Q.1 **Question 1**

1. Consider a large aluminium plate of thickness 0.12 m with initial uniform temperature of 85°C. Suddenly, the temperature of one of the faces is lowered to 20°C, while the other face is perfectly insulated. Assuming that the plate can be modelled as a 1D problem, the following thermal energy conservative equation can be used,

$$\rho C_p \frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial x^2} + \mathcal{S},$$

where ρ, C_p, t are density, heat capacity and time, respectively. κ and $\mathcal{S}$ are thermal conductivity coefficient and source term. Using the finite difference method (FDM) with spatial (Δx) and temporal (Δt) increments of 0.03 m and 300 s, respectively,

a) Determine the number of nodes necessary to discretise the problem; [2 marks]

Solution:

[2/2] *For a length $L = 0.12$ m and spatial increment $\Delta x = 30 \times 10^{-3}$ m,*

$$\Delta x = \frac{L}{M-1} \implies M = 5 \textit{ nodes}$$

b) Describe the boundary and initial conditions for this problem; [5 marks]

Solution:

For 5 nodes, $i = 0, \cdots, 4$

(i) Boundary conditions:

[1/5]
- *Dirichlet BC at $x = L =$ 0.12 m;*

$$T\left(x = L, t\right) = T_4^j = 20^\circ \textit{C}, \quad (j = 0, 1, \cdots)$$

[3/5]
- *Adiabatic plane at $x = 0$ can be treated as a symmetric plane, i.e.,*

$$T_{i+1}^j = T_{i-1}^j \quad \textit{for } i = 0 \implies T_1^j = T_{-1}^j$$

[1/5] *(ii) Initial condition: the plate has uniform temperature of 85°C, i.e.,*

$$T\left(x < L, t = 0s\right) = T_i^0 = 85^\circ \textit{C}, \quad (i = 0, 1, 2, 3)$$

c) Determine the temperature distribution of the plate at $t = 10$ minutes using the FDM, [16 marks]

Solution:

The discretised thermal energy equation,

$$\rho C_p \frac{T_i^{j+1} - T_i^{j+1}}{\Delta t} = \kappa \frac{T_{i+1}^j - 2T_i^j + T_{i-1}^j}{\left(\Delta x\right)^2} + \mathcal{S}_i^j,$$

[1/16] *can be re-arranged to*

$$\frac{\rho C_p}{\kappa}\frac{(\Delta x)^2}{\Delta t}\left(T_i^{j+1}-T_i^{j+1}\right)=\left(T_{i+1}^j-2T_i^j+T_{i-1}^j\right)+\frac{(\Delta x)^2}{\kappa}\mathcal{S}_i^j.$$

Defining the mesh Fourier number,

$$\tau=\frac{\alpha\Delta t}{(\Delta x)^2}$$

and the thermal diffusivity, $\left(\alpha=\kappa\rho^{-1}C_p^{-1}\right)$*,*

$$T_i^{j+1}=T_i^j+\tau\left(T_{i+1}^j-2T_i^j+T_{i-1}^j\right)+\frac{\tau(\Delta x)^2}{\kappa}\mathcal{S}_i^j.$$

[2/16] *For this problem, the source term is null,* $\mathcal{S}_i^j=0$*,*

$$T_i^{j+1}=T_i^j+\tau\left(T_{i+1}^j-2T_i^j+T_{i-1}^j\right).$$

The first step to solve FDM problems is to check if the stability criteria is met,

[1/16] *i.e.,* $\tau\leq 0.5$

$$\tau=\frac{\alpha\Delta t}{(\Delta x)^2}=0.5 \quad \textit{with} \quad \Delta t=300\ \textit{s}.$$

[2/16] *Boundary and initial conditions are:*

$$\begin{cases}\textit{BC1: } T_4^j=20, & j=0,1,\cdots\\ \textit{BC2: } T_i^j=T_{-1}^j, & j=0,1,\cdots\\ \textit{IC: } T_i^0=85, & i=0,\cdots,3\end{cases}$$

Now solving the problem for $j=0,1,2$

[5/16] **j=0** $\rightarrow\ T_i^{j+1}=T_i^1\ (t=300s)$*:*

$$\begin{cases}i=0, & T_0^1=T_0^0+\tau\left(T_1^0-2T_0^0+T_{-1}^0\right)=85\\ i=1, & T_1^1=T_1^0+\tau\left(T_2^0-2T_1^0+T_0^0\right)=85\\ i=2, & T_2^1=T_2^0+\tau\left(T_3^0-2T_2^0+T_1^0\right)=85\\ i=3, & T_3^1=T_3^0+\tau\left(T_4^0-2T_3^0+T_2^0\right)=52.50\\ i=4, & T_4^1=20\end{cases}$$

[5/16] **j=1** $\rightarrow\ T_i^{j+1}=T_i^2\ (t=600s)$*:*

$$\begin{cases}i=0, & T_0^2=T_0^1+\tau\left(T_1^1-2T_0^1+T_{-1}^1\right)=85\\ i=1, & T_1^2=T_1^1+\tau\left(T_2^1-2T_1^1+T_0^1\right)=85\\ i=2, & T_2^2=T_2^1+\tau\left(T_3^1-2T_2^1+T_1^1\right)=68.75\\ i=3, & T_3^2=T_3^1+\tau\left(T_4^1-2T_3^1+T_2^1\right)=52.50\\ i=4, & T_4^2=20\end{cases}$$

Thermal diffusivity $\left(\alpha=\kappa\rho^{-1}C_p^{-1}\right)$ of the plate is 1.5×10^{-6} $m^2.s^{-1}$. The discretised form of the thermal energy equation is

$$\rho C_p\frac{T_i^{j+1}-T_i^{j+1}}{\Delta t}=\kappa\frac{T_{i+1}^j-2T_i^j+T_{i-1}^j}{(\Delta x)^2}+\mathcal{S}_i^j,$$

where i and j are spatial and temporal indices.

2. A double-pipe (shell-and-tube) heat exchanger is constructed of a stainless steel ($\kappa = 15.1$ W/(m.°C) inner tube of inner diameter $D_i = 1.5$ cm and outer diameter $D_o = 1.9$ cm and an outer shell of inner diameter 3.2 cm. The convective heat transfer coefficient is $h_i = 800$ W/(m^2.°C) on the inner surface of the tube and $h_o = 1200$ W/(m^2.°C) on the outer surface. For a fouling factor $R_{f,i} = 0.0004$ m^2.°C/W on the tube side and $R_{f,o} = 0.0001$ m^2.°C/W on the shell side, determine:

(a) The thermal resistance of the heat exchanger per unit length (in °C/W) and; [3 marks]

Solution:

[1/3] *The total thermal resistance, R, of the heat exchanger through the pipes per unit length is*

$$R = \frac{1}{h_i A_i} + \frac{R_{fi}}{A_i} + \frac{\ln\left(\frac{D_0}{D_i}\right)}{2\pi\kappa L} + \frac{R_{f0}}{A_0} + \frac{1}{h_0 A_0},$$

[2/3] *where $A_i = 0.04712$ m^2 and $A_0 = 0.5969$ m^2, resulting in* $\mathbf{R = 5.3145 \times 10^{-2}}$**°C.W**$^{-1}$

(b) The overall heat transfer coefficients, U_i and U_o (in W/(m^2.°C)) based on the inner and outer surface areas of the tube, respectively. [7 marks]

Solution:

[1/7] *The overall heat transfer coefficient based on the inner and the outter surface areas of the tube per length are*

$$R = \frac{1}{UA} = \frac{1}{U_i A_i} = \frac{1}{U_0 A_0},$$

[3/7] *thus for U_i*

$$U_i = \frac{1}{RA_i} = 399.3303 \frac{W}{m^2.^\circ C}$$

[3/7] *and for U_0*

$$U_0 = \frac{1}{RA_0} = 315.2461 \frac{W}{m^2.^\circ C}$$

Total Question Marks:33

Q.2 **Question 2**

1. A new material is to be developed for bearing balls in a new rolling-element bearing. For annealing (heat treatment) each bearing ball, a sphere of radius $r_o = 5$ mm, is heated in a furnace until it reaches to the equilibrium temperature of the furnace at 400°C. Then, it is suddenly removed from the furnace and subjected to a two-step cooling process.

Stage 1: Cooling in an air flow of 20°C for a period of time t_{air} until the center temperature reaches 335°C. For this situation, the convective heat transfer coefficient of air is assumed constant and equal to $h = 10$ W/(m^2.K). After the sphere has reached this specific temperature, the second step is initiated.

Stage 2: Cooling in a well-stirred water bath at 20°C, with a convective heat transfer coefficient of water $h = 6000$ W/(m^2.K).

The thermophysical properties of the material are $\rho = 3000$ kg/m^3, $\kappa = 20$ W/(m.K), $C_p = 1000$ J/(kg.K). Determine:

(a) The time t_{air} required for *Stage 1* of the annealing process to be completed; [5 marks]

Solution:

[1/5] *The first step is to check if the lumped-capacitance method can be used:*

$$\mathbf{Bi} = \frac{hL_c}{\kappa} = \mathbf{8.33 \times 10^{-4}}$$

[1/5] *As* $\mathbf{Bi} < 0.1$ *, the lumped-capacitance method can be used in this stage, i.e.,*

[3/5] *temperature changes uniformily throughout the sphere,*

$$\frac{T(t) - T_\infty}{T_0 - T_\infty} = \exp\left[-\frac{h}{L_c \rho C_p} t\right]$$

$$\frac{335 - 20}{400 - 20} = \exp\left[-\frac{10t}{\frac{5\times10^{-3}}{3} \times 3000 \times 1000}\right] \quad \Rightarrow \quad \mathbf{t = 93.80} \ s$$

(b) The time t_{water} required for *Stage 2* of the annealing process during which the center of the sphere cools from 335°C (the condition at the completion of *Stage 1*) to 50°C. [6 marks]

Solution:

Checking if the lumped-capacitance method can be used:

$$\mathbf{Bi} = \frac{\mathbf{hL_c}}{\kappa} = \mathbf{0.5 > 0.1},$$

[1/6] *therefore the* **lumped-capacitance method can not be used** . *In order to use the Tables for the analytical solution, we need to calculate the Bi number based on r_0,*

$$Bi = \frac{hr_0}{\kappa} = 1.5$$

[2/6] *From the Table with coefficients for the one-term approximate solution of 1D transient conduction in spheres, we can obtain* $\lambda_1 = \mathbf{1.7998}$ *and* $\mathbf{A_1} = \mathbf{1.3763}$ *to be applied into*

$$\theta_{\textit{centre}} = \frac{T_0 - T_\infty}{T_i - T_\infty} = \frac{50-20}{335-20} = A_1 \exp\left[-\lambda_1^2 \tau\right],$$

[1/6] *where* $\tau = \alpha t . r_0^{-2}$ *with* $\alpha = \kappa.\left(\rho \mathbf{C_p}\right)^{-1} = \mathbf{6.67 \times 10^{-6}}$ $m^2.s^{-1}$.

[1/6] *Solving the equation above results in* $\tau = \mathbf{0.8245}$ *, and* t *is obtained from the*

[1/6] *Fourier number,*

$$\tau = \frac{\alpha t}{r_0^2} \quad \Rightarrow \quad \mathbf{t = 3.09\ s}$$

2. Water at the rate of 68 kg/min is heated from 35 to 75°C by an oil having a specific heat of 1.9 kJ/(kg.°C). The fluids are used in a counterflow double-pipe HE, and the oil enters the exchanger at 110°C and leaves at 75°C. The overall heat-transfer coefficient is 320 W/(m^2.°C). Given heat capacity of water (at constant pressure) of 4.18 kJ/(kg.°C),

   (a) Calculate the HE area; [5 marks]

   Solution:

   [1/5] *The total heat transfer can be obtained from*

$$Q = \dot{m}_w C_{p,w} \Delta T_w = 189.49 \ \textit{kJ.s}^{-1}.$$

   And the heat exchange surface area can be calculated from

$$Q = UA\Delta T_{lm},$$

   [2/5] *where* $U = 320$ *W.m*$^{-2}.^\circ$*C*$^{-1}$ *and*

$$\Delta T_{lm} = \frac{(T_{h,in} - T_{c,out}) - (T_{h,out} - T_{c,in})}{\ln \frac{T_{h,in} - T_{c,out}}{T_{h,out} - T_{c,in}}} = 37.44^\circ C,$$

   [2/5] *thus*

$$\begin{aligned} Q &= UA\Delta T_{lm} = 189.49 \ \textit{kJ.s}^{-1} \\ A &= 15.82 \ m^2 \end{aligned}$$

   (b) Now assume that the heat exchanger is a shell-and-tube with water making one shell pass and the oil making two tube passes. Calculate the area of the new heat exchanger. Assume that the overall heat-transfer coefficient remains the same. [4 marks]

   Solution:

   For cross-flow and multi-pass shell-and-tubes heat exchangers,

$$Q = UAF\Delta T_{lm}.$$

   [2/4] *From Fig. 10.8 with,*

$$\begin{cases} R = \frac{T_{c,in}-T_{c,out}}{T_{h,out}-T_{h,in}} = 1.1429, \\ P = \frac{T_{h,out}-T_{h,in}}{T_{c,in}-T_{h,in}} = 0.4667, \end{cases}$$

[2/4] *leads to* $F \sim 0.8$ *and*

$$Q = UAF\Delta T_{lm} \implies A = 19.77 \ m^2$$

Total Question Marks:20

Q.3 **Question 3**

1. Hot oil is to be cooled in a double-tube counter-flow heat exchanger. The copper inner tubes have diameter of 2 cm and negligible thickness. The inner diameter of the outer tube (shell) is 3 cm. Water flows through the tube at a rate of 0.5 kg.s^{-1}, and the oil through the shell at a rate of 0.8 kg.s^{-1}. Taking the average temperatures of the water and the oil to be 45°C and 80°C, respectively, determine the overall heat transfer coefficient of this heat exchanger. Given,

   (a) Water at 45°C: $\rho = 990$ kg.m^{-3}, $\kappa = 0.637$ W.(m.K)$^{-1}$, $Pr = 3.91$, $\nu = \mu/\rho = 0.602 \times 10^{-6}$ m^2.s^{-1};

   (b) Oil at 80°C: $\rho = 852$ kg.m^{-3}, $\kappa = 0.138$ W.(m.K)$^{-1}$, $Pr = 490$, $\nu = 37.5 \times 10^{-6}$ m^2.s^{-1}.

   The overall heat transfer coefficient can be expressed as,

$$U^{-1} = h_i^{-1} + h_0^{-1}$$

   The inner convective heat transfer coefficient, h_i, can be obtained from

$$Nu = \frac{h_i D_h}{\kappa} = \begin{cases} 4.36 & \text{(for laminar flows),} \\ 0.023 Re^{0.8} Pr^{0.4} & \text{(for turbulent flows),} \end{cases}$$

   where D_h is the hydraulic diameter. The outter convective heat transfer coefficient, h_0 is 75.2 W.(m^2.K). [17 marks]

   Solution:

   In order to solve this problem, we need to calculate h_i through the dimensionless Nusselt number with $D_h = D_i = 2 \times 10^{-2}$ m,

$$Nu = \frac{h_i D_h}{\kappa} = \begin{cases} 4.36 & \textit{(for laminar flows),} \\ 0.023 Re^{0.8} Pr^{0.4} & \textit{(for turbulent flows).} \end{cases}$$

   However, we first need to assess the flow regime in the inner tube through the dimensionless Reynolds number

$$Re_D = \frac{\rho v D}{\mu} = \frac{vD}{\nu}$$

   The flow velocity, v, can be obtained from the mass flow rate, diameter of the
[4/17] *tube and density of the fluid,*

$$\dot{m}_w = v\rho A \quad \Longrightarrow \quad v = 1.6077 \text{ m.s}^{-1},$$

[2/17] *with Re_D*

$$Re_D = \frac{vD}{\nu} = 53411.96$$

[6/17] *As $Re >>> 4000$, the water flow is turbulent and we can use*

$$\begin{aligned} Nu &= 0.023Re^{0.8}Pr^{0.4} = 240.2754 \\ Nu &= \frac{h_i D_h}{\kappa} = 240.2754 \implies h_i = 7652.77 \text{ W.} \left(m^2.K\right)^{-1} \end{aligned}$$

[5/17] *Now, calculating U*

$$U^{-1} = h_i^{-1} + h_0^{-1} \implies U = 74.47 \text{ W.} \left(m^2.K\right)^{-1}$$

2. A long rod of 60 mm diameter and thermophysical properties $\rho = 8000$ kg/m^3, $C_p = 500$ J/(kg.K), and $k = 50$ W/(m.K) is initially at a uniform temperature and is heated in a forced convection furnace maintained at 750 K. The convection coefficient is estimated to be 1000 W/(m^2.K). Calculate the centerline temperature of the rod when the surface temperature is 550 K. [16 marks]

Solution:

[3/16] *Assuming that the Fourier number for this problem is larger than 0.2, we can use the 1D analytical solution for the transient conductive equation,*

$$\theta_{cyl} = \frac{T(r,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \mathbf{J_0}\left(\frac{\lambda_1 r}{r_0}\right)$$

[3/16] *and at the centre of the geometry,*

$$\theta_{cyl} = \frac{T(0,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}$$

[3/16] *We need to obtain $T(0,t)$ at time $t = t_i$ such that $T(r_0, t_i) =$ 500 K. Merging both equations,*

$$\frac{T(r_0,t_i) - T_\infty}{T_i - T_\infty} = \frac{T(0,t_i) - T_\infty}{T_i - T_\infty} \mathbf{J_0}\left(\frac{\lambda_1 r_0}{r_0}\right)$$

[4/16] *In order to solve this expression, $\mathbf{J_0}\left(\frac{\lambda_1 r}{r_0}\right)$ and λ_1 need to be obtained from the Table of coefficients for the approximate solution of the transient 1D heat conduction based on,*

$$Bi = \frac{h r_0}{\kappa} = 0.60 \Rightarrow \lambda_1 = 1.0184 \text{ and } \mathbf{J_0}\left(\frac{\lambda_1 r_0}{r_0}\right) = \mathbf{J_0}(\lambda_1) = 0.7571$$

[3/16] *Replacing $\mathbf{J_0}$ in the merged equation,*

$$(550 - 750) = [T(0,t_i) - 750] \times 0.7571 \Rightarrow T(0,t_i) = 485.83 \text{ K}$$

Total Question Marks:33

Q.4 **Question 4**

1. In a counterflow double-pipe heat exchanger, water $\left(C_p = 4.18 \text{ kJ.kg}^{-1}.°\text{C}\right)$ at 35°C is heated by oil $\left(C_p = 1.9 \text{ kJ.kg}^{-1}.°\text{C}\right)$. The mass flow rate of the water stream is 40 kg.min^{-1} and 170.97 kg.min^{-1} of oil enters the heat exchanger at 110°C. The overall heat-transfer coefficient is 320 W.(m^2.°C)$^{-1}$. Calculate:

   (a) Exit water and oil temperatures; [5 marks]
   Solution:
   [1/5] *The energy balance for both fluid streams (oil and water), where $T_{oil,in} = T_{h,in} = 110°C$ and $T_{w,in} = T_{c,in} = 35°C$,*

$$Q_w + Q_{oil} = 0$$
$$\dot{m}_w C_{p,w}\left(T_{c,out} - T_{c,in}\right) = -\dot{m}_o C_{p,o}\left(T_{h,out} - T_{h,in}\right)$$
$$\frac{T_{c,out} - T_{c,in}}{T_{h,in} - T_{h,out}} = \frac{\dot{m}_o C_{p,o}}{\dot{m}_w C_{p,w}} \implies T_{c,out} = T_{c,in} + \frac{\dot{m}_o C_{p,o}}{\dot{m}_w C_{p,w}} T_{h,in} - T_{h,out},$$

   [1/5] *$T_{c,out} = T_{w,out}$ can be obtain as a function of $T_{h,out}$. However, with $Q = UA\Delta T_{lm}$,*

$$\dot{m}_w C_{p,w}\left(T_{c,out} - T_{c,in}\right) = UA\frac{\left(T_{h,in} - T_{c,out}\right) - \left(T_{h,out} - T_{c,in}\right)}{\ln \frac{T_{h,in} - T_{c,out}}{T_{h,out} - T_{c,in}}}$$

   [1/5] *and substituting $T_{c,out}$ in this expression we obtain $T_{h,out} = 79.70°C$. Now,*

   [2/5]
$$\frac{T_{c,out} - T_{c,in}}{T_{h,in} - T_{h,out}} = \frac{\dot{m}_o C_{p,o}}{\dot{m}_w C_{p,w}} = 1.9427 \implies T_{c,out} = T_{w,out} = 93.86°C$$

   (b) Total heat transfer (in kW); [1 marks]
   Solution:
   [1/1] *The heat transferred between streams can be obtained from*

$$Q = \dot{m}_w C_{p,w}\left(T_{c,out} - T_{c,in}\right) = 164023.2 \text{ J.s}^{-1} = 164.02 \text{ kW}$$

2. Consider three consecutive nodes $n-1$, n, $n+1$ in a plane wall. Using the finite difference form of the first derivative at the midpoints, show that the finite difference form of the second derivative can be expressed as, [4 marks]

$$\frac{T_{n-1} - 2T_n + T_{n+1}}{\Delta x^2} = \left.\frac{\partial^2 T}{\partial x^2}\right|_N .$$

   Hint: You should start the demonstration from the 1D expansion in Taylor series of a continuous and real function $f(x)$ about a point $x = a$,

$$f(x) = f(a) + f^{'}(a)\left(x-a\right) + \frac{f^{''}(a)}{2!}\left(x-a\right)^2 + \frac{f^{'''}(a)}{3!}\left(x-a\right)^3 + \cdots + \frac{f^n(a)}{n!}\left(x-a\right)^n$$

Solution:

[3/4] *In a 1D plane wall divided in M nodes with an internal node N. Spatial increment is given by Δx. The expansion in Taylor series around node N truncating in the second derivative leads to:*

$$\begin{cases} T_{N-1} = T_N - \Delta x \left(\frac{dT}{dx}\right)_N + \frac{(\Delta x)^2}{2!}\left(\frac{d^2T}{dx^2}\right)_N + \mathcal{O}\left[(\Delta x)^3\right], \\ T_{N+1} = T_N + \Delta x \left(\frac{dT}{dx}\right)_N + \frac{(\Delta x)^2}{2!}\left(\frac{d^2T}{dx^2}\right)_N + \mathcal{O}\left[(\Delta x)^3\right], \end{cases}$$

[1/4] *Summing these expressions leads to*

$$\frac{T_{n-1} - 2T_n + T_{n+1}}{\Delta x^2} = \left.\frac{\partial^2 T}{\partial x^2}\right|_N .$$

3. Heavy oil at 150°C is pumped into a storage tank before being transported to distillation columns. The tank is insulated with a layer of polyisocyanurate of 10 cm thick. The tank's wall is at 150°C and the initial temperature of the insulation layer is 20°C. Assuming that the tank wall-insulation layer-environment system can be modelled as a 1-D finite difference method (FDM) problem, calculate the temperature profile of the insulated layer, $T\left(\underline{x}, t\right)$, at $t = 5$ seconds with $\underline{x} = [0.0, 2.5, 5.0, 7.5, 10.0]$. The outer layer of the insulation material is subjected to the environment with temperature of 15°C and convective heat transfer coefficient of 10 W(m^2.°C)$^{-1}$. Given for polyisocyanurate insulation layer:

   - Conductive heat transfer coefficient: 5.40 W.(m.°C)$^{-1}$;
   - Heat capacity at constant pressure: 0.1 kJ.(kg.°C)$^{-1}$;
   - Density: 550 kg.m^{-3}.

   The discretised thermal energy equation is

   $$T_i^{j+1} = T_i^j + \alpha \frac{\Delta t}{(\Delta x)^2}\left(T_{i+1}^j - 2T_i^j + T_{i-1}^j\right)$$

   where $\alpha = \kappa\left(\rho C_p\right)^{-1}$ is the thermal diffusivity, Δx and Δt are the spatial-interval and time-step size, respectively. i and j are the spatial- and time-indices. For this problem, use $\Delta t = 5$ seconds. [10 marks]

Solution:

[1/10] *The thermal diffusivity,*

$$\alpha = \frac{\kappa}{\rho C_p} = \left(5.40 \frac{W}{m.^\circ C}\right)\left(\frac{1}{550}\frac{m^3}{kg}\right)\left(\frac{1}{0.1\times 10^3}\frac{kg.^\circ C}{J}\right) = \mathbf{9.8182\times 10^{-5}\frac{m^2}{s}},$$

[1/10] *and the Fourier number,*

$$\tau = \frac{\alpha\Delta t}{(\Delta x)^2} = \left(9.8182\times 10^{-5}\frac{m^2}{s}\right)(5s)\left(\frac{1}{2.5\times 10^{-2}}\frac{1}{m}\right)^2 = \mathbf{0.7855}.$$

Boundary and initial conditions are:

- *Dirichlet BC at node* $i = 0$ $(x = 0.0$*cm*$)$ *:* $T_0^0 = T_0^1 = T_0^2 = \cdots =$ **150**$^\circ$**C***;*
- *Newmann BC at node* $i = 4$ $(x = 10.0$*cm*$)$*:*

$$-\kappa \frac{\partial T}{\partial x} = h\left(T - T_\infty\right) \Longrightarrow -\kappa \frac{T_{i+1}^j - T_{i-1}^j}{2\Delta x} = h\left(T_i^j - T_\infty\right)$$
$$\mathbf{T_{i+1}^j = T_{i-1}^j - \frac{2h\Delta x}{\kappa}\left(T_i^j - T_\infty\right)},$$

- *Initial conditions:* $T_1^0 = T_2^0 = T_3^0 = T_4^0 =$ **20**$^\circ$**C***.*

[2/10] *The discretised thermal equation,*

$$T_i^{j+1} = T_i^j + \alpha \frac{\Delta t}{(\Delta x)^2}\left(T_{i+1}^j - 2T_i^j + T_{i-1}^j\right) \quad \textit{with} \quad \tau = \frac{\alpha \Delta t}{(\Delta x)^2}$$
$$\mathbf{T_i^{j+1} = (1 - 2\tau)\, T_i^j + \tau\left(T_{i+1}^j + T_{i-1}^j\right)}$$

Thus for $j = 0$*:*

$$\begin{array}{ll} \mathbf{i = 1} & \mathbf{T_1^1 = (1-2\tau)\,T_1^0 + \tau\left(T_2^0 + T_0^0\right)} \\ \vdots & \vdots \\ \mathbf{i = 4} & \mathbf{T_4^1 = (1-2\tau)\,T_4^0 + \tau\left(T_5^0 + T_3^0\right)} \end{array}$$

[2/10] *with ghost-cell,* T_5^0 *defined through the Newmann BC:*

$$\mathbf{T_5^0 = T_3^0 - \frac{2h\Delta x}{\kappa}\left(T_4^0 - T_\infty\right)}$$

[4/10] *Thus:*

t (s)	T_0	T_1	T_2	T_3	T_4
0.0	*150.00*	*20.00*	*20.00*	*20.00*	*20.00*
5.0	**150.00**	**122.12**	**20.00**	**20.00**	**20.00**

Total Question Marks:20

END OF PAPER

Sum of all question's marks:106