

Graph Machine Learning

Artificial Intelligence Summer Camp - 2025



Inteligencia Artificial PUCP (IA-PUCP)

January 14, 2025

1 Introduction

2 Traditional Method

3 Graph Neural Network (GNN)

4 Resources

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Graphs

Graph is a data structure and is defined by $G = (V, E)$, where V is the set of nodes and E is the set of edges.

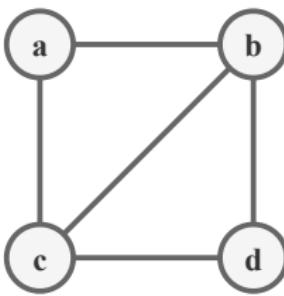
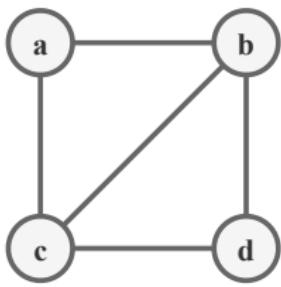


Figure 1: Graph example.

Graph representations



(a) Input graph.

	a	b	c	d
a	0	1	1	0
b	1	0	1	1
c	1	1	0	1
d	0	1	1	0

(b) Adjacency matrix.

a	b	c	
b	a	c	d
c	a	b	d
d	b	c	

(c) Adjacency list.

a	b
a	c
b	c
b	d
c	d

(d) Edge list.

Figure 2: Graph representations.

Other definitions/concepts

- Homogeneous graph
- Heterogeneous graph
- Directed graph
- Undirected graph
- Weighted graph
- Bipartite graph
- Complete graph
- Degree
- Density
- Diameter

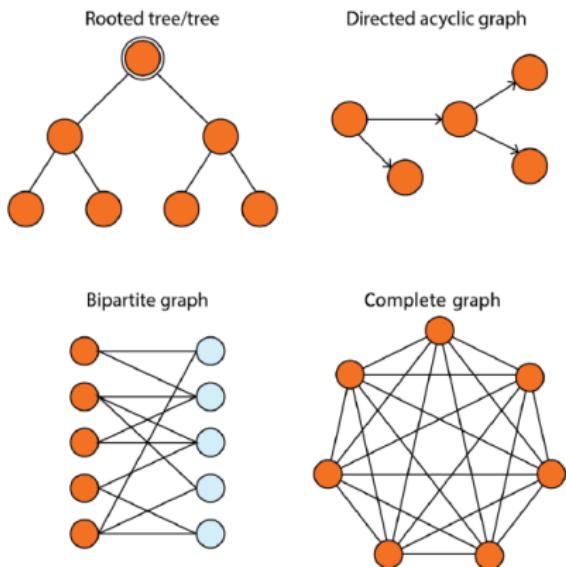


Figure 3: Common type of graphs.
Adapted from [1].

Graphs are everywhere!

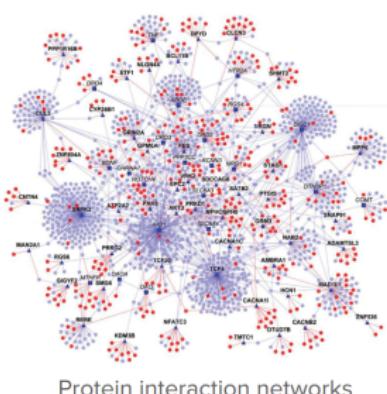
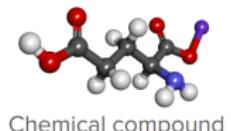


Figure 4: Graphs are everywhere.

Graph embedding

It is a function $f: A \rightarrow Z$ that maps each node of the graph $G = (V, E)$ into a d -dimensional vector. Where,
 $Z = \{z_1, z_2, \dots, z_N\}$, $z_i \in \mathbb{R}^d$ with $d \ll N$.

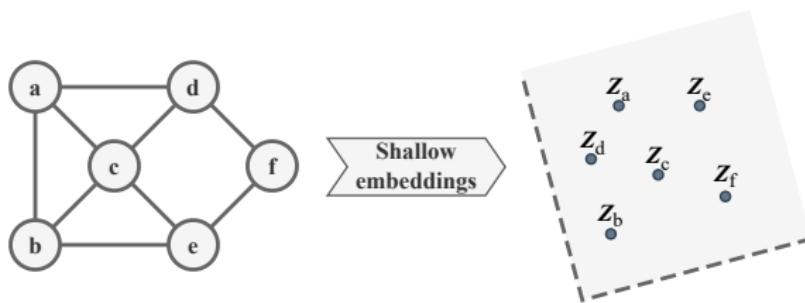


Figure 5: Shallow embeddings scheme.

More about embeddings

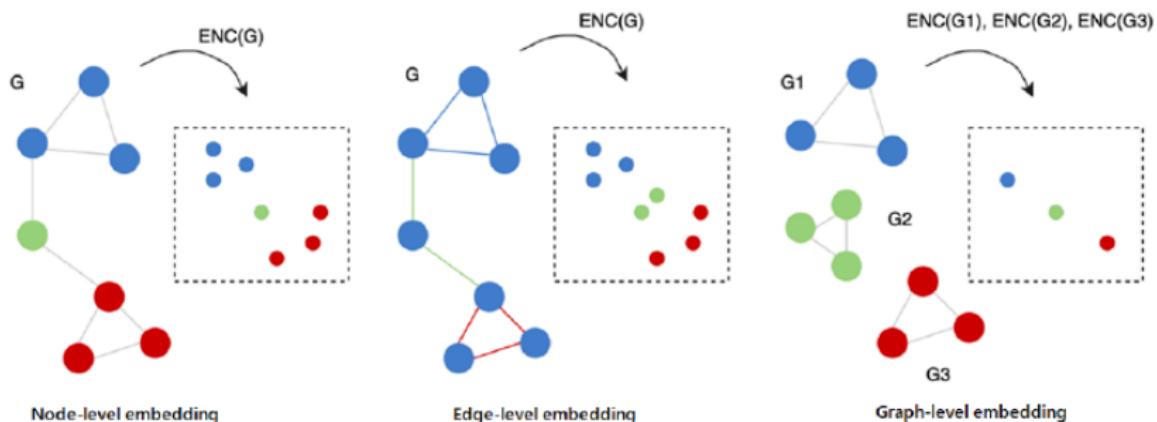


Figure 6: Visual representation of the three different levels of granularity in graphs.
Adapted from [2].

Tasks

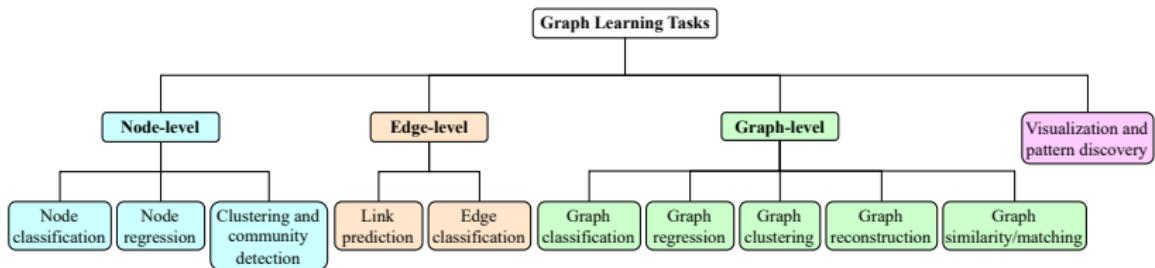
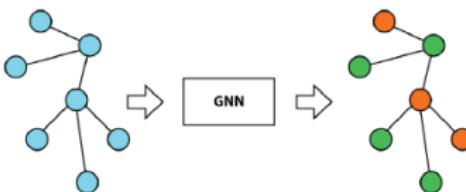


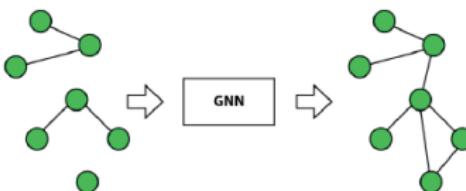
Figure 7: Overview of graph learning tasks.

Tasks

a. Node classification



b. Link prediction



c. Graph classification

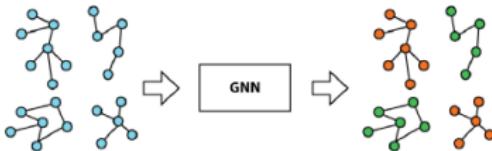


Figure 8: Examples of graph learning tasks. Adapted from [1].

Applications

- Computer vision
- Natural language processing
- Recommender systems
- Program analysis
- Knowledge graphs
- Bioinformatics
- Chemistry
- Biology
- Physic
- Anomaly detection
- Urban intelligence
- Financial transaction
- ...

Taxonomy of graph machine learning methods.

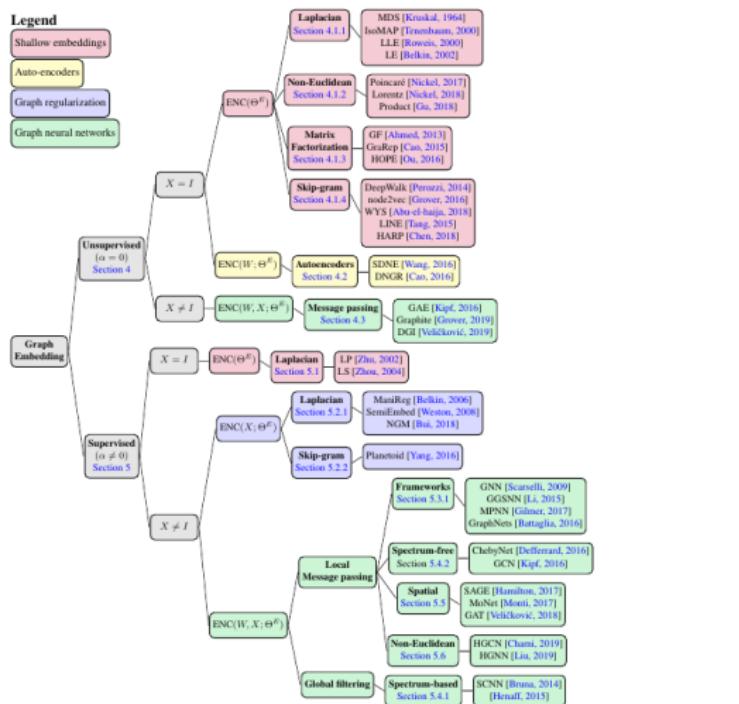


Figure 9: Taxonomy of graph machine learning. Adapted from [3].

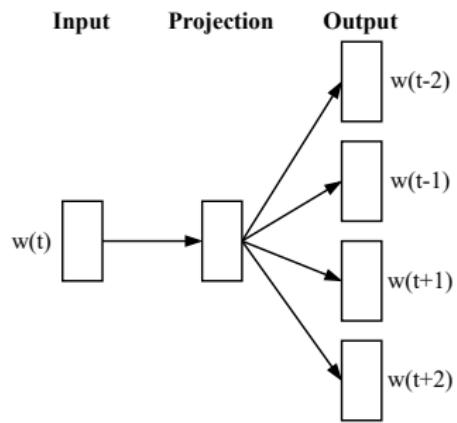
1 Introduction

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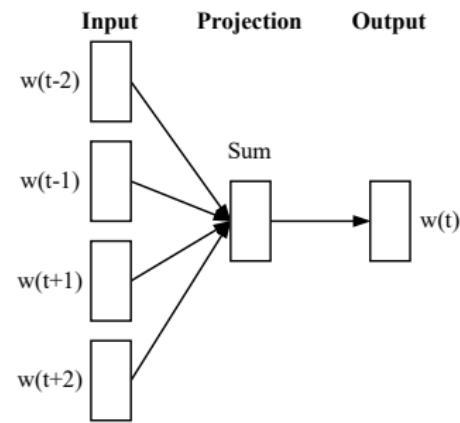
3 Graph Neural Network (GNN)

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Word2vec [4]



(a) Skip-gram.



(b) CBOW.

Figure 10: Word embedding architectures. Adapted from [4].

Word2vec

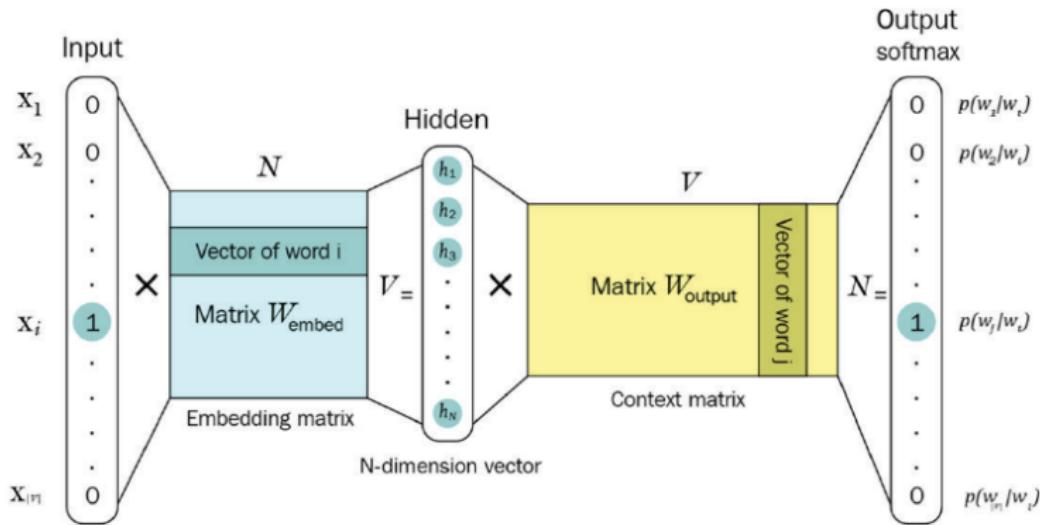


Figure 11: Word2vec architecture. Adapted from [1].

Word2vec in action

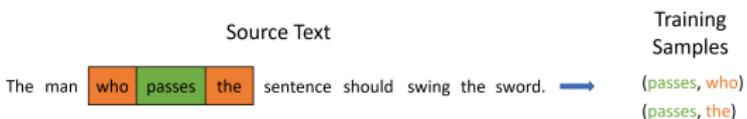


Figure 12: Train window.

Input Layer (x)
(V-dim)

Center Word (w_t)	v=0	v=1	v=2	v=3	v=4	v=5	v=6	v=7
t=0 The	0	0	0	0	0	0	1	0
t=1 man	1	0	0	0	0	0	0	0
t=2 who	0	0	0	0	0	0	0	1
	aegis4048.github.io							
	⋮					⋮		
t=9 sword	0	0	0	0	0	1	0	0

Figure 13: One-hot encoded input vector.

Word2vec in action

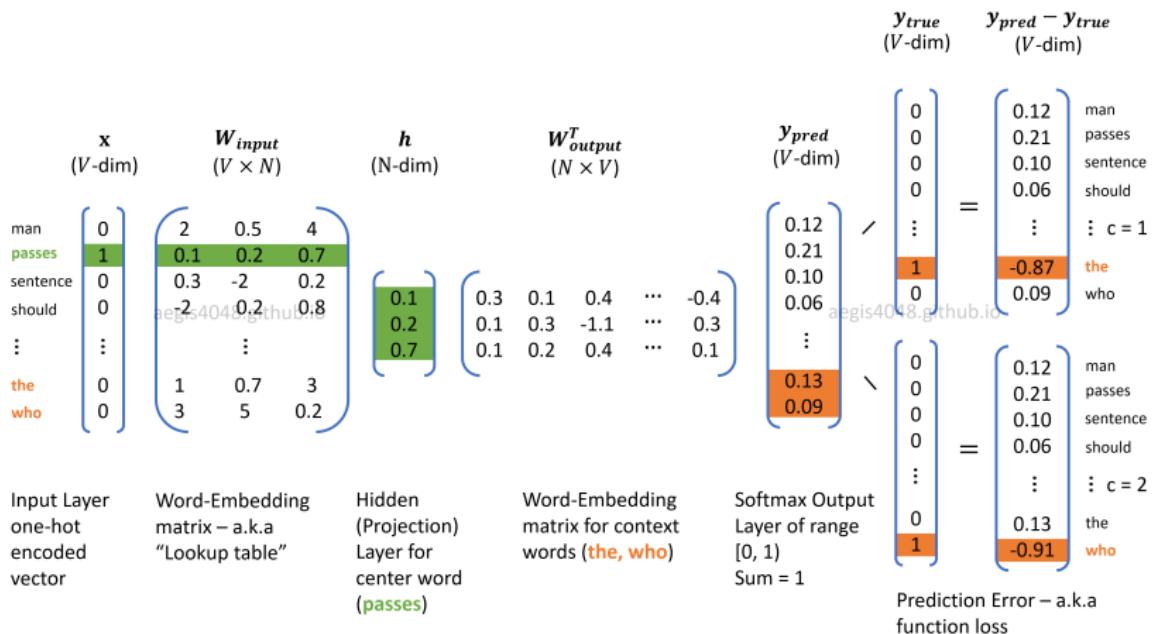


Figure 14: Skip-Gram model structure. Current center word is “passes”. From https://aegis4048.github.io/demystifying_neural_network_in_skip_gram_language_modeling.

Word2vec in action

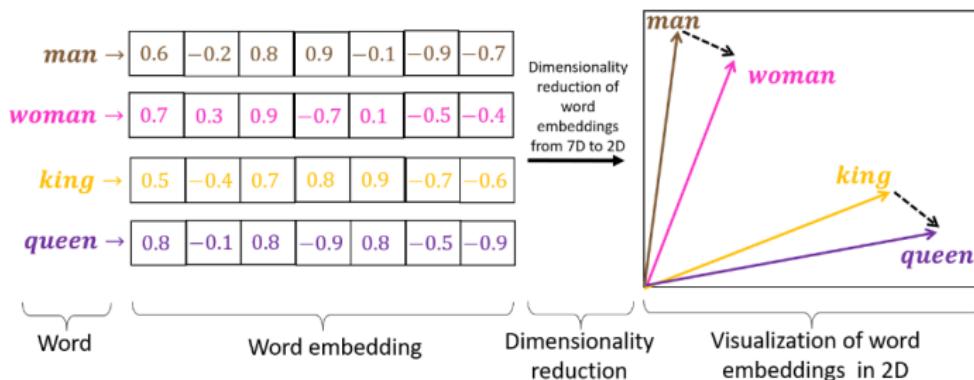


Figure 15: Word embeddings.

From <https://doi.org/10.1371/journal.pone.0231189.g008>.

Cosine similarity

$$\cos(\theta) = \text{sim}(z_i, z_j) = \frac{z_i \cdot z_j}{\|z_i\| \|z_j\|} \quad (1)$$

DeepWalk [5]

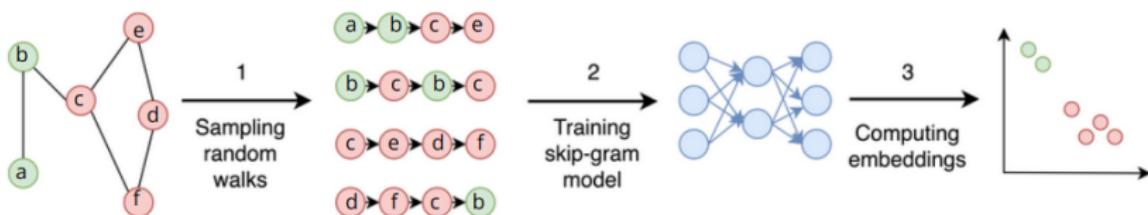


Figure 16: DeepWalk workflow.

Parameters:

- $\text{length} = 4$
- $n = 1$

Node2vec [6]

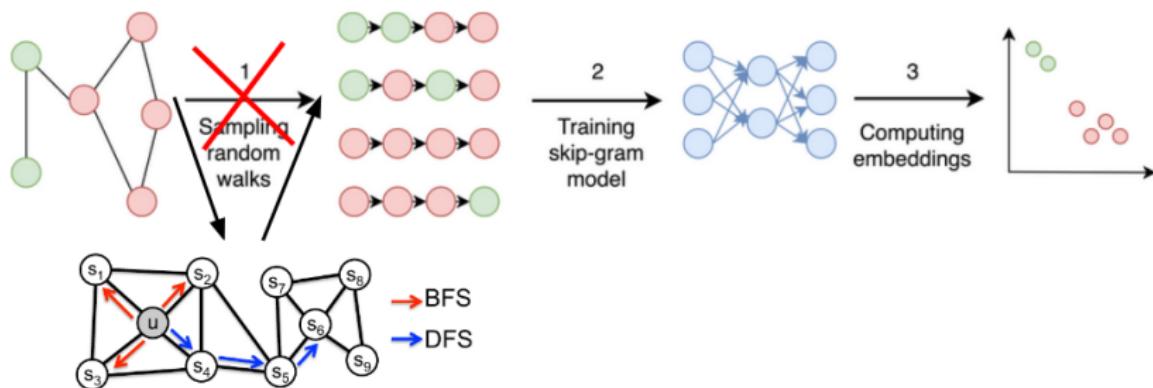


Figure 17: Node2vec workflow.

Parameters:

- $\text{length} = 4$
- $n = 1$
- $p = 1, q = 2$ (BFS: micro-view) or
- $p = 1, q = 0.5$ (DFS: macro-view)

Application on biomedical data

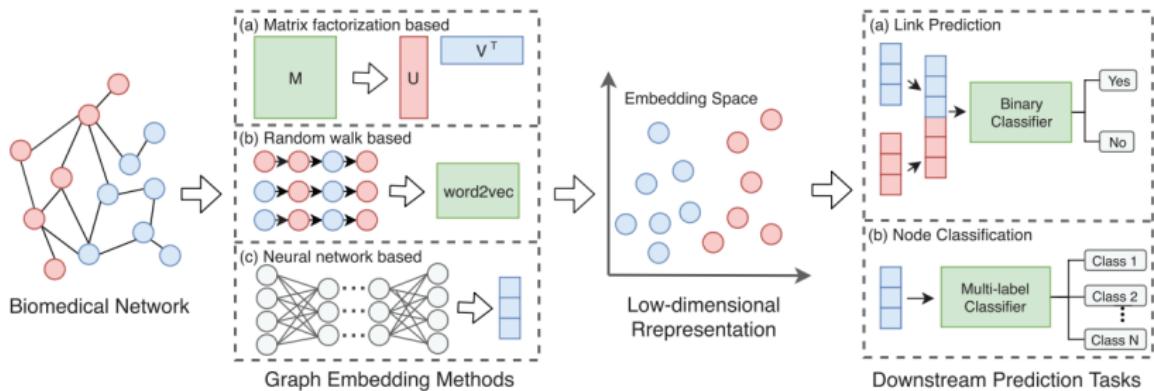


Figure 18: Applying graph embedding methods to biomedical tasks. Adapted from [7].

Practical example: Node2vec

https://github.com/win7/Presentations-Hub/blob/main/SummerCamp-2025_IA-PUCP/Community_Detection.ipynb

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Graph Neural Network (GNN)

GNN = Neural networks + Graph theory

Basics of Neural networks (NN)

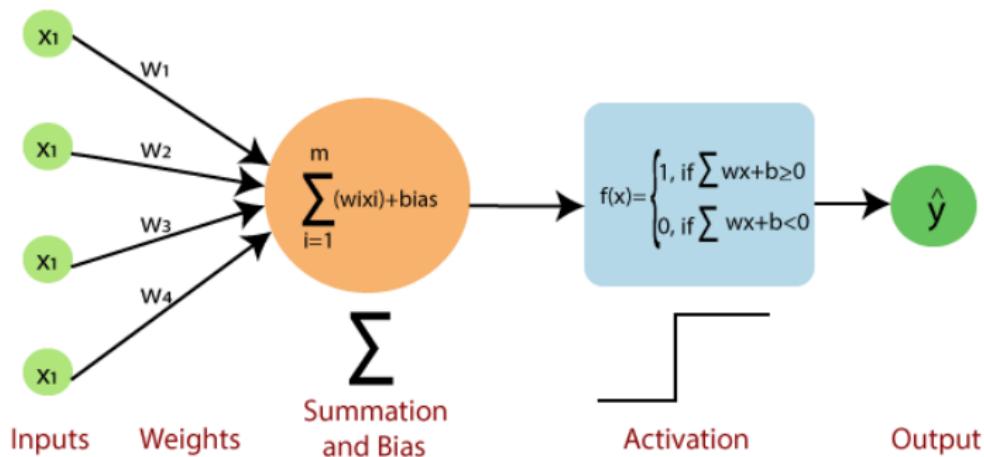


Figure 19: Basic NN.

From <https://www.javatpoint.com/single-layer-perceptron-in-tensorflow>.

Neural networks (NN)

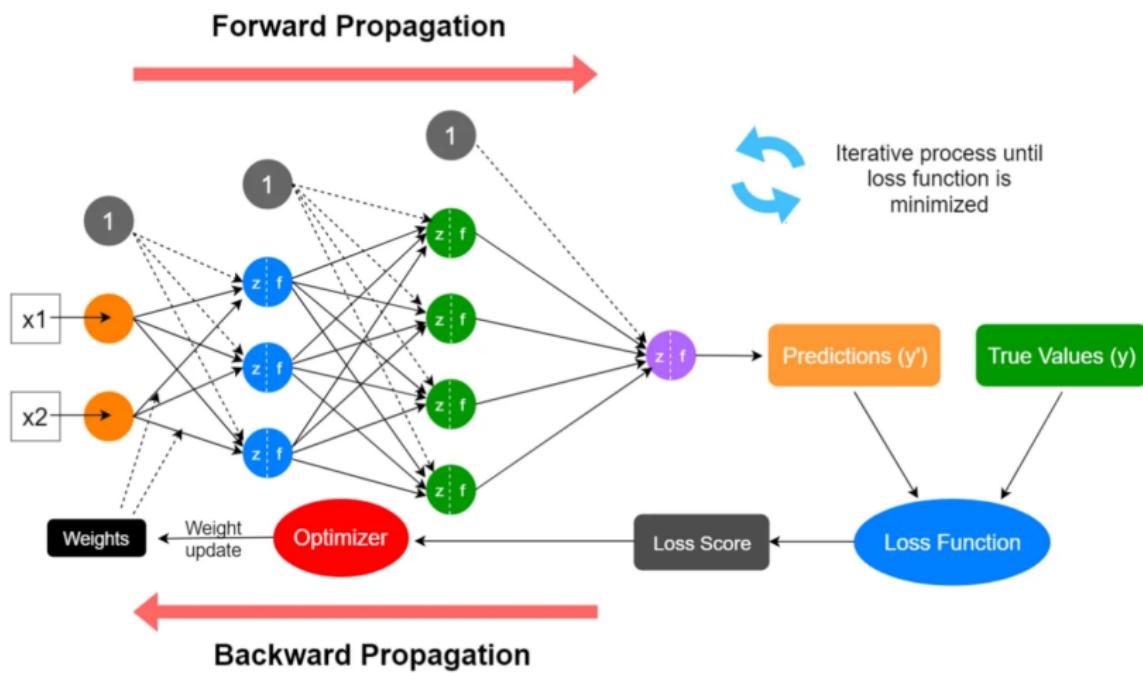


Figure 20: Basic NN. From <https://medium.com/data-science-365/overview-of-a-neural-networks-learning-process-61>.

Example of NN

Figure 21: Image classification. From
<https://www.3blue1brown.com/lessons/neural-networks>.

Graph Neural Networks (GNN)

It is a function $f: (A, X) \rightarrow H$ that maps each node of the graph $G = (V, E)$ into a F -dimensional vector.

Here, $A \in \mathbb{R}^{N \times N}$, $X \in \mathbb{R}^{N \times C}$, $H^l \in \mathbb{R}^{N \times F}$ with $F \ll N$,
 $l \in \{1, 2, \dots, L\}$.

$$H = \text{GNN}(A, X)$$

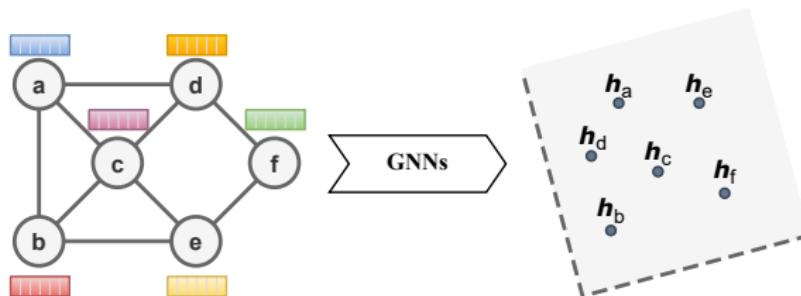


Figure 22: GNNs scheme.

How GNNs works?

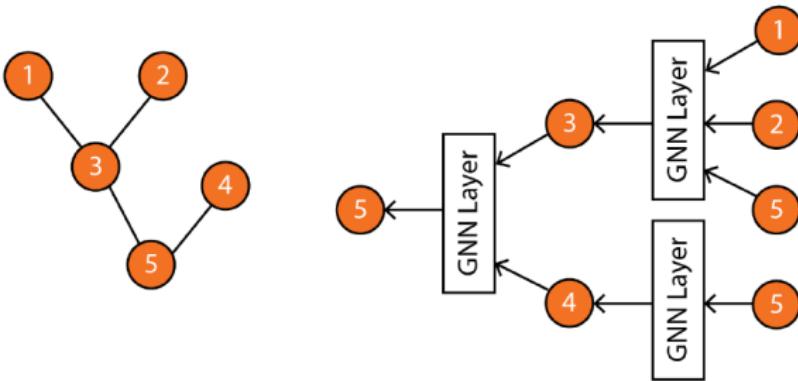


Figure 23: Computation graph representation. Adapted from [1].

Tasks

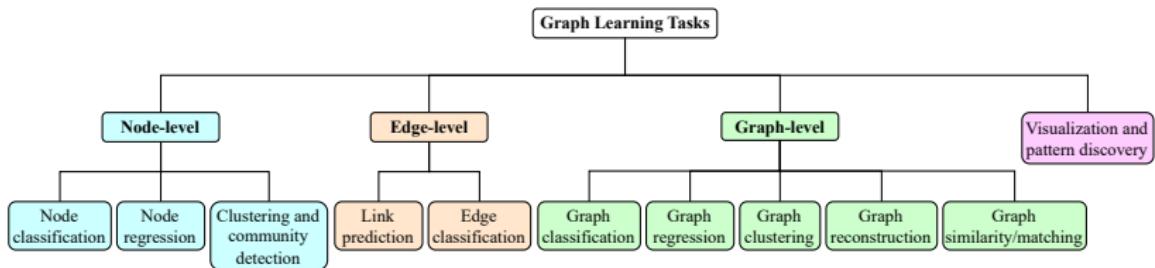


Figure 24: Overview of graph learning tasks.

Taxonomy of graph machine learning methods.

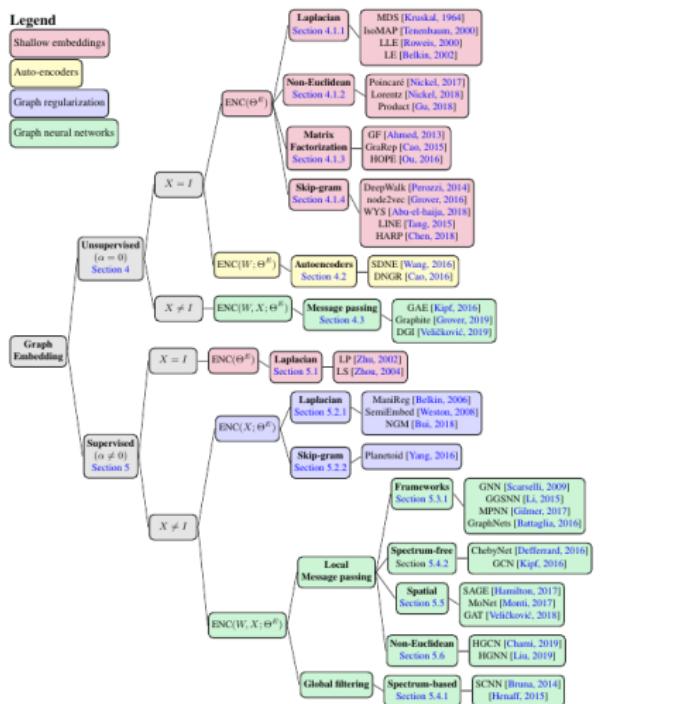


Figure 25: Taxonomy of graph machine learning. Adapted from [3].

Variants of GNNs

- Graph Convolutional Networks (GCN) [8]
- Graph Attention Networks (GAT) [9]
- Graph Isomorphism Networks (GIN) [10]
- Variational Graph Auto-Encoder (VGAE) [11]
- Deep Graph Infomax (DGI) [12]
- Adversarially Regularized Variational Graph Autoencoder (ARVGA) [13]
- Linear Graph Variational Autoencoder (LVAE) [14]
- ...

Graph Convolutional Network (GCN)

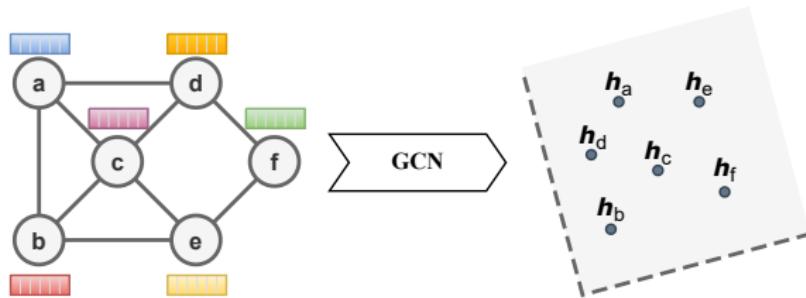


Figure 26: GCN scheme.

$$H^{(l+1)} = \sigma(\tilde{D}^{-\frac{1}{2}} \tilde{A} \tilde{D}^{-\frac{1}{2}} H^{(l)} W^{(l)}) \quad (2)$$

Here, $\tilde{A} = A + I_N$, $\tilde{D}_{ii} = \sum_j \tilde{A}_{ij}$, $W^{(l)}$ is trainable weight matrix, $\sigma(\cdot)$ is an activation function.

Application in computer vision

SuperGlue: Learning Feature Matching with Graph Neural Networks [15].

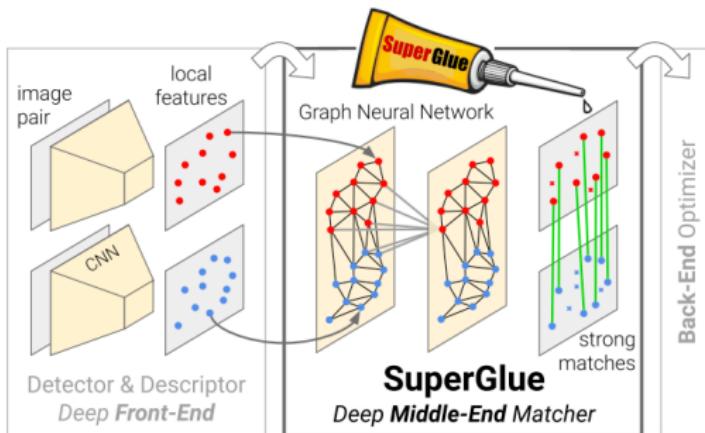


Figure 27: SuperGlue scheme.

SuperGlue

Figure 28: Matching example (indoor).

SuperGlue

SuperGlue
Keypoints: 250:250
Matches: 52

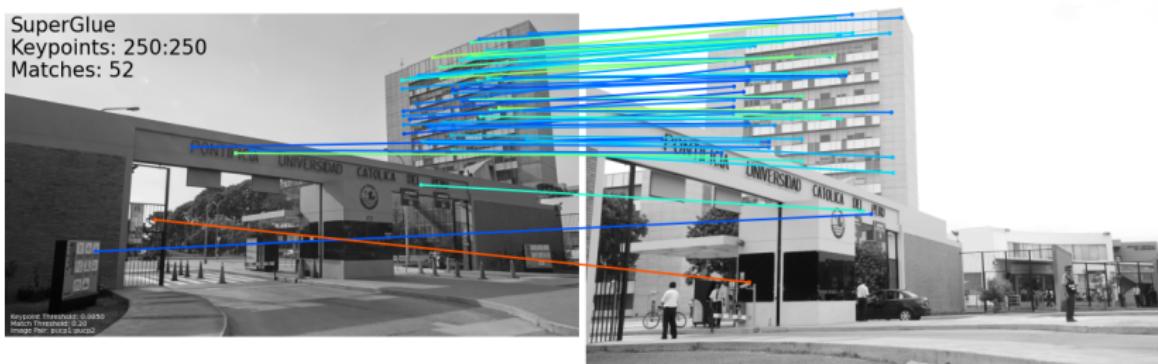


Figure 29: Matching example (outdoor).

Application in recommender systems

Uber Eats

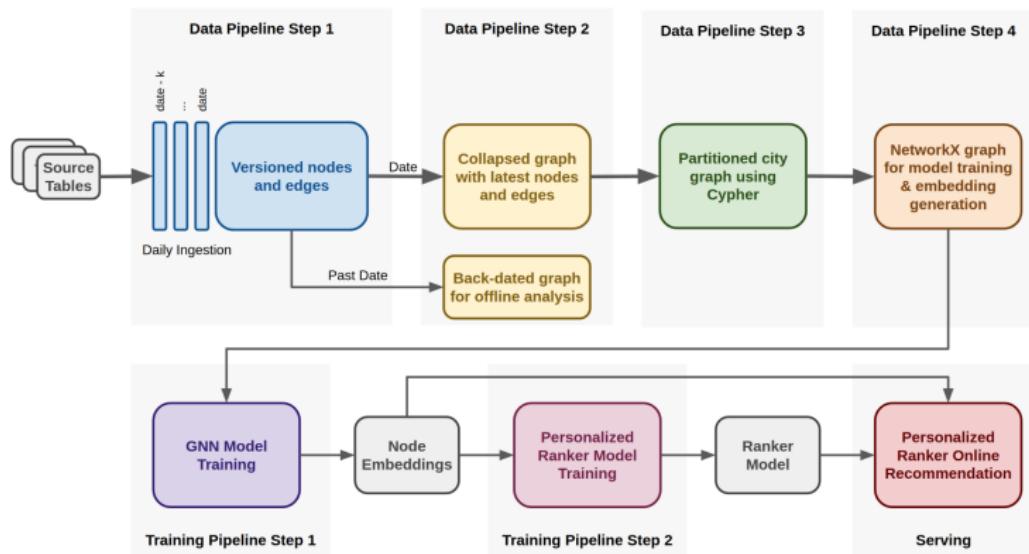


Figure 30: Pipeline Uber eats. From
<https://www.uber.com/en-US/blog/uber-eats-graph-learning/>.

Uber Eats

Figure 31: Recommender systems example.

Practical example: GCN to node classification

Dataset: Karate club

Number of graphs: 1
 Number of nodes: 34
 Number of edges: 156
 Number of features: 34
 Number of classes: 4

Data(x=[34, 34], edge_index=[2, 156], y=[34], train_mask=[34])

`x=[34, 34]`

```
tensor([[1., 0., 0., ..., 0., 0., 0.],
       [0., 1., 0., ..., 0., 0., 0.],
       [0., 0., 1., ..., 0., 0., 0.],
       ...,
       [0., 0., 0., ..., 1., 0., 0.],
       [0., 0., 0., ..., 0., 1., 0.],
       [0., 0., 0., ..., 0., 0., 1.]])
```

`y=[34]`

```
tensor([1, 1, 1, 1, 3, 3, 1, 0, 1, 3, 1, 1, 0, 0, 0, 3, 1, 0, 1, 0, 1, 0, 0,
       2, 2, 0, 0, 2, 0, 0, 2, 0, 0])
```

`train_mask=[34]`

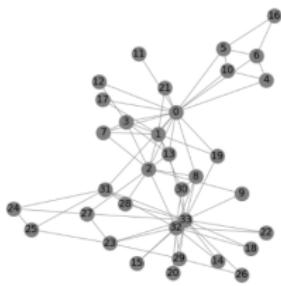
```
tensor([ True, False, False, False,  True, raise, raise, False, False,  True,
        False, False, False, False, False, False, False, False, False, False,
        False, False, False, False,  True, False, False, False, False, False,
        False, False, False, False])
```

`edge_index=[2, 156]`

```
tensor([[ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
         1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 3,
         3, 3, 3, 3, 3, 4, 4, 4, 5, 5, 5, 5, 6, 6, 6, 6, 7, 7,
         7, 7, 8, 8, 8, 8, 9, 9, 10, 10, 10, 11, 12, 12, 13, 13, 13,
         13, 13, 14, 14, 15, 15, 16, 16, 17, 17, 17, 18, 18, 19, 19, 19, 20, 20,
         21, 21, 22, 22, 23, 23, 23, 23, 24, 24, 24, 25, 25, 26, 26, 27, 27,
         27, 27, 28, 28, 28, 29, 29, 29, 30, 30, 30, 30, 31, 31, 31, 31,
         31, 32, 32, 32, 32, 32, 32, 32, 32, 32, 32, 32, 33, 33, 33, 33,
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         33, 33, 33, 33, 33, 33, 33, 33, 33, 33, 33, 33, 33, 33, 33, 33,
         33, 33, 33, 33, 33, 33, 33, 33, 33, 33, 33, 33, 33, 33, 33, 33,
         1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 17, 19, 21, 31, 0, 2,
         3, 7, 13, 17, 19, 21, 30, 0, 1, 3, 7, 8, 9, 13, 27, 28, 32, 0,
         1, 2, 7, 12, 13, 0, 6, 10, 0, 6, 10, 16, 0, 4, 5, 16, 0, 1,
         2, 3, 0, 2, 30, 32, 33, 2, 33, 0, 4, 5, 0, 0, 3, 0, 1, 2,
         3, 33, 32, 33, 32, 33, 5, 6, 0, 1, 32, 33, 0, 1, 33, 32, 33, 0,
         1, 32, 33, 25, 27, 29, 32, 33, 25, 27, 31, 23, 24, 31, 29, 33, 2, 23,
         24, 33, 2, 31, 33, 23, 26, 32, 33, 1, 8, 32, 33, 0, 24, 25, 28, 32,
         33, 2, 8, 14, 15, 18, 20, 22, 23, 29, 30, 31, 33, 8, 9, 13, 14, 15,
         18, 19, 20, 22, 23, 26, 27, 28, 29, 30, 31, 32]])
```

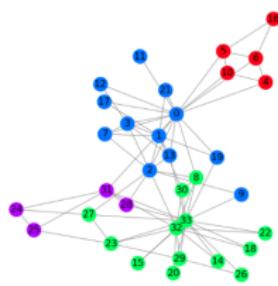
Figure 32: Karate club dataset.

Practical example: GCN to node classification



(a) Input graph.

(b) Train GCN.



(c) Output graph.

Figure 33: GCN to node classification.

Practical example: GCN to node classification

https://github.com/win7/Presentations-Hub/blob/main/SummerCamp-2025_IA-PUCP/Node_Classification.ipynb

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Package, library, and framework

Graph manipulation



Create models



Figure 34: Package, library, and framework.

Books

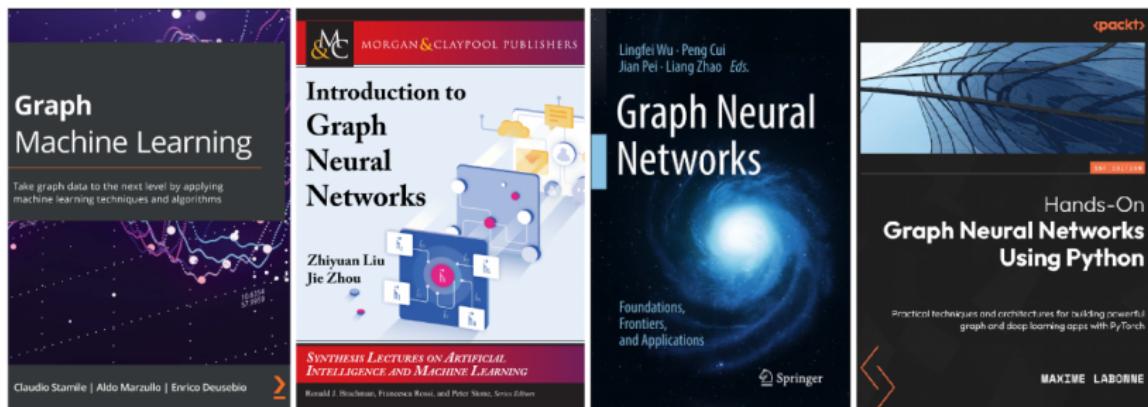


Figure 35: GNNs books.

More learning

- Courses
 - CS224W: Machine Learning with Graphs
(<http://web.stanford.edu/class/cs224w/>)
- Tutorials
 - PyTorch Geometric: Introduction by Example
(<https://pytorch-geometric.readthedocs.io/en/latest/notes/introduction.html>)
- Conferences
 - Learning on Graphs
(<https://logconference.org/>)

[1] Maxime Labonne.

Hands-On Graph Neural Networks Using Python: Practical techniques and architectures for building powerful graph and deep learning apps with PyTorch.

Packt Publishing Ltd, 2023.

[2] Claudio Stamile, Aldo Marzullo, and Enrico Deusebio.

Graph Machine Learning: Take graph data to the next level by applying machine learning techniques and algorithms.

Packt Publishing Ltd, 2021.

[3] Ines Chami, Sami Abu-El-Haija, Bryan Perozzi, Christopher Ré, and Kevin Murphy.

Machine learning on graphs: A model and comprehensive taxonomy.

Journal of Machine Learning Research, 23(89):1–64, 2022.

- [4] Tomas Mikolov, Kai Chen, Greg Corrado, and Jeffrey Dean.
Efficient estimation of word representations in vector space.
arXiv preprint arXiv:1301.3781, 2013.
- [5] Tomas Mikolov, Ilya Sutskever, Kai Chen, Greg S Corrado, and Jeff Dean.
Distributed representations of words and phrases and their compositionality.
Advances in neural information processing systems, 26, 2013.
- [6] Aditya Grover and Jure Leskovec.
node2vec: Scalable feature learning for networks.
In *Proceedings of the 22nd ACM SIGKDD international conference on Knowledge discovery and data mining*, pages 855–864, 2016.

- [7] Xiang Yue, Zhen Wang, Jingong Huang, Srinivasan Parthasarathy, Soheil Moosavinasab, Yungui Huang, Simon M Lin, Wen Zhang, Ping Zhang, and Huan Sun.
Graph embedding on biomedical networks: methods, applications and evaluations.
Bioinformatics, 36(4):1241–1251, 2020.
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Semi-supervised classification with graph convolutional networks.
arXiv preprint arXiv:1609.02907, 2016.
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Graph attention networks.
stat, 1050(20):10–48550, 2017.

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How powerful are graph neural networks?
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- [11] Thomas N Kipf and Max Welling.
Variational graph auto-encoders.
arXiv preprint arXiv:1611.07308, 2016.
- [12] Petar Veličković, William Fedus, William L Hamilton, Pietro Liò, Yoshua Bengio, and R Devon Hjelm.
Deep graph infomax.
arXiv preprint arXiv:1809.10341, 2018.

- [13] Shirui Pan, Ruiqi Hu, Guodong Long, Jing Jiang, Lina Yao, and Chengqi Zhang.
Adversarially regularized graph autoencoder for graph embedding.
arXiv preprint arXiv:1802.04407, 2018.
- [14] Guillaume Salha, Romain Hennequin, and Michalis Vazirgiannis.
Simple and effective graph autoencoders with one-hop linear models.
In *Machine Learning and Knowledge Discovery in Databases: European Conference, ECML PKDD 2020, Ghent, Belgium, September 14–18, 2020, Proceedings, Part I*, pages 319–334. Springer, 2021.

- [15] Paul-Edouard Sarlin, Daniel DeTone, Tomasz Malisiewicz, and Andrew Rabinovich.

Superglue: Learning feature matching with graph neural networks.

In *Proceedings of the IEEE/CVF conference on computer vision and pattern recognition*, pages 4938–4947, 2020.

Thank you for your attention!

∞

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<https://github.com/win7>

@articlegoyal2018graph, title=Graph embedding techniques, applications, and performance: A survey, author=Goyal, Palash and Ferrara, Emilio, journal=Knowledge-Based Systems, volume=151, pages=78–94, year=2018, publisher=Elsevier
@bookaggarwal2020machine, title=Machine learning in social networks: embedding nodes, edges, communities, and graphs, author=Aggarwal, Manasvi and Murty, M Narasimha, year=2020, publisher=Springer Nature

@articlemakarov2021survey, title=Survey on graph embeddings and their applications to machine learning problems on graphs, author=Makarov, Ilya and Kiselev, Dmitrii and Nikitinsky, Nikita and Subelj, Lovro, journal=PeerJ Computer Science, volume=7, pages=e357, year=2021, publisher=PeerJ Inc.

@booklabonne2023hands, title=Hands-On Graph Neural Networks Using Python: Practical techniques and architectures for building powerful graph and deep learning apps with PyTorch, author=Labonne, Maxime, year=2023, publisher=Packt Publishing



Ltd

@bookstamile2021graph, title=Graph Machine Learning: Take graph data to the next level by applying machine learning techniques and algorithms, author=Stamile, Claudio and Marzullo, Aldo and Deusebio, Enrico, year=2021, publisher=Packt Publishing Ltd, isbn=1800204493,9781800204492

@articlemikolov2013efficient, title=Efficient estimation of word representations in vector space, author=Mikolov, Tomas and Chen, Kai and Corrado, Greg and Dean, Jeffrey, journal=arXiv preprint arXiv:1301.3781, year=2013

@articlemikolov2013distributed, title=Distributed representations of words and phrases and their compositionality, author=Mikolov, Tomas and Sutskever, Ilya and Chen, Kai and Corrado, Greg S and Dean, Jeff, journal=Advances in neural information processing systems, volume=26, year=2013

@inproceedingsgrover2016node2vec, title=node2vec: Scalable feature learning for networks, author=Grover, Aditya and Leskovec,



Jure, booktitle=Proceedings of the 22nd ACM SIGKDD international conference on Knowledge discovery and data mining, pages=855–864, year=2016

@article{leyue2020graph, title=Graph embedding on biomedical networks: methods, applications and evaluations, author=Yue, Xiang and Wang, Zhen and Huang, Jingong and Parthasarathy, Srinivasan and Moosavinasab, Soheil and Huang, Yungui and Lin, Simon M and Zhang, Wen and Zhang, Ping and Sun, Huan, journal=Bioinformatics, volume=36, number=4,

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