February 18, 2014

Example 1 *In a turbine, steam at 20 bar, 360°C is expanded to 0.08 bar. The steam is condensed into saturated liquid water. The pump feeds the water back into the boiler. Assume ideal processes, calculate the net work per kg of steam and the cycle efficiency.*

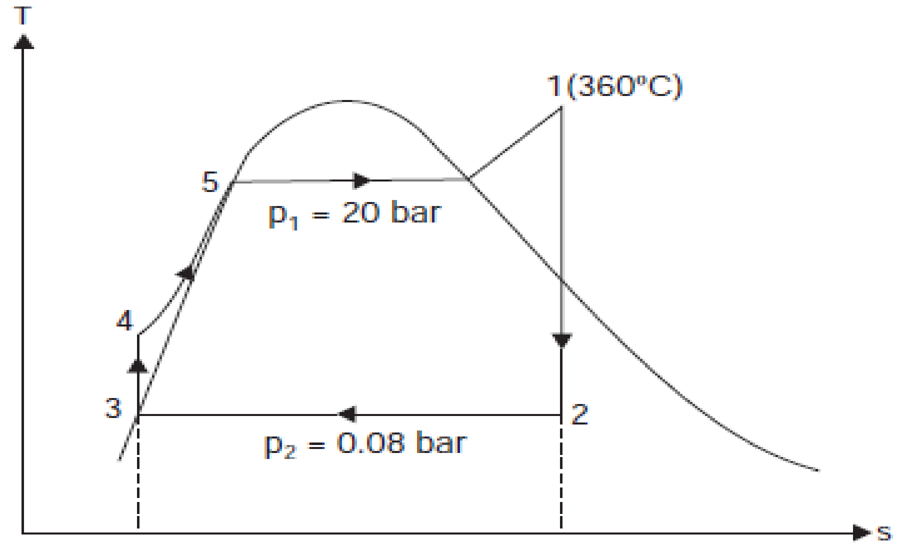


Figure 1: Carnot and Rankine Cycles: Ts diagram – **Example 1**.

As the process *1-2* is isentropic (Fig. 1), $s_1 = s_2$ and using the data in Table 1,

$$6.9917 = s_{f,2} + x_2 s_{fg,2} = 0.5926 + 7.6361 x_2 \implies x_2 = 0.838$$

$$h_2 = h_{f,2} + x_2 h_{fg,2} \implies h_2 = 2187.68\ kJ/kg$$

	P (bar)	T (K)	h_f (kJ/kg)	h_{fg} (kJ/kg)	s_f (kJ/(kg.K))	s_{fg} (kJ/(kg.K))	s_g (kJ/(kg.K))	v_f (m^3/kg)
Boiler	20	633.15	3159.3	–	6.9917	–	–	–
Condenser	0.08	–	173.88	2403.1	0.5926	7.6361	8.2287	1.008×10^{-3}

Table 1: Carnot and Rankine Cycles: Steam tables – **Example 1**.

The net work (W_{net}) is calculated from, $W_{\text{net}} = W_{\text{turbine}} - W_{\text{pump}}$. The work required by the pump is

$$W_{\text{pump}} = h_{f,4} - h_{f,3} = v_{f,2}\left(P_1 - P_2\right) = 1.008 \times 10^{-3}\left[\frac{m^3}{kg}\right] \times (20 - 0.08)[bar] \times \left[\frac{100\ kJ/kg}{m^3.bar/kg}\right] = 2.008\ kJ/kg$$
$$h_{f,4} = 2.008 + h_{f,2} = 175.89\ kJ/kg$$

And the work produced by the turbine is defined as

$$W_{\text{turbine}} = h_1 - h_2 = 3159.3 - 2187.68 = 971.62\ kJ/kg$$

And the net work is

$$W_{\text{net}} = W_{\text{turbine}} - W_{\text{pump}} = 971.62 - 2.008$$
$$W_{\text{net}} = 969.61 \frac{kJ}{kg}$$

The efficiency of the cycle is obtained by the relationship $\eta_{\text{cycle}} = \frac{W_{\text{net}}}{Q_1}$. The heat added into the boiler (Q_1) is given by,

$$Q_1 = h_1 - h_{f,4} = 3159.3 - 175.89 = 2983.41\ kJ/kg$$

and the efficiency is

$$\eta_{\text{cycle}} = \frac{969.61}{2983.41} = 0.325 \text{ or } 32.5\%$$

Example 2 *A Rankine cycle operates between pressures of 80 and 0.1 bar and the maximum temperature is 873.15 K. Assuming that the steam turbine and condensate pump efficiencies are 90% and 80%, respectively, calculate the specific work and thermal efficiency.*

From the saturated water and steam tables:

P (bar)	T (K)	Specific Volume (m^3/kg)		Specific Enthalpy (kJ/kg)			Specific Entropy (kJ/(kg.K)		
		v_f	v_g	h_f	h_{fg}	h_g	s_f	s_{fg}	s_g
0.1	318.99	1.0103×10^{-3}	0.1468×10^{2}	191.9	2392.3	2584.2	0.6488	7.5006	8.1494
80	568.25	1.385×10^{-3}	0.0235	1317.0	1440.5	2757.5	3.2073	2.5351	5.7424

Table 2: Carnot and Rankine Cycles: From Saturated water and steam tables.

Since $s_1 = s_2$, the steam quality is calculated from,

$$s_2 = s_{f,2} + x_2 s_{fg,2}$$
$$7.0206 = 0.6488 + 7.5006 x_2$$
$$x_2 = 0.85 \tag{1}$$

Superheated Steam		
	v	0.486 m^3/kg
$P = 80\ bar\ ,\ T = 600^oC$	h	3642 kJ/kg
	s	7.0206 kJ/(kg.K)

Table 3: Carnot and Rankine Cycles: From Superheated Steam.

and the enthalpy is obtained from

$$h_2 = h_{f,2} + x_2 h_{fg,2} = 191.9 + 0.85 \times 2392.3 = 2225.36\ kJ/kg$$

And the work

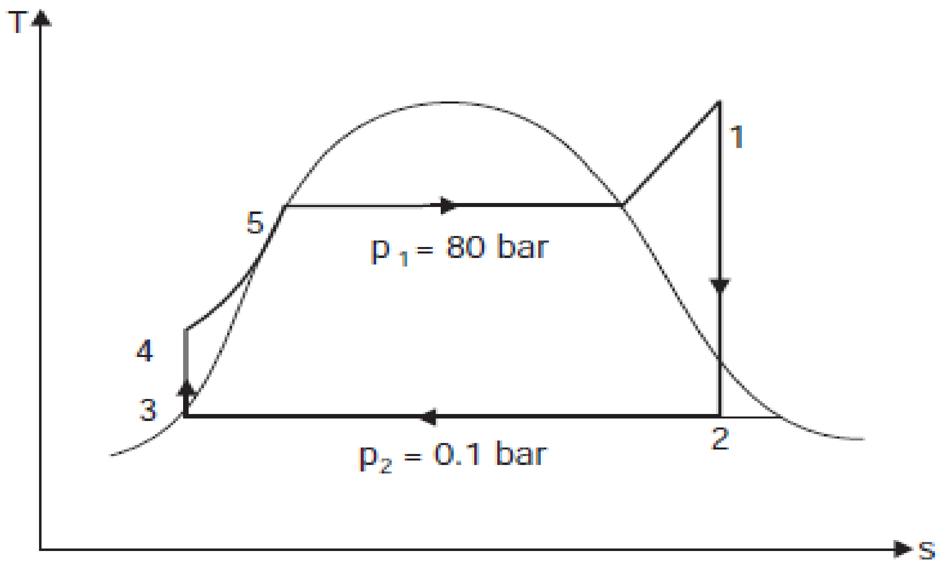


Figure 2: Carnot and Rankine Cycles: Ts diagram – **Example 2**.

Example 3 *In a steam power plant operating on the Rankine cycle, steam enters the turbine at 30 bar and 623.15 K and later condensed at 0.1 bar. Calculate: (a) thermal efficiency; (b) thermal efficiency assuming that the steam is superheated to 873.15 K before entering the turbine; (c)thermal efficiency assuming that the boiler pressure is raised to 150 bar while the inlet temperature at turbine is maintained at 623.15 K.*

(a) The *Ts* diagrams of the cycle for all three cases can be seen in Fig. 3, and the original phase states can be assigned as

$P_1 = 0.1\ bar$ (saturated liquid)	$P_2 = 30\ bar$	$P_3 = 30\ bar$	$P_4 = 0.1\ bar$
$h_1 = 191.81\ kJ/kg$	$s_2 = s_1$	$T_3 = 623.15\ K$	$s_4 = s_3$
$v_1 = 1.01 \times 10^{-3}\ m^3/kg$		$h_3 = 3116.1\ kJ/kg$	
		$s_3 = 6.7450\ kJ/(kg.K)$	

Enthalpy of the fluid leaving the pump can be calculated as

$$h_2 = h_1 + W_{\text{in}}$$

where the pump work can be computed as

$$W_{\text{in}} = v_1 (P_2 - P_1) = 3.02\ kJ/kg$$

therefore the fluid's enthalpy is $h_2 = 194.83\ kJ/kg$. The fluid quality at the turbine output is (assuming isentropic process),

$$x_4 = \frac{s_4 - s_f}{s_{fg}} = \frac{6.7450 - 0.6492}{7.4996} = 0.8128$$

Thus,

$$\begin{aligned} h_4 &= h_f + x_4 h_{fg} = 191.81 + 0.8128 \times 2392.1 = 2136.1\ kJ/kg \\ q_{\text{in}} &= h_3 - h_2 = 3116.1 - 194.83 = 2921.3\ kJ/kg \\ q_{\text{out}} &= h_4 - h_1 = 2136.1 - 191.81 = 1944.3\ kJ/kg \end{aligned} \tag{2}$$

and the cycle efficiency is given by,

$$\eta_{\text{(a)}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{1944.3}{2921.3} = 0.334 \text{ or } 33.4\%$$

(b) For this case, phase-states (1) and (2) remain the same whereas the enthapies of (2) and (4) need to be updated

$P_1 = 0.1\ bar$ (saturated liquid)	$P_2 = 30\ bar$	$P_3 = 30\ bar$	$P_4 = 0.1\ bar$
$h_1 = 191.81\ kJ/kg$	$s_2 = s_1$	$T_3 = 873.15\ K$	$s_4 = s_3$
$v_1 = 1.01 \times 10^{-3}\ m^3/kg$		$h_3 = 3682.8\ kJ/kg$	
		$s_3 = 7.509\ kJ/(kg.K)$	

and using the same procedure as **Example 3a**, $x_4 = 0.915$ and $h_4 = 2380.3\ kJ/kg$. Added and removed heats are

$$\begin{aligned} q_{\text{in}} &= h_3 - h_2 = 3682.8 - 194.83 = 3488.0\ kJ/kg \\ q_{\text{out}} &= h_4 - h_1 = 2380.3 - 191.81 = 2188.5\ kJ/kg \end{aligned} \tag{3}$$

and the cycle efficiency is given by,

$$\eta_{\text{(b)}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{2188.5}{3488.0} = 0.373 \text{ or } 37.3\%$$

	Case (a)	**Case (b)**	**Case (c)**
	$P_3 = 30\ bar$	$P_3 = 30\ bar$	$P_3 = 150\ bar$
	$T_3 = 623.15\ K$	$T_3 = 873.15\ K$	$T_3 = 623.15\ K$
	$P_1 = 0.1\ bar$	$P_1 = 0.1\ bar$	$P_1 = 0.1\ bar$
	$x_4 = 0.8128$	$x_4 = 0.915$	$x_4 = 0.804$
$\eta\,(\%)$	33.4	37.3	43.0

Table 4: Carnot and Rankine Cycles: Improving the efficiency of Rankine cycles, **Example 3**.

The thermal efficiency increases from 33.4% to 37.3% (Table 4) as superheated steam temperature is raised from 623.15 K to 873.15 K with a smaller amount of moisture – from 18.7% to 8.5 % (i.e., steam quality increases from 81.3% to 91.5%).

(c) For this case, phase-state (1) remain the same but all other phase-states changes. Enthalpies are determined in a similar way as shown in **Example 3**a – $h_2 = 206.95\ kJ/kg$, $h_3 = 3583.1\ kJ/kg$ and $h_4 = 2115.3\ kJ/kg$ with steam quality as $x_4 = 0.804$. Added and removed heats then become

$$\begin{aligned} q_{\text{in}} &= h_3 - h_2 = 3583.1 - 206.95 = 3376.2\ kJ/kg \\ q_{\text{out}} &= h_4 - h_1 = 2115.3 - 191.81 = 1923.5\ kJ/kg \end{aligned} \tag{4}$$

and the cycle efficiency is given by,

$$\eta_{(c)} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{1923.5}{3376.2} = 0.430 \text{ or } 43.0\%$$

The thermal efficiency increases from 37.3% to 43.0 % as the boiler pressure is raised from 30 to 150 bar with constant turbine inlet temperature, 623.15 K (Table 4).

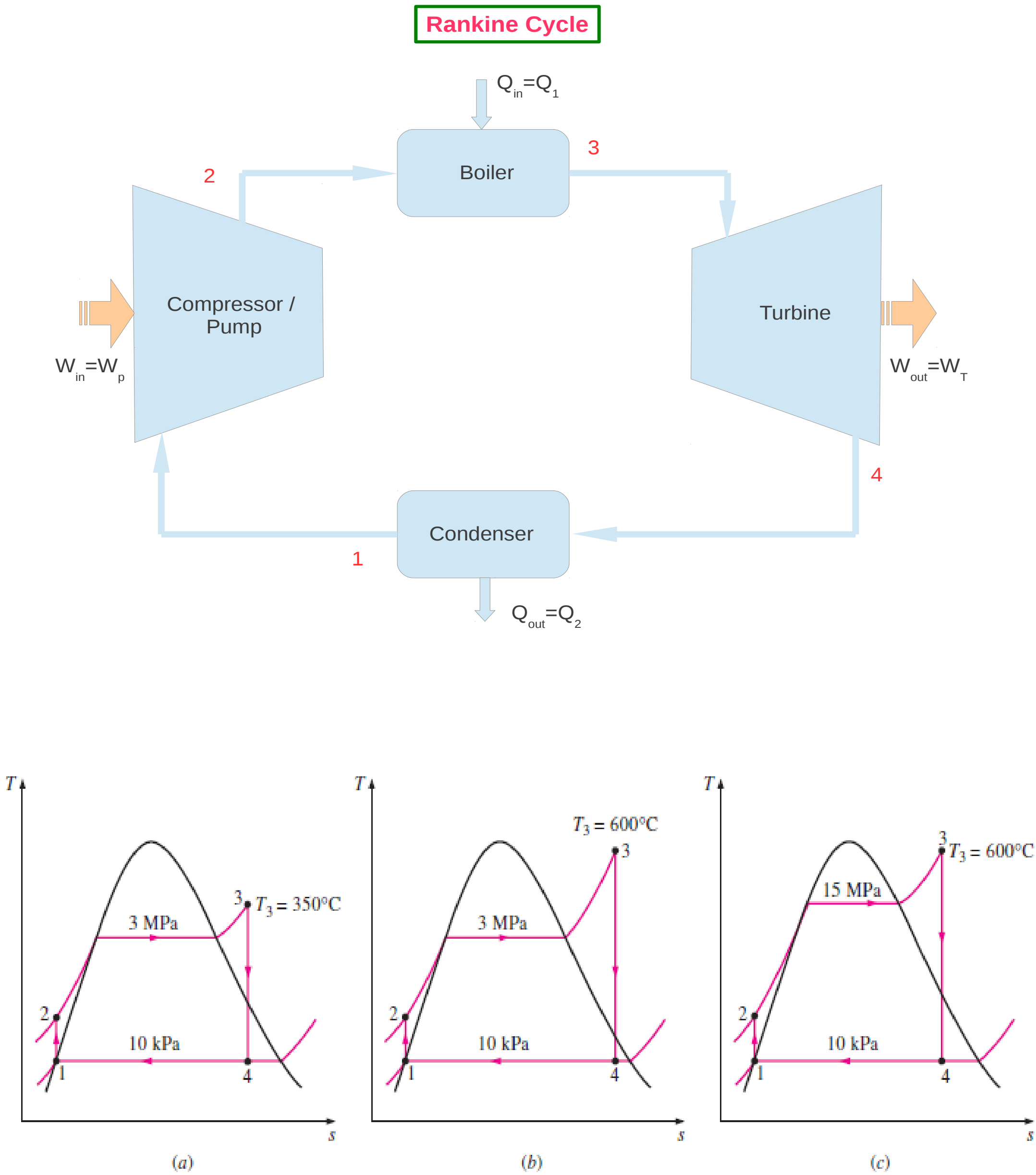


Figure 3: Carnot and Rankine Cycles: Schematic of a Rankine cycle and Ts diagrams – **Example 3**.

Problem 1 *A Carnot engine with water/steam (1 kg/s) as the working fluid operates on the cycle shown in Fig. 4. For $T_1 = 475\ K$ and $T_2 = 300\ K$, determine: (a) pressures at states 1, 2, 3, and 4; (b) quality x^{vapour} at states 2 and 3; (c) rate of heat addition; (d) rate of heat rejection; (e) mechanical power for each of the four steps; (f) thermal efficiency (η) of the cycle.*

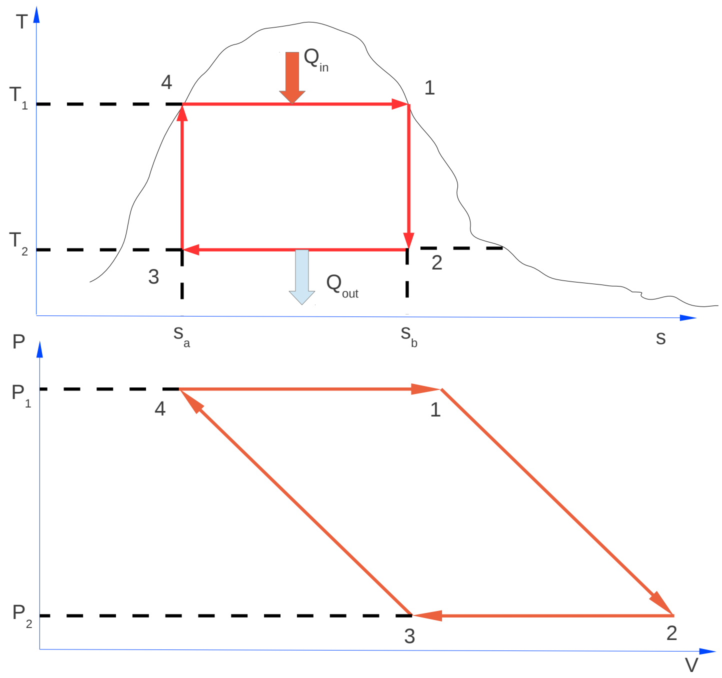


Figure 4: Ts and Pv diagrams for Carnot cycle (**Problem 1**).

Problem 2 *Water is the working fluid in an ideal Rankine cycle. Saturated vapour enters the turbine at 16 MPa, and the condenser pressure is 8 kPa. The mass flow rate of steam entering in the turbine is 120 kg/s. Calculate:*

(a) the net power developed (in MW);

(b) rate of heat transfer to the steam passing through the boiler (in MW);

(c) thermal efficiency;

(d) mass flow rate of the condenser cooling water (in kg/s(, if the cooling water undergoes a temperature increase of 18^{o}C with negligible pressure change in passing through the condenser.

Problem 3 *A power plant based on the Rankine cycle is under development to provide a net power output of 10MW. Solar collectors are to be used to generate Refrigerant 22 vapour at 1.6MPa and 50^{o}C, for expansion through the turbine. Cooling water is available ar 20^{o}C. Specify he preliminary design of the cycle and estimate the thermal efficiency and the refrigerant and cooling water flow rates (in kg/s).*

Problem 4 *Steam enters the turbine of a simple vapour power plant with a pressure of 10 MPa and temperature T, and expands adiabatically to 6 KPa. The isentropuc turbine efficiency is 85%. Saturated liquid exits the condenser at 6KPa and the isentropic pump efficiency is 82%.*

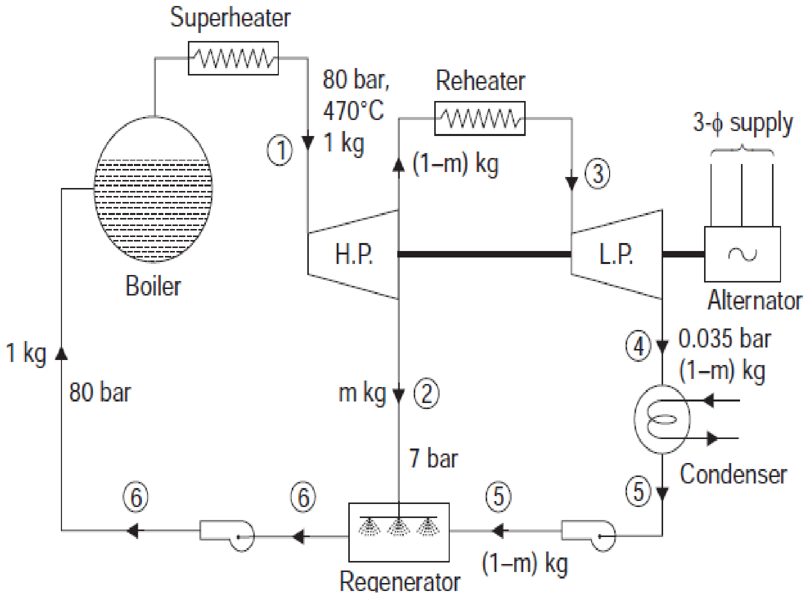
(a) For $T = 580^oC$, determine the turbine exit quality and the cycle thermal efficiency;

(b) Compute the same of (a) for $T = 700^oC$.

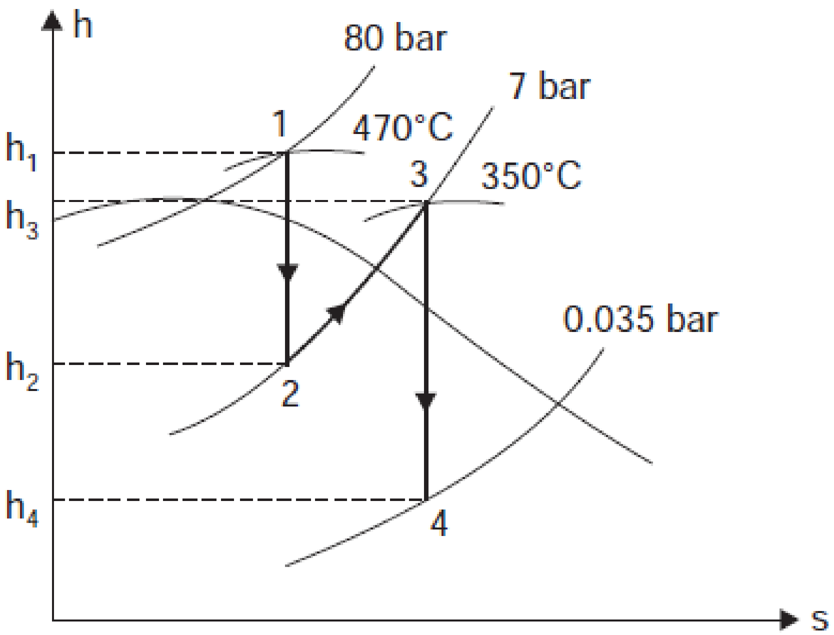
Problem 5 *A steam power plant operates with with regenerative and reheat arrangement cycles. Steam is supplied to the H.P. turbine (Fig. 5a) at 80 bar and 470°C. For feed heating, a part of steam is extracted at 7 bar and remainder of the steam is reheated to 350°C in a reheater and then expanded in L.P. turbine down to 0.035 bar. Determine: (a) amount of steam bled-off for feed heating; (b) amount of steam supplied to L.P. turbine; (c) heat supplied to the boiler and reheater; (d) cycle efficiency, and (e) power developed by the system. The steam supplied by the boiler is 50 kg/s.*

Problem 6 *Steam at 32 MPa and 520°C enters the first stage of a supercritical reheat cycle including three turbine stages. Steam exiting the first-stage turbine at pressure P is reheated at constant pressure to 440°C, and steam exiting the second-stage turbine at 0.5 MPa is reheated at constant pressure to 360°C. Each turbine stage and the pump has an isentropic effciency of 85%. The condenser pressure is at 8 kPa. For $P = 4\ MPa$, determine the net work per unit mass of steam flowing (kJ/kg) and the thermal efficiency.*

Problem 7 *Steam power plant operating on a regenerative cycle, includes just one feedwater heater. Steam enters the turbine at 4500 kPa and 773.15 K and exhausts at 20 kPa. Steam for the feedwater heater is extracted from the turbine at 350 kPa, and in condensing raises the temperature of the feedwater to within 6 K of its condensation temperature at 350 kPa. If the turbine and pump efficiencies are both 0.78, what is the thermal efficiency of the cycle and what fraction of the steam entering the turbine is extracted for the feedwater heater?*



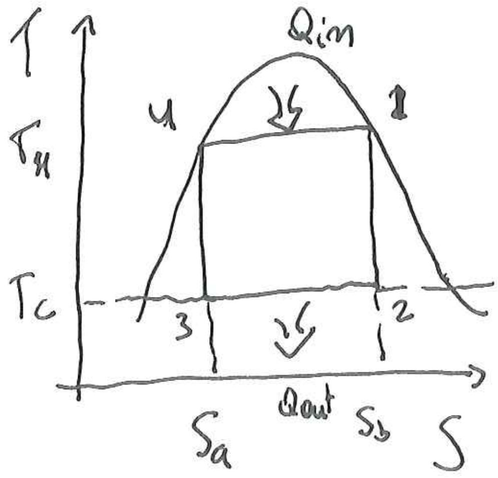
(a)



(b)

Figure 5: (a) Schematics and (b) hs diagram of a steam power cycle (**Problem 5**).

Problem 01



Carnot cycle : water / steam

given
$$\begin{cases} \dot{m} = 1 \text{ kg/s} \\ T_H = 475 \text{ K} \\ T_c = 300 \text{ K} \end{cases}$$

$$\begin{cases} P_i = ? & i = 1, \dots, 4 \\ x_2^v, x_3^v = ? \\ \dot{Q}_{14}, \dot{Q}_{23} = ? \\ \dot{W}_{i,j} = ? & i = 1, \dots, 4 \\ & j = 1, \dots, 4 \\ \eta = ? \end{cases}$$

In order to calculate all required fields, we first need to know $\underline{H}$, $\underline{S}$, and $\underline{P}$ for the given temperature assuming that the fluid can be either saturated liquid (SL), saturated vapour (SV) or wet vapor (WV). Therefore from the saturated steam tables (through linear interpolation):

State	Condition	T (K/°C)	P (bar)	H (kJ/kg)	S (kJ/kg.K)
01	SV	475/201.85			
02	WV	300/26.85			
03	WV	300/26.85			
04	SL	475/201.85			

For State 01 (from saturated steam table):

P (bar)	T (°C)	H_f	H_g	S_f	S_g
16.0	201.4	858.6	2791.7	2.3436	6.4175
16.5	202.8	865.3	2792.6	2.3576	6.4065
		(KJ/Kg)		(KJ/Kg.K)	

At 201.85 °C, we can calculate the saturated pressure:

$\Delta P = 16.5 - 16.0$ ——— $\Delta T = 202.8 - 201.4$

ΔP^* ——— $\Delta T^* = 202.8 - 201.85$

$$\Delta P^* = 16.5 - P^* = \frac{(16.5-16)(202.8-201.85)}{202.8-201.4}$$

$\Delta P^* = 16.5 - P^* = 0.3393$ ∴ ~~P(T=201.85°C)~~

$$\boxed{P(T = 201.85\,°C) = 16.16 \text{ bar}}$$

Remember that at State 01, the fluid is SV, thus

$\Delta H_g = 2792.6 - 2791.7$ ——— $\Delta T = 202.8 - 201.4$

$\Delta H_g^* = 2792.6 - H_g^*$ ——— $\Delta T^* = 202.8 - 201.85$

$$\boxed{H_g^* = H_g(T = 201.85\,°C) = 2791.99 \text{ KJ/Kg}}$$

And similarly for entropy (still State 01):

$$\Delta S_g^* = 6.4065 - 6.4175 \quad \text{———} \quad \Delta T = 202.8 - 201.4$$

$$\Delta S_g^* = 6.4065 - S_g^* \quad \text{———} \quad \Delta T^* = 202.8 - 201.85$$

$$6.4065 - S_g^* = \frac{(6.4065 - 6.4175)(202.8 - 201.85)}{(202.8 - 201.4)}$$

$$\boxed{S_g^* = S_g(T = 201.85°C) = 6.4140 \ kJ/kg.K}$$

We can use the same methodology as before to calculate $\underline{P}$, $\underline{H}$ and $\underline{S}$ for State 04. However, as we know that the fluid at this state is SL, our calculations are based on H_f and S_f.

State	Condition	T (°C)	P (bar)	H (kJ/kg)	S (kJ/kg K)
01	SV	201.85	16.16	2791.99	6.4140
02	WV	26.85			
03	WV	26.85			
04	SL	201.85	16.16	860.85	2.3481

Fluid at State 02 and 03 are wet vapour, i.e., 2 phase fluid at pressure P_2 (see Fig. 4). From the saturated steam table at 26.85 °C :

P(bar)	T (°C)	H_f	H_g	S_f	S_g
0.035	26.70	111.9	2550.3	0391	8.523
0.040	29.00	121.5	2554.4	0423	8.475
0.0353	26.85	112.53	2550.57	0.393	8.520

From linear interpolation, we can calculate the pressure — $\boxed{P_2 = 0.0353 \text{ bar}}$

We know that processes/stages 3-4 and 1-2 are isentropic, $S_3 = S_4$ and $S_1 = S_2$. We can then calculate the quality of the fluids at states 02 and 03 :

From Example 01 (Module 2.1), the quality of the fluid can be defined as

$$x^* = \frac{\Psi^* - \Psi_f}{\Psi_g - \Psi_f}$$

where Ψ stands for any thermodynamic property, eg., H or S or...

Thus at state 02:

$S_2 = S_1$

$$x_2 = \frac{S_2 - S_f}{S_g - S_f} = \frac{6.4140 - 0.393}{8.520 - 0.393}$$

$$\boxed{x_2 = 0.741}$$

And for state 03:

$S_3 = S_4$

$$x_3 = \frac{S_3 - S_f}{S_g - S_f} = \frac{2.3481 - 0.393}{8.520 - 0.393}$$

$$\boxed{x_3 = 0.241}$$

Now that we know the quality, we are able to calculate the enthalpies H_2 and H_3

$$x_2 = 0741 = \frac{H_2 - H_f}{H_g - H_f} = \frac{H_2 - 112.53}{2550.57 - 112.53}$$

$$\boxed{H_2 = 1919.12 \text{ KJ/Kg}}$$

$$x_3 = 0241 = \frac{H_3 - H_f}{H_g - H_f} = \frac{H_3 - 112.53}{2550.57 - 112.53}$$

$$\boxed{H_3 = 700.10 \text{ KJ/Kg}}$$

Now calculating the heat $\underbrace{\text{addition}}_{4-1}$ and $\underbrace{\text{removal}}_{2-3}$

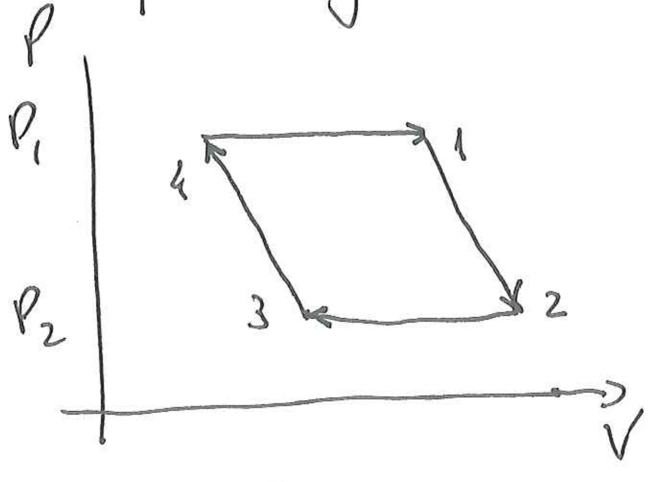
$$\dot{Q}_{41} = \dot{m}(H_1 - H_4) = 1 \frac{Kg}{s} (2791.99 - 860.85) \frac{KJ}{Kg}$$

$$\boxed{\dot{Q}_{41} = 1931.14 \text{ KJ/s}}$$

$$\boxed{\dot{Q}_{23} = -1219.02 \text{ KJ/s}}$$

$$\dot{Q}_{23} = \dot{m}(H_3 - H_2) = 1 \frac{Kg}{s} (700.10 - 1919.12) \text{KJ/Kg}$$

The mechanical power for each stage can be analysed by the PV diagram:



As there is no variation in pressure in both 1-4 and 2-3:

$$\dot{W}_{14} = 0 = \dot{W}_{23}$$

For stages 3-4 and 1-2:

$$\dot{W}_{34} = \dot{m}\,(H_4 - H_3) = 1\,\frac{Kg}{s}\,(860.85 - 700.10)\,\frac{KJ}{Kg}$$

$$\boxed{\dot{W}_{34} = 160.75\ KJ/s} \Rightarrow \boxed{160.75\ KW}$$

$$\dot{W}_{12} = \dot{m}\,(H_2 - H_1) = 1\,\frac{Kg}{s}\,(1919.12 - 2791.99)\,\frac{KJ}{Kg}$$

$$\boxed{\dot{W}_{12} = -872.87\ KJ/s} \Rightarrow \boxed{-872.87\ KW}$$

And the efficiency is

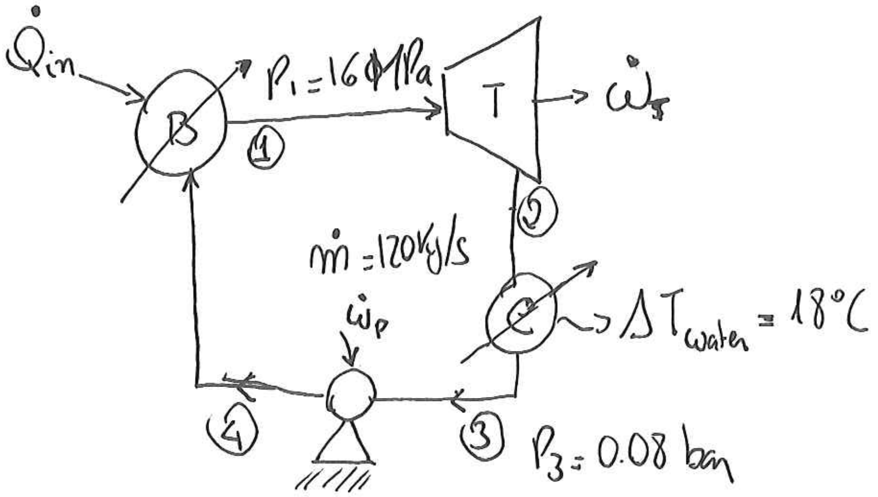
$$\eta = \frac{\text{Net Work}}{\text{Added Heat}} = \frac{|\dot{W}_{12} - \dot{W}_{34}|}{\dot{Q}_{41}} = \boxed{03687}$$

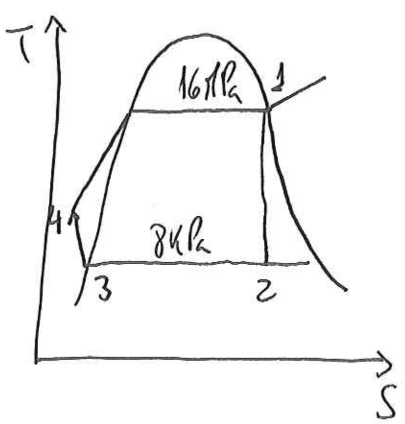
The 1st Law is STILL satisfied:

$$\sum Q = \dot{Q}_{41} + \dot{Q}_{23} \qquad \sum W = \dot{W}_{12} + \dot{W}_{34}$$

$$\sum Q + \sum W = 0$$

Problem 02:





As usual, let's first obtain the fluid properties at each stage of the cycle:

(1) Saturated vapor at $P_1 = 16$ MPa

$$H_1 = 2580.6 \text{ kJ/kg}$$
$$S_1 = 5.2455 \text{ kJ/(kg.K)}$$

(2) $P_2 = 8$ kPa and $S_2 = S_1$

↳ calculating the quality x_2

$$x_2 = \frac{S_2 - S_{f2}}{S_{g2} - S_{f2}} = \frac{5.2455 - 0593}{8.229 - 0593} = 06091$$

and the enthalpy H_2:

$$H_2 = H_{f2} + x_2 (H_{g2} - H_{f2}) = 0.6091(2577.0 - 173.9) + 173.9$$
$$H_2 = 1637.6 \text{ kJ/kg}$$

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(3) $P_3 = 0.08$ bar (saturated liquid)

$\hookrightarrow H_3 = H_{f3} = 173.9$ KJ/Kg

(4) $P_4 = 16$ MPa

$\hookrightarrow H_4 \cong H_3 + v_3 (P_4 - P_3)$

$H_4 = 190.01$ KJ/Kg

notice that we ~~[illegible]~~ neglected compressibility here!!!

The net power developed by the cycle is,

$$\dot{W}_{cycle} = \dot{W}_T - \dot{W}_P = \dot{m}\left[(H_1 - H_2) - (H_4 - H_3)\right]$$

$$\dot{W}_{cycle} = 1.112 \times 10^5 \text{ KW} \therefore \boxed{\dot{W}_{cycle} = 1.112 \times 10^2 \text{ MW}}$$

The rate of HT to the steam through the boiler is,

$$\dot{Q}_{in} = \dot{m}(H_1 - H_4) \therefore \boxed{\dot{Q}_{in} = 2.869 \times 10^2 \text{ MW}}$$

Thermal efficiency can be readily calculated from

$$\boxed{\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}} = 0.388}$$

If we define a control volume enveloping the condenser

$$\dot{m}_w \Delta H_w = \dot{m}(H_2 - H_3)$$

$$\dot{m}_w (C_w \Delta T_w) = \dot{m}(H_2 - H_3)$$

with $C_w = 4.179$ KJ/Kg K

$$\boxed{\dot{m}_w = 2335 \text{ Kg/s}}$$

Problem 03: Rankine cycle (R22)

01/02

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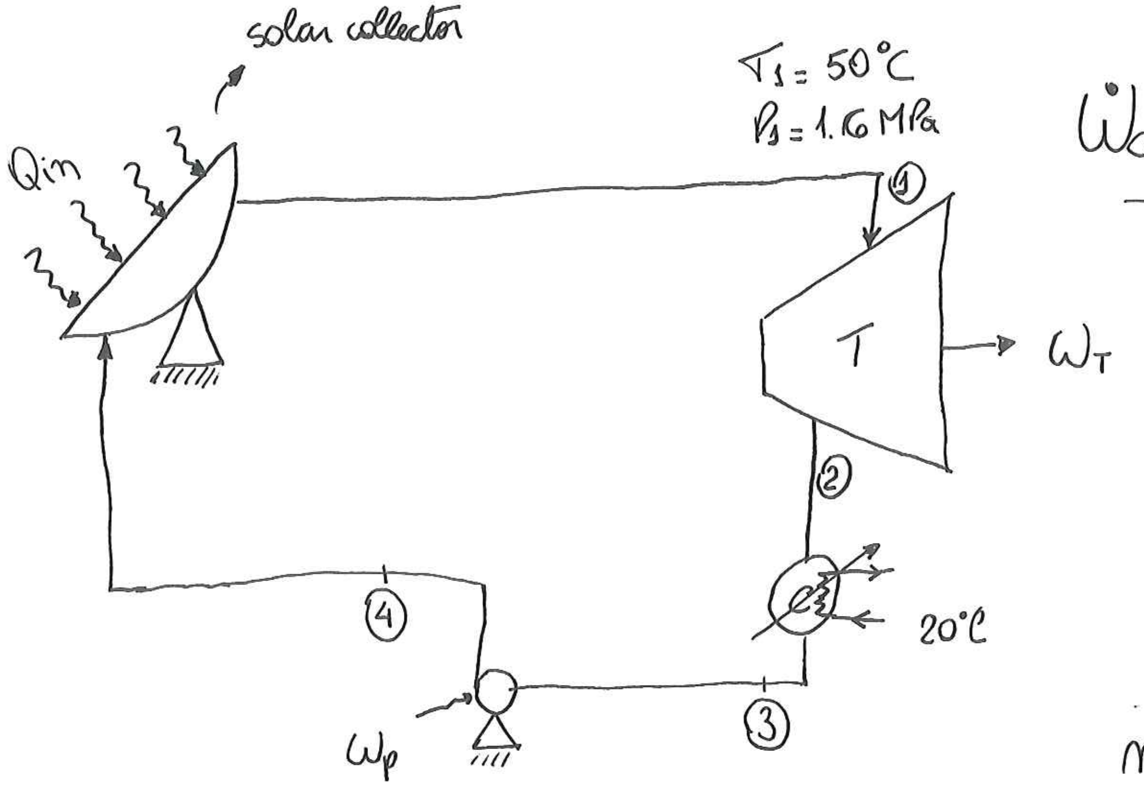

$\dot{W}_{cycle} = 10$ MW

$\underbrace{\eta, \dot{m}, \dot{m}_w = ?}$

max. performance

In order to maximise the performance of the cycle, the pressure in the condenser should be as low as possible. For $P_3 = 12$ bar (using the "properties of R22 tables") $\Rightarrow T_{sat} = 30.25°C$, the rise in the temperature of the cooling water would be approx. 8°C.

We can further assume that $\eta_T \cong 85\%$ is a reasonable approximation for such preliminary design. Similarly we can also assume that $\eta_P \cong 70\%$. Now we can set up the properties for each state:

~~10~~
12

① $P_1 = 16$ bar
$T_1 = 323.15 K$ } superheated ~~saturated~~ R22 tables

↳ $H_1 = 269.18$ KJ/Kg
$S_1 = 0.8971$ KJ/(Kg.K)

↳ $T_{sat}(P_1) = 314.88 K$, therefore the fluid is at superheated state.

② $P_2 = 12$ bar
$S_{2s} = S_1$ } from linear interpolation of Superheated R22 tables:

H (KJ/Kg)	S (KJ/(Kg K))
258.94	0.8864
267.62	0.9146

$H_{2s} = 262.2$ KJ/Kg

Now taking into account the turbine efficiency

$$H_2 = H_1 - \eta_T (H_1 - H_{2s}) \therefore H_2 = 263.2 \frac{KJ}{Kg}$$

③ $P_3 = 12$ bar is a saturated liquid

↳ $h_3 = 81.90$ KJ/Kg $V_3 = 0.8546 \times 10^{-3}$ m³/Kg

④ $h_{4s} \cong h_3 + V_3 (P_4 - P_3) \therefore h_{4s} = 82.24$ KJ/Kg

And with the pump efficiency

$$h_4 = h_3 + \frac{(h_{4s} - h_3)}{\eta_p} \quad \therefore h_4 = 82.39 \text{ KJ/Kg}$$

With all these properties (and assumptions) we can now calculate the efficiency of the cycle

$$\eta = \frac{W_T - W_P}{Q_{in}} = \frac{\dot{W}_T/\dot{m} - \dot{W}_P/\dot{m}}{\dot{Q}_{in}/\dot{m}}$$

with

$$W_T = \frac{\dot{W}_T}{\dot{m}} = h_1 - h_2 = 5.98 \text{ KJ/Kg}$$

$$W_P = \frac{\dot{W}_P}{\dot{m}} = h_4 - h_3 = 0.49 \text{ KJ/Kg}$$

$$Q_{in} = \frac{\dot{Q}_{in}}{\dot{m}} = h_1 - h_4 = 186.8 \text{ KJ/Kg}$$

The efficiency is then

$$\boxed{\eta = 0.0294 \quad (2.94\%)}$$

For a net power output of 10MW, we can now compute $\dot{m}$

$$\dot{W}_{cycle} = \dot{W}_T - \dot{W}_P = \dot{m}\left(\frac{\dot{W}_T}{\dot{m}} - \frac{\dot{W}_P}{\dot{m}}\right)$$

$$\dot{m} = \dot{W}_{cycle}\left(\frac{\dot{W}_T}{\dot{m}} - \frac{\dot{W}_P}{\dot{m}}\right)^{-1} = 10000\ \frac{KJ}{s}\left[(5.98-0.49)\frac{KJ}{Kg}\right]^{-1}$$

$$\boxed{\dot{m} = 6.56 \times 10^6\ Kg/h}$$

Calculating the mass flow rate of the cooling water, $\dot{m}_\omega$

$$Q_\omega = \dot{m}_\omega C_{p_\omega} \Delta T_\omega = \dot{m}\, \Delta h_\omega$$

$$\left.\begin{array}{l} H_\omega^{in}(T_\omega^{in} = 20°C) = 83.96\ KJ/Kg \\ H_\omega^{out}(T_\omega^{out} = 20+8°C) = 117.43\ KJ/Kg \end{array}\right\}$$

ΔT_ω

$$\dot{m}_\omega = \frac{\dot{m}(H_2 - H_3)}{H_\omega^{out} - H_\omega^{in}}$$

$$\boxed{\dot{m}_\omega = 35.53 \times 10^6\ Kg/h}$$

We can now improve our design by: (a) changing P_{cond}, (b) Changing ΔT_ω

Problem 04:

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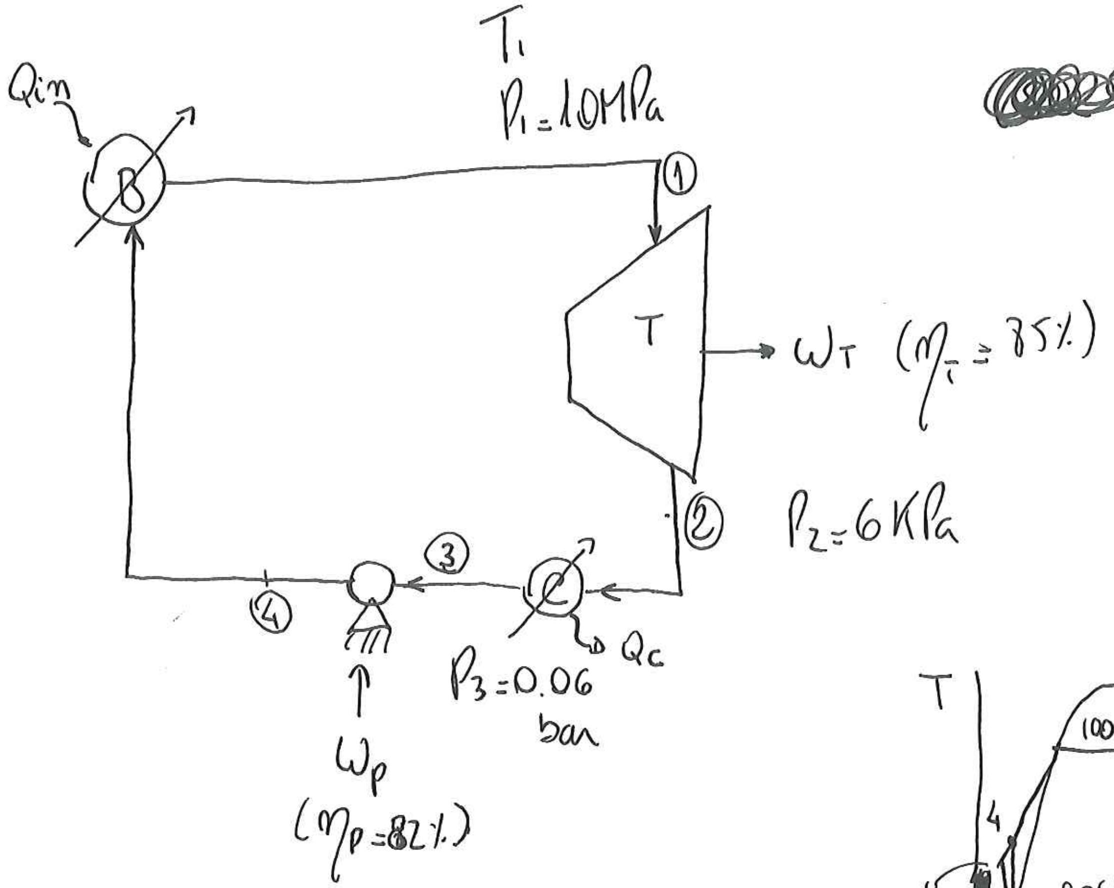

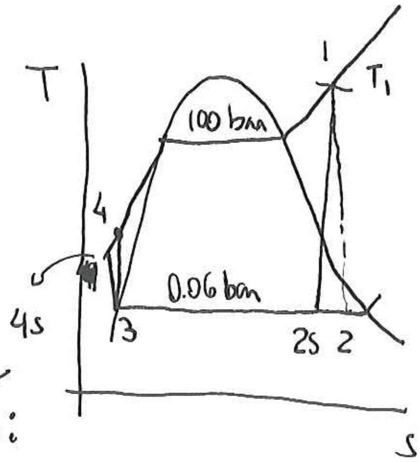

Calculating the properties at each state:

① $P_1 = 100$ bar, $T_1 = 580°C$ } from superheated steam tables

$\hookrightarrow H_1 = 3575.7$ KJ/Kg

$S_1 = 6.8447$ KJ/(Kg K)

② $P_2 = 6$ KPa, $S_{2s} = S_1$

$$x_{2s} = \frac{S_{2s} - S_l}{S_v - S_l} = \frac{6.8447 - 0.5211}{8.3304 - 0.5211} = 0.80975$$

$$x_{2s} = \frac{H_{2s} - H_l}{H_v - H_l} = \frac{H_{2s} - 151.53}{2567.4 - 151.53} \Rightarrow H_{2s} = 2107.8 \text{ KJ/Kg}$$

with $\eta_T = 085$

$$H_2 = H_1 - \eta_T (H_1 - H_{2s})$$

$H_2 = 2328.0$ KJ/Kg and we can calculate the quality

$$x_2 = 0.901$$

(3) $P_3 = 0.06$ bar (saturated liquid)

$$H_3 = H_{liq} = 151.53 \text{ KJ/Kg}$$

$$V_3 = V_{liq} = 1.0064 \times 10^{-3} \text{ m}^3/\text{Kg}$$

(4) $H_4 = H_3 + \dfrac{V_3 (P_4 - P_3)}{\eta_P} \quad \therefore \quad H_4 = 163.8$ KJ/Kg

And

$$\eta = \frac{(H_1 - H_2) - (H_4 - H_3)}{H_1 - H_4} \quad \therefore \quad \boxed{\eta = 0362}$$

Now, the same procedure for $T_1 = 800°C$

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① $P_1 = 100$ bar ; $T_1 = 700°C$

$h_1 = 3870.5$ KJ/Kg

$S_1 = 7.1687$ KJ/(Kg.K)

② $P_2 = 6$ KPa

$S_{2s} = 0.8512$

$h_{2s} = 2208.02$ KJ/Kg

$h_2 = 2457.39$ KJ/Kg

$x_2 = 0.9544$

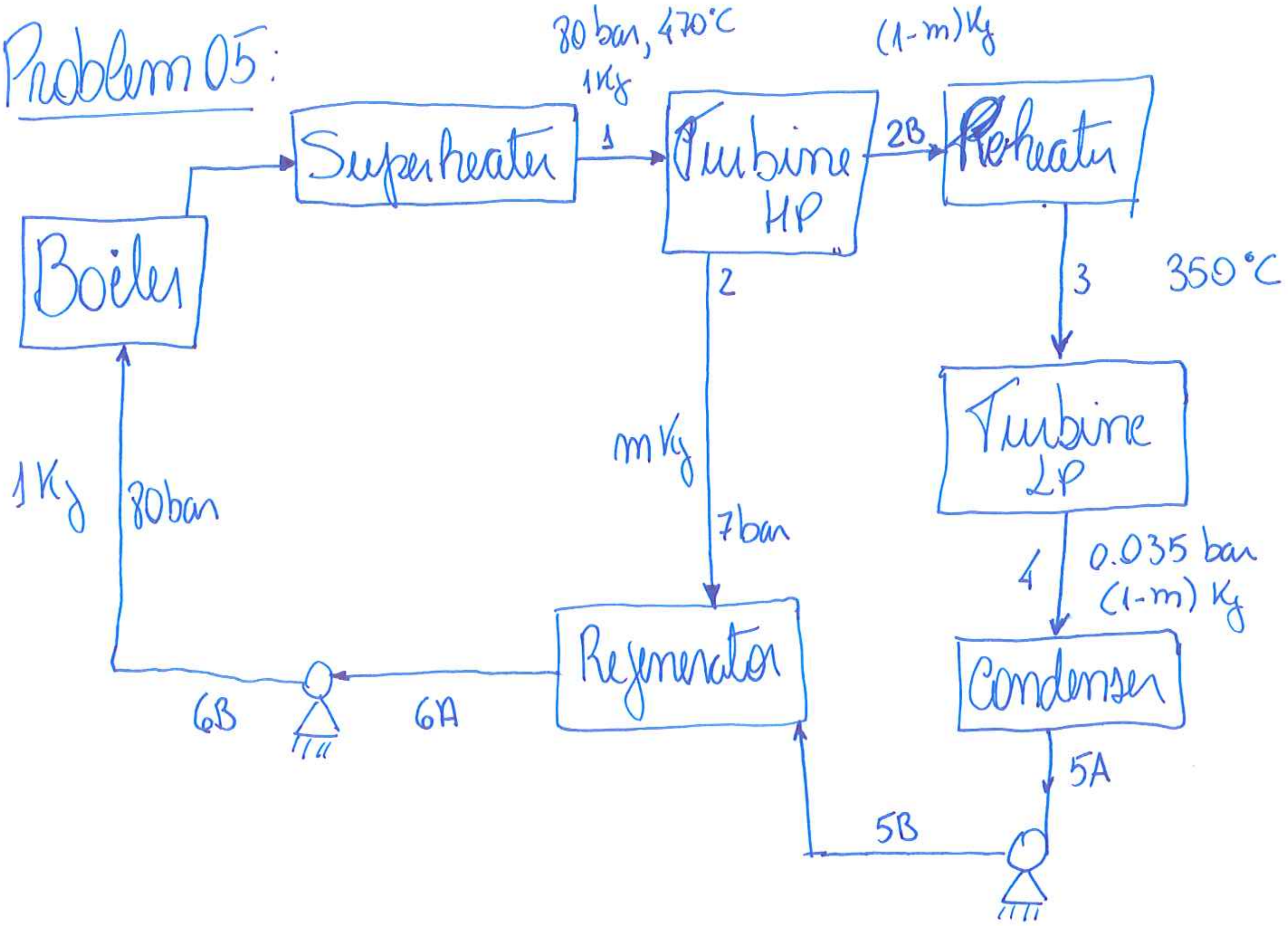
③ $P_3 = 0.06$ bar

$h_3 = 151.53$ KJ/Kg ; $V_3 = 1.0064 \times 10^{-3}$ m³/Kg

④ $h_4 = 163.8$ KJ/Kg

$$\boxed{\eta_{T_1 = 700°C} = 0.3779}$$

Problem 05:



1: $P_1 = 80$ bar, $T_1 = 470°C$, $m_1 = 1$ Kg

At P_1, the saturation temperature $T_1^{sat} = 295.1°C \lll T_1$, therefore the fluid is superheated vapour:

$$H_1 = 3322.83 \text{ KJ/Kg}$$

$$S_1 = 6.6237 \text{ KJ/(Kg.K)}$$

2: Isentropic expansion at 7 bar $(S_2 = S_1)$

P(bar)	H_f	H_{fg}	H_g	S_f	S_g
7.00	697.22	2066.3	2763.5	1.9922	6.7080

The quality of the steam is

$$x_2 = \frac{S_2 - S_f}{S_g - S_f} = \frac{6.6237 - 1.9922}{6.7080 - 1.9922}$$

$$x_2 = 0.9821$$

Now calculating the enthalpy:

$$x_2 = \frac{H_2 - H_f}{H_g - H_f} = \frac{H_2 - 697.22}{2763.5 - 697.22} = 0.9821$$

$$H_2 = 2726.51 \text{ KJ/Kg}$$

3: $T_3 = 350°C$, $P_3 = P_2 = 7$ bar { At $P_3 = 7$ bar, the saturation temp. is 165°C <<<< T_3 — thus the fluid is a superheated steam:

$$H_3 = 3163.75 \text{ KJ/Kg}$$

$$S_3 = 7.4722 \text{ KJ/Kg.K}$$

4: Isentropic expansion at 0.035 bar ($S_4 = S_3$)

P (bar)	T_{sat} (°C)	H_f	H_{fg}	H_g	S_f	S_g
0.30	69.10	289.23	2336.1	2625.3	0.9439	7.7686
0.40	75.87	317.58	2319.2	2636.8	1.0259	7.6700

~~Through linear interpolation:~~

(from saturated table – Rajput – Engineering Thermodynamics)

P (bar)	T_{sat} (°C)	H_f	H_{fg}	H_g	S_f	S_g	V_f (m^3/Kg)
0.035	26.7	111.9	2438.4	2550.3	0.391	8.523	1.003×10^{-3}

$$x_4 = \frac{S_4 - S_f}{S_g - S_f} = \frac{7.4722 - 0.391}{8.523 - 0.391} = 0.8708$$

And the enthalpy

$$x_4 = \frac{H_4 - H_f}{H_g - H_f} = \frac{H_4 - 111.9}{2550.3 - 111.9} = 0.8708$$

$$H_4 = 2235.26 \ KJ/Kg$$

5A: The flow leaving the condenser is saturated liquid at $P_{5A} = P_4$

$$H_{5A} = 111.9 \ KJ/Kg$$

5B: Saturated liquid leaving the pump with $P_{5B} = P_2$ (assuming incompressible flow)

$$H_{5B} \cong H_{5A} + V_{5A}(P_{5B} - P_{5A})$$

$$H_{5B} = 111.9 \ KJ/Kg + \underbrace{1.003\times10^{-3}}_{m^3/Kg}\underbrace{(7 - 0.035)}_{bar} \times \frac{10^5 \ Kg/(ms^2)}{1 \ bar} \times \frac{10^{-3} \ KJ/Kg}{1 \ m^2/s^2}$$

$H_{5B} = 112.59$ KJ/Kg (So you can assume $H_{5B} = H_{6A}$)

• **6A**: Saturated liquid at 7bar

$$H_{6A} = H_l(P=7\,bar) = 697.22\ KJ/Kg$$

• **6B**: Saturated liquid at 80 bar

$$H_{6B} \cong H_{6A} + V_{6A}\,(P_{6B} - P_{6A})$$

$$H_{6B} = \underbrace{697.22}_{KJ/Kg} + \underbrace{1.1080\times10^{-3}}_{m^3/Kg}\,(\underbrace{80-7}_{bar}) \times \underbrace{10^{+5}}_{\frac{Kg/(m.s^2)}{bar}} \times \underbrace{10^{-3}}_{\frac{KJ/Kg}{m^2/s^2}}$$

$$H_{6B} = 705.31\ KJ/Kg$$

Energy balance in the regenerator:

$$m_T \times H_{6A} = m \times H_2 + (1-m)\,H_{5B}$$

$$\underbrace{1}_{1Kg} \times 697.22 = m \times 2726.51 + (1-m)\,112.59$$

$$\boxed{m = 0.2237\ Kg} \quad (a)$$

Thus 22.37% of the steam generated by the boiler is "bled off". Therefore 77.63% of the steam is supplied to the LP turbine.

The heat supplied by the boiler is (assuming 1 kg of H_2O):

$$\dot{Q}_{6B-1} = H_1 - H_{6B} = 3322.83 - 705.31$$

$$\boxed{\dot{Q}_{6B-1} = 2617.52 \text{ kJ/kg}} \quad (c)$$

And now, by the reheater (per kg of steam generated)

$$\dot{Q}_{3-2B} = (1-m)(H_3 - H_{2B})$$

$$\boxed{H_{2B} = H_2}$$

$$\dot{Q}_{3-2B} = (1-0.2237)(3163.75 - 2726.51)$$

$$\boxed{\dot{Q}_{3-2B} = 339.43 \text{ kJ/kg}} \quad (c)$$

Thus the total heat supplied by the boiler and reheater is (per kg of steam generated)

$$\dot{Q}_S = \dot{Q}_{6B-1} + \dot{Q}_{3-2B} = 2617.52 + 339.43$$

$$\dot{Q}_S = 2956.95 \text{ kJ/kg}$$

Thus the thermal efficiency is

$$\eta_{cycle} = \frac{W_{TOTAL}}{\dot{Q}_S}$$

The work done (per Kg of steam generated) is:

- HP turbine: $W_{T_{HP}} = m_T H_1 - [m H_2 + (1-m) H_{2B}]$

  with $H_{2B} = H_2$ and $m_T = 1$ Kg

$$W_{T_{HP}} = H_1 - H_2$$

- LP turbine: $W_{T_{LP}} = (1-m)(H_3 - H_4)$

- Pump 1: $W_{P_1} = (1-m)(H_{5B} - H_{5A})$

- Pump2: $W_{P_2} = m_T (H_{6B} - H_{6A})$

$$W_{TOTAL} = W_{T_{HP}} + W_{T_{LP}} - (W_{P_1} + W_{P_2})$$

$$W_{TOTAL} = (3322.83 - 2726.51) + (1 - 0.2237)(-2235.26 + 3163.75) - [(1 - 0.2237)(112.59 - 111.9) + 1(705.31 - 697.22)]$$

$$W_{TOTAL} = 1308.48 \text{ KJ/Kg}$$

And

$$\eta_{cycle} = \frac{1308.48}{2956.95} = 0.4425 \therefore \boxed{44.25\%} \quad (d)$$

The power developed by the ~~sy~~ system:

$$\dot{W}_s = \dot{m}_s \, \dot{W}_{TOTAL}$$

$$\dot{W}_s = 50\,\frac{Kg}{s}\; 1308.48\,\frac{KJ}{Kg}$$

$$\dot{W}_s = 65424\ KJ/s \therefore \boxed{65.4\ MW} \quad (e)$$

Problem 06: Reheat Rankine

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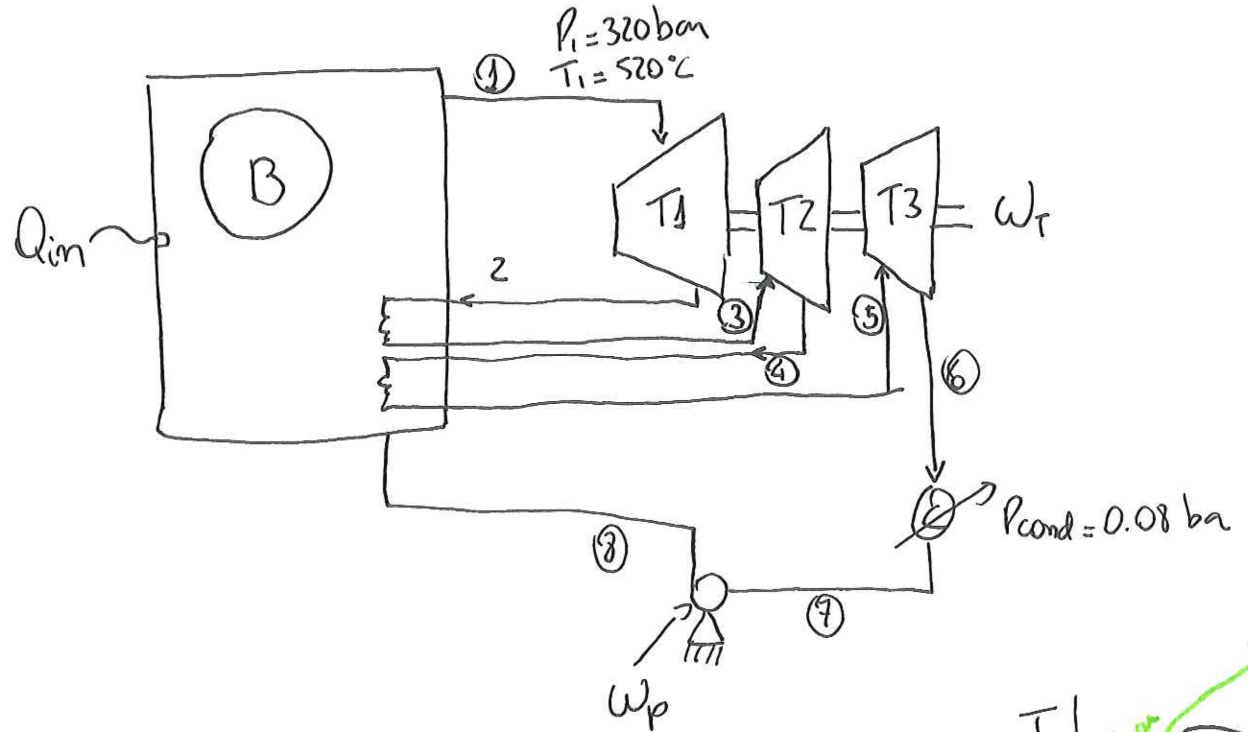

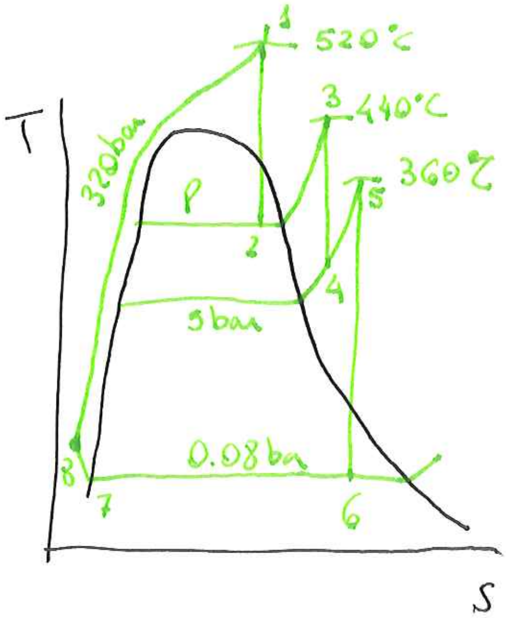

If $P = 4\,MPa$ $\left\{ \begin{array}{c} \dot{W}_{cycle}/\dot{m} \\ \eta \end{array} \right\}$?

For each state:

① $P_1 = 320$ bar, $T_1 = 520$°C $\Big\{$ $H_1 = 3133.7$ KJ/Kg, $S_1 = 5.8357$ KJ/(Kg K)

② $P_2 = P = 40$ bar, $S_2 = S_1$ $\Big\{$ $x_2 = 0.9284 \Rightarrow H_2 = 2678.7$ KJ/Kg

③ $P_3 = P = 40$ bar, $T_3 = 440$°C $\Big\{$ $H_3 = 3307.1$ KJ/Kg, $S_3 = 6.9041$ KJ/(Kg K)

(4) $P_4 = 5$ bar, $S_4 = S_3$ $\Big\}$ from superheated table $H_4 = 2785.0$ KJ/Kg

(5) $P_5 = 5$ bar, $T_5 = 360°C$ $\Big\}$ $H_5 = 3188.4$ KJ/Kg, $S_5 = 7.666$ KJ/(Kg.K)

(6) $P_6 = 0.08$ bar, $S_6 = S_5$ $\Big\}$ $x_6 = 092631 \Rightarrow H_6 = 2399.9$ KJ/Kg

(7) $P_7 = 0.08$ bar (saturated liquid) $\Big\}$ $H_7 = 173.88$ KJ/Kg

(8) $H_8 \cong H_7 + V_7 (P_8 - P_7) \therefore H_8 = 206.14$ KJ/Kg

The net work : $\dfrac{\dot{W}_{cycle}}{\dot{m}} = \dfrac{\dot{W}_T}{\dot{m}} + \dfrac{\dot{W}_P}{\dot{m}}$

Thus the work associated with the set of turbines

$$\frac{\dot{W}_T}{\dot{m}} = (H_1 - H_2) + (H_3 - H_4) + (H_5 - H_6) = 1765.6 \text{ KJ/Kg}$$

And for the pump

$$\frac{\dot{W}_P}{\dot{m}} = H_8 - H_7 = 32.26 \text{ KJ/Kg}$$

$$\boxed{\frac{\dot{W}_{cycle}}{\dot{m}} = 1733.3 \text{ KJ/Kg}}$$

In order to calculate the thermal efficiency

$$\eta = \frac{(\dot{W}_{cycle}/\dot{m})}{(\dot{Q}_{in}/\dot{m})}$$

we need to calculate

$$\frac{\dot{Q}_{in}}{\dot{m}} = (h_1-h_8)+(h_3-h_2)+(h_5-h_4)$$

$$\dot{Q}_{in}/\dot{m} = 3959.4 \text{ KJ/Kg}$$

Then

$$\boxed{\eta = 0.438}$$

Problem ~~10~~ 07: Steam in a regenerative cycle

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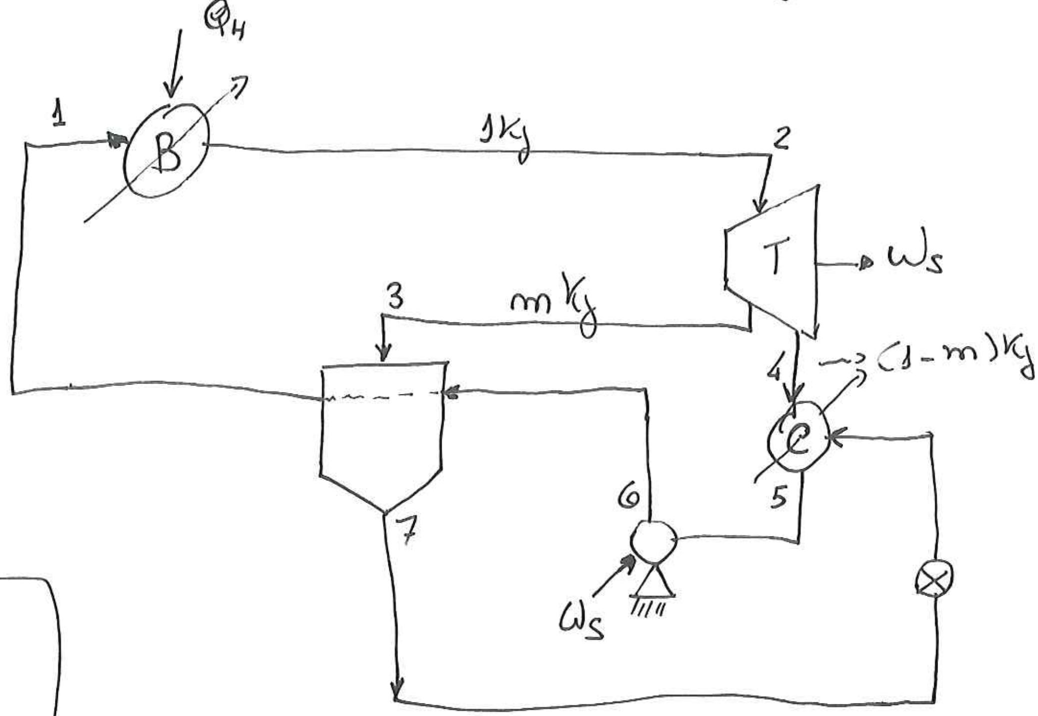

$P_2 = 4500$ KPa

$T_2 = 773.15$ K

from superheated steam table

$H_2 = 3499.3$ KJ/Kg

$S_2 = 7.0311$ KJ/Kg.K

$\eta_T = \eta_P = 0.78$

$\eta = ?$

$m = ?$

Because it's an isentropic expansion $S'_3 = S_2$

$P_3 = 350$ KPa → $H'_3 = 2770.6$ KJ/Kg ; with $\eta_T = 0.78$

$W_I = \eta(H'_3 - H_2)$

$W_I = -521.586$ KJ/Kg

Thus to calculate the actual enthalpy in 3, H_3:

$H_3 = H_2 + W_I = \underline{2917.714 \text{ KJ/Kg}}$

the isentropic expansion to 20 KPa ($S_4' = S_2$) results in a saturated liquid + vapour:

P(KPa)	H_l (KJ/Kg)	H_v	S_l (KJ/KgK)	S_v
19.92	251.1	2609.7	0.8310	7.9108
20.86	255.3	2611.4	0.8435	7.8948

Linear interpolation leads to

$H_l = 251.435$ KJ/Kg $\qquad S_l = 0.8321$ KJ/(Kg K)

$H_v = 2609.9$ Kj/Kg $\qquad S_v = 7.9094$ KJ/(Kg K)

To calculate actual H_4 and H_5, we need to calculate the quality x_4': $S_4' = S_l + x_4'(S_v - S_l)$ ∴ $\underline{S_4' = 0.876}$

Similarly for H_4': $H_4' = H_l + x_4'(H_v - H_l)$

$$H_4' = 2317 \text{ KJ/Kg}$$

H_4 is then

$$H_4 = H_2 + \eta_T (H_4' - H_2) = \underline{2564 \text{ KJ/Kg}}$$

The flow leaving the condenser is a saturated liquid:

$H_5 = H_{liq}$

And V_5 (from the saturated water-steam table): $\begin{cases} P_5 = 20\text{KPa} \\ V_5 = 1.017 \frac{cm^3}{g} \end{cases}$

Pressure out of the pump: $P_6 = 4500 KPa$
and the work is

$$W_{pump} = \frac{V_5 (P_6 - P_5)}{\eta_p} = 5.841 KJ/Kg$$

$$H_6 = H_5 + W_{pump}$$

$$H_6 = \underline{257.294 KJ/Kg}$$

The fluid ~~being fed~~ feeding the condenser from the FWH is a saturated liquid at <u>350 KPa</u> (water-steam table):

$H_7 = 584.270 KJ/Kg$ and $T_7 = 412.02K$ ($= 138.87°C$)

We now need to compute the enthalpy of compressed liquid at 1 with

$$T_1 = T_7 - 6K = 406.02K$$

and $H_{liq}^{sat} = 558.5 KJ/Kg$; $P_{sat} = 294.261$

$V_{liq}^{sat} = 1.0730 cm^3/g$

But we actually need H_1 which is a bulk mixing of <u>saturated liquid</u> and <u>vapour</u> in thermodynamic equilibrium.

Thus from the "properties of fluids":

$$H_1 = H_{liq}^{sat} + V_{liq}^{sat}(1-\beta T_1)(P_1 - P_{sat}) \quad (*)$$

where β is the volume expansivity is defined as,

$$\beta \equiv \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$$

We can approximate from the values obtained from the water-steam table ($P_1 = P_6$)

$$\beta = \frac{1}{V_{liq}^{sat}}\left(\frac{1.083 - 1.063 \ (cm^3/g)}{20 \text{ K}}\right)$$

$$\underline{\beta = 9.32 \times 10^{-4} \ K^{-1}}$$

Back to (*), replacing β:

$$\underline{H_1 = 561.305 \ KJ/Kg}$$

The energy balance in the feedwater heater:

$$m = \frac{H_1 - H_6}{H_3 - H_7} \quad \therefore \quad \boxed{m = 0.13028 \ Kg}$$

Now, in order to calculate the ~~of~~ efficiency of the cycle,

$$\eta = \frac{W_{net}}{Q_H}$$

the work in the second section of the turbine:

$$W_{II} = (1-m)(H_4 - H_3) \therefore W_{II} = -307.567 \, KJ$$

and net work

$$W_{net} = (W_I + W_{pump}) \times 1Kg + W_{II}$$

$$W_{net} = -823.3 \, KJ$$

and heat in the boiler,

$$Q_H = (H_2 - H_1) \times 1Kg \therefore Q_H = 2878 \, KJ$$

The efficiency is then

$$\eta = \frac{W_{net}}{Q_H} = \boxed{0.2861 \ (or \ 28.61\%)}$$