

Problem 1: Steam entering a turbine at 4MPa and 400°C expands reversibly and adiabatically.

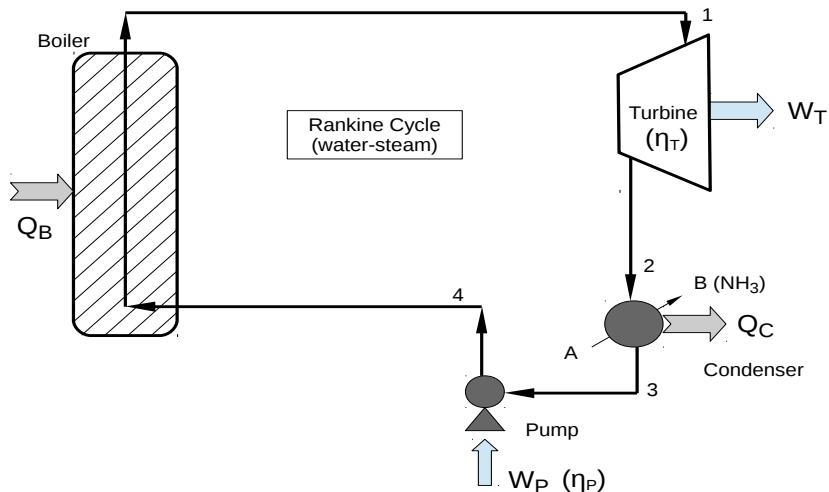
- For what discharge pressure is the exit stream a saturated vapour?
- Determine the steam quality for a discharge pressure of 250 kPa.
- Sketch both processes in a *TS* diagram.

Problem 2: Superheated steam originally at $P_1 = 1000\text{kPa}$ and $T_1 = 250^\circ\text{C}$ expands through a nozzle to an exhaust pressure $P_2 = 200 \text{ kPa}$. What is the downstream state of the steam and the change in enthalpy assuming a reversible and adiabatic process?

Problem 3: A liquid-vapour water system is in equilibrium at 80 bar. The system consists of equal volumes of liquid and vapour with total volume (V^T) of 0.15 m^3 . Calculate the total enthalpy (H^T) and (S^T).

Problem 4: For $\beta = \frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_P$ (volume expansivity) and $\kappa = -\frac{1}{V} \left(\frac{\partial V}{\partial P} \right)_T$ (isothermal compressibility), demonstrate that $\left(\frac{\partial \beta}{\partial P} \right)_T = - \left(\frac{\partial \kappa}{\partial T} \right)_P$. Hint: consider $V = V(T, P)$.

Problem 5: Steam is produced in a boiler at 60 bar and 335°C . The fluid is driven into a turbine where an isentropic expansion take place with efficiency of 75%. Before re-vaporisation in the boiler, the fluid is condensed in a heat exchanger, where the extracted energy is used to heat up a stream of NH_3 at 15°C . The mass flow rate of water and NH_3 are 20 and 220 kg.s^{-1} , respectively. Assume that the pump has 100% of efficiency.



- Calculate (a-n) in the table below.
- Calculate the power produced in the turbine (\dot{W}_T) and required in the pump (\dot{W}_P).

- (c) Calculate the heat given to the system in the boiler (\dot{Q}_B) and extracted in the condenser (\dot{Q}_C).
- (d) Calculate the efficiency of the cycle ($\eta_{\text{cycle}} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_B}$)

Stage	P (bar)	T (°C)	State	H (kJ.kg ⁻¹)	S (kJ.(kg.K) ⁻¹)	Quality of the vapour (x_i)
1	60	335	(a)	(b)	(c)	–
2	4.50	–	wet vapour	(d)	(e)	(f)
3	(g)		sat. liquid	(h)	(i)	–
4	(j)		(k)	(l)	(m)	–
A	–	15	–	–	–	–
B	–	(n)	–	–	–	–

Problem 6: Evaluate $\left(\frac{\partial S}{\partial V}\right)_T$ for water-vapour at 240°C and specific volume of 0.4646 m³.kg⁻¹. Use,

- (a) Peng-Robinson EOS;
- (b) Redlich-Kwong EOS.

Given: T_c 647.096 K, P_c = 220.6 bar and ω = 0.344.

P₁

Turbine (Steam)

$$\begin{aligned} P_1 &= 4 \text{ MPa} = 40 \text{ bar} && \left. \begin{array}{l} \text{reversibly} \\ \text{adiabatically} \end{array} \right\} \\ T_1 &= 400^\circ\text{C} && \end{aligned}$$

At P₁ $\Rightarrow T_{\text{sat}} = 250.4^\circ\text{C} \ll T_1$

↳ Fluid is superheated state:

$$S_1 = 6.7690 \text{ kJ/kg.K} = S_2 \quad (\text{reversible process})$$

(a) Exit as saturated vapour at S₂. Now
from saturated water table

P (bar)	S (kJ/kg.K)
5.00	6.8212
6.00	6.7600

Linear interpolation:

$$\frac{(6.00 - 5.00) \text{ bar}}{(6.00 - P_2) \text{ bar}} = \frac{(6.7600 - 6.8212) \text{ kJ/kg.K}}{(6.7600 - 6.7690) \text{ kJ/kg.K}}$$

$$\underline{P_2 = 5.85 \text{ bar}}$$

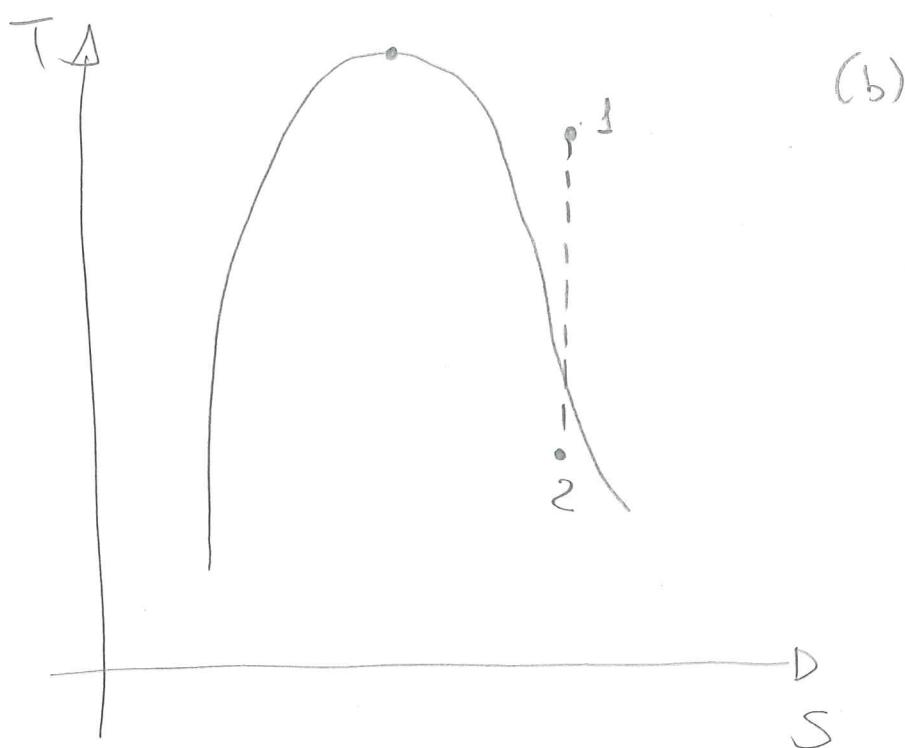
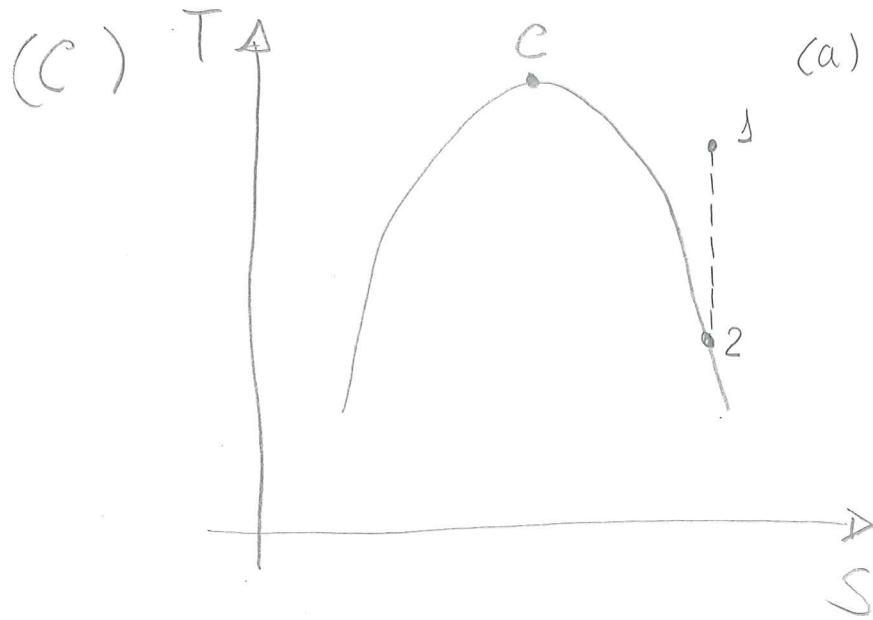
(b) For P₂ = 250 kPa = 2.50 bar (2 phase region)

$$\begin{aligned} S_g &= 1.6072 \quad \left. \begin{array}{l} \text{kJ/kg.K} \\ \text{v} \end{array} \right\} \\ S_g &= 7.0527 \quad \left. \begin{array}{l} \text{kJ/kg.K} \\ \text{l} \end{array} \right\} \end{aligned}$$

Ques

Durability is given by

$$\eta_2 = \frac{S_2 - S_f}{S_g - S_f} = \frac{6.7690 - 1.6072}{7.0527 - 1.6072} = \underline{\underline{0.9479}}$$



P2

Steam

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$$\begin{array}{l} \text{super-heated steam} \\ \left. \begin{array}{l} P_1 = 1000 \text{ kPa} = 10 \text{ bar} \\ T_1 = 250^\circ\text{C} \end{array} \right\} \Rightarrow P_2 = 200 \text{ kPa} = 2 \text{ bar} \end{array}$$

Reversible & adiabatic $\left\{ \begin{array}{l} = \text{isentropic} \\ \Delta S = 0 \end{array} \right.$

From the superheated steam table at $P_1 = 10 \text{ bar}$

$T (\text{ }^\circ\text{C})$	$H (\text{kJ/kg})$	$S (\text{kJ/kg.K})$
240	2920.4	6.8817
280	3008.2	7.0465

$$\begin{array}{l} \text{(linear)} (280 - 240)^\circ\text{C} = (3008.2 - 2920.4) \text{ kJ/kg} \\ \text{(interpolation)} (280 - 250)^\circ\text{C} = (3008.2 - H_1) \text{ kJ/kg} \\ H_1 = 2942.35 \text{ kJ/kg} \end{array}$$

Similar procedure for entropy $S_1 = 6.9229 \text{ kJ/kg.K}$

$$S_2 = S_1 \quad (\text{isentropic})$$

Entropy of saturated vapour at $P_2 = 2 \text{ bar}$ (from the saturated table, $S_g = 7.1271 \text{ kJ/kg.K}$) is larger than S_2 ,

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thus it is a 2-phase system in which the
saturation temperature (T_{sat}) is 120.4°C . We can
calculate the quality of the vapour

$$x_2 = \frac{s_2 - s_f}{s_g - s_f} = \frac{6.9229 - 1.5301}{7.1271 - 1.5301} = 0.9635$$

96.35% of mass : vapour
3.65% of mass : liquid

Now we can obtain H_2

$$x_2 = \frac{H_2 - H_f}{H_g - H_f} = \frac{H_2 - 504.70}{2706.7 - 504.70}$$

$$\underline{H_2 = 2626.33 \text{ kJ/kg}}$$

$$\underline{\Delta H = H_2 - H_1 = -316.02 \text{ kJ/kg}}$$

P3: Water-vapour at 80 bar

$$V_g = 1.3842 \times 10^{-3} \left\{ \text{m}^3/\text{kg} \right.$$

$$V_g = 0.02352$$

$$H_g = 1316.6 \left\{ \text{kJ/kg} \right.$$

$$H_g = 2758.0 \left\{ \text{kJ/kg} \right.$$

$$S_g = 3.2068 \left\{ \text{kJ/kg.K} \right.$$

$$S_g = 5.7432$$

For 0.15m^3 of water, we can obtain the mass of liquid and vapour phases (equally partitioned in both phases)

$$m_g = \frac{\sqrt{V_g}}{V_g} = \frac{0.15\text{m}^3/2}{1.3842 \times 10^{-3}\text{m}^3/\text{kg}} = 54.18\text{ kg}$$

$$m_g = \frac{\sqrt{V_g}}{V_g} = \frac{0.15\text{m}^3/2}{0.02352\text{m}^3/\text{kg}} = 3.19\text{ kg}$$

$$\left\{ H^T = m_g H_g + m_g H_g = 80131.41\text{ kJ} \right.$$

$$\left\{ S^T = m_g S_g + m_g H_g = 192.07\text{ kJ/K} \right.$$

P4

Show That

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$$\left(\frac{\partial \beta}{\partial P}\right)_T = -\left(\frac{\partial \kappa}{\partial T}\right)_P$$

$$\beta = 1/V \left(\frac{\partial V}{\partial T}\right)_P$$

$$\kappa = -V \left(\frac{\partial V}{\partial P}\right)_T$$

For $V = V(T, P)$:

$$dV = \left(\frac{\partial V}{\partial T}\right)_P dT + \left(\frac{\partial V}{\partial P}\right)_T dP$$

$$dV = \underbrace{(\beta V) dT}_M + \underbrace{(-\kappa V) dP}_N$$

This is an exact differential and assuming M & N are continuous and differentiable through all domain,
we can assume that

$$\left(\frac{\partial M}{\partial P}\right)_T = \left(\frac{\partial N}{\partial T}\right)_P$$

$$\left[\frac{\partial(\beta V)}{\partial P}\right]_T = \left[\frac{\partial(-\kappa V)}{\partial T}\right]_P$$

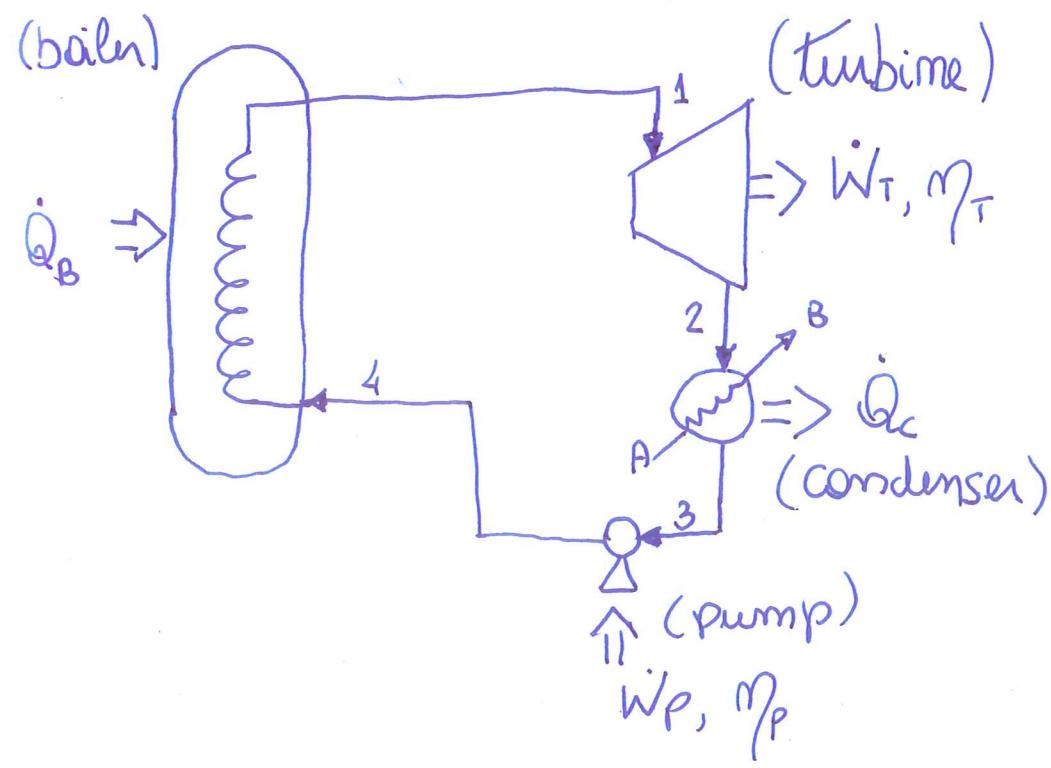
$$\beta \left(\frac{\partial V}{\partial P}\right)_T + V \left(\frac{\partial \beta}{\partial P}\right)_T = -\kappa \left(\frac{\partial V}{\partial T}\right)_P - V \left(\frac{\partial \kappa}{\partial T}\right)_P$$

Replacing β & κ

$$\cancel{\frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_P \left(\frac{\partial V}{\partial P} \right)_T} + \cancel{V \left(\frac{\partial \beta}{\partial P} \right)_T} = - \frac{1}{V} \left(\frac{\partial V}{\partial P} \right)_T \left(\frac{\partial V}{\partial T} \right)_P + \\ - V \left(\frac{\partial \kappa}{\partial T} \right)_P$$

$$\left(\frac{\partial \beta}{\partial P} \right)_T = - \left(\frac{\partial \kappa}{\partial T} \right)_P$$

Vapour is produced in a boiler at 60 bar and 335°C. The fluid is driven into a turbine where an isentropic expansion take place with efficiency of 75%. Before re-vaporisation in the boiler, the fluid is condensed in a heat exchanger, where the extracted energy is used to heat up a stream of NH₃ at 15°C.



$$\begin{cases} P_1 = 60 \text{ bar} \\ T_1 = 335^\circ\text{C} \end{cases}$$

$$\begin{cases} P_2 = 4.50 \text{ bar} \\ \eta_T = 75\% \\ \eta_P = 100\% \end{cases}$$

$$T_{NH_3}^A = 15^\circ\text{C}$$

$$\dot{m}_{NH_3} = 220 \frac{\text{kg}}{\text{s}}$$

$$C_{p,NH_3} = 2.17 \frac{\text{kJ}}{\text{kg.K}}$$

$$\dot{m}_\omega = 20 \frac{\text{kg}}{\text{s}}$$

$$(i) \left\{ H_i, S_i \quad i=1,2,3,4 \right\} \quad ?$$

$$\textcircled{2} \quad \left\{ x_2 \right\}$$

$$(ii) \dot{W}_T \quad (\checkmark) \dot{Q}_B \quad ?$$

$$(iii) \dot{Q}_C \quad T_{NH_3}^B \quad ? \quad (vi) \eta_{cycle} \quad ?$$

$$(iv) \dot{W}_P \quad ?$$

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Flowing the energy streams in the cycle:

$$\# \text{ Stage 1: } \left\{ \begin{array}{l} P_1 = 60 \text{ bar} \\ T_1 = 335^\circ\text{C} \end{array} \right\} \left\{ \begin{array}{l} T_{\text{sat}}(P_1) = 275.6^\circ\text{C} < T_1 \end{array} \right.$$

Superheated steam (SHS)

From the SHS table at $P_1 = 60 \text{ bar}$:

T(°C)	H (kJ/kg)	S (kJ/(kg.K))
320	2952.6	6.1846
360	3071.1	6.3782

For $T_s = 335^\circ\text{C}$ \Rightarrow linear interpolation

$$(360-320)^\circ\text{C} = (3071.1-2952.6)\text{KJ/Kg}$$

$$(360 - 335)^\circ \text{C} = (3071.1 - H_s) \text{ KJ/Kg}$$

$$H_1 = \frac{3071.1 \text{ kJ}}{\text{kg}} \cdot \frac{(360 - 335)^\circ\text{C} \times (3071.1 - 2952.6) \text{ kJ/kg}}{(360 - 320)^\circ\text{C}}$$

$$H_1 = 2997.04 \text{ KJ/Kg}$$

Similar procedure for entropy:

$$S_3 = 6.2572 \text{ KS/Kg.K}$$

Stage 2: Isentropic expansion in the turbine $\Rightarrow S_{2s} = S_1$

Ideal (i.e., reversible)

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At $P_2 = 4.50 \text{ bar}$:

$$S_f = 1.8207 \quad \left\{ \text{KS/Kg.K} \right.$$

$$S_g = 6.8562$$

$S_{2s} < S_g \Rightarrow$ Fluid is saturated! Now

Calculating the quality of the vapour (2 phase system):

$$\chi_i = \frac{\Psi_i - \Psi_g}{\Psi_g - \Psi_f} \quad \Psi = \{ H, S \}$$

$$\chi_{2s} = \frac{S_{2s} - S_f}{S_g - S_f} = \frac{6.2572 - 1.8207}{6.8562 - 1.8207} = 0.8810$$

We can now calculate H_{2s}

$$\chi_{2s} = \frac{H_{2s} - H_f}{H_g - H_f} = \frac{H_{2s} - 623.25}{2743.9 - 623.25} = 0.8810$$

$$H_{2s} = 2491.54 \text{ KS/Kg}$$

For an efficiency (turbine) of 75%

$$\eta_T = \frac{H_2 - H_s}{H_{2s} - H_s} = 0.75 = \frac{H_2 - 2997.04}{2491.54 - 2997.04}$$

$$H_2 = 2617.92 \text{ kJ/kg}$$

With actual enthalpy H_2 , we can calculate the actual quality of the vapour, x_2

$$x_2 = \frac{H_2 - H_f}{H_g - H_f} = \frac{2617.92 - 623.25}{2743.9 - 623.25}$$

$$x_2 = 0.9406$$

And the actual enthalpy

$$x_2 = \frac{s_2 - s_f}{s_g - s_f} \therefore s_2 = 6.5573 \text{ kJ/kg.K}$$

Stage 3 : Water left the condenser at liquid state. There is no pressure drop in Condenser:

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$$\underline{P_3 = P_2 = 4.50 \text{ bar}}$$

$$\left\{ \begin{array}{l} H_3 = H_g(P_3) = 623.25 \text{ KS/Kg} \\ S_3 = S_g(P_3) = 1.8207 \text{ KS/Kg.K} \\ V_3 = V_g(P_3) = 1.0882 \times 10^{-3} \text{ m}^3/\text{Kg} \end{array} \right.$$

Stage 4: Assuming the liquid fluid is incompressible. $P_4 = P_1$

$$dH \approx \sqrt{dP}$$

$$H_4 - H_3 = V_3 (P_4 - P_3) \quad 6.03951$$

$$H_4 = 623.25 \frac{\text{KS}}{\text{Kg}} + 1.0882 \times 10^{-3} \frac{\text{m}^3}{\text{Kg}} (60 - 4.50) \text{ bar} \times \left[\frac{10^5 \text{ Pa}}{1 \text{ bar}} \times \frac{1 \text{ N/m}^2}{1 \text{ Pa}} \times \frac{1 \text{ J}}{1 \text{ Nm}} \times \frac{1 \text{ KS}}{1000 \text{ J}} \right]$$

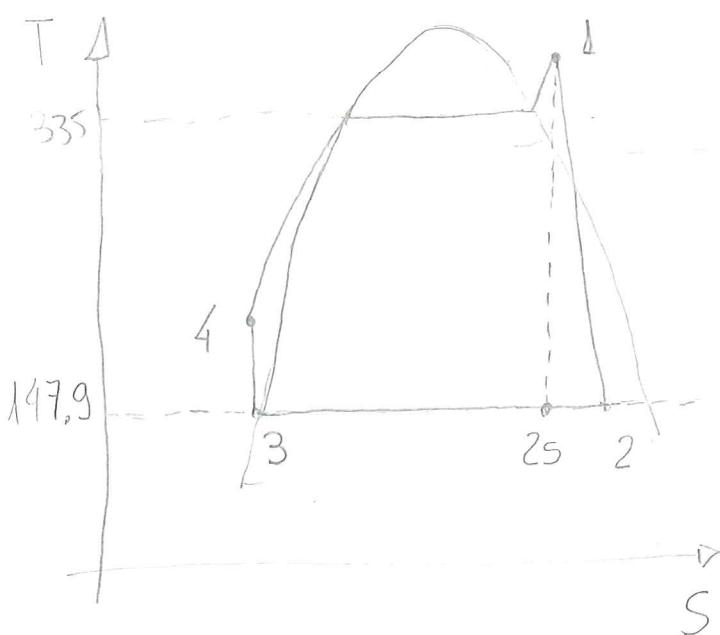
$$H_4 = 629.29 \text{ KS/Kg} > (H_3 = H_g) \therefore \underline{\text{Subcooled Fluid}}$$

As the pump undertakes an isentropic compression with 100% efficiency:

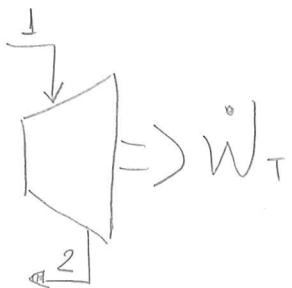
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$$\underline{S_4 = S_3 = 1.8207 \text{ kJ/kg.K}}$$

Stage	State	H (kJ/kg)	S (kJ/kg.K)	Quality
1	Superheated steam	2997.04	6.2572	—
2	Saturated vapour (wet vapour)	2637.92	6.5571	0.9406
3	Saturated liquid	623.25	1.8207	—
4	Subcooled fluid	629.29	1.8207	—



TS Diagram

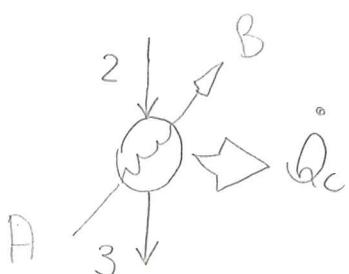
(ii) Calculating \dot{W}_T :

$$\dot{W}_T = \dot{m}_w (H_2 - H_1)$$

$$\dot{W}_T = \frac{20 \text{ kg}}{\text{s}} (2617.92 - 2997.04) \frac{\text{kJ}}{\text{kg}}$$

$$\dot{W}_T = -7582.4 \frac{\text{kJ}}{\text{s}} \Rightarrow 7.58 \text{ MW}$$

↳ produced by the turbine and transferred to the system

(iii) Calculating \dot{Q}_c 

$$\dot{Q}_c = \dot{m}_w (H_3 - H_2)$$

$$\dot{Q}_c = \frac{20 \text{ kg}}{\text{s}} (623.25 - 2617.92) \frac{\text{kJ}}{\text{kg}}$$

$$\dot{Q}_c = -39893.40 \frac{\text{kJ}}{\text{s}} \Rightarrow 39.9 \text{ MW}$$

↳ heat extracted and transferred to the outer system water.

$$-\dot{Q}_c = \dot{m}_{NH_3} C_{P,NH_3} (T_{NH_3}^B - T_{NH_3}^A)$$

$$39893.40 \frac{\text{kJ}}{\text{s}} = 220 \frac{\text{kg}}{\text{s}} \times 2.17 \frac{\text{kJ}}{\text{kgK}} (T_{NH_3}^B - 15)$$

$$T_{NH_3}^B = 98.56^\circ\text{C}$$

(iv) Power supplied to the pump

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$$\dot{W}_p = \dot{m}_w (H_4 - H_3) = 20 \frac{\text{kg}}{\text{s}} (629.29 - 623.25) \frac{\text{kJ}}{\text{kg}}$$

$$\dot{W}_p = 120.8 \frac{\text{kJ}}{\text{s}} = 0.12 \text{MW}$$

(v) Heat supplied to the boiler

$$\dot{Q}_B = \dot{m}_w (H_1 - H_4) = 20 \frac{\text{kg}}{\text{s}} (2997.04 - 629.29) \frac{\text{kJ}}{\text{kg}}$$

$$\dot{Q}_B = 47355 \frac{\text{kJ}}{\text{s}} \Rightarrow 47.4 \text{MW}$$

(vi) Thermal Efficiency of the cycle (η_{cycle})

$$\eta_{\text{cycle}} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_B} = \frac{|\dot{W}_T + \dot{W}_p|}{\dot{Q}_B} = \frac{|-7582.4 + 120.8|}{47355} = 0.1576$$

$$\boxed{\eta_{\text{cycle}} = 15.76\%}$$

Evaluate $(\frac{\partial S}{\partial V})_T$ for water-vapour at $240^\circ C$ and specific volume of $0.4646 \text{ m}^3/\text{kg}$. Use

(a) Peng-Robinson EOS

(b) Redlich-Kwong EOS

Given: $T_c = 647.096 \text{ K}$; $P_c = 220.6 \text{ bar}$; $\omega = 0.344$

From the Maxwell's relations

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V \quad (1)$$

We can apply the rhs of this relation in the EOS:

(a) PR EOS

$$P = \frac{RT}{V-b} - \frac{a\alpha}{\sqrt{(V+b)+b(V-b)}} \quad (2)$$

$$a = 0.45724 (RT_c)^2 / P_c$$

$$b = 0.07780 (RT_c) / P_c$$

$$\alpha = [1 + \gamma(1 - T_n)]^{1/2} = [1 + \gamma - \gamma T/T_c]^{1/2} \quad (3)$$

$$\gamma = 0.37464 + 1.5422\omega - 0.26992\omega^2$$

Deriving the first term in rhs of (2) wrt T

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$$\left[\frac{\partial}{\partial T} \left(\frac{RT}{V-b} \right) \right]_V = \frac{R}{V-b}$$

Now, the 2nd term in the rhs of (2), where

$$\alpha = \alpha(T),$$

$$\left\{ \frac{\partial}{\partial T} \left[\frac{a(1+\gamma-\delta T/T_c)^{1/2}}{V(V+b)+b(V-b)} \right] \right\}_V =$$

$$\frac{1/2 a (1+\gamma-\delta T/T_c)^{-1/2} (-\delta/T_c)}{[V(V+b)+b(V-b)]^2} [V(V+b)+b(V-b)]$$

$$\left(\frac{\partial P}{\partial T} \right)_V = \frac{R}{V-b} - \frac{-1/2 a \delta}{T_c (1+\gamma-\delta T/T_c)^{1/2} [V(V+b)+b(V-b)]}$$

$$\gamma = 0.8732$$

$$T_n = T/T_c = (240+273.15)/647.096 = 0.7930$$

$$\alpha = [1 + \gamma (1 - T_n)]^{1/2} = 1.0866$$

$$b = 0.07780 \frac{RT_c}{P_c} = 0.07780 \times 8134 \frac{J}{JmolK} \times \frac{647.096 K}{220 bar} \times \left[\frac{1 bar}{10^5 N/m^2} \right]$$

$$\left[\frac{JNm}{J} \right] = 1.8614 \times 10^{-5} \frac{m^3}{gmol}$$

$$a = 0.45724 \frac{(RT_c)^2}{P_c}$$

$$a = 0.45724 \left[8.134 \frac{J}{g\text{mol}\text{K}} \times 647.096 \text{K} \times \frac{1 \text{ JNm}}{1 \text{ J}} \right]^2 \frac{1}{220 \text{ bar}} \times \frac{1 \text{ bar}}{10^5 \text{ N/m}^2}$$

$$a = 0.57733 \frac{\text{Nm}^4}{\text{gmol}^2}$$

$$\sqrt{V} = 0.4646 \frac{\text{m}^3}{\text{kg}} \times \frac{18 \text{ KJ}}{\text{Kmol}} \times \frac{1 \text{ KJmol}}{1000 \text{ J/K}}$$

$$\sqrt{V} = 0.0083628 \frac{\text{m}^3}{\text{gmol}}$$

$$\frac{R}{V-b} = 974.8194 \frac{\text{N}}{\text{m}^2 \text{K}}$$

$$\frac{-1/2 a \gamma}{T_c (1 + \gamma - \gamma T/T_c)^{1/2} [V(\sqrt{V}+b) + b(\sqrt{V}-b)]} = -5.1031 \frac{\text{N}}{\text{m}^2 \text{K}}$$

$$\left(\frac{\partial P}{\partial T} \right)_V = 979.91 \frac{\text{N}}{\text{m}^2 \text{K}} = 0.979 \frac{\text{KJ}}{\text{m}^3 \text{K}} = \left(\frac{\partial S}{\partial V} \right)_T$$

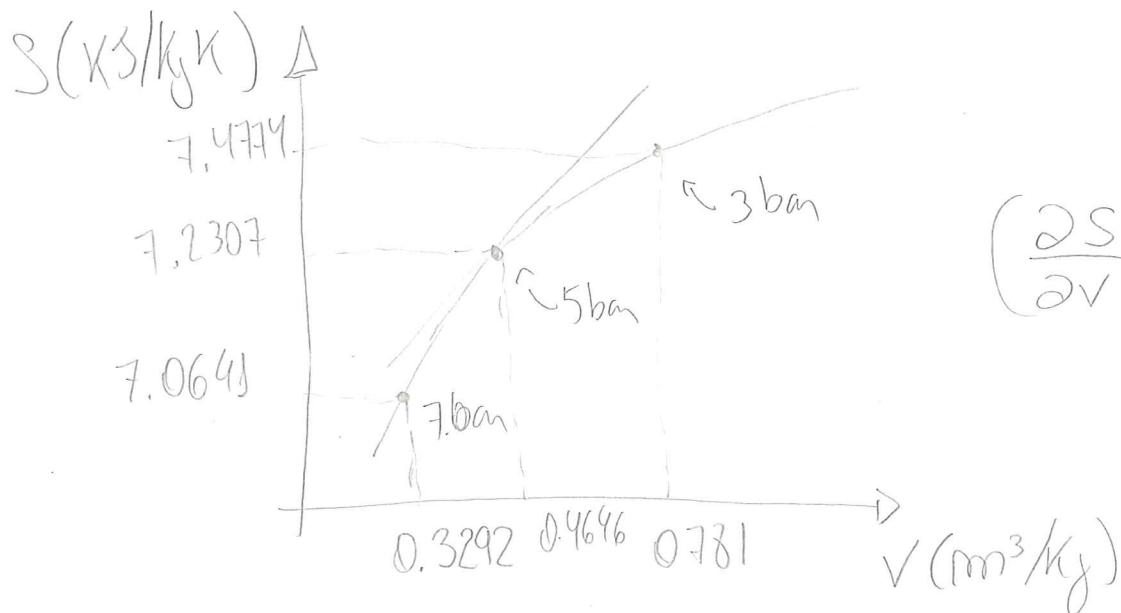
(b) With a similar procedure for RK EOS

$$\left(\frac{\partial P}{\partial T} \right)_V = 1.0043 \frac{\text{KJ}}{\text{m}^3 \text{K}} = \left(\frac{\partial S}{\partial V} \right)_T$$

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At 240°C, the superheated steam tables give:

P (bar)	S (kS/kgK)	V (m³/kg)
1.0	7.9949	2.359
1.5	7.8052	1.570
3.0	7.4774	0.781
5.0	7.2307	0.4646 \Leftarrow
7.0	7.0641	0.3292
10.0	6.8817	0.2275



$$\left(\frac{\partial S}{\partial V}\right)_T \approx 1 \text{ kS/m}^3\text{K}$$