The Drift-Diffusion Model

Master for Brain & Cognition, Data Analysis course

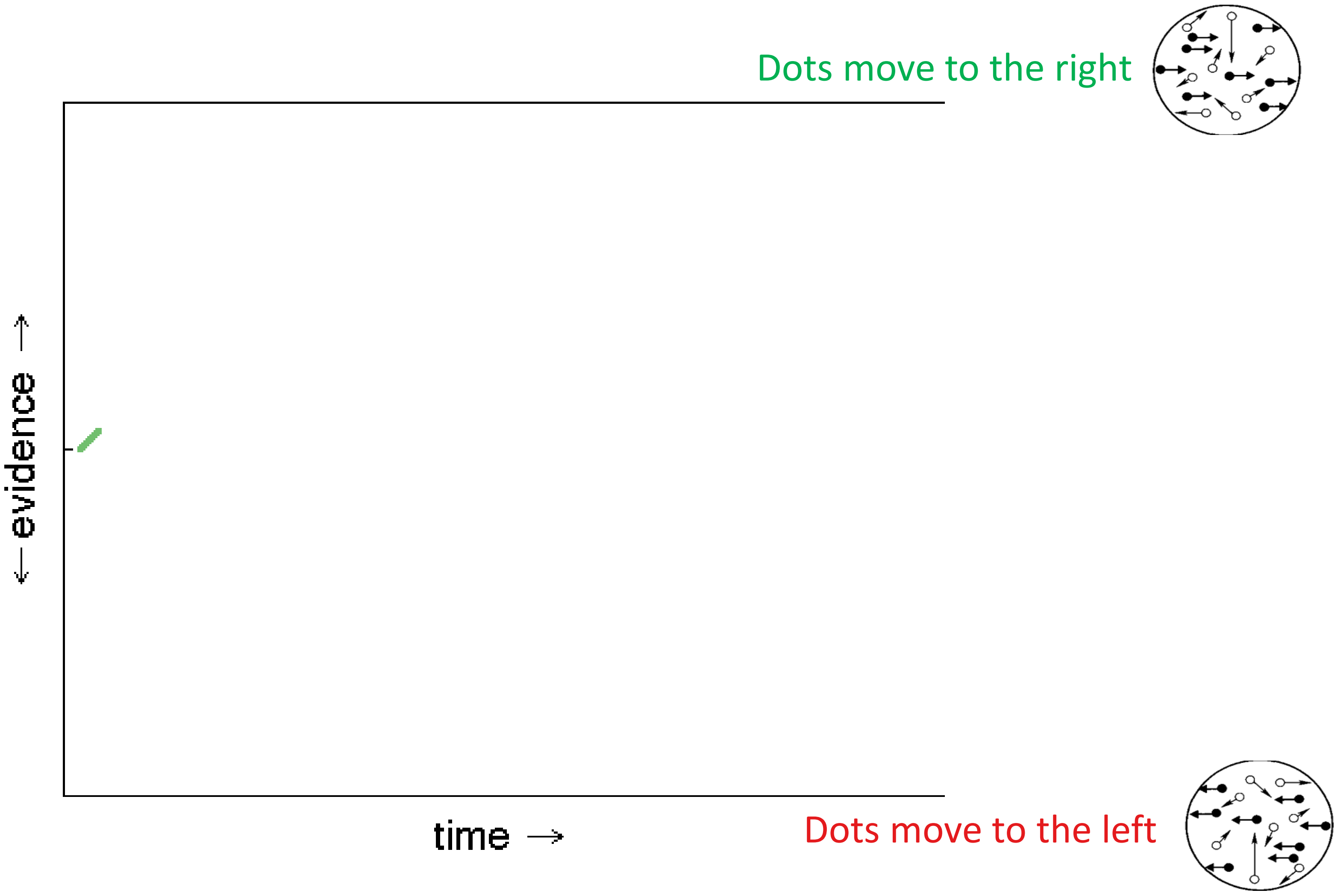
Heavily based on material from M. Shinn, J. de la Rocha, A. Renart, A. Urai

Sequential Sampling Analysis

Decisions are *not* instantaneous but unfold in time

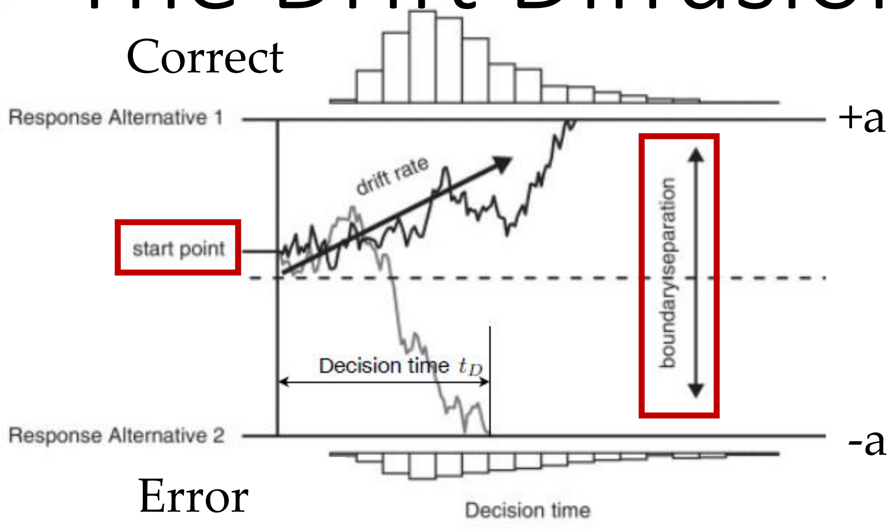
Why do we care?

- Changes the task fundamentally as there are 2 things to decided: **When** to decide and **What** to do.
- The timing of decisions is a very valuable ‘diagnostic’ observable: *when* they are made can inform about *how* they are made.
- Some tasks show no variability in response accuracy (i.e. no errors)
- Sequential sampling theories describe situations where stimuli arrive ‘in time’ (i.e., sensory streams) and agents decide both When and What.



Dots move to the right
evidence
time
Dots move to the left

The Drift Diffusion Model



Accumulation of stochastic, temporally uncorrelated sensory evidence:

$$dX_t = \mu dt + \sigma dW_t$$

'decision variable'

Evidence accumulation

Zero-mean Gaussian noise with variance $\sigma^2 dt$ (diffusion)

Simple Diffusion Model Parameters

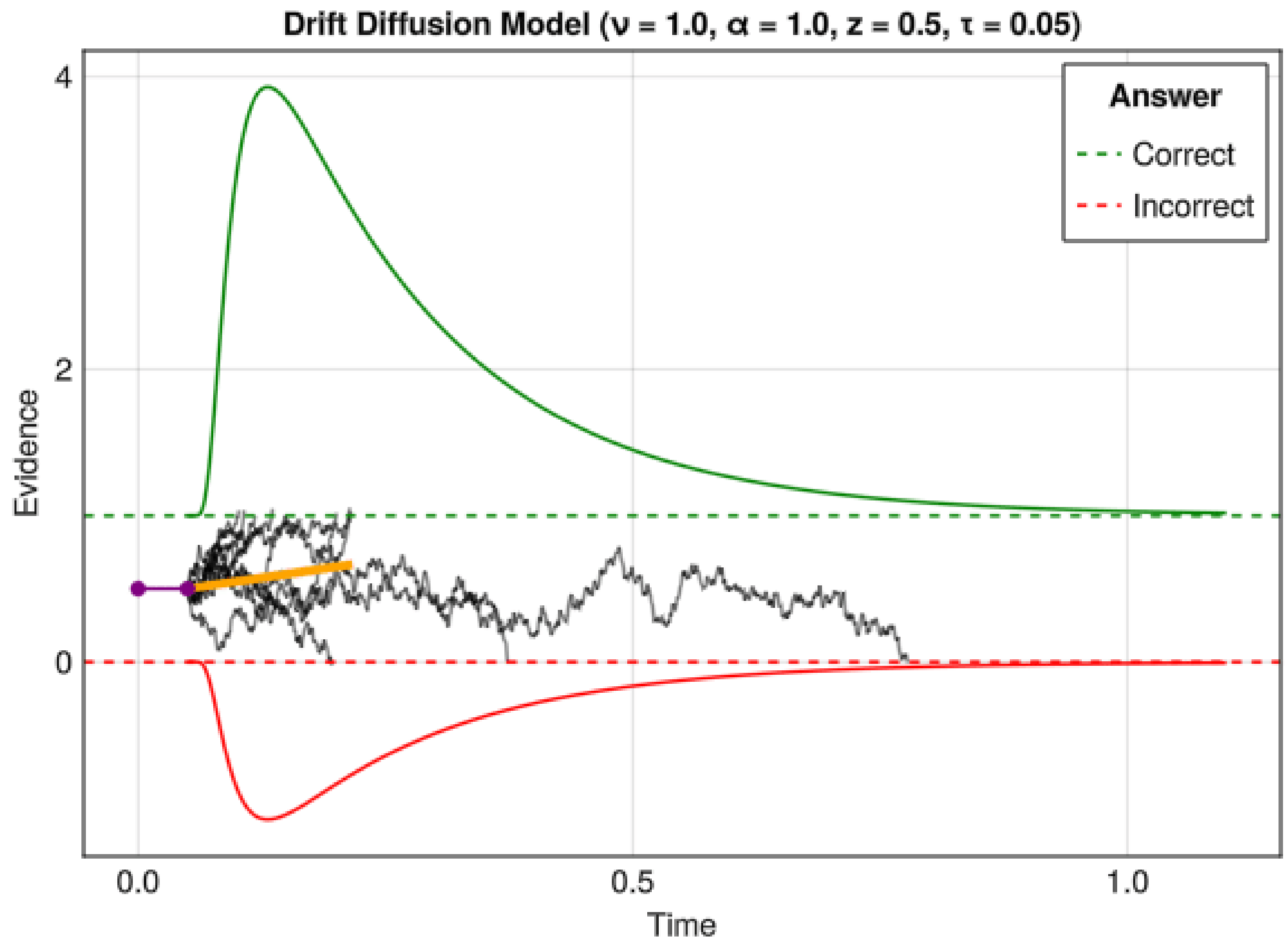
- μ: Drift rate (strength of evidence)
- σ : Noise infinitesimal standard deviation (square root of diffusion coefficient). Often set to 1 by convention (scaling parameter).
- a : boundary separation
- z : start point (= a/2 for an unbiased decision)
- T_{er} : non-decision time

$$RT = t_D + t_{ND}$$

'non-decision time' sensory encoding and motor delays

The Drift Diffusion Model

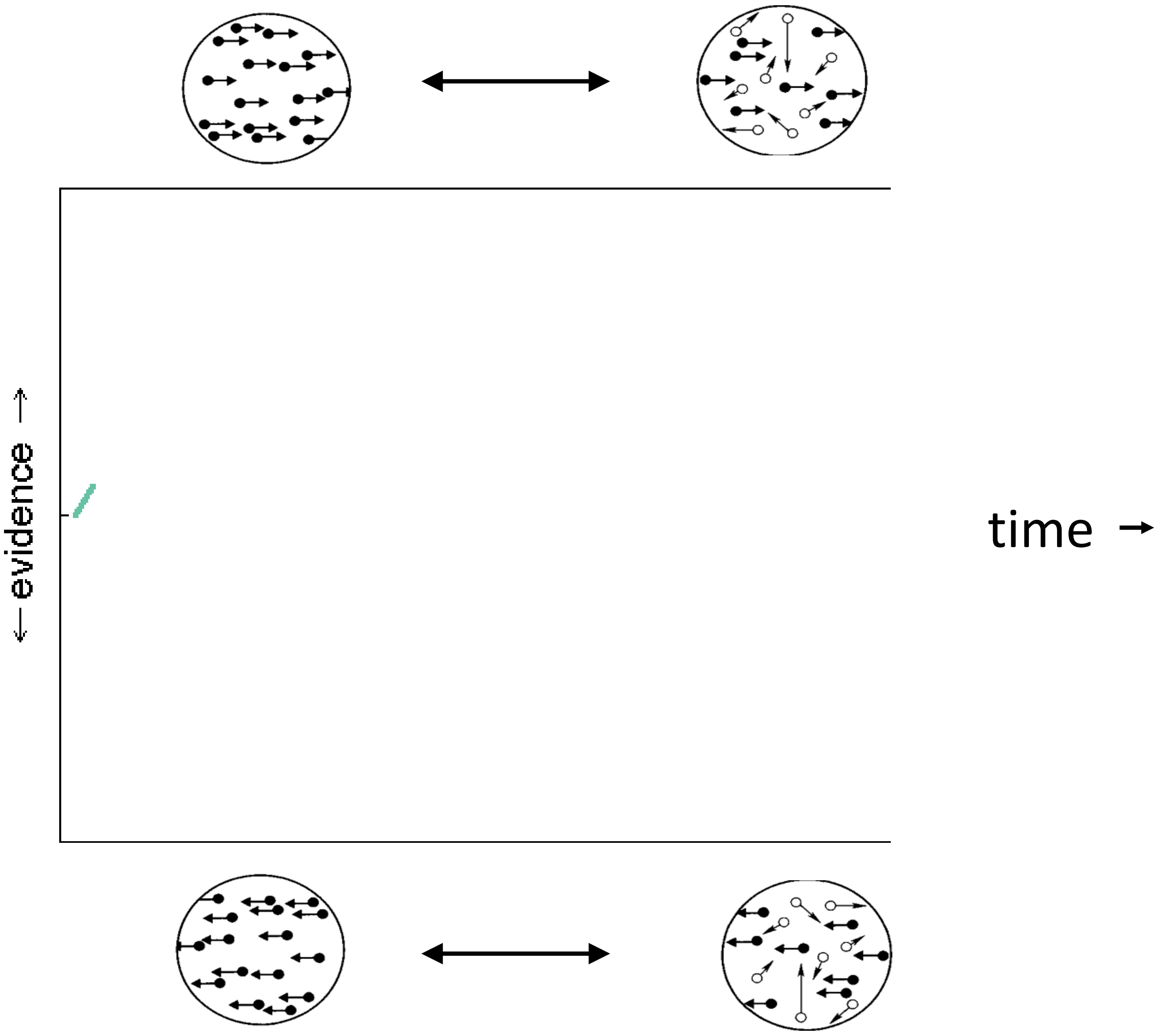
- Each parameter *uniquely* affects RT distributions and accuracies.
- For each set of parameters, we obtain a choice probability P(Correct) and two predicted RT distributions: correct and error responses (or left vs right responses).



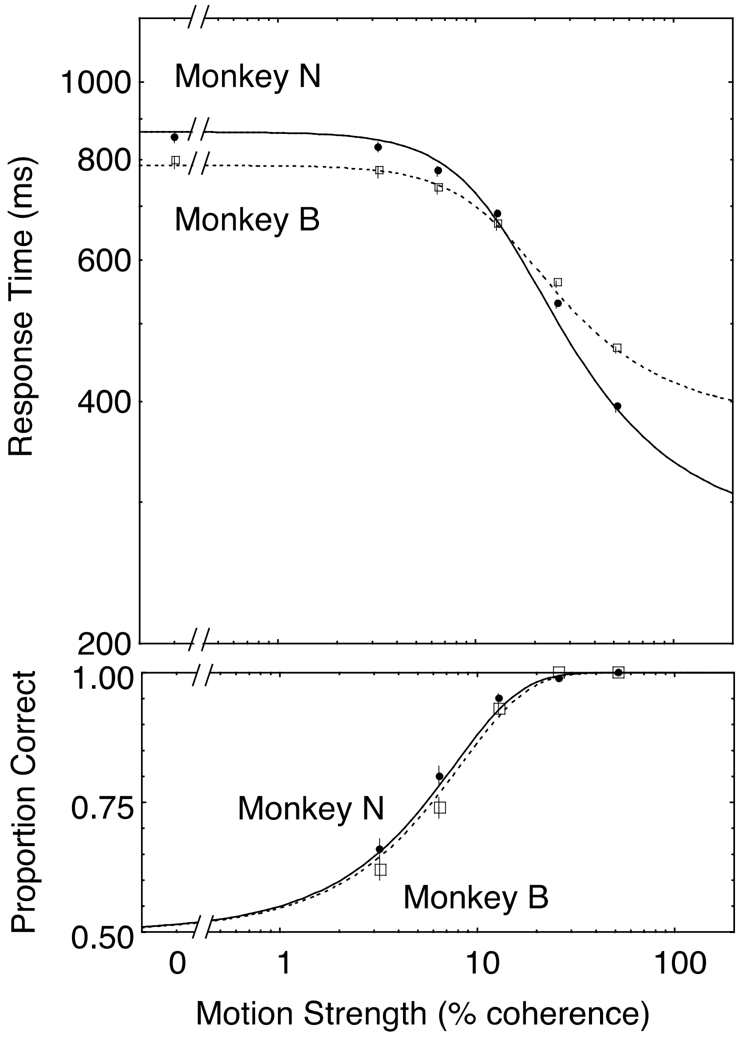
https://dominiquemakowski.github.io/CognitiveModels/4b_rt_generative.html

The DDM's ingredients

DDM ingredients: evidence strength

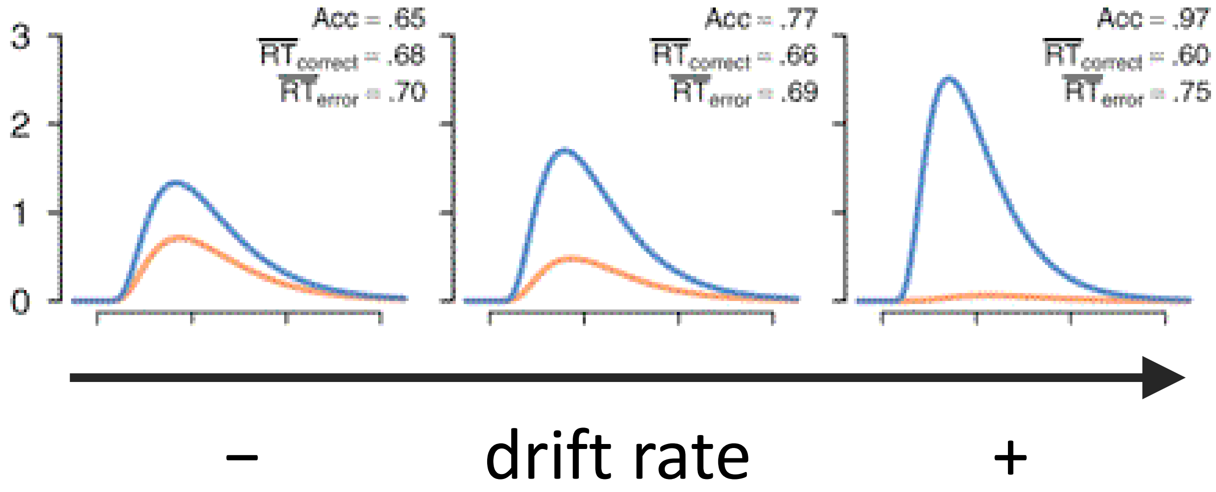


Changing evidence strength



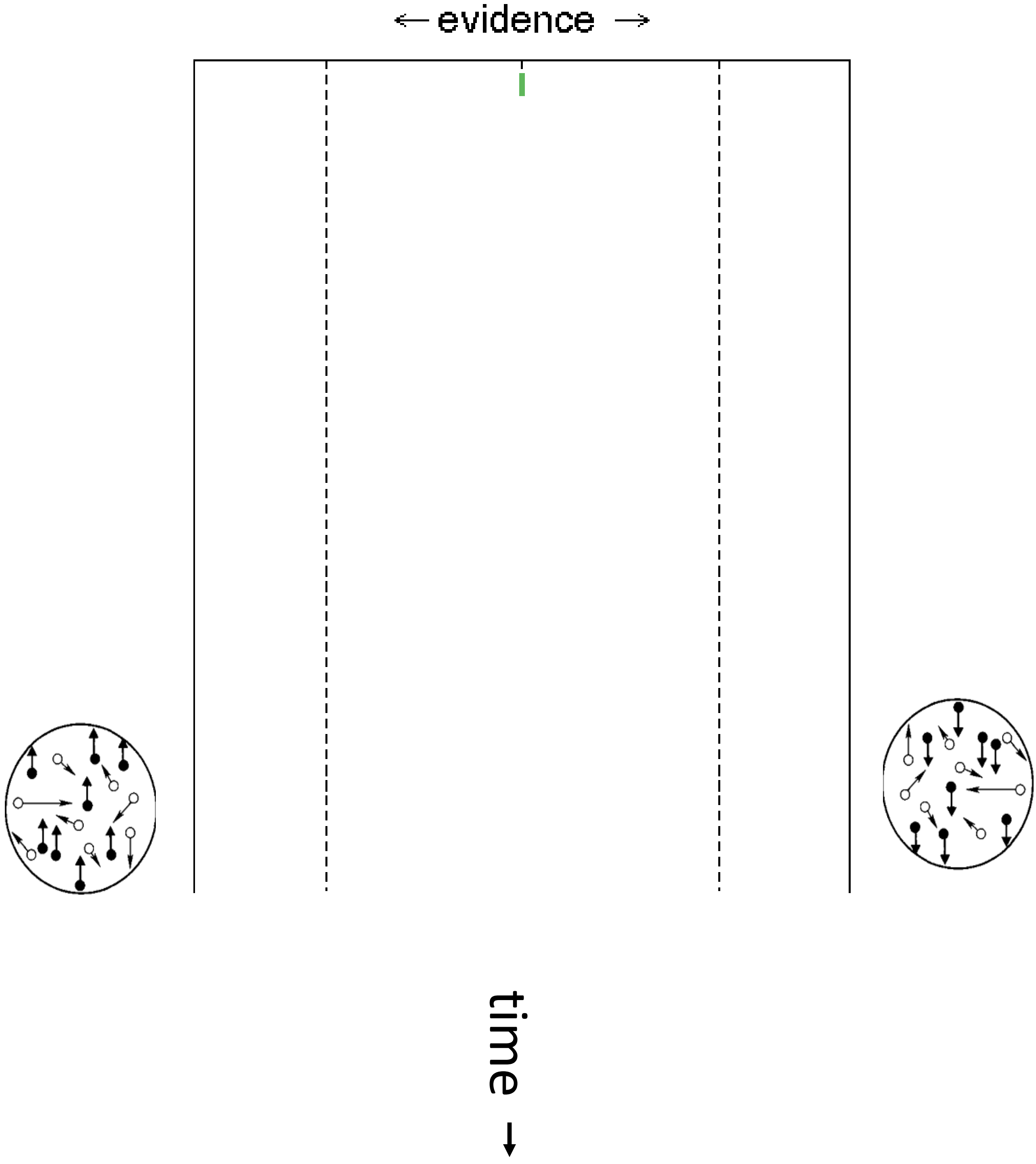
Increasing the drift rate:

- Higher accuracy
- Faster RT for both correct and error choices.

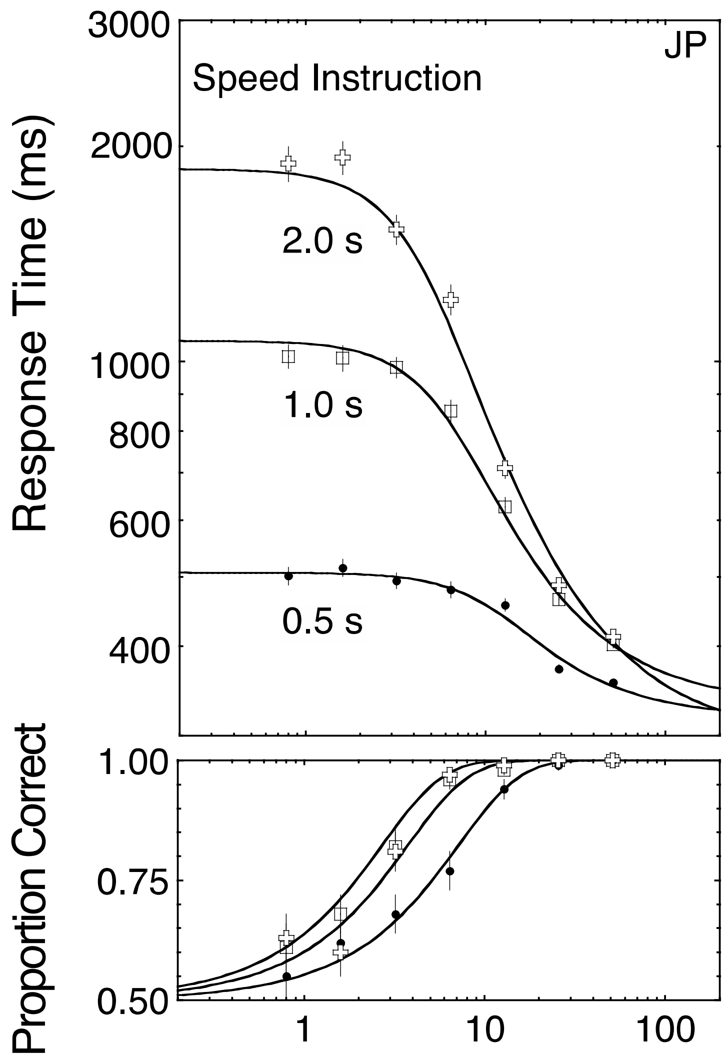


Palmer et al. 2005

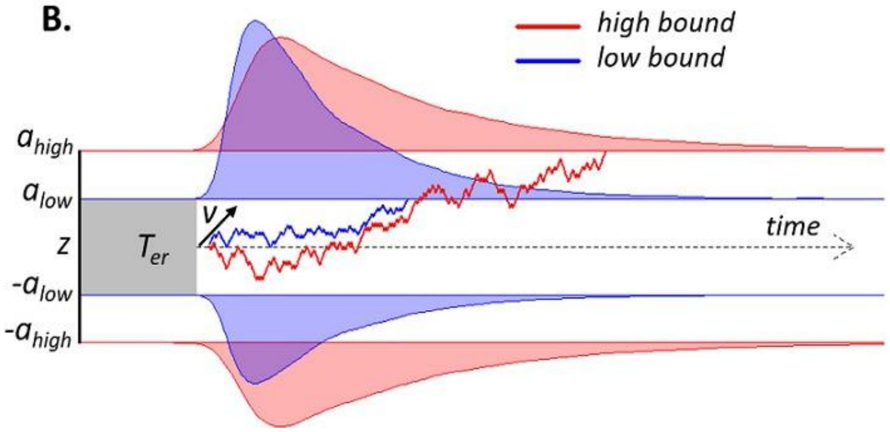
DDM ingredients: boundary adjustment



Speed-accuracy trade-off



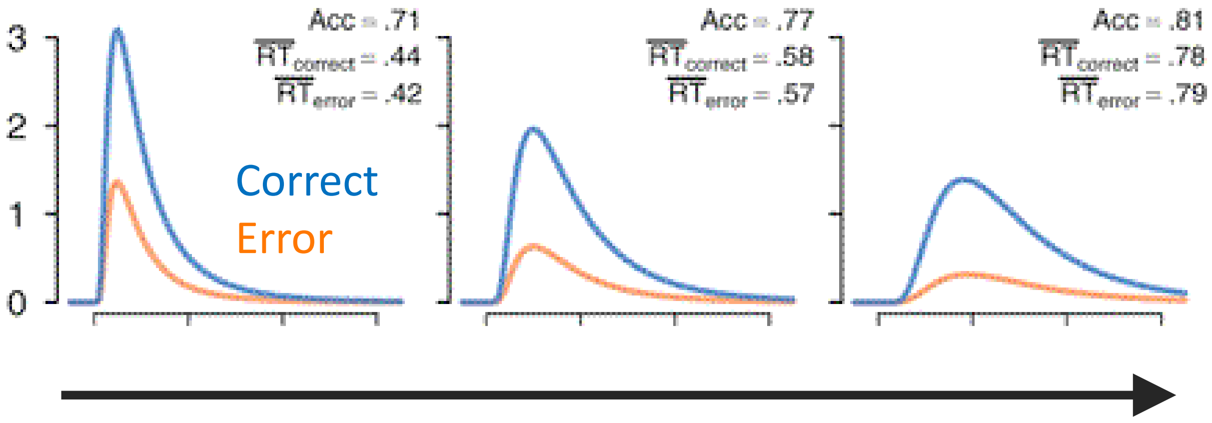
Palmer et al. 2005



Desender et al 2019

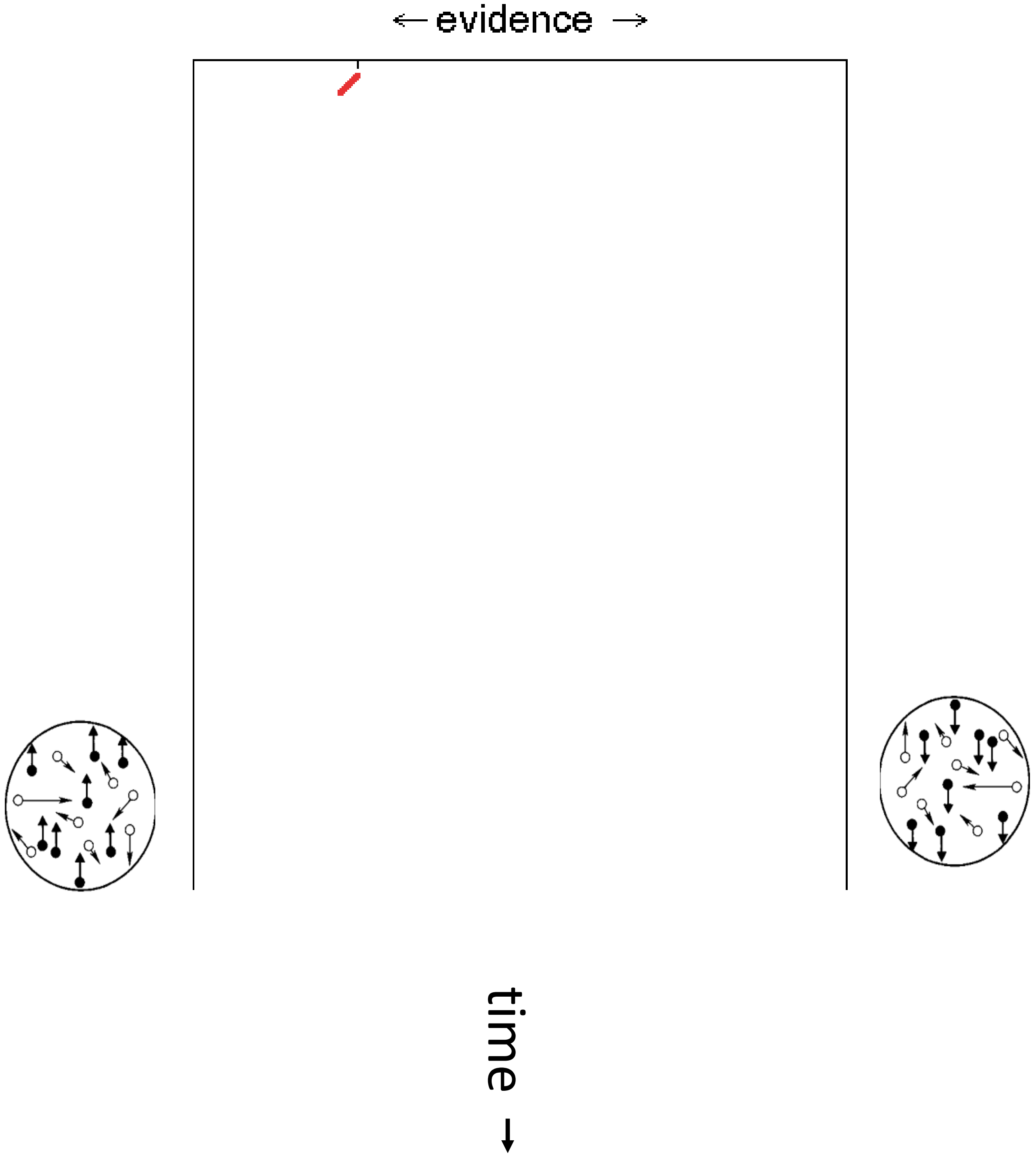
Increasing the bound:

- Higher accuracy
- Slower RT
- Increasing RT variance
- Decreasing skew

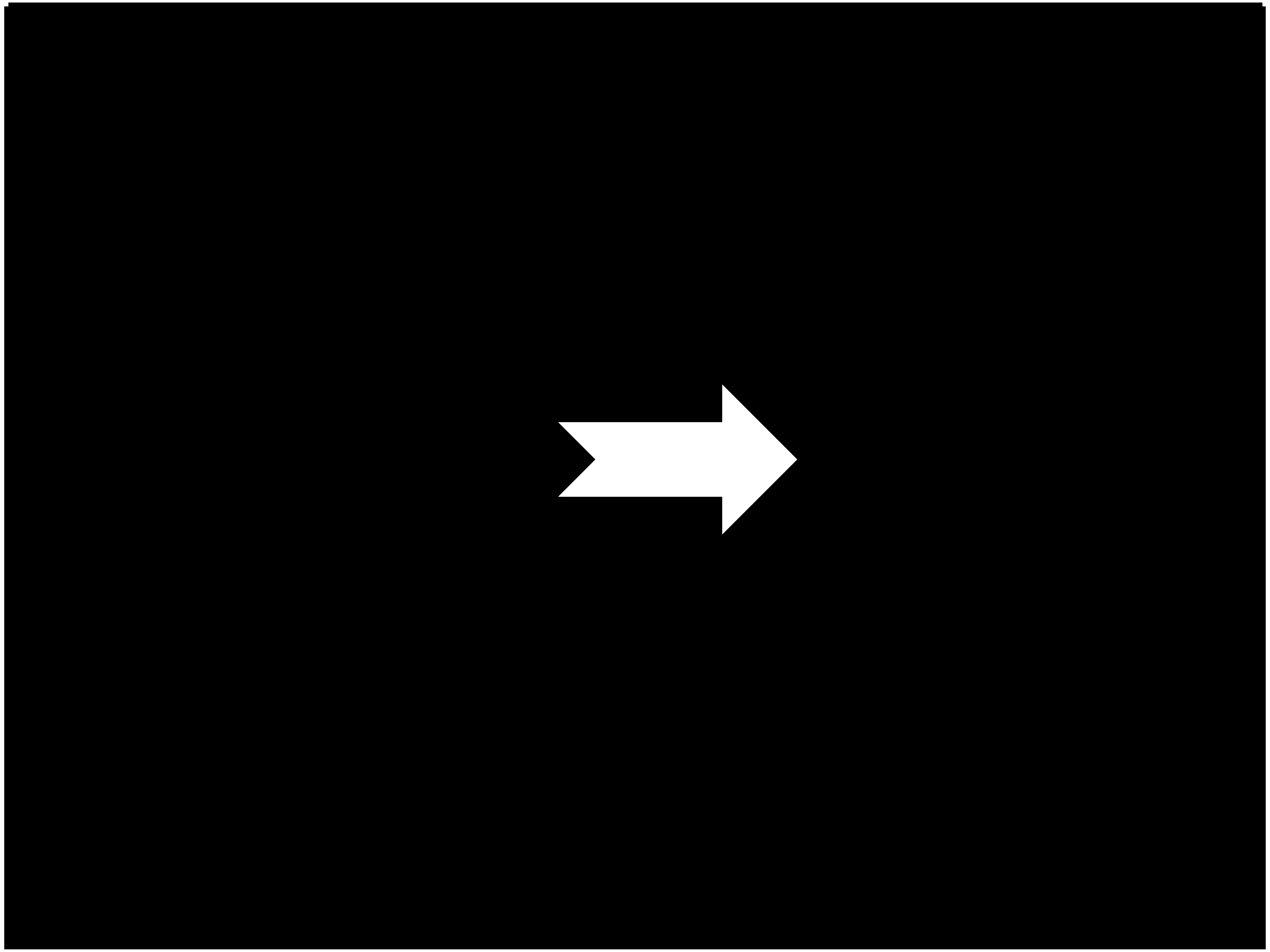


– decision bound +

DDM ingredients: starting point

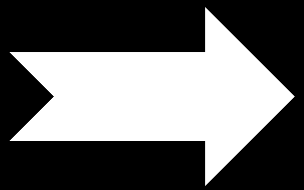


Motion discrimination task



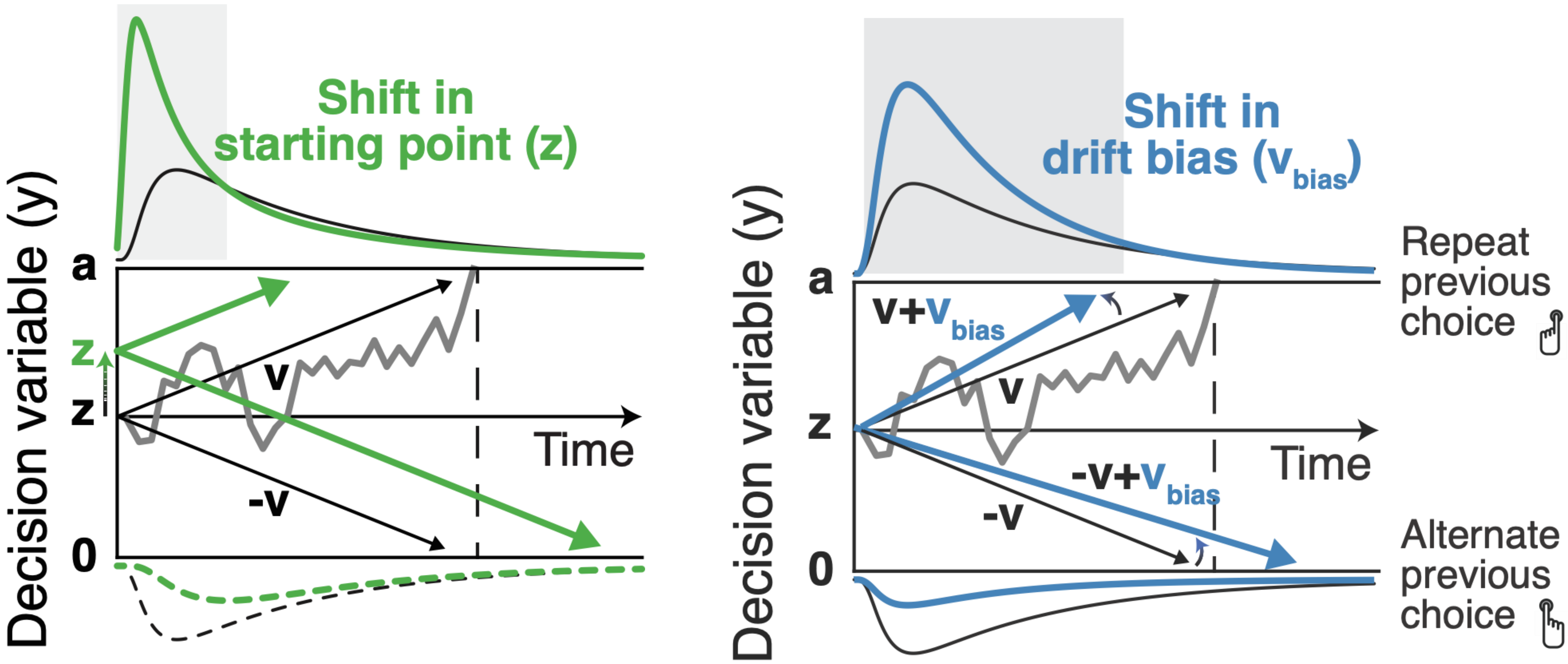
Coherence = 0.1

Motion discrimination task



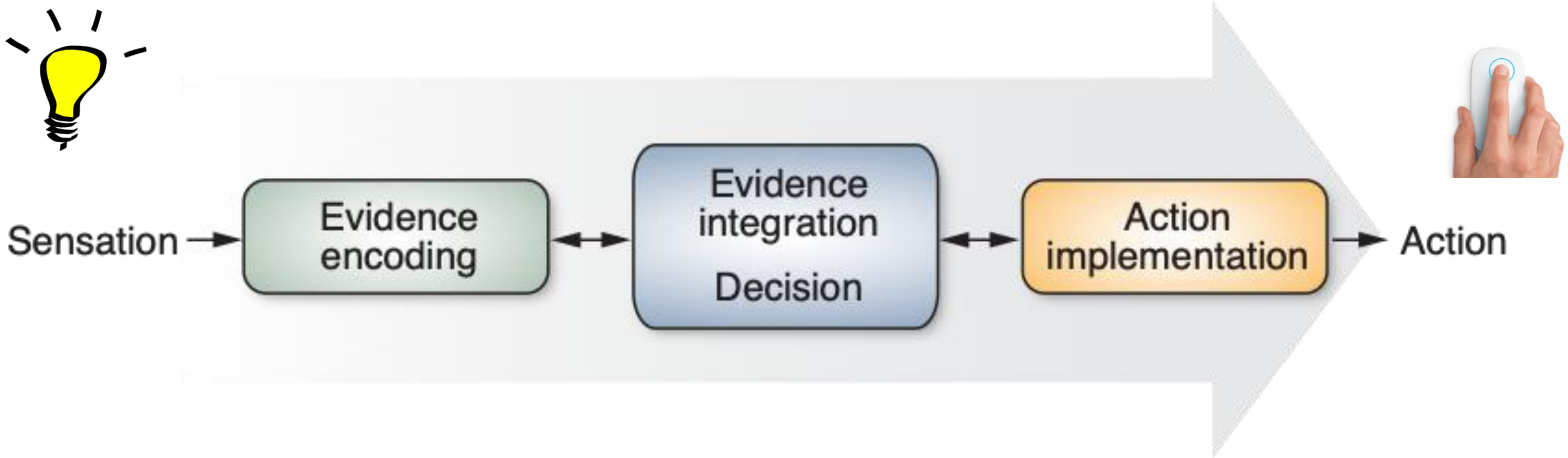
- This arrow (stimulus precue) predicts the motion direction *only* in 75% of the trials.
- It creates an unbalanced prior expectation in the subject that can be leverared to increase response accuracy.

Decision biases



Urai, de Gee, Tsetsos & Donner, eLife (2019)

Response times: arising from processing stages

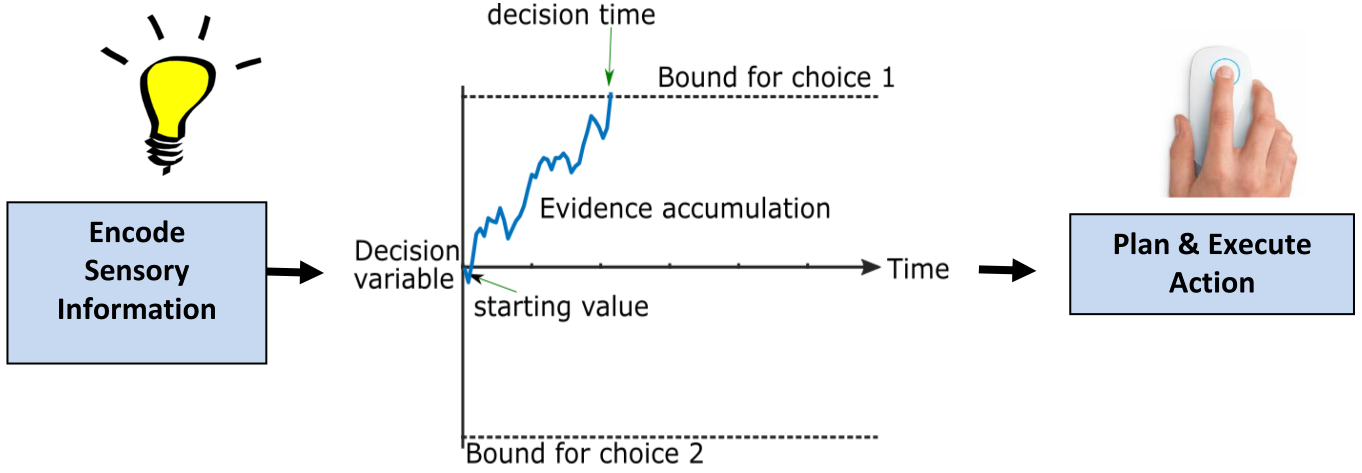


(Donders 1869, Sternberg 1969)

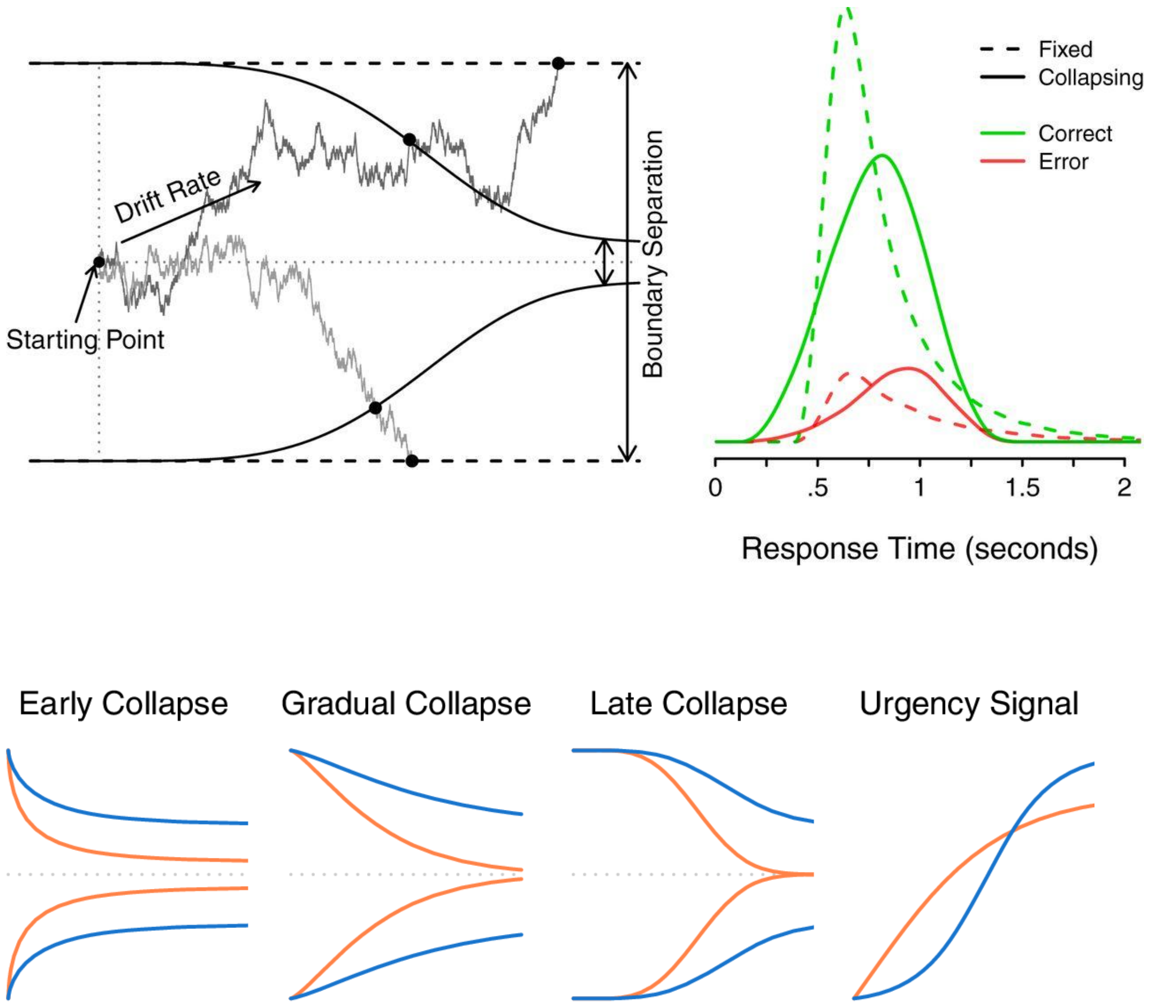
DDM ingredients: non-decision time

- In the simple diffusion model, non-decision time is determined by a single parameter: $t_{ND} = T_{er}$ (time for encoding and response).
- $RT = t_D + t_{ND}$

- Usually assumed to be constant across stimulus discriminability levels, speed-accuracy instructions (but not always! See Donkin et al)
- Having a *high discriminability condition* can be important for accurate estimation



Collapsing bounds / urgency



Hawkins et al., 2015

- Another mechanism to produce slow errors
- Controversial in the literature.. requires additional parameters to be estimated
- But, collapsing bounds are inconsistent with the neuroscience (build to threshold signals)
- In the context of racing accumulators, additive urgency is mathematically equivalent to a collapsing bound (Ditterich 2006)
- Several studies have found neurophysiological evidence for urgency (Churchland et al, 2008, Thura & Cisek 2014, Corbett et al, 2023)

Sequential sampling models (SSM)

- SSMs posit that the brain makes decisions by sampling noisy evidence over time, up to a threshold.
- Multiple model variants:
  - relative vs absolute evidence accumulation
  - Discrete vs continuous time
  - Leak vs perfect accumulation

Fitting behavioral data with the DDM

From descriptive modelling to statistical modelling

- Linear regression: captures reaction time
- Logistic regression: captures binary choices
- DDM: captures reaction time + binary choices

Descriptive vs mechanistic models, yet same framework (statistical, parameter based) so same tools (MLE, model selection, confidence intervals)

When to use a DDM

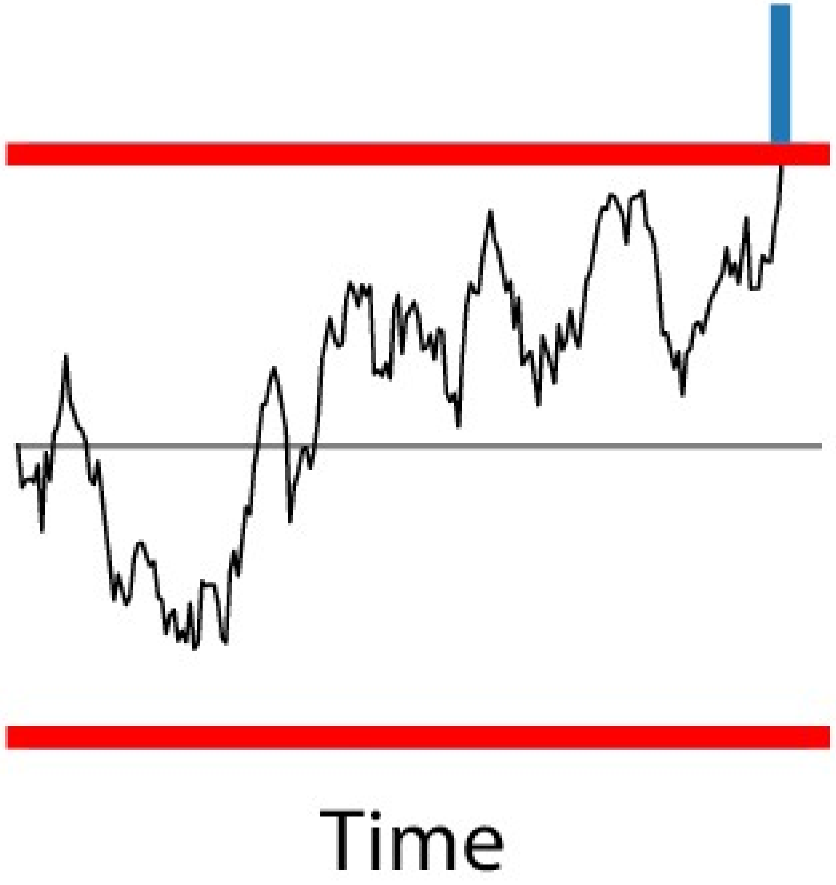
- You want to incorporate both choice and response time
  - If you don’t care about RT, there are simpler models (GLM, reinforcement learning, etc.)
- You have two alternatives
  - If you have only one alternative, you can fit a Wald distribution instead
  - If you have more than two alternatives, you (probably) need to use a race model

Part 1: Simulating the DDM by hand

- Basic algorithm
  - 1. Set x to starting point
  - 2. Set:

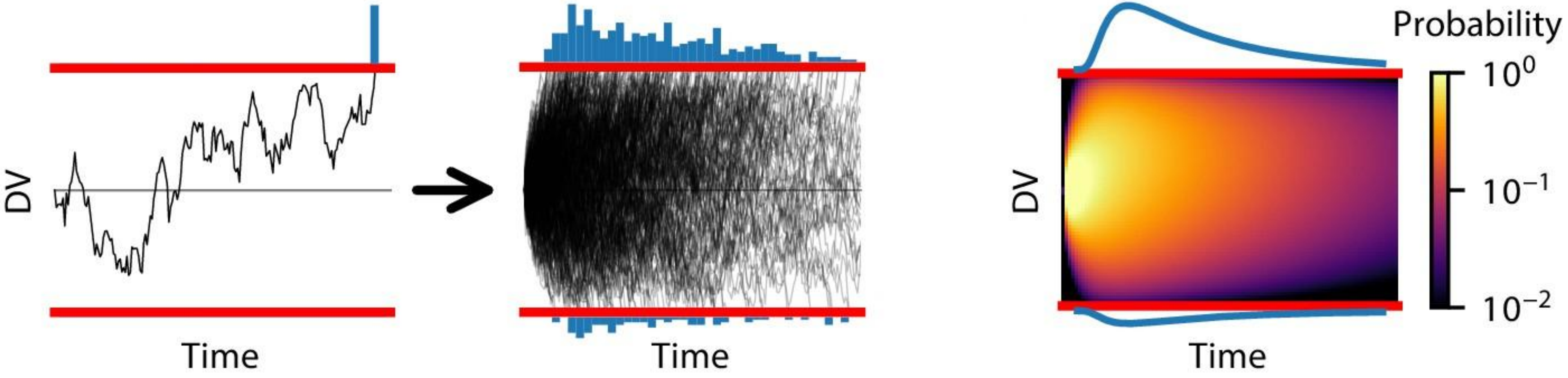
$$x_{t+1} = x_t + [\text{drift}]\Delta t + [\text{noise}] z_t \sqrt{\Delta t}$$
$$z_i \sim N(0,1)$$

  - 3. Check if x crosses a boundary. If so, you are done
  - 4. Otherwise, go to (2)



Part 2: Simulating the DDM using PyDDM

- Use more efficient methods to simulate the probability distribution of a trajectory's position instead of one trial at a time



How PyDDM works:

- Construct a Model from its components
- Model components:
  - Drift rate
  - Noise
  - Bound
  - Initial Condition
  - Non-decision time
  - Mixture model

Many model components are built-in:

- Each component can be:
  - A constant value (e.g., `3`)
  - A fittable parameter, given by the the name (e.g., `param1`)
  - A function which depends on:
    - Parameters
    - Task conditions
    - Magic arguments

Parameters and conditions

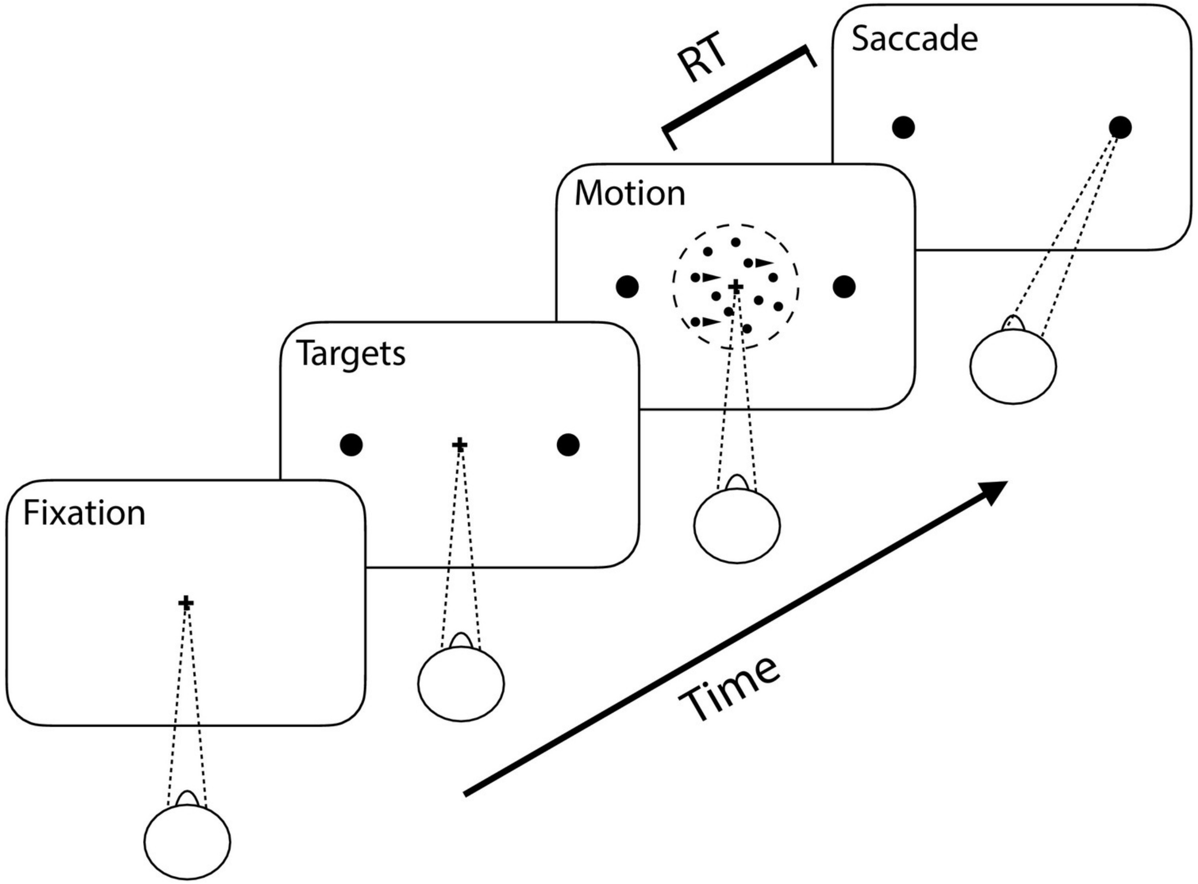
- Parameters: Have the same value for the entire dataset
  - E.g. bound height
- Conditions: May change from trial to trial
  - E.g. strength of motion coherence

Three objects to remember in PyDDM

- Model: created by the gddm() function
  - May need to call model.fit() before using if there are parameters
- Solution: Created using model.solve(conditions={...})
- Sample: RT and choice data, either experimental or simulated data

Part 3: Fitting the DDM to data

- Dataset: Monkeys performing the random dot motion task (Roitman and Shadlen, 2002)
- Several levels of motion coherence

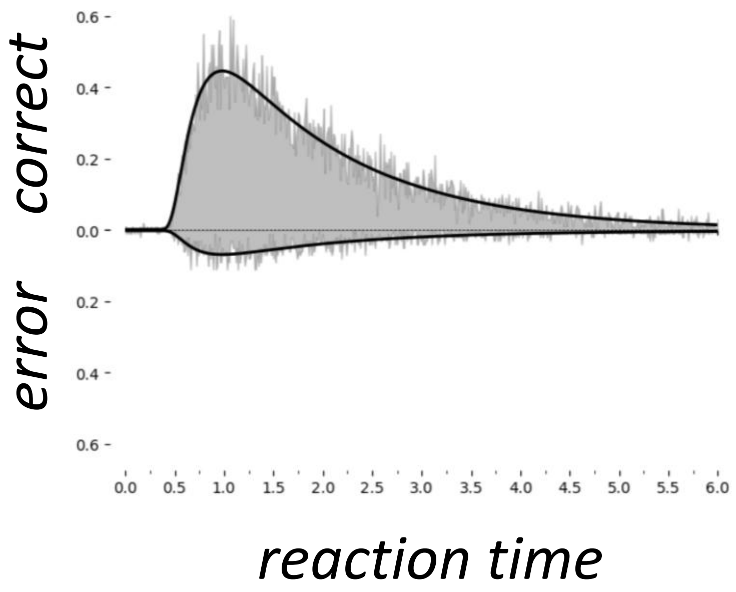


https://shadlenlab.columbia.edu/resources/RoitmanDataCode.html

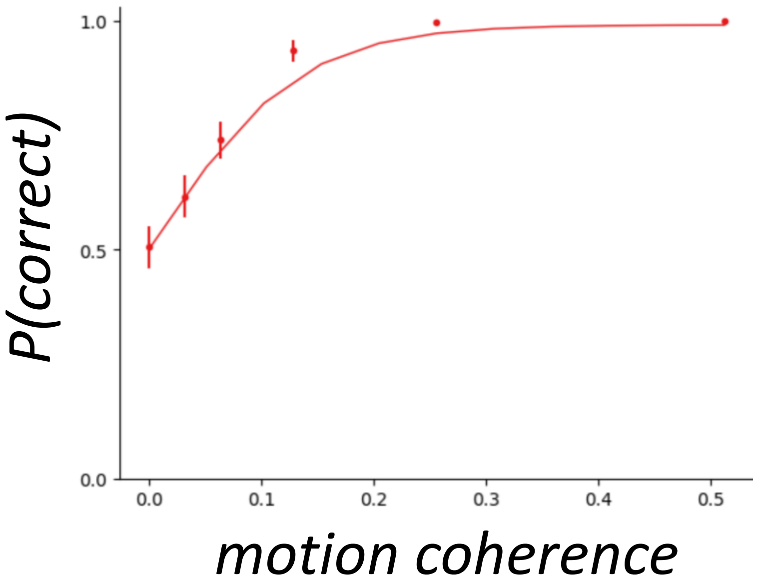
Validating the DDM

It is **crucial** to check that the fitted model captures the important features in the behaviour!

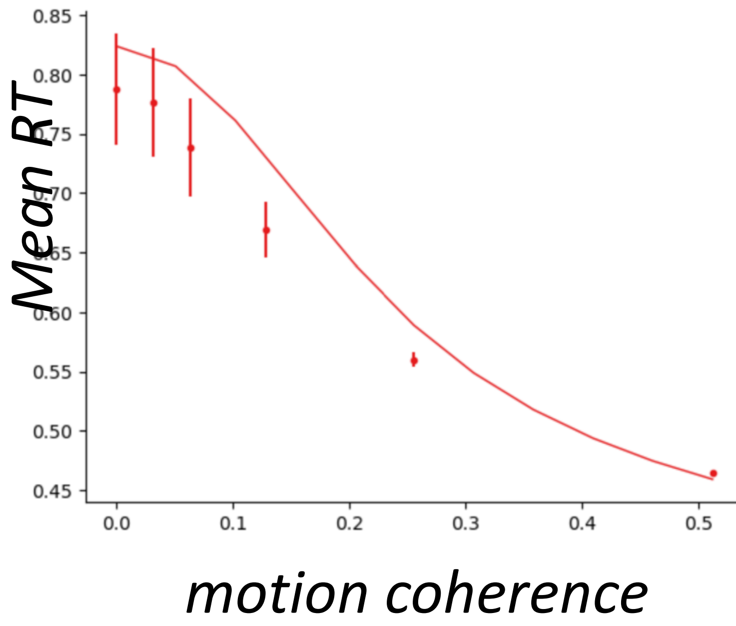
RT densities for correct/incorrect responses



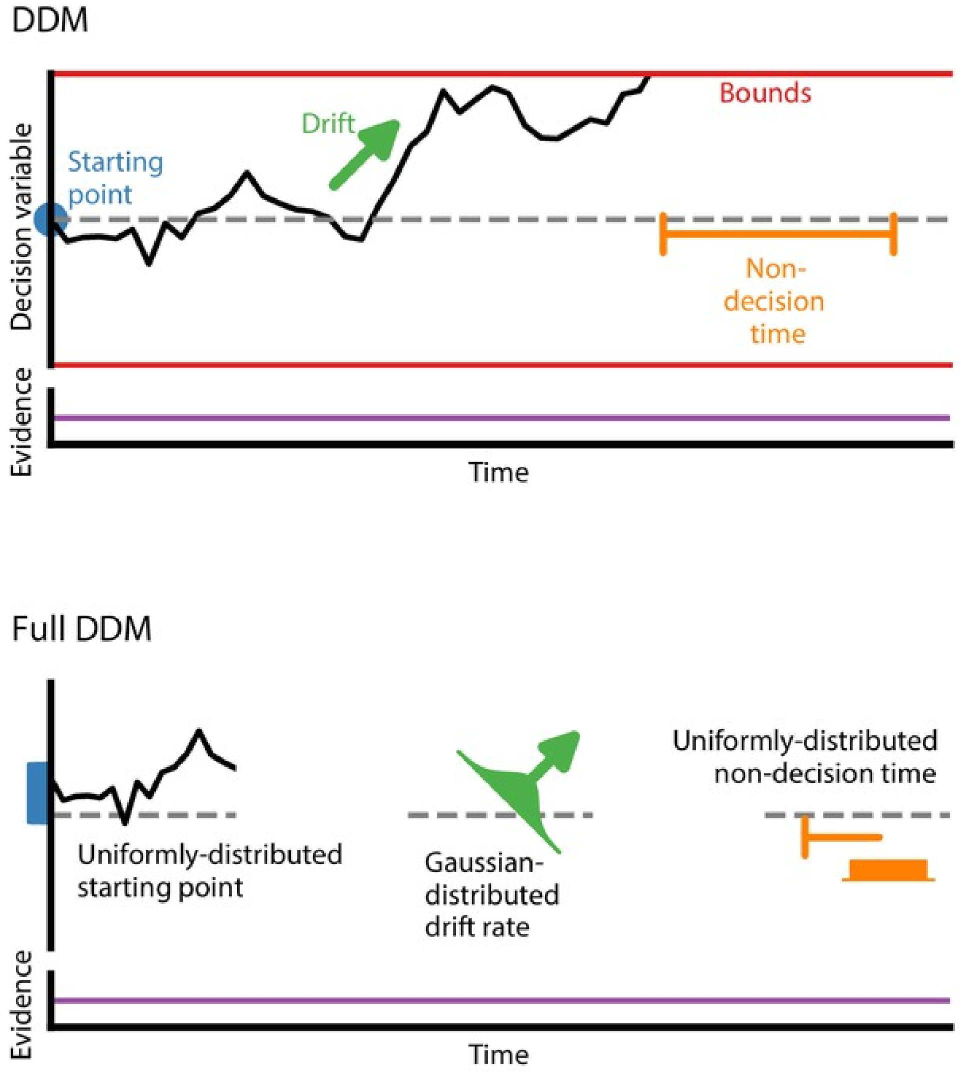
Psychometric curve

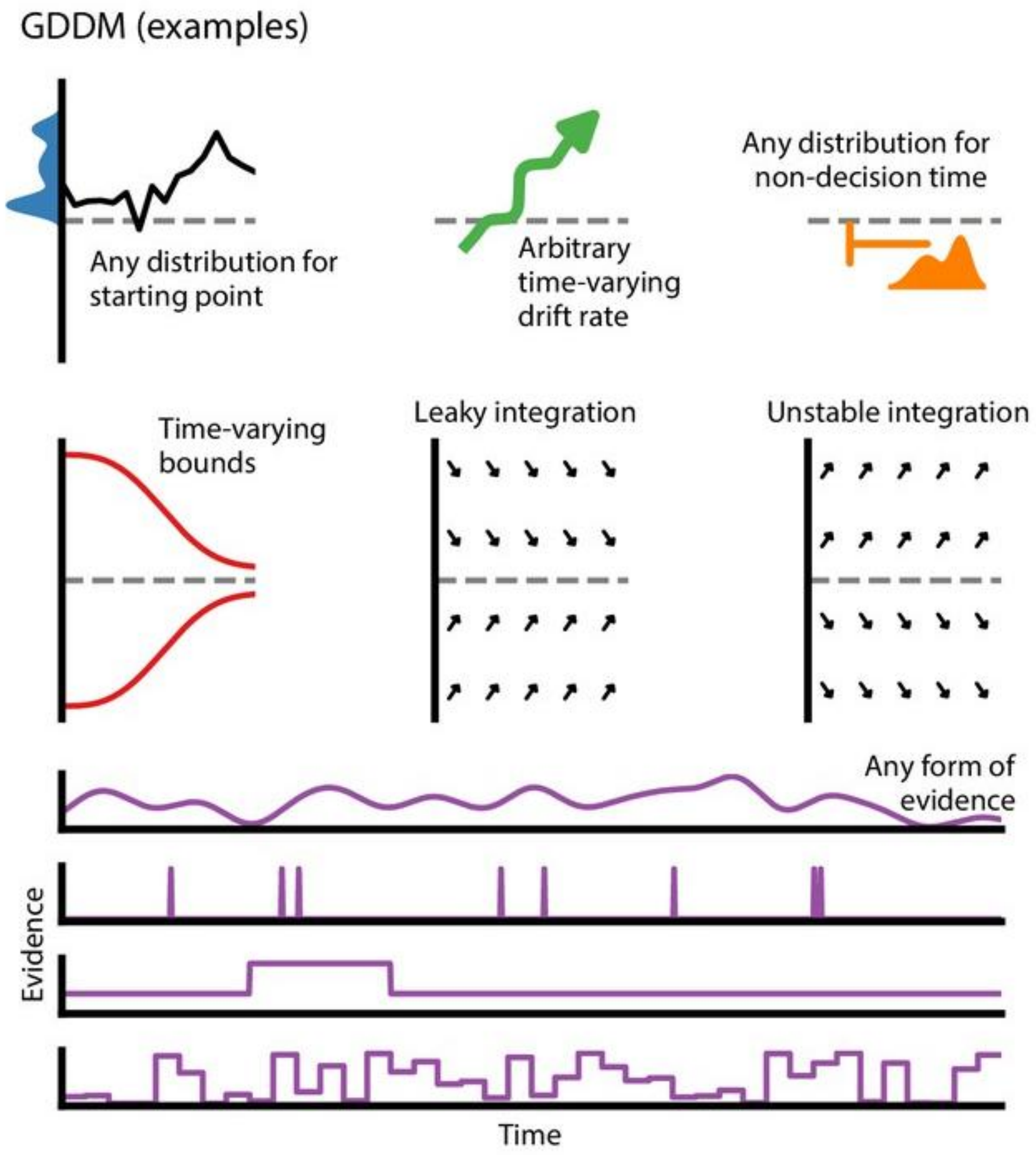


Chronometric curve



Generalized DDM (GDDM)





When should you use these GDDMs?

- Response time distribution is not skewed
  - Consider leaky integration
- The speed-accuracy tradeoff may change across the trial
  - Consider collapsing bounds
- I think the agent may be more likely to choose one choice over another or have a prior
  - Consider a starting point or drift bias
- Evidence is not constant in my task or it requires multisensory integration
  - Consider a more complex drift rate function
- There is a large variability in motor actions
  - Consider non-decision time variability *(but be careful! This can make the model non-recoverable)*