

## A novel method for forecasting time series based on fuzzy logic and visibility graph

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**Abstract** Time series attracts much attention for its remarkable forecasting potential. This paper discusses how fuzzy logic improves accuracy when forecasting time series using visibility graph and presents a novel method to make more accurate predictions. In the proposed method, historical data is firstly converted into a visibility graph. Then, the strategy of link prediction is utilized to preliminarily forecast the future data. Eventually, the future data is revised based on fuzzy logic. To demonstrate the performance, the proposed method is applied to forecast Construction Cost Index, Taiwan Stock Index and student enrollments. The results show that fuzzy logic is able to improve the accuracy by designing appropriate fuzzy rules. In addition, through comparison, it is proved that our method has high flexibility and predictability. It is expected that our work will not only make contributions to the theoretical study of time series forecasting, but also be beneficial to practical areas such as economy and engineering by providing more accurate predictions.

**Keywords** Forecasting · Time series · Fuzzy logic · Visibility graph · Link prediction

**Mathematics Subject Classification** 40B99

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## 1 Introduction

Time series is a sequence of numerical values that describe the statistical index of one certain phenomenon and is ordered in chronological. During our daily life, many data exists in the form of time series, such as air temperature (Yang et al. 2015), Consumer Price Index (CPI) (Tiwari et al. 2014), stock prices (Wang et al. 2011) and so on (McDowell 2014; Derde et al. 2014; Hayes et al. 2014). Via time series analysis, the inner relationships reflected from known data will be obtained, so that possible future tendency can be forecasted. Therefore, this paper is aimed at forecasting time series through proper analysis.

The ability to make more accurate predictions is of great utility value. For instance, by forecasting time series constructed by seismic waves, people are able to reduce the loss of catastrophe (Michas et al. 2014). To improve the accuracy of estimations, early mathematicians developed models like Exponential Smoothing (ES) and Holt-ES (Brown 1957; Holt 2004). With the development of probability and statistics, a number of statistical methods have been put forward and they can refine predictions in a large degree, such as Auto-Regression (AR) and moving average (MA) (Schuster 1906; Yule 1927; Box and Jenkins 1970). Based on ES method and ARIMA models, a software package is developed, which is able to forecast time series automatically and efficiently (Hyndman and Khandakar 2018). Lately, many novel methods have been proposed to improve predictability, such as maximum entropy method (Chliamovitch et al. 2015) and modified Grey Model (Kayacan et al. 2010). Even though these existing methods have a good performance, they are not accurate enough for all the complex situations, because of complicated data structure and an increasing demand for accuracy. This paper will develop time series forecasting models with the motivation to improve the accuracy.

Recently, many scientists have shown a great interest in complex networks (Hu et al. 2016; Liu et al. 2016). One important point of view is that complex networks is valuable for mining information in time series (Zhang et al. 2016; Wang et al. 2017; Zhang et al. 2016). Many researchers have claimed that time series is closely related to networks (Donner et al. 2011; Gao et al. 2015). Hence, analyzing time series by networks becomes a unique perspective to make predictions. Especially, Lacasa et al. proposed an effective method called visibility graph algorithm, which converts time series into a network (Lacasa et al. 2008). In this way, statistical characteristics will appear in a geometric form so that more details of time series can be achieved. Taking Hurst exponent as an example, Lacasa et al. made use of visibility graph to quantify the long-range dependence in Fractional Brownian motion series (Lacasa et al. 2009). By means of the visibility graph method, Telesca investigated the seismicity of Italy (Telesca and Lovallo 2012), Donner found out the fractal properties of geophysical time series (Donner and Donges 2012), and Jiang achieved success in produced water management (Jiang et al. 2016, 2017). Therefore, our research will be based on a network converted by the visibility graph algorithm.

On the other hand, the problems of link prediction, which makes predictions in a network, have been studied. At first, node similarity is defined to estimate the likelihood of the existence of a link between two nodes (Liu and Lü 2010). Furthermore, a universal structural consistency index is proposed to explain the relationships between

the regular organization of networks and the ability of link prediction (Lü and Zhou 2011). Their research reveals that link prediction can uncover future or missing links of networks. Based on link prediction, many models are proposed to study the development of networks, such as in time-evolving graphs (Richard et al. 2012) and a disease spreading network (Kaya and Poyraz 2015). The results of their models show that link prediction is effective for probing network organizing mechanisms. In this paper, we will conduct link prediction in a visibility graph in order to forecast the possible future data.

Nevertheless, due to the fact that uncertainties exist inevitably, inaccurate estimations occur from time to time. To properly handle uncertainties, there are many alternative theories. For example, generalized evidence theory (GET) is able to deal with uncertain information by assuming that the general situation is an open world (Deng 2015). Inspired by GET, researchers have managed uncertainties in emergency management (Zhou et al. 2017), human reliability (Zhou et al. 2017) and linguistic decision making (Mo and Yong 2016). Their work indicates that basic probability assessment (BPA) is indispensable in evidence theory. However, it is difficult to establish BPAs from time series, which makes GET not applicable for forecasting time series.

Another feasible theory to cope with uncertainties is fuzzy logic that is first proposed by Zadeh (1965). According to Zadeh's fuzzy set theory, one concrete object can be corresponding to an abstract extent, during which uncertainties can be well addressed. So far, fuzzy logic has been widely applied in both theoretical research and engineering field, such as Dijkstra algorithm (Deng et al. 2012), pattern recognition (Melin and Castillo 2014), load frequency control (Sabahi et al. 2014) and flight network's vulnerability (Zhang et al. 2016). More importantly, some researches imply that fuzzy logic is able to make predictions as well. For example, Tseng and Tzeng proposed a fuzzy seasonal ARIMA model to predict the total production value of machinery industry and soft drink time series (Tseng and Tzeng 2002). Considering the advantages of fuzzy logic, it will be adopted to revise the preliminary forecasting results in this paper.

This paper is structured as follows: The background of this research is given in Sect. 2. Section 3 introduces basic theories regarding forecasting and fuzzy logic. In Sect. 4, the fuzzy logic combined forecasting model is proposed. Section 5 illustrates the effectiveness of the proposed method. And Sect. 6 ends the paper with a short conclusion.

## 2 Research background

As one vital index, Construction Cost Index (CCI) is monthly published by Engineering News Record (ENR), a magazine that provides valuable information in the field of construction industry. According to official website, CCI is defined as the weighted aggregate of average price of constant quantities of common labor, standard structural steel, portland cement and lumber in 20 cities (ENR 2011). Cost estimators try their best to get more accurate predictions so that they can well evaluate cost for projects and make good preparations for budgets. Additionally, a wise cost estimator not only

focuses on the next month's CCI (short term), but also cares about CCIs in the following several months (long term). Thus, there are two kinds of forecasting models to provide short-term and long-term predictions respectively, which is specified as follows.

In-sample forecasting models: also called one-step-ahead forecasting models, are designed to forecast one observed value at the next one time point, based on historical data, through one iteration.

Out-of-sample forecasting models: also called multi-step-ahead forecasting models, are designed to forecast the observed values at the next time period that includes more than one time point, through multiple iterations. However, in each iteration, one already forecasted data, instead of actual data, will be added to the historical data set.

However, uncertain fluctuations usually cause under or over estimations, which results in decision failures and capital loss. So this paper will propose a new method to forecast time series more accurately and CCI will be used as a major example to illustrate the proposed method. In addition, to verify the advantages, the proposed method will also be applied to the following data sets.

1. Taiwan Stock Index (TAIEX) is a stock market index for companies traded on the Taiwan Stock Exchange. Daily values of TAIEX can be obtained from its web site.
2. Student enrollments of the University of Alabama are widely considered as a target of fuzzy time series models. The historical data is assembled from Song and Chissom (1993, 1994).

### 3 Preliminaries

In this section, fundamental concepts used in the proposed method will be introduced, including visibility graph, link prediction and fuzzy logic.

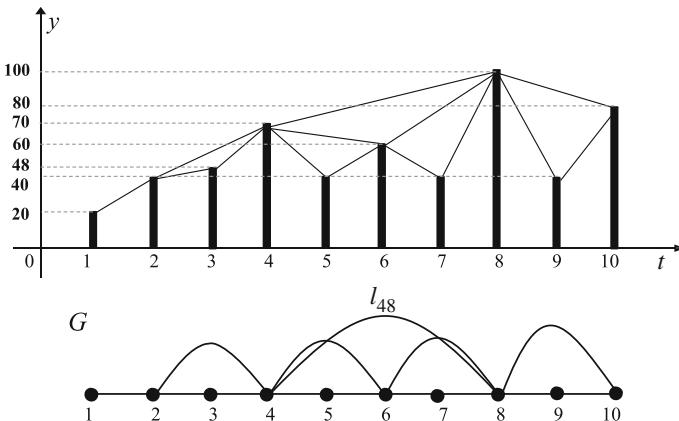
#### 3.1 Visibility graph

For discrete time series  $U = \{(t_1, y_1), (t_2, y_2), \dots, (t_m, y_m), \dots, (t_N, y_N)\}$ ,  $y_m$  is the observed value and  $t_m$  represents time point. In a visibility graph, one element  $(t_m, y_m)$  is defined as node, denoted by node  $m$ . The links among nodes that satisfy visibility algorithm are defined as edges. The visibility algorithm is described in Def. 1 (Lacasa et al. 2008).

**Definition 1** Two arbitrary elements in  $U$ , i.e.  $(t_a, y_a)$  and  $(t_b, y_b)$ , will be visible and connected by an edge as two nodes, if any other element, i.e.  $(t_c, y_c)$ , placed between them satisfies Eq. (1).

$$y_c < y_b + (y_a - y_b) \frac{t_b - t_c}{t_b - t_a} \quad (1)$$

The newly-generated visibility graph is denoted by  $G$ . It should be pointed out that visibility graph is considered as an efficient tool that converts time series into a network, where the edges indicate the relationships among time series according to observed values and the nodes arranged on the time scale indicate the time orientation.



**Fig. 1** The illustration of visibility algorithm: the time series is firstly pictured in a diagram, where the observed values are represented by the height of the vertical bars (shown by dotted lines) and the time points are represented by the time scale on the horizontal axis. Then, it is converted into a visibility graph by generating links according to Eq. (1) (the links are shown by solid lines in the diagram)

In Fig. 1, the process of transforming time series into a visibility graph is detailed. As is seen from the picture, 10 elements are transformed into 10 nodes and 14 edges. For example, (4, 70) and (8, 100) have visibility with each other and they are connected by edge  $l_{48}$ . Here, the visibility is translated by a link between two nodes. At the same time, there are no other elements between them whose observed value is larger than both of the two nodes. As the existence of this link, it is supposed that there is certain relationship between the two nodes, i.e. (4, 70) has an impact on future node (8, 100) and (8, 100) can date back to past node (4, 70).

Although there are other kinds of visibility graph, such as horizontal visibility graph (HVG) (Luque et al. 2009) and limited penetrable visibility graph (LPVG) (Zhou et al. 2012), Eq. (1) will be utilized to convert time series into a network in this paper. There are several reasons.

1. The graph converted by Eq. (1) is invariant under rescaling of both horizontal and vertical axes, and under horizontal and vertical translations (Lacasa et al. 2008), which guarantees original attributes of time series.
2. Horizontal lines are allowed in HVG, but HVG is limited in subgraph of the visibility graph defined by Eq. (1), which means HVG is one particular case of the visibility graph. Besides, HVG will result in smaller average node degree. In other words, the relationships among nodes are limited in the local nodes.
3. Outliers and noise are diminished by LPVG. However LPVG also precludes neighbor nodes that are important for prediction, which means LPVG does not have the complete information of original time series and hence may cause inaccurate predictions.

### 3.2 Link prediction

Link prediction is used to generate potential edges in a network by linking two certain nodes based on node similarity that indicates interrelations of nodes.

According to Liu and Lü (2010), the node similarity is calculated based on the results of a random walk process that describes the process of a walker walking randomly in a network. Given a network with  $N$  nodes, the probability transfer matrix  $P$  is used to represent the probability that a random walker departs from one node and arrives at another node within one step time. The element in  $P$ , i.e.  $P_{xy}$ , is calculated by Eq. (2).

$$P_{xy} = \frac{a_{xy}}{k_x} \quad (2)$$

where  $1 \leq x \leq N$  denotes the departure node and  $1 \leq y \leq N$  denotes the arrival node.  $a_{xy} = 1$  if node  $x$  is linked to node  $y$ , otherwise  $a_{xy} = 0$ .  $k_x$  is the node degree of node  $x$ .

In initial state, a random walker that is located at node  $x$  is represented by an  $N \times 1$  row vector, denoted by  $\pi_x(0)$ . The  $x$ -th element in  $\pi_x(0)$  equals to 1 and others equal to 0, which means the random walker will start walking from node  $x$ . Likewise, a random walker that is located at node  $y$  is initialized by  $\pi_y(0)$ .

After  $t$  steps time, the probabilities that the random walker reaches certain node are summarized in row vectors  $\pi_x(t)$  and  $\pi_y(t)$ , calculated by Eqs. (3) and (4).

$$\pi_x(t) = P^T \pi_x(t-1) \quad (3)$$

$$\pi_y(t) = P^T \pi_y(t-1) \quad (4)$$

where  $T$  represents the transpose of matrix.

Meanwhile, in each time step, the similarity between node  $x$  and node  $y$  is calculated by Eq. (5).

$$S_{xy}^{LRW}(t) = \frac{k_x}{2|E|} \times \pi_{xy}(t) + \frac{k_y}{2|E|} \times \pi_{yx}(t) \quad (5)$$

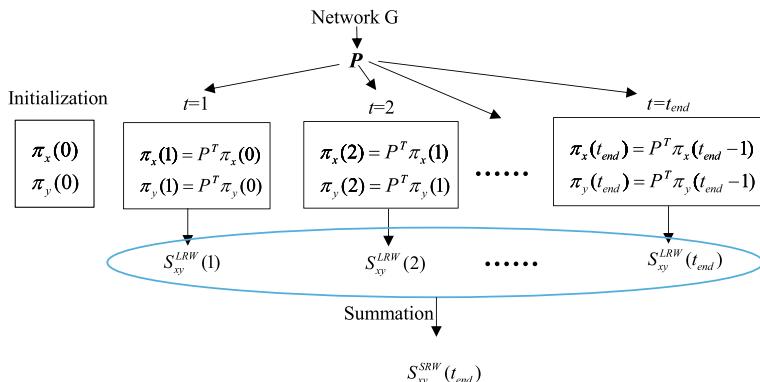
where  $|E|$  is the number of edges in the network.  $k_y$  represents the node degree of node  $y$ .  $\pi_{xy}(t)$  is the  $y$ -th element in  $\pi_x(t)$ .  $\pi_{yx}(t)$  is the  $x$ -th element in  $\pi_y(t)$ .  $S^{LRW}$  denotes the similarity based on local random walk LRW.

Finally, to prevent a random walker from walking too far away from both node  $x$  and node  $y$ , Eq. (6) defines similarity between node  $x$  and node  $y$  by superposing results of previous random walk process.

$$S_{xy}^{SRW}(t) = \sum_{l=1}^t S_{xy}^{LRW}(l) \quad (6)$$

where  $S^{SRW}$  denotes the similarity based on superposed random walk.

The process of calculating the similarity between node  $x$  and node  $y$  is illustrated in Fig. 2. Note that the time  $t_{end}$  is set to be a number that ensures a random walker is able to walk enough randomly in the network. Mathematically,  $t_{end}$  should satisfy



**Fig. 2** The illustration of calculating the similarity between node  $x$  and node  $y$

$\pi_x(t_{end}) \approx \pi_x(t_{end} - 1)$ . In the proposed method, we set  $t_{end}$  ensuring that the elements in  $\pi_x(t) - \pi_x(t - 1)$  are less than  $10^{-5}$ .

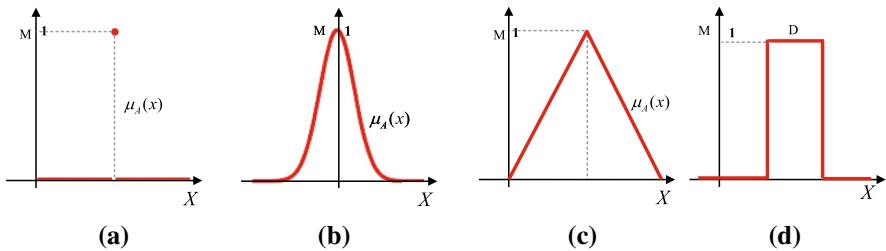
As is discussed in Liu and Lü (2010), Eq. (6) has a much lower computation complexity and can give better predictions. So it is adopted to calculate the node similarity and forecast time series in the proposed method. The reasons are detailed as follows:

1. The time complexity in calculating the inverse or pseudoinverse of an  $N \times N$  matrix is  $O(N^3)$ , while the time complexity of n-step LRW (or SRW) is approximately  $O(N\langle k \rangle^n)$ , where  $\langle k \rangle^n$  is the average degree of the network. Since in most networks  $\langle k \rangle$  is much smaller than  $N$ , LRW and SRW run much faster than other random-walk-based similarity indices, such as average commute time (ACT) and random walk with restart (RWR) that are based on global information.
2. By using local information, methods such as common neighbours, resource allocation and local path indices are also efficient for computation. However, due to incomplete information, these methods have lower prediction accuracy. By superposing similarity based on local random walk at each time step (Eqs. 5, 6) accumulates local information and hence has higher prediction accuracy.

### 3.3 Fuzzy logic

According to Zadeh's fuzzy set theory, fuzzy logic involves fuzzy sets, membership functions and fuzzy rules. In Def. 2, the mathematical description is detailed (Zadeh 1965).

**Definition 2** Let  $X$  be a space of points (objects), with a generic element of  $X$  denoted by  $x$ , i.e.  $X = \{x\}$ . One fuzzy set  $A$  defined on  $X$  is characterized by a membership function  $\mu_A$  that maps each point in  $X$  to a real number in the interval  $[0, 1]$ . The value of  $\mu_A(x)$  represents the grade of membership of  $x$  in  $A$ , where  $\mu_A(x) = 1$  means  $x$  belongs to  $A$  and  $\mu_A(x) = 0$  means  $x$  does not belong to  $A$ . The fuzzy set  $A$  defined on  $X$  can be denoted by:



**Fig. 3** Examples of membership functions: **a** a single value function, **b** a gaussian function, **c** a triangular function, **d** a classical set

$$A = \{(x, \mu_A(x)) | x \in X\} \quad (7)$$

where  $x$  is called fuzzy variable.  $X$  is called the domain of  $x$ , which is a classical set.

Figure 3 shows three common membership functions of fuzzy sets, as well as a classical set. In order to simplify the computation work, triangular functions will be adopted in this paper.

By using fuzzy sets, a simple fuzzy proposition is defined to describe the memberships of  $x$  in  $A$ , in the form of “ $x$  is  $A$ ”, where  $A$  is a fuzzy set defined on the domain of  $x$ . This term is denoted by  $\mu_A(x)$ , which quantitatively specifies the degree of  $x$  belonging to  $A$ . So fuzzy set  $A$  is able to flexibly assign each  $x$  with different membership degrees from 0 to 1. However, as is indicated in Fig. 3, a classical set only maps a variable to 0 or 1.

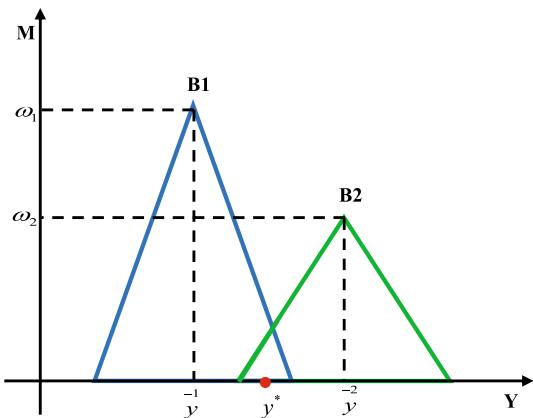
Besides, simple fuzzy propositions can be compounded by logical operators, such as AND operator. For example, a compound fuzzy proposition is expressed by “ $x$  is  $A$  AND  $y$  is  $B$ ”, where  $B$  is a fuzzy set defined on the domain of  $y$ . The AND operator is denoted by  $\cup$ , which represents the union set of two fuzzy sets in the simple fuzzy propositions. The membership function of a fuzzy proposition compounded by AND operator is calculated by Eq. (8).

$$\mu_{A \cup B} = \max[\mu_A(x), \mu_B(y)] \quad (8)$$

Def. 3 gives the definition of fuzzy rules. Suppose we have a fuzzy rule that goes “IF  $x$  is  $A$  THEN  $y$  is  $B$ ”. It is explained as “if we have a fuzzy variable  $x$  with membership  $\mu_A(x)$ , then we will have a fuzzy variable  $y$  with membership  $\mu_B(y)$ . To illustrate the relationship between  $\mu_A(x)$  and  $\mu_B(y)$ , a fuzzy set  $Q$  is defined on the Cartesian product of two classical sets, i.e.  $X$  and  $Y$ , denoted by  $X \times Y$ . The fuzzy set  $Q$  defined on  $X \times Y$  can be written by Eq. (9), according to Eq. (7). Since the fuzzy rules are independent with each other, Eq. (10) is used to determine the membership function of  $Q$  in order to simplify the computation process.

**Definition 3** Fuzzy rules are usually called IF-THEN rules. One fuzzy rule is a statement that is composed of two fuzzy propositions (FP), i.e.  $IF < FP1 > THEN < FP2 >$ . The fuzzy rule associates  $FP1$  with  $FP2$  by considering  $FP1$  is a condition and  $FP2$  is a conclusion.

**Fig. 4** The illustration of Eq. (13):  $B_1$  and  $B_2$  are two fuzzy sets defined on  $Y$ .  $\bar{y}^{B_1}$  and  $\bar{y}^{B_2}$  are the center of  $B_1$  and  $B_2$ .  $\omega_1$  and  $\omega_2$  are their heights. So  $y^*$  is calculated by  $(\bar{y}^{B_1}\omega_1 + \bar{y}^{B_2}\omega_2)/(\omega_1 + \omega_2)$



$$Q = \{(x, y), \mu_Q(x, y)) | (x, y) \in X \times Y\} \quad (9)$$

$$\mu_Q(x, y) = \mu_A(x)\mu_B(y) \quad (10)$$

Thus, given a fuzzy set  $A$  defined on  $X$  and a fuzzy rule whose relationship is defined by Eq. (10), we can infer a fuzzy set  $B$  defined on  $Y$  through Eq. (11).

$$\mu_{B(y)} = \sup_{x \in X} [\mu_A(x)\mu_Q(x, y)] \quad (11)$$

If the number of fuzzy rules is  $m$ , rules will be logically combined by Eq. (8). So Eq. (11) will be rewritten by Eq. (12).

$$\mu_{B(y)} = \max_{l=1}^m [\sup_{x \in X} (\mu_{A^l}(x)\mu_{Q^l}(x, y))] \quad (12)$$

where  $l$  represents the fuzzy sets in  $l$ -th fuzzy rule.

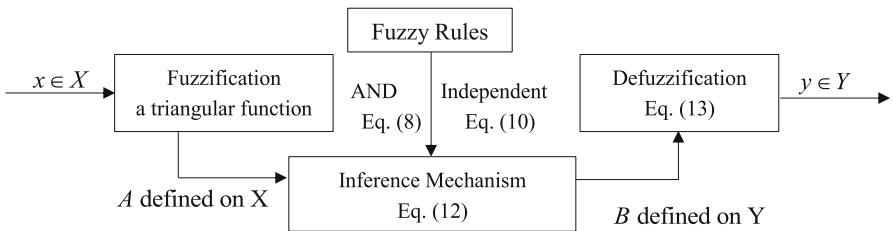
Finally, to realize the equivalent transformation from fuzzy sets to real numbers, Eq. (13) will be adopted in this paper. As is shown in Fig. 4, Eq. (13) is used to choose a real number  $y^*$  that can best stand for the fuzzy sets.

$$y^* = \frac{\sum_{l=1}^M \bar{y}^l \omega_l}{\sum_{l=1}^L \omega_l} \quad (13)$$

where  $\bar{y}^l$  is the center of  $l$ -th fuzzy set, and  $\omega_l$  is its height.

Considering the following advantages of fuzzy logic, it is adopted to deal with time series in this paper.

1. Fuzzy logic well translates variables in the form of linguistic terms into fuzzy sets, so that human experience can be intelligently integrated into the models. That's to say, for a fuzzy rule  $IF < FP1 > THEN < FP2 >$ ,  $FP1$  is a specific condition and  $FP2$  is a possible result. Hence, by designing appropriate fuzzy rules, the



**Fig. 5** A simple fuzzy system

causal relationship between  $FP1$  and  $FP2$  is established, which associates the future tendency of time series. And it is ensured by people's knowledge of the properties of time series.

- Fuzzy logic is simple in computation but rigorous in logical. As shown in Fig. 5, a simple fuzzy system is able to map a real number  $x$  to a real number  $y$  through fuzzy sets  $A$  and  $B$ . To be specific, a triangular function fuzzifies each fuzzy variable by linearly assigning a membership degrees. AND operator easily links two fuzzy sets in a fuzzy rule by algebraic product. In defuzzification process, only arithmetic summation is used to determine the output real number.

## 4 Proposed method

In this section, the proposed method is divided into two phases. Phase 1 makes predictions by using visibility graph and link prediction. Phase 2 revises the predictions based on fuzzy logic. To elaborate the proposed method clearly, CCI will be used as an example.

The proposed method is aimed at forecasting the future value based on historical data that detailed below.

Future value:  $y_{N+1}$  at  $t_{N+1}$

Historical data:  $\{(t_1, y_1), (t_2, y_2), \dots, (t_N, y_N)\}$

### 4.1 Phase 1: Initial forecasting

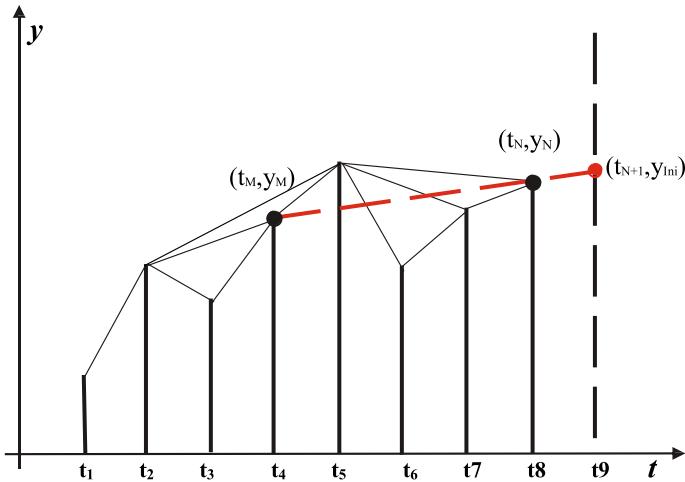
- *Step 1* Convert the time series into a visibility graph

The original data is denoted by time series  $T = \{(t_1, y_1), (t_2, y_2), \dots, (t_N, y_N)\}$ . And it is converted into a graph according to Eq. (1).

- *Step 2* Calculate the similarities

Firstly, based on Eq. (2), the probability transfer matrix  $P$  is calculated according to the visibility graph obtained in *Step 1*.

Secondly,  $\pi_x(t)$  is initialized at time  $t = 0$ , for  $x \in [1, N]$ . Then, based on the matrix  $P$  and vector  $\pi_x(0)$ , a random process is conducted by iterating Eq. (3) from  $t = 1$  to  $t = t_{end}$ . In the meantime, Eq. (5) is used to calculate the similarities based on local random walk between node  $N$  and previous  $(N - 1)$  nodes, at each time step.



**Fig. 6** Illustration of initial forecasting:  $(t_M, y_M)$  is the node with highest similarity.  $(t_N, y_N)$  is the last known node. They are linked by the red line that is used to determine the future node  $(t_{N+1}, y_{\text{ini}})$

Finally, by using the results of Eq. (3) iterated from  $t = 1$  to  $t = t_{\text{end}}$ , Eq. (6) is used to calculate the similarities based on superposed random walk between node  $N$  and previous  $(N - 1)$  nodes, denoted by  $\mathbf{S}^{\text{SRW}} = [S_{1N}^{\text{SRW}}(t_{\text{end}}), S_{2N}^{\text{SRW}}(t_{\text{end}}), \dots, S_{(N-1)N}^{\text{SRW}}(t_{\text{end}})]$

– *Step 3 Make initial predictions*

Find out the maximum similarity in  $S^{\text{SRW}}$ , which is denoted by  $S_{MN} = \max\{S_{1N}^{\text{SRW}}(t_{\text{end}}), S_{2N}^{\text{SRW}}(t_{\text{end}}), \dots, S_{(N-1)N}^{\text{SRW}}(t_{\text{end}})\}$ . And the corresponding node  $(t_M, y_M)$  is considered as the node with highest similarity. The initial prediction is determined by Eq. (14).

$$y_{\text{ini}} = \frac{y_N - y_M}{t_N - t_M}(t_{N+1} - t_N) + y_N \quad (14)$$

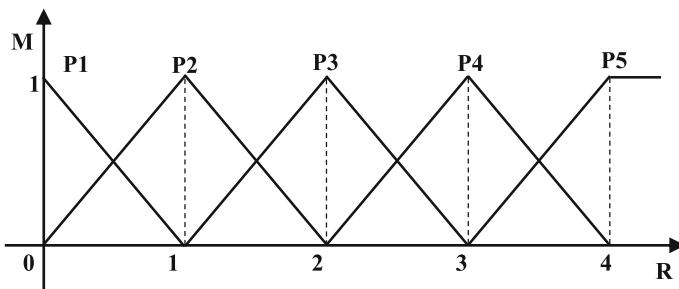
Figure 6 describes the process of initial forecasting. As can be seen, node  $(t_M, y_M) = (t_4, y_4)$  is the node with highest similarity. It is linked to the last known node, i.e.  $(t_N, y_N) = (t_8, y_8)$ . Thus, they are together to determine the initial forecasting value at  $t_9$ , i.e.  $(t_{N+1}, y_{\text{ini}}) = (t_9, y_{\text{ini}})$ .

## 4.2 Phase 2: Amendatory forecasting

– *Step 4 Determine the fuzzy variable*

The distance between the node  $(t_N, y_N)$  and the node  $(t_M, y_M)$  on the time scale, denoted by  $d_{M \rightarrow N}$ , is computed by Eq. (15). Based on the distance, Eq. (16) is used to denote the fuzzy variable  $r$ .

$$d_{M \rightarrow N} = |t_N - t_M| \quad (15)$$



**Fig. 7** Membership functions of  $P$

$$r = \log_2 d_{M \rightarrow N} \quad (16)$$

#### – Step 5 Design the fuzzy rules

Before designing the fuzzy rules, three important factors should be taken into consideration.

##### (1) The range of visibility

The range of visibility determines the domain of fuzzy variable  $r$ . Due to increasing amounts of historical data, some data becomes too old to predict future tendency. Although some past nodes are visible to the current nodes, they are not valuable to make predictions. Therefore, the range of visibility guarantees the effectiveness of visibility, which means only useful nodes will be used in the proposed method. For example, referring the research by [Ashuri and Lu \(2010\)](#), it is proved that seasonality is one important property that CCI possesses. To be specific, both box plots and autocorrelation function (ACF) graph of the incremental monthly changes in CCI show that CCI has the period of 12 months. For convenience of calculations and consideration of margin,  $r = 4$  is set to a boundary, i.e.  $d_{M \rightarrow N} = 16$ . That is to say, if the node  $(t_M, y_M)$  makes  $1 \leq d_{M \rightarrow N} \leq 16$  hold, it is counted in the visible scope and will be used to forecast  $(t_{N+1}, y_{N+1})$ . Otherwise,  $(t_M, y_M)$  will be omitted because it may not have much value to forecast  $(t_{N+1}, y_{N+1})$ .

##### (2) The level of visibility

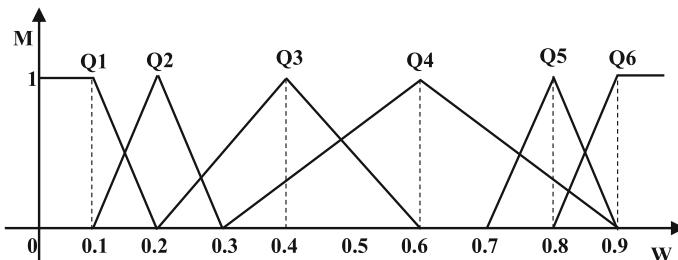
The level of visibility determines the number of fuzzy sets and each fuzzy set represents the distance between  $(t_M, y_M)$  and  $(t_N, y_N)$ . Taking CCI as an example, the 16 months are classified into 5 levels. In linguistic terms, we use “Very close”, “Close”, “Moderate”, “Far” and “Very far” to describe the levels. Accordingly, 5 fuzzy sets are designed for the 5 terms in the proposed method, and 16 months are allocated to the corresponding fuzzy sets as well, as is indicated in Fig. 7 and Table 1.

##### (3) The effect on future data

The last known node  $(t_N, y_N)$  is regarded as the direct effect on future data  $\hat{y}_{N+1}$ , since node  $N$  will be directly linked to node  $N + 1$  if node  $N + 1$  is added into the visibility graph. The initial forecasting results  $y_{Ini}$  is considered as the indirect effect, since  $y_{Ini}$  is obtained by a linear approximation by the line between node  $N$  (the last known node) and  $M$  (the node with highest similarity).

**Table 1** Definitions of fuzzy variables in the proposed method

Fuzzy variable	Fuzzy sets	Linguistic terms	Real numbers of months/weights
r	P1	Very close	1
	P2	Close	2, 3
	P3	Moderate	3, 4, 5, 6, 7
	P4	Far	5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15
	P5	Very far	9, 10, 11, 12, 13, 14, 15, 16, ...
f,g	Q1	Very very unimportant	[0, 0.2]
	Q2	Very unimportant	[0.1, 0.3]
	Q3	Unimportant	[0.2, 0.6]
	Q4	Important	[0.3, 0.9]
	Q5	Very important	[0.7, 0.9]
	Q6	Very very important	[0.8, 1]

**Fig. 8** Membership functions of  $Q$ 

With the distance  $d_{M \rightarrow N}$  getting smaller, the indirect effect tends to weaken and the direct effect tends to heighten, because node M plays the same role as node N does, which means past information is not useful for the future value. Whereas, if the the distance  $d_{M \rightarrow N}$  becomes larger, the indirect effect will heighten and the direct effect will weaken, because node N is more related to the previous nodes that carry important information for the future value. The intensity of the effects are described by linguistic terms from “Very very important” to “Very very unimportant”, and they are represented by the fuzzy sets pictured in Fig. 8.

To quantify the intensity of effects, different weight coefficients will be adopted.  $f$  denotes the weight of indirect effect and  $g$  denotes the weight of direct effect. To be specific, when the distance  $d_{M \rightarrow N}$  is getting smaller ( $r$  becomes smaller),  $f$  should be smaller and  $g$  should be larger, which means indirect effect becomes unimportant and direct effect becomes important. When the distance  $d_{M \rightarrow N}$  is getting larger ( $r$  becomes larger),  $emphf$  should be larger and  $emphg$  should be smaller, which means indirect effect becomes important and direct effect becomes unimportant. The range of the weights for each term is detailed in Table 1.

In conclusion, the above factors are generalized in the following fuzzy rules:

(1) IF  $r$  is  $P1$ , THEN  $f$  is  $Q1$  and  $g$  is  $Q6$

- (2) IF  $r$  is  $P_2$ , THEN  $f$  is  $Q_2$  and  $g$  is  $Q_5$
- (3) IF  $r$  is  $P_3$ , THEN  $f$  is  $Q_3$  and  $g$  is  $Q_4$
- (4) IF  $r$  is  $P_4$ , THEN  $f$  is  $Q_5$  and  $g$  is  $Q_2$
- (5) IF  $r$  is  $P_5$ , THEN  $f$  is  $Q_6$  and  $g$  is  $Q_1$

where  $P_i (1 \leq i \leq 5)$  and  $Q_j (1 \leq j \leq 6)$  are fuzzy sets defined on  $R$  and  $W$  respectively.

Then, based on Eq. (13) in Sect. 3, Eqs. (17) and (18) are used to obtain real numbers of  $f$  and  $g$ . Take  $d_{M \rightarrow N} = 3$  as an example. Firstly, we will obtain  $r = \log_2 3$ . Secondly, according to fuzzy sets in Fig. 7,  $r = \log_2 3$  belongs to  $P_2$  and  $P_3$  at the same time, with the grade of membership degree  $\mu_{P_2}(r)$  and  $\mu_{P_3}(r)$  respectively. Thirdly, rule (2) and rule (3) are utilized to infer the fuzzy sets of  $f$  and  $g$ , based on Eq. (12). Finally,  $Q_2$ ,  $Q_3$ ,  $\mu_{P_2}(r)$  and  $\mu_{P_3}(r)$  are used to calculate the real number of  $f$  and  $g$  according to Eq. (17) and (18).

$$\begin{aligned} f &= \frac{\sum_{l=1}^5 Q_l \mu_{P_l}(r)}{\sum_{l=1}^5 \mu_{P_l}(r)} \\ &= \frac{Q_1 \mu_{P_1}(r) + Q_2 \mu_{P_2}(r) + Q_3 \mu_{P_3}(r) + Q_5 \mu_{P_4}(r) + Q_6 \mu_{P_5}(r)}{\mu_{P_1}(r) + \mu_{P_2}(r) + \mu_{P_3}(r) + \mu_{P_4}(r) + \mu_{P_5}(r)} \end{aligned} \quad (17)$$

$$\begin{aligned} g &= \frac{\sum_{l=1}^5 Q_l \mu_{P_l}(r)}{\sum_{l=1}^5 \mu_{P_l}(r)} \\ &= \frac{Q_6 \mu_{P_1}(r) + Q_5 \mu_{P_2}(r) + Q_4 \mu_{P_3}(r) + Q_2 \mu_{P_4}(r) + Q_1 \mu_{P_5}(r)}{\mu_{P_1}(r) + \mu_{P_2}(r) + \mu_{P_3}(r) + \mu_{P_4}(r) + \mu_{P_5}(r)} \end{aligned} \quad (18)$$

#### – Step 6 Make final predictions

Based on the initial prediction obtained in Step 3, the final prediction is determined by Eq. (19).

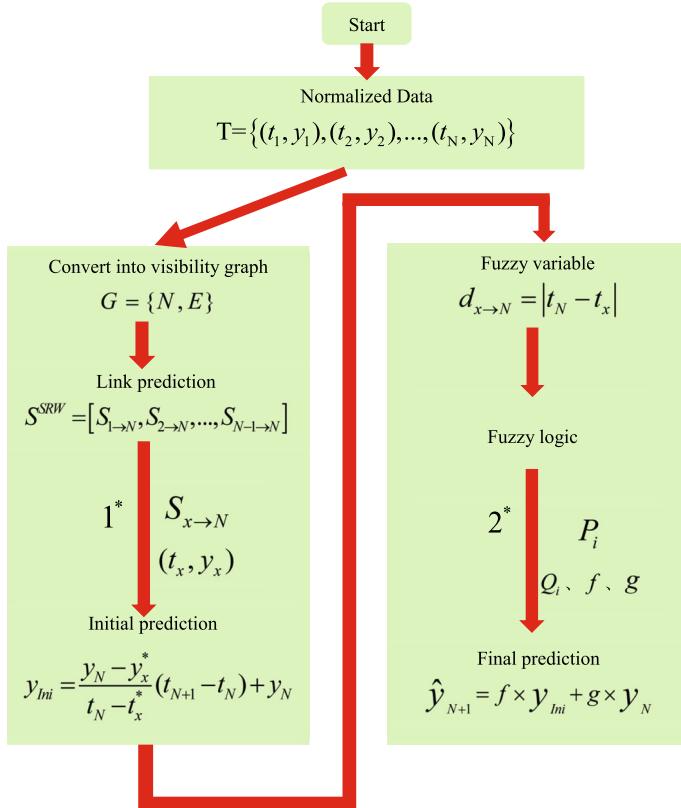
$$\hat{y}_{N+1} = f \times y_{Ini} + g \times y_N \quad (19)$$

### 4.3 Flow chart

In all, the flow chart of the proposed method is given in Fig. 9.

## 5 Results and discussions

In this section, the proposed method will be applied to forecast time series introduced in Sect. 2. In order to measure the accuracy and predictability, Eqs. (20–24) are adopted to calculate the statistical errors, i.e. mean absolute difference (MAD), mean absolute percentage error (MAPE), symmetric mean absolute percent error (SMAPE) and normalized root mean square error (NRMSE). Additionally, some necessary comparisons and analysis will be provided to demonstrate the effectiveness of the proposed method.



**Fig. 9** The flow chart of the proposed method (1\*: Find the node with highest similarity. 2\*: Design fuzzy rules and membership functions)

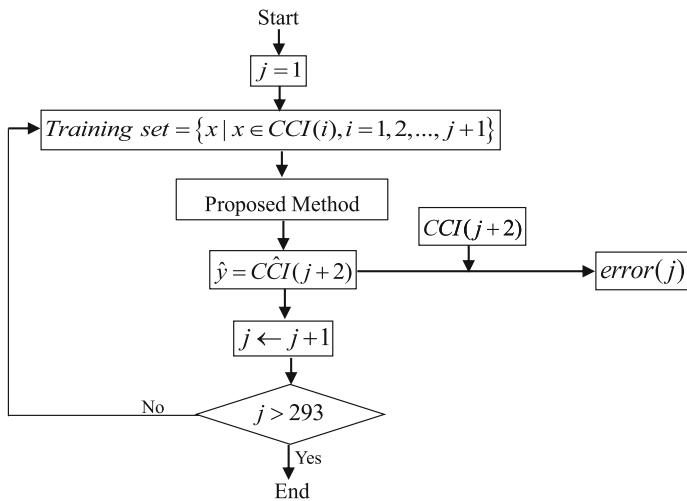
$$MAD = \frac{1}{M} \sum_{t=1}^M |\hat{Y}(t) - \tilde{Y}(t)| \quad (20)$$

$$MAPE = \frac{1}{M} \sum_{t=1}^M \frac{|\hat{Y}(t) - \tilde{Y}(t)|}{\tilde{Y}(t)} \times 100 \quad (21)$$

$$SMAPE = \frac{2}{M} \sum_{t=1}^M \frac{|\hat{Y}(t) - \tilde{Y}(t)|}{\hat{Y}(t) + \tilde{Y}(t)} \quad (22)$$

$$RMSE = \sqrt{\frac{1}{M} \sum_{t=1}^M [\hat{Y}(t) - \tilde{Y}(t)]^2} \quad (23)$$

$$NRMSE = \frac{\sqrt{\frac{1}{M} \sum_{t=1}^M [\hat{Y}(t) - \tilde{Y}(t)]^2}}{\tilde{Y}_{\max} - \tilde{Y}_{\min}} \quad (24)$$



**Fig. 10** Process of CCI in-sample forecasting

**Table 2** Error measurements of CCI in-sample forecasting in Phase 1 and Phase 2

	MAD	MAPE (%)	SMAPE (%)	RMSE	NRMSE (%)
Phase 1	20.0	0.2889	0.2894	29.3	0.5690
Phase 2	19.6	0.2845	0.2851	29.1	0.5659

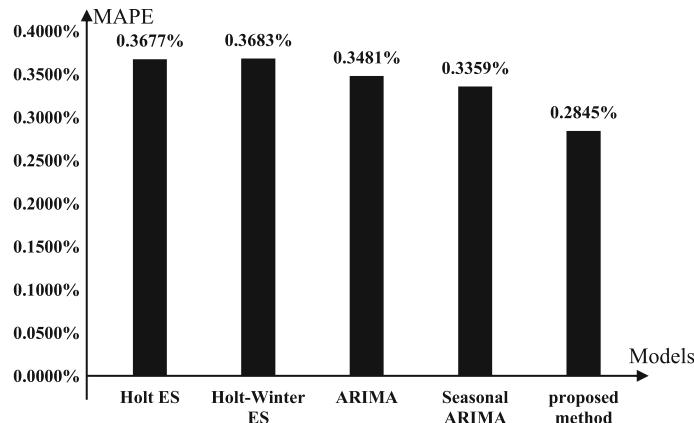
where  $\hat{Y}(t)$  is the forecasted value,  $\tilde{Y}(t)$  is the actual value and  $M$  is the total number of forecasted values.

## 5.1 Forecasting CCI

Firstly, an in-sample forecasting experiment is conducted by inputting CCI data from January 1990 to July 2014. As is illustrated in Fig. 10, the observed values of 295 CCIs are saved in a data set  $CCI(i)$ , where  $1 \leq i \leq 295$  denotes the time point. Then, by iterating the proposed method, CCIs from January 1990 to June 2014 are used to forecast CCIs from March 1990 to July 2014.

Table 2 lists the error measurements of forecasting results in Phase 1 and Phase 2. According to the decreased errors, it is concluded that Phase 2 indeed revises the initial forecasting results in Phase 1.

Based on the research in Ashuri and Lu (2010), the proposed method is compared with other models, including Holt ES, Holt-Winter ES, ARIMA, and seasonal ARIMA, in the aspect of MAPE. As is shown in Fig. 11, the proposed method has improved the accuracy by 0.0514%, compared with the MAPE of seasonal ARIMA. By conducting the  $t$  test of MAD obtained by the proposed method and ARIMA, it shows there exists significant difference since the  $p$  value equals to  $8.40e-23$ .



**Fig. 11** Comparison of MAPE in CCI in-sample forecasting

Secondly, the proposed method is used to make out-of-sample predictions. In this step, a sliding window over data is set to verify the forecasting range of the proposed method. To test roundly, the length of the sliding window equals to 3, 6 and 12. For example, Algorithm 1 describes the process of the sliding window ( $length = 12$ ) moving from CCI(3) to CCI(283), which indicates that 12 CCIs will be forecasted in each iteration. Figure 12 presents MAPE of the forecasting results, the average of which are  $Error_{L=3} = 0.48\%$ ,  $Error_{L=6} = 0.75\%$  and  $Error_{L=12} = 1.27\%$ . Obviously, the error becomes larger with the sliding window getting longer and it turns out that the proposed method is able to forecasting the following three months' CCI most accurately. So the average MAPE  $Error_{L=3}$  is compared with that of Holt ES, Holt-Winter ES, ARIMA, seasonal ARIMA and ENR. The results are presented by the histogram in Fig. 13. It indicates that the proposed method have the smallest MAPE, which reduces the error by 0.17% when compared with MAPE of Holt-Winter ES model.

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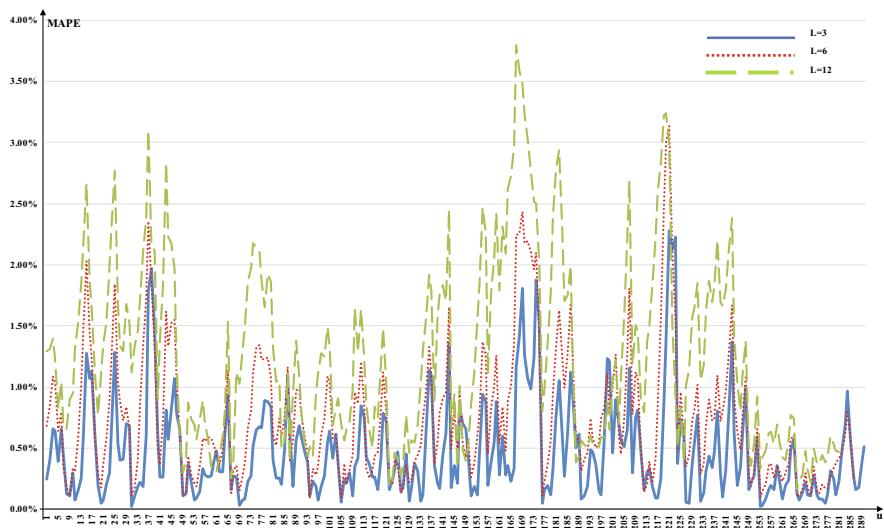
#### Algorithm 1 Out-of-sample forecasting with a sliding window

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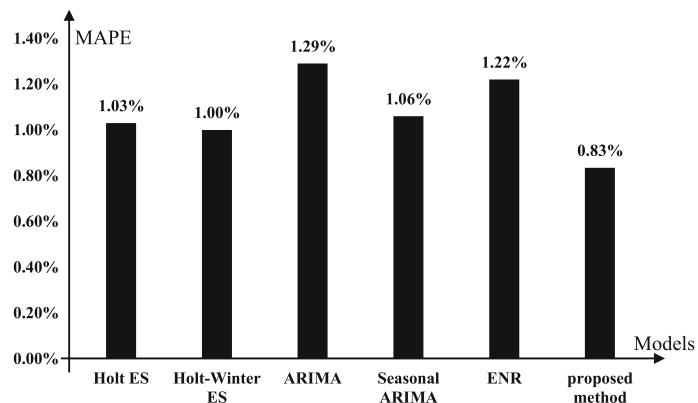
**Input:** Data set: 295 CCIs. Variables: length of the sliding window  $L = 12$

**Output:**  $Error$

- 1: **for**  $u = 1$  to  $(295 - L - 2)$  **do**
  - 2:    $Trainingset = \{x | x \in CCI(i), i = 1, 2, \dots, u + 1\}$
  - 3:   **for**  $v = 1$  to  $L$  **do**
  - 4:     Conduct the proposed method to obtain the forecasted value  $\hat{CCI}(u + v + 1)$
  - 5:     Calculate error, i.e.  $e(v) = f[\hat{CCI}(u + v + 1), \tilde{CCI}(u + v + 1)]$
  - 6:     Add  $\hat{CCI}(u + v + 1)$  to  $Trainingset$
  - 7:   **end for**
  - 8:    $Error(u) = Average(e)$
  - 9: **end for**
-



**Fig. 12** Results of MAPE in CCI out-of-sample forecasting with a sliding window

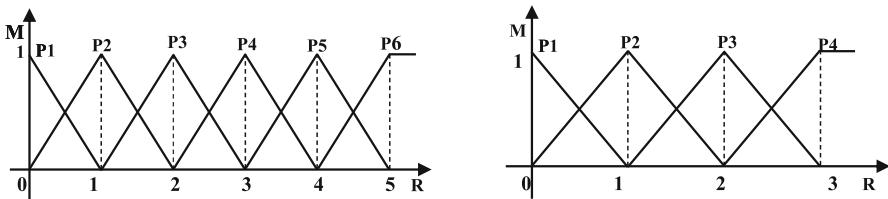


**Fig. 13** Comparison of MAPE in CCI out-of-sample forecasting with sliding window  $L = 12$

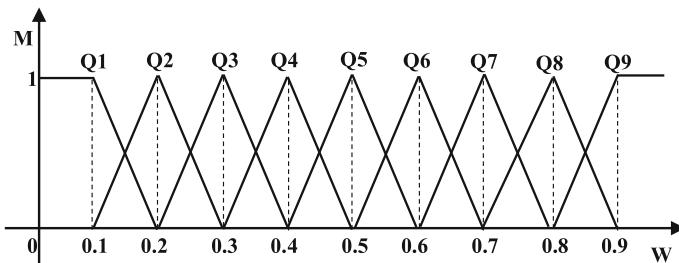
## 5.2 Forecasting TAIEX

First of all, considering that TAIEX is a daily index, it has a period of one month (30 days). To calculate easily, the visibility range is extended to 32 days. As a result, fuzzy set P6 is added to make fuzzy rules. Accordingly, the membership functions of P and Q, together with fuzzy rules, are presented in Figs. 14, 15 and Table 3.

Then, the proposed method is applied to make in-sample predictions with TAIEX data from 1991 to 1999. The TAIEX data in each year is considered as a separate group. For each group, the TAIEX data in last 2 months will be forecasted based on the data in preceding 10 months. By the error measurement of RMSE, the forecasting results of the proposed method, as well as the method in Cheng et al. (2008), are detailed in



**Fig. 14** P membership functions of TAIEX (left) and enrollments (right)



**Fig. 15** Q membership functions of weights in TAIEX and enrollments

**Table 3** Fuzzy rules of TAIEX

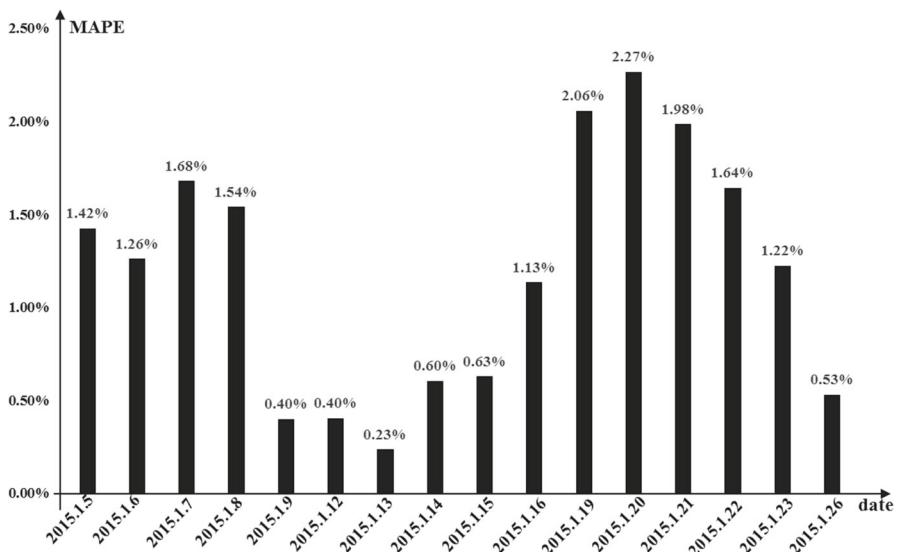
	P1	P2	P3	P4	P5	P6
f	Q1	Q2	Q4	Q6	Q8	Q9
g	Q9	Q8	Q6	Q4	Q2	Q1

**Table 4** Results of RMSE in TAIEX in-sample forecasting

Group	1	2	3	4	5	6	7	8	9	Average
Cheng et al. (2008)	42	43	105	75	53	51	134	113	109	81
Proposed method	46	43	40	93	57	53	151	117	106	78

Table 4. Although the average RMSE manifests that the proposed method is better, the error obtained by Cheng's method is lower in most years. Then,  $t$  test of the errors is performed, the result of which shows that there is no significant difference between the RMSE obtained by the proposed method and Cheng's method ( $p$  value = 0.80). Thus, the forecasting errors of the proposed method are proved to be acceptable.

In addition, to test the performance of out-of-sample forecasting, data from 2013 to 2014 are used to predict the data in January 2015. Similar to the process in Algorithm 1, a sliding window is set to 5, which means next week's TAIEX data will be forecasted in each iteration. The MAPE of the forecasting results is presented in Fig. 16. Although the errors at certain points are a little bit large, the averaged MAPE (1.19%) is much lower than 10%. According to Wong et al. (2005), the forecasting error is acceptable.



**Fig. 16** Results of MAPE in TAIEX in out-of-sample forecasting

**Table 5** Fuzzy rules of student enrollments

	P1	P2	P3	P4
f	Q6	Q7	Q8	Q9
g	Q4	Q3	Q2	Q1

### 5.3 Forecasting student enrollments

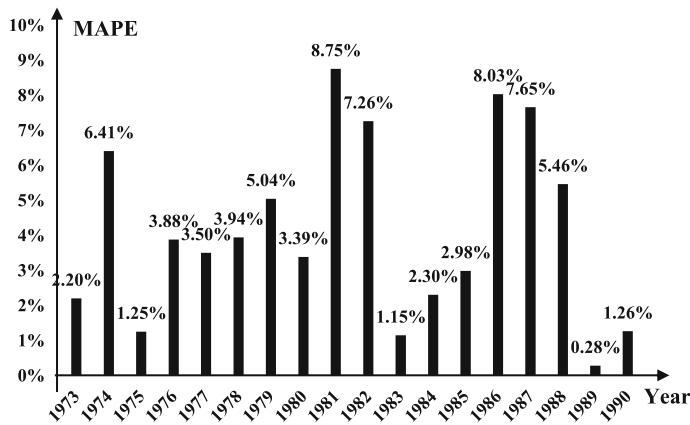
In the first place, since student enrollments of the university are counted every year, only four fuzzy sets are designed in the proposed method. Namely, the enrollments from the past eight years are included in the visible range. The membership functions and fuzzy rules are shown in Figs. 14, 15 and Table 5.

In the second place, enrollments from 1971 to 1992 are used by the proposed method to make in-sample predictions. The error measurements of the proposed method are compared with the methods in Cheng et al. (2008), which is summarized in Table 6. It reveals that the errors of the proposed method are lower than Chen's and Lee's methods, but a little bit higher than Cheng's method. Likewise, by conducting *t* test on the MAD and RMSE of the errors obtained by Cheng's method and the proposed method, the high *p* values (0.22 for MAD and 0.21 for RMSE) suggest that there is no significant difference between the two methods. Hence, the predictions of the proposed method is proved to be accurate.

Finally, the proposed method is used to forecast the following two years' enrollments. Similar to Algorithm 1, a sliding window is set to 2 in order to perform an out-of-sample experiment. The MAPE of the forecasting results is shown in Fig. 17, the average of which equals to 4.64%. Even though the errors in some years are a little

**Table 6** Results of student enrollments in-sample forecasting

Error	Chen (1996)	Hwang (1998)	cheng et al. (2008)	Proposed method
MAD	499	488	341	448
MAPE	3.110%	2.936%	2.087%	2.741%
SMAPE	0.0310	0.0298	0.0210	0.0275
RMSE	638	611	438	554
NRMSE	0.1106	0.1117	0.0759	0.1012

**Fig. 17** Results of MAPE in student enrollments out-of-sample forecasting

bit large, the error measurements of the proposed method are acceptable since they are below 10%, according to Wong et al. (2005).

#### 5.4 Analysis

1. The proposed method takes advantages of visibility graph and node similarity, so that the inner relationships among time series are visualized by potential links and they are quantified by similarities. Moreover, the initial forecasting is obtained by the last known node and the node with highest similarity, which is considered to be the most representative nodes from the original data set.
2. Fuzzy logic has played an important role in modifying initial forecasting by adding the effect of the last known data. In the proposed method, Phase 2 combines  $y_{Ini}$  and  $y_N$  by assigning weights. In essential, the final forecasting result is a weighted mean of  $y_{Ini}$  and  $y_N$ , however, the way of calculating the coefficients of the weights is novel. The novelty lies in designing the fuzzy rules based on the distances on time scale, which associates past time series with future tendency. In Table 2, although the bias between Phase 1 and Phase 2 seems small, it goes without doubt that even 0.01% is crucial in economical and engineering fields. As far as CCI is concerned, it is frequently used in managing capital and preparing budget. So more accurate

CCI predictions will be of great significance for cost estimators. Especially, for those projects over millions or even billions, the errors will absolutely become greater. To avoid risks or capital loss, it is worth emphasizing the accuracy at any level. Even though the errors have been reduced by 0.01% when compared with other method, it means hundreds of or even thousands of money can be saved in large or huge projects.

3. In speaking of the forecasting results, the proposed method is able to provide rather accurate in-sample forecasts. Within limited range, it is applicable for out-of-sample forecasting as well. In Sects. 5.1, 5.2 and 5.3, since the data is recorded by day, month and year, the number of fuzzy sets and the level of visibility have been adjusted. Based on the forecasting results, the error are declined by the proposed method. Through comparison, the proposed method is able to provide more accurate predictions than many other models. Although it does not have the lowest error in all the cases, the accuracy of the proposed method is acceptable, according to the  $p$  values of  $t$  test.
4. The proposed method differs from the models based on fuzzy time series. Firstly, the proposed method makes predictions in a visibility graph, which makes use of the geometrical properties. While in fuzzy time series, the data is fuzzified in the first step. That is to say, the fuzzy rules and fuzzy sets are designed based on the original data. Consequently, the forecasting accuracy mainly depends on the divided intervals that are used to determine fuzzy sets. For example, Lu et al. (2015) designed more fuzzy sets by using interval information granules and the accuracy has been improved prominently. What's more, our approach can be used for both in-sample and out-of-sample forecasting. The experiment shows that the proposed method is able to accurately forecast the following 3-months CCI, 5-days TAIEX and 2-years enrollments. But methods based on fuzzy times fail to achieve out-of-sample predictions.

## 6 Conclusion

In this paper, a fuzzy logic based method, combining visibility graph and link prediction, is proposed to forecast time series. In the proposed method, time series is converted into a network at first. Utilizing link prediction, a strategy is designed to make initial predictions. Then, based on fuzzy logic, the initial forecasting is amended by fusing the direct and indirect effect of historical data. The key notion is to determine the fuzzy sets and fuzzy rules according to interrelationship among the historical data. Through comparison, the proposed method achieves better predictability by testing the effectiveness with various data set.

As is indicated in the experiments, the proposed method has been improved the accuracy of time series estimations. It is believed that cost estimators, investors and administrators will benefit from the proposed method in capital and personal management. Besides, it also provides a new idea for theoretical research about time series.

Even though the proposed method performs well, it is still difficult to satisfy the forecasting precision in some particular situation. Hence, to guarantee that the accuracy and predictability remain the same under any circumstances, our future work will

emphasize the following points: (1) the process of converting the time series into networks should be more useful for forecasting; (2) more useful information from known data should be excavated and adopted to make predictions; (3) more universal revision strategies should be designed in order to fit more cases.

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