Circuit Series 1

Author:
PROBLEM WRITING
CLUB

February 21, 2014

RULES

1) At the start, every team should have an envelope with 5 problems in it (this will be provided).
2) All teachers should have on a desk empty envelopes for every team. (Please have the names of the teams on these envelopes)
3) In addition, under these empty envelopes for every team should be a stapled set of problems. (Please do not mix envelopes or problem sets).
4) Each team can be comprised of up to 4 members.
5) When a member of team thinks that they have solved a problem, bring up the slip of paper with the answer written on the back.
6) The teacher will check if the answer is correct or not. If it is incorrect, then an 'X' will be placed on the slip, and the team will be able to resolve the problem.
7) If a problem has more than 3 'X' marks, then the team will not get any points from that question.
8) If the answer is correct, the problem slip will be placed into the envelope, and the teacher will rip the next problem from the problem set and hand it to the student.
9) This continues for **30** minutes.
10) The problems start easy, but get progressively difficult. It is normal not to solve all 30 problems provided.
11) At the end of the 30 minutes, the students may keep everything they have. The teachers must seal every envelope. The extra problems from the stapled sets may be distributed to the students.
12) Whenever the teacher has free time, please hand the envelopes to Mr.Jaishankar.
13) The rankings will be given to the teachers after all of the envelopes have been collected.

Problem 1

How many factors does 2014 have?

Problem 2

If $a + \frac{1}{b} = 20$ and $b + \frac{1}{a} = 14$, find the value of $ab + \frac{1}{ab}$.

Problem 3

$2014_x = 133132_4$. Find the value of x.

Problem 4

Line segment AB has points A at $(3, 1)$ and B at $(2, 4)$. It is rotated about a point such that A maps to $(4, 2)$ and B maps to $(7, 3$. Find the sum of the coordinates of the center of rotation.

Problem 5

How many positive two-digit integers have exactly 8 positive factors?

Problem 6

What percent of the interval with endpoints 5 and 5 consists of real numbers x satisfying the inequality $x + 1 > \frac{8}{x-1}$?

Problem 7

Find the greatest common factor of 1357 and 4897

Problem 8

8 runners are going to run a marathon, but only 3 can be chosen. How many ways can you do this?

Problem 9

A pythagorean triplet is a set of integers such that $a^2+b^2+c^2$, where a,b,c, are integers. In this set of all pythagorean triplets, the following are proposed:
(i) the hypotenuse always has a remainder of 1 when divided by 4 (4 points)
(ii) At least one leg must be even (2 points)
(iii) Both legs can be divisible by 3 (3 points)
(iv) Both the leg and hypotenuse can be divisible by 5 (5 points)
(v) Either a leg is divisible by 7 or the sum of the difference of the legs is. (7 points)
What is the sum of the point values of all the true statments?

Problem 10

The sum of the coefficients of $(2014x-1)^{2014}$ is A. Find the units digit of A

Problem 11

If $a(x-1)(x+1)+b(x-1)(x-3)+c(x+1)(x-3)=2x-20$, find $a+b+c$.

Problem 12

If A36415489812706444B is divisible by 99, find A+B

Problem 13

If x, y, z are all positive integers, how many ordered triples of (x, y, z) make $x+y+z=20$ true?

Problem 14

A rectangle is inscribed in an isosceles right triangle. (That is, one corner of the rectangle rests of the right angle of the triangle. The two sides of the rectangle that intersect at that corner run along the sides of the triangle. The opposite corner of the rectangle from the corner resting on the right angle of the triangle lies on the triangles hypotenuse.) If each leg of the triangle is 2014, compute the perimeter of the rectangle.

Problem 15

If $a^2 + b^2 + c^2 = 25$, $x^2 + y^2 + z^2 = 36$, and $ax + by + cz = 30$, find $\frac{a+b+c}{x+y+z}$.

Problem 16

Find the sum of all n in which $n^2 - 19n + 99$ is a perfect square.

Problem 17

Triangle ABC is equilateral with area equal to 1. A point D is on AC such that BD cuts the triangle into two sections of equal area. E is chosen on AB such that CE splits the side into a ratio of BE:EA=1:4. Let P be the intersection of BD and CE. What is the area of triangle BPC?

Problem 18

Find (x,y,z) if $xy + x + y = 11$, $xz + x + z = 17$, and $yz + y + z = 24$.

Problem 19

Let $f(x) = x^2 + x\sqrt{2} - k = 0$, where k is real. If k is chosen from $[-7, -2]$, what is the probability the roots of f are real?

Problem 20

$\frac{1}{9!1!} + \frac{1}{7!3!} + \frac{1}{5!5!} + \frac{1}{3!7!} + \frac{1}{1!9!}$ can be expressed as $\frac{2^a}{b!}$. Find $a + b$. (Note that $n! = n \cdot (n-1) \cdot (n-2) ... \cdot 3 \cdot 2 \cdot 1$)

Problem 21

There exists a point P inside an equilateral triangle ABC such that $PA = 3$, $PB = 4$, and $PC = 5$. Find the side length of the equilateral triangle.

Problem 22

On Sesame Street, a mailman is delivering 9 postcards of different colors to 5 mailboxes (A, B, C, D, E) such that no mailbox gets more than 2 letters. Find the probability that mailbox A receives 2 letters.

Problem 23

Compute the last 3 digits of 9^{2014}

Problem 24

How many terms are there in the expansion of $(a + b + c + d)^{2014}$?

Problem 25

F_n is the nth term of the Fibonacci sequence (where $F_1 = 1, F_2 = 1, F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$). Find $\frac{F_1}{5^1} + \frac{F_2}{5^2} + \frac{F_3}{5^3} + ...$

Problem 26

The number of ordered triples of x,y,z such that x,y,z are nonnegative that satisfy $x + y + z \leq 20$ is X. Find the sum of the digits of X.

Problem 27

Triangle ABC is a $3-4-5$ right triangle. Then, $cosA+cosB+cosC=1+a$. Find the value of a. (In a right angled triangle, the cosine of an angle is the length of the adjacent side divided by the length of the hypotenuse.)

Problem 28

A bracelet has 11 beads, each of which can be red, blue, or green. How many different bracelets can there be? A rotation/reflection is considered equivalent.

Problem 29

Let A be the sum of the decimal digits of 4444^{4444}, and B be the sum of the decimal digits of A. Find the sum of the decimal digits of B.

Problem 30

Let ABC be a triangle with $AB=AC$. The angle bisectors of angle CAB and ABC meet the sides BC and CA at D and E repsectively. Let K be the incenter of triangle ADC. Suppose that angle BEK is 45 degrees. Find all possible values of angle CAD

Answer Key

1) 8

2) 278

3) 10

4) 5

5) 10

6) 60

7) 59

8) 56

9) 16

10) 9

11) 0

12) 9

13) 231

14) 4028

15) $\frac{5}{6}$

16) 38

17) $\frac{1}{6}$

18) $(\frac{6\sqrt{6}}{5} - 1, \frac{5\sqrt{6}}{3} - 1, \frac{5\sqrt{6}}{2} - 1)$

19) $\frac{3}{5}$

20) 19

21) $\sqrt{25+12\sqrt{3}}$

22) $\frac{4}{5}$

23) 961

24) 1365589680

25) $\frac{6}{19}$

26) 16

27) $\frac{2}{5}$

28) 8418

29) 7

30) 60 or 90 degrees