February 25, 2014

Example 1 *A reversible engine converts one-sixth of the heat input into work. When the temperature of the sink is reduced by 70 °C, its efficiency is doubled. Determine the temperature of the source and the sink.*

Let's first stablish that T_1 and T_2 [K] are the source and sink temperatures, respectively. For a **reversible engine**, converting 1/6 of the heat into work means

$$\frac{T_1 - T_2}{T_1} = \frac{1}{6} \quad \Longrightarrow \quad T_1 = 1.2T_2$$

Now if the sink temperature is reduced to *70 °C = 343.15 K*, ie, $T_2' = T_2 - 343.15$ then the efficiency of the cycle is doubled

$$\frac{T_1 - T_2'}{T_1} = 2 \times \frac{1}{6}$$
$$2T_1 = 3T_2 - 1029.45 \quad \Longrightarrow \quad T_2 = 1715 \text{ K and } T_1 = 2058.90 \text{ K}$$

Example 2 *The minimum pressure and temperature in an Otto cycle are 100 kPa and 27 °C. The amount of heat added to the air per cycle is 1500 kJ/kg. Calculate:*

(a) Pressures and temperatures at all stages of the air standard Otto cycle;

(b) Specific work and thermal efficiency of the cycle for a compression ratio of 8 : 1.

Assuming an isentropic compression stage 1–2 and an isentropic expansion stage 3–4 with a compression ratio $V_1/V_2 = 8$. Initial temperature of 300.15 K and pressure of 100 kPa.

- *adiabatic compression (1–2):*

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = r^{\gamma-1} = 8^{1.4-1} = 2.297 \quad \Longrightarrow \quad T_2 = 689.10 \text{ K}$$

  and

$$\frac{P_1}{P_2} = \left(\frac{V_1}{V_2}\right)^{\gamma} = 18.379 \Longrightarrow P_2 = 18.379 \text{ bar}$$

- *constant volume (2–3):* heat added: $C_v\,(T_3 - T_2) = 1500 \Longrightarrow T_3 = 2772.4$ K and

$$\frac{P_2}{T_2} = \frac{P_3}{T_3} \Longrightarrow P_3 = 73.94 \text{ bar}$$

- *adiabatic expansion (3–4):*

$$\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} = r^{\gamma-1} \Longrightarrow T_4 = 1206.9 \text{ K}$$

  and

$$P_3V_3^{\gamma} = P_4V_4^{\gamma} \Longrightarrow P_4 = 4.023 \text{ bar}$$

- *Specific work* = heat added - heat rejected

$$W = C_v\left(T_3 - T_2\right) - C_v\left(T_4 - T_1\right) = 847\ \text{kJ/kg}$$

- *Thermal efficiency:*

$$\eta_{\text{Otto}} = 1 - \frac{1}{r^{\gamma-1}} = 0.5647$$

Problem 1 *An ideal engine operates on the Carnot cycle using a perfects gas as the working fluid. The ratio of the greatest to the least volume is fixed as $x:1$, the lower temperature of the cycle is also fixed, but the volume compression ratio r of the reversible adiabatic compression is variable. The ratio of the specific heats is γ. Show that if the work done in the cycle is a maximum then,*

$$(\gamma - 1)\ln\frac{x}{r} + \frac{1}{r^{\gamma-1}} - 1 = 0$$

Problem 2 *An ideal Otto cycle has a volumetric compression ratio of 6, the lowest cycle pressure of 0.1 MPa and operates between temperature limits of 300.15 and 1842.15 K.*

(a) Calculate the temperature and pressure after the isentropic expansion;

*(b) Since the values in (**Problem 2a**) are well above the lowest cycle operating conditions, the expansion process was allowed to continue down to a pressure of 0.1 MPa. Which process is required to complete the cycle ?*

(c) Determine the percentage in which the cycle efficiency has improved.

Problem 3 *The volume ratios of compression and expansion for a diesel engine are 15.3 and 7.5, respectively. The pressure and temperature at the beginning of the compression are 1 bar and 27 ^{o}C. Assuming an ideal engine, determine the (a) MEP, (b) ratio of maximum pressure to MEP and (c) cycle efficiency. Also find the fuel consumption per kWh if the indicated thermal efficiency is 0.5 of ideal efficiency, mechanical efficiency is 0.8 and the calorific value of oil 42000 kJ/kg.*

Problem 4 *An air-standard dual cycle has a compression ratio of 9. At the beginning of compression, $P_1 = 100$ kPa, $T_1 = 300$ K and $V_1 = 14$ liters. The heat addition is 22.7 KJ with one-half added at constant volume and one-half added at constant pressure. Determine: (a) the temperatures at the end of each heat addition process; (b) the net work of the cycle per unit mass of air; (c) the thermal efficiency and; (d) MEP.*

Appendix 1

PROPERTY TABLES AND CHARTS (SI UNITS)

TABLE A–17

Ideal-gas properties of air

T K	h kJ/kg	P_r	u kJ/kg	v_r	$s°$ kJ/kg · K	T K	h kJ/kg	P_r	u kJ/kg	v_r	$s°$ kJ/kg · K
200	199.97	0.3363	142.56	1707.0	1.29559	580	586.04	14.38	419.55	115.7	2.37348
210	209.97	0.3987	149.69	1512.0	1.34444	590	596.52	15.31	427.15	110.6	2.39140
220	219.97	0.4690	156.82	1346.0	1.39105	600	607.02	16.28	434.78	105.8	2.40902
230	230.02	0.5477	164.00	1205.0	1.43557	610	617.53	17.30	442.42	101.2	2.42644
240	240.02	0.6355	171.13	1084.0	1.47824	620	628.07	18.36	450.09	96.92	2.44356
250	250.05	0.7329	178.28	979.0	1.51917	630	638.63	19.84	457.78	92.84	2.46048
260	260.09	0.8405	185.45	887.8	1.55848	640	649.22	20.64	465.50	88.99	2.47716
270	270.11	0.9590	192.60	808.0	1.59634	650	659.84	21.86	473.25	85.34	2.49364
280	280.13	1.0889	199.75	738.0	1.63279	660	670.47	23.13	481.01	81.89	2.50985
285	285.14	1.1584	203.33	706.1	1.65055	670	681.14	24.46	488.81	78.61	2.52589
290	290.16	1.2311	206.91	676.1	1.66802	680	691.82	25.85	496.62	75.50	2.54175
295	295.17	1.3068	210.49	647.9	1.68515	690	702.52	27.29	504.45	72.56	2.55731
298	298.18	1.3543	212.64	631.9	1.69528	700	713.27	28.80	512.33	69.76	2.57277
300	300.19	1.3860	214.07	621.2	1.70203	710	724.04	30.38	520.23	67.07	2.58810
305	305.22	1.4686	217.67	596.0	1.71865	720	734.82	32.02	528.14	64.53	2.60319
310	310.24	1.5546	221.25	572.3	1.73498	730	745.62	33.72	536.07	62.13	2.61803
315	315.27	1.6442	224.85	549.8	1.75106	740	756.44	35.50	544.02	59.82	2.63280
320	320.29	1.7375	228.42	528.6	1.76690	750	767.29	37.35	551.99	57.63	2.64737
325	325.31	1.8345	232.02	508.4	1.78249	760	778.18	39.27	560.01	55.54	2.66176
330	330.34	1.9352	235.61	489.4	1.79783	780	800.03	43.35	576.12	51.64	2.69013
340	340.42	2.149	242.82	454.1	1.82790	800	821.95	47.75	592.30	48.08	2.71787
350	350.49	2.379	250.02	422.2	1.85708	820	843.98	52.59	608.59	44.84	2.74504
360	360.58	2.626	257.24	393.4	1.88543	840	866.08	57.60	624.95	41.85	2.77170
370	370.67	2.892	264.46	367.2	1.91313	860	888.27	63.09	641.40	39.12	2.79783
380	380.77	3.176	271.69	343.4	1.94001	880	910.56	68.98	657.95	36.61	2.82344
390	390.88	3.481	278.93	321.5	1.96633	900	932.93	75.29	674.58	34.31	2.84856
400	400.98	3.806	286.16	301.6	1.99194	920	955.38	82.05	691.28	32.18	2.87324
410	411.12	4.153	293.43	283.3	2.01699	940	977.92	89.28	708.08	30.22	2.89748
420	421.26	4.522	300.69	266.6	2.04142	960	1000.55	97.00	725.02	28.40	2.92128
430	431.43	4.915	307.99	251.1	2.06533	980	1023.25	105.2	741.98	26.73	2.94468
440	441.61	5.332	315.30	236.8	2.08870	1000	1046.04	114.0	758.94	25.17	2.96770
450	451.80	5.775	322.62	223.6	2.11161	1020	1068.89	123.4	776.10	23.72	2.99034
460	462.02	6.245	329.97	211.4	2.13407	1040	1091.85	133.3	793.36	23.29	3.01260
470	472.24	6.742	337.32	200.1	2.15604	1060	1114.86	143.9	810.62	21.14	3.03449
480	482.49	7.268	344.70	189.5	2.17760	1080	1137.89	155.2	827.88	19.98	3.05608
490	492.74	7.824	352.08	179.7	2.19876	1100	1161.07	167.1	845.33	18.896	3.07732
500	503.02	8.411	359.49	170.6	2.21952	1120	1184.28	179.7	862.79	17.886	3.09825
510	513.32	9.031	366.92	162.1	2.23993	1140	1207.57	193.1	880.35	16.946	3.11883
520	523.63	9.684	374.36	154.1	2.25997	1160	1230.92	207.2	897.91	16.064	3.13916
530	533.98	10.37	381.84	146.7	2.27967	1180	1254.34	222.2	915.57	15.241	3.15916
540	544.35	11.10	389.34	139.7	2.29906	1200	1277.79	238.0	933.33	14.470	3.17888
550	555.74	11.86	396.86	133.1	2.31809	1220	1301.31	254.7	951.09	13.747	3.19834
560	565.17	12.66	404.42	127.0	2.33685	1240	1324.93	272.3	968.95	13.069	3.21751
570	575.59	13.50	411.97	121.2	2.35531						

TABLE A–17

Ideal-gas properties of air (*Concluded*)

T K	h kJ/kg	P_r	u kJ/kg	v_r	$s°$ kJ/kg · K	T K	h kJ/kg	P_r	u kJ/kg	v_r	$s°$ kJ/kg · K
1260	1348.55	290.8	986.90	12.435	3.23638	1600	1757.57	791.2	1298.30	5.804	3.52364
1280	1372.24	310.4	1004.76	11.835	3.25510	1620	1782.00	834.1	1316.96	5.574	3.53879
1300	1395.97	330.9	1022.82	11.275	3.27345	1640	1806.46	878.9	1335.72	5.355	3.55381
1320	1419.76	352.5	1040.88	10.747	3.29160	1660	1830.96	925.6	1354.48	5.147	3.56867
1340	1443.60	375.3	1058.94	10.247	3.30959	1680	1855.50	974.2	1373.24	4.949	3.58335
1360	1467.49	399.1	1077.10	9.780	3.32724	1700	1880.1	1025	1392.7	4.761	3.5979
1380	1491.44	424.2	1095.26	9.337	3.34474	1750	1941.6	1161	1439.8	4.328	3.6336
1400	1515.42	450.5	1113.52	8.919	3.36200	1800	2003.3	1310	1487.2	3.994	3.6684
1420	1539.44	478.0	1131.77	8.526	3.37901	1850	2065.3	1475	1534.9	3.601	3.7023
1440	1563.51	506.9	1150.13	8.153	3.39586	1900	2127.4	1655	1582.6	3.295	3.7354
1460	1587.63	537.1	1168.49	7.801	3.41247	1950	2189.7	1852	1630.6	3.022	3.7677
1480	1611.79	568.8	1186.95	7.468	3.42892	2000	2252.1	2068	1678.7	2.776	3.7994
1500	1635.97	601.9	1205.41	7.152	3.44516	2050	2314.6	2303	1726.8	2.555	3.8303
1520	1660.23	636.5	1223.87	6.854	3.46120	2100	2377.7	2559	1775.3	2.356	3.8605
1540	1684.51	672.8	1242.43	6.569	3.47712	2150	2440.3	2837	1823.8	2.175	3.8901
1560	1708.82	710.5	1260.99	6.301	3.49276	2200	2503.2	3138	1872.4	2.012	3.9191
1580	1733.17	750.0	1279.65	6.046	3.50829	2250	2566.4	3464	1921.3	1.864	3.9474

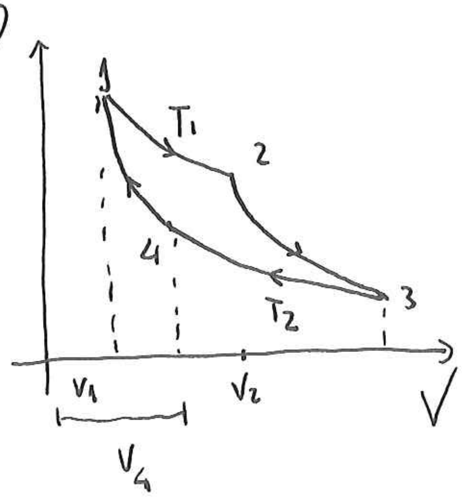
Note: The properties P_r (relative pressure) and v_r (relative specific volume) are dimensionless quantities used in the analysis of isentropic processes, and should not be confused with the properties pressure and specific volume.

Source: Kenneth Wark, *Thermodynamics,* 4th ed. (New York: McGraw-Hill, 1983), pp. 785–86, table A–5. Originally published in J. H. Keenan and J. Kaye, *Gas Tables* (New York: John Wiley & Sons, 1948).

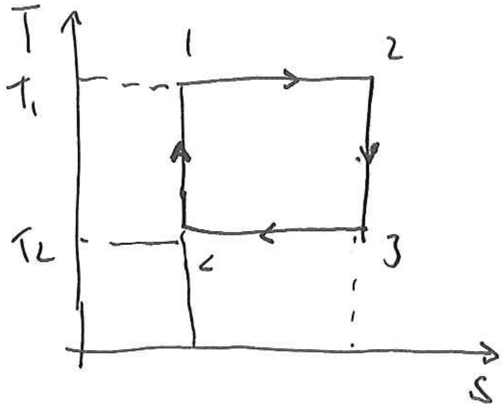
Gas Power Cycles

Problem 01

$$\frac{V_3}{V_1} = \tau \quad \text{and}$$



compression ratio $r = V_4/V_1$



In the isotherms, since compression ratio = expansion ratio

$$\frac{V_3}{V_4} = \frac{V_2}{V_1}$$

But we can redefine V_3/V_4 with the known variables:

$$\frac{V_3}{V_4} = \frac{V_3}{V_1} \times \frac{V_1}{V_4} = \tau \cdot \frac{1}{r} = \tau / r$$

The work done per unit mass of gas is

$W =$ heat supplied - heat rejected

$$= RT_1 \ln(\kappa/r) - RT_2 \ln(\kappa/r)$$

$$= R(T_1 - T_2)\ln(\kappa/r)$$

However we know that the isentropic change

$$\frac{T_1}{T_2} = \left(\frac{V_4}{V_1}\right)^{\gamma-1} = r^{\gamma-1}$$

Thus

$$W = RT_2\,(r^{\gamma-1} - 1)\ln(\kappa/r)$$

The maximum work is obtained by differentiating

$\underline{W}$ wrt $\underline{r}$:

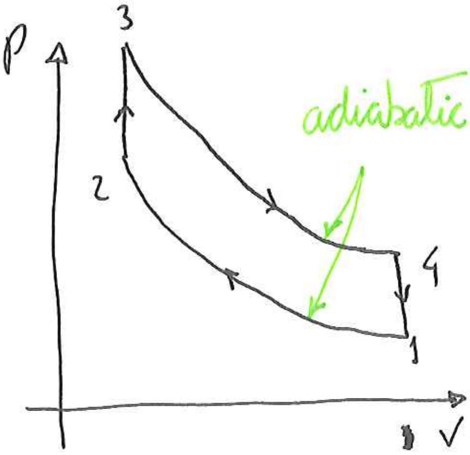
$$\frac{dW}{dr} = RT_2\left[(r^{\gamma-1} - 1)\,\frac{r}{\kappa}\,(-\kappa r^{-2}) + \ln\frac{\kappa}{r}\,(\gamma-1)r^{\gamma-2}\right]$$

$$= 0$$

This equation becomes

$$\boxed{(\gamma-1)\ln(\kappa/r) + \frac{1}{r^{\gamma-1}} - 1 = 0}$$

Problem 04:



Given: $\frac{V_1}{V_2} = \frac{V_4}{V_3} = r = 6$

$P_1 = 0.1$ MPa

$T_1 = 300.15\,K$; $T_3 = 1842.15\,K$

(a) T_4, P_4 : ?

For the compression 1-2

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma} \quad \therefore \quad P_2 = P_1 \left(\frac{V_1}{V_2}\right)^{\gamma} = 12.29 \text{ bar}$$

and

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma - 1} = 6^{\gamma-1} = 2.0477$$

$$T_2 = 614.61\,K$$

For the constant-volume process 2-3:

$$\frac{P_2}{T_2} = \frac{P_3}{T_3} \quad \therefore \quad P_3 = \frac{P_2 T_3}{T_2} = 36.82 \text{ bar}$$

And for the expansion 3-4:

$$\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} = 2.0477$$

$$T_4 = T_3 \left(\frac{V_3}{V_4}\right)^{\gamma-1} = \boxed{899.63\ K = T_4}$$

and

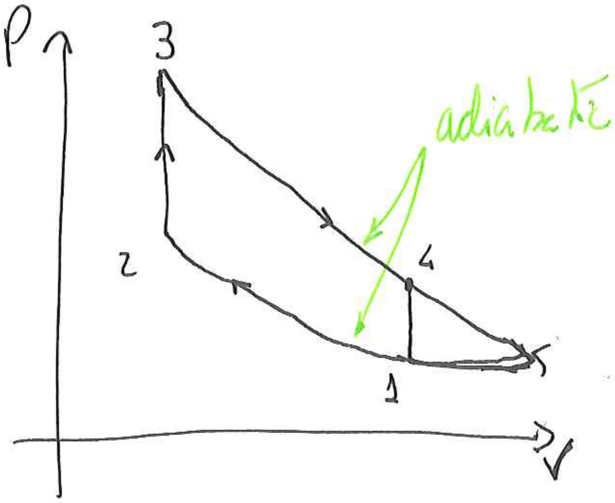
$$P_3 V_3^{\gamma} = P_4 V_4^{\gamma} \therefore P_4 = P_3 \left(\frac{V_3}{V_4}\right)^{\gamma}$$

$$\boxed{P_4 = 2.997\ \text{bar}}$$

(b) We can use a heat rejection at constant pressure after further expansion –

We saw this as the

Atkinson cycle



(c)

$$\eta_{otto} = 1 - \frac{1}{2^{\gamma-1}} = 0.5116 \therefore \boxed{51.16\%}$$

$$\frac{01/03}{05}$$

$$\eta_{Atkinson} = \frac{\text{Heat Supplied} - \text{Heat Rejected}}{\text{Heat Supplied}}$$

$$\eta_{Atkinson} = \frac{C_v(T_3 - T_2) - C_p(T_5 - T_1)}{C_v(T_3 - T_2)}$$

$$= 1 - \frac{C_p}{C_v}\frac{(T_5 - T_1)}{(T_3 - T_2)} = 1 - \gamma \frac{T_5 - T_1}{T_3 - T_2}$$

The isentropic process 3-5

$$\frac{T_5}{T_3} = \left(\frac{P_5}{P_3}\right)^{\frac{\gamma-1}{\gamma}} \quad \therefore T_5 = 657.47\,K$$

$$\eta_{Atkinson} = 1 - 1.4\,\frac{657.47 - 300.15}{1842.15 - 614.61}$$

$$\eta_{Atkinson} = 0.5925 \therefore \boxed{59.25\%}$$

↪ Improvement of $\boxed{8.08\%}$

Problem 03

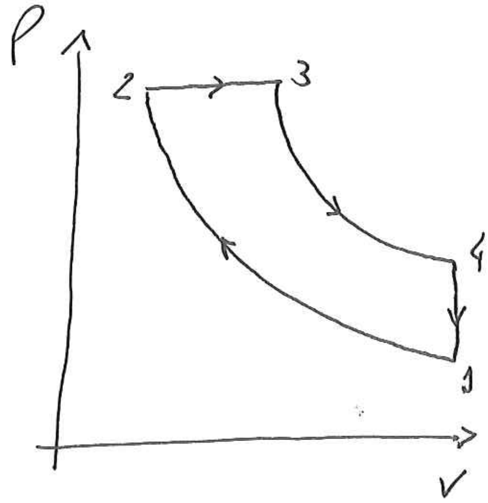
01/03
06

Given: $V_1/V_2 = 15.3$

$V_4/V_3 = 7.5$

$P_1 = 1$ bar

$T_1 = 300.15\,K$



(a) MEP : ?

$$MEP = \frac{\text{Work done by the cycle}}{\text{Swept Volume}} = \frac{\omega}{V_1 - V_2}$$

$$= \frac{m C_p (T_3 - T_2) - m C_v (T_4 - T_1)}{V_1 - V_2}$$

Before wes are able to calculate MEP, we need to determine all properties of the cycle, i.e., T_i, V_i, and T_i $(i = 1, 4)$. Thus

(i) adiabatic compression 1-2:

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma - 1} \therefore T_2 = 893.75\,K$$

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma} \therefore P_2 = 45.56 \text{ bar}$$

(ii) Constant pressure (2-3):

$$\frac{V_2}{T_2} = \frac{V_3}{T_3} \therefore T_3 = \frac{V_3}{V_2} T_2 = \frac{V_1}{V_2} \frac{V_3}{V_4} T_2$$

$$T_3 = \frac{15.3}{7.5} T_2 \therefore T_3 = 1823.25 \text{ K}$$

Let's assume that $V_2 = 1 m^3$, thus the mass of air is (assuming ideal gas behaviour)

$$m = \frac{P_2 V_2}{R T_2} = \frac{45.56 \text{ bar} \times 1 m^3}{8.314 \times 10^{-5} \frac{m^3 . bar}{K . gmol} \times 893.75 K \times \frac{1 gmol}{29 g}}$$

$$m = 17792.512 \text{ g} = 17.79 \text{ Kg}$$

(iii). For adiabatic expansion (3-4):

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1} = \left(\frac{1}{7.5}\right)^{\gamma-1} = 04467$$

$$T_4 = 814.37 K$$

Thus

$$MEP = \frac{m\left[C_p(T_3 - T_2) - C_v(T_4 - T_1)\right]}{V_1 - V_2}$$

$C_p \to 1.005 \frac{KJ}{Kg.K}$ $\quad C_v \to 0.718 \frac{KJ}{Kg.K}$

$$MEP = \frac{10051.64 \text{ KJ}}{14.3 V_2} = 702.91 \frac{KJ}{m^3} = \boxed{7.029 \text{ bar}}$$

$V_2 \to 1 m^3$

(b) $\frac{P_2}{MEP}$: ?

$$\boxed{\frac{P_2}{MEP} = 6.48}$$

(c)

$$\eta = \frac{\text{Work done}}{\text{heat supplied}} = \frac{10051.64}{mC_p(T_3 - T_2)}$$

$$\boxed{\eta = 0.6049 \therefore 60.49\%}$$

(d) Fuel consumption : ? $\quad \eta_T = 0.5\,\eta_{ideal}$

$$\eta_T = 0.5 \times 0.6049 = 0.3024 \therefore 30.24\%$$

However if the mechanical efficiency is 0.8, the "real" thermal efficiency is

$$\eta_T' = \eta_T \times \eta_M = 0.3024 \times 0.8 = 0.2419$$

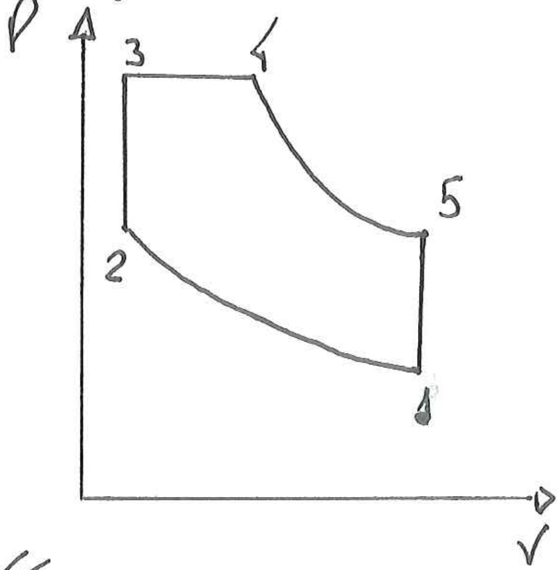
$$\eta_T' = \frac{\text{Brake Power}}{\dot{m}_f \times C}$$

→ effective power output of the engine.

↳ calorific heat of the oil

$$0.2419 = \frac{1}{\frac{\dot{m}_f}{3600}\, 42000} \quad \therefore \quad \boxed{\dot{m}_f = 0.354\ \text{Kg/KWh}}$$

Problem 04: Air-Standard Dual-Cycle



Given: • compression ratio : $V_1/V_2 = 9$

• $P_1 = 100\ KPa$

$T_1 = 300K$

$V_1 = 14\ l = 0.014 m^3$

• $Q_{34} = Q_{23} = \dfrac{22.7}{2} KJ = 11.35\ KJ$

Compute: T_i ? MEP_{dual} ?

W_{cycle}/m ?

η_{dual}

(a) Thermal analysis of all stages:

• Stage 01: from "ideal-gas properties of air" (table in attachment)

$T_1 = 300K$, $P_1 = 100 KPa$ $\begin{cases} u_1 = 214.07\ KJ/Kg \\ v_{r_1} = 621.2 \quad \text{(dimensionless)} \end{cases}$

↳ relative specific volume (see note)

• Stage 02 : isentropic compression

$$\frac{V_{r_2}}{V_{r_1}} = \frac{V_2}{V_1} \therefore V_{r_2} = V_{r_1}\left(\frac{V_2}{V_1}\right) = 621.2 \times \frac{1}{9}$$

$$V_{r_2} = 69.022$$

From the table :

T(K)	U(KJ/Kg)	V_r
700	512.33	69.76
710	520.23	67.07

With linear interpolation:

$$T_2 = 702.74\ K$$

$$U_2 = 514.50\ KJ/Kg$$

• Stage 03 : For 2-3 , heat addition at constant volume : $U_3 = \frac{Q_{23}}{m} + U_2$

$$Q_{23} = m(U_3 - U_2) \therefore$$

As we know all relevant variables at Stage 01, we can calculate the mass, m, of the fluid:

$$PV = nRT = \frac{m}{\overline{MW}} RT$$

$\overline{MW} \longrightarrow$ molecular weight

$\hookrightarrow$ 28.97 Kg/Kgmole

$$m = \frac{P_1 V_1}{R T_1} \overline{MW}$$

$$m = (100\ KPa)(0.014\, m^3)\left(\frac{1}{300K}\right)\left(\frac{Kgmole.K}{8.314\ KJ}\right) \times \left(28.97\ \frac{Kg}{Kgmole}\right)$$

$$\times \left(\frac{10^3\ N/m^3}{1\ KPa}\right)\left(\frac{1\ KJ}{10^3\ Nm}\right)$$

$$\boxed{m = 0.01626\ Kg}$$

Thus in

$$U_3 = \frac{Q_{23}}{m} + U_2 = \frac{11.35\ KJ}{0.01626\ Kg} + 514.50\ KJ/Kg = 1212.53\ KJ/Kg$$

From the Table:

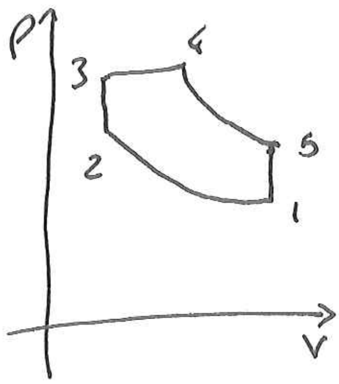
T(K)	H(KJ/Kg)	U(KJ/Kg)
1500	1635.97	1205.41
1520	1660.23	1223.87

Through linear interpolation:

$T_3 = 1507.71\ K$

$H_3 = 1645.33\ KJ/Kg$

- **State 04**: For constant pressure heat addition



$$Q_{34} = m\,(u_4 - u_3)$$

$$u_4 = \frac{Q_{34}}{m} + u_3$$

$$u_4 = \frac{11.35\ kJ}{0.01626\ kg} + 1645.33\ \frac{kJ}{kg} = \underline{2343.36\ kJ/kg}$$

~~[illegible]~~ From the table:

T(K)	u (kJ/kg)	V_r
2050	2314.6	2.555
2100	2377.7	2.356

Through linear interpolation:

$T_4 = 2072.79\,K$

$V_{r4} = 2.4643$

- **State 05**: isentropic expansion

$$\frac{V_{r5}}{V_{r4}} = \frac{V_5}{V_4} \therefore V_{r5} = V_{r4}\frac{V_5}{V_4} = V_{r4}\frac{V_1}{V_4} = V_{r4}\left(\frac{V_1}{V_2}\,\frac{V_3}{V_4}\right)$$

$$\text{As } P_3 = P_4 \Rightarrow \frac{V_3}{V_4} = \frac{T_3}{T_4}$$

$$V_{r5} = V_{r4}\left(\frac{V_1}{V_2}\frac{T_3}{T_4}\right) = 2.4643\left(9 \times \frac{1507.71}{2072.79}\right)$$

$$\boxed{V_{r5} = 16.13}$$

From the table:

T (K)	U (KJ/Kg)	V_r
1140	880.35	16.946
1160	897.91	16.064

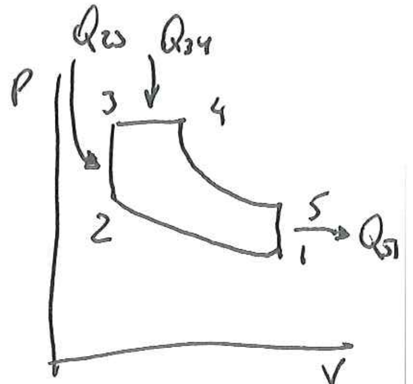
Through linear interpolation

$T_5 = 1158.50\,K$

$U_5 = 896.60$ KJ/Kg

Now all $\underline{T_i}$ are calculated, and W_{cycle}/m:

$$\frac{W_{cycle}}{m} = \frac{Q_{cycle}}{m} = \frac{Q_{23}+Q_{34}-Q_{51}}{m}$$



$$\frac{W_{cycle}}{m} = \frac{22.70 - m(U_5 - U_1)}{m}$$

$$= \frac{22.70 - 0.01626\,(896.60 - 214.07)}{0.01626} = \underline{\underline{713.53\ \text{KJ/Kg}}}$$

And the efficiency:

$$\eta = \frac{W_{cycle}}{Q_{in}} = \frac{11.60}{22.70} = 0.511 \hookrightarrow 51.1\%$$

$Q_{in} \downarrow Q_{34} + Q_{23}$

And the Mean Effective Pressure (MEP):

$$MEP = \frac{W_{cycle}}{V_1(1 - V_2/V_1)} = \frac{11.60\ KJ}{0.014m^3(1-1/9)} \times \frac{10^3 Nm}{1KJ}\ \frac{1KPa}{10^3\ N/m^2}$$

$$MEP = 932.14\ KPa$$

Back from the Past - Or defining $\underline{\underline{V_2}}$

From the second law we know that for ideal gas:

$$\begin{cases} dU = TdS - PdV & (1) \\ dH = TdS + VdP & (2) \end{cases}$$

Replacing $dU = C_v dT$, $dH = C_p dT$ and $PV = RT$ in (1) and (2):

$$\begin{cases} dS = C_v \dfrac{dT}{T} + R\dfrac{dV}{V} & (3) \\ dS = C_p \dfrac{dT}{T} - R\dfrac{dP}{P} & (4) \end{cases}$$

with: $C_p + C_v = R$. Integrating (3-4) from (T_1, P_1) to (T_2, P_2) conditions

$$\begin{cases} S(T_2, V_2) - S(T_1, V_1) = \displaystyle\int_{T_1}^{T_2} C_v \frac{dT}{T} + R \ln\left(\frac{V_2}{V_1}\right) & (5) \\ S(T_2, P_2) - S(T_1, P_2) = \displaystyle\int_{T_1}^{T_2} C_p \frac{dT}{T} - R \ln\left(\frac{P_2}{P_1}\right) & (6) \end{cases}$$

Now, if we define

$$S^{\circ}(T) = \int_0^T C_p \frac{dT}{T}$$

as the entropy at 1 atm and temperature T.
Also assuming that temperature is continuous and differentiable through all domain:

$$\int_{T_1}^{T_2} C_p \frac{dT}{T} = \int_0^{T_2} C_p \frac{dT}{T} - \int_0^{T_1} C_p \frac{dT}{T} = S^{\circ}(T_2) - S^{\circ}(T_1)$$

Thus, replacing in ~~(5) and~~ (6):

$$S(T_2, P_2) - S(T_1, P_1) = S^{\circ}(T_2) - S^{\circ}(T_1) - R \ln\left(\frac{P_2}{P_1}\right) \quad (7)$$

~~And using the same as above in (5)~~):

~~S(T_2, P_2)-~~

Eqn (7) is valid if we assume that C_p is a function of T, i.e. $C_p = C_p(T)$ and we only need to integrate the $C_p(T)\frac{dT}{T}$ term. However if C_p is constant, the (5) and (6) can be easily integrated resulting in:

$$\begin{cases} S(T_2, V_2) - S(T_1, V_1) = C_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{V_2}{V_1}\right) & (8) \\ S(T_2, P_2) - S(T_1, P_1) = C_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right) & (9) \end{cases}$$

Now for isentropic processes — $\underline{S_2 = S_1}$, Eqn. (8) becomes

$$C_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{V_2}{V_1}\right) = 0 \qquad (\cdot 1/C_v)$$

$$\ln\left(\frac{T_2}{T_1}\right) = -\frac{R}{C_v} \ln\left(\frac{V_2}{V_1}\right) = \ln\left(\frac{V_1}{V_2}\right)^{R/C_v}$$

Since: $R = c_p - c_v$, $\gamma = c_p / c_v$ $\Big\} \; R/c_v = \gamma - 1$

$$\left(\frac{T_2}{T_1}\right)_{S=\text{const}} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \qquad (10)$$

Applying the same procedure to Eqn. (9):

$$\left(\frac{T_2}{T_1}\right)_{S=\text{Const}} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \qquad (11)$$

Now (10)=(11):

$$\left(\frac{P_2}{P_1}\right)_S = \left(\frac{V_1}{V_2}\right)^{\gamma} \qquad (12)$$

Eqns (10-12) - isentropic relations for ideal gas may also be rewritten as:

$$\left| \begin{array}{l} T V^{\gamma-1} = \text{constant} \\ T P^{\frac{1-\gamma}{\gamma}} = \text{constant} \\ P V^{\gamma} = \text{constant} \end{array} \right| \qquad (13)$$

In most cases specific heats are not constant, thus we need to use Eqn (7),

$$S(T_2,P_2)-S(T_1,P_1)=S^\circ(T_2)-S^\circ(T_1)-R\ln\left(\frac{P_2}{P_1}\right) \quad (7)$$

and for an isentropic process 1-2:

$$0=S^\circ(T_2)-S^\circ(T_1)-R\ln(P_2/P_1)$$

$$S^\circ(T_2)=S^\circ(T_1)+R\ln(P_2/P_1) \quad (14)$$

Now ~~taking~~ making P_2/P_1 explicit:

$$\exp\left[\frac{S^\circ(T_2)-S^\circ(T_1)}{R}\right]=\frac{P_2}{P_1}$$

$$P_2/P_1=\frac{\exp[S^\circ(T_2)/R]}{\exp[S^\circ(T_1)/R]} \quad (15)$$

The term $\exp[S^\circ/R]$ is the relative pressure P_r; therefore

$$\boxed{\left(\frac{P_2}{P_1}\right)_s=\frac{P_{r_2}}{P_{r_1}}} \quad (16)$$

So $\underline{S^\circ}$ is a function of the temperature only, $\underline{P_r}$ is also a function of temperature only. $\underline{P_r}$ can be obtained from tabulated tables.

Now using the ideal-gas equation:

$$\frac{PV}{T} = \text{constant}$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \therefore \frac{V_2}{V_1} = \frac{T_2}{T_1}\frac{P_1}{P_2} = \frac{T_2}{T_1}\frac{P_{r1}}{P_{r2}}$$

$$V_2/V_1 = \frac{T_2/P_{r2}}{T_1/P_{r1}}$$

The term T/P_r is called relative specific volume $\underline{V_r}$, and just as the relative pressure is a function of the temperature only:

$$\boxed{\left(\frac{V_2}{V_1}\right)_s = \frac{V_{R2}}{V_{R1}}} \qquad (17)$$