UNIVERSITY OF ABERDEEN SESSION 2017–18

EX3030

Degree Examination in EX3030 Heat, Mass, & Momentum Transfer

13th December 2017 2 pm – 5 pm

PLEASE NOTE THE FOLLOWING

(i) You **must not** have in your possession any material other than that expressly permitted in the rules appropriate to this examination. Where this is permitted, such material **must not** be amended, annotated or modified in any way.

(ii) You **must not** have in your possession any material that could be determined as giving you an advantage in the examination.

(iii) You **must not** attempt to communicate with any candidate during the exam, either orally or by passing written material, or by showing material to another candidate, nor must you attempt to view another candidate's work.

(iv) You **must not** take to your examination desk any electronic devices such as mobile phones or other smart devices. The only exception to this rule is an approved calculator.

Failure to comply with the above will be regarded as cheating and may lead to disciplinary action as indicated in the Academic Quality Handbook Section 7 and particularly Appendix 7.1

Notes:

(i) Candidates ARE permitted to use an approved calculator.

(ii) Candidates ARE permitted to use the Engineering Mathematics Handbook.

(iii) Data sheets are attached to the paper.

Candidates should attempt *all* questions.

Question 1

1. Consider a large aluminium plate of thickness 0.12 m with initial uniform temperature of 85°C. Suddenly, the temperature of one of the faces is lowered to 20°C, while the other face is perfectly insulated. Assuming that the plate can be modelled as a 1D problem, the following thermal energy conservative equation can be used,

$$\rho C_p \frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial x^2} + \mathcal{S},$$

where ρ, C_p, t are density, heat capacity and time, respectively. κ and $\mathcal{S}$ are thermal conductivity coefficient and source term. Using the finite difference method (FDM) with spatial (Δx) and temporal (Δt) increments of 0.03 m and 300 s, respectively,

   a) Determine the number of nodes necessary to discretise the problem; [2 marks]

   b) Describe the boundary and initial conditions for this problem; [5 marks]

   c) Determine the temperature distribution of the plate at $t = 10$ minutes using the FDM, [16 marks]

   Thermal diffusivity $\left(\alpha = \kappa \rho^{-1} C_p^{-1}\right)$ of the plate is 1.5×10^{-6} m^2.s^{-1}. The discretised form of the thermal energy equation is

$$\rho C_p \frac{T_i^{j+1} - T_i^{j+1}}{\Delta t} = \kappa \frac{T_{i+1}^j - 2T_i^j + T_{i-1}^j}{\left(\Delta x\right)^2} + \mathcal{S}_i^j,$$

   where i and j are spatial and temporal indices.

2. A double-pipe (shell-and-tube) heat exchanger is constructed of a stainless steel (κ = 15.1 W/(m.°C) inner tube of inner diameter D$_i$ = 1.5 cm and outer diameter D$_o$ = 1.9 cm and an outer shell of inner diameter 3.2 cm. The convective heat transfer coefficient is h$_i$ = 800 W/(m^2.°C) on the inner surface of the tube and h$_o$ = 1200 W/(m^2.°C) on the outer surface. For a fouling factor R$_{f,i}$ = 0.0004 m^2.°C/W on the tube side and R$_{f,o}$ = 0.0001 m^2.°C/W on the shell side, determine:

   (a) The thermal resistance of the heat exchanger per unit length (in °C/W) and; [3 marks]

   (b) The overall heat transfer coefficients, U$_i$ and U$_o$ (in W/(m^2.°C)) based on the inner and outer surface areas of the tube, respectively. [7 marks]

Question 2

1. A new material is to be developed for bearing balls in a new rolling-element bearing. For annealing (heat treatment) each bearing ball, a sphere of radius $r_o = 5$ mm, is heated in a furnace until it reaches to the equilibrium temperature of the furnace at 400°C. Then, it is suddenly removed from the furnace and subjected to a two-step cooling process.

   Stage 1: Cooling in an air flow of 20°C for a period of time t_{air} until the center temperature reaches 335°C. For this situation, the convective heat transfer coefficient of air is assumed constant and equal to $h = 10$ W/(m^2.K). After the sphere has reached this specific temperature, the second step is initiated.

   Stage 2: Cooling in a well-stirred water bath at 20°C, with a convective heat transfer coefficient of water $h = 6000$ W/(m^2.K).

   The thermophysical properties of the material are $\rho = 3000$ kg/m^3, $\kappa = 20$ W/(m.K), $C_p = 1000$ J/(kg.K). Determine:

   (a) The time t_{air} required for *Stage 1* of the annealing process to be completed; [5 marks]

   (b) The time t_{water} required for *Stage 2* of the annealing process during which the center of the sphere cools from 335°C (the condition at the completion of *Stage 1*) to 50°C. [6 marks]

2. Water at the rate of 68 kg/min is heated from 35 to 75°C by an oil having a specific heat of 1.9 kJ/(kg.°C). The fluids are used in a counterflow double-pipe HE, and the oil enters the exchanger at 110°C and leaves at 75°C. The overall heat-transfer coefficient is 320 W/(m^2.°C). Given heat capacity of water (at constant pressure) of 4.18 kJ/(kg.°C),

   (a) Calculate the HE area; [5 marks]

   (b) Now assume that the heat exchanger is a shell-and-tube with water making one shell pass and the oil making two tube passes. Calculate the area of the new heat exchanger. Assume that the overall heat-transfer coefficient remains the same. [4 marks]

Question 3

1. Hot oil is to be cooled in a double-tube counter-flow heat exchanger. The copper inner tubes have diameter of 2 cm and negligible thickness. The inner diameter of the outer tube (shell) is 3 cm. Water flows through the tube at a rate of 0.5 kg.s^{-1}, and the oil through the shell at a rate of 0.8 kg.s^{-1}. Taking the average temperatures of the water and the oil to be 45°C and 80°C, respectively, determine the overall heat transfer coefficient of this heat exchanger. Given,

   (a) Water at 45°C: $\rho = 990$ kg.m^{-3}, $\kappa = 0.637$ W.(m.K)$^{-1}$, $Pr = 3.91$, $\nu = \mu/\rho = 0.602 \times 10^{-6}$ m^2.s^{-1};

   (b) Oil at 80°C: $\rho = 852$ kg.m^{-3}, $\kappa = 0.138$ W.(m.K)$^{-1}$, $Pr = 490$, $\nu = 37.5 \times 10^{-6}$ m^2.s^{-1}.

   The overall heat transfer coefficient can be expressed as,

   $$U^{-1} = h_i^{-1} + h_0^{-1}$$

   The inner convective heat transfer coefficient, h_i, can be obtained from

   $$Nu = \frac{h_i D_h}{\kappa} = \begin{cases} 4.36 & \text{(for laminar flows),} \\ 0.023 Re^{0.8} Pr^{0.4} & \text{(for turbulent flows),} \end{cases}$$

   where D_h is the hydraulic diameter. The outter convective heat transfer coefficient, h_0 is 75.2 W.(m^2.K). [17 marks]

2. A long rod of 60 mm diameter and thermophysical properties $\rho =$ 8000 kg/m^3, $C_p =$ 500 J/(kg.K), and $k =$ 50 W/(m.K) is initially at a uniform temperature and is heated in a forced convection furnace maintained at 750 K. The convection coefficient is estimated to be 1000 W/(m^2.K). Calculate the centerline temperature of the rod when the surface temperature is 550 K. [16 marks]

Question 4

1. In a counterflow double-pipe heat exchanger, water $\left(C_p = 4.18 \text{ kJ.kg}^{-1}.^\circ\text{C}\right)$ at 35°C is heated by oil $\left(C_p = 1.9 \text{ kJ.kg}^{-1}.^\circ\text{C}\right)$. The mass flow rate of the water stream is 40 kg.min^{-1} and 170.97 kg.min^{-1} of oil enters the heat exchanger at 110°C. The overall heat-transfer coefficient is 320 W.(m^2.°C)$^{-1}$. Calculate:

   (a) Exit water and oil temperatures; [5 marks]

   (b) Total heat transfer (in kW); [1 marks]

2. Consider three consecutive nodes $n-1$, n, $n+1$ in a plane wall. Using the finite difference form of the first derivative at the midpoints, show that the finite difference form of the second derivative can be expressed as, [4 marks]

$$\frac{T_{n-1} - 2T_n + T_{n+1}}{\Delta x^2} = \left.\frac{\partial^2 T}{\partial x^2}\right|_N .$$

   Hint: You should start the demonstration from the 1D expansion in Taylor series of a continuous and real function $f(x)$ about a point $x = a$,

$$f(x) = f(a) + f^{'}(a)(x-a) + \frac{f^{''}(a)}{2!}(x-a)^2 + \frac{f^{'''}(a)}{3!}(x-a)^3 + \cdots + \frac{f^n(a)}{n!}(x-a)^n$$

3. Heavy oil at 150°C is pumped into a storage tank before being transported to distillation columns. The tank is insulated with a layer of polyisocyanurate of 10 cm thick. The tank's wall is at 150°C and the initial temperature of the insulation layer is 20°C. Assuming that the tank wall-insulation layer-environment system can be modelled as a 1-D finite difference method (FDM) problem, calculate the temperature profile of the insulated layer, $T\left(\underline{x}, t\right)$, at $t = 5$ seconds with $\underline{x} = [0.0, 2.5, 5.0, 7.5, 10.0]$. The outer layer of the insulation material is subjected to the environment with temperature of 15°C and convective heat transfer coefficient of 10 W(m^2.°C)$^{-1}$. Given for polyisocyanurate insulation layer:

   - Conductive heat transfer coefficient: 5.40 W.(m.°C)$^{-1}$;
   - Heat capacity at constant pressure: 0.1 kJ.(kg.°C)$^{-1}$;
   - Density: 550 kg.m^{-3}.

   The discretised thermal energy equation is

$$T_i^{j+1} = T_i^j + \alpha \frac{\Delta t}{(\Delta x)^2}\left(T_{i+1}^j - 2T_i^j + T_{i-1}^j\right)$$

   where $\alpha = \kappa\left(\rho C_p\right)^{-1}$ is the thermal diffusivity, Δx and Δt are the spatial-interval and time-step size, respectively. i and j are the spatial- and time-indices. For this problem, use $\Delta t = 5$ seconds. [10 marks]

END OF PAPER

DATASHEET

General balance equations:

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \rho\, \boldsymbol{v} \qquad \text{(Mass/Continuity)} \quad (1)$$

$$\frac{\partial C_A}{\partial t} = -\nabla \cdot \boldsymbol{N}_A + \sigma_A \qquad \text{(Species)} \quad (2)$$

$$\rho \frac{\partial \boldsymbol{v}}{\partial t} = -\rho\, \boldsymbol{v} \cdot \nabla \boldsymbol{v} - \nabla \cdot \boldsymbol{\tau} - \nabla\, p + \rho\, \boldsymbol{g} \qquad \text{(Momentum)} \quad (3)$$

$$\rho\, C_p \frac{\partial T}{\partial t} = -\rho\, C_p\, \boldsymbol{v} \cdot \nabla\, T - \nabla \cdot \boldsymbol{q} - \boldsymbol{\tau} : \nabla\, \boldsymbol{v} - p\, \nabla \cdot \boldsymbol{v} + \sigma_{energy} \qquad \text{(Heat/Energy)} \quad (4)$$

In Cartesian coordinate systems, ∇ can be treated as a vector of derivatives. In curvelinear coordinate systems, the directions $\hat{\boldsymbol{r}}$, $\hat{\boldsymbol{\theta}}$, and $\hat{\boldsymbol{\phi}}$ depend on the position. For convenience in these systems, look-up tables are provided for common terms involving ∇.

Cartesian coordinates (with index notation examples)
where s is a scalar, $\boldsymbol{v}$ is a vector, and $\boldsymbol{\tau}$ is a tensor.

$$\nabla s = \nabla_i s = \left[\frac{\partial\, s}{\partial x}, \frac{\partial\, s}{\partial y}, \frac{\partial\, s}{\partial z}\right]$$

$$\nabla^2 s = \nabla_i \nabla_i s = \frac{\partial^2\, s}{\partial x^2} + \frac{\partial^2\, s}{\partial y^2} + \frac{\partial^2\, s}{\partial z^2}$$

$$\nabla \cdot \boldsymbol{v} = \nabla_i v_i = \frac{\partial\, v_x}{\partial x} + \frac{\partial\, v_y}{\partial y} + \frac{\partial\, v_z}{\partial z}$$

$$\nabla \cdot \boldsymbol{\tau} = \nabla_i\, \tau_{ij}$$

$$[\nabla \cdot \boldsymbol{\tau}]_x = \frac{\partial\, \tau_{xx}}{\partial x} + \frac{\partial\, \tau_{yx}}{\partial y} + \frac{\partial\, \tau_{zx}}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_y = \frac{\partial\, \tau_{xy}}{\partial x} + \frac{\partial\, \tau_{yy}}{\partial y} + \frac{\partial\, \tau_{zy}}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_z = \frac{\partial\, \tau_{xz}}{\partial x} + \frac{\partial\, \tau_{yz}}{\partial y} + \frac{\partial\, \tau_{zz}}{\partial z}$$

$$\boldsymbol{v} \cdot \nabla \boldsymbol{v} = v_i\, \nabla_i\, v_j$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_x = v_x \frac{\partial\, v_x}{\partial x} + v_y \frac{\partial\, v_x}{\partial y} + v_z \frac{\partial\, v_x}{\partial z}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_y = v_x \frac{\partial\, v_y}{\partial x} + v_y \frac{\partial\, v_y}{\partial y} + v_z \frac{\partial\, v_y}{\partial z}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_z = v_x \frac{\partial\, v_z}{\partial x} + v_y \frac{\partial\, v_z}{\partial y} + v_z \frac{\partial\, v_z}{\partial z}$$

Cylindrical coordinates

where s is a scalar, $\boldsymbol{v}$ is a vector, and $\boldsymbol{\tau}$ is a tensor. All expressions involving $\boldsymbol{\tau}$ are for symmetrical $\boldsymbol{\tau}$ only.

$$\nabla s = \left[\frac{\partial s}{\partial r}, \frac{1}{r}\frac{\partial s}{\partial \theta}, \frac{\partial s}{\partial z}\right]$$

$$\nabla^2 s = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial s}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 s}{\partial \theta^2} + \frac{\partial^2 s}{\partial z^2}$$

$$\nabla \cdot \boldsymbol{v} = \frac{1}{r}\frac{\partial}{\partial r}(r\,v_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_r = \frac{1}{r}\frac{\partial}{\partial r}(r\,\tau_{rr}) + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} - \frac{1}{r}\tau_{\theta\theta} + \frac{\partial \tau_{rz}}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_\theta = \frac{1}{r}\frac{\partial \tau_{\theta\theta}}{\partial \theta} + \frac{\partial \tau_{r\theta}}{\partial r} + \frac{2}{r}\tau_{r\theta} + \frac{\partial \tau_{\theta z}}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_z = \frac{1}{r}\frac{\partial}{\partial r}(r\,\tau_{rz}) + \frac{1}{r}\frac{\partial \tau_{\theta z}}{\partial \theta} + \frac{\partial \tau_{zz}}{\partial z}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_r = v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_\theta = v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r\,v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_z = v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}$$

Spherical coordinates

where s is a scalar, $\boldsymbol{v}$ is a vector, and $\boldsymbol{\tau}$ is a tensor. All expressions involving $\boldsymbol{\tau}$ are for symmetrical $\boldsymbol{\tau}$ only.

$$\nabla s = \left[\frac{\partial s}{\partial r}, \frac{1}{r}\frac{\partial s}{\partial \theta}, \frac{1}{r\,\sin\theta}\frac{\partial s}{\partial \phi}\right]$$

$$\nabla^2 s = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial s}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial s}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 s}{\partial \phi^2}$$

$$\nabla \cdot \boldsymbol{v} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\,v_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(v_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$$

$$[\nabla \cdot \boldsymbol{\tau}]_r = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\,\tau_{rr}\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\tau_{r\theta}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial \tau_{r\phi}}{\partial \phi} - \frac{\tau_{\theta\theta} + \tau_{\phi\phi}}{r}$$

$$[\nabla \cdot \boldsymbol{\tau}]_\theta = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\,\tau_{r\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\tau_{\theta\theta}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial \tau_{\theta\phi}}{\partial \phi} + \frac{\tau_{r\theta}}{r} - \frac{\cot\theta}{r}\tau_{\phi\phi}$$

$$[\nabla \cdot \boldsymbol{\tau}]_\phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\,\tau_{r\phi}\right) + \frac{1}{r}\frac{\partial \tau_{\theta\phi}}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial \tau_{\phi\phi}}{\partial \phi} + \frac{\tau_{r\theta}}{r} + \frac{2\cot\theta}{r}\tau_{\theta\phi}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_r = v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_\theta = v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{v_r\,v_\theta - v_\phi^2\cot\theta}{r}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_\phi = v_r\frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r\,v_\phi + v_\theta\,v_\phi\cot\theta}{r}$$

Rectangular		Cylindrical		Spherical	
q_x	$-k\frac{\partial T}{\partial x}$	q_r	$-k\frac{\partial T}{\partial r}$	q_r	$-k\frac{\partial T}{\partial r}$
q_y	$-k\frac{\partial T}{\partial y}$	q_θ	$-k\frac{1}{r}\frac{\partial T}{\partial \theta}$	q_θ	$-k\frac{1}{r}\frac{\partial T}{\partial \theta}$
q_z	$-k\frac{\partial T}{\partial z}$	q_z	$-k\frac{\partial T}{\partial z}$	q_ϕ	$-k\frac{1}{r\,\sin\theta}\frac{\partial T}{\partial \phi}$
τ_{xx}	$-2\,\mu\frac{\partial v_x}{\partial x} + \mu^B\,\nabla\cdot\boldsymbol{v}$	τ_{rr}	$-2\,\mu\frac{\partial v_r}{\partial r} + \mu^B\,\nabla\cdot\boldsymbol{v}$	τ_{rr}	$-2\,\mu\frac{\partial v_r}{\partial r} + \mu^B\,\nabla\cdot\boldsymbol{v}$
τ_{yy}	$-2\,\mu\frac{\partial v_y}{\partial y} + \mu^B\,\nabla\cdot\boldsymbol{v}$	$\tau_{\theta\theta}$	$-2\,\mu\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right) + \mu^B\,\nabla\cdot\boldsymbol{v}$	$\tau_{\theta\theta}$	$-2\,\mu\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right) + \mu^B\,\nabla\cdot\boldsymbol{v}$
τ_{zz}	$-2\,\mu\frac{\partial v_z}{\partial z} + \mu^B\,\nabla\cdot\boldsymbol{v}$	τ_{zz}	$-2\,\mu\frac{\partial v_z}{\partial z} + \mu^B\,\nabla\cdot\boldsymbol{v}$	$\tau_{\phi\phi}$	$-2\,\mu\left(\frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r+v_\theta\cot\theta}{r}\right)$ $+\mu^B\,\nabla\cdot\boldsymbol{v}$
τ_{xy}	$-\mu\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)$	$\tau_{r\theta}$	$-\mu\left(r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right)$	$\tau_{r\theta}$	$-\mu\left(r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right)$
τ_{yz}	$-\mu\left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right)$	$\tau_{\theta z}$	$-\mu\left(\frac{1}{r}\frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z}\right)$	$\tau_{\theta\phi}$	$-\mu\left(\frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi}\right)$
τ_{xz}	$-\mu\left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right)$	τ_{zr}	$-\mu\left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right)$	$\tau_{\phi r}$	$-\mu\left(\frac{1}{r\sin\theta}\frac{\partial v_r}{\partial \phi} + r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right)\right)$

Table 1: *Fourier's law for the heat flux and Newton's law for the stress in several coordinate systems. Please remember that the stress is symmetric, so* $\tau_{ij} = \tau_{ji}$.

Viscous models:
Power-Law Fluid:

$$|\tau_{xy}| = k\left|\frac{\partial v_x}{\partial y}\right|^n \tag{5}$$

Bingham-Plastic Fluid:

$$\frac{\partial v_x}{\partial y} = \begin{cases} -\mu^{-1}\left(\tau_{xy} - \tau_0\right)) & \text{if } \tau_{xy} > \tau_0 \\ 0 & \text{if } \tau_{xy} \le \tau_0 \end{cases}$$

Dimensionless Numbers

$$\text{Re} = \frac{\rho\,\langle v\rangle\,D}{\mu} \qquad \text{Re}_H = \frac{\rho\,\langle v\rangle\,D_H}{\mu} \qquad \text{Re}_{MR} = -\frac{16\,L\,\rho\,\langle v\rangle^2}{R\,\Delta p} \tag{6}$$

The hydraulic diameter is defined as $D_H = 4\,A/P_w$.

Single phase pressure drop calculations in pipes:
Darcy-Weisbach equation:

$$\frac{\Delta p}{L} = -\frac{C_f\,\rho\,\langle v\rangle^2}{R} \tag{7}$$

where $C_f = 16/Re$ for laminar Newtonian flow. For turbulent flow of Newtonian fluids in smooth pipes, we have the Blasius correlation:

$$C_f = 0.079\,\mathrm{Re}^{-1/4} \qquad \text{for } 2.5\times10^3 < \mathrm{Re} < 10^5 \text{ and smooth pipes.}$$

Otherwise, you may refer to the Moody diagram.

Laminar Power-Law fluid:

$$\dot{V} = \frac{n\,\pi\,R^3}{3\,n+1}\left(\frac{R}{2\,k}\right)^{\frac{1}{n}}\left(-\frac{\Delta p}{L}\right)^{\frac{1}{n}}$$

Two-Phase Flow:
Lockhart-Martinelli parameter:

$$X^2 = \frac{\Delta p_{liq.-only}}{\Delta p_{gas-only}}$$

Pressure drop calculation:

$$\Delta p_{two-phase} = \Phi^2_{liq.}\,\Delta p_{liq.-only} = \Phi^2_{gas}\,\Delta p_{gas-only}$$

Chisholm's relation:

$$\Phi^2_{gas} = 1 + c\,X + X^2$$

$$\Phi^2_{liq.} = 1 + \frac{c}{X} + \frac{1}{X^2} \qquad c = \begin{cases} 20 & \text{turbulent liquid \& turbulent gas} \\ 12 & \text{laminar liquid \& turbulent gas} \\ 10 & \text{turbulent liquid \& laminar gas} \\ 5 & \text{laminar liquid \& laminar gas} \end{cases}$$

Farooqi and Richardson expression for liquid hold-up in co-current flows of Newtonian fluids and air in horizontal pipes:

$$h = \begin{cases} 0.186 + 0.0191\,X & 1 < X < 5 \\ 0.143\,X^{0.42} & 5 < X < 50 \\ 1/\left(0.97 + 19/X\right) & 50 < X < 500 \end{cases}$$

Heat Transfer Dimensionless numbers:

$$\mathrm{Nu} = \frac{h\,L}{k} \qquad \mathrm{Pr} = \frac{\mu\,C_p}{k} \qquad \mathrm{Gr} = \frac{g\,\beta\,(T_w - T_\infty)\;L^3}{\nu^2}$$

where $\beta = V^{-1}(\partial V/\partial T)$.
Heat transfer: Resistances

$$Q = U_T\,A_T\,\Delta T = R_T^{-1}\,\Delta T$$

Figure 1: *Hewitt-Taylor flow pattern map for multiphase flows in vertical pipes.*

Figure 2: *Chhabra and Richardson flow pattern map for horizontal pipes.*

	Conduction Shell Resistances			Radiation
	Rect.	Cyl.	Sph.	
R	$\dfrac{X}{k\,A}$	$\dfrac{\ln\left(R_{outer}/R_{inner}\right)}{2\,\pi\,L\,k}$	$\dfrac{R_{inner}^{-1}-R_{outer}^{-1}}{4\,\pi\,k}$	$\left[A\,\varepsilon\,\sigma\left(T_j^2+T_i^2\right)\left(T_j+T_i\right)\right]^{-1}$

Radiation Heat Transfer:
Stefan-Boltzmann constant $\sigma = 5.6703\times10^{-8}$ W m^{-2} K^{-4}.
Summation relationship, $\sum_j F_{i\to j} = 1$, and reciprocity relationship, $F_{i\to j}\,A_i = F_{j\to i}\,A_j$.
Radiation shielding factor $1/(N+1)$.

$$Q_{rad.,i\to j} = \sigma\,\varepsilon\,F_{i\to j}\,A_i\left(T_j^4 - T_i^4\right) = h_{rad.}\,A\left(T_\infty - T_w\right)$$

Natural Convection

Ra $=$ Gr Pr	C	m
$< 10^4$	1.36	1/5
10^4–10^9	0.59	1/4
$> 10^9$	0.13	1/3

Table 2: *Natural convection coefficients for isothermal vertical plates in the empirical relation Nu $\approx C\,(\text{Gr}\,\text{Pr})^m$.*

For isothermal vertical cylinders, the above expressions for isothermal vertical plates may be used but must be scaled by a factor, F (i.e., $\mathrm{Nu}_{v.cyl.} = F\,\mathrm{Nu}_{v.plate}$):

$$F = \begin{cases} 1 & \text{for } (D/H) \geq 35\,\mathrm{Gr}_H^{-1/4} \\ 1.3\left[H\,D^{-1}\,\mathrm{Gr}_D^{-1}\right]^{1/4} + 1 & \text{for } (D/H) < 35\,\mathrm{Gr}_H^{-1/4} \end{cases}$$

where D is the diameter and H is the height of the cylinder. The subscript on Gr indicates which length is to be used as the critical length to calculate the Grashof number.

Churchill and Chu expression for natural convection from a horizontal pipe:

$$\mathrm{Nu}^{1/2} = 0.6 + 0.387\left\{\frac{\mathrm{Gr}\,\mathrm{Pr}}{\left[1 + (0.559/\mathrm{Pr})^{9/16}\right]^{16/9}}\right\}^{1/6} \qquad \text{for } 10^{-5} < \mathrm{Gr}\,\mathrm{Pr} < 10^{12}$$

Forced Convection:
Laminar flows:

$$\mathrm{Nu} \approx 0.332\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}$$

Well-Developed turbulent flows in smooth pipes:

$$\mathrm{Nu} \approx \frac{(C_f/2)\mathrm{Re}\,\mathrm{Pr}}{1.07 + 12.7(C_f/2)^{1/2}\left(\mathrm{Pr}^{2/3} - 1\right)}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

Boiling:
Forster-Zuber pool-boiling coefficient:

$$h_{nb} = 0.00122\frac{k_L^{0.79}\,C_{p,L}^{0.45}\,\rho_L^{0.49}}{\gamma^{0.5}\,\mu_L^{0.29}\,h_{fg}^{0.24}\,\rho_G^{0.24}}\,(T_w - T_{sat})^{0.24}\,(p_w - p_{sat})^{0.75}$$

Mostinski correlations:

$$h_{nb} = 0.104\,p_c^{0.69}\,q^{0.7}\left[1.8\left(\frac{p}{p_c}\right)^{0.17} + 4\left(\frac{p}{p_c}\right)^{1.2} + 10\left(\frac{p}{p_c}\right)^{10}\right]$$

$$q_c = 3.67\times 10^4\,p_c\left(\frac{p}{p_c}\right)^{0.35}\left[1 - \frac{p}{p_c}\right]^{0.9}$$

(**Note**: for the Mostinski correlations, the pressures are in units of bar)
Condensing:
Horizontal pipes

$$h = 0.72\left(\frac{k^3\,\rho^2\,g_x\,E_{latent}}{D\,\mu\,(T_w - T_\infty)}\right)^{1/4}$$

Lumped capacitance method:

$$\mathrm{Bi} = \frac{h\,L_c}{\kappa}$$
$$L_c = V/A \qquad\qquad \text{for Bi} < 0.1$$

$$\frac{T(t) - T_\infty}{T_0 - T_\infty} = e^{-bt} \qquad\qquad b = \frac{hA_s}{\rho V C_p}$$

1-D Transient Heat Conduction:

$$Fo = \frac{\alpha \Delta t}{(\Delta x)^2} = \tau, \qquad \alpha = \kappa\,(\rho C_p)^{-1}$$

$$\theta_{\text{wall}} = \frac{T(x,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \cos\left(\frac{\lambda_1 x}{L}\right), \qquad \theta_{\text{cyl}} = \frac{T(r,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \mathbf{J_0}\left(\frac{\lambda_1 r}{r_0}\right)$$

$$\theta_{\text{sph}} = \frac{T(r,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin\left(\frac{\lambda_1 r}{r_0}\right)}{\frac{\lambda_1 r}{r_0}}$$

$$\theta_{0,\text{wall}} = \theta_{0,\text{cyl}} = \theta_{0,\text{sph}} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}$$

$$\left(\frac{\mathcal{Q}}{\mathcal{Q}_{\text{max}}}\right)_{\text{wall}} = 1 - \theta_{0,\text{wall}} \frac{\sin \lambda_1}{\lambda_1}, \quad \left(\frac{\mathcal{Q}}{\mathcal{Q}_{\text{max}}}\right)_{\text{cyl}} = 1 - 2\theta_{0,\text{cyl}} \frac{\mathbf{J_1}(\boldsymbol{\lambda_1})}{\lambda_1}$$

$$\left(\frac{\mathcal{Q}}{\mathcal{Q}_{\text{max}}}\right)_{\text{sph}} = 1 - 3\theta_{0,\text{sph}} \frac{\sin \lambda_1 - \lambda_1 \cos \lambda_1}{\lambda_1^3}$$

Finite-Difference Method:

$$\frac{\partial}{\partial t}(\rho\phi) + \frac{\partial}{\partial x}(\rho \boldsymbol{v} \phi) = \nabla \cdot (\Gamma \nabla \phi) + \mathcal{S} \qquad \text{(1D transport equation)}$$

$$\left(\frac{d\phi}{dx}\right)_i = \frac{\phi_{i+1} - \phi_{i-1}}{2\Delta x} \quad \text{and} \quad \left(\frac{d^2\phi}{dx^2}\right)_i = \frac{\phi_{i-1} + \phi_{i+1} - 2\phi_i}{(\Delta x)^2}$$

Figure 3: *Coefficients for the 1D transient equations.*

Figure 4: *Heisler and Gröber Charts*

$$T_i^{j+1} = (1-2\tau)\, T_i^j + \tau\left(T_{i+1}^j + T_{i-1}^j\right) + \frac{\tau\,(\Delta x)^2}{\kappa}\mathcal{S}_i^j$$

Overall Heat Transfer Coefficient:

$$\dot{\mathcal{Q}} = \frac{\Delta T}{\mathcal{R}} = UA\Delta T = U_i A_i \Delta T = U_o A_o \Delta T$$

$$\mathcal{R} = R_i + R_{\text{wall}} + R_o = \frac{1}{h_i A_i} + \frac{\ln D_o/D_i}{2\pi\kappa L} + \frac{1}{h_o A_o}$$

Fouling Factor:

$$\mathcal{R} = \frac{1}{h_i A_i} + \frac{R_{\text{f,i}}}{A_i} + R_{\text{wall}} + \frac{R_{\text{f,o}}}{A_o} + \frac{1}{h_o A_o}$$

LMTD Method:

$$\dot{\mathcal{Q}} = UA_s\Delta T_{\text{lm}} \quad \text{with} \quad \Delta T_{\text{lm}} = \frac{\Delta T_2 - \Delta T_1}{\ln\frac{\Delta T_2}{\Delta T_1}} = \frac{\Delta T_1 - \Delta T_2}{\ln\frac{\Delta T_1}{\Delta T_2}}$$

$$\text{Parallel flows:}\begin{cases}\Delta T_1 = T_{\text{hot,in}} - T_{\text{cold,in}}\\ \Delta T_2 = T_{\text{hot,out}} - T_{\text{cold,out}}\end{cases}$$

$$\text{Counter flows:}\begin{cases}\Delta T_1 = T_{\text{hot,in}} - T_{\text{cold,out}}\\ \Delta T_2 = T_{\text{hot,out}} - T_{\text{cold,in}}\end{cases}$$

ϵ-NTU Method:

$$\epsilon = \frac{\dot{\mathcal{Q}}}{\dot{\mathcal{Q}}_{\text{max}}}, \quad \text{with } \dot{\mathcal{Q}}_{\text{max}} = \mathcal{C}_{\text{min}}\left(T_{\text{hot,in}} - T_{\text{cold,in}}\right) \quad \text{and} \quad \mathcal{C}_{\text{min}} = Min\left\{\dot{m}_{\text{hot}} C_{p,\text{hot}}, \dot{m}_{\text{cold}} C_{p,\text{cold}}\right\}$$

$$\text{NTU} = \frac{UA_s}{C_{\text{min}}}$$

Figure 5: *Correction-factors for LMTD Method, extracted from Y. A. Cengel, "Heat transfer:A practical approach", 2nd Ed.*

Figure 6: *Correction-factors for LMTD Method, extracted from Y. A. Cengel, "Heat transfer:A practical approach", 2nd Ed.*

Figure 7: *NTU relations extracted from Y. A. Cengel, "Heat transfer:A practical approach", 2nd Ed.*

Figure 8: *NTU plots extracted from Y. A. Cengel, "Heat transfer:A practical approach", 2nd Ed.*

Diffusion Dimensionless Numbers

$$\text{Sc} = \frac{\mu}{\rho\, D_{AB}} \qquad\qquad \text{Le} = \frac{k}{\rho\, C_p\, D_{AB}}$$

Diffusion
General expression for the flux:

$$\boldsymbol{N}_A = \boldsymbol{J}_A + x_A \sum_B \boldsymbol{N}_B$$

Fick's law:

$$\boldsymbol{J}_A = -D_{AB}\, \nabla C_A$$

Stefan's law:

$$N_{s,r} = -D \frac{c}{1-x} \frac{\partial x}{\partial r}$$

Misc

$$P\,V = n\,R\,T \qquad\qquad R \approx 8.314598 \text{ J K}^{-1} \text{ mol}^{-1}$$