COMP1321 : Mathematics II

Calculus - Problem Sheet 3

Integral Calculus

Question 1 Given the sum definition of A, express and compute B in terms of A.

$$A = \sum_{i=1}^{10} \frac{1}{i+3}$$

$$B = \sum_{i=1}^{10} \frac{i-13}{14-i} \quad = \quad ?$$

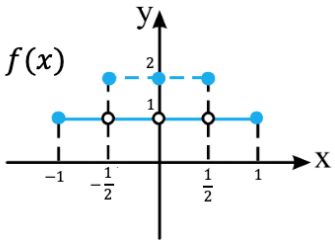
Question 2 Compute the difference between the upper sum and the lower sum of $f(x)$ on the interval $[1, 4]$ with $n = 180$ subintervals.

$$f(x) = x^2$$

Question 3 Compute the difference between the upper sum and the lower sum of $f(x)$ on the interval $[-8, 8]$ with $n = 100$ subintervals.

$$f(x) = \sqrt[3]{x^2}$$

Question 4 Based on the graph of $f(x)$, compute the lower sum of $f(x)$ on the interval $[-1, 1]$ with n subintervals.



Question 5 Using the Comparison Theorem, determine a reasonable range for S, where:

$$S = \int_1^2 \frac{dx}{1 + x^4}.$$

Provide a justification for your answer based on appropriate upper and lower bound functions.

Question 6 Based on the following equation, compute $f(x)$.

$$\int \frac{1 - 3x}{\sqrt{x}}\, dx = \sqrt{x} f(x) + c$$

Question 7 Based on the following equation, compute $f(x)$.

$$\int \frac{x + 1}{x^3}\, dx = \frac{f(x)}{2x^2} + c$$

Question 8 Based on the following equation, determine the values of m and k.

$$\int \frac{1}{\sqrt[3]{(2x + 1)^2}}\, dx = m(2x + 1)^k + c$$

Question 9 Based on the following two equations, compute $g(x)$.

$$\int (f(x) + g(x))\,dx = (x^2+1)^2 + c$$

$$\int (f(x) - g(x))\,dx = (x^2-1)^2 + c$$

Question 10 Assuming $f(2) = 2f(0) = 1$, compute:

$$\int_0^{\sqrt{2}} xf'(x^2)\,dx.$$

Question 11 Based on $f(x)$, compute the derivative of $f(f(x))$ at $x = 1$.

$$f(x) = \int_1^x \sqrt{t^2+3}\,dt$$

Question 12 Given that the average value of the function f over the interval $[1, 3]$ is 5, and over the interval $[1, 9]$ is 6.5, determine the average value of f on the interval $[3, 9]$.

Question 13 Determine the area enclosed by the curves:

$$y = x^2 - 1 \quad \text{and} \quad y = -x^2 + x.$$

Question 14 Determine the area enclosed by the curves:

$$y = x^2|x| \quad \text{and} \quad y = 8.$$