Problem 1: Two spheres are removed from a furnace and let to cool with air at 25°C under relatively low convection coefficient of 15 W.$\left(\text{m}^2.°\text{C}\right)$. The spheres are made of copper, $\kappa_{\text{Cu}} = 401$ W. $(\text{m}.°\text{C})^{-1}$, and coal, $\kappa_{\text{Coal}} = 0.2$ W. $(\text{m}.°\text{C})^{-1}$. Can we apply the lumped method to both spheres?

Problem 2: A steel ball of 5 cm in diameter and at uniform temperature of 450°C is suddenly placed in a controlled environment where temperature is kept at 100°C. The prescribed convection heat transfer coefficient is 10 W.$\left(\text{m}^2.°\text{C}\right)^{-1}$. Calculate the time required for the ball to reach 150°C.
Given $\text{C}_{p,\text{steel}} = 0.46$ kJ.kg^{-1}, $\kappa_{\text{steel}} = 35$ W. (m.°C) and $\rho_{\text{steel}} = 7.8 \times 10^3$kg.m^{-3}.

Problem 3: A long 20 cm diameter cylindrical shaft made of stainless steel 304 comes out of an oven at a uniform temperature of 600°C. The shaft is then allowed to cool slowly in an environment chamber at 200°C with an average heat transfer coefficient of (h) of 80 W/$\left(\text{m}^2.°\text{C}\right)$. Determine the temperature at the center of the shaft 45 min after the start of the cooling process. Also, determine the heat transfer per unit length of the shaft during this time period. Given, for stainless steel 304 at room temperature:

κ	=	14.9	W/(m.°C)	ρ	=	7900	kg/m^3
C_p	=	477	J/(kg.°C)	α	=	3.95×10^{-6}	m^2/s

Problem 4: A new material is to be developed for bearing balls in a new rolling-element bearing. For annealing (heat treatment) each bearing ball, a sphere of radius $r_o = 5$ mm, is heated in a furnace until it reaches the thermal equilibrium at 400°C. Then, it is suddenly removed from the furnace and subjected to a two-step cooling process.

Stage 1: Cooling in an air flow of 20°C for a period of time t_{air} until the center temperature reaches 335°C. For this situation, the convective heat transfer coefficient of air is assumed constant and equal to $h = 10$ W/(m^2.K). After the sphere has reached this specific temperature, the second step is initiated.

Stage 2: Cooling in a well-stirred water bath at 20°C, with a convective heat transfer coefficient of water $h = 6000$ W/(m^2.K).

The thermophysical properties of the material are $\rho = 3000$ kg/m^3, $\kappa = 20$ W/(m.K), $C_p = 1000$ J/(kg.K). Determine:

(a) The time t_{air} required for *Stage 1* of the annealing process to be completed;

(b) The time t_{water} required for *Stage 2* of the annealing process during which the center of the sphere cools from 335°C (the condition at the completion of *Stage 1*) to 50°C.

Problem 5: Carbon steel balls ($\rho = 7833$ kg/m^3, $\kappa = 54$ W/(m.°C), $C_p = 0.465$ kJ/(kg.°C), and $\alpha = 1.474{\times}10^{-6}$ m^2/s) of 8 mm in diameter are annealed by heating them first to 900°C in a furnace and then allowing them to cool slowly to 100°C in ambient air at 35°C. If the average heat transfer coefficient is 75 W/(m^2.°C), determine how long the annealing process will take. If 2500 balls are to be annealed per hour, determine the total rate of heat transfer from the balls to the ambient air.

Problem 6: A long 35-cm-diameter cylindrical shaft made of stainless steel 304 (κ = 14.9 W/(m.°C), ρ = 7900 kg/m^3, C_p = 477 J/(kg.°C), and α = 3.95×10^{-6} m^2/s) comes out of an oven at a uniform temperature of 400°C. The shaft is then allowed to cool slowly in a chamber at 150°C with an average convection heat transfer coefficient of h = 60 W/(m^2.°C. Determine the temperature at the center of the shaft 20 min after the start of the cooling process. Also, determine the heat transfer per unit length of the shaft during this time period.

Problem 7: Apple are left in the freezer at -15°C to cool from an initial uniform temperature of 20°C. The heat transfer coefficient at the surfaces is 8 W/(m^2.°C). Treating the apples as 9 cm diameter sphere and taking their properties to be ρ = 840 kg/m^3, C_p = 3.81 kJ/(kg.°C), κ = 0.418 W/(m.°C), and α = 1.3×10^{-7} m^2/s, determine the center and surface temperatures of thes apples in 1 h. Also, determine the amount of heat transfer from each apple.

Problem 8: Consider a large uranium plate of thickness L = 4 cm, κ = 28 W/(m.°C), and α = 12.5×10^{-6} m^2/s that is initially at a uniform temperature of 200°C. Heat is generated uniformly in the plate at a constant rate of 5×10^6 W/m^3. At time t = 0, one side of the plate is brought into contact with iced water and is maintained at 0°C at all times, while the other side is subjected to convection to an environment at T$_\infty$ = 30°C with a heat transfer coefficient of h = 45 W/(m^2.°C). Considering a total of three equally spaced nodes in the medium, two at the boundaries and one at the middle, estimate the exposed surface temperature of the plate 2.5 min after the start of cooling using finite difference method.

Problem 9: Consider three consecutive nodes $n-1$, n, $n+1$ in a plane wall. Using the finite difference form of the first derivative at the midpoints, show that the finite difference form of the second derivative can be expressed as

$$\frac{T_{n-1} - 2T_n + T_{n+1}}{\Delta x^2} = \left.\frac{\partial^2 T}{\partial x^2}\right|_N$$

Problem 10: Consider a large uranium plate of thickness 5 cm and thermal conductivity of 28 W/(m.°C) in which heat is generated uniformly at a constant rate of 6×10^5 W/m^3. One side of the plate is insulated while the other side is subjected to convection to an environment at 30°C with a heat transfer coefficient of h = 60 W/(m^2.°C). Considering six equally spaced nodes with a nodal spacing of 1 cm,

(a) obtain the finite difference formulation of this problem and,

(b) determine the nodal temperatures under steady conditions by solving those equations.

Problem 11: Hot oil is to be cooled in a double-tube counter-flow HE. The copper inner tubes have diameter of 2cm and negligible thickness. The inner diameter of the outer tube (shell) is 3cm. Water flows through the tube at a rate of 0.5 kg/s, and the oil through the shell at a rate of 0.8 kg/s. Taking the average temperatures of the water and the oil to be 45°C and 80°C, respectively, determine the overall heat transfer coefficient of this HE. Given,

(a) Water at 45°C: $\rho = 990\text{kg/m}^3$, $\kappa = 0.637\text{W/(m.K)}$, $Pr = 3.91$, $\nu = \mu/\rho = 0.602 \times 10^{-6}\text{m}^2\text{/s}$;

(b) Oil at 80°C: $\rho = 852\text{kg/m}^3$, $\kappa = 0.138\text{W/(m.K)}$, $Pr = 490$, $\nu = 37.5 \times 10^{-6}\text{m}^2\text{/s}$

The inner convective heat transfer coefficient, h_i, can be obtained from

$$Nu = \frac{h_i D_h}{\kappa} = \begin{cases} 4.36 & \text{(for laminar flows),} \\ 0.023\text{Re}^{0.8}\text{Pr}^{0.4} & \text{(for turbulent flows),} \end{cases}$$

and the outter convective heat transfer coefficient, h_0 is 75.2 W.$\left(\text{m}^2\text{.K}\right)$.

Problem 12: A double-pipe (shell-and-tube) heat exchanger is constructed of a stainless steel ($\kappa = 15.1$ W/(m.°C) inner tube of inner diameter D$_i$ = 1.5 cm and outer diameter D$_o$ = 1.9 cm and an outer shell of inner diameter 3.2 cm. The convective heat transfer coefficient is h$_i$ = 800 W/(m^2.°C) on the inner surface of the tube and h$_o$ = 1200 W/(m^2.°C) on the outer surface. For a fouling factor R$_{f,i}$ = 0.0004 m^2.°C/W on the tube side and R$_{f,o}$ = 0.0001 m^2.°C/W on the shell side, determine:

(a) The thermal resistance of the heat exchanger per unit length and;

(b) The overall heat transfer coefficients, U$_i$ and U$_o$ based on the inner and outer surface areas of the tube, respectively.

Problem 13: Water at the rate of 68 kg/min is heated from 35 to 75°C by an oil having a specific heat of 1.9 kJ/(kg.°C). The fluids are used in a counterflow double-pipe HE, and the oil enters the exchanger at 110°C and leaves at 75°C. The overall heat-transfer coefficient is 320 W/(m^2.°C). Given heat capacity of water (at constant pressure) of 4.18 kJ/(kg.°C),

(a) Calculate the HE area;

(b) Now assume that the HE is a shell-and-tube with water making one shell pass and the oil making two tube passes. Calculate the new HE. Assume that the overall heat-transfer coefficient remains the same.

Problem 14: For the HE of **Problem 13:** with the same entering-fluid temperatures, calculate the exit water temperature when only 40 kg/min of water is heated but the same quantity of oil is used. Also calculate the total heat transfer under these new conditions.

Problem 15: A finned-tube heat exchanger (Fig. 1) is used to heat 2.36 m^3/s of air at 1 atm from 15.55 to 29.44°C. Hot water enters the tubes at 82.22°C, and the air flows across the tubes, producing an average overall heat-transfer coefficient of 227 $W/(m^2.°C)$. The total surface area of the exchanger 9.29 m^2. Calculate the exit water temperature and the heat-transfer rate. Assume air behaves as an ideal gas with molar mass of 28.97 $g.mol^{-1}$ and heat capacity at constant pressure of 1005 $J.(kg.K)^{-1}$.

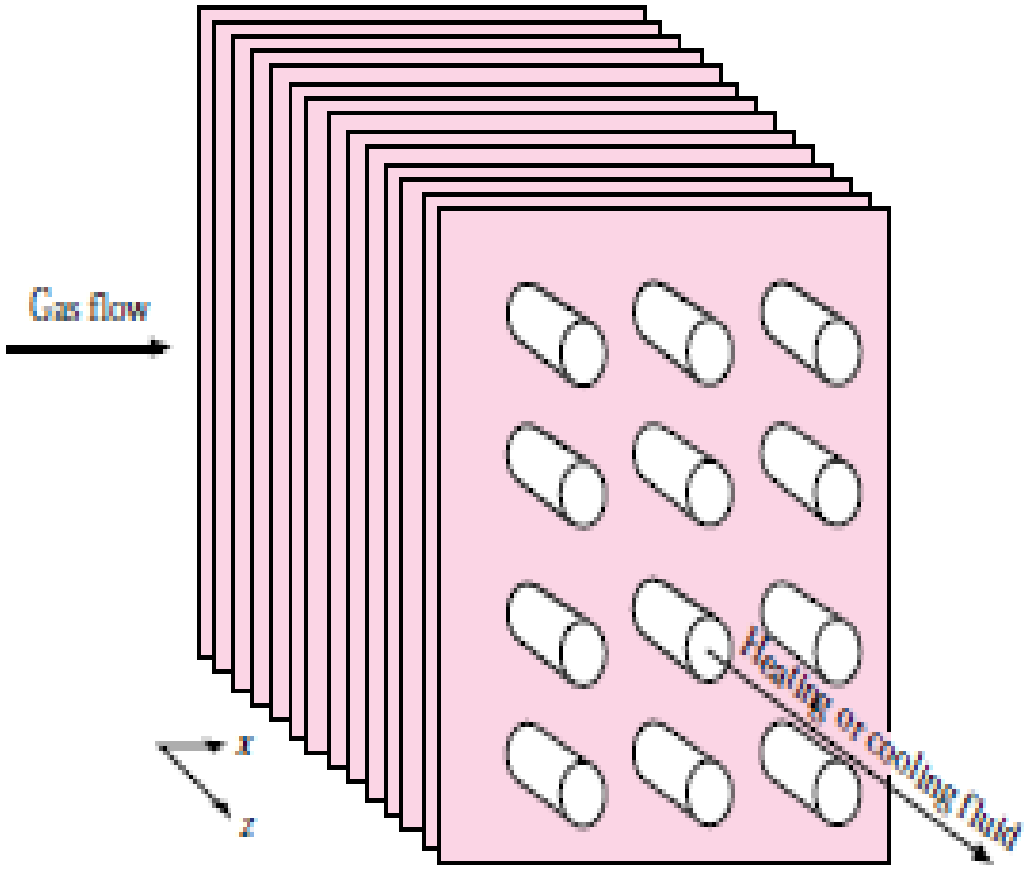


Figure 1: Cross-flow HE with Unmixed fluids (**Problem 15:**).

Problem 16: A double-pipe heat exchanger is constructed of a copper ($\kappa = 380$ W/(m.°C) inner tube of internal diameter $D_i = 1.2$ cm and external diameter $D_o = 1.6$ cm and an outer tube of diameter 3.0 cm. The convection heat transfer coefficient is $h_i = 700$ $W/(m^2.°C)$ on the inner surface of the tube and $h_o = 1400$ $W/(m^2.°C)$ on its outer surface. For a fouling factor $R_{f,i} = 0.0005$ $m^2.°C/W$ on the tube side and $R_{f,o} = 0.0002$ m^2.°C/W on the shell side. Determine

(a) the thermal resistance of the heat exchanger per unit length and,

(b) the overall heat transfer coefficients U_i and U_o based on the inner and outer surface areas of the tube, respectively.

Problem 17: In a binary geothermal power plant, the working fluid isobutane is condensed by air in a condenser at 75°C $\left(h_{\text{fg}} = 255.7 \text{ kJ/kg}\right)$ at a rate of 2.7 kg/s. Air enters the condenser at 21°C and leaves at 28°C. The heat transfer surface area based on the isobutane side is 24 m^2. Determine the mass flow rate of air and the overall heat transfer coefficient.

Problem 18: A cross-flow air-to-water heat exchanger with an effectiveness of 0.65 is used to heat water ($C_p =$ 4180 J/(kg.°C) with hot air ($C_p = 1010$ J/(kg.°C). Water enters the heat exchanger at 20°C at a rate of 4 kg/s, while air enters at 100°C at a rate of 9 kg/s. If the overall heat transfer coefficient based on the water side is 260 W/(m^2.°C), determine the heat transfer surface area of the heat exchanger on the water side. Assume both fluids are unmixed.

Problem 19: A shell-and-tube process heater is to be selected to heat water ($C_p = 4180$ J/(kg.°C) from 20°C to 90°C by steam flowing on the shell side. The heat transfer load of the heater is 600 kW. If the inner diameter of the tubes is 1 cm and the velocity of water is not to exceed 3 m/s, determine how many tubes need to be used in the heat exchanger.

P1:

$$L_c = \frac{V}{A_s} = \frac{4/3 \pi r^3}{4 \pi r^2} = \frac{r}{3}$$

$$Bi_{Cu} = \frac{h L_c}{K_{Cu}} = \frac{15\ r_{Cu}/3}{401}$$

$$Bi_{coal} = \frac{h L_c}{K_{coal}} = \frac{15\ r_{coal}/3}{0.2}$$

To use lumped method $Bi < 0.1$

$$Bi_{Cu} = \frac{15\ r_{Cu}/3}{401} \leqslant 0.1$$

$$\boxed{r_{Cu} \leqslant 8.02 \text{ m}}$$

$$Bi_{coal} = \frac{15\ r_{coal}/3}{0.2} \leqslant 0.1$$

$$\boxed{r_{coal} \leqslant 4 \times 10^{-3} \text{ m}}$$

P2: First, let's check if lumped-capacitance method can be used in this problem:

$$Bi = \frac{h L_c}{k} = \frac{h(\ell/3)}{k} = \frac{h(d/6)}{k}$$

$$Bi = \frac{10\,(5\times10^{-2}/6)}{35} = 2.38\times10^{-3} <<< 0.1$$

thus we can use lumped-~~capacitity~~ capacitance method!

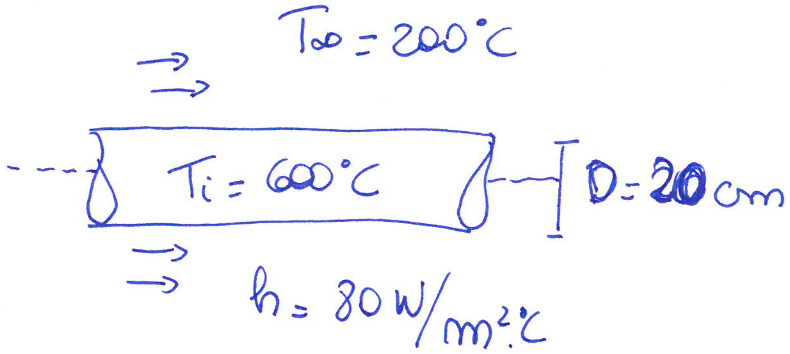
Therefore,

$$\frac{T(t)-T_\infty}{T_0-T_\infty} = e^{-bt} \qquad \text{where } b = \frac{hA_s}{\rho V C_p} = \frac{h}{L_c \rho C_p}$$

$$\frac{150-100}{450-100} = \exp\left[\frac{-10t}{(5\times10^{-2}/6)\times7.8\times10^{3}\times0.46\times10^{3}}\right]$$

$$\boxed{t = 5818.27\ s \simeq 1.62\ h}$$

P3 :



As the problem deals with a long geometry and the thermal symmetry is in the centerline, we can assume the problem as 1-D. The radius of the cylinder is 0.1 m, thus

$$Bi = \frac{h r_o}{k} = \frac{80 \times 0.1}{14.9} = 0.5369$$

$\hookrightarrow > 0.1$

Thus using Heisler chart for cylindrical geometry with

(thus lumped method cannot be used here!)

$$\begin{cases} 1/Bi = 1.86 \\ Fo = \dfrac{\alpha t}{r_o^2} = \tau = \dfrac{3.95 \times 10^{-6} \times (45 \times 60)}{0.1^2} = 1.07 \end{cases}$$

$$\hookrightarrow \frac{T_0 - T_\infty}{T_i - T_\infty} = 0.40 = \frac{T_0 - 200}{600 - 200} \therefore \underline{T_0 = 360°C}$$

The actual heat transfer can be obtained from the Gröber chart with

$$\left\{ \begin{array}{l} Bi^2\tau = 0.31 \\ Bi = 0.54 \end{array} \right. \Rightarrow Q/Q_{MAX} = 0.62$$

Calculating Q_{MAX} for $L = 1\,m$

$$Q_{MAX} = m C_p (T_\infty - T_i)$$

with $m = \rho V = \rho \pi r_o^2 L = 248.18\ Kg$

$$Q_{MAX} = 248.18 \times 477 \times (600 - 200) = 47352744\ J$$
$$\cong 47.35\ MJ$$

$$\boxed{Q = 29358701.28\ J \cong 29.36\ MJ}$$

— // — // —

We can also solve this problem using analytical methods, With $Bi = 0.5369$ & $\tau = 1.07$

$\hookrightarrow$ $\lambda_1 = 0.9694$ \
$A_1 = 1.1218$ } Coef & Bessel Functions

$$\theta_0 = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = 1.1218 e^{-(0.9694^2)\tau}$$

$$\theta_0 = \frac{T_0 - T_\infty}{T_i - T_\infty} = 0.4104 \quad \therefore \quad \underline{T_0 = 364.16\,°C}$$

Now

$$\left(\frac{Q}{Q_{MAX}}\right)_{cyl} = 1 - 2\theta_0 \frac{J_1(\lambda_1)}{\lambda_1}$$

where $J_1(\lambda_1) = 0.4296$

$$\left(\frac{Q}{Q_{MAX}}\right)_{cyl} = 1 - 2 \times 0.4104 \times \frac{0.4296}{0.9694} = 0.636$$

$$\boxed{Q = 30116345.18\,J \cong 30.12\,MJ}$$

P4:

Stage 1: check if we can use lumped-capacitance method:

$$Bi = \frac{h L_c}{k} = \frac{10 \times \overset{5\times10^{-3}\,m}{\cancel{R}}/3}{20} = 8.33\times10^{-4} < 0.1$$

Thus lumped method can be used for ~~the~~ this stage, i.e., temperature changes uniformly throughout the sphere,

$$\frac{T(t)-T_\infty}{T_0-T_\infty} = \exp\left[-\frac{h}{L_c \rho C_p} t\right]$$

$$\frac{335-20}{400-20} = \exp\left[-\frac{10\,t}{5\times10^{-3}/3 \times 3000 \times 1000}\right] \therefore \boxed{t = 93.80\,s}$$

(a)

Stage 2: checking Bi:

$$Bi = \frac{6000 \times (5\times10^{-3}/3)}{20} = 0.5 > 0.1$$

Lumped method can not be used!

6

In order to use the Tables for the analytical solution, we need to calculate Bi number based on r_0:

$$Bi = \frac{h r_0}{k} = \frac{6000 \times 5\times10^{-3}}{20} = 1.5$$

$$\hookrightarrow \lambda_1 = 1.7998$$
$$A_1 = 1.3763$$

$$\theta_{center} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 \exp\left[-\lambda_1^2 \tau\right]$$

Where $\tau = \frac{\alpha t}{r_0^2}$ with $\alpha = \frac{k}{\rho C_p} = 6.67\times10^{-6} \frac{m^2}{s}$

$$\frac{50-20}{335-20} = 1.3763 \exp\left[-1.7998^2 \tau\right]$$

$$\tau = 0.8245 = \frac{\alpha t}{r_0^2} = \frac{6.67\times10^{-6} t}{(5\times10^{-3})^2}$$

$$\boxed{t = 3.09 s}$$

P5

4-23 A number of carbon steel balls are to be annealed by heating them first and then allowing them to cool slowly in ambient air at a specified rate. The time of annealing and the total rate of heat transfer from the balls to the ambient air are to be determined.

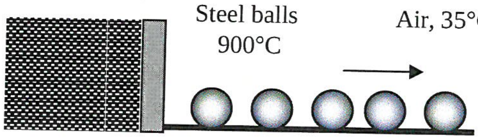
Assumptions **1** The balls are spherical in shape with a radius of $r_0 = 4$ mm. **2** The thermal properties of the balls are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The thermal conductivity, density, and specific heat of the balls are given to be $k = 54$ W/m.°C, $\rho = 7833$ kg/m^3, and $C_p = 0.465$ kJ/kg.°C.

Analysis The characteristic length of the balls and the Biot number are

$$L_c = \frac{V}{A_s} = \frac{\pi D^3/6}{\pi D^2} = \frac{D}{6} = \frac{0.008\text{ m}}{6} = 0.0013\text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(75\text{ W/m}^2.°\text{C})(0.0013\text{ m})}{(54\text{ W/m.°C})} = 0.0018 < 0.1$$



Therefore, the lumped system analysis is applicable. Then the time for the annealing process is determined to be

$$b = \frac{hA_s}{\rho C_p V} = \frac{h}{\rho C_p L_c} = \frac{75\text{ W/m}^2.°\text{C}}{(7833\text{ kg/m}^3)(465\text{ J/kg.°C})(0.0013\text{ m})} = 0.01584\text{ s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{100-35}{900-35} = e^{-(0.01584\text{ s}^{-1})t} \longrightarrow t = \mathbf{163\text{ s} = 2.7\text{ min}}$$

The amount of heat transfer from a single ball is

$$m = \rho V = \rho\frac{\pi D^3}{6} = (7833\text{ kg/m}^3)\frac{\pi(0.008\text{ m})^3}{6} = 0.0021\text{ kg}$$

$$Q = mC_p[T_f - T_i] = (0.0021\text{ kg})(465\text{ J/kg.°C})(900-100)°\text{C} = 781\text{ J} = 0.781\text{ kJ (per ball)}$$

Then the total rate of heat transfer from the balls to the ambient air becomes

$$\dot{Q} = \dot{n}_{\text{ball}}Q = (2500\text{ balls/h}) \times (0.781\text{ kJ/ball}) = 1{,}953\text{ kJ/h} = \mathbf{543\text{ W}}$$

P6

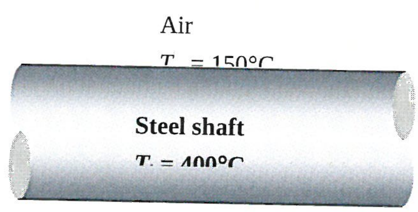
4-38 A long cylindrical shaft at 400°C is allowed to cool slowly. The center temperature and the heat transfer per unit length of the cylinder are to be determined.

Assumptions **1** Heat conduction in the shaft is one-dimensional since it is long and it has thermal symmetry about the center line. **2** The thermal properties of the shaft are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of stainless steel 304 at room temperature are given to be k = 14.9 W/m.°C, ρ = 7900 kg/m^3, C_p = 477 J/kg.°C, $\alpha = 3.95\times10^{-6}$ m^2/s

Analysis First the Biot number is calculated to be

$$Bi = \frac{hr_o}{k} = \frac{(60\ \text{W/m}^2.°\text{C})(0.175\ \text{m})}{(14.9\ \text{W/m.°C})} = 0.705$$



The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-1,

$$\lambda_1 = 1.0935 \quad \text{and} \quad A_1 = 1.1558$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(3.95\times10^{-6}\ \text{m}^2/\text{s})(20\times60\ \text{s})}{(0.175\ \text{m})^2} = 0.1548$$

which is very close to the value of 0.2. Therefore, the one-term approximate solution (or the transient temperature charts) can still be used, with the understanding that the error involved will be a little more than 2 percent. Then the temperature at the center of the shaft becomes

$$\theta_{0,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.1558)e^{-(1.0935)^2(0.1548)} = 0.9605$$

$$\frac{T_0 - 150}{400 - 150} = 0.9605 \longrightarrow T_0 = \mathbf{390\ °C}$$

The maximum heat can be transferred from the cylinder per meter of its length is

$$m = \rho V = \rho \pi r_o^2 L = (7900\ \text{kg/m}^3)[\pi(0.175\ \text{m})^2(1\ \text{m})] = 760.1\ \text{kg}$$

$$Q_{\max} = mC_p[T_\infty - T_i] = (760.1\ \text{kg})(0.477\ \text{kJ/kg.°C})(400-150)°\text{C} = 90{,}638\ \text{kJ}$$

Once the constant J_1 = 0.4689 is determined from Table 4-2 corresponding to the constant λ_1=1.0935, the actual heat transfer becomes

$$\left(\frac{Q}{Q_{\max}}\right)_{cyl} = 1 - 2\left(\frac{T_0 - T_\infty}{T_i - T_\infty}\right)\frac{J_1(\lambda_1)}{\lambda_1} = 1 - 2\left(\frac{390-150}{400-150}\right)\frac{0.4689}{1.0935} = 0.177$$

$$Q = 0.177(90{,}638\ \text{kJ}) = \mathbf{16{,}015\ kJ}$$

P7

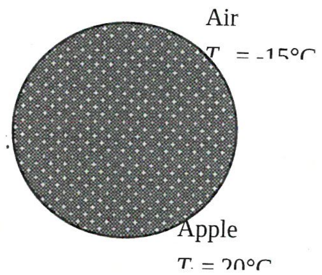
4-50 A person puts apples into the freezer to cool them quickly. The center and surface temperatures of the apples, and the amount of heat transfer from each apple in 1 h are to be determined.

Assumptions **1** The apples are spherical in shape with a diameter of 9 cm. **2** Heat conduction in the apples is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the apples are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of the apples are given to be $k = 0.418$ W/m.°C, $\rho = 840$ kg/m^3, $C_p = 3.81$ kJ/kg.°C, and $\alpha = 1.3\times10^{-7}$ m^2/s.

Analysis The Biot number is

$$Bi = \frac{hr_o}{k} = \frac{(8 \text{ W/m}^2.°\text{C})(0.045 \text{ m})}{(0.418 \text{ W/m.°C})} = 0.861$$



The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-1,

$$\lambda_1 = 1.476 \quad \text{and} \quad A_1 = 1.2390$$

The Fourier number is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(1.3\times10^{-7} \text{ m}^2/\text{s})(1 \text{ h}\times 3600 \text{ s/h})}{(0.045 \text{ m})^2} = 0.231 > 0.2$$

Then the temperature at the center of the apples becomes

$$\theta_{o,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{T_0 - (-15)}{20-(-15)} = (1.239)e^{-(1.476)^2(0.231)} = 0.749 \longrightarrow T_0 = \mathbf{11.2°C}$$

The temperature at the surface of the apples is

$$\theta(r_o,t)_{sph} = \frac{T(r_o,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}\frac{\sin(\lambda_1 r_o/r_o)}{\lambda_1 r_o/r_o} = (1.239)e^{-(1.476)^2(0.231)}\frac{\sin(1.476 \text{ rad})}{1.476} = 0.505$$

$$\frac{T(r_o,t)-(-15)}{20-(-15)} = 0.505 \longrightarrow T(r_o,t) = \mathbf{2.7°C}$$

The maximum possible heat transfer is

$$m = \rho V = \rho\frac{4}{3}\pi r_o^3 = (840 \text{ kg/m}^3)\left[\frac{4}{3}\pi(0.045 \text{ m})^3\right] = 0.3206 \text{ kg}$$

$$Q_{max} = mC_p(T_i - T_\infty) = (0.3206 \text{ kg})(3.81 \text{ kJ/kg.°C})[20-(-15)]°\text{C} = 42.76 \text{ kJ}$$

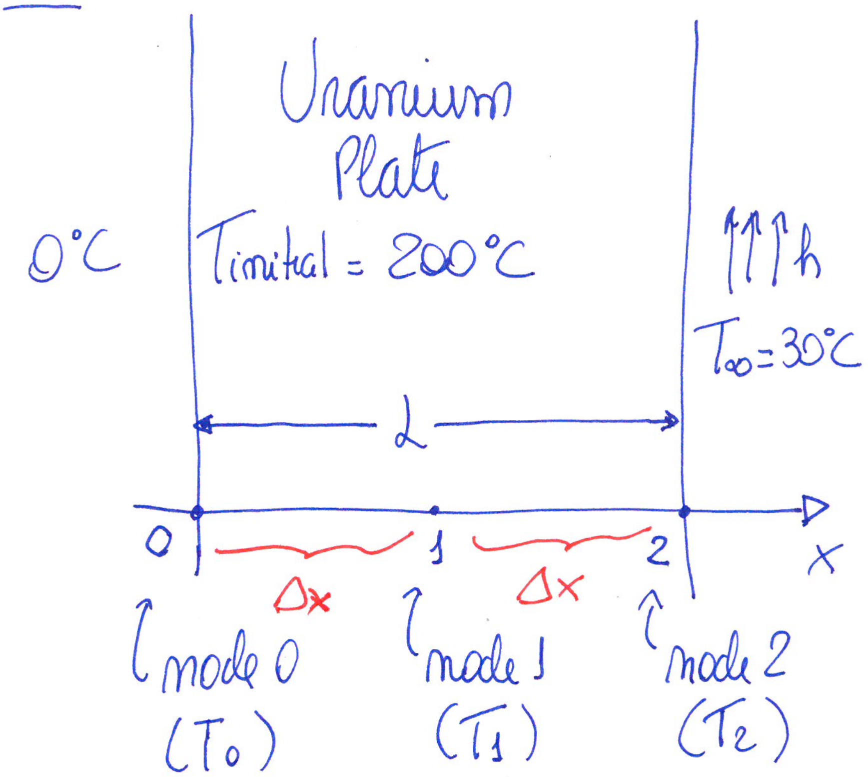
Then the actual amount of heat transfer becomes

$$\frac{Q}{Q_{max}} = 1 - 3\theta_{o,sph}\frac{\sin(\lambda_1) - \lambda_1\cos(\lambda_1)}{\lambda_1^3} = 1 - 3(0.749)\frac{\sin(1.476 \text{ rad}) - (1.476)\cos(1.476 \text{ rad})}{(1.476)^3} = 0.402$$

$$Q = 0.402Q_{max} = (0.402)(42.76 \text{ kJ}) = \mathbf{17.2 \text{ kJ}}$$

rad → degrees
$* \, 180/\pi$

P3:



Dividing the domain in 3 nodes with spacing of

$$\Delta x = \frac{L}{M-1} = \frac{0.04}{3-1}$$

$$\Delta x = 0.02\,m$$

With boundary conditions:

$$T(x=0,t) = T_0(t)$$

Our original thermal equation is

$$\rho C_p \frac{\partial T}{\partial t} = K \frac{\partial^2 T}{\partial x^2} + S$$

assuming K is constant

source/sink term

Discretising with FDM in space and time

$$\rho C_p \frac{T_i^{j+1} - T_i^{j}}{\Delta t} = K \frac{T_{i+1}^{j} - 2T_i^{j} + T_{i-1}^{j}}{(\Delta x)^2} + S_i^{j}$$

$\times \frac{1}{K}$

$$\frac{\rho C_p}{k} \frac{T_i^{j+1} - T_i^j}{\Delta t} = \frac{T_{i+1}^j - 2T_i^j + T_{i-1}^j}{(\Delta x)^2} + \frac{S_i^j}{k}$$

$\to \alpha^{-1}$ (thermal diffusivity)

$$\underbrace{\frac{(\Delta x)^2}{\alpha \Delta t}}_{\tau^{-1}} \left(T_i^{j+1} - T_i^j\right) = \left[T_{i+1}^j - 2T_i^j + T_{i-1}^j\right] + \frac{(\Delta x)^2}{k} S_i^j$$

$\tau^{-1} \to$ mesh Fourier number

$$T_i^{j+1} = T_i^j + \tau\left(T_{i+1}^j - 2T_i^j + T_{i-1}^j\right) + \frac{\tau(\Delta x)^2}{k} S_i^j$$

$$\boxed{T_i^{j+1} = (1-2\tau) T_i^j + \tau\left(T_{i+1}^j + T_{i-1}^j\right) + \frac{\tau(\Delta x)^2}{k} S_i^j \qquad (1)}$$

with $\tau = \dfrac{\alpha \Delta t}{(\Delta x)^2} \leq 0.5$ (stability criterion)

$\Delta t \leq 16s$

$\to$ using $\Delta t = 15s$ in the above equation

$\downarrow$

$\tau = 0.4688$

~~At time $t = 0s$:~~ ~~$T_1^0 = 200°C = T_2^0$~~

Equation 1 is designed for the central node(s),

1. For nodes in the borders of the domain:

(a) Imposed Dirichlet BC at $i=0$

$$T_0^0 = T_0^1 = T_0^2 = \dots = 0°C$$

(b) Imposed Newman BC at $x = L$ (i.e., $i=2$)

$$-K \frac{\partial T}{\partial x} = h(T - T_\infty)$$

using a 2nd order accurate FD approximation at $x = L$:

$$-K \frac{T_{i+1}^j - T_{i-1}^j}{2\Delta x} = h(T_i^j - T_\infty)$$

$$\boxed{T_{i+1}^j = T_{i-1}^j - \frac{2\Delta x h}{K}(T_i^j - T_\infty)} \quad (2)$$

And initial conditions of:

$$T_1^0 = T_2^0 = \dots = 200°C$$

For time $t = 0s$ $(j=0)$:

- node $i=1$:

$5\times10^{6}\ W/m^{3}$

$$T_1^1 = (1-2\tau)T_1^0 + \tau(T_2^0 + T_0^0) + \frac{\tau(\Delta x)^2}{k} S_1^0$$

$$\boxed{T_1^1 = 139.73\,°C}$$

- node $i=2$:

$$T_2^1 = (1-2\tau)T_2^0 + \tau(T_3^0 + T_1^0) + \frac{\tau(\Delta x)^2}{k} S_2^0$$

T_3^0 is not within the mesh, but we can replace it from Eqn. (2):

$$T_3^0 = T_1^0 - 2\frac{h\Delta x}{k}(T_2^0 - T_\infty)$$

Leading to

$$T_2^1 = (1-2\tau)T_2^0 + \tau\left[T_1^0 - 2\frac{h\Delta x}{k}(T_2^0 - T_\infty) + T_1^0\right] + \frac{\tau(\Delta x)^2}{k} S_2^0$$

$$\boxed{T_2^1 = 228.51\,°C}$$

For time $t = 15s$ ($j=1$):

- node $i = 1$:

$$T_1^2 = (1-2\tau)T_1^1 + \tau(T_2^1 + T_0^1) + \frac{\tau(\Delta x)^2}{k} S_1^1$$

$$\boxed{T_1^2 = 149.33°C}$$

- node $i = 2$:

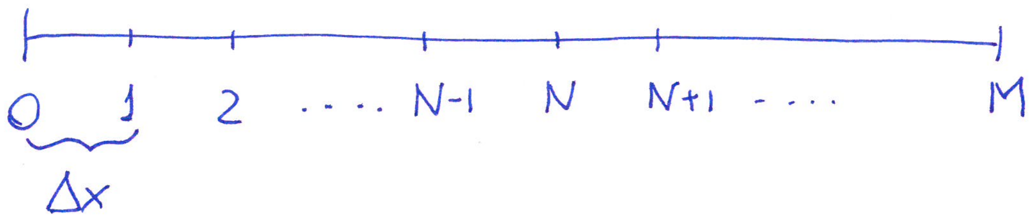
$$T_2^2 = (1-2\tau)T_2^1 + \tau\left[T_1^1 - 2h\Delta x/k\,(T_2^1 - T_\infty) + T_1^1\right] + \frac{\tau(\Delta x)^2}{k} S_2^1$$

$$\boxed{T_2^2 = 172.77°C}$$

Continuing with the same ~~procedure~~ procedure until $t = 2.5\,min = 150\,s$ leads to

$$\boxed{T_2^{50} = 139°C}$$

P9: In a 1D plane wall divided in M nodes:



The expansion in Taylor series around node N leads to:

$$T_{N-1} = T_N - \Delta x \left(\frac{dT}{dx}\right)_N + \frac{(\Delta x)^2}{2!}\left(\frac{d^2T}{dx^2}\right)_N + O\left[(\Delta x)^3\right]$$

$$T_{N+1} = T_N + \Delta x \left(\frac{dT}{dx}\right)_N + \frac{(\Delta x)^2}{2!}\left(\frac{d^2T}{dx^2}\right)_N + O\left[(\Delta x)^3\right]$$

Summings these expansions and truncating in the second derivative:

$$\frac{T_{N+1} - 2T_N + T_{N-1}}{(\Delta x)^2} = \left.\frac{\partial^2 T}{\partial x^2}\right|_N$$

D10

5-24 A uranium plate is subjected to insulation on one side and convection on the other. The finite difference formulation of this problem is to be obtained, and the nodal temperatures under steady conditions are to be determined.

Assumptions **1** Heat transfer through the wall is steady since there is no indication of change with time. **2** Heat transfer is one-dimensional since the plate is large relative to its thickness. **3** Thermal conductivity is constant. **4** Radiation heat transfer is negligible.

Properties The thermal conductivity is given to be k = 28 W/m·°C.

Analysis The number of nodes is specified to be M = 6. Then the nodal spacing Δx becomes

$$\Delta x = \frac{L}{M-1} = \frac{0.05\,\text{m}}{6\text{-}1} = 0.01\,\text{m}$$

This problem involves 6 unknown nodal temperatures, and thus we need to have 6 equations to determine them uniquely. Node 0 is on insulated boundary, and thus we can treat it as an interior note by using the mirror image concept. Nodes 1, 2, 3, and 4 are interior nodes, and thus for them we can use the general finite difference relation expressed as

$$\frac{T_{m-1} - 2T_m + T_{m+1}}{\Delta x^2} + \frac{\dot{g}_m}{k} = 0, \quad \text{for } m = 0, 1, 2, 3, \text{and } 4$$

Finally, the finite difference equation for node 5 on the right surface subjected to convection is obtained by applying an energy balance on the half volume element about node 5 and taking the direction of all heat transfers to be towards the node under consideration:

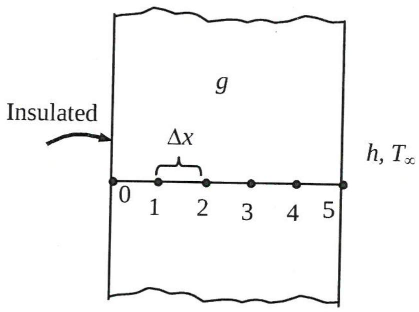
Node 0 (Left surface - insulated): $\dfrac{T_1 - 2T_0 + T_1}{\Delta x^2} + \dfrac{\dot{g}}{k} = 0$

Node 1 (interior): $\dfrac{T_0 - 2T_1 + T_2}{\Delta x^2} + \dfrac{\dot{g}}{k} = 0$

Node 2 (interior): $\dfrac{T_1 - 2T_2 + T_3}{\Delta x^2} + \dfrac{\dot{g}}{k} = 0$

Node 3 (interior): $\dfrac{T_2 - 2T_3 + T_4}{\Delta x^2} + \dfrac{\dot{g}}{k} = 0$

Node 4 (interior): $\dfrac{T_3 - 2T_4 + T_5}{\Delta x^2} + \dfrac{\dot{g}}{k} = 0$



Node 5 (right surface - convection): $h(T_\infty - T_5) + k\dfrac{T_4 - T_5}{\Delta x} + \dot{g}(\Delta x/2) = 0$

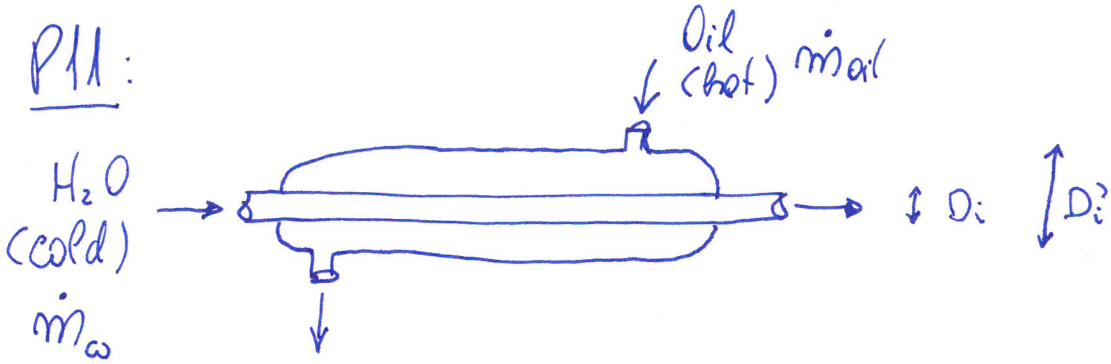
where $\Delta x = 0.01\,\text{m}$, $\dot{g} = 6\times10^5$ W/m^3, $k = 28$ W/m·°C, $h = 60$ W/m^2·°C, and $T_\infty = 30$°C. This system of 6 equations with six unknown temperatures constitute the finite difference formulation of the problem.

(*b*) The 6 nodal temperatures under steady conditions are determined by solving the 6 equations above simultaneously with an equation solver to be

T_0 **=556.8°C,** T_1 **=555.7°C,** T_2 **=552.5°C,** T_3 **=547.1°C,** T_4 **=539.6°C, and** T_5 **=530.0°C**

Discussion This problem can be solved analytically by solving the differential equation as described in Chap. 2, and the analytical (exact) solution can be used to check the accuracy of the numerical solution above.

P11:



$D_i = 2\times10^{-2}$ m
$D_i' = 3\times10^{-2}$ m
$\dot{m}_w = 0.5$ Kg/s
$\dot{m}_{oil} = 0.8$ Kg/s
$T_w^{aver} = 45°C$
$T_{oil}^{aver} = 80°C$
$h_o = 75.2$ W/m²K
$\boxed{U = ?}$

The overall HT coefficient is expressed as:

$$U^{-1} = h_i^{-1} + h_o^{-1}$$

h_o: convective HT coef. outside the tube

h_i: convective HT coef. inside the tube

We need to calculate $\underline{h_i}$ from the Nusselt number definition:

$$Nu = \frac{h_i D_h}{K}$$

D_h: hydraulic diameter $= D_i = 2\times10^{-2}$ m

In order to calculate $\underline{Nu}$, we first need to know the ~~flow~~ water flow regime in the inner tube through the Reynolds number:

$$Re_D = \frac{\rho v D}{\mu} = \frac{v D}{\nu}$$

ν: kinematic viscosity

The flow velocity can be obtained from

the mass flow rate, diameter of the tube and density of the fluid:

$$\dot{m}_\omega = v \rho A$$

$$v = \frac{\dot{m}_\omega}{\rho A} = \frac{\dot{m}_\omega}{\rho \pi D^2/4}$$

$$v = \frac{0.5 \text{ Kg/s}}{990 \text{ Kg/m}^3 \times \pi \times (2\times10^{-2} \text{ m})^2/4} = 1.6077 \text{ m/s}$$

with Re_D:

$$Re_D = \frac{1.6077 \times 2\times10^{-2}}{0.602\times10^{-6}} = 53411.96$$

As $Re_D >>> 4000$, the water flow is turbulent and we can use any expression to determine the Nu number, e.g., Dittus-Boelter:

$$Nu = 0.023\, Re_D^{0.8}\, Pr^{0.4}$$

$$Nu = 0.023 (53411.96)^{0.8} (3.91)^{0.4} = 240.2754$$

Therefore

$$Nu = \frac{h_i D_h}{K} \therefore h_i = \frac{K\,Nu}{D_h} = \frac{0.637 \times 240.2754}{2 \times 10^{-2}}$$

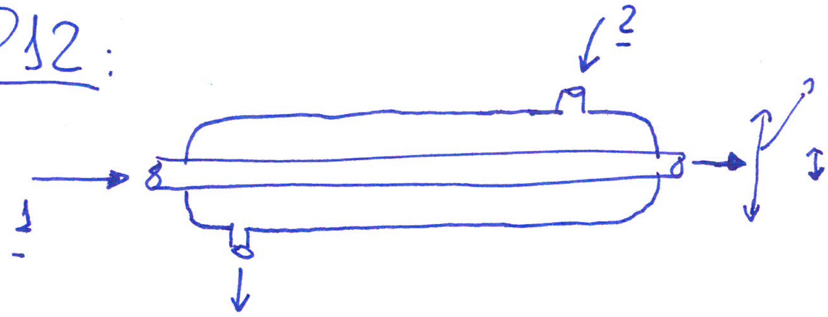
$$h_i = 7652.77 \ W/m^2 \cdot K$$

Now, calculating $\underline{U}$:

$$U^{-1} = h_i^{-1} + h_o^{-1} = (7652.77)^{-1} + (75.2)^{-1}$$

$$\boxed{U = 74.47 \ W/m^2 K}$$

P12:



$D_o' = 3.2\times10^{-2}\,m$

$D_i = 1.5\times10^{-2}\,m$
$D_o = 1.9\times10^{-2}\,m$

$k = 15.1\ W/m.K$

$h_i = 800\ W/m^2.K$

$h_o = 1200\ W/m^2.K$

$R_{f,i} = 4\times10^{-4}\ m^2.K/W$

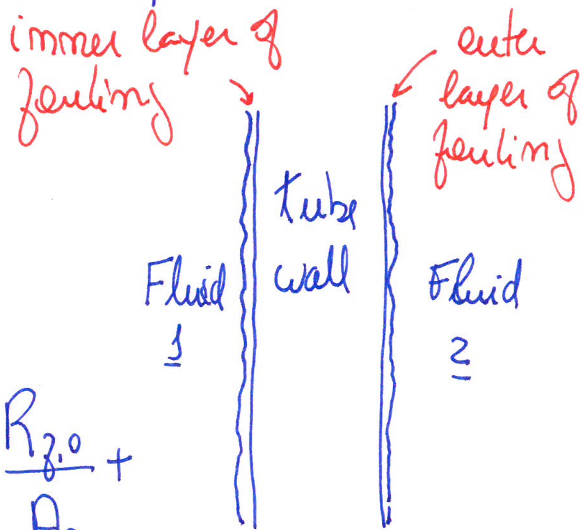
$R_{f,o} = 10^{-4}\ m^2.K/W$

(a) R : ?

(b) U_i & U_o : ?

The thermal resistance for a shell-and-tube HE with fouling on both HT surfaces is expressed as:

$$R = \frac{1}{UA_s} = \frac{1}{U_iA_i} = \frac{1}{U_oA_o}.$$



$$R = \frac{1}{h_iA_i} + \frac{R_{f,i}}{A_i} + \frac{\ln(D_o/D_i)}{2\pi kL} + \frac{R_{f,o}}{A_o} + 1/h_oA_o$$

Assuming the tube has a length of 1m,

$$A_i = \pi D_i L = 0.04712\ m^2$$

$$A_o = \pi D_o L = 0.05969\ m^2$$

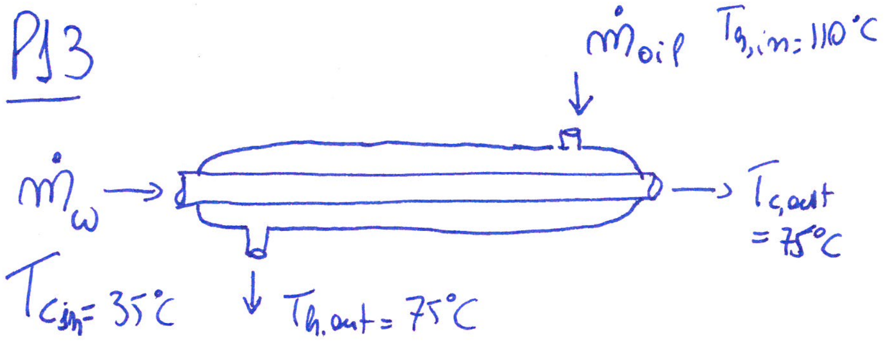
$$R = \frac{1}{800 \times 0.04712} + \frac{4\times10^{-4}}{0.04712} + \frac{\ln\left(1.9\times10^{-2}/1.5\times10^{-2}\right)}{2\pi \times 15.1 \times 1} +$$

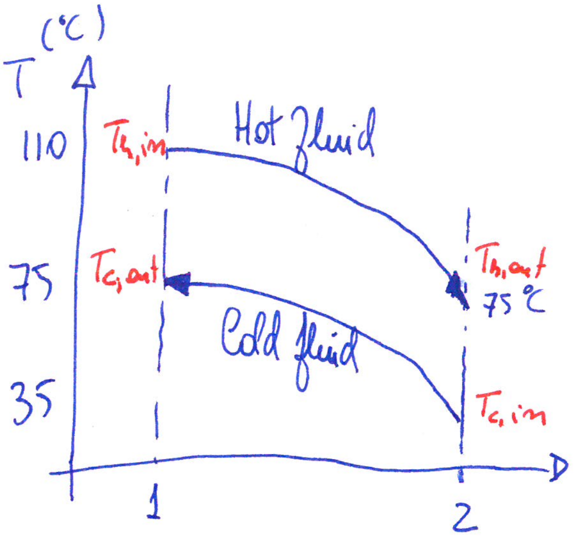
$$+ \frac{10^{-4}}{0.05969} + \frac{1}{1200\times 0.05969} = \underline{5.3145\times10^{-2}\ K/W}$$

(b) $$U_i = \frac{1}{RA_i} = 399.33\ W/m^2.K$$

$$U_o = \frac{1}{RA_o} = 315.24\ W/m^2.K$$

P13





The total heat transfer can be obtained from:

$$Q = \dot{m}_w \, C_{p_w} \, \Delta T_w$$

$$Q = 68 \frac{Kg}{min} \times 4.18 \times 10^3 \frac{J}{Kg.°C} \times (75-35)°C = 189.49 \, KJ/s$$

And the HE surface area is

$Q = UA \Delta T_{lm}$ where $U = 320 \, W/m^2 °C$, and

$$\Delta T_{lm} = \frac{(T_{h,out} - T_{c,out}) - (T_{h,in} - T_{c,in})}{\ln \left(\frac{T_{h,out} - T_{c,out}}{T_{h,in} - T_{c,in}} \right)} \quad \text{parallel-flow}$$

$$\Delta T_{lm} = \frac{(T_{h,in} - T_{c,out}) - (T_{h,out} - T_{c,in})}{\ln \left(\frac{T_{h,in} - T_{c,out}}{T_{h,out} - T_{c,in}} \right)} \quad \text{Counter-flow}$$

For counter-flow HE:

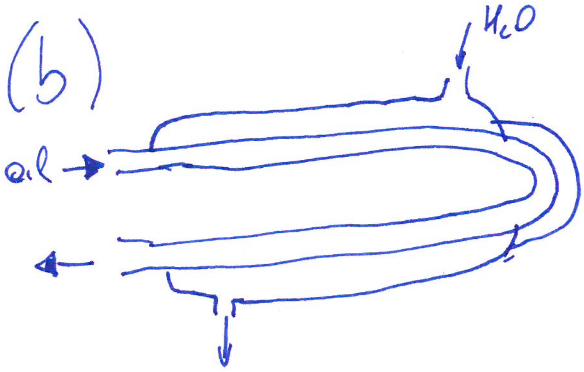
$$\Delta T_{lm} = \frac{(110-75)-(75-35)}{\ln\left(\frac{110-75}{75-35}\right)} = 37.44°C$$

Thus,

$$Q = UA\,\Delta T_{lm} = 189.49$$

$$A = \frac{189.49 \times 10^3\ J/s}{320\ W/m^2 °C \times 37.44°C} = 15.82\ m^2$$

(b)



For cross-flow & multipass shell-and-tube HE:

$$Q = UA_s F \Delta T_{lm}$$

From Fig 10.8 (Appendix of Lecture Notes), with

$$R = \frac{T_{c,in} - T_{c,out}}{T_{h,out} - T_{h,in}} = \frac{35-75}{75-110} = 1.1489$$

$$P = \frac{T_{h,out} - T_{h,in}}{T_{c,in} - T_{h,in}} = \frac{75 - 110}{35 - 110} = 0.4667$$

From the plot: $F \cong 0.8$

$$Q = U A F \Delta T_{lm}$$

$$A = 19.77 m^2$$

P14: From P13 with the original water flow rate of 68 kg/min,

$$Q = 189.49 \ \frac{kJ}{s} = -\dot{m}_{oil} \, C_{p_{oil}} \, \Delta T_{oil}$$

$$\dot{m}_{oil} = -\frac{189.49 \times 10^3 \ J/s}{1.9 \times 10^3 \ J/kg^\circ C \times (75 - 110)^\circ C} = 2.8495 \ kg/s$$

For this problem: $T_{h,in} = 110^\circ C$
$T_{c,in} = 35^\circ C$
$\dot{m}_w = 40 \ kg/min$

The energy balance for both fluid streams:

$$Q_w + Q_{oil} = 0$$

$$\dot{m}_w C_{p_w} (T_{c,out} - T_{c,in}) = -\dot{m}_{oil} C_{p_{oil}} (T_{h,out} - T_{h,in})$$

$$\frac{T_{c,out} - T_{c,in}}{T_{h,in} - T_{h,out}} = \frac{\overbrace{\dot{m}_{oil} C_{p_{oil}}}^{\phi}}{\dot{m}_w C_{p_w}}$$

$$\boxed{T_{c,out} = T_{c,in} + \phi (T_{h,in} - T_{h,out})}$$

↳ we have $T_{c,out}$ as a function of $T_{h,out}$!

With $Q = UA\Delta T_{lm}$

$$\dot{m}_w C_{p_w} (T_{c,out} - T_{c,in}) = UA \frac{(T_{h,in} - T_{c,out}) - (T_{h,out} - T_{c,in})}{\ln\left[\frac{T_{h,in} - T_{c,out}}{T_{h,out} - T_{c,in}}\right]}$$

Substituting $T_{c,out}$ in this expression by

$$T_{c,out} = T_{c,in} + \phi (T_{h,in} - T_{h,out})$$

and solving for $T_{h,out}$

$$\hookrightarrow \boxed{T_{h,out} = 79.70\,^\circ C}$$

Now

$$\frac{T_{c,out} - T_{c,in}}{T_{h,in} - T_{h,out}} = \frac{m_{oil} C_{p_{oil}}}{m_w C_{p_w}} = 1.9427$$

$$\boxed{T_{c,out} = 93.86\,^\circ C}$$

And the HT:

$$Q = \dot{m}_w C_{p_w} (T_{c,out} - T_{c,in})$$

$$Q = 164023.2 \text{ J/s} = 164.02 \text{ KJ/s} = 164.02 \text{ KW}$$

Alternative method using NTU method:

Calculating the capacity rates, C_{min}/C_{max}

$$\phi = \frac{\dot{m}_{oil}\, C_{p\,oil}}{\dot{m}_{w}\, C_{pw}} = 1.9427$$

$$\frac{C_{min}}{C_{max}} = \frac{1}{\phi} = 0.5147$$

$$NTU_{max} = \frac{UA}{C_{MIN}} = \frac{320 \times 15.82}{0.6667 \times 4.18 \times 10^{3}} = 1.8167$$

From Fig. 10-13 (Appendix of Lecture Notes):

↳ $\varepsilon \cong 0.75$

Or from Table 10.3 (also in Appendix of Lecture Notes):

↳ $\varepsilon = 0.745$

Because the cold fluid is the minimum,

$$\varepsilon = \frac{\Delta T_{cold}}{\Delta T_{max}} = \frac{\Delta T_{cold}}{110 - 35} = 0.745$$

$\Delta T_{cold} = 55.875 \therefore T_{c,out} = 90.88°C$

And the heat transfer is

$$\dot{Q} = \dot{m}_w C_{p_w} \Delta T_{COLD} = 155.71 \text{ KW}$$

P15: The cold fluid is air at 1atm with temperature ranging from 15.55°C to 29.44°C. The heat transfer rate is,

$$\dot{Q}_{air} = \dot{m}_{air} C_{p_{air}} \Delta T_{air} = \dot{m}_c C_{p_c} \Delta T_c$$

We know the volumetric flow rate of air - 2.36 m³/s, but we need to obtain the mass flow rate, $\dot{m}_{air}$,

volumetric flow rate

$$\dot{m}_{air} = \rho_{air} \dot{V}_{air}$$

Assuming ideal gas behavior

$$\rho_{air} = \frac{P}{RT} = 4.2212 \times 10^{-5} \text{ mol/cm}^3$$

molar mass

$$\rho_{air} = 4.2212 \times 10^{-5} \frac{\text{mol}}{\text{cm}^3} * 28.97 \frac{\text{g}}{\text{mol}}$$

$$\rho_{air} = 1.223 \text{ Kg/m}^3$$

Thus

$$\dot{m}_{air} = 2.8863 \text{ Kg/s}$$

The heat transfer rate is:

$$\dot{Q}_{air} = 2.8863 \times 1005 \times (29.44 - 15.55)$$

$$\dot{Q}_{air} = 40291.16 \text{ J/s} = 40.29 \text{ KW}$$

The problem does not state which fluid (air or water) can be considered minimum. If air the minimum fluid we can calculate NTU and with the help of Fig. 10-15 (Appendix of the Lecture Notes) calculate $\dot{m}_w$ and the exit water temperature. However if the water is the minimum fluid, we may need to use an iterative method.

Assumption 1: Air the minimum fluid:

$$C_{MIN} = \dot{m}_c C_{p_c} = 2.8863 \times 1005 = 2900.73 \text{ W/°C}$$

and

$$NTU_{MAX} = \frac{UA}{C_{MIN}} = \frac{227 \times 9.29}{2900.73} = 0.7270$$

And the effectiveness is

$$\varepsilon = \frac{\Delta T \text{ (MINIMUM FLUID)}}{\text{Max Temp Diff in HE}}$$

$$\varepsilon = \frac{\Delta T_{air}}{\Delta T_{MAX}} = \frac{29.44 - 15.55}{82.22 - 15.55} = 0.2083$$

Using ε & NTU_{MAX} in Fig. 10-15, we can not find a match with the existing curves, therefore water is the minimum fluid.

Assumption 2: Water is the minimum fluid.

Therefore we need to estimate C_{MIN}; calculate $NTU_{MAX} \Rightarrow \Delta T_h$

Thus:

$$C_{MAX} = \dot{m}_{air} c_{p_{air}} = 2900.73 \text{ W/°C}$$

$$NTU_{MAX} = UA / C_{MIN}$$

$$\varepsilon = \frac{\Delta T_w}{\Delta T_{MAX}} = \frac{\left| T_w^{out} - T_w^{in} \right|}{82.22 - 15.55} \quad (1)$$

with

$$\dot{Q}_{air} = -\dot{Q}_w = \underbrace{\dot{m}_w C_{pw}}_{C_{MIN}} \Delta T_w \therefore \Delta T_w = -\frac{40291.16}{C_{MIN}} \quad (2)$$

Eqn. 2 — from Fig 10-15 a Table 10-3 — from Eqn. 1

C_{MIN}/C_{MAX}	C_{MIN}	NTU_{MAX}	$\lvert\Delta T_w\rvert$	ε_{TABLE}	ε_c	Error
0.5	1450.37	1.4540	27.78	0.65	0.4167	35.89
0.25	725.18	2.9080	55.56	0.89	0.8334	6.36
0.22	638.16	3.3045	63.14	0.92	0.9471	2.95

CONVERGED !

$$\text{Error } (\%) : \frac{\lvert \varepsilon_{TABLE} - \varepsilon_c \rvert}{\varepsilon_{TABLE}} \times 100$$

Thus for $C_{MIN} = \dot{m}_w C_{pw} = 638.16$ (C_{pw}: 4180 J/kg K)

$$\dot{m}_w = 0.1527 \text{ kg/s}$$

$$\Delta T_w = T_w^{OUT} - T_w^{IN} = -\frac{40291.16}{638.16} \therefore \boxed{T_w^{OUT} = 19.08\,°C}$$

P16

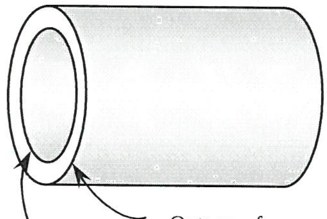
13-18 The heat transfer coefficients and the fouling factors on tube and shell side of a heat exchanger are given. The thermal resistance and the overall heat transfer coefficients based on the inner and outer areas are to be determined.

Assumptions **1** The heat transfer coefficients and the fouling factors are constant and uniform.

Analysis (*a*) The total thermal resistance of the heat exchanger per unit length is

$$R = \frac{1}{h_i A_i} + \frac{R_{fi}}{A_i} + \frac{\ln(D_o/D_i)}{2\pi kL} + \frac{R_{fo}}{A_o} + \frac{1}{h_o A_o}$$

$$R = \frac{1}{(700\ \text{W/m}^2.°\text{C})[\pi(0.012\ \text{m})(1\ \text{m})]} + \frac{(0.0005\ \text{m}^2.°\text{C/W})}{[\pi(0.012\ \text{m})(1\ \text{m})]}$$
$$+ \frac{\ln(1.6/1.2)}{2\pi(380\ \text{W/m}.°\text{C})(1\ \text{m})} + \frac{(0.0002\ \text{m}^2.°\text{C/W})}{[\pi(0.016\ \text{m})(1\ \text{m})]}$$
$$+ \frac{1}{(700\ \text{W/m}^2.°\text{C})[\pi(0.016\ \text{m})(1\ \text{m})]}$$
$$= \mathbf{0.0837°C/W}$$



(*b*) The overall heat transfer coefficient based on the inner and the outer surface areas of the tube per length are

$$R = \frac{1}{UA} = \frac{1}{U_i A_i} = \frac{1}{U_o A_o}$$

$$U_i = \frac{1}{RA_i} = \frac{1}{(0.0837\ °\text{C/W})[\pi(0.012\ \text{m})(1\ \text{m})]} = \mathbf{317\ W/m^2.°C}$$

$$U_o = \frac{1}{RA_o} = \frac{1}{(0.0837\ °\text{C/W})[\pi(0.016\ \text{m})(1\ \text{m})]} = \mathbf{238\ W/m^2.°C}$$

P 17

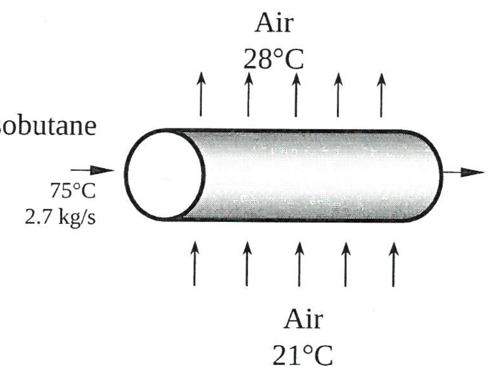
13-64 Isobutane is condensed by cooling air in the condenser of a power plant. The mass flow rate of air and the overall heat transfer coefficient are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** There is no fouling. **5** Fluid properties are constant.

Properties The heat of vaporization of isobutane at 75°C is given to be h_{fg} = 255.7 kJ/kg and specific heat of air is given to be C_p = 1005 J/kg.°C.

Analysis First, the rate of heat transfer is determined from

$$\dot{Q} = (\dot{m}h_{fg})_{\text{isobutane}} = (2.7\ \text{kg/s})(255.7\ \text{kJ/kg}) = 690.39\ \text{kW}$$



The mass flow rate of air is determined from

$$\dot{Q} = [\dot{m}C_p(T_{\text{out}} - T_{\text{in}})]_{\text{air}}$$

$$\dot{m}_{\text{air}} = \frac{\dot{Q}}{C_p(T_{\text{out}} - T_{\text{in}})}$$

$$= \frac{690.39\ \text{kJ/s}}{(1.005\ \text{kJ/kg.°C})(28°\text{C} - 21°\text{C})}$$

$$= \mathbf{98.14\ kg/s}$$

The temperature differences between the isobutane and the air at the two ends of the condenser are

$$\Delta T_1 = T_{\text{h,in}} - T_{\text{c,out}} = 75°\text{C} - 21°\text{C} = 54°\text{C}$$

$$\Delta T_2 = T_{\text{h,out}} - T_{\text{c,in}} = 75°\text{C} - 28°\text{C} = 47°\text{C}$$

and

$$\Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{54 - 47}{\ln(54/47)} = 50.4°\text{C}$$

Then the overall heat transfer coefficient is determined from

$$\dot{Q} = UA_s\Delta T_{\text{lm}} \longrightarrow 690{,}390\ \text{W} = U(24\ \text{m}^2)(50.4°\text{C}) \longrightarrow U = \mathbf{571\ W/m^2.°C}$$

Q18

13-89 Water is heated by hot air in a heat exchanger. The mass flow rates and the inlet temperatures are given. The heat transfer surface area of the heat exchanger on the water side is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** The overall heat transfer coefficient is constant and uniform.

Properties The specific heats of the water and air are given to be 4.18 and 1.01kJ/kg.°C, respectively.

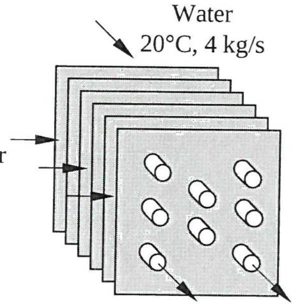
Analysis The heat capacity rates of the hot and cold fluids are

$$C_h = \dot{m}_h C_{ph} = (4\text{ kg/s})(4.18\text{ kJ/kg.°C}) = 16.72\text{ kW/°C}$$

$$C_c = \dot{m}_c C_{pc} = (9\text{ kg/s})(1.01\text{ kJ/kg.°C}) = 9.09\text{ kW/°C}$$

Therefore, $C_{\min} = C_c = 9.09\text{ kW/°C}$

and $C = \dfrac{C_{\min}}{C_{\max}} = \dfrac{9.09}{16.72} = 0.544$



Then the NTU of this heat exchanger corresponding to $C = 0.544$ and $\varepsilon = 0.65$ is determined from Fig. 13-26 to be

$$\text{NTU} = 1.5$$

Then the surface area of this heat exchanger becomes

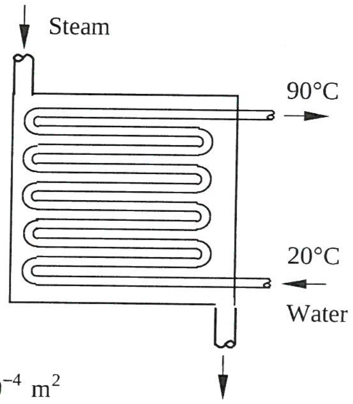
$$\text{NTU} = \frac{UA_s}{C_{\min}} \longrightarrow A_s = \frac{\text{NTU}\,C_{\min}}{U} = \frac{(1.5)(9.09\text{ kW/°C})}{0.260\text{ kW/m}^2\text{.°C}} = \mathbf{52.4\ m^2}$$

P19

3-107 Water is to be heated by steam in a shell-and-tube process heater. The number of tube passes need to be used is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible.

Properties The specific heat of the water is given to be 4.19 kJ/kg.°C.



Analysis The mass flow rate of the water is

$$\dot{Q} = \dot{m}_c C_{pc}(T_{c,out} - T_{c,in})$$

$$\dot{m} = \frac{\dot{Q}}{C_{pc}(T_{c,out} - T_{c,in})}$$

$$= \frac{600 \text{ kW}}{(4.19 \text{ kJ/kg.°C})(90°\text{C} - 20°\text{C})}$$

$$= 2.046 \text{ kg/s}$$

The total cross-section area of the tubes corresponding to this mass flow rate is

$$\dot{m} = \rho V A_c \rightarrow A_c = \frac{\dot{m}}{\rho V} = \frac{2.046 \text{ kg/s}}{(1000 \text{ kg/m}^3)(3 \text{ m/s})} = 6.82 \times 10^{-4} \text{ m}^2$$

Then the number of tubes that need to be used becomes

$$A_s = n\frac{\pi D^2}{4} \longrightarrow n = \frac{4A_s}{\pi D^2} = \frac{4(6.82 \times 10^{-4} \text{ m}^2)}{\pi(0.01 \text{ m})^2} = 8.68 \cong \mathbf{9}$$

Therefore, we need to use at least 9 tubes entering the heat exchanger.