## Minimum wavelength of phonons in fixed material

In general, if we have an incoming ultralight DM signal, we want it to affect the array and the sensor as a whole, we want them to be considered as rigid blocks. However, at certain frequencies, it could be possible that instead of creating a visible acceleration on the accelerometer in use, the acceleration between accelerometer and array is just used to create a phonon within the array, which then travels through the array while interfering with our incoming signal and therefore making detection problematic. Therefore, we have to consider if there can be phonons emitted at the frequencies we are discussing in our current model.

When solving the equations of motion in a grid of atoms (or, for simplification, a chain), we get solutions of the form $x_n = Ae^{-i\omega t - ikna}$. For these to actually solve the equations of motion for a system of finite length Na (with N being the number of layers and a being the distance between layers) with circular boundary condition $x_1 = x_{N+1}$, k has to be of the form $k = \frac{2\pi}{Na} l = \frac{2\pi}{L} l$ with $n \in \mathbb{N}$ and $-\frac{N}{2} \leq l \leq \frac{N}{2}$ (first Brillouin zone). Therefore:

- There is a minimum wavelength (which corresponds to a maximum wavenumber $k = \frac{2\pi}{L}$)
$$k \geq \frac{2\pi}{L} \quad \Leftrightarrow \quad \frac{2\pi}{\lambda} \geq \frac{2\pi}{L} \quad \Leftrightarrow \quad \lambda > L$$
Therefore, there is minimum frequency for acoustic phonons in the material: $f \geq \frac{c_s}{L}$
- For aluminum this corresponds to about 3kHz (shear wave, $c_s \approx 3600 \frac{m}{s}$) for our current array with a length of 1.2m.

Options to solve this problem:

a) Reduce the length of the array by cutting it into pieces. For $L \leq 30cm$, we will have no problems with phonons being created from incoming ultralight dark matter forces. If we then put the arrays one after the other, separated by a sufficient layer of air, we can still get practically the same array length in total and therefore the same number of sensors. In this case for example, cutting it into pieces of each 30cm would allow us to go up till 10kHz.
b) Ignore certain discrete frequencies. As we know that phonons can only appear at certain discrete frequencies, if we remove these with a sufficient surrounding from our analysis (in this case, that would be 3kHz, 6kHz, …), we could still do the analysis for the rest of the frequencies, as we know there would be no phonons occurring there, our assumption would still hold.