# Mock ARML I- Individual

*Author:*
Problem Writing
Club

March 7, 2014

# Problem 1

Given circle $\omega$, and that chords $A_0A_1$ and $A_2A_3$ intersect at $A_4$, find $\frac{A_0A_1}{A_2A_3}$ if $\frac{A_0A_4}{A_1A_4} = \frac{1}{4}$ and $\frac{A_2A_4}{A_3A_4} = \frac{4}{9}$.

# Problem 2

Brice has a sphere with radius $\frac{2014}{2013}$ and decides to magically place an octahedron with maximal volume in it. Let $R$ be the ratio of the volume of the sphere to the volume of the octahedron. Compute $\lfloor R \rfloor$. ($\lfloor x \rfloor$ is the greatest integer less than or equal to $x$)

# Problem 3

Let $N$ be $\prod_{n=0}^{89}(1 - cot^5(\frac{n\pi}{180}))$. Compute the value of $2014N$.

# Problem 4

The maximum value attained by the equation $\frac{18(x^2+x+1)^2}{x^2+2x+1}$ can be expressed as $A\sqrt[R]{M \cdot L}$. Compute the value $A \cdot R \cdot M \cdot L$.

# Problem 5

A sector of a circle with radius 15 and central angle 216 degrees is formed into a cone. The cone is then cut parallel to its base by a plane. Find the probability that the area of the intersection of the plane and the cone is less than or equal to $9\pi$.

# Problem 6

In a parallel universe, the game of football begins with a coin flip. Each of the two teams sends out 4 players for the coin toss. Each of these eight players shakes hands with two other players (the players can be from the same or opposing team). A student watching TV sees this, and decides to write a list of all unordered pairs of players that shook hands with each other. How many different lists can this student come up with, given that order does not matter?

# Problem 7

100! can be expressed as $p_1^{e_1} \cdot p_2^{e_2} ... \cdot p_{25}^{e_{25}}$, where $p_1, p_2 ... p_{25}$ are the $n^{th}$ prime numbers, where $n$ is the subscript of $p_n$. For instance, $p_1 = 2$, since the first prime number is 2. Compute $\sum\limits_{i=1}^{25} e_i$.

# Problem 8

Define an ARML-ish number as a number whose digits are all one's. For instance, $1, 11, 111$ are all ARML-ish numbers. Let $\omega$ be the smallest ARML-ish number such that $\omega$ is divisible by 97. Compute the sum of the digits of $\omega$.

# Problem 9

The machine ARMLBOX was built in the year 2014. The ARMLBOX takes any real number $x$ and returns the value $\frac{1}{\sqrt[2014]{1-x^{2014}}}$. A boy punches in 2014 into the ARMLBOX, and gets a value. He finds this value is a mess, and punches it again to get another value. He repeats this 2013 times. What is the value the ARMLBOX returns after the $2013^{th}$ time he repeats this?

# Problem 10

Let $ABCDE$ be a regular pentagon and point $M$ in its interior such that $\angle MBA = \angle MEA = 42°$. Compute the measure of $\angle CMD$.

**Answer Key**

1) $\frac{15}{13}$

2) 3

3) 0

4) 1080

5) $\frac{1}{3}$

6) 3507

7) 239

8) 96

9) 2014

10) $60^\circ$