement	Critical sections	v_c		
Stirrups	All	$2\lambda_s\lambda\sqrt{f_c'}$		(a)
Headed shear stud reinforcement	According to 22.6.4.1	Least of (b), (c), and (d):	$3\lambda_s\lambda\sqrt{f_c'}$	(b)
			$\left(2+\frac{4}{\beta}\right)\lambda_s\lambda\sqrt{f_c'}$	(c)
			$\left(2+\frac{\alpha_s d}{b_o}\right)\lambda_s\lambda\sqrt{f_c'}$	(d)
	According to 22.6.4.2		$2\lambda_s\lambda\sqrt{f_c'}$	(e)

Notes: (i) λ_s is the size effect factor given in 22.5.5.1.3. (ii) β is the ratio of long to short sides of the column, concentrated load, or reaction area. (iii) α_s is given in 22.6.5.3.

22.6.6.2 It shall be permitted to take λ_s as 1.0 if (a) or (b) is satisfied:

(a) Stirrups are designed and detailed in accordance with 8.7.6 and $A_v/s \geq 2\sqrt{f_c'}b_o/f_{yt}$.
(b) Smooth headed shear stud reinforcement with stud shaft length not exceeding 10 in. is designed and detailed in accordance with 8.7.7 and $A_v/s \geq 2\sqrt{f_c'}b_o/f_{yt}$.

22.6.6.3 For members with shear reinforcement, effective depth shall be selected such that two-way shear stress v_u calculated at critical sections does not exceed the values in Table 22.6.6.3.

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R22.6.6.2 The size effect in slabs with $d > 10$ in. can be mitigated if a minimum amount of shear reinforcement is provided. The ability of ordinary (smooth) headed shear stud reinforcement to effectively mitigate the size effect on the two-way shear strength of slabs may be compromised if studs longer than 10 in. are used. Until experimental evidence becomes available, it is not permitted to use λ_s equal to 1.0 for slabs with $d > 10$ in. without headed shear stud reinforcement with stud shaft length not exceeding 10 in. Stacking or "piggybacking" of headed shear studs, as shown in Fig. R22.6.6.2, introduces an intermediate head that contributes to further anchor the stacked stud.

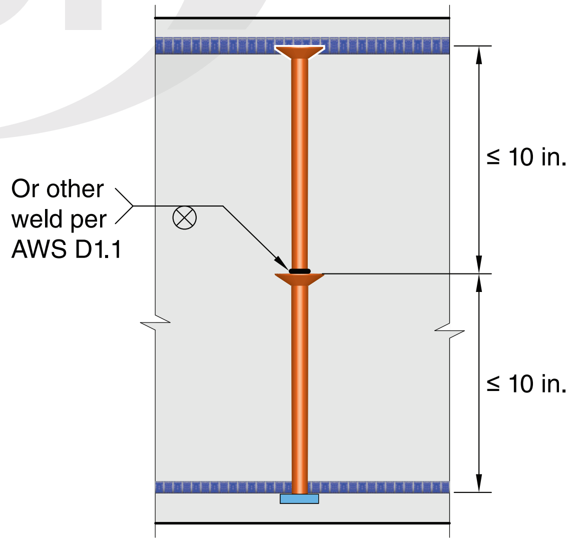

Fig. R22.6.6.2—Stacking (piggybacking) of headed shear stud reinforcement.

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Table 22.6.6.3—Maximum two-way shear stress v_u for members with shear reinforcement

Type of shear reinforcement	Maximum v_u at critical sections defined in 22.6.4.1	
Stirrups	$\phi 6\sqrt{f_c'}$	(a)
Headed shear stud reinforcement	$\phi 8\sqrt{f_c'}$	(b)

22.6.7 *Two-way shear strength provided by single- or multiple-leg stirrups*

22.6.7.1 Single- or multiple-leg stirrups fabricated from bars or wires shall be permitted to be used as shear reinforcement in slabs and footings satisfying (a) and (b):

(a) d is at least 6 in.
(b) d is at least $16d_b$, where d_b is the diameter of the stirrups

22.6.7.2 For members with stirrups, v_s shall be calculated by:

$$v_s = \frac{A_v f_{yt}}{b_o s} \quad (22.6.7.2)$$

where A_v is the sum of the area of all legs of reinforcement on one peripheral line that is geometrically similar to the perimeter of the column section, and s is the spacing of the peripheral lines of shear reinforcement in the direction perpendicular to the column face.

22.6.8 *Two-way shear strength provided by headed shear stud reinforcement*

22.6.8.1 Headed shear stud reinforcement shall be permitted to be used as shear reinforcement in slabs and footings if the placement and geometry of the headed shear stud reinforcement satisfies 8.7.7.

22.6.8.2 For members with headed shear stud reinforcement, v_s shall be calculated by:

$$v_s = \frac{A_v f_{yt}}{b_o s} \quad (22.6.8.2)$$

where A_v is the sum of the area of all shear studs on one peripheral line that is geometrically similar to the perimeter of the column section, and s is the spacing of the peripheral

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R22.6.7 *Two-way shear strength provided by single- or multiple-leg stirrups*

R22.6.7.2 Because shear stresses are used for two-way shear in this chapter, shear strength provided by transverse reinforcement is averaged over the cross-sectional area of the critical section.

R22.6.8 *Two-way shear strength provided by headed shear stud reinforcement*

Tests (ACI PRC-421.1) show that headed shear stud reinforcement mechanically anchored as close as practicable to the top and bottom of slabs is effective in resisting punching shear. The critical section beyond the shear reinforcement is generally assumed to have a polygonal shape (refer to Fig. R22.6.4.2a, R22.6.4.2b, and R22.6.4.2c). Equations for calculating shear stresses on such sections are given in ACI PRC-421.1.

R22.6.8.2 Because shear stresses are used for two-way shear in this chapter, shear strength provided by transverse reinforcement is averaged over the cross-sectional area of the critical section.

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lines of headed shear stud reinforcement in the direction perpendicular to the column face.

22.6.8.3 If headed shear stud reinforcement is provided, A_v/s shall satisfy:

$$\frac{A_v}{s} \geq 2\sqrt{f_c'}\frac{b_o}{f_{yt}} \quad (22.6.8.3)$$

22.7—Torsional strength

COMMENTARY

R22.7—Torsional strength

The design for torsion in this section is based on a thin-walled tube space truss analogy. A beam subjected to torsion is idealized as a thin-walled tube with the core concrete cross section in a solid beam neglected as shown in Fig. R22.7(a). Once a reinforced concrete beam has cracked in torsion, its torsional strength is provided primarily by closed stirrups and longitudinal bars located near the surface of the member. In the thin-walled tube analogy, the strength is assumed to be provided by the outer skin of the cross section roughly centered on the closed stirrups. Both hollow and solid sections are idealized as thin-walled tubes both before and after cracking.

In a closed thin-walled tube, the product of the shear stress τ and the wall thickness t at any point in the perimeter is known as the shear flow, $q = \tau t$. The shear flow q due to torsion acts as shown in Fig. R22.7(a) and is constant at all points around the perimeter of the tube. The path along which it acts extends around the tube at midthickness of the walls of the tube. At any point along the perimeter of the tube, the shear stress due to torsion is $\tau = T/(2A_ot)$, where A_o is the gross area enclosed by the shear flow path, shown shaded in Fig. R22.7(b), and t is the thickness of the wall at the point where τ is being calculated. For a hollow member with continuous walls, A_o includes the area of the hole.

The concrete contribution to torsional strength is ignored, and in cases of combined shear and torsion, the concrete contribution to shear strength does not need to be reduced. The design procedure is derived and compared with test results in MacGregor and Ghoneim (1995) and Hsu (1997). Detailed information on the thin-walled tube space truss analogy is provided in ACI PRC-445.1.

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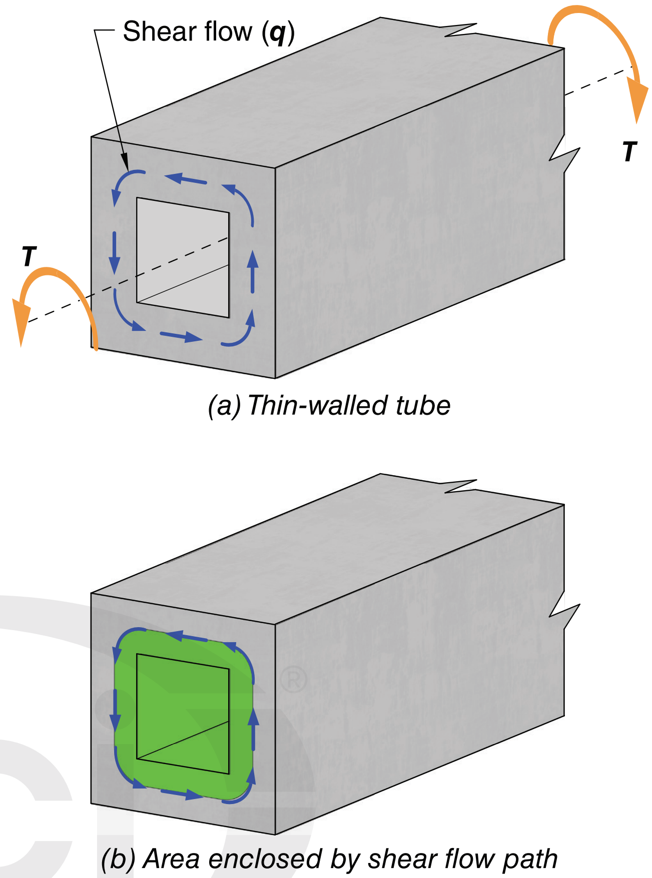

Fig. R22.7—(a) Thin-walled tube; and (b) area enclosed by shear flow path.

22.7.1 *General*

22.7.1.1 This section shall apply to members if $T_u \geq \phi T_{th}$, where ϕ is given in Chapter 21 and threshold torsion T_{th} is given in 22.7.4. If $T_u < \phi T_{th}$, it shall be permitted to neglect torsional effects.

22.7.1.2 Nominal torsional strength shall be calculated in accordance with 22.7.6.

22.7.1.3 For calculation of T_{th} and T_{cr}, λ shall be in accordance with 19.2.4.

R22.7.1 *General*

R22.7.1.1 Torsional moments that do not exceed the threshold torsion T_{th} will not cause a structurally significant reduction in either flexural or shear strength and can be ignored.

22.7.2 *Limiting material strengths*

22.7.2.1 The value of $\sqrt{f_c'}$ used to calculate T_{th} and T_{cr} shall not exceed 100 psi.

22.7.2.2 The values of f_y and f_{yt} for longitudinal and transverse torsional reinforcement shall not exceed the limits in 20.2.2.4.

R22.7.2 *Limiting material strengths*

R22.7.2.1 Because of a lack of test data and practical experience with concretes having compressive strengths greater than 10,000 psi, the Code imposes a maximum value of 100 psi on $\sqrt{f_c'}$ for use in the calculation of torsional strength.

R22.7.2.2 The upper limit of 60,000 psi on the value of f_y and f_{yt} used in design is intended to control diagonal crack width.

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22.7.3 *Factored design torsion*

COMMENTARY

R22.7.3 *Factored design torsion*

In designing for torsion in reinforced concrete structures, two conditions may be identified (Collins and Lampert 1973; Hsu and Burton 1974):

(a) The torsional moment cannot be reduced by redistribution of internal forces (22.7.3.1). This type of torsion is referred to as equilibrium torsion because the torsional moment is required for the structure to be in equilibrium. For this condition, illustrated in Fig. R22.7.3(a), torsional reinforcement must be provided to resist the total design torsional moments.

(b) The torsional moment can be reduced by redistribution of internal forces after cracking (22.7.3.2) if the torsion results from the member twisting to maintain compatibility of deformations. This type of torsion is referred to as compatibility torsion.

For this condition, illustrated in Fig. R22.7.3(b), the torsional stiffness before cracking corresponds to that of the uncracked section according to St. Venant's theory. At torsional cracking, however, a large twist occurs under an essentially constant torsional moment, resulting in a large redistribution of forces in the structure (Collins and Lampert 1973; Hsu and Burton 1974). The cracking torsional moment under combined shear, moment, and torsion corresponds to a principal tensile stress somewhat less than the $4\lambda\sqrt{f_c'}$ used in R22.7.5.

If the torsional moment exceeds the cracking torsional moment (22.7.3.2), a maximum factored torsional moment equal to the cracking torsional moment may be assumed to occur at critical sections. The maximum factored torsional moment has been established to limit the width of torsional cracks.

Provision 22.7.3.2 applies to typical and regular framing conditions. With layouts that impose significant torsional rotations within a limited length of the member, such as a large torsional moment located close to a stiff column, or a column that rotates in the reverse directions because of other loading, a more detailed analysis is advisable.

If the factored torsional moment from an elastic analysis based on uncracked section properties is between ϕT_{th} and ϕT_{cr}, torsional reinforcement should be designed to resist the calculated torsional moments.

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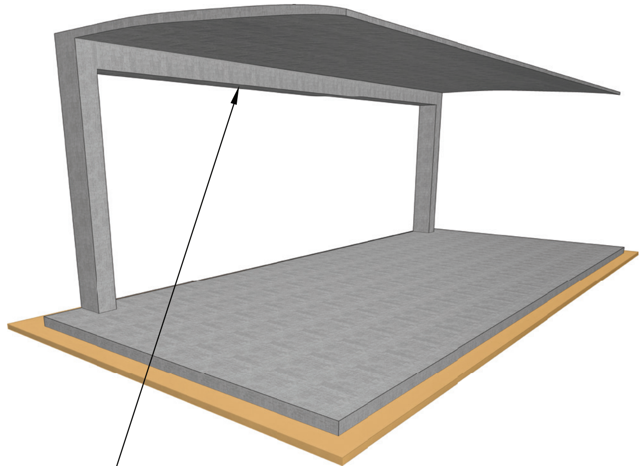

Fig. R22.7.3a—Equilibrium torsion, the design torsional moment may not be reduced (22.7.3.1).

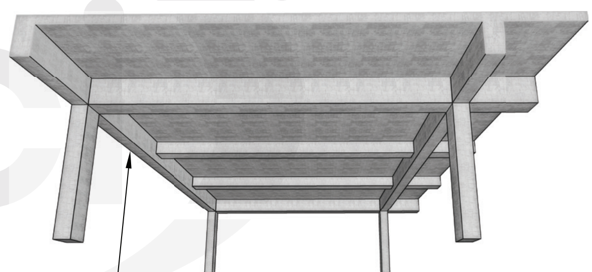

Fig. R22.7.3b—Compatibility torsion, the design torsional moment may be reduced (22.7.3.2).

22.7.3.1 If $T_u \geq \phi T_{cr}$ and T_u is required to maintain equilibrium, the member shall be designed to resist T_u.

22.7.3.2 In a statically indeterminate structure where $T_u \geq \phi T_{cr}$ and a reduction of T_u can occur due to redistribution of internal forces after torsional cracking, it shall be permitted to reduce T_u to ϕT_{cr}, where the cracking torsion T_{cr} is calculated in accordance with 22.7.5.

22.7.3.3 If T_u is redistributed in accordance with 22.7.3.2, the factored moments and shears used for design of the adjoining members shall be in equilibrium with the reduced torsion.

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22.7.4 *Threshold torsion*

22.7.4.1 Threshold torsion T_{th} shall be calculated in accordance with Table 22.7.4.1(a) for solid cross sections and Table 22.7.4.1(b) for hollow cross sections, where N_u is positive for compression and negative for tension.

Table 22.7.4.1(a)—Threshold torsion for solid cross sections

Type of member	T_{th}	
Nonprestressed member	$\lambda\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)$	(a)
Prestressed member	$\lambda\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)\sqrt{1+\frac{f_{pc}}{4\lambda\sqrt{f_c'}}}$	(b)
Nonprestressed member subjected to axial force	$\lambda\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)\sqrt{1+\frac{N_u}{4A_g\lambda\sqrt{f_c'}}}$	(c)

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R22.7.4 *Threshold torsion*

The threshold torsion is defined as one-fourth the cracking torsional moment T_{cr}. For sections of solid members, the interaction between the cracking torsional moment and the inclined cracking shear is approximately circular or elliptical. For such a relationship, a threshold torsional moment of T_{th}, as used in 22.7.4.1, corresponds to a reduction of less than 5% in the inclined cracking shear, which is considered negligible.

For torsion, a hollow section is defined as having one or more longitudinal voids, such as a single-cell or multiple-cell box girder. Small longitudinal voids, such as ungrouted post-tensioning ducts that result in $A_g/A_{cp} \geq 0.95$, can be ignored when calculating T_{th}. The interaction between torsional cracking and shear cracking for hollow sections is assumed to vary from the elliptical relationship for members with small voids, to a straight-line relationship for thin-walled sections with large voids. For a straight-line interaction, a torsional moment of T_{th} would cause a reduction in the inclined cracking shear of approximately 25%, which was considered to be significant. Therefore, the expressions for solid sections are modified by the factor $(A_g/A_{cp})^2$ to develop the expressions for hollow sections. Tests of solid and hollow beams (Hsu 1968) indicate that the cracking torsional moment of a hollow section is approximately (A_g/A_{cp}) times the cracking torsional moment of a solid section with the same outside dimensions. An additional multiplier of (A_g/A_{cp}) reflects the transition from the circular interaction between the inclined cracking loads in shear and torsion for solid members, to the approximately linear interaction for thin-walled hollow sections.

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Table 22.7.4.1(b)—Threshold torsion for hollow cross sections

Type of member	T_{th}	
Nonprestressed member	$\lambda\sqrt{f_c'}\left(\frac{A_g^2}{p_{cp}}\right)$	(a)
Prestressed member	$\lambda\sqrt{f_c'}\left(\frac{A_g^2}{p_{cp}}\right)\sqrt{1+\frac{f_{pc}}{4\lambda\sqrt{f_c'}}}$	(b)
Nonprestressed member subjected to axial force	$\lambda\sqrt{f_c'}\left(\frac{A_g^2}{p_{cp}}\right)\sqrt{1+\frac{N_u}{4A_g\lambda\sqrt{f_c'}}}$	(c)

22.7.5 *Cracking torsion*

22.7.5.1 Cracking torsion T_{cr} shall be calculated in accordance with Table 22.7.5.1 for solid and hollow cross sections, where N_u is positive for compression and negative for tension.

Table 22.7.5.1—Cracking torsion

Type of member	T_{cr}	
Nonprestressed member	$4\lambda\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)$	(a)
Prestressed member	$4\lambda\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)\sqrt{1+\frac{f_{pc}}{4\lambda\sqrt{f_c'}}}$	(b)
Nonprestressed member subjected to axial force	$4\lambda\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)\sqrt{1+\frac{N_u}{4A_g\lambda\sqrt{f_c'}}}$	(c)

22.7.6 *Nominal torsional strength*

22.7.6.1 For nonprestressed and prestressed members, T_n shall be the lesser of (a) and (b):

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R22.7.5 *Cracking torsion*

R22.7.5.1 The cracking torsional moment under pure torsion, T_{cr}, is derived by replacing the actual section with an equivalent thin-walled tube with a wall thickness $t$prior to cracking of $0.75A_{cp}/p_{cp}$ and an area enclosed by the wall centerline A_o equal to $2A_{cp}/3$. Cracking is assumed to occur when the principal tensile stress reaches $4\lambda\sqrt{f_c'}$. The stress at cracking, $4\lambda\sqrt{f_c'}$, has purposely been taken as a lower-bound value. In a nonprestressed beam loaded with torsion alone, the principal tensile stress is equated to the torsional shear stress, $\tau = T/(2A_ot)$. Thus, cracking occurs when τ reaches $4\lambda\sqrt{f_c'}$, giving the cracking torsional moment T_{cr} as defined by expression (a) in Table 22.7.5.1.

For prestressed members, the torsional cracking load is increased by the prestress given by expression (b) in Table 22.7.5.1. A Mohr's Circle analysis based on average stresses indicates the torsional moment required to cause a principal tensile stress equal to $4\lambda\sqrt{f_c'}$ is $\sqrt{1+f_{pc}/\left(4\lambda\sqrt{f_c'}\right)}$times the corresponding torsional cracking moment in a nonprestressed beam. A similar modification is made in expression (c) in Table 22.7.5.1 for members subjected to axial force and torsion.

If the factored torsional moment exceeds ϕT_{cr} in a statically indeterminate structure, a maximum factored torsional moment equal to ϕT_{cr} may be assumed to occur at critical sections. This limit has been established to control the width of the torsional cracks. The replacement of A_{cp} with A_g, as in the calculation of T_{th} for hollow sections in 22.7.4.1, is not applied here. Thus, the torsional moment after redistribution is larger and, hence, more conservative.

R22.7.6 *Nominal torsional strength*

In the calculation of T_n, all the torsion is assumed to be resisted by stirrups and longitudinal reinforcement, neglecting any concrete contribution to torsional strength. At the same time, the nominal shear strength provided by concrete, V_c, is assumed to be unchanged by the presence of torsion.

R22.7.6.1 Equation (22.7.6.1a) is based on the space truss analogy shown in Fig. R22.7.6.1a with compression diago-

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(a) $T_n = \dfrac{2A_oA_tf_{yt}}{s}\cot\theta$ (22.7.6.1a)

(b) $T_n = \dfrac{2A_oA_\ell f_y}{p_h}\tan\theta$ (22.7.6.1b)

where A_o shall be determined by analysis; θ shall not be taken less than 30 degrees nor greater than 60 degrees; A_t is the area of one leg of a closed stirrup resisting torsion; A_ℓ is the area of longitudinal torsional reinforcement; and p_h is the perimeter of the centerline of the outermost closed stirrup.

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nals at an angle θ, assuming the concrete resists no tension and the reinforcement yields. After torsional cracking develops, the torsional strength is provided mainly by closed stirrups, longitudinal reinforcement, and compression diagonals. The concrete outside these stirrups is relatively ineffective. For this reason A_o, the gross area enclosed by the shear flow path around the perimeter of the tube, is defined after cracking in terms of A_{oh}, the area enclosed by the centerline of the outermost closed transverse torsional reinforcement.

The shear flow q in the walls of the tube, discussed in R22.7, can be resolved into the shear forces V_1 to V_4 acting in the individual sides of the tube or space truss, as shown in Fig. R22.7.6.1a.

As shown in Figure R22.7.6.1b, on a given wall of the tube, the shear flow V_i is resisted by a diagonal compression component, $D_i = V_i/\sin\theta$, in the concrete. An axial tension force, $N_i = V_i(\cot\theta)$, is required in the longitudinal reinforcement to complete the resolution of V_i.

Because the shear flow due to torsion is constant at all points around the perimeter of the tube, the resultants of D_i and N_i act through the midheight of side i. As a result, half of N_i can be assumed to be resisted by each of the top and bottom chords as shown. Longitudinal reinforcement with a strength $A_\ell f_y$ is required to resist the sum of the N_i forces, $\sum N_i$, acting in all of the walls of the tube.

In the derivation of Eq. (22.7.6.1b), axial tension forces are summed along the sides of the area A_o. These sides form a perimeter length p_o approximately equal to the length of the line joining the centers of the bars in the corners of the tube. For ease in calculation, this has been replaced with the perimeter of the closed stirrups, p_h.

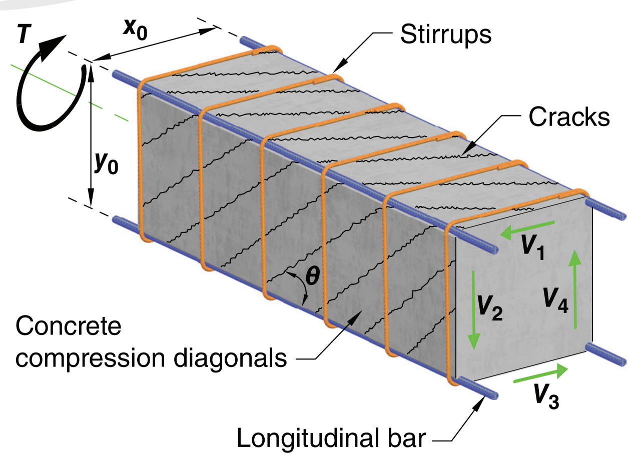

Fig. R22.7.6.1a—Space truss analogy.

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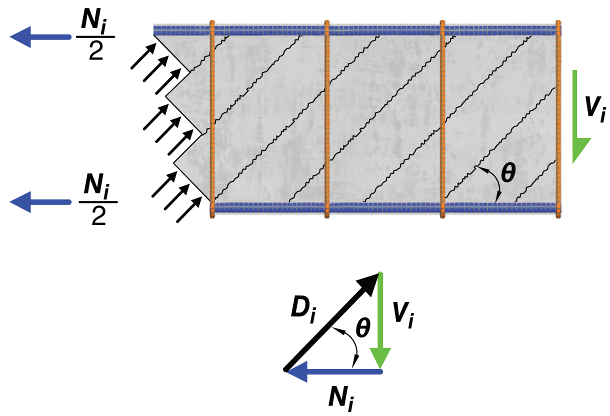

Fig. R22.7.6.1b—Resolution of shear force V_i *into diagonal compression force* D_i *and axial tension force* N_i *in one wall of tube.*

22.7.6.1.1 In Eq. (22.7.6.1a) and (22.7.6.1b), it shall be permitted to take A_o equal to $0.85A_{oh}$.

R22.7.6.1.1 The area A_{oh} is shown in Fig. R22.7.6.1.1 for various cross sections. In I-, T-, L-shaped, or circular sections, A_{oh} is taken as that area enclosed by the outermost transverse reinforcement.

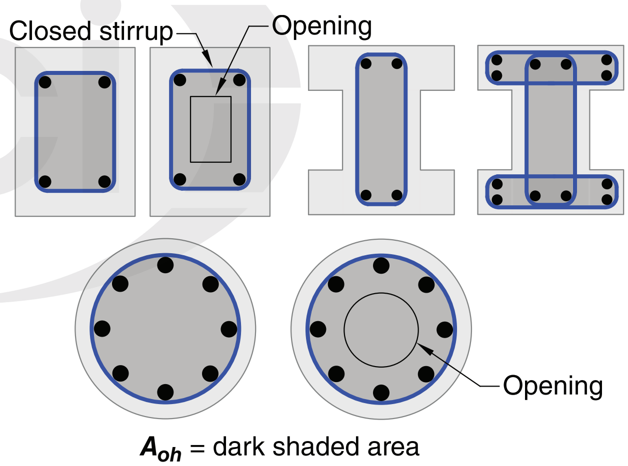

Fig. R22.7.6.1.1—Definition of A_{oh}.

22.7.6.1.2 In Eq. (22.7.6.1a) and (22.7.6.1b), it shall be permitted to take θ equal to (a) or (b):

(a) 45 degrees for nonprestressed members or members with $A_{ps}f_{se} < 0.4(A_{ps}f_{pu} + A_sf_y)$
(b) 37.5 degrees for prestressed members with $A_{ps}f_{se} \geq 0.4(A_{ps}f_{pu} + A_sf_y)$

R22.7.6.1.2 The angle θ can be obtained by analysis (Hsu 1990) or may be taken equal to the values given in 22.7.6.1.2(a) or (b). The same value of θ is required to be used in both Eq. (22.7.6.1a) and (22.7.6.1b). With smaller values of θ, the amount of stirrups required by Eq. (22.7.6.1a) decreases. At the same time, the amount of longitudinal reinforcement required by Eq. (22.7.6.1b) increases.

22.7.7 *Cross-sectional limits*

R22.7.7 *Cross-sectional limits*

22.7.7.1 Cross-sectional dimensions shall be selected such that (a) or (b) is satisfied:

(a) For solid sections

R22.7.7.1 The size of a cross section is limited for two reasons: first, to reduce excessive cracking, and second, to minimize the potential for crushing of the surface concrete due to inclined compressive stresses due to shear and torsion. In Eq. (22.7.7.1a) and (22.7.7.1b), the two terms on the left-

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$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right)^2} \leq \phi\left(\frac{V_c}{b_w d} + 8\sqrt{f_c'}\right) \qquad (22.7.7.1a)$$

(b) For hollow sections

$$\left(\frac{V_u}{b_w d}\right) + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right) \leq \phi\left(\frac{V_c}{b_w d} + 8\sqrt{f_c'}\right) \qquad (22.7.7.1b)$$

22.7.7.1.1 For prestressed members, the value of d used in 22.7.7.1 need not be taken less than $0.8h$.

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hand side are the shear stresses due to shear and torsion. The sum of these stresses may not exceed the stress causing shear cracking plus $8\sqrt{f_c'}$, similar to the limiting strength given in 22.5.1.2 for shear without torsion. The limit is expressed in terms of V_c to allow its use for nonprestressed or prestressed concrete. It was originally derived on the basis of crack control. It is not necessary to check against crushing of the web because crushing occurs at higher shear stresses.

In a hollow section, the shear stresses due to shear and torsion both occur in the walls of the box as shown in Fig. R22.7.7.1(a) and hence are directly additive at Point A as given in Eq. (22.7.7.1b). In a solid section, the shear stresses due to torsion act in the tubular outside section while the shear stresses due to V_u are spread across the width of the section, as shown in Fig. R22.7.7.1(b). For this reason, stresses are combined in Eq. (22.7.7.1a) using the square root of the sum of the squares rather than by direct addition.

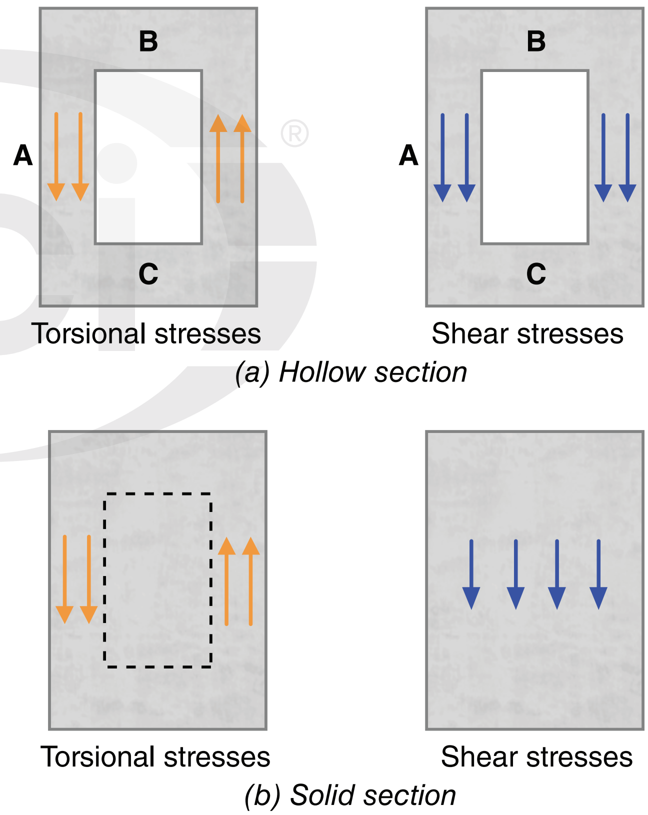

Fig. R22.7.7.1—Addition of torsional and shear stresses.

R22.7.7.1.1 Although the value of d may vary along the span of a prestressed beam, studies (MacGregor and Hanson 1969) have shown that, for prestressed concrete members, d need not be taken less than $0.8h$. The beams considered had some straight prestressed reinforcement or reinforcing bars at the bottom of the section and had stirrups that enclosed the longitudinal reinforcement.

22.7.7.1.2 For hollow sections where the wall thickness varies around the perimeter, Eq. (22.7.7.1b) shall be evaluated at the location where the term

$$\left(\frac{V_u}{b_w d}\right) + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right)$$

is a maximum.

R22.7.7.1.2 Generally, the maximum torsional stress will be on the wall where the torsional and shearing stresses are additive (Point A in Fig. R22.7.7.1(a)). If the top or bottom flanges are thinner than the vertical webs, it may be necessary to evaluate Eq. (22.7.7.1b) at Points B and C in Fig. R22.7.7.1(a). At these points, the stresses due to the shear are usually negligible

22.7.7.2 For hollow sections where the wall thickness is less than A_{oh}/p_h, the term $(T_u p_h/1.7A_{oh}^2)$ in Eq. (22.7.7.1b) shall be taken as $(T_u/1.7A_{oh}t)$, where t is the thickness of the wall of the hollow section at the location where the stresses are being checked.

22.8—Bearing

22.8.1 *General*

22.8.1.1 Section 22.8 shall apply to the calculation of bearing strength of concrete members.

22.8.1.2 Bearing strength provisions in 22.8 shall not apply to post-tensioned anchorage zones.

22.8.2 *Required strength*

22.8.2.1 Factored compressive force transferred through bearing shall be calculated in accordance with the factored load combinations defined in Chapter 5 and analysis procedures defined in Chapter 6.

22.8.3 *Design strength*

22.8.3.1 Design bearing strength shall satisfy:

$$\phi B_n \geq B_u \tag{22.8.3.1}$$

for each applicable factored load combination.

22.8.3.2 Nominal bearing strength B_n shall be calculated in accordance with Table 22.8.3.2, where A_1 is the loaded area, and A_2 is the area of the lower base of the largest frustum of a pyramid, cone, or tapered wedge contained wholly within the support and having its upper base equal to the loaded area. The sides of the pyramid, cone, or tapered wedge shall be sloped 1 vertical to 2 horizontal.

R22.8—Bearing

R22.8.1 *General*

R22.8.1.2 Because post-tensioned anchorage zones are usually designed in accordance with 25.9, the bearing strength provisions in 22.8 are not applicable.

R22.8.3 *Design strength*

R22.8.3.2 The permissible bearing stress of $0.85f_c'$ is based on tests reported in Hawkins (1968). Where the supporting area is wider than the loaded area on all sides, the surrounding concrete confines the bearing area, resulting in an increase in bearing strength. No minimum depth is given for the support, which will most likely be controlled by the punching shear requirements of 22.6.

A_1 is the loaded area but not greater than the bearing plate or bearing cross-sectional area.

Where the top of the support is sloped or stepped, advantage may still be taken of the condition that the supporting member is larger than the loaded area, provided the supporting member does not slope at too great an angle. Figure R22.8.3.2 illustrates the application of the frustum to find A_2 for a support under vertical load transfer.

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Table 22.8.3.2—Nominal bearing strength

Geometry of bearing area	B_n		
Supporting surface is wider on all sides than the loaded area	Lesser of (a) and (b)	$\sqrt{A_2/A_1}(0.85f_c'A_1)$	(a)
		$2(0.85f_c'A_1)$	(b)
Other cases	$0.85f_c'A_1$		(c)

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Adequate bearing strength needs to be provided for cases where the compression force transfer is in a direction other than normal to the bearing surface. For such cases, this section applies to the normal component and the tangential component needs to be transferred by other methods, such as anchor bolts or shear lugs.

The frustum should not be confused with the path by which a load spreads out as it progresses downward through the support. Such a load path would have steeper sides. However, the frustum described has somewhat flat side slopes to ensure that there is concrete immediately surrounding the zone of high stress at the bearing.

Where tensile forces occur in the plane of bearing, it may be desirable to reduce the allowable bearing stress, provide confinement reinforcement, or both. Guidelines are provided in the *PCI Design Handbook* for precast and prestressed concrete (PCI MNL 120).

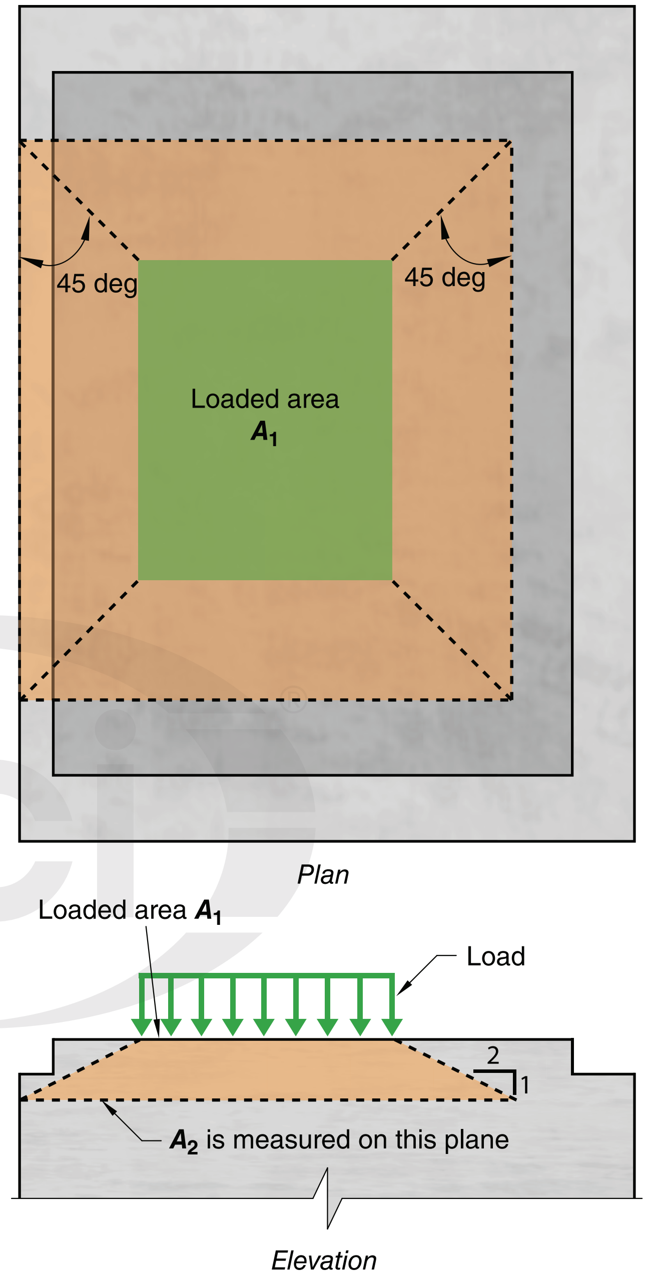

Fig. R22.8.3.2—Application of frustum to find A_2 *in stepped or sloped supports.*

22.9—Shear friction

22.9.1 *General*

22.9.1.1 This section shall apply where it is appropriate to consider shear transfer across any given plane, such as an existing or potential crack, an interface between dissimilar materials, or an interface between two concretes cast at different times.

R22.9—Shear friction

R22.9.1 *General*

R22.9.1.1 The purpose of this section is to provide a design method to address possible failure by shear sliding on a plane. Such conditions include a plane formed by a crack in monolithic concrete, an interface between concrete and steel, and an interface between concretes cast at different times (Birkeland and Birkeland 1966; Mattock and Hawkins 1972).

Although uncracked concrete is relatively strong in direct shear, there is always the possibility that a crack will form in an unfavorable location. The shear-friction concept

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assumes that such a crack will form, and that reinforcement is provided across the crack to resist relative displacement along it. When shear acts along a crack, one crack face slips relative to the other. If the crack faces are rough and irregular, this slip is accompanied by separation of the crack faces. At nominal strength, the separation is sufficient to stress, in tension, the reinforcement crossing the crack to its specified yield strength. The reinforcement in tension provides a clamping force $A_{vf}f_y$ across the crack faces. The applied shear is then resisted by friction between the crack faces, by resistance to the shearing off of protrusions on the crack faces, and by dowel action of the reinforcement crossing the crack. Successful application of this section depends on proper selection of the location of an assumed crack (PCI MNL 120; Birkeland and Birkeland 1966).

The requirements of 22.9 were developed based on monotonic testing and may be unconservative for interfaces that are part of the seismic-force-resisting system and experience strength degradation due to force and displacement reversals. Palieraki et al. (2022) provides design guidance for interfaces subject to cyclic loading that could cause sliding.

22.9.1.2 The required area of shear-friction reinforcement across the assumed shear plane, A_{vf}, shall be calculated in accordance with 22.9.4. Alternatively, it shall be permitted to use shear transfer design methods that result in prediction of strength in substantial agreement with results of comprehensive tests.

R22.9.1.2 The relationship between shear-transfer strength and the reinforcement crossing the shear plane can be expressed in various ways. Equations (22.9.4.2) and (22.9.4.3) are based on the shear-friction model and provide a conservative estimate of the shear-transfer strength.

Other relationships that provide a more accurate estimate of shear-transfer strength can be used under the requirements of this section. Examples of such procedures can be found in the *PCI Design Handbook* (PCI MNL 120), Mattock et al. (1976b), and Mattock (1974).

22.9.1.3 The value of f_y used to calculate V_n for shear friction shall not exceed the limit in 20.2.2.4.

22.9.1.4 Surface preparation of the shear plane assumed for design shall be specified in the construction documents.

R22.9.1.4 For concrete cast against hardened concrete or structural steel, 26.5.6.1 requires the licensed design professional to specify the surface preparation in the construction documents.

22.9.2 *Required strength*

22.9.2.1 Factored forces across the assumed shear plane shall be calculated in accordance with the factored load combinations defined in Chapter 5 and analysis procedures defined in Chapter 6.

22.9.3 *Design strength*

22.9.3.1 Design shear strength across the assumed shear plane shall satisfy:

$$\phi V_n \geq V_u \qquad (22.9.3.1)$$

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for each applicable factored load combination.

22.9.4 *Nominal shear strength*

22.9.4.1 Value of V_n across the assumed shear plane shall be calculated in accordance with 22.9.4.2 or 22.9.4.3. V_n shall not exceed the value calculated in accordance with 22.9.4.4.

22.9.4.2 If shear-friction reinforcement is perpendicular to the shear plane, nominal shear strength across the assumed shear plane shall be calculated by:

$$V_n = \mu\left(A_{vf}f_y + N_u\right) \quad (22.9.4.2)$$

where μ is the coefficient of friction in accordance with Table 22.9.4.2, and N_u is the minimum factored compressive force acting concurrently with V_u. It shall be permitted to take N_u equal to zero even if compression across the interface is present.

Table 22.9.4.2—Coefficients of friction

Contact surface condition	Coefficient of friction μ[1]	
Concrete placed monolithically	1.4λ	(a)
Concrete placed against hardened concrete that is clean, free of laitance, and intentionally roughened to a trough-to-peak amplitude of approximately 1/4 in.[2]	1.0λ	(b)
Concrete placed against hardened concrete that is clean, free of laitance, and not intentionally roughened	0.6	(c)
Concrete placed against as-rolled structural steel that is clean, free of paint, and with shear transferred across the contact surface by headed studs or by welded deformed bars or wires.	0.7λ	(d)

[1]λ = 1.0 for normalweight concrete. For lightweight concrete, λ is calculated as given in 19.2.4, but shall not exceed 0.85.
[2]Refer to 26.5.6.2(e) for compliance requirements for intentional roughening.

22.9.4.3 If shear-friction reinforcement is inclined to the shear plane and the shear force induces tension in the shear-friction reinforcement, nominal shear strength across the assumed shear plane shall be calculated by:

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R22.9.4 *Nominal shear strength*

R22.9.4.2 The required area of shear-friction reinforcement, A_{vf}, is calculated using:

$$A_{vf} = \frac{V_u - \phi\mu N_u}{\phi f_y \mu} \quad (R22.9.4.2)$$

Only compressive normal force is considered in Eq. (22.9.4.2); 22.9.4.5 requires reinforcement across the interface to resist net factored tension. Normal force U is factored in accordance with the load combinations of Chapter 5. All applicable load combinations should be considered to determine the most critical design condition, recognizing that non-permanent loads should only be considered if they add to V_u and load combinations that include a 0.9 factor on dead load should be considered.

The upper limit on shear strength that can be achieved using Eq. (22.9.4.2) is given in 22.9.4.4.

In the shear-friction method of calculation, it is assumed that all the shear resistance is due to the friction between the crack faces. It is therefore necessary to use artificially high values of the coefficient of friction in the shear-friction equations so that the calculated shear strength will be in reasonable agreement with test results.

For concrete cast against hardened concrete not roughened in accordance with 22.9.4.2, shear resistance is primarily due to dowel action of the reinforcement. Test results (Mattock 1977) indicate that the reduced value of μ = 0.6 specified for this case is appropriate.

Beginning with the 2025 Code, the λ factor was removed from (c) of Table 22.9.4.2 based on research by Krc et al. (2016), which examined the use of λ for different concrete interface conditions. For contact surfaces not intentionally roughened, the lower strength of lightweight aggregate does not reduce shear transfer strength because the interfacial crack does not propagate through the aggregate.

For concrete placed against as-rolled structural steel, the shear-transfer reinforcement may be either reinforcing bars or headed studs. The design of shear connectors for composite action of concrete slabs and steel beams is not covered by these provisions. ANSI/AISC 360 contains design provisions for these systems.

R22.9.4.3 Inclined shear-friction reinforcement is illustrated in Fig. R22.9.4.3a (Mattock 1974), where α is the acute angle between the bar and the shear plane. Equation (22.9.4.3) applies only when the shear force component parallel to the reinforcement produces tension in the rein-

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$$V_n = A_{vf}f_y(\mu\sin\alpha + \cos\alpha) + \mu N_u \quad (22.9.4.3)$$

where α is the angle between shear-friction reinforcement and assumed shear plane, and μ and N_u are as defined in 22.9.4.2.

22.9.4.4 The value of V_n across the assumed shear plane shall not exceed the limits in Table 22.9.4.4. Where concretes of different strengths are cast against each other, the lesser value of f_c' shall be used in Table 22.9.4.4.

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forcement and the force component parallel to the shear plane resists part of the shear, as shown in Fig. R22.9.4.3a.

If the shear-friction reinforcement is inclined such that the shear force component parallel to the reinforcement produces compression in the reinforcement, as shown in Fig. R22.9.4.3b, then shear friction does not apply ($V_n = 0$).

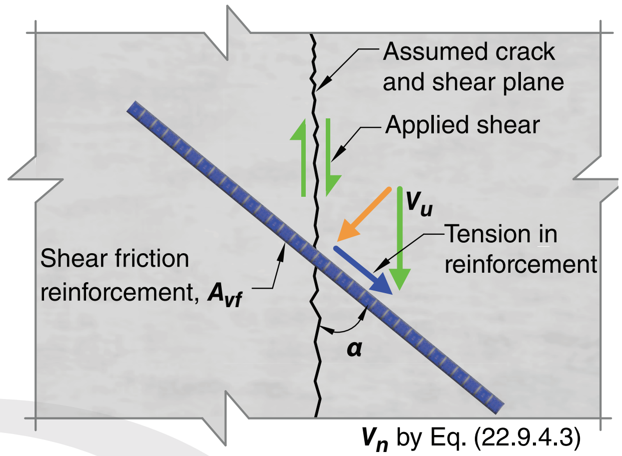

Fig. R22.9.4.3a—Tension in shear friction reinforcement.

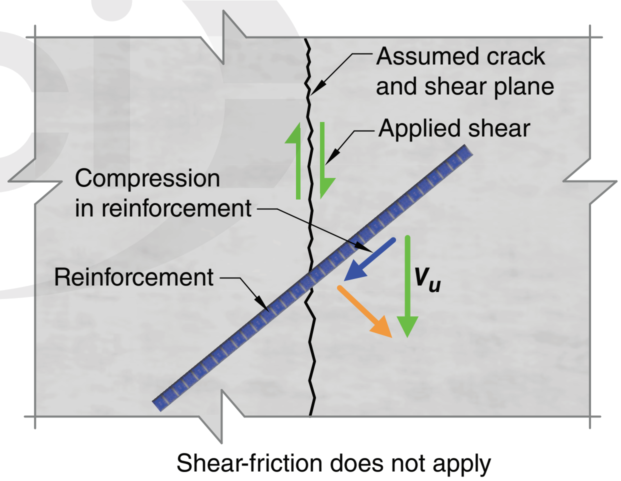

Fig. R22.9.4.3b—Compression in reinforcement.

R22.9.4.4 Upper limits on shear friction strength are necessary, as Eq. (22.9.4.2) and (22.9.4.3) may become unconservative for some cases (Kahn and Mitchell 2002; Mattock 2001).

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Table 22.9.4.4—Maximum V_n across the assumed shear plane

Condition	Maximum V_n		
Normalweight concrete placed monolithically or placed against hardened concrete that is clean, free of laitance, and intentionally roughened to a trough-to-peak amplitude of approximately 1/4 in.[1]	Least of (a), (b), and (c)	$0.2f_c'A_c$	(a)
		$(480 + 0.08f_c')A_c$	(b)
		$1600A_c$	(c)
Other cases	Lesser of (d) and (e)	$0.2f_c'A_c$	(d)
		$800A_c$	(e)

[1]Refer to 26.5.6.2(e) for compliance requirements for intentional roughening.

22.9.4.5 Area of reinforcement required to resist a net factored tension across an assumed shear plane shall be added to the area of reinforcement required for shear friction crossing the assumed shear plane.

22.9.5 *Detailing for shear-friction reinforcement*

22.9.5.1 Reinforcement crossing the shear plane to satisfy 22.9.4 shall develop f_y in tension on both sides of the shear plane.

22.9.5.2 It shall be permitted to consider inverted U-bars extending into composite topping slabs sufficient to develop f_y in tension at the interface if specified details satisfy (a) through (e):

(a) U-bar size does not exceed No. 7.
(b) Cross-leg of each U-bar extends at least $5d_b$ above the interface, where d_b is the diameter of the U-bar leg.
(c) Upper corners of each U-bar enclose a bar or strand.

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R22.9.4.5 Tension across the shear plane may be caused by restraint of deformations due to temperature change, creep, and shrinkage.

Where moment acts on a shear plane, the flexural compression and tension forces are in equilibrium and do not change the resultant compression $A_{vf}f_y$ acting across the shear plane or the shear-friction resistance. It is therefore not necessary to provide additional reinforcement to resist the flexural tension stresses, unless the required flexural tension reinforcement exceeds the amount of shear-transfer reinforcement provided in the flexural tension zone (Mattock et al. 1975).

R22.9.5 *Detailing for shear-friction reinforcement*

R22.9.5.1 If no moment acts across the shear plane, reinforcement should be uniformly distributed along the shear plane to minimize crack widths. If a moment acts across the shear plane, the shear-transfer reinforcement should be placed primarily in the flexural tension zone.

Anchorage may be developed by bond, by a mechanical device, or by threaded dowels and screw inserts. Space limitations often require the use of mechanical anchorage devices. For anchorage of headed studs in concrete, refer to *PCI Design Handbook* for precast and prestressed concrete (PCI MNL 120).

The shear-friction reinforcement anchorage should engage the primary reinforcement; otherwise, a potential crack may pass between the shear-friction reinforcement and the body of the concrete. This requirement applies particularly to welded headed studs used with steel inserts.

R22.9.5.2 Detailing requirements of this provision are based on tests by Mattock (1987) and Waweru et al. (2018) and are illustrated in Fig. R22.9.5.2. Although inverted U-bars are typically used, this provision is intended to also apply to rectangular ties. The top leg of the U-bar should extend as close to the top surface as cover requirements permit.

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(d) Spacing between U-bar legs in both directions is at least $8d_b$.

(e) Below the interface, vertical legs of the U-bars develop f_y in tension in accordance with 25.4.2.

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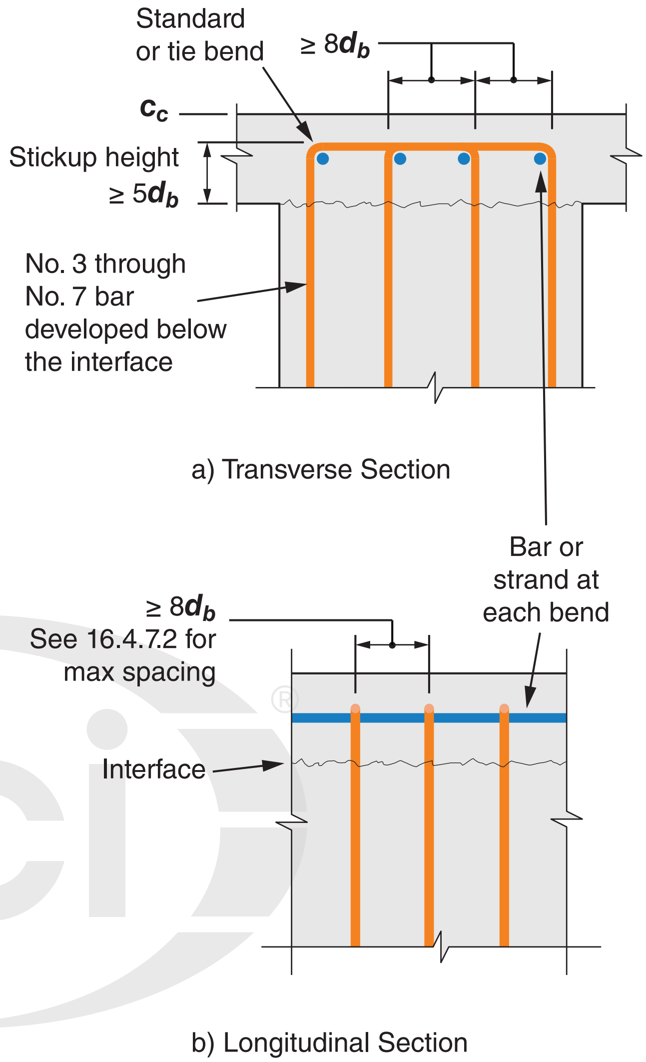

Fig. R22.9.5.2—Details for development of U-bars across a shear-friction interface.

Notes