Mock ARML Team

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PROBLEM WRITING
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Problem 1

$\arctan 3 + \arctan 4 = \arctan x$. Compute the value of x.

Problem 2

Compute the value of $2014 + (\lfloor\sqrt{2014}\rfloor - 1 + \lfloor\sqrt[3]{2014}\rfloor - 1 + ...)$. ($\lfloor x \rfloor$ denotes the greatest integer less than or equal to x).

Problem 3

Convex quadrilateral $ABCD$ exists such that $AD = 7, CD = 9, BC = 11$, and $AB = 13$. There exists a point E on AD such that $CE \perp AD$. Diagonals AC and BD intersect at F and CE intersects BD at G. $\angle ECD = 38°$, $\angle BAC = 7°$, $\angle BCA = \frac{1}{2}\angle GEA$. If $AF = FB$, and $CF = FD$, compute the value of AC.

Problem 4

Let $p(x) = x^3 - 6x^2 + 9x - 4$. The range of α, for which α is real and $p(x) = \alpha$ has only one solution, is the set $\alpha \in (-\infty, -a) \cup (b, \infty)$. Find the value of $a + b$.

Problem 5

The Fibonacci sequence is a sequence that is defined as: $F_0 = 0, F_1 = 1, F_n = F_{n-1} + F_{n-2}$. A cool-sumish set is a subset of $1, 2, ..., 2013$, such that the sum of the elements is equal to $F_{2014} - 1$. Let γ be the number of cool-sumish sets. Compute the value of γ.

Problem 6

2014 circles are drawn on the plane. Let N be the maxmimum number of regions that the 2014 circles can divide the plane. Find the number of divisors of N. Note that 506773 is prime.

Problem 7

Let the set a_0, a_1, a_2, a_3 be the root of $a^4 - 2014a + 1$. Let x be $\max(a_0, a_1, a_2, a_3)$, where $\max(b_0, b_1, ...)$ returns the largest value of $b_0, b_1, ...$ Compute the value of $\lfloor \frac{1}{x^3 - 2014} \rfloor$. ($\lfloor x \rfloor$ denotes the greatest integer less than or equal to x).

Problem 8

Compute $\sum_{\alpha=2014}^{\infty} \frac{1}{\binom{\alpha}{2014}}$

Problem 9

There exists a 2014×2015 rectangle on the cartesian plane, with one corner being $(0, 0)$, and the other at $(2014, 2015)$. Define a Swag-tile as a 1×2 tile. A monster comes and decides to destroy a pair of squares (remove a pair of squares). Let k be the number of ways the monster can do this such that you can still tile the rest of the region with Swag-tiles. Compute the value of $\lfloor \sqrt{k} \rfloor$. ($\lfloor x \rfloor$ denotes the greatest integer less than or equal to x).

Problem 10

Let $f = lnx$. Let the equation of a function g is $sin(f) + 2sin(\frac{\pi}{2} - 3f) \cdot sin(2f)$. Bob graphs this equation on a paper that extends indefinitely. Let S be the set of all α, where $\alpha > 1$, that are roots of g. Let find(x,S) be the function that finds the x^{th} smallest element in the set of S, and if there is no such element, it returns -2014. If $e^{\frac{k\pi}{m}} =$ find(2014,S), find the value of $k + m$.

Answers

1) $-\frac{7}{11}$
2) 2082
3) $\sqrt{194}$
4) 5
5) 1007
6) 8
7) -13
8) $\frac{2014}{2013}$
9) 2029105
10) 2019