EECE 5698

Assignment 3: Parallel Classification

 $\label{lem:preparation.} \begin{tabular}{ll} \textbf{Preparation.} Create a folder on discovery named after your username under the directory $$/$gs_gpfs_scratch. $$ Copy the directory $$$

```
/gss_gpfs_scratch/EECE5698/Assignment3
```

to this folder, by typing:

```
cp -r /qss_qpfs_scratch/EECE5698/Assignment3 /qss_qpfs_scratch/$USER/
```

Make the contents of this directory private, by typing:

```
chmod -R o-rx /gss_gpfs_scratch/$USER/Assignment3
```

The directory contains the following python files:

```
LogisticRegression.py
ParallelLogisticRegression.py
SparseVector.py
helpers.py
```

as well as two directories called mushrooms and newsgroups.

The mushrooms dataset¹ contains the features of edible (label:+1) and poisonous (label:-1) mushrooms. The newsgroups dataset² contains the most indicative words in messages posted in two newsgroups: sci.med (label:+1) and rec.sports.baseball (label:-1). In this assignment, all features are *binarized*: the presense of a feature is indicated with a value 1.0, while the absense is indicated with an (implicit) value 0.0 in the corresponding feature vector.

You must:

- 1. Provide a report, in pdf format, outlining the answers of the questions below. The report should be type-written in a word processor of your choice (e.g., MS Word, Latex, etc.).
- 2. Provide the final code of files LogisticRegression.py and ParallelLogisticRegression.py.

The report along with your final code should be uploaded to Blackboard. Executions of the code to generate the requested output can be run on "local" mode, on a single compute node which you have reserved on the Discovery cluster. Use, e.g., a node in ser-par-10g-3 or ser-par-10g-4, with 40 logical cores.

¹https://archive.ics.uci.edu/ml/datasets/Mushroom

²http://ana.cachopo.org/datasets-for-single-label-text-categorization

Question 1: File SparseVector.py defines a new class called SparseVector.

- (1a) Describe how this class relates to a standard python dictionary.
- (1b) Give a few examples of functionalities/methods it shares with a standard dictionary, as well as ones that it has that are not present in a dictionary.
- (1c) Launch the python interpreter, create two sparse vectors, add and multiply them with each other and with a scalar. Include the operations that you executed and the outcome of each execution in your report.

Question 2: Consider dataset \mathcal{D} of n pairs $(x_i, y_i) \in \mathbb{R}^d \times \{-1, +1\}$, $i = 1, \dots n$, where $x_i \in \mathbb{R}^d$ are *features* and $y_i \in \{-1, +1\}$ are *binary labels*. Logistic regression attempts to find a vector $\beta \in \mathbb{R}^d$ such that, given an x_i , the *prediction*:

$$\hat{f}(x_i) = \begin{cases} +1, & \text{if } \beta^\top x_i \ge 0. \\ -1, & \text{if } \beta^\top x_i < 0. \end{cases}$$

agrees with the values y_i in the data set. Regularized logistic regression trains such a β from \mathcal{D} by minimizing the loss function:

$$L(\beta) = \sum_{i=1}^{n} \ell(\beta; x_i, y_i) + \lambda \|\beta\|_2^2$$

over $\beta \in \mathbb{R}^d$, where

$$\ell(\beta; x_i, y_i) = \log\left(1 + e^{-y\beta^{\top}x}\right)$$

is known as the logistic loss.

- **2(a)** Prove that, for all $x \in \mathbb{R}^d$ and all $y \in \{-1, +1\}$, the logistic loss $\ell(\beta; x, y)$ is a convex function of $\beta \in \mathbb{R}^d$.
- **2(b)** Prove that

$$\nabla \ell(\beta; x, y) = -\frac{yx}{1.0 + e^{y\beta^{\top} x}}.$$

- **2(c)** Suppose that $x_j = 0$ for some coordinate $j \in \{1, \dots, d\}$ of vector $x \in \mathbb{R}^d$. Prove that $\ell(\beta; x, y)$ and $\nabla \ell(\beta; x, y)$ do not depend on β_j , the j-th coordinate of $\beta \in \mathbb{R}^d$.
- **2(d)** Prove that $L(\beta)$ is a convex function of $\beta \in \mathbb{R}^d$.

Question 3: (3a) Implement the functions:

logisticLoss
gradLogisticLoss
gradTotalLoss

in program LogisticRegression.py, so that they act as specified in their docstring comments. **Do not use spark in this implementation**. You may use estimateGrad from file helpers.py to test the correctness of your gradient code. Include the three function definitions you wrote for these three functions in your report.

- **3(b)** Implement the function test in LogisticRegression.py, so that it computes the accuracy, precision, and recall of β over a dataset. Include the final function definition in your report.
- 3(c) Run the code LogisticRegression.py on the mushrooms train and test datasets with three different values of λ , for a maximum of 20 iterations. Include the output of your execution in your report.
- **3(d)** Recall that label +1 indicates edible mushrooms and label -1 indicates poisonous mushrooms. If you were to choose between (a) high precision and low recall, vs. (b) high recall and low precision, which would you prefer?

Question 4: Complete the missing code in program ParallelLogisticRegression.py in Spark so that it replicates exactly the same functionality as LogisticRegression.py (messages printed can differ in style/format, but should contain the same information). You may import and reuse any of the following functions from LogisticRegression.py:

readBeta, writeBeta, gradLogisticLoss, logisticLoss, lineSearch

but you may not import any other function implemented in LogisticRegression.py. In particular, you should not include any for loop or list comprehension that iterates over the data in dataRDD.

- (a) Include the code of every function you wrote in your report. All functions should contain docstring comments describing the the function in a single succinct sentence, followed by a detailed description of what the function computes, what are its inputs and outputs, etc.
- (b) Compare the output of your program to the output of LogisticRegression.py when executed with $\lambda=0$ over the mushrooms train and test datasets for at most 20 iterations. Include the output in your report. Which one is faster?
- (c) Compare the output of your program to the output of LogisticRegression.py when executed with $\lambda=0$ over the newsgroups train and test datasets for at most 20 iterations. Include the output in your report. Which one is faster?

Question 5: In what follows, run your code over the newsgroups train and test datasets. You need not run this until convegence: report how many iterations you used.

- (a) Produce a plot for $\lambda=0$ with time on the x-axis and the norm of the gradient $\|\nabla L\|_2$ on the y-axis, showing two curves: one for LogisticRegression.py and one for ParallelLogisticRegression.py, when executed over newsgroups. Produce 3 similar plots showing accuracy, precision, and recall on the test set, respectively, on the y-axis, for $\lambda=0$.
- (b) Produce a single plot showing accuracy on the test set as a function of time, for three different values of λ .
- (c) For the value of λ and the number of iterations that gives you the highest accuracy, store the resulting β . Either in a table or a bar plot, report the 10 features of β with the most positive values. Similarly, report the 10 features of β with the most negative values.