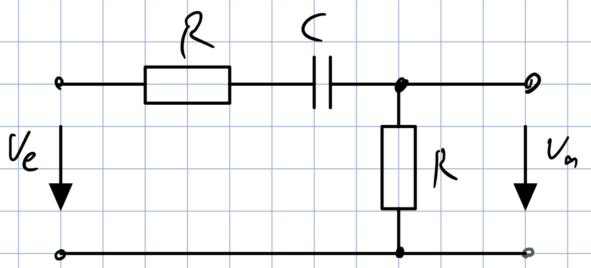
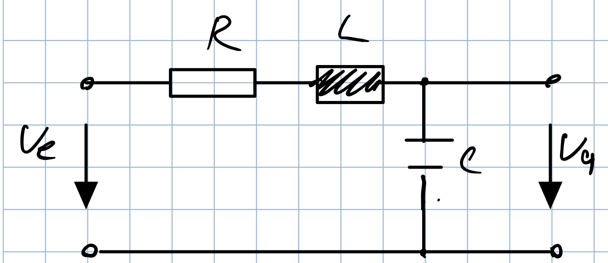
Filter 1. Ordnung:



$$G(j\omega) = \frac{V_a}{V_e} = \frac{R}{R+R+\frac{1}{j\omega C}} = \frac{R}{2R+\frac{1}{j\omega C}} = \frac{\frac{R}{2R}}{1+\frac{1}{j\omega C\cdot 2R}} = \frac{\cancel{R}}{2\cancel{R}} \cdot \frac{1}{1+\frac{1}{j2\omega RC}}$$



$$G(j\omega) = \frac{V_a}{V_e} = \frac{\frac{1}{j\omega C}}{R+j\omega L+\frac{1}{j\omega C}} \cdot \frac{j\omega C}{j\omega C} = \frac{1}{1+j\omega RC+j^2\omega^2 LC}$$

$$= \frac{1}{1+j\omega_n\omega_g RC+j^2\omega_n^2\omega_g^2 LC}$$

$$= \frac{1}{1 + \underbrace{s_n\omega_g RC}_{a_1} + \underbrace{s_n^2\omega_g^2 LC}_{b_1}}$$

$\omega_n = \frac{\omega}{\omega_g}$ normierte Frequenz

$\omega = \omega_n \cdot \omega_g$

z.B. $\omega = 4 \cdot 10\,kHz$

Laplace:

$j\omega \rightarrow s$

$j\omega_n \rightarrow s_n$

Bsp:

$\omega_g = 10\,s^{-1}$

$C = 300\,nF$

Bessel Filter

$a_1 = 1{,}3617$

$b_1 = 0{,}6180$

$$G(j\omega) = \frac{1}{1+1{,}3617+0{,}6180}$$

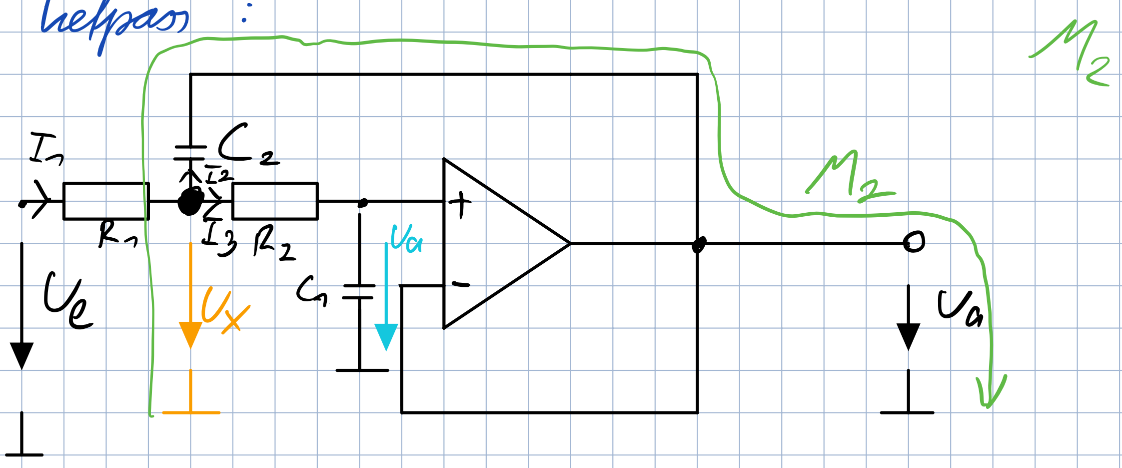
$$a_1 = \omega_g RC \Rightarrow R = \frac{a_1}{(\omega_g\cdot C)} = \frac{1{,}3617}{(10\,s^{-1}\cdot 300\,nF)} \approx 453{,}9\,k\Omega$$

$$b_1 = \omega_g^2 LC \Rightarrow L = \frac{b_1}{(\omega_g^2\cdot C)} = \frac{0{,}6180}{(10^2\,s^{-1}\cdot 300\,nF)} \approx 20{,}6\,kH$$

Aktive Filter

Filter mit Einfachmitkopplung (Sallen-Key) (2. Ordnung)

Tiefpass :



1) Knotenregel

$I_1 = I_2 + I_3$

$\rightarrow$ $I = \frac{U}{R}$ einsetzen

$$\frac{U_x}{U_a} = \frac{R_2 + X_{C1}}{X_{C1}} = \frac{R_2 + \frac{1}{j\omega C_1}}{\frac{1}{j\omega C_1}}$$

$$I_1 = \frac{U_1}{R_1} = \frac{U_e - U_x}{R_1} = \frac{U_e}{R_1} - \frac{U_a(1 + j\omega R_2 C_1)}{R_1}$$

$$I_2 = \frac{U_2}{X_{C2}} = \frac{U_x - U_a}{X_{C2}} = \frac{U_x - U_a}{\frac{1}{j\omega C_2}} = j\omega C_2 (U_x - U_a) - j\omega C_2 U_a ([1 + j\omega R_2 C_1] - \cancel{1})$$

$$I_2 = U_a \cdot j^2 \omega^2 R_2 C_1 C_2$$

$$I_3 = \frac{U_a}{\frac{1}{j\omega C_1}} = U_a \, j\omega C_1$$

$$\frac{U_a}{U_e} = ?$$

$$\overbrace{\frac{U_e}{R_1} - \frac{U_a(1 + j\omega R_2 C_1)}{R_1}}^{I_1} = \overbrace{U_a \cdot j^2 \omega^2 R_2 C_1 C_2}^{I_2} + \overbrace{U_a \cdot j\omega C_1}^{I_3} \quad | \cdot R_1$$

$$U_e = U_a (1 + j\omega R_2 C_1 + j\omega R_1 C_1 + j^2 \omega^2 R_1 R_2 C_1 C_2)$$

$$\frac{U_a}{U_e} = \frac{1}{(1 + j\omega R_2 C_1 + j\omega R_1 C_1 + j^2 \omega^2 R_1 R_2 C_1 C_2)}$$

$$\frac{U_a}{U_e} = \frac{1}{1 + j\omega C_1(R_1 + R_2) + j^2\omega^2 R_1R_2C_1C_2}$$

$$s_n \underbrace{\omega_g C_1(R_1+R_2)}_{a_1} \quad s_n^2 \underbrace{\omega_g^2 R_1R_2C_1C_2}_{b_1}$$

$\omega = \omega_n \cdot \omega_g$

$s_n = j\omega_n$

$j\omega \Rightarrow \underbrace{j\omega_n}_{s_n} \omega_g$

$j\omega = s_n \cdot \omega_g$

$a_1 = \omega_g C_1 (R_1 + R_2)$

$b_1 = \omega_g^2 R_1R_2C_1C_2$

aus a_1: $R_2 = \frac{a_1}{(\omega_g C_1)} - R_1$

R_2 in b_1 einsetzen:

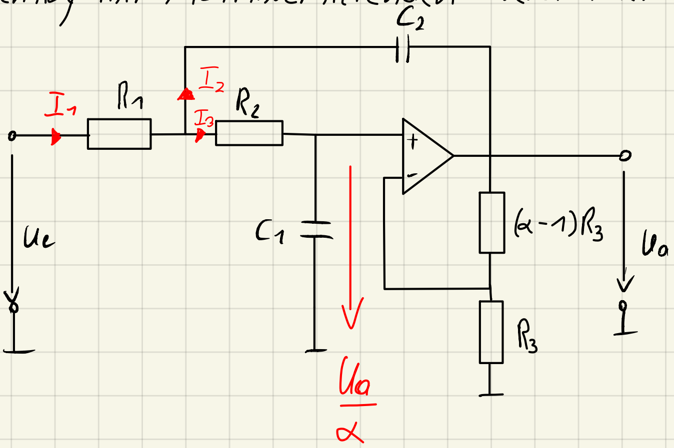
$$b_1 = \frac{\cancel{\omega_g^2} R_1R_2\cancel{C_1}C_2\,a_1}{\cancel{\omega_g C_1}} - \omega_g^2 R_1^2 R_2 C_1 C_2$$

$$0 = -R_1^2 \omega_g R_2 C_1 C_2 + \omega_g^2 R_1 R_2 C_2 a_1 - b_1$$

$$0 = R_1^2 - \frac{\cancel{\omega_g} R_1 \cancel{C_2} a_1}{\omega_g^2 C_1 \cancel{C_2}} + \frac{b_1}{\omega_g^2 R_2 C_1 C_2}$$

$$= R_1^2$$

Sallen key mit Nichtinvertierenden Verstärker



$$V_u = 1 + \frac{R_1}{R_2}$$

$$U_x = \frac{U_a}{\alpha} \cdot (1 + j\omega R_2 C_1)$$

$$I_1 = I_2 + I_3$$

$$I_1 = \frac{U_1}{R_1} = \frac{U_e - U_x}{R_1}$$

$$I_2 = \frac{U_2}{X_{C2}} = \frac{U_x - U_a}{\frac{1}{j\omega C_2}}$$

$$I_3 = \frac{U_a}{\frac{1}{j\omega C_1}} = U_a \cdot j\omega C_1$$

$$I_1 = \frac{U_e - \frac{U_a}{\alpha} \cdot (1 + j\omega R_2 C_1)}{R_1}$$

$$I_2 = \frac{U_x - U_a}{X_{C2}} = \overbrace{\frac{U_a}{\alpha} \cdot \frac{1 + j\omega C_1 R_2}{\frac{1}{j\omega C_2}}}^{U_x/X_{C2}} \overbrace{- U_a j\omega C_2}^{U_a/X_{C2}}$$

$$= \frac{U_a}{\alpha} \cdot (j\omega C_2 + j^2\omega^2 R_2 C_1 C_2) - U_a \cdot j\omega C_2$$

$$= \frac{U_a}{\alpha} \cdot (j\omega C_2 + j^2\omega^2 R_2 C_1 C_2 - \alpha \cdot j\omega C_2)$$

$$= \frac{U_a}{\alpha} \cdot ((1-\alpha) j\omega C_2 + j^2\omega^2 R_2 C_1 C_2)$$

$$I_3 = \frac{\frac{U_a}{\alpha}}{\frac{1}{j\omega C_1}} = \frac{U_a}{\alpha} \cdot j\omega C_1$$

$$\overbrace{\frac{U_e}{R_1} - \frac{\frac{U_a}{\alpha} \cdot (1 + j\omega R_2 C_1)}{R_1}}^{I_1} = \overbrace{\frac{U_a}{\alpha} \cdot ((1-\alpha) j\omega C_2 + j^2\omega^2 R_2 C_1 C_2)}^{I_2} + \overbrace{\frac{U_a}{\alpha} \cdot j\omega C_1}^{I_3} \quad | \cdot R_1$$

$$U_e - \frac{U_a}{\alpha} \cdot (1 + j\omega R_2 C_1) = \frac{U_a}{\alpha} \cdot [(1-\alpha) j\omega R_1 C_2 + j^2\omega^2 R_1 R_2 C_1 C_2] + \frac{U_a}{\alpha} \cdot j\omega C_1$$

$$U_e = \frac{U_a}{\alpha} \cdot ((1 + j\omega C_1 R_2) + (1-\alpha) j\omega R_1 C_2 + j^2\omega^2 R_1 R_2 C_1 C_2 + j\omega C_1)$$

$$\frac{U_e}{(\ )} = \frac{U_a}{\alpha}$$

$$\frac{U_e}{(\ )} \cdot \alpha = U_a$$

$$\frac{U_a}{U_e} = \frac{\alpha}{1 + j\omega C_1 R_2 + (1-\alpha) j\omega R_1 C_2 + j^2\omega^2 R_1 R_2 C_1 C_2 + j\omega C_1}$$

$$\frac{U_a}{U_e} = \frac{\alpha}{1 + j\omega(C_1 \cdot (R_1 + R_2) + (1-\alpha) R_1 C_2) + j^2\omega^2 R_1 R_2 C_1 C_2}$$

$\alpha \rightarrow$ Verstärkung

$$H(s_n) = \frac{\alpha}{1 + \omega_g [C_1 (R_1 + R_2) + (1-\alpha) R_1 C_2] s_n + \omega_g^2 R_1 R_2 C_1 C_2 s_n^2}$$

$$a_1 = \omega_g C_1 R_1 + \omega_g C_1 R_2 + (1-\alpha) \cdot \omega_g R_1 C_1$$

$$b_1 = \omega_g^2 R_1 R_2 C_1 C_2$$

$$R_1 = R_2, \quad C_1 = C_2$$

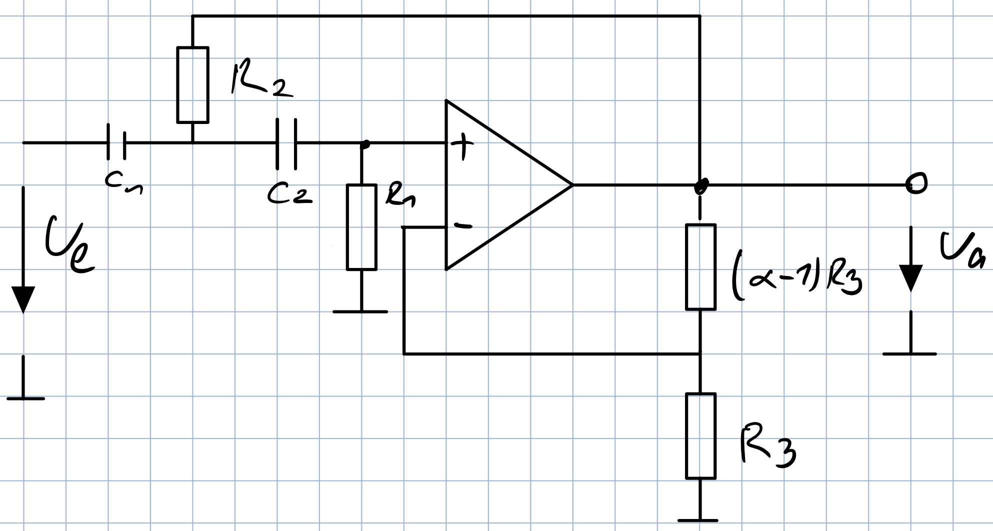
$$b_1 = \omega_g^2 R^2 C^2 \rightarrow \sqrt{b_1} = \omega_g RC$$

$$a_1 = \underbrace{\omega_g \cdot RC}_{\sqrt{b_1}} + \underbrace{\omega_g RC}_{\sqrt{b_1}} + (1-\alpha) \cdot \underbrace{\omega_g RC}_{\sqrt{b_1}}$$

$$a_1 = 2\sqrt{b_1} + \sqrt{b_1} - \alpha\sqrt{b_1} = 3\sqrt{b_1} - \alpha\sqrt{b_1}$$

$$\alpha = 3 - \frac{a_1}{\sqrt{b_1}}$$

Hochpass



$$R_1 \rightarrow \frac{1}{j\omega C_1}$$

$$\frac{1}{j\omega C_1} \rightarrow R_1$$

$$j\omega \rightarrow s_n \omega_g$$

$$s_n = j\frac{\omega}{\omega_g}$$

$$s_n \omega_g C_1 \rightarrow j\omega C_1 = \frac{1}{X_C}$$

$$j\omega C_1 = \frac{1}{R_1}$$

$$j\omega C_1 \cdot j\omega C_2 \rightarrow \frac{1}{R_1}\,\frac{1}{R_2}$$

$$R_1 R_2 \rightarrow \frac{1}{j\omega C_1}\,\frac{1}{j\omega C_2}$$

$$\frac{1}{s_n^2 \omega_g^2 C_1 C_2 R_1 R_2}$$

$$\frac{U_a}{U_e} = \frac{\alpha}{1 + j\omega\left(C_1(R_1+R_2) + [1-\alpha]R_1C_2\right) + j^2\omega^2 R_1R_2C_1C_2}$$

$$\frac{\alpha}{1 + j\omega C_1 R + j\omega C_1 R_2 + j\omega[1-\alpha]R_1C_2 + \frac{1}{j^2\omega^2 R_1R_2C_1C_2}}$$

$$\frac{\alpha}{1 + \frac{1}{j\omega R_1 C_1} + \frac{1}{j\omega R_1 C_2} + \frac{[1-\alpha]}{j\omega C_1 R_2} + \frac{1}{j^2\omega^2 R_1R_2C_1C_2}}$$

$$= \frac{\alpha}{1 + \frac{1}{s_n}\cdot\frac{R_2C_2 + R_2C_1 + [1-\alpha]R_1C_2}{\omega_g(R_1R_2C_1C_2)} + \frac{1}{s_n^2}\underbrace{\frac{1}{\omega_g^2 R_1R_2C_1C_2}}_{b_1}}$$

$$\frac{\alpha}{1 + \frac{1}{s_n}\cdot\frac{R_1(C_1+C_2) + [1-\alpha]R_1C_2}{\omega_g(R_1R_2C_1C_2)} + \frac{1}{s_n^2}\underbrace{\frac{1}{\omega_g^2 R_1R_2C_1C_2}}_{b_1}}$$