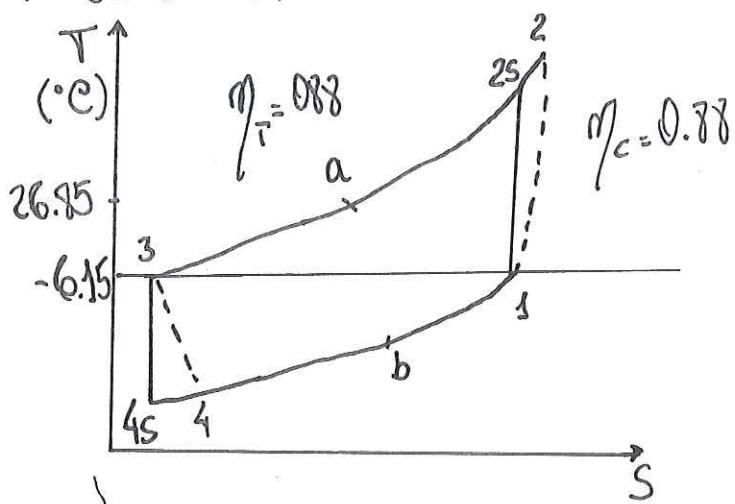


## u 04: Refrigeration and Liquefaction

Problem 02: same as P01 but with compressor and turbine isentropic efficiencies of 88%. The TS diagram then become:



In a similar way we have done in P01, we should use the ideal gas properties of air tables to compute the properties

of each state: From P01, we know that:

- State 1:  $U_1 = 267.10 \text{ kJ/kg}$
- State 2s:  $U_{2s} = 355.17 \text{ kJ/kg}$
- State a:  $U_a = 300.19 \text{ kJ/kg}$
- State 3:  $U_3 = 267.10 \text{ kJ/kg}$
- State 4s:  $U_{4s} = 200.88 \text{ kJ/kg}$
- State b:  $U_b = 234.01 \text{ kJ/kg}$

The efficiencies of the turbine and compressor lead to:

• State 2 :

$$\eta_c = \frac{H_{2s} - H_1}{H_2 - H_1} \therefore H_2 = H_1 + \frac{H_{2s} - H_1}{\eta_c} = 267.10 + \frac{355.17 - 267.10}{0.88}$$

$$H_2 = 367.18 \text{ kJ/kg}$$

• State 4 :

$$\eta_t = \frac{H_3 - H_4}{H_3 - H_{4s}} \therefore H_4 = H_3 - \eta_t (H_3 - H_{4s})$$

$$H_4 = 267.10 - 0.88 (267.10 - 200.88)$$

$$H_4 = 208.83 \text{ kJ/kg}$$

(a) Now calculating the updated air mass flow rate

$$\dot{m} = \frac{\dot{Q}_{in}}{H_b - H_4} = \frac{15 \text{ tons}}{234.01 - 208.83 \text{ kJ/kg}} \times \frac{1.4 \times 10^4 \text{ kJ/h}}{1 \text{ ton}}$$

$$\dot{m} = 8339.95 \text{ kg/h}$$

and the volumetric flow rate :

$$\dot{V} = \frac{\dot{m} R T_1}{P_s} = 8339.95 \frac{\text{kg}}{\text{h}} \times \frac{8.314 \times 10^{-3} \text{ m}^3 \cdot \text{bar}}{\text{mol} \cdot \text{K}} \times 267 \text{ K} \times \frac{1 \text{ mol}}{28.97 \text{ g}} \times \frac{1000 \text{ g}}{1 \text{ kg}}$$

$$\times \frac{1 \text{ h}}{60 \text{ min}} \times \frac{1}{1 \text{ bar}}$$

$$\dot{V} = 106.51 \text{ m}^3/\text{min}$$

(b) The COP for the new cycle is :

$$\begin{aligned} \text{COP} &= \frac{\text{Refrigerant Effect}}{\text{Work done}} = \frac{H_b - H_a}{(H_2 - H_1) - (H_3 - H_4)} \\ &= \frac{234.01 - 208.83}{(367.18 - 267.10) - (267.10 - 208.83)} \end{aligned}$$

$$\boxed{\text{COP} = 0602}$$

Comparing P01 with P02 :

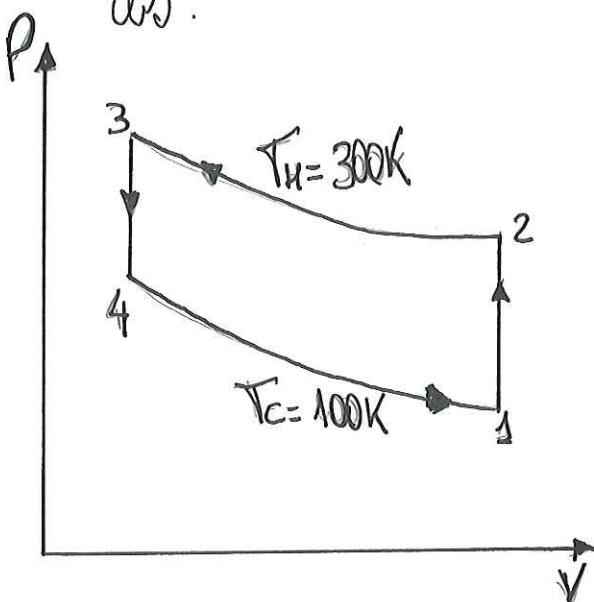
	P01	P02
$\dot{V}_t$	80.95	106.51
COP	1.516	0602

The volumetric flow rate is larger in P02 with lower COP. This is effectively due to the irreversibilities in the compressor and turbine.

04

Problem 03: Stirling refrigeration cycle with known compression ratio. The PV diagram can be drawn

as:



Given:  $P_2 = 100 \text{ kPa}$

$$\frac{\sqrt{V_1}}{\sqrt{V_4}} = \frac{\sqrt{V_2}}{\sqrt{V_3}} = 6$$

(a) Heat transfer during ~~and then~~ isothermal expansion (4  $\rightarrow$  1)

(b) Net work for the cycle

(c) COP

(a) The energy balance for 4-1 reduces to (from the 1st law):

$$\frac{Q_{41}}{m} = \cancel{(U_1 - U_4)} + \frac{W_{41}}{m}$$

(isothermal)

$$\int_4^1 \frac{RT_c}{V} dV$$

The work is calculated from:

$$\frac{W_{41}}{m} = \int_4^1 P dV = RT_c \ln \frac{\sqrt{V_1}}{\sqrt{V_4}}$$

ln (6)

$$= 8.314 \frac{J}{kg \cdot K} \times \frac{1 \text{ kgol}}{28.97 \text{ g}} \times 100 \text{ K} \times \frac{1000 \text{ g}}{1 \text{ kg}} \times \frac{1 \text{ KJ}}{1000 \text{ J}}$$

$$W_{41}/m = 51.42 \text{ KJ/kg} = Q_{41}/m$$

(b) The work for process 2-3 is:

$$\begin{aligned}\frac{\underline{W_{23}}}{\dot{m}} &= \int_2^3 P dV = RT_u \ln \frac{\sqrt{V_3}}{\sqrt{V_2}} \\ &= 8.314 \frac{J}{K \cdot \text{mol}} \times \frac{1 \text{ mol}}{28.97 \text{ g}} \times \frac{1000 \text{ g}}{1 \text{ kg}} \times \frac{3 \text{ K}}{1000 \text{ J}} \times 300 \text{ K} \times \ln(1/6)\end{aligned}$$

$$\underline{W_{23}/\dot{m}} = -154.26 \text{ KS/Kg}$$

Therefore  $\frac{\underline{W_{\text{cycle}}}}{\dot{m}} = \left| \frac{\underline{W_{23}}}{\dot{m}} \right| - \frac{\underline{W_{41}}}{\dot{m}} = 154.26 - 51.42$

$$\frac{\underline{W_{\text{cycle}}}}{\dot{m}} = 102.84 \text{ KS/Kg}$$

(c)

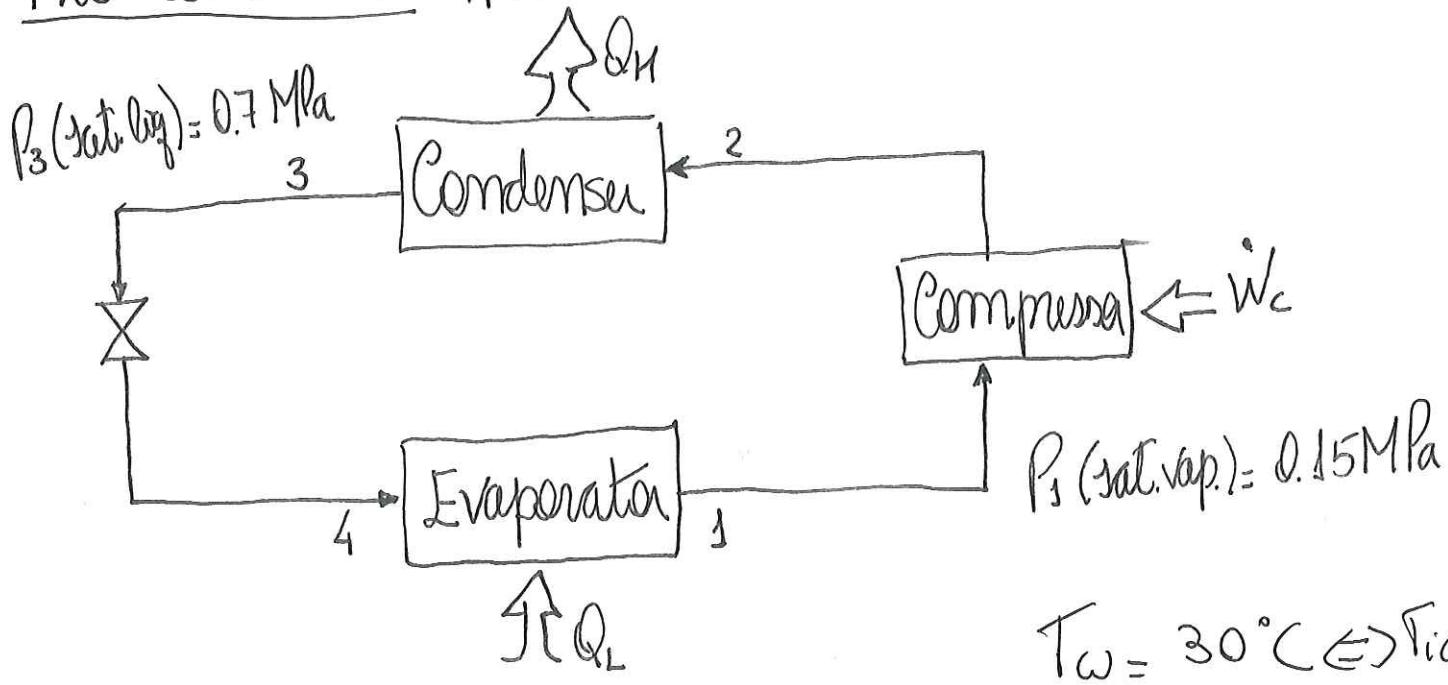
$$\text{COP} = \frac{\text{Refrigerant Effect}}{\text{Work done}} = \frac{\underline{Q_{41}/\dot{m}}}{\underline{W_{\text{cycle}}/\dot{m}}} = \frac{51.42}{102.84}$$

$$\text{COP} = 0.5$$

Alternatively, we could calculate the COP as (similar to reversed Carnot cycle):

$$\text{COP} = \frac{T_c}{T_h - T_c} = \frac{100}{300 - 100} = 0.5$$

# Problem 05: R134a



We need to remove heat to convert ~~kg~~ kg of water into ice at  $-5^\circ\text{C}$

$$Q = \dot{m} C_{p,w} (T_w - \theta) + \dot{m} L_{f,ice} + \dot{m} C_{p,ice} (0 - T_{ia})$$

$\hookrightarrow$  latent heat (kJ/kg)  
 $\hookrightarrow$  of ice  
 $\hookrightarrow$  mass of water/ice (kg)

$$Q = 1 \times 4.18 (30 - 0) + 1 \times 334 + 1 \times 2.1 \times (0 - (-5))$$

$$Q = 469.90 \text{ kJ/kg}$$

If we want to produce 10 kg/h of ice, the <sup>required</sup> refrigerant effect is

$$R_n = 10 \frac{\text{kg}}{\text{h}} \times 469.90 \frac{\text{kJ}}{\text{kg}} \times \frac{1\text{h}}{3600\text{s}} = 1.31 \frac{\text{kJ}}{\text{s}} = \underline{\underline{1.31 \text{ kW}}}$$

Now calculating the enthalpies

- $\underline{H_1} : ? \quad P_1 = 1.5 \text{ bar} \therefore \text{from the saturated}$  ~~gased~~ <sup>vapour</sup>

table:

$P(\text{bar})$	$T_{\text{sat}} (\text{°C})$	$H (\text{kJ/kg})$		$S (\text{kJ/kgK})$	
		Sat. Liq.	Evap.	Sat. Vap	Sat. Liq
1.4	-18.80	25.77	210.27	236.04	0.1055
1.6	-15.62	29.78	208.19	237.97	0.1211

From linear interpolation:

P

$$(1.6 - 1.4) \quad - \quad (237.97 - 236.04)$$

$$(1.5 - 1.4) \quad - \quad y$$

$$y = 0.965$$

$$\hookrightarrow H_1 = 237.00 \text{ kJ/kg}$$

$$S_1 = 0.9309 \text{ kJ/kgK}$$

- $\underline{H_2} : ?$

(1-2) is isentropic process  $\Rightarrow S_1 = S_2$

and  $P_2 = P_3 = 0.7 \text{ MPa} = 7 \text{ bar}$ .

The fluid is superheated therefore:

$T(\text{°C})$	$H (\text{kJ/kg})$	$S (\text{kJ/kgK})$
30	265.37	0.9197
40	275.93	0.9539

and from linear interpolation

$$(275.93 - 265.37) = (0.9539 - 0.9197)$$

$$y = (0.9539 - 0.9309)$$

$$y = 0.1018$$

$$\therefore h_2 = 268.83 \text{ kJ/kg}$$

$h_3$  : ? Saturated liquid ( $P_3 = 7 \text{ bar}$ )

$P(\text{bar})$	$T_{\text{sat}}(\text{C})$	$h(\text{kJ/kg})$		$s(\text{kJ/kg K})$	
		Sat. Lig.	Evap.	Sat. Vap.	Sat. Lig.
7.0	26.72	86.78	175.07	265.85	0.3242 0.9080

$$h_3 = 86.78 \text{ kJ/kg}$$

$h_4$  : ?  $\therefore$  Process 3-4 is isenthalpic (expansion valve)  $\therefore h_3 = h_4$

The refrigerating effect is given by:

$$R_m = 1.31 \text{ kW} = 1.31 \frac{\text{kJ}}{\text{s}} = \dot{m}_R (h_3 - h_4)$$

$$\dot{m}_R = \left( \frac{237.00 - 86.78}{1.31} \right)^{-1} \frac{(\text{kJ/kg})}{(\text{kJ/s})}$$

$$\dot{m}_R = 0.00872 \text{ kg/s}$$

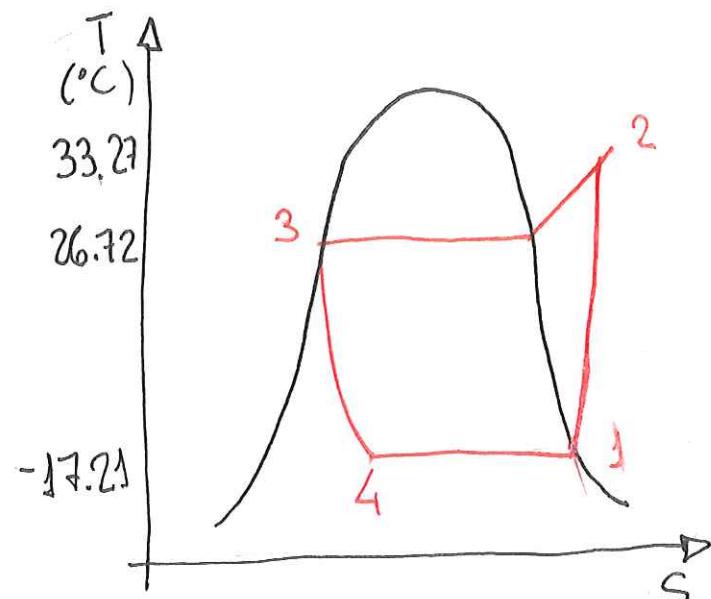
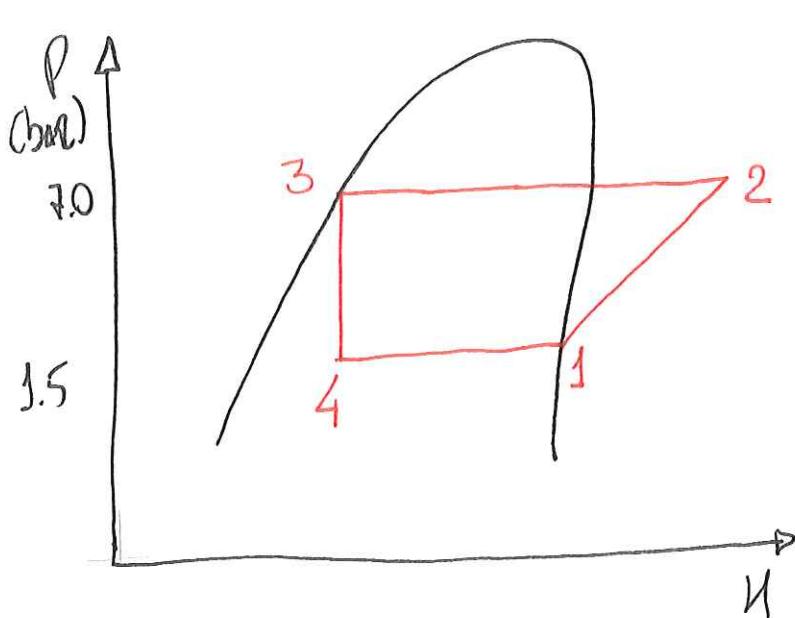
The compression work -  $\dot{W}_c$

$$\dot{W}_c = \dot{m}_R (H_2 - H_1) = 0.00872 \frac{\text{kg}}{\text{s}} \left[ \frac{268.83 - 237.0}{\text{kJ/kg}} \right]$$

$$\dot{W}_c = 0.2775 \frac{\text{kJ}}{\text{s}} = \underline{\underline{0.2775 \text{ kW}}}$$

The coefficient of performance :

$$\text{COP} = \frac{R_n}{\dot{W}_c} = \frac{1.35}{0.2775} = \underline{\underline{4.72}}$$



If we use  $\text{NH}_3$  instead of R134a as refrigerant,  
 we can calculate the whole cycle to produce the  
 same refrigerant effect

$\text{H}_1$ ( $P_1 = 1.5 \text{ bar} \therefore \text{from saturated vapour table}$ )					
$P(\text{bar})$	$T_{\text{sat}} (\text{°C})$	Sat. Liq.	Evap.	Sat. Vap.	S
1.50	-25.22	65.32	345.28	1410.63	0.2712 5.6973

$$H_1 = 1410.63$$

$$S_1 = 5.6973$$

- $H_2$ : 1-2 is isentropic process ( $S_1 = S_2$ ) with  $P_2 = P_3 = 0.7 \text{ MPa} = 7 \text{ bar}$  and the ~~superheat~~ superheated fluid:

$T(\text{°C})$	$H$	$S$
80	1625.56	5.6939
100	1673.46	5.8258

$$\hookrightarrow H_2 = (1673.46 - 1625.56) = 5.8258 - 5.6939 \\ y = (5.6973 - 5.6939)$$

$$H_2 = 1626.79 \text{ kJ/kg} \quad \begin{matrix} \leftarrow \\ (T_2 = 80.52^\circ\text{C}) \end{matrix} \quad \begin{matrix} \leftarrow \\ 1.2347 \end{matrix}$$

- $\underline{H_3}$  : saturated liquid ( $P_3 = 7 \text{ bar}$ )

$P(\text{bar})$	$T_{\text{sat}}(^\circ\text{C})$	$H$			$S$	
		Sat. Liq.	Evap.	Sat. Vap.	Sat. Liq.	Sat. Vap.
7.0	13.79	244.69	1410.38	1455.07	0.9394	5.1576

$$H_3 = 244.69 \text{ kJ/kg}$$

- $\underline{H_4}$  : 3-4 is isenthalpic (expansion valve)

$$H_4 - H_3 = 244.69 \text{ kJ/kg}$$

The refrigeration effect is given by

$$R_m = 1.31 \text{ kW} = 1.31 \frac{\text{kJ}}{\text{s}} = \dot{m}_R (1410.61 - 244.69)$$

$$\dot{m}_R = 1.12 \times 10^{-3} \text{ kg/s}$$

And the compression work:

$$\dot{W}_c = \dot{m}_R (H_2 - H_1) = 1.12 \times 10^{-3} (1626.79 - 1410.61)$$

$$\dot{W}_c = \underline{0.2428 \text{ kW}}$$

And the COP

$$COP = \frac{1.31}{0.2428} = 5.39$$

Now for propane :

$$U_1 (P_1 = 1.5 \text{ bar}) = 430.35 \text{ kJ/kg}$$

$$g_s (P_1 = 1.5 \text{ bar}) = 1.802 \text{ kJ/(kg.K)}$$

$$U_2 (P_2 = 7 \text{ bar}) = 504.42 \text{ kJ/kg}$$

$$U_3 (P_3 = 7 \text{ bar}) = 129.6 \text{ kJ/kg}$$

$$U_4 = U_3 = 129.6 \text{ kJ/kg}$$

$$\dot{m}_R = \frac{1000}{430.35 - 129.6} \cdot \frac{1.31}{1.31} = 4.36 \times 10^{-3}$$

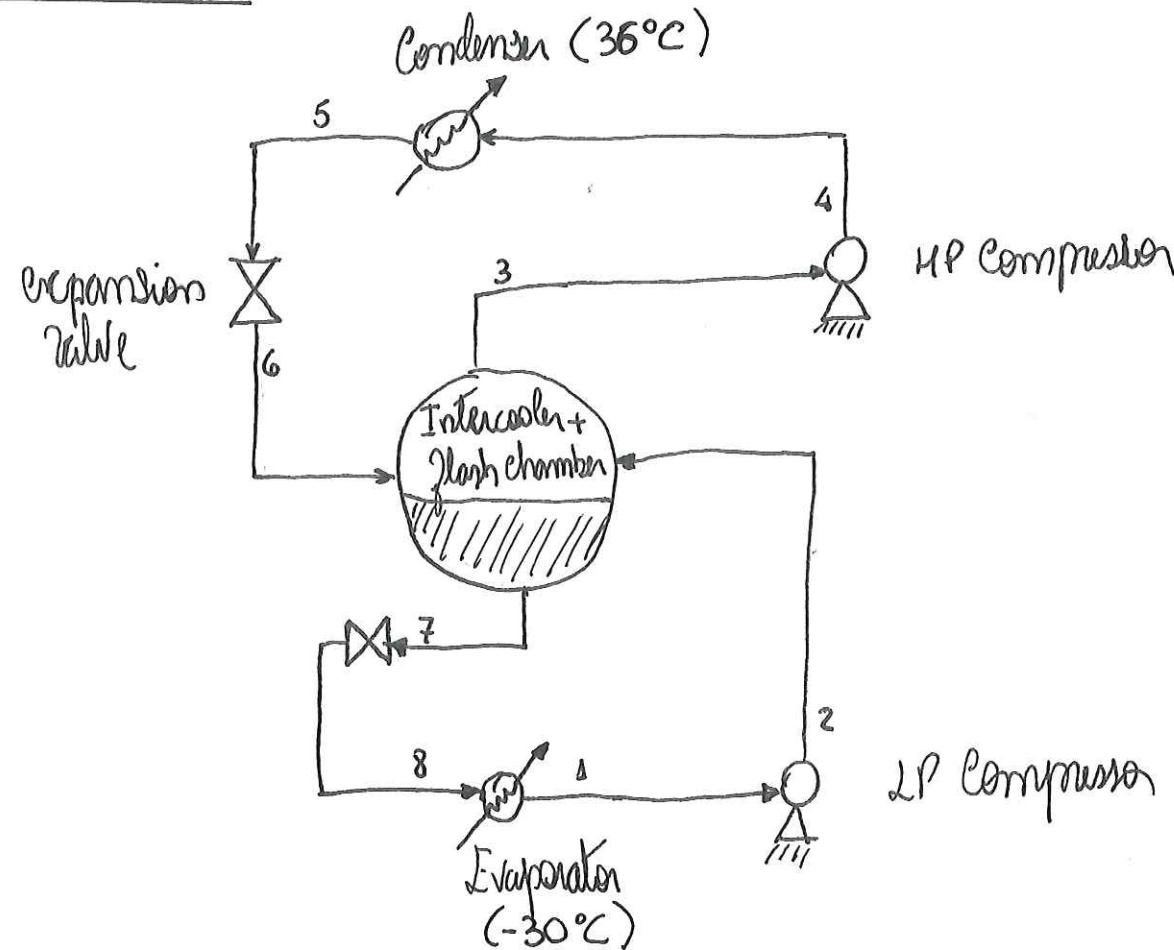
$$\dot{\omega}_c = \frac{0.3226}{1000} \text{ kW}$$

$$COP = 4.06$$

	R-134a	NH <sub>3</sub>	Propane
$\mu_1$	237.00	1410.61	430.35
$\mu_2$	268.83	1626.79	504.42
$\mu_3$	86.78	244.69	129.6
$\mu_4$	86.78	244.26	129.6
$\dot{m}_n$	$8.72 \times 10^{-3}$	$1.12 \times 10^{-3}$	$4.36 \times 10^{-3}$
$\dot{\omega}_c$	0.2775	0.2428	0.3226
COP	4.72	5.39	4.06

# Problem 06:

NH<sub>3</sub>



At 30°C (243.15K), the Saturated ammonia Table

T(°C)	P(bar)	H(kJ/kg)		S(kJ/kg, K)	
		sat. liq.	evap.	sat. vap	sat. liq.
-30	1.1950	44.10	1359.65	1403.75	0.1849
36°C	13.896	351.69	1117.00	1468.70	5.7767

Thus:  $P_s$  (saturation pressure at -30°C) = 1.1950 bar  
 $P_d$  (saturation pressure at 36°C) = 13.896 bar

The intermediate pressure is given by

$$P_i = (P_s \cdot P_d)^{1/2} = 4.0750 \text{ bar}$$

The enthalpies of each stage are

- $H_1 = H_g(T = -30^\circ\text{C}) = 1403.75 \text{ kJ/kg}$
- $\underline{H_2} : ? \therefore$  The fluid at (2) is reheatated vapour at  $P_2 = P_i = 4.0750 \text{ bar}$ . ~~100000 Pa~~ As  $1-2$  is an isentropic compression  $S_1 = S_2 = 5.7767 \text{ kJ/kg.K}$ , the superheated vapour table gives

$T(\text{ }^\circ\text{C})$	$H(\text{kJ/kg})$	$S(\text{kJ/kg.K})$
40	1543.38	5.7101
60	1590.17	5.8549

The linear interpolation results in

$$\frac{T}{(60-40)} = \frac{H}{(1590.17 - 1543.38)} = \frac{S}{(5.8549 - 5.7101)}$$

$$y = 21.52$$

$$\therefore \underline{H_2} = 1564.90 \text{ kJ/kg}$$

- $\underline{H_3} : ? \therefore H_3 = H_g (P_3 = P_i = 4.0750 \text{ bar})$

From saturated ammonia table:

P(bar)	$T(^{\circ}\text{C})$	$\mu(\text{kJ/kg})$		$s(\text{kJ/kg.K})$		$\frac{12}{\text{kg}}$
		Sat. Liq.	Evap.	Sat. Vap.	Sat. Liq.	
4.00	-1.90	171.18	1268.71	1439.89	0.6776	5.3548
4.25	-0.29	178.62	1263.01	1441.63	0.7048	5.3336

with linear interpolation:

$$\frac{P}{(4.25 - 4.00)} = \frac{H}{(1441.63 - 1439.89)} = \frac{5.3548 - 5.3336}{y}$$

$$\frac{(4.0750 - 4.00)}{y} = \frac{-}{y'} = \frac{y}{y'}$$

$$y = \cancel{0.000} .0522$$

$$\hookrightarrow \mu_3 = 1440.41 \text{ kJ/kg}$$

$$y' = 6.36 \times 10^{-3} \quad \frac{\text{kJ}/(\text{kg.K})}{S_3 = 5.3484 \cancel{0.00000}}$$

•  $\underline{\mu_4} : ? \quad P_4 = P_d = 13.896 \text{ bar} \quad \left\{ \text{superheated vapour} \right.$   
 $S_4 = S_3 = 5.3484 \text{ kJ/kg.K} \quad \left. \right\}$

For simplicity, let's assume the pressure is  $\sim 14 \text{ bar}$  and  
 from the superheated table:

$T(^{\circ}\text{C})$	$\mu(\text{kJ/kg})$	$s(\text{kJ/kg.K})$
80	1598.59	5.2984
100	1651.20	5.4433

13

with linear interpolation

$$H \quad S$$
$$(-1598.59 + 1651.20) = (5.4433 - 5.2984)$$

$$y = (5.4433 - 5.2984)$$

$$y = 34.4561$$

$$\hookrightarrow H_4 = 1633.46 \text{ kJ/kg}$$

•  $H_5 = H_2 (T = 36^\circ\text{C}) = 351.69 \text{ kJ/kg}$

•  $H_6 = H_5 = 351.69 \text{ kJ/kg}$

•  $H_7 = H_f (P_f = 4.0750 \text{ bar})$

~~saturated liquid~~

$$P \quad H$$
$$(4.25 - 4.00) = (178.62 - 171.18)$$

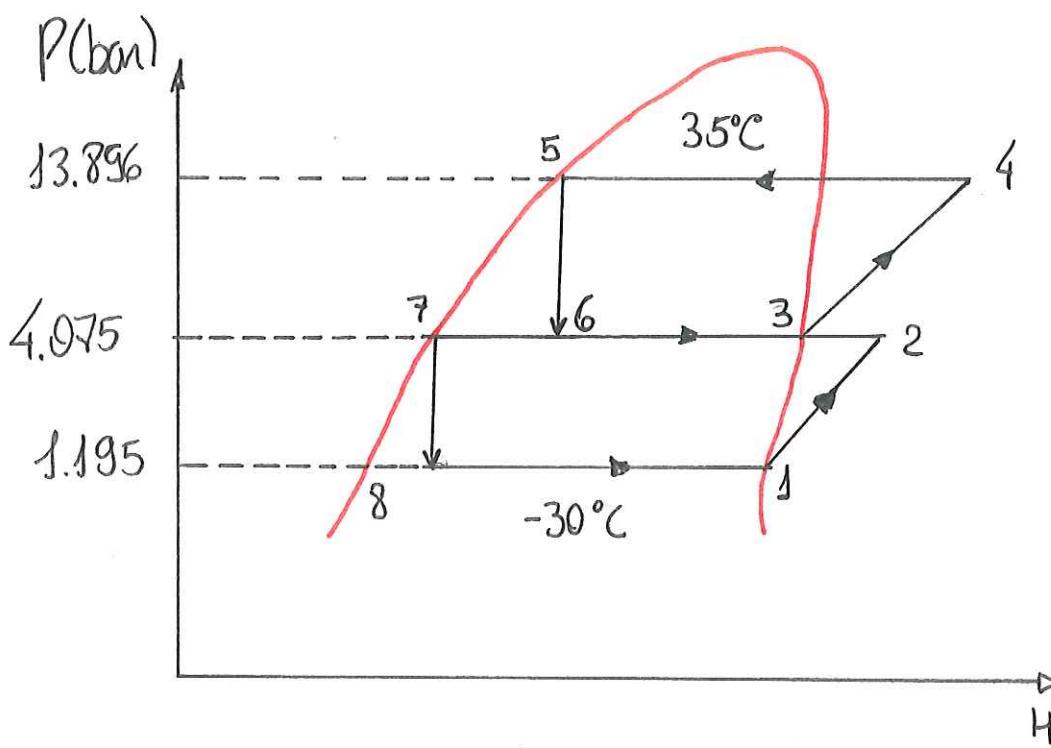
$$(4.0750 - 4.00) =$$

$$y = 2.232$$

$$\hookrightarrow H_7 = 173.41 \text{ kJ/kg}$$

•  $H_8 = H_7 = 173.41 \text{ kJ/kg}$

The PH diagram becomes



The energy balance in the evaporator (1-8)

$$\dot{Q} = \dot{m}_1 (H_1 - H_8) = 300 \text{ kW} = 300 \frac{\text{kJ}}{\text{s}}$$

$$\dot{m}_1 = \frac{\dot{Q}}{H_1 - H_8} = \frac{300 \text{ (kJ/s)}}{(1403.75 - 173.41) \text{ kJ/kg}}$$

$$\boxed{\dot{m}_1 = 0.2438 \text{ kg/s}}$$

$$\hookrightarrow \dot{m}_1 = \dot{m}_2 = \dot{m}_7 = \dot{m}_8$$

The heat and mass balance in the intercooler :

$$\dot{m}_2 H_2 + \dot{m}_6 H_6 = \dot{m}_7 H_7 + \dot{m}_3 H_3$$

↳ with  $\dot{m}_3 = \dot{m}_6$

$$0.2438 \times 1564.90 + \dot{m}_3 \times 351.69 = 0.2438 \times 173.41 + \dot{m}_3 \times 1440.41$$

$$\boxed{\dot{m}_3 = 0.3116 \text{ kg/s}}$$

(a) Power for the compressors:

$$\text{LP: } P_{LP} = \dot{m}_2 (U_2 - U_1) = 0.2438 (1564.90 - 1493.75)$$

$$P_{LP} = 39.28 \frac{\text{KJ}}{\text{s}} = \boxed{39.28 \text{ KW}}$$

$$\text{HP: } P_{HP} = \dot{m}_3 (U_4 - U_3) = 0.3116 (1633.46 - 1490.41)$$

$$\boxed{P_{HP} = 60.15 \text{ KW}}$$

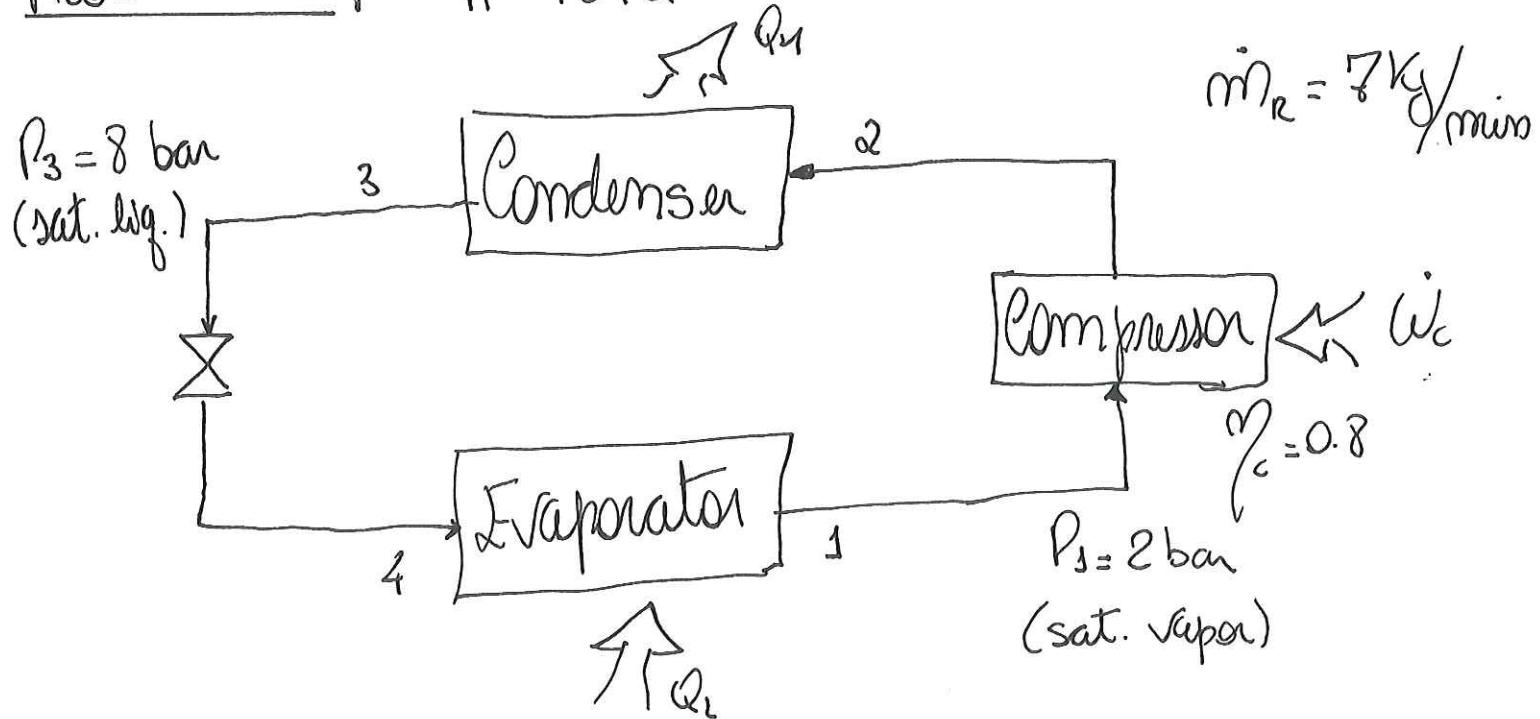
$$\text{Total Power: } P_T = P_{LP} + P_{HP}$$

$$\boxed{P_T = 99.43 \text{ KW}}$$

(b)  $\text{COP} = \frac{\text{Refrigerant Effect}}{\text{Work Done}}$

$$\text{COP} = \frac{300}{99.43} \quad \therefore \boxed{\text{COP} = 3.03}$$

Problem 07: R-134a



• 1:  $H_1 (P_1 = 2 \text{ bar}, \text{sat. vap.}) = 241.30 \text{ kJ/kg}$

$$S_1 = 0.9253 \text{ kJ/kg.K}$$

• 2:  $P_2 = P_3 = 8 \text{ bar}$

$$S_{2s} = S_1 \quad (\text{isentropic compression})$$

↳ from superheated fluid table

$T(^{\circ}\text{C})$	$H(\text{kJ/kg})$	$S(\text{kJ/kg.K})$
31.33	264.55	0.9066
40.00	273.66	0.9374

linear interpolation ↳  $H_{2s} = 269.62 \text{ kJ/kg}$

Updating using the compression efficiency :  $\eta_c = \frac{H_{2s} - H_1}{H_2 - H_1} \therefore H_2 = 277.08 \text{ kJ/kg}$

• 3 :  $P_3 = 8 \text{ bar (sat. liq)}$

$$U_3 = 93.42 \text{ kJ/kg}$$

• 4 : Isenthalpic throttling process :  $U_4 = U_3 = 93.42 \text{ kJ/kg}$

The compressor power can be calculated as

$$\dot{W}_c = \dot{m}_R (U_2 - U_1) = 7 \frac{\text{kg}}{\text{min}} (277.08 - 241.80) \frac{\text{kJ}}{\text{kg}} \times \frac{1 \text{ min}}{60 \text{ s}}$$

$$\dot{W}_c = 4.17 \frac{\text{kJ}}{\text{s}} = 4.17 \text{ kW}$$

The refrigeration effect is (in tons)

$$R_m = \dot{m}_R (U_1 - U_4) = 7 \frac{\text{kg}}{\text{min}} (241.30 - 93.42) \frac{\text{kJ}}{\text{kg}} \times \frac{60 \text{ min}}{1 \text{ h}}$$

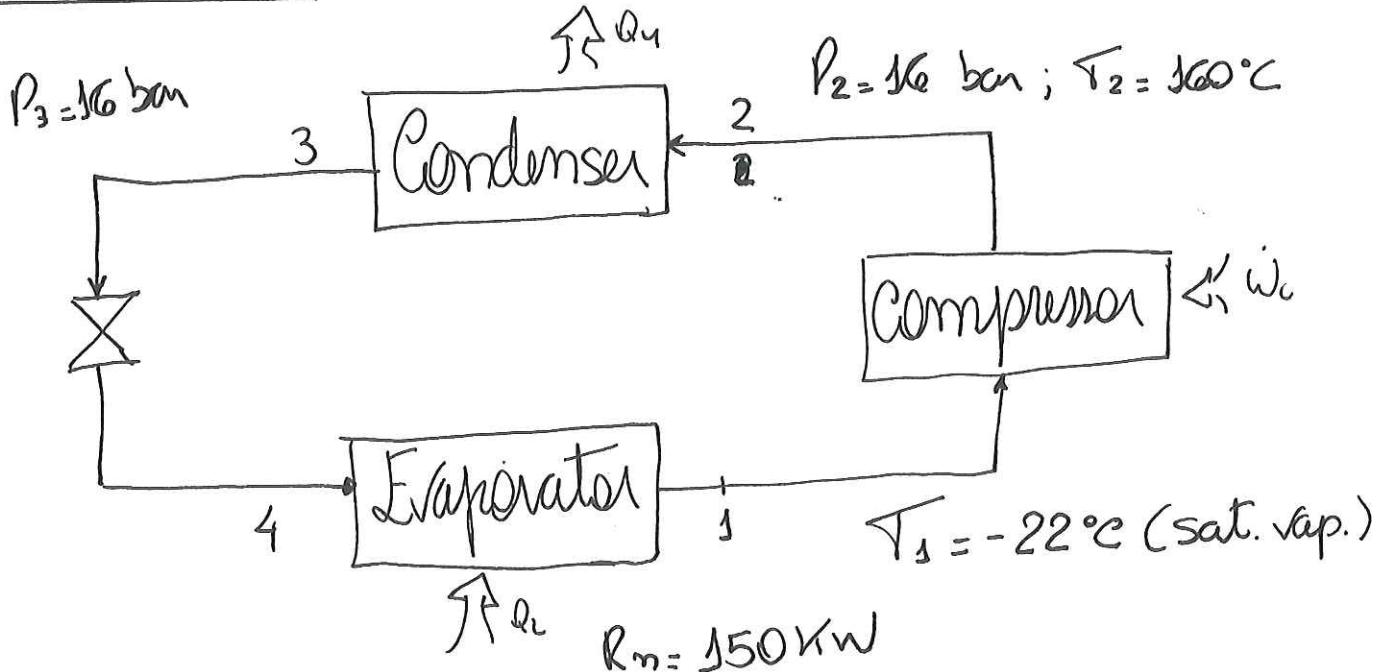
$$R_m = 62109.60 \frac{\text{kg}}{\text{h}} \times \frac{1 \text{ ton}}{0.4 \times 10^4 \text{ kJ/h}}$$

$$R_m = 4.44 \text{ tons} \quad (= 17.25 \text{ kJ/s})$$

And the COP :

$$COP = \frac{R_m}{\dot{W}_c} = 4.14$$

## Problem 08: Ammonia



- 1:  $T_1 = -22^\circ\text{C}$  {  $H_1 = 1415.08 \text{ kJ/kg}$   
 $(\text{Sat. vap})$  {  $S_1 = 5.6457 \text{ kJ/kg.K}$

- 2:  $P_2 = 16 \text{ bar}$  {  $T_{\text{sat}}(P=16 \text{ bar}) = 41.03^\circ\text{C} \ll T_2 = 160^\circ\text{C}$   
 $T_2 = 160^\circ\text{C}$  { thus the fluid is superheated  
 $U_2 = 1798.45 \text{ kJ/kg}$

- 3:  $P_3 = 16 \text{ bar}$  {  $U_3 = 376.46 \text{ kJ/kg}$   
 $(\text{sat. lig.})$  {

- 4: Isenthalpic throttling process:  $U_4 = U_3 = 376.46 \text{ kJ/kg}$

The mass flow rate of the refrigerant can be obtained from the energy balance around the evaporator:

$$R_m = 150 \frac{KJ}{s} = \dot{m}_R (H_2 - H_1)$$

$$\dot{m}_R = 0.1444 \text{ Kg/s}$$

The compressor power is

$$\dot{W}_c = \dot{m}_R (H_2 - H_1)$$

$$\dot{W}_c = 0.1444 \frac{KJ}{s} (1798.45 - 1415.08) \frac{KJ}{Kg}$$

$$\dot{W}_c = 55.36 \text{ KW}$$

COP :

$$COP = \frac{R_m}{\dot{W}_c} = \frac{150}{55.36} = 2.71$$

For isentropic compression, }  $P_{2s} = 16 \text{ bar}$  }  
 $S_{2s} = S_1$  }

$T(^{\circ}\text{C})$	$H(KJ/Kg)$	$S(KJ/KgK)$	
140	1747.72	5.6276	
160	1798.45	5.7478	

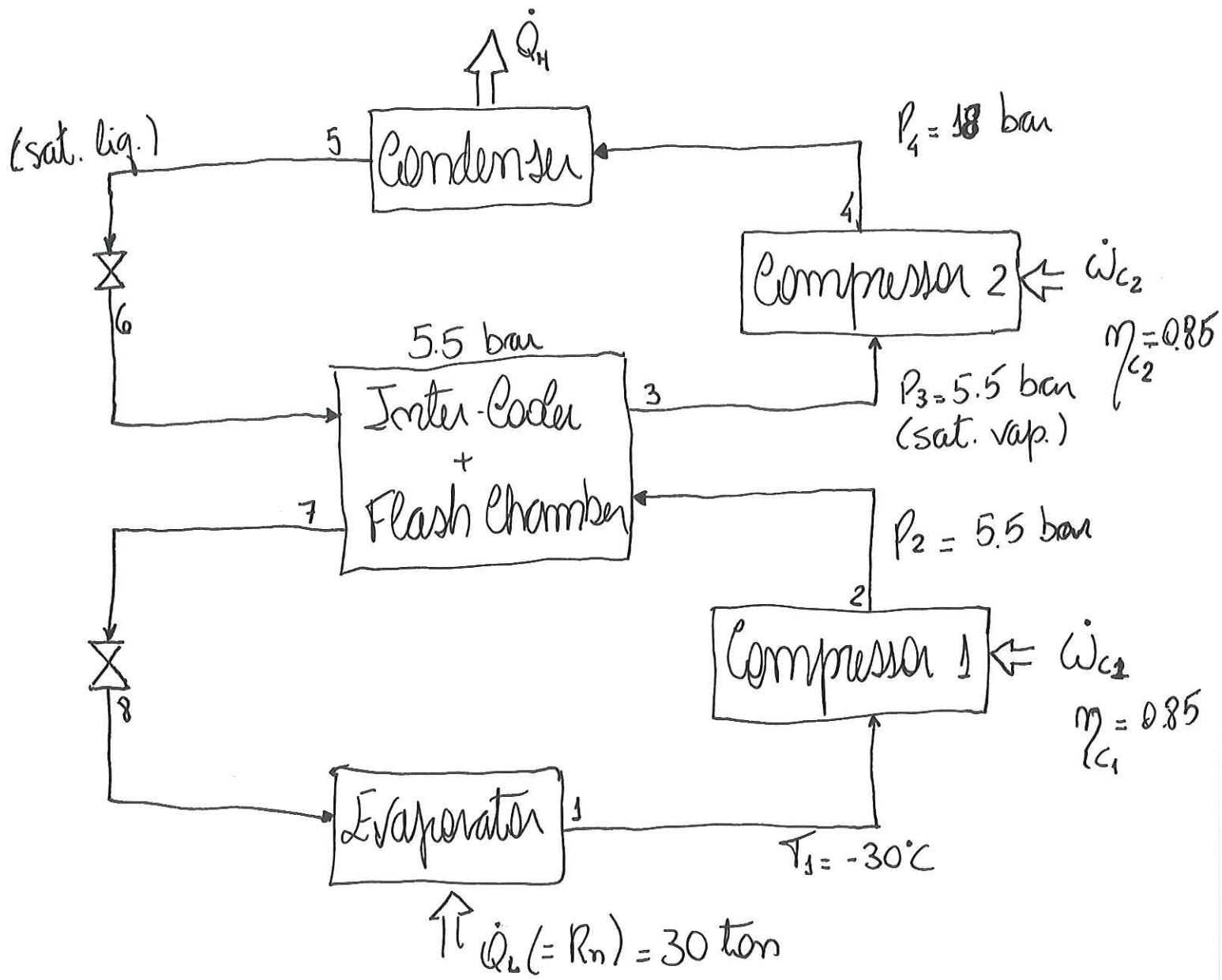


$$\text{Is } H_{2s} = 1755.38 \text{ KJ/Kg}$$

$$\eta_c = \frac{H_{2s} - H_1}{H_2 - H_1} = \frac{1755.38 - 1415.08}{1798.45 - 1415.08}$$

$$\eta_c = 0.89 \therefore 89\%$$

# Problem 09: Ammonia (2 stage FC refrigeration cycle)



As in the previous problems, we first need to compute  $\underline{\mu}$  of all stages of the cycle:

- 1:  $T_1 = -30^\circ\text{C}$  (sat. vap.)  $\rightarrow \underline{\mu}_1 = 1403.75 \text{ kJ/kg}$   
 $S_1 = 5.7767 \text{ kJ/kg.K}$
- 2:  $P_2 = 5.5 \text{ bar}$ , where 1-2 is an isentropic compression  
 with  $\eta_{c_1} = 0.85 \therefore S_{2s} = S_1$  (superheated fluid)

$$H_{2S} = 1615.15 \text{ kJ/kg} \quad (\text{through linear interpolation})^{21}$$

Now we can calculate  $\underline{H_2}$  from

$$\eta_{C_1} = 0.85 = \frac{H_{2S} - H_1}{H_2 - H_1} \therefore H_2 = 1652.46 \frac{\text{kJ}}{\text{kg}}$$

- 3: Saturated vapour at  $P_3 = 5.5 \text{ bar}$

$$H_3 = 1448.80 \text{ kJ/kg}$$

$$S_3 = 5.2430 \text{ kJ/kg.K}$$

- 4:  $P_4 = 18 \text{ bar}$  - The fluid underwent an isentropic compression with  $\eta_{C_2} = 0.85$ :

$$S_{4S} = S_3 \therefore$$

$$H_{4S} = 1619.38 \text{ kJ/kg}$$

$$\eta_{C_2} = 0.85 = \frac{H_{4S} - H_3}{H_4 - H_3} \therefore H_4 = 1649.48 \frac{\text{kJ}}{\text{kg}}$$

- 5:  $P_5 = P_4 = 18 \text{ bar} \Rightarrow$  saturated liquid

$$H_5 = 398.00 \text{ kJ/kg}$$

- 6: Throttling process:  $H_6 = H_5$

• 7:

$P_7 = 5.5 \text{ bar}$  (saturated liquid)

$$H_7 = 211.65 \text{ kJ/kg}$$

• 8 : Throttling process  $\therefore H_8 = H_7$

The mass flow rate ~~is~~ through the evaporator :

$$R_n = \dot{m}_{R_1} (H_1 - H_8)$$

$$\dot{m}_{R_1} = 30 \text{ ton} \frac{1.4 \times 10^4 \text{ kJ/h}}{1 \text{ ton}} \frac{1}{1403.75 - 211.65} \frac{\text{kg}}{\text{kJ}}$$

$$\dot{m}_{R_1} = 352.32 \text{ kg/h} = 0.09787 \text{ kg/s}$$

The mass flow rate in the second compressor can be calculated from an energy balance in the intercooler + flash chamber, ie.,

$$\dot{m}_6 H_6 + \dot{m}_2 H_2 = \dot{m}_7 H_7 + \dot{m}_3 H_3$$

We know that  $\dot{m}_7 = \dot{m}_2 = \dot{m}_{R_1}$  and  
 $\dot{m}_6 = \dot{m}_3 = \dot{m}_{R_2}$ , thus

$$\dot{m}_{R2} (H_6 - H_3) = \dot{m}_{R1} (H_7 - H_2)$$

$$\dot{m}_{R2} = \dot{m}_{R1} \frac{H_7 - H_2}{H_6 - H_3}$$

$$\dot{m}_{R2} = 0.1342 \text{ Kg/s}$$

Thus the ratio  $IR = \frac{\dot{m}_3}{\dot{m}_1} = \frac{\dot{m}_{R2}}{\dot{m}_{R1}} = 1.371$

The power input to the first ~~or~~ compressor stage is:

$$\dot{W}_{C1} = \dot{m}_{R1} (H_2 - H_1) = 24.34 \frac{\text{KJ}}{\text{s}} = 24.34 \text{ KW}$$

and for the second compressor:

$$\dot{W}_{C2} = \dot{m}_{R2} (H_4 - H_3) = \frac{26.93}{\cancel{24.34}} \text{ KW}$$

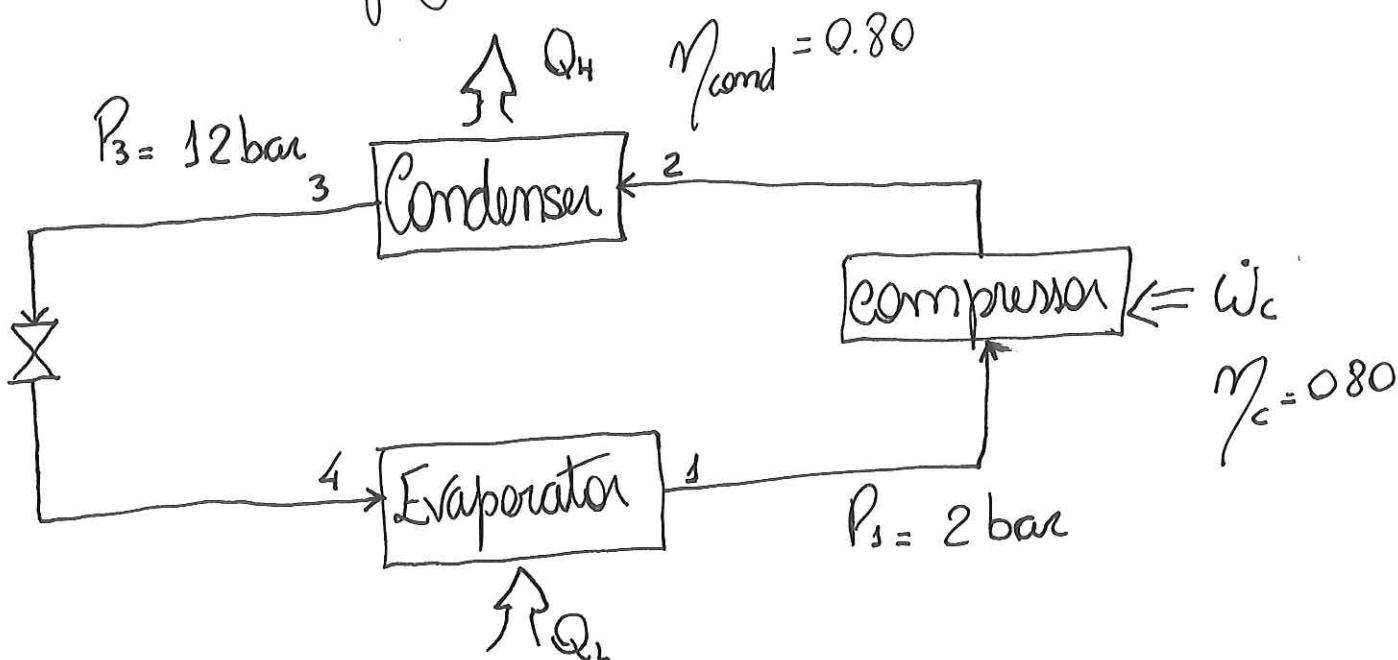
And the COP:

$$COP = \frac{\dot{Q}_L}{\sum \dot{W}_{Ci}} = \frac{\dot{Q}_L}{\dot{W}_{C1} + \dot{W}_{C2}} = \frac{30 \text{ tons}}{(24.34 + 26.93) \text{ KW}}$$

$$COP = 30 \text{ tons} \times \frac{1.4 \times 10^4 \text{ KJ/h}}{1 \text{ ton}} \times \frac{1 \text{ h}}{3600 \text{ s}} \times \frac{1}{51.27 \text{ KJ/s}}$$

$$COP = 2.28$$

Problem 10: Vapour-compression heat pump  
Refrigerant 22



Heat rejected in the heat pump cycle is

$$Q_H = \frac{500 \text{ KJ/min}}{0.8} = 10.416 \frac{\text{KJ}}{\text{s}} = 10.416 \text{ KW}$$

Now calculating the enthalpies:

- 1 :  $P_1 = 2 \text{ bar}$  (saturated vapour)

$$H_1 = 239.88 \text{ KJ/Kg}$$

$$S_1 = 0.9693 \text{ KJ/Kg.K}$$

- 2 : 1-2 is an isentropic process ( $S_1 = S_{2s}$ ) with  $P_2 = P_3 = 12 \text{ bar}$ . The fluid in 2 is superheated vapour:

T	H	S	
60	284.43	0.9666	{ linear interpolation for $S_{2s} = 0.9691 \frac{KJ}{Kg \cdot K}$
70	292.58	0.9907	

$$(292.58 - 284.43) - (0.9907 - 0.9666)$$

$$y = (0.9691 - 0.9666)$$

$$y = 0.8454$$

$$\rightarrow H_{2s} = 285.28 \frac{KJ}{Kg}$$

Updating using the  
~~compressor efficiency~~  
compressor efficiency

$$\eta_c = 0.80 = \frac{H_{2s} - H_2}{H_2 - H_3}$$

$$0.80 = \frac{285.28 - 239.88}{H_2 - 239.88}$$

$$H_2 = 296.63 \frac{KJ}{Kg}$$

• 3 : Saturated liquid at  $P_3 = 12 \text{ bar}$

$$H_3 = 81.90 \frac{KJ}{Kg}$$

• 4 : 3-4 is an isenthalpic expansion :

$$H_4 = H_3 = 81.90 \frac{KJ}{Kg}$$

Now, to calculate the mass flow rate of the refrigerant in the Heat Pump cycle, let's calculate the energy balance around the condenser :

$$\dot{Q}_H = \dot{m}_R (H_2 - H_3)$$

$$10.416 \frac{\text{kJ}}{\text{s}} = \dot{m}_R (296.63 - 81.90) \frac{\text{kJ}}{\text{kg}}$$

$$\dot{m}_R = 4.85 \times 10^{-2} \frac{\text{kg}}{\text{s}} = 2.91 \frac{\text{kg}}{\text{min}}$$

The work in the compressor is calculated as

$$\dot{W}_C = \frac{\dot{m}_R (H_2 - H_1)}{\eta} = \frac{4.85 \times 10^{-2} \text{ kg}}{0.8} \frac{(296.63 - 239.88)}{\text{s}} \frac{\text{kJ/kg}}{\text{s}}$$

$$\dot{W}_C = 3.44 \text{ kW}$$

And the COP :

$$\text{COP} = \frac{\dot{Q}_H}{\dot{W}_C} = \frac{10.416}{3.44} = 3.03$$

	P(bar)	H(kJ/kg)	S(kJ/kg.K)	State
1	2.0	239.88 <sup>A</sup>	0.9691 <sup>B</sup>	Saturated vapour (sv)
2	12.0	296.63 <sup>C</sup>	—	Superheated vapour <sup>D</sup> (shv)
3	12.0	81.90 <sup>E</sup>	—	Saturated liquid <sup>F</sup>
4	—	81.90 <sup>G</sup>	—	Saturated liquid <sup>H</sup>