

 注 带通型限带白噪声的自相关函数过程为

$$\begin{aligned}
 R_Y(\tau) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} G_X(\omega) e^{j\omega\tau} d\omega \\
 &= \frac{1}{2\pi} \left[\int_{-\omega_0 - \frac{\Omega}{2}}^{-\omega_0 + \frac{\Omega}{2}} G_0 e^{j\omega\tau} d\omega + \int_{\omega_0 - \frac{\Omega}{2}}^{\omega_0 + \frac{\Omega}{2}} G_0 e^{j\omega\tau} d\omega \right] \\
 &= \frac{G_0}{2\pi} \cdot \frac{1}{j\tau} \cdot \left[e^{j\omega\tau} \Big|_{-\omega_0 - \frac{\Omega}{2}}^{-\omega_0 + \frac{\Omega}{2}} + e^{j\omega\tau} \Big|_{\omega_0 - \frac{\Omega}{2}}^{\omega_0 + \frac{\Omega}{2}} \right] \\
 &= \frac{G_0}{2\pi} \cdot \frac{1}{j\tau} \cdot \left[\left[e^{j(-\omega_0 + \frac{\Omega}{2})\tau} - e^{j(-\omega_0 - \frac{\Omega}{2})\tau} \right] + \left[e^{j(\omega_0 + \frac{\Omega}{2})\tau} - e^{j(\omega_0 - \frac{\Omega}{2})\tau} \right] \right] \\
 &= -\frac{jG_0}{2\pi\tau} \left[\cos(-\omega_0 + \frac{\Omega}{2})\tau + j \sin(-\omega_0 + \frac{\Omega}{2})\tau - \cos(-\omega_0 - \frac{\Omega}{2})\tau - j \sin(-\omega_0 - \frac{\Omega}{2})\tau \right. \\
 &\quad \left. \cos(\omega_0 + \frac{\Omega}{2})\tau + j \sin(\omega_0 + \frac{\Omega}{2})\tau - \cos(\omega_0 - \frac{\Omega}{2})\tau + j \sin(\omega_0 - \frac{\Omega}{2})\tau \right] \\
 &= -\frac{jG_0}{2\pi\tau} \left[-2j \sin(\omega_0 - \frac{\Omega}{2})\tau + 2j \sin(\omega_0 + \frac{\Omega}{2})\tau \right] \\
 &= \frac{2G_0}{2\pi\tau} \left[\sin(\omega_0 + \frac{\Omega}{2})\tau - \sin(\omega_0 - \frac{\Omega}{2})\tau \right] \\
 &= \frac{\Omega G_0}{\pi} \cdot \frac{\sin(\Omega\tau/2)}{\Omega\tau/2} \cos \omega_0 \tau \\
 &= 2R_X(\tau) \cos \omega_0 \tau.
 \end{aligned}$$

(3.96)