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Dynamic stabilization of non-spherical bodies against unlimited collapse

G. S. Bisnovatyi-Kogan[1,2,3⋆] and O. Yu. Tsupko[1,3⋆]
[1] *Space Research Institute of Russian Academy of Science, Profsoyuznaya 84/32, Moscow 117997, Russia*
[2] *Joint Institute for Nuclear Research, Dubna, Russia*
[3] *Moscow Engineering Physics Institute, Moscow, Russia*

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ABSTRACT
We solve equations, describing in a simplified way the newtonian dynamics of a self-gravitating nonrotating spheroidal body after loss of stability. We find that contraction to a singularity happens only in a pure spherical collapse, and deviations from the spherical symmetry stop the contraction by the stabilising action of nonlinear non-spherical oscillations. A real collapse happens after damping of the oscillations due to energy losses, shock wave formation or viscosity. Detailed analysis of the nonlinear oscillations is performed using a Poincaré map construction. Regions of regular and chaotic oscillations are localized on this map.

Key words: gravitation – instabilities.

1 INTRODUCTION

Dynamic stability of spherical stars is determined by an average adiabatic power $\gamma = \frac{\partial \log P}{\partial \log \rho}|_S$. For a density distribution $\rho = \rho_0 \varphi(m/M)$, the star in the newtonian gravity is stable against dynamical collapse when $\int_0^R (\gamma - \frac{4}{3}) P \frac{dm}{\varphi(m/M)} > 0$ (Zeldovich & Novikov 1967; Bisnovatyi-Kogan 1989). This approximate criterium becomes exact for adiabatic stars with constant γ. Here ρ_0 is a central density, M is a stellar mass, m is the mass inside a Lagrangian radius r, so that $m = 4\pi \int_0^r \rho r^2 dr$, $M = m(R)$, R is a stellar radius. Collapse of a spherical star may be stopped only by a stiffening of the equation of state, like neutron star formation at late stages of evolution, or formation of fully ionized stellar core with $\gamma = \frac{5}{3}$ at collapse of clouds during star formation. Without a stiffening a spherical star in the newtonian theory would collapse into a point with an infinite density (singularity).

In the presence of a rotation a star is becoming more dynamically stable against collapse. Due to the more rapid increase of a centrifugal force during contraction, in comparison with the newtonian gravitational force, collapse of a rotating star will be always stopped at finite density by centrifugal forces. Here we show, that deviation from the spherical symmetry in a non-rotating star with zero angular momentum leads to a similar stabilization, and non-spherical star without dissipative processes never will reach a singularity. Therefore collapse to a singularity is connected with a secular type of instability, even without rotation.

When a uniform sphere of cold gas is set into free-fall collapse under its self-gravity its shape is unstable. The linear instability of large-scale modes (second-order harmonics) during the collapse was discovered by Lynden-Bell (1964) and for the case of non-rotating sphere by Lynden-Bell (1979) (see also Lynden-Bell (1996)). Numerical investigations of collapsing pressureless spheroids have been done in the papers of Lynden-Bell (1964) and Lin, Mestel & Shu (1965). In Lynden-Bell (1979) it was also shown that the pressure prevents the development of large-scale shape instability if initially the gravity is more than three-fifths pressure resisted. Fujimoto (1968) derived the equations of motion for a rotating ellipsoid from a system of hydrodynamical equations. However, the accounting for pressure effects in his approach was inconsistent with thermal processes, leading to the wrong results for the dynamical behaviour of a system with radiative losses (see Bisnovatyi-Kogan & Tsupko (2005)). Using the variational principle and deriving the equation for the entropy function from energy balance give the correct relations for the pressure and the total energy. In Rosensteel & Tran (1991) the virial equations for rotating Riemann ellipsoids of incompressible fluid are demonstrated to form a Hamiltonian dynamical system.

We calculate a dynamical behavior of a non-spherical, non-rotating star after its loss of a linear stability, and investigate nonlinear stages of contraction. We use approximate system of dynamic equations, describing 3 degrees of freedom of a uniform self-gravitating compressible ellipsoidal body (Bisnovatyi-Kogan (2004), Bisnovatyi-Kogan & Tsupko (2005)). We obtain that the development of instability leads to the formation of a regularly or chaotically oscillating body, in which dynamical motion

⋆ E-mail: gkogan@iki.rssi.ru (GSBK); tsupko@iki.rssi.ru (OYuT)

prevents the formation of the singularity. We find regions of chaotic and regular pulsations by constructing a Poincaré diagram for different values of the initial eccentricity and initial entropy. For simplicity we restrict ourself by calculating only spheroidal figures with $\gamma = \frac{4}{3}$, and only briefly represent results for $\gamma = \frac{6}{5}$. At the end we discuss qualitatively effects of general relativity in a non-spherical collapse of a non-rotating body.

2 EQUATIONS OF MOTION

Let us consider 3-axis ellipsoid with semi-axes $a \neq b \neq c$:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, \tag{1}$$

and uniform density ρ. A mass m of the uniform ellipsoid is written as (V is the volume of the ellipsoid)

$$m = \rho V = \frac{4\pi}{3}\rho\, abc. \tag{2}$$

Let us assume a linear dependence of velocities on coordinates

$$\upsilon_x = \frac{\dot{a}x}{a}, \quad \upsilon_y = \frac{\dot{b}y}{b}, \quad \upsilon_z = \frac{\dot{c}z}{c}. \tag{3}$$

The gravitational energy of the uniform ellipsoid is defined as (Landau & Lifshitz 1993):

$$U_g = -\frac{3Gm^2}{10}\int\limits_0^\infty \frac{du}{\sqrt{(a^2+u)(b^2+u)(c^2+u)}}. \tag{4}$$

The equation of state $P = K\rho^\gamma$ is considered here, with $\gamma = 4/3$. Note that the case $\gamma = 5/3$ was considered by Bisnovatyi-Kogan (2004) and Bisnovatyi-Kogan & Tsupko (2005), but this case is not interesting for the present work, because isentropic spherical star with $\gamma = 5/3$ always stops contraction, and never suffers collapse to singularity. A spherical star with $\gamma = 4/3$ collapses to singularity at small enough K, and we show here, how deviations from a spherical form prevent formation of any singularity. For $\gamma = 4/3$, the thermal energy of the ellipsoid is $E_{th} \sim V^{-1/3} \sim (abc)^{-1/3}$, and the value

$$\varepsilon = E_{th}(abc)^{1/3} = 3\left(\frac{3m}{4\pi}\right)^{1/3} K$$

remains constant in time. A Lagrange function of the ellipsoid is written as

$$L = U_{kin} - U_{pot}, \quad U_{pot} = U_g + E_{th}, \tag{5}$$

$$U_{kin} = \frac{1}{2}\rho\int\limits_V (\upsilon_x^2+\upsilon_y^2+\upsilon_z^2)\,dV = \frac{m}{10}(\dot{a}^2+\dot{b}^2+\dot{c}^2), \tag{6}$$

$$E_{th} = \frac{\varepsilon}{(abc)^{1/3}}. \tag{7}$$

Equations of motion describing behavior of 3 semiaxes (a, b, c) is obtained from the Lagrange function (5) in the form

$$\ddot{a} = -\frac{3Gm}{2}a\int\limits_0^\infty \frac{du}{(a^2+u)\Delta} + \frac{5}{3m}\frac{1}{a}\frac{\varepsilon}{(abc)^{1/3}}, \tag{8}$$

$$\ddot{b} = -\frac{3Gm}{2}b\int\limits_0^\infty \frac{du}{(b^2+u)\Delta} + \frac{5}{3m}\frac{1}{b}\frac{\varepsilon}{(abc)^{1/3}}, \tag{9}$$

$$\ddot{c} = -\frac{3Gm}{2}c\int\limits_0^\infty \frac{du}{(c^2+u)\Delta} + \frac{5}{3m}\frac{1}{c}\frac{\varepsilon}{(abc)^{1/3}}, \tag{10}$$

$$\Delta^2 = (a^2+u)(b^2+u)(c^2+u).$$

3 DIMENSIONLESS EQUATIONS AND NUMERICAL RESULTS

To obtain a numerical solution of equations we write them in non-dimensional variables. Let us introduce the variables

$$\tilde{t} = \frac{t}{t_0},\ \tilde{a} = \frac{a}{a_0},\ \tilde{b} = \frac{b}{a_0},\ \tilde{c} = \frac{c}{a_0},$$

$$\tilde{m} = \frac{m}{m_0},\ \tilde{\rho} = \frac{\rho}{\rho_0},\ \tilde{U} = \frac{U}{U_0},\ \tilde{E}_{th} = \frac{E_{th}}{U_0},\ \tilde{\varepsilon} = \frac{\varepsilon}{\varepsilon_0}.$$

The scaling parameters $t_0,\ a_0,\ m_0,\ \rho_0,\ U_0,\ \varepsilon_0$ are connected by the following relations

$$t_0^2 = \frac{a_0^3}{Gm_0},\ U_0 = \frac{Gm_0^2}{a_0},\ \rho_0 = \frac{m_0}{a_0^3},\ \varepsilon_0 = U_0 a_0. \tag{11}$$

System of non-dimensional equations:

$$\ddot{a} = -\frac{3m}{2}a\int\limits_0^\infty \frac{du}{(a^2+u)\Delta} + \frac{5}{3m}\frac{1}{a}\frac{\varepsilon}{(abc)^{1/3}}, \tag{12}$$

$$\ddot{b} = -\frac{3m}{2}b\int\limits_0^\infty \frac{du}{(b^2+u)\Delta} + \frac{5}{3m}\frac{1}{b}\frac{\varepsilon}{(abc)^{1/3}}, \tag{13}$$

$$\ddot{c} = -\frac{3m}{2}c\int\limits_0^\infty \frac{du}{(c^2+u)\Delta} + \frac{5}{3m}\frac{1}{c}\frac{\varepsilon}{(abc)^{1/3}}, \tag{14}$$

$$\Delta^2 = (a^2+u)(b^2+u)(c^2+u).$$

In equations (12)-(14) only non-dimensional variables are used, and "tilde" sign is omitted for simplicity in this section. The non-dimensional Hamiltonian (or non-dimensional total energy) is:

$$H = U_{kin} + U_g + E_{th} = \frac{m}{10}(\dot{a}^2+\dot{b}^2+\dot{c}^2) -$$

$$-\frac{3m^2}{10}\int\limits_0^\infty \frac{du}{\sqrt{(a^2+u)(b^2+u)(c^2+u)}} + \frac{\varepsilon}{(abc)^{1/3}}. \tag{15}$$

In case of the sphere ($a = b = c$, $\dot{a} = \dot{b} = \dot{c}$) the non-dimensional Hamiltonian and non-dimensional equations of motion reduce to:

$$H = \frac{3}{10}m\dot{a}^2 - \frac{3}{5a}\left(m^2 - \frac{5}{3}\varepsilon\right), \tag{16}$$

$$\ddot{a} = -\frac{1}{ma^2}\left(m^2 - \frac{5}{3}\varepsilon\right). \tag{17}$$

As follows from (16), (17) for the given mass there is only one equilibrium value of ε

$$\varepsilon_{eq} = \frac{3m^2}{5}, \tag{18}$$

at which the spherical star has zero total energy, and it may have an arbitrary radius. For smaller $\varepsilon < \varepsilon_{eq}$ the sphere should contract to singularity, and for $\varepsilon > \varepsilon_{eq}$ there will be a total disruption of the star with an expansion to infinity. We solve here numerically the equations of motion for a spheroid with $a = b \neq c$, which, using (12)-(14), are written for the oblate spheroid with $k = c/a < 1$ as

$$\ddot{a} = \frac{3}{2}\frac{m}{a^2(1-k^2)}\left[k - \frac{\arccos k}{\sqrt{1-k^2}}\right] + \frac{5}{3m}\frac{1}{a}\frac{\varepsilon}{(a^2c)^{1/3}}, \tag{19}$$

$$\ddot{c} = -3\frac{m}{a^2(1-k^2)}\left[1 - \frac{k\arccos k}{\sqrt{1-k^2}}\right] + \frac{5}{3m}\frac{1}{c}\frac{\varepsilon}{(a^2c)^{1/3}}; \tag{20}$$

and for the prolate spheroid $k = c/a > 1$ as

$$\ddot{a} = -\frac{3}{2}\frac{m}{a^2(k^2-1)}\left[k - \frac{\cosh^{-1} k}{\sqrt{k^2-1}}\right] + \frac{5}{3m}\frac{1}{a}\frac{\varepsilon}{(a^2c)^{1/3}}, \tag{21}$$

$$\ddot{c} = 3\frac{m}{a^2(k^2-1)}\left[1 - \frac{k\cosh^{-1} k}{\sqrt{k^2-1}}\right] + \frac{5}{3m}\frac{1}{c}\frac{\varepsilon}{(a^2c)^{1/3}}. \tag{22}$$

It is convenient to introduce variables

$$\varepsilon_* = \frac{5}{3}\frac{\varepsilon}{m^2}, \quad t_* = t\sqrt{m}. \tag{23}$$

In these variables the equations (19)-(22), (17) are written as (omitting subscript "*")

$$\ddot{a} = \frac{3}{2a^2(1-k^2)}\left[k - \frac{\arccos k}{\sqrt{1-k^2}}\right] + \frac{1}{a}\frac{\varepsilon}{(a^2c)^{1/3}}, \tag{24}$$

$$\ddot{c} = -\frac{3}{a^2(1-k^2)}\left[1 - \frac{k\arccos k}{\sqrt{1-k^2}}\right] + \frac{1}{c}\frac{\varepsilon}{(a^2c)^{1/3}} \tag{25}$$

for the oblate spheroid $k = c/a < 1$,

$$\ddot{a} = -\frac{3}{2a^2(k^2-1)}\left[k - \frac{\cosh^{-1} k}{\sqrt{k^2-1}}\right] + \frac{1}{a}\frac{\varepsilon}{(a^2c)^{1/3}}, \tag{26}$$

$$\ddot{c} = \frac{3}{a^2(k^2-1)}\left[1 - \frac{k\cosh^{-1} k}{\sqrt{k^2-1}}\right] + \frac{1}{c}\frac{\varepsilon}{(a^2c)^{1/3}} \tag{27}$$

for the prolate spheroid $k = c/a > 1$, and

$$\ddot{a} = -\frac{1-\varepsilon}{a^2} \tag{28}$$

for the sphere, where the equilibrium corresponds to $\varepsilon_{eq} = 1$. Near the spherical shape we should use expansions around $k = 1$, what leads to equations of motion valid for both oblate and prolate cases

$$\ddot{a} = -\frac{1-\varepsilon}{a^2} + \left(\frac{\varepsilon}{3} + \frac{3}{5}\right)\frac{1-k}{a^2},$$
$$\ddot{c} = -\frac{1-\varepsilon}{a^2} + \left(\frac{4\varepsilon}{3} - \frac{4}{5}\right)\frac{1-k}{a^2}. \tag{29}$$

In these variables the total energy is written as

$$H_* = \frac{H}{m^2}, \tag{30}$$

and omitting "*" we have

$$H = \frac{\dot{a}^2}{5} + \frac{\dot{c}^2}{10} - \frac{3}{5a}\frac{\arccos k}{\sqrt{1-k^2}} + \frac{3}{5}\frac{\varepsilon}{(a^2c)^{1/3}}, \quad \text{(oblate)}$$

$$H = \frac{\dot{a}^2}{5} + \frac{\dot{c}^2}{10} - \frac{3}{5a}\frac{\cosh^{-1} k}{\sqrt{k^2-1}} + \frac{3}{5}\frac{\varepsilon}{(a^2c)^{1/3}}, \quad \text{(prolate)}$$

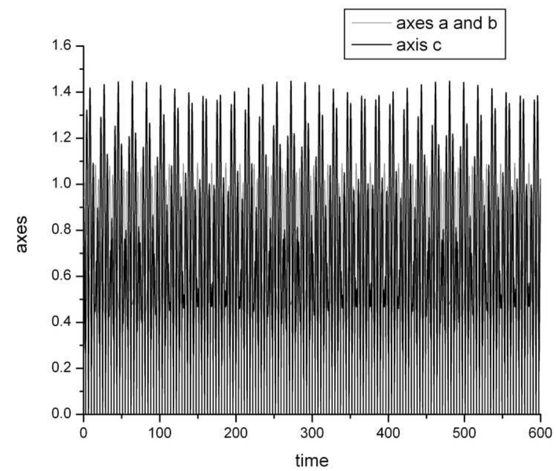

Figure 1. Example of regular motion of spheroid with $\gamma = 4/3$, $H = -1/5$, $\varepsilon = 2/3$. This motion corresponds to full line on the Poincaré map in Fig.4.

$$H = \frac{3\dot{a}^2}{10} - \frac{3}{5a}(1-\varepsilon), \quad \text{(sphere)} \tag{31}$$

$$H = \frac{\dot{a}^2}{5} + \frac{\dot{c}^2}{10} - \frac{3}{5a}\left(1 + \frac{\delta}{3} + \frac{2\delta^2}{15}\right) + \frac{3\varepsilon}{5a}\left(1 + \frac{\delta}{3} + \frac{2\delta^2}{9}\right),$$

$\delta = 1 - k$, (around the sphere), $|\delta| \ll 1$.

Solution of the system of equations (19)-(22) was performed for initial conditions at $t = 0$: $\dot{c}_0 = 0$, different values of initial a_0, $\dot{a}_0$, k_0, and different values of the constant parameter ε. Evidently, at $k_0 = 1$, $\dot{a}_0 = 0$, $\varepsilon < 1$ we have the spherical collapse to singularity. The most interesting result was obtained at $k_0 \neq 1$, and all other cases with deviations from spherical symmetry. No singularity was reached in this case for any $\varepsilon > 0$. It is clear, that at $\varepsilon = 0$ a weak singularity is reached during formation of a pancake with infinite volume density, and finite gravitational force. At $\varepsilon > 0$ the behavior depends on the value of the total energy H: at $H > 0$ we obtain a full disruption of the body, and at $H < 0$ the oscillatory regime is established at any value of $\varepsilon < 1$. At $\varepsilon \geqslant 1$ the total energy of spheroid is positive, $H > 0$, determining the full disruption. The case with $H = 0$ is described separately below.

At $H < 0$ the type of oscillatory regime depends on initial conditions, and may be represented either by regular periodic oscillations, or by chaotic behavior. Examples of two types of such oscillations are represented in Figs.1-2 (regular, periodic), and in Fig.3 (chaotic). For rigorous separation between these kinds of oscillations we use a method developed by Poincaré (Lichtenberg & Lieberman 1983).

3.1 The case H=0

In the case of a sphere with zero initial radial velocity we have equilibrium state with $\varepsilon = \varepsilon_{eq} = 1$. Let us consider the case of $\dot{a} \neq 0$ and $\varepsilon < 1$. It follows from (31), that for non-zero initial velocity $\dot{a}$ entropy function ε is strictly less than unity. The fate of such sphere depends on the sigh of $\dot{a}$. At $\dot{a} < 0$ the star collapses to singularity, and at $\dot{a} > 0$ there total disruption happens, with zero velocity at infinity.

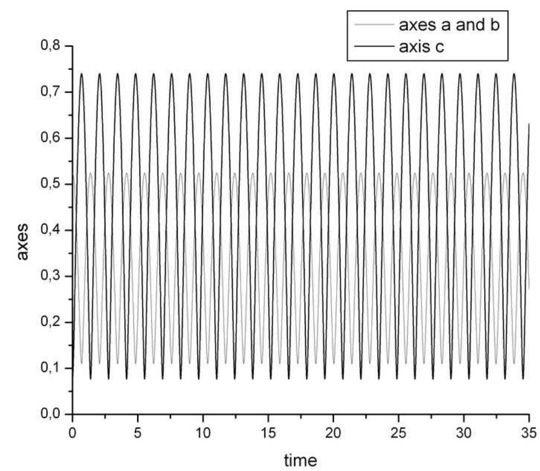

Figure 2. Example of regular motion of spheroid with $\gamma = 4/3$, $H = -1/5$, $\varepsilon = 2/3$. This motion corresponds to the point inside the regular region on the Poincaré map in Fig.4.

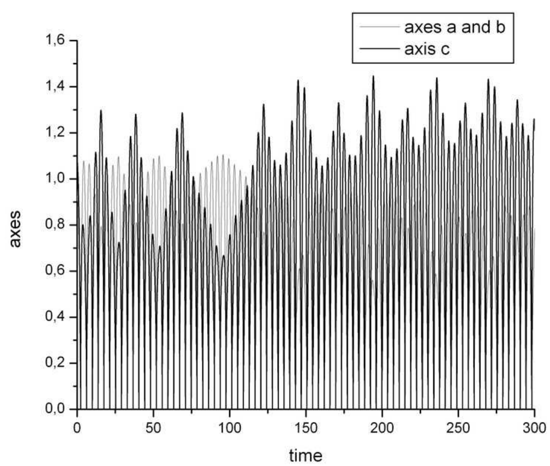

Figure 3. Example of chaotic motion of spheroid with $\gamma = 4/3$, $H = -1/5$, $\varepsilon = 2/3$. This motion corresponds to gray points on the Poincaré map in Fig.4.

For the initial spheroid with $a = b \neq c$, and zero initial velocities $\dot{a} = \dot{c} = 0$ the entropy function is also less than 1. In this case the value of the entropy function ε is uniquely determined by the deformation $\delta = (a-c)/a$. This dependence can be found explicitly from (31); for small δ we obtain

$$\varepsilon = 1 - \frac{4}{45}\,\delta^2. \tag{32}$$

Thus even in the case of zero initial velocities, ε is less than unity for a spheroid, and reaches unity only for the sphere. For the same ε the deformation δ of the zero energy body at rest have two states with $\delta = \pm\frac{3}{2}\sqrt{5(1-\varepsilon)}$, corresponding to oblate and prolate spheroids. If we set non-zero initial velocities in case of spheroid, entropy function will be even less. Calculation of the motion in this case leads finally to the expansion of the body, with oscillating behavior of δ around zero (oblate - prolate oscillations). It takes place both under the condition of the initial contraction or initial expansion.

4 THE POINCARÉ SECTION

To investigate regular and chaotic dynamics we use the method of Poincaré section (Lichtenberg & Lieberman 1983) and obtain the Poincaré map for different values of the total energy H. Let us consider a spheroid with semi-axes $a = b \neq c$. This system has two degrees of freedom. Therefore in this case the phase space is four-dimensional: a, $\dot{a}$, c, $\dot{c}$. If we choose a value of the Hamiltonian H_0, we fix a three-dimensional energy surface $H(a, \dot{a}, c, \dot{c}) = H_0$. During the integration of the equations (19)-(22) which preserve the constant H, we fix moments t_i, when $\dot{c} = 0$. At these moments there are only two independent values (i.g. a and $\dot{a}$), because the value of c is determined uniquely from the relation for the hamiltonian at constant H. At each moment t_i we put a dot on the plane $(a, \dot{a})$.

For the same values of H and ε we solve equations of motion (19)-(22) at initial $\dot{c} = 0$, and different a, $\dot{a}$. For each integration we put the points on the plane $(a, \dot{a})$ at the moments t_i. These points are the intersection points of the trajectories on the three-dimensional energy surface with a two-dimensional plane $\dot{c} = 0$, called the Poincaré section.

For each fixed combination of ε, H we get the Poincaré map, represented in Figs. 3-6. Condition $\dot{c} = 0$ splits in two cases, of a minimum and of a maximum of c. The Poincaré maps are drawing separately, either for the minimum, or for the maximum of c, and both maps lead to identical results. The regular oscillations are represented by closed lines on the Poincaré map, and chaotic behavior fills regions of finite square with dots. These regions are separated from the regions of the regular oscillations by separatrix line.

4.1 The bounding curve

Actually, the variables a and $\dot{a}$ cannot occupy the whole plane $(a, \dot{a}):\ 0 < a < \infty, -\infty < \dot{a} < +\infty$. Let us obtain a curve bounding the area of the values a and $\dot{a}$. Let a function $\Phi(a, \dot{a}, c)$ in the variables (23), (30) be

$$\Phi(a, \dot{a}, c) = \frac{1}{10}\,(\dot{a}^2 + \dot{b}^2 + \dot{c}^2) -$$

$$-\frac{3}{10}\int\limits_0^\infty \frac{du}{\sqrt{(a^2+u)(b^2+u)(c^2+u)}} + \frac{3\varepsilon}{5(abc)^{1/3}} - H \tag{33}$$

at $a = b$, $\dot{a} = \dot{b}$, $\dot{c} = 0$, and fixed value of ε and H. The equation for the bounding curve $f(a, \dot{a}) = 0$ at given ε and H is determined from the following system of equation

$$\Phi(a, \dot{a}, c) = 0\,, \quad \frac{\partial}{\partial c}\Phi(a, \dot{a}, c) = 0\,. \tag{34}$$

In case of $c < a$ this system of equations has a form

$$\frac{1}{5}\,\dot{a}^2 - \frac{3}{5}\,\frac{\arccos(c/a)}{\sqrt{a^2-c^2}} + \frac{3\varepsilon}{5(a^2c)^{1/3}} - H = 0\,, \tag{35}$$

$$-\frac{3}{5}\,c\,\frac{\arccos(c/a)}{(a^2-c^2)^{3/2}} + \frac{3}{5}\,\frac{1}{a^2-c^2} - \frac{1}{3}\,\frac{1}{c}\,\frac{3\varepsilon}{5(a^2c)^{1/3}} = 0\,. \tag{36}$$

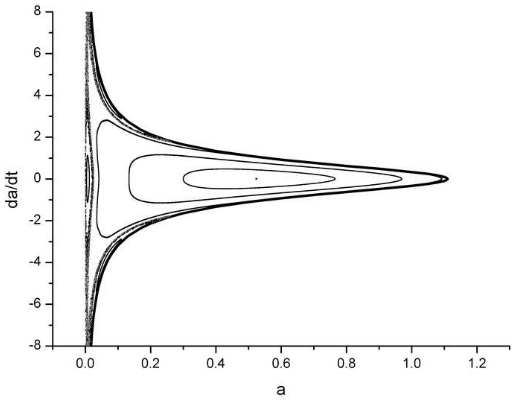

Figure 4. The Poincaré map for five regular and two chaotic trajectories in case of $\gamma = 4/3$, $H = -1/5$, $\varepsilon = 2/3$. The $(a, \dot{a})$ values are taken in the minimum of c. Full black line is the bounding curve. The point inside the regular region corresponds to coherent oscillations with the same period for a and c values, represented in Fig.2.

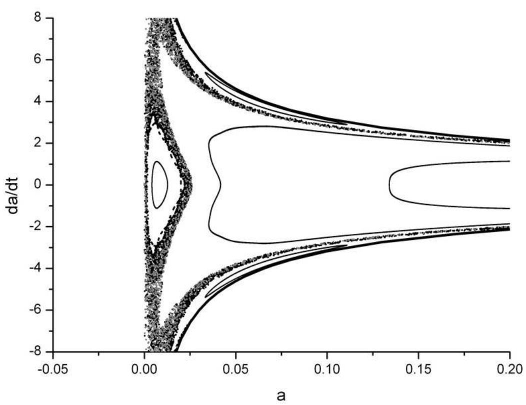

Figure 5. Zoom of previous figure.

This system is solved numerically. The second equation doesn't depend on $\dot{a}$. We set a and obtain corresponding value of c from the second equation. Then we substitute a and c into the first equation and find $\dot{a}$. Thus we obtain the point $(a, \dot{a})$. Changing a, we obtain numerically the curve $f(a, \dot{a}) = 0$. This bounding curve is painted in Figs.4-8 by a heavy line.

5 DISCUSSION

The main result following from our calculations is the indication to a degenerate nature of formation of a singularity in unstable newtonian self-gravitating gaseous bodies. Only pure spherical models can collapse to singularity, but any kind of nonsphericity leads to nonlinear stabilization of the collapse by a dynamic motion, and formation of regularly or chaotically oscillating body. This conclusion is valid for all unstable equations of state, namely, for adiabatic with

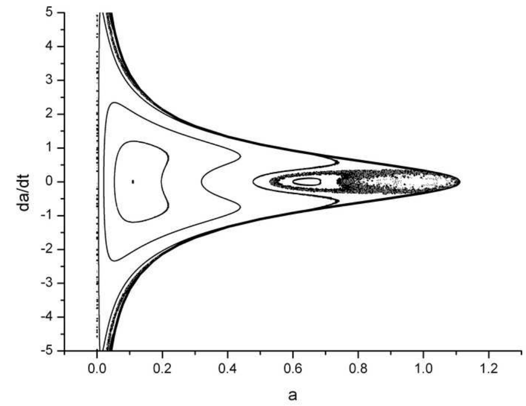

Figure 6. The Poincaré map for five regular and two chaotic trajectories in case of $\gamma = 4/3$, $H = -1/5$, $\varepsilon = 2/3$. The $(a, \dot{a})$ values are taken in the maximum of c. Full black line is the bounding curve. The point inside the regular region corresponds to coherent oscillations with the same period for a and c values, represented in Fig.2.

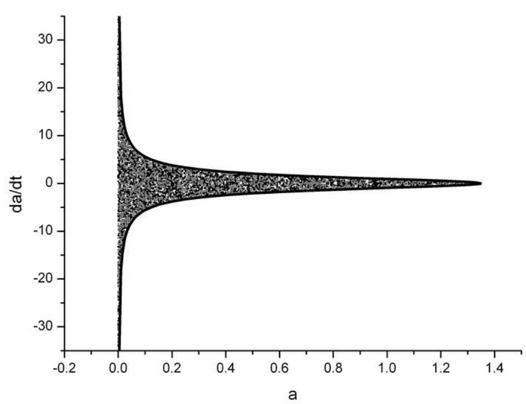

Figure 7. The Poincaré map for two chaotic trajectories in case of $\gamma = 4/3$, $H = -1/2$, $\varepsilon = 1/6$. The $(a, \dot{a})$ values are taken in the minimum of c. Full black line is the bounding curve.

$\gamma < 4/3$. In addition to the case with $\gamma = 4/3$, we have calculated the dynamics of the model with $\gamma = 6/5$, and have obtained similar results. The Poincaré map for this case is represented in Fig.9. For $\gamma = 6/5$ we have the entropy function $\varepsilon = E_{th}(abc)^{1/5}$, nondimensional entropy function $\tilde{\varepsilon} = \varepsilon / U_0 a_0^{3/5}$, see (11). For the spherical star with $\gamma = 6/5$ the nondimensional Hamiltonian instead of (16) is

$$\tilde{H} = \frac{3m}{10}\dot{a}^2 - \frac{3}{5a}m^2 + \frac{\tilde{\varepsilon}}{a^{3/5}}. \qquad (37)$$

Note, that region of chaotic behavior on the Poincaré map is gradually increasing for $\gamma = 4/3$ with decreasing of the entropy ε and the total energy H. At $\varepsilon = 9/10$ and $H = -3/50$ we have found only regular oscillations (Fig.8), at $\varepsilon = 2/3$ and $H = -1/5$ both kind of oscillations are present (Figs. 4-6), and only chaotic behavior is found at $\varepsilon = 1/6$ and $H = -1/2$ (Fig.7). We connect

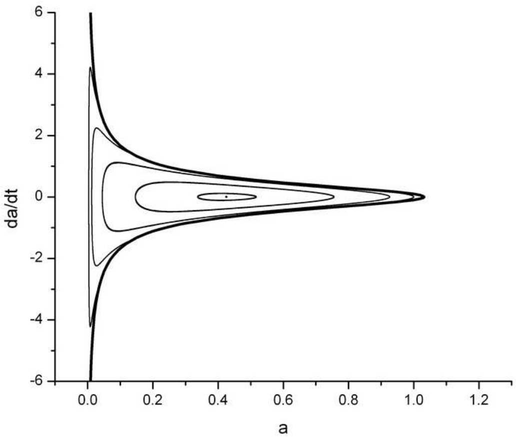

Figure 8. The Poincaré map for six regular trajectories in the case of $\gamma = 4/3$, $H = -3/50$, $\varepsilon = 9/10$. The $(a, \dot{a})$ values are taken in the minimum of c. Full black line is the bounding curve. The point inside the regular region corresponds to coherent oscillations with the same period for a and c values, similar to those represented in Fig.2.

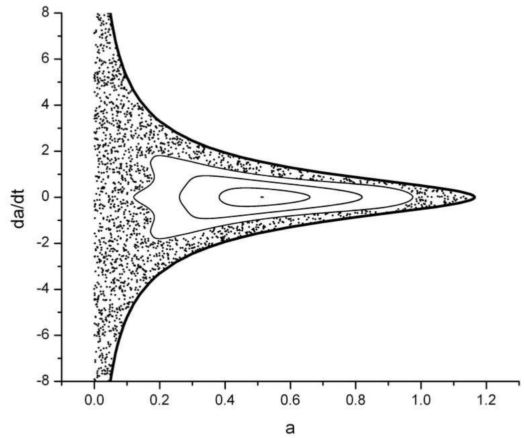

Figure 9. The Poincaré map for one chaotic and four regular trajectories in case of $\gamma = 6/5$, $m = 1$, $\tilde{H} = -3/50$, $\tilde{\varepsilon} = 27/50$, see (37). The $(a, \dot{a})$ values are taken in the minimum of c. Full black line is the bounding curve. The point inside the regular region corresponds to coherent oscillations with the same period for a and c values, similar to those represented in Fig.2.

(Bisnovatyi-Kogan & Tsupko 2005) this chaotic behavior with development of anisotropic instability, when radial velocities strongly exceed the transversal ones (Antonov 1973; Fridman & Polyachenko 1985).

In reality a presence of dissipation leads to damping of these oscillations, and to final collapse of nonrotating model, when total energy of the body is negative. In the case of core-collapse supernova the main dissipation is due to emission of neutrino. The time of the neutrino losses is much larger than the characteristic time of the collapse, so we may expect that the collapse leads to formation of a neutron star where nonspherical modes are excited and exist during several seconds after the collapse. In addition to the damping due to neutrino emission the shock waves will be generated, determining highly variable energy losses during the oscillations. Besides viscosity and radiation which can damp the ellipsoid-like motions and allow collapse, there is the possibility that inertial interactions with higher order modes that must be present in the real stratified bodies may cause an inertial cascade and drain the energy from the second order modes an a non-secular way that does not depend on dissipative coefficients. It is very seductive to connect chaotic oscillations with highly variable emission observed in the prompt gamma ray emission of cosmic gamma ray bursts (Mazets et al. 1981). A presence of rotation and magnetic field strongly complicate the picture of the core-collapse supernova explosion (Ardeljan, Bisnovatyi-Kogan & Moiseenko 2005).

In this paper we consider in details only spheroidal bodies. In reality the spheroid will become a triaxial ellipsoid during the motion. In addition to spheroids we calculate many variants with triaxial figures (see also Bisnovatyi-Kogan & Tsupko (2005)). Qualitatively we obtain the same results for ellipsoids: no singularity was reached for any $\varepsilon > 0$ and establishing oscillatory (regular or chaotic) regime under negative total energy prevents the collapse. However, in the case of the ellipsoid with semi-axes $a \neq b \neq c$ we have a system with three degrees of freedom and six-dimensional phase space. Therefore we could not carry out rigorous investigation of regular and chaotic types of motion by the constructing Poincaré map as it was done for spheroid with two degrees of freedom, and restricted ourselfs by a description of the spheroidal case.

In the frame of a general relativity dynamic stabilization against collapse by nonlinear nonspherical oscillations cannot be universal. When the size of the body approaches gravitational radius no stabilization is possible at any γ. Nevertheless, the nonlinear stabilization may happen at larger radii, so after damping of the oscillations the star would collapse to the black hole. Due to development of nonspherical oscillations there is a possibility for emission of gravitational waves during the collapse of nonrotating stars with the intensity similar to rotating bodies, or even larger.

Account of general relativity will introduce a new non-dimensional parameter, which can be written as $p_g = \frac{2Gm_0}{c^2 a_0}$. The fate of the gravitating body will depend on the value of this parameter, and we may expect a direct relativistic collapse to a black hole at increasing p_g, approaching unity. It is known, that a nonrotating black hole is characterized only by its mass (Zeldovich & Novikov 1967). In absence of other dissipative processes, the excess of energy, connected with a nonspherical motion should be emitted by gravitational waves during a formation of the black hole.

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