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A novel visibility graph transformation of time series into weighted networks

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ABSTRACT

Analyzing time series from the perspective of complex network has interested many scientists. In this paper, based on visibility graph theory a novel method of constructing weighted complex network from time series is proposed. The first step is to determine the weights of vertices in time series, which linearly combines the weights generated by induced ordered averaging aggregation operator (IOWA) and visibility graph aggregation operator (VGA). Then, two strategies, averaging strategy and gravity strategy, are proposed to construct weighted network. To testify the validity of proposed method, an artificial case is adopted, in which link prediction is used to evaluate the performance of the weighted network. It is shown that the weighted network constructed by proposed method greatly outperforms the unweighted network obtained by traditional visibility graph theory.

1. Introduction

Time series is a model based on the information provided by a finite number of observations [1], or more commonly, a sequence taken at successive equally spaced points in time order. Time series is widely applied in statistics [2], finance [3], biology [4] and so forth [5,6]. Recently, it arouses many scientists' interest to analyze time series from the perspective of complex network [7–9]. Zhang et al. [10] introduced a method to deal with the pseudoperiodic time series and found that the structure of the corresponding network depends on the dynamics of the series. Wang et al. [11] proposed the phase space coarse graining algorithm to convert time series into directed network which inherits several properties of the series in its structure. Gao and Jin [12] found that time series with different dynamics exhibit distinct topological properties.

Visibility graph [16] is considered as the bridge between time series and complex network, in which the values of a time series are plotted as vertical bars, and two bars are connected if they can see each other from the top. The structure of time series conserves when it is converted to topological graph [10,16]. Visibility graph theory is wildly applied to many fields, such as economy [17–19], geology [20,21], biology [22–24], transportation [25] and so on [26–28]. Serval developments were made based on traditional visibility graph, such as horizontal visibility graph (HVG) [29]. and multi-scale limited penetrable visibility graph (LPVG) [30], which mainly focused on different ways of constructing visibility graph. Aggregation operators based on visibility graph is another application, visibility graph aggregation (VGA) operator [31] and visibility graph power averaging aggregation (VGPA) operator [32] were proposed. The VGA operator is able to generate weights for vertices in time series based on their degree distribution in the corresponding visibility graph.

To determine weights for the vertices in time series in the first place, the Induced ordered weighted averaging (IOWA) operator and visibility graph aggregation (VGA) operator are adopted. IOWA operator [33] is developed from ordered weighted averaging (OWA) aggregation operator [34]. It is a more general type of OWA operator, in which one component is used to induce an ordering over the second components. OWA operator and IOWA operator are widely used in uncertainty management [35–38], decision-making process [14,39–43], failure analysis [45], and so on[44]. Weight determination is an essential issue for IOWA operator. Many approaches, such as learning the weighting vector, best yesterday model, and so forth [46]. Fuller et al. [47] proposed a method to obtain the weights for OWA operator based on maximal entropy.

In this paper, A novel method of constructing time series into a weighted complex network based on visibility graph theory is proposed. The first step is to generate weights of vertices in the time series, i.e., weights of nodes in the corresponding visibility graph. The weights consist of two parts which are obtained by IOWA op-

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erator and VGA operator separately. Both time decay and vertices' relative importance are taken into consideration when determining weights. Then two different strategies, averaging strategy and gravity strategy, are applied to distribute the weights of nodes to the weights of edges, so that a weighted network is established.

To testify the validity of the weight determination in proposed method, link prediction is introduced. Link prediction aims at estimating the likelihood of the existence of a link between two nodes [48]. A simulated time series case from [49] is adopted to show the effectiveness of proposed method. It is shown that the weighted network generated by proposed method greatly outperforms the unweighted network obtained by the traditional visibility graph.

The structure of this paper is organized as follows: Section 2 briefly introduces some basic preliminaries of IOWA operator, link prediction, and visibility graph theory. Then, the proposed method is detailed in Section 3 with a simple numerical example. Section 4 demonstrates the validity of proposed method by an artificial case. Finally, Section 5 briefly summarise this paper.

2. Preliminary

2.1. IOWA operator

OWA operator and IOWA operator are both mappings $F_W = R^n \to R$, characterized by n-dimensional vector W called the weighting vector. In IOWA operator, the tuples need to be aggregated are in the form $(v_i,\ a_i)$, where v_i represents order inducing value and a_i for the argument value. The IOWA aggregation is performed as [33],

$$F_W(\langle v_1, a_1\rangle, \ldots, \langle v_i, a_i\rangle) = W^T B_V. \tag{1}$$

The elements in B_V are the argument values a_i, ordered by their order inducing value v_i.

Traditionally, IOWA operator is used for aggregation. However, in this paper, IOWA operator is introduced to generate weights for vertices in the time series. We adopt Fuller and Majlenders approach [47] to obtain the weights with maximal entropy. The main procedure is listed as follow,

1. If $n = 2$, then $w_1 = \alpha, w_2 = 1 - \alpha$.
2. If $\alpha = 0$ or $\alpha = 1$, then $w = (0, 0, \cdots, 1)^T$ or $w = (1, 0, \cdots, 0)^T$, respectively.
3. If $n \geq 3$ and $0 < \alpha < 1$ then

$$w_j = \sqrt[n-1]{w_1^{n-j} w_n^{j-1}} \tag{2}$$

$$w_n = \frac{((n-1)\alpha - n)w_1 + 1}{(n-1)\alpha + 1 - nw_1} \tag{3}$$

$$w_1[(n-1)\alpha + 1 - nw_1]^n = ((n-1)\alpha)^{n-1}[((n-1)\alpha - n)w_1 + 1], \tag{4}$$

where α characterizes the degree to which the aggregation is like an *or* operation.

Through the procedure above, we can obtain the first part of the weights for each vertex in time series, which is denoted by ω^{IOWA}.

2.2. Visibility graph averaging aggregation operator

2.2.1. Visibility graph

The visibility graph method was first proposed by Lacasa in [16]. As illustrated in Fig. 1, only if the two bars meet the following requirement, can they be linked. Two data values $(t_1,\ y_1)$ and $(t_2,\ y_2)$ will have visibility and become two connected nodes in the constructed graph, if any other value $(t_3,\ y_3)$ is placed between them satisfies [16],

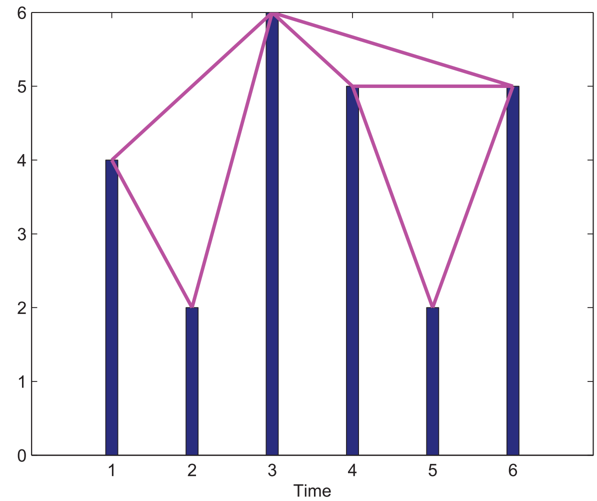


Fig. 1. A simple example of time series and its corresponding visibility graph. The values of time series are plotted by vertical bars. The pink lines represent the edges in the visibility graph if two nodes fulfill the required relationship in Eq. (5). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

$$y_3 < y_2 + (y_1 - y_2)\frac{t_2 - t_3}{t_2 - t_1}. \tag{5}$$

The visibility graph constructed by visibility algorithm above has following properties [16]:

1. **Connected:** each vertex sees at least its nearest neighbors.
2. **Undirected:** there is no direction in the links built up by the algorithm.
3. **Invariant under affine transformation of the series data:** though rescale either horizontal or vertical axes, the visibility criterion is invariant.

2.2.2. Visibility graph averaging aggregation operator

The VGA operator [31] generates weights according to the distribution of nodes' degree in the visibility graph. The VGA operator is a mapping: $F_W = R^n \to R$, to be more specific:

$$F(a_1, a_2, \ldots, a_n) = \omega_1 a_1 + \omega_2 a_2 + \cdots \omega_n a_n, \tag{6}$$

where a_i is the i^{th} argument value in the time series. ω_i is the corresponding weight of a_i which satisfies,

$$\omega_i = \frac{k(n_i)}{\sum_{i=1}^{n} k(n_i)}, \tag{7}$$

where k_{n_i} is the degree of the node n_i.

2.3. Link prediction

2.3.1. Complex network

Complex network has received many attention from researchers in different fields, such as identifying important nodes [50,51], network dynamics[52], graph mining [53–55] and so on [13,15,56]. A network is represented as an ordered pair $G = (N, E)$, in which set $N = \{n_1, n_2, \cdots, n_n\}$ is for nodes and set $E = \{e_1, e_2, \cdots, e_k\}$ is for edges or arcs in the graph. Any edge e_x in the network can be represented as $e_x = e_{ij} = (n_i, n_j)$. Weighted network $G^\omega = (N, E, W)$ has an additional set of weights $W = \{\omega_1, \omega_2, \cdots, \omega_k\}$ assigned to corresponding edges [57]. Node strength measures the strength of the node n_i in terms of the total edge weight of its connections. According to [58], in most real-world weighted networks in which the edge weights are related to the structure of network, there exists power law relationship between node strength and degree.

We follow Ref. [59]. to define the undirected weighted network with following properties. (1) Node cannot link to itself, which means edge in the form of $e_x = (n_i, n_i)$ is not allowed. (2) There's one edge between two vertices at most. The form of $e_x = e_y = \left(n_i, n_j\right)$ shall not exist. (3) Edges are not directive, which means $(n_i, n_j) = (n_j, n_i)$. It is important to point out the weighted network constructed based on visibility theory meets above-mentioned properties.

2.3.2. Similarity-based indices in link prediction

Link prediction is a hot topic in the field of complex network, which aims at estimating the likelihood of the existence of a link between two nodes [48]. It is commonly accepted that two nodes are more likely to be connected if they are more similar, or they have higher "proximity". There are three representative classes of similarity-based indices, Local similarity indices such as common neighbours (CN) index [60], Adamic-Adar (AA) index [61], and resource allocation (RA) index [62]. Global similarity indices, for example, random walk with restart (RWR) index [63] and average commute time (ACT) index. Typical Quasi-local indices includes local random walk (LRW) index and superposed random walk (SRW) index [48]. In this paper, we adopt CN index as the evaluation of nodes' similarity for its simplicity and effectiveness [62].

For a node n_x, let $\Gamma(x)$ denote the set of its neighbours, the similarity of CN index between nodes n_x and n_y is defined as,

$$S_{xy}^{CN} = |\Gamma(n_x) \cap \Gamma(n_y)|, \tag{8}$$

where $|Q|$ is the cardinality of the set Q.

To cope with nodes' similarity in weighted network, Murata and Moriyasu [64] proposed weighted common neighbours (WCN) index. For nodes n_x and n_y in a weighted network, n_z is the common neighbour of n_x and n_y, ω_{xz} denotes the weight of edge e_{xz}, the S_{xy}^{WCN} is calculated by

$$S_{xy}^{WCN} = \sum_{z\in\Gamma(x)\cap\Gamma(y)} \frac{\omega_{xz} + \omega_{zy}}{2}. \tag{9}$$

Notice that "2" in the denominator does not matter when sorting the vertices, so it can be ignored when calculating in practice.

2.3.3. Evaluation metrics

For an undirected network G, all the possible edges are in the universal set U. The number of elements in U is denoted as $\frac{|N|\cdot(|N|-1)}{2}$ [59]. To test the accuracy of the algorithm, the network G is randomly divided into training set E^T and probe set E^P. For a training set, the edges are considered as the known information. While in the probe set, the edges in this set shall not be a reference for prediction, they are used for testing. Then edges in E^P are considered as the elements in the set $U - E$. Obviously, the sets E^T and E^P fulfill:

$$E^T \cup E^P = E \quad and \quad E^T \cap E^P = \emptyset. \tag{10}$$

After establishing the training set E^T and probe set E^P, a method named area under the receiver operating characteristic curve (AUC) [65] is adopted to measure prediction algorithms's accuracy. The AUC can be interpreted as the probability that a randomly chosen missing link (in the set E^P) is given a higher score than a randomly chosen nonexistent link (in the set $U - E$). Among m independent comparisons, when there are m_1 times the missing link having a higher score and m_2 times being of the same score, then the AUC value is:

$$AUC = \frac{m_1 + m_2}{m}. \tag{11}$$

Note that if all the scores are generated from an independent and identical distribution, the AUC value should be about 0.5. Hence, the degree to which the AUC exceeds 0.5 suggests how much better the algorithm performs than pure chance.

3. The proposed method

In this section, the proposed method for constructing a weighted network from corresponding time series is detailed, and a simple numerical example is presented for illustrative purpose. The procedure can be divided into three major steps as follows.

Step 1: Determine the weights of vertices in time series

First, IOWA operator is adopted to generate ω_i^{IOWA}. Considering a time series $T = \{(a_1, v_1), (a_2, v_2), \cdots, (a_n, v_n)\}$ with inducing value v_i and augment value a_i, sorted by inducing value, ω^{IOWA} is obtained by Eq. (2), Eq. (3) and Eq. (4). Second, ω^{VGA} is generated by VGA operator introduced in Section 2.2. Then linearly combine ω^{IOWA} and ω^{VGA} to obtain ω as the weight for vertex, denoted as:

$$\omega_i = \lambda\omega_i^{IOWA} + (1-\lambda)\omega_i^{VGA}, \tag{12}$$

where λ is determined based on specific dataset. That is, λ is set to balance the relative importance of time decay factor in given time series T, whose value can be determined by cross validation.

Step 2: Transform time series into visibility graph

Apply the visibility graph theory to transform T into corresponding visibility graph G. The weight ω obtained in Step 1 is regarded as the weights for nodes in the visibility graph.

Step 3: Construct an undirected weighted network

To distribute the weights of nodes to the connected edges, two different strategies are proposed. The first strategy, averaging strategy, is to consider ω_i as vertex strength in the network, which means averagely distributing ω_i to the edges that are connected to node n_i in G. The weight of edge μ_{ij}^a is obtained by,

$$\mu_{ij}^a = \frac{\omega_i}{k_{n_i}} + \frac{\omega_j}{k_{n_j}}, \tag{13}$$

where k_{n_i} refers to the degree of n_i.

The other strategy, gravity strategy, is adopted, which is also widely used in human movement [66,67] and traffic flow [68] to generate the probability of transportation. μ_{ij}^g then can be obtained by

$$\mu_{ij}^g = \frac{A\omega_i\omega_j}{{d_{ij}}^2}, \tag{14}$$

where d_{ij} is the Euclidean metric between nodes n_i and n_j, ω_i and ω_j are their weights respectively. Note when applying to link prediction, the coefficient A in Eq. (14) doesn't affect the final outcome obviously. Therefore it is not a concern to determine the value of A.

For illustrative purpose, a simple example is adopted to demonstrate the procedure of proposed method which is also summarized in Algorithm 1.

Example 3.1. A short time series $S = \{(1, 4), (2, 2), (3, 6), (4, 5), (5, 2), (6, 5)\}$ (Fig. 1) is known, the detailed procedure of transforming it to a weighted network is presented as follows.

First, determine the weights for every vertex in the time series by adopting the IOWA operator and VGA operator, the results are shown in Table 1. Transform time series into the visibility graph with weights on the nodes, shown in the Fig. 2.

In the last step, we assign the weights of vertices to the edges in two different methods, averaging strategy (Eq. (13)) and gravity strategy (Eq. (14)) separately. And then present them in the form of adjacency matrix for further application. The demonstration of the two simple weighted networks are shown in Fig. 3. Note that edges' weights in Fig. 3(a) are smaller than those in Fig. 3(b). However, what really matters in the network is the relative magnitude of these edges' weights.

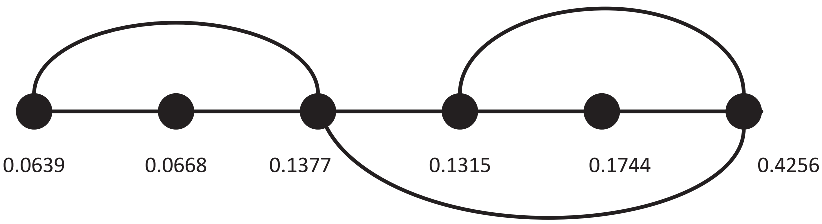
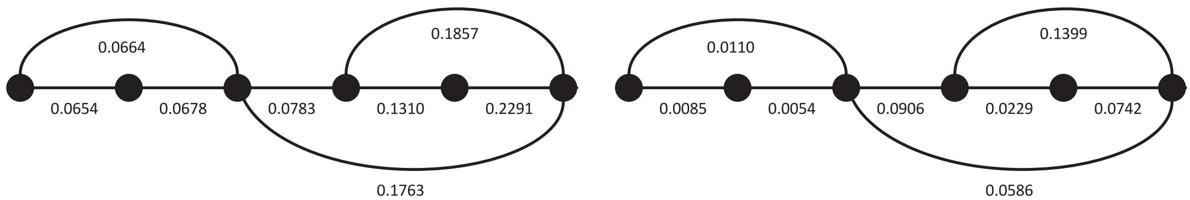


Fig. 2. The graph with weights of nodes.



(a) Averaging strategy (b) Gravity strategy ($G = 10$)

Fig. 3. The weighted network derived from two weight distribution strategies, the weight of each edge is beneath the corresponding edge.

Algorithm 1 Visibility Graph Transformation of Time Series into Weighted Networks.

Input: Time series with order inducing value v_i and argument value a_i, $T = \{(v_1, a_1), (v_2, a_2), \cdots, (v_m, a_m)\}$
Output: Weighted network G^{ω}
1: Adopt IOWA operator to generate ω^{IOWA} (Eq. (2), Eq. (3), and Eq. (4)).
2: Transform T into network G based on visibility graph theory.
3: Determine ω^{VGA} based on the degree distribution in G, $\omega_i^{VGA} = \frac{k(n_i)}{\sum_{i=1}^{n} k(n_i)}$ (Eq.(7)).
4: Linearly combine ω^{IOWA} and ω^{VGA} to obtain ω for vertex in time series, i.e., node in G, with $\omega_i = \lambda\omega_i^{IOWA} + (1-\lambda)\omega_i^{VGA}$ (Eq.(12)).
5: Adopt averaging strategy, $\mu_{ij}^{a} = \frac{\omega_i}{k_{n_i}} + \frac{\omega_j}{k_{n_j}}$ (Eq.(13)), and dummyTXdummy-(gravity strategy, $\mu_{ij}^{g} = \frac{G\omega_i\omega_j}{{d_{ij}}^2}$ (Eq.(14)) to obtain μ^a and μ^g as the weight of edge in weighted network G^{ω} respectively.
6: **return** G^{ω}

Table 1
The weights for vertices in the time series and visibility graph described in Example 3.1. ($\alpha = 0.1$, $\lambda = 0.5$).

Time	1	2	3	4	5	6
ω^{IOWA}	0.0029	0.0086	0.0255	0.0755	0.2238	0.6637
ω^{VGA}	0.1250	0.1250	0.2500	0.1875	0.1250	0.1875
ω	0.0639	0.0668	0.1377	0.1315	0.1744	0.4256

4. Case study

To demonstrate the rationality of weight determination in constructed network, an artificial time series case from [49] is adopted, in which we use link prediction to evaluate the proposed method. In Section 4.1, the basic information of the artificial case and simulation results are presented. Sensitivity analysis is presented with brief discussion in Section 4.2.

4.1. Data and results

In [49], an artificial case is built by simulating 114 trajectories of length $T = 100$ time steps. The signal's behaviour is defined as:

Table 2
Distribution of random parameters a, b, v, d, e, μ, ω, α_1, α_2 and α_3.

Parameter	Distribution
a	Uniform[0.45,0.55]
b	Uniform[0.3,0.4]
c	Uniform[1.1,1.3]
d	Uniform[1.2,1.3]
e	Uniform[0,2]
α_1	Uniform[1,1.5]
α_2	Uniform[0.8,1]
α_3	Uniform[0.6,0.8]
μ	Uniform[2.2,2.7]
ω	Uniform[0,11]

Table 3
Equations used to simulate the 114 trajectories.

Trajectories	100–105	106–112	113–114
Equation	Eq. (15)	Eq. (16)	Eq. (17)

$$x_j = 2\alpha\alpha_1\left[1 + erf\frac{t-e-u}{sqrt2}\right] + 10^{-3\omega} \tag{15}$$

$$x_j = \alpha_2\left(c^{d^{t-e}} - c\right) + 10^{-3\omega} \tag{16}$$

$$x_j = \alpha_3 b(t-e) + 10^{-3\omega} \tag{17}$$

where a, b, v, d, e, μ, ω, α_1, α_2 and α_3 are values randomly sampled from the probability distributions listed in Table 2.

A total of 105 trajectories are simulated by Eq. (15). They are intended to produce the nominal behaviour of a system. To artificially generate some possible anomalous behaviours, the other 9 trajectories are simulated by Eqs. (16) and (17). The specific distribution of the 114 trajectories is in the Table 3. The projections of 114 trajectories simulated by the rules listed above are plotted in Fig. 4.

To test the performance of the proposed method in link prediction. The following steps is taken: (1) Consider every trajectory as a time series with 100 time steps, then construct the corresponding weighted network for every trajectory as introduced in Algorithm 1. For the sake of comparison, we generate a group of unweighted networks using the traditional visibility graph theory,

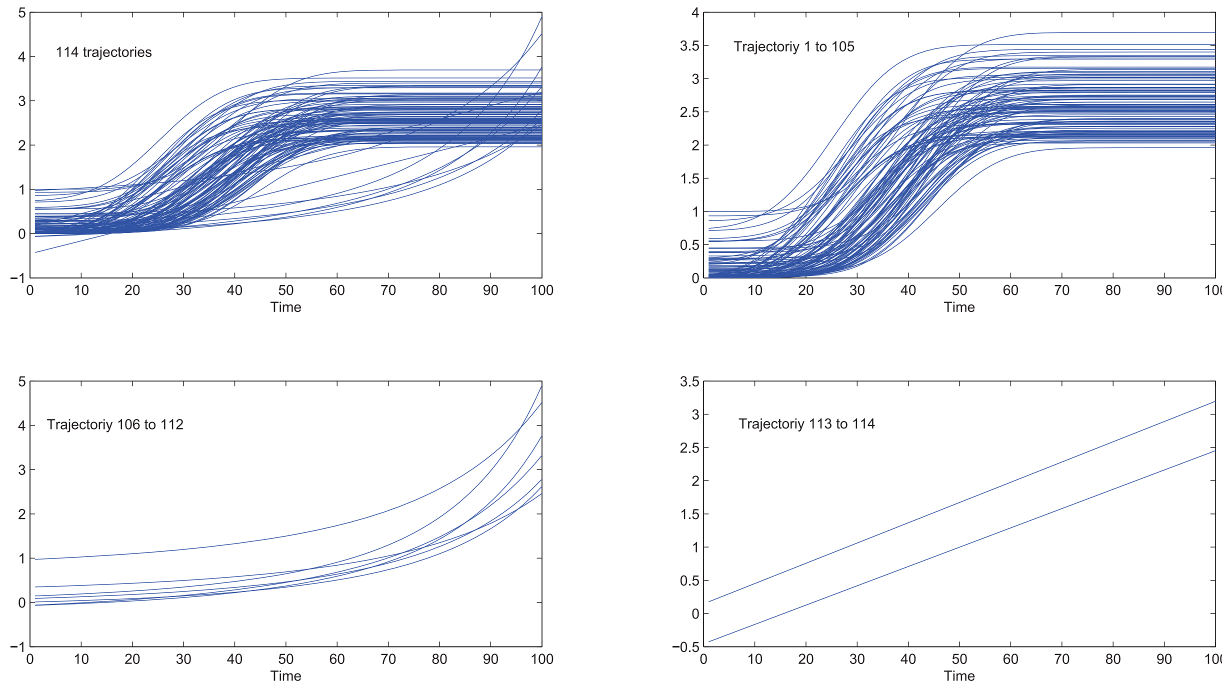


Fig. 4. Time series trajectories generated in the artificial case.

Table 4
Average AUC for different operators when using averaging strategy with $\alpha = 0.1$ and $\lambda = 0.5$. The simulation results are obtained by 100 independent implementations.

Method	Unweighted network	IOWA operator only	VGA operator only	Proposed method
Average AUC	0.5211	0.5530	0.7202	0.7226

Table 5
Average AUC for averaging strategy and gravity strategy when using the proposed method with $\alpha = 0.1$ and $\lambda = 0.5$. The simulation results are obtained by 100 independent implementations.

Method	Averaging strategy	Gravity strategy
Average AUC	0.7226	0.7215

and two other groups of weighted networks by VGA operator and IOWA operator separately. (2) As introduced in Section 2.3.3, randomly select 50 edges in every network as the probe set E^P, then the rest of network is regarded as the training set E^T. (4) Use the weighted CN index, i.e., Eq. (9), to calculate the similarity of every possible edge in the network and sort the edges according to the similarity. Then obtain the AUC value for each group, which are listed in Table 4 and Table 5.

From Table 4, the proposed method and VGA operator perform much better than other groups in the field of link prediction. However, performance of weighted network constructed by the proposed method and VGA operator are similar to each other. Table 5 shows that the two different edge's weight-distribution method, averaging strategy and gravity strategy, have basically the same performance.

Degree distribution is analyzed in the unweighted networks to comprehensively reflect the local information of the nodes in the networks. When it comes to weighted network, node strength is considered, according to [11]. The node strength distribution of three different kinds of simulated signal trajectories are presented in Fig. 5 respectively. It indicates that both networks generated by two strategies satisfy a power-law distribution, which means the simulated signal network is a scale-free network and have some properties of real-word networks introduced.

4.2. Sensitivity analysis

In this section, we run some sensitivity analysis on hyperparameters introduced in proposed method to demonstrate its validity.

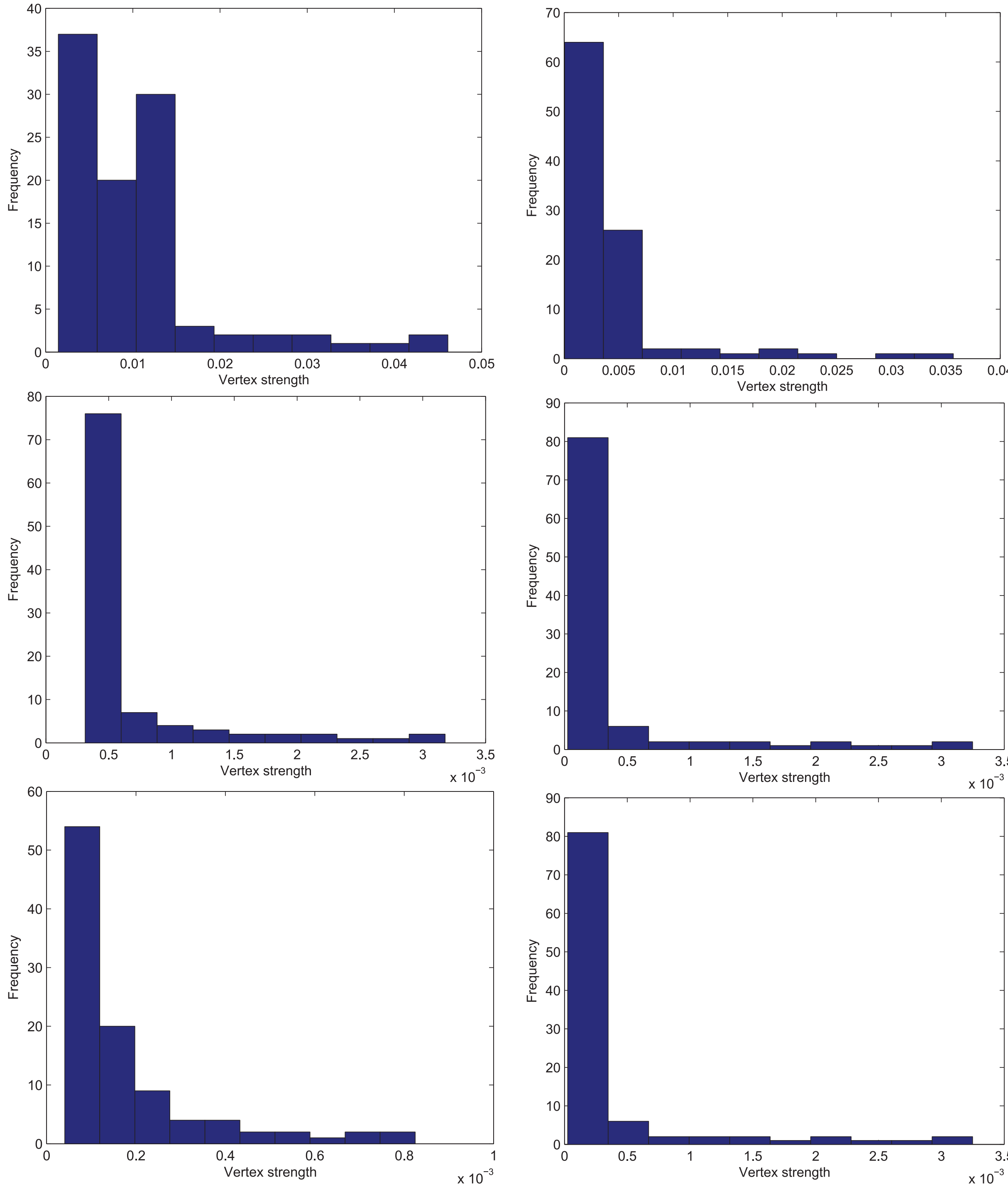
First, parameter α and λ in the proposed method, are analyzed. The results are presented in Table 6. The change of α and λ do not affect the AUC value, i.e., the performance of proposed method. The reason for VGA operator and the proposed method to perform well in link prediction is that they have take the network's structural property into consideration, that is, the edges' weights are concentrated on the edges with higher degree.

The amount of independent comparisons n in the calculation of the AUC value, are changed to see whether it affects the final results. According to Table 7, it's safe to say that the value of n doesn't have a big influence on the prediction accuracy.

4.3. Discussion

μ_{ij}^a obtained by averaging strategy only lies on the weights assigned to n_i and n_j in the first place, while for μ_{ij}^g obtained by gravity strategy, it takes amplitude of variation of time series into consideration when determining weights. Though in the case adopted in this paper, the performance of two strategies have no obvious difference. It requires further study to determine whether the gravity strategy can outperform the averaging strategy in some certain cases.

Weak link theory is proposed in Ref. [69]. and mentioned in Ref. [48]., in which the weak links are considered as the more



(a) Averaging strategy (b) Gravity strategy

Fig. 5. Node strength distribution in the network generated by averaging strategy and gravity strategy (from top to bottom, trajectory 1–105, trajectory 106 to 112, trajectory 113–114).

Table 6
Average AUC values in the sensitivity analysis of α and λ. The simulation results are obtained by 100 independent implementations.

Parameter values	α, $\lambda = 0.5$		λ, $\alpha = 0.1$
	IOWA operator only	Proposed method	Proposed method
0.1	0.5526	0.7226	0.7209
0.3	0.5690	0.7215	0.7249
0.5	0.5698	0.7197	0.7205
0.7	0.5693	0.7191	0.7174
0.9	0.5735	0.7197	0.7237

Table 7
Sensitivity analysis of n in the AUC calculation. Proposed method is adopted with $\alpha = 0.1$ and $\lambda = 0.5$. The simulation results are obtained by 100 independent implementations.

n	20	40	60	80	100	120	140	160	180
AUC	0.7151	0.7167	0.7250	0.7237	0.7329	0.7259	0.7314	0.7326	0.7358

significant role compared to the links with higher weights. However, Murata et al. suggest that links with higher weights are more important in predicting missing links [64]. The argument on whether opinion is right is still an open issue. Artificially constructing weighted networks to test the importance of edges with different weights presents a possible way to study in this field.

5. Conclusion

Traditionally, time series are converted into unweighted networks. This paper proposed a novel method to construct weighted networks from the time series. First of all, the traditional visibility graph is adopted to generate a unweighted network. Then the proposed method is utilized to determine weights for vertices in the time series, i.e. nodes in the corresponding visibility graph. The weights are obtained by combining the weights generated by IOWA operator and VGA operator, which is able to take factors like time decay and nodes' relative importance into consideration when in weight determination. Finally, two different strategies, averaging strategy and gravity strategy, are introduced to distribute the weights of nodes to the connected edges, so that a weighted network is established. The gravity strategy is able to take more factors into consideration when distributing the weights. However, the performance of these two methods are alike in the artificial case applying to the field of link prediction. weighted networks constructed by both averaging strategy and gravity strategy greatly outperform the unweighted network constructed by traditional visibility graph method, which testify the rationality of the proposed method.

In conclusion, the weighted networks generated by proposed method are able to be of great potential in the application of time series analysis. Furthermore, time decay and the relative importance of vertices in the time series are both taken into consideration when determining the weights, the proposed method can be applied to the field of prediction with more precise results.

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