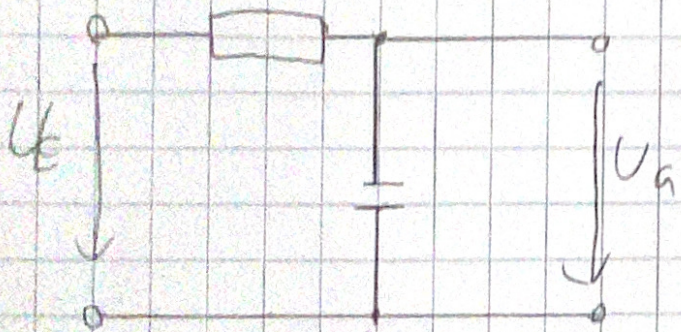
Analoge Filter

2.1.1 Amplitudengang

Amplitudengang $|G(j\omega)|$

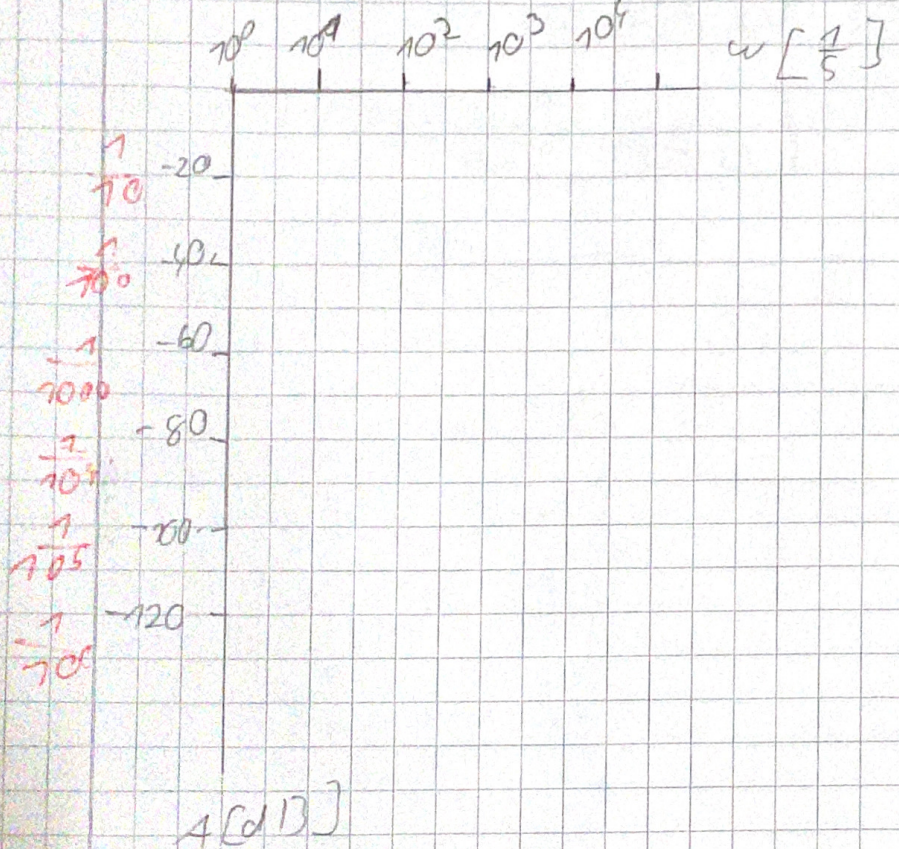
← Tiefpass (Filtert hohe Frequenzen heraus)



$$G(j\omega) = \frac{U_a}{U_e} = \frac{X_C}{R + X_C} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{j\omega C}{j\omega C} = \frac{1}{1 + j\omega RC}$$

$$|G(j\omega)| = \frac{\sqrt{1^2 + 0}}{\sqrt{1^2 + (\omega RC)^2}}$$

$$A = 20 \log (|G(j\omega)|)$$



Phasengang von $G(j\omega)$

$$G(j\omega) = \frac{1}{1 + j\omega RC}$$

$$\varphi = \arctan\frac{0}{1} - \arctan\frac{\omega Rc}{1}$$

$$\varphi = 0 - \arctan(\omega Rc)$$

$$\varphi = -\arctan(\omega Rc)$$

$R = 10\,k\Omega$

$C = 10\,nF$

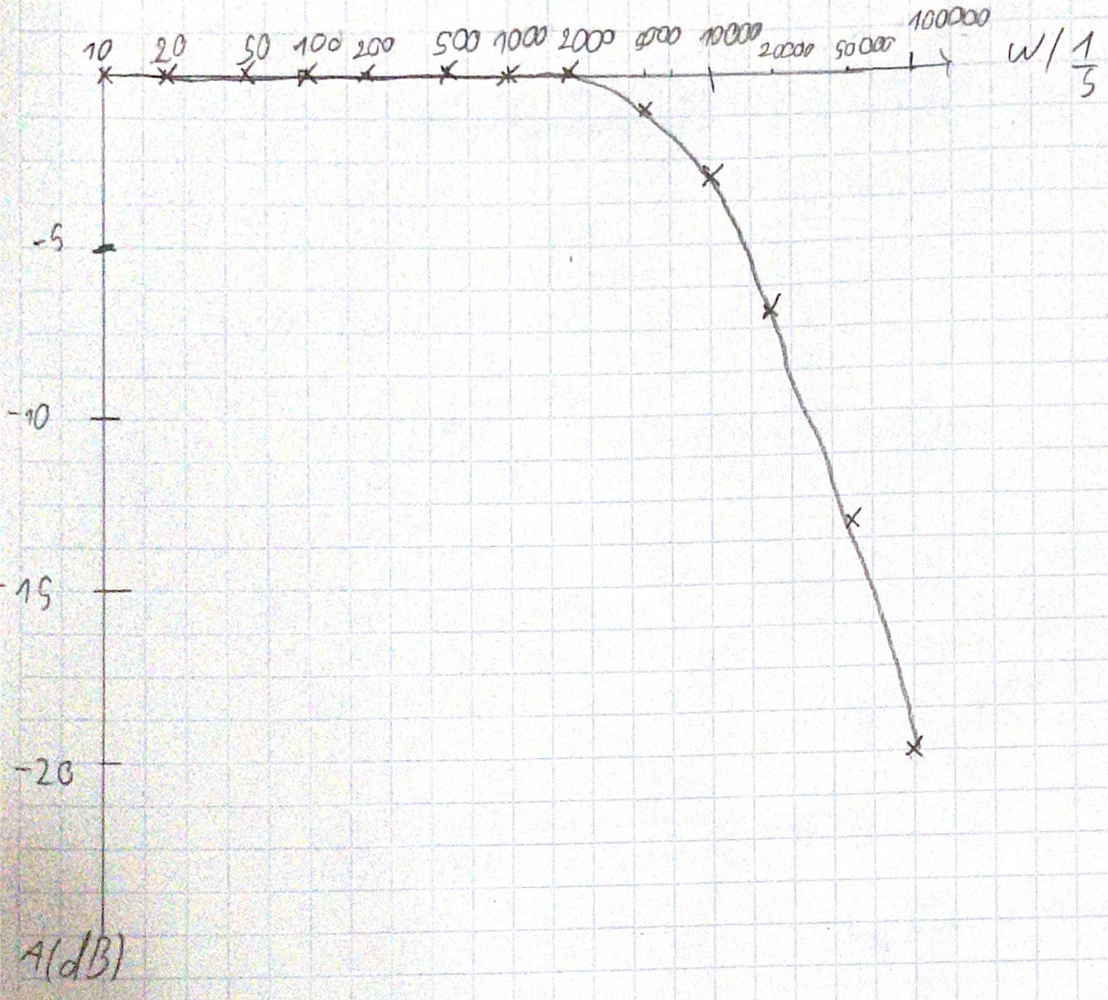
Amplitudengang

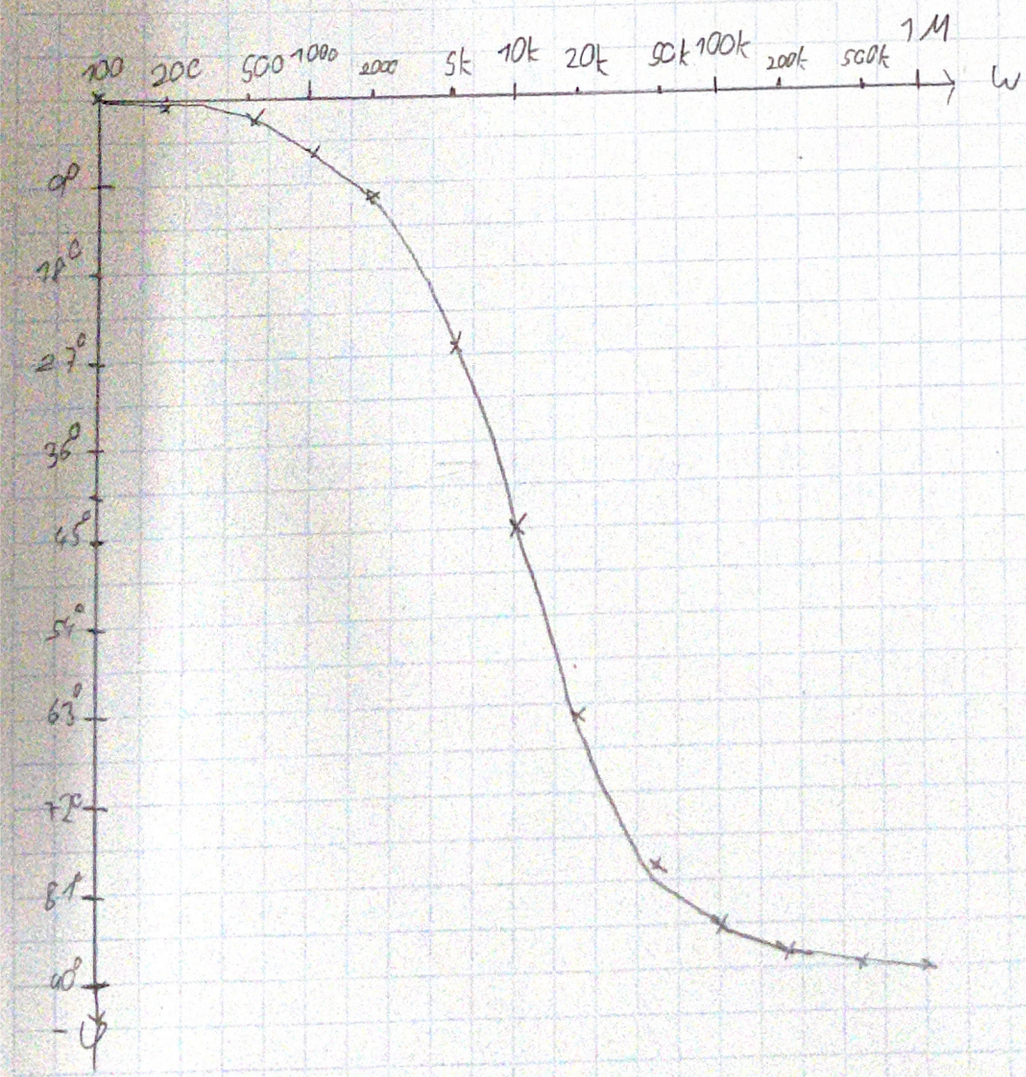
$\omega = 10, 20, 50, 100, 200, 500, 1000, \ldots, 100\,000$

$$A = 20\log\left|\frac{1}{\sqrt{1+\omega^2 R^2 C^2}}\right|$$

$$\varphi = -\arctan(\omega RC)$$

ω	A	φ
10	$-4,3 \cdot 10^{-6}$	
20	$-17,37 \cdot 10^{-6}$	
50	$-108 \cdot 10^{-6}$	
100	$-434 \cdot 10^{-6}$	-0,57
200	$-1,73 \cdot 10^{-3}$	-1,15
500	$-10,84 \cdot 10^{-3}$	-2,86
1000	$-43,21 \cdot 10^{-3}$	-5,7
2000	$170,3 \cdot 10^{-3}$	-11,3
5000	$-969,1 \cdot 10^{-3}$	-26,57
10000	-3,01	-45
20000	-6,99	-63,43
50000	-14,15	-78,69
100000	-20,04	-84,29
200k		-87,14
500k		-88,86
1M		-89,43

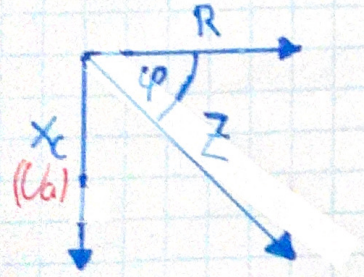

10
20
50
100
200
500
1000
2000
5000
10000
20000
50000
100000
w/1/s
-5
-10
-15
-20
A(dB)


100
200
500
1000
2000
5k
10k
20k
50k
100k
200k
500k
1M
w
0°
18°
27°
36°
45°
54°
63°
72°
81°
90°
-Φ

Grenzfrequenz.

-3dB Abschwächung $U_a = \frac{1}{\sqrt{2}} \cdot U_E = 0{,}707 \cdot U_E$

$\sim 70\%$ von U_E



$\frac{U_a}{U_e} = \frac{1}{\sqrt{2}}$

$\boxed{\text{Im} = \text{Re}}$

$\Rightarrow |\varphi| = 45°$

$G(j\omega) = \frac{1}{1 + j\omega RC} \Rightarrow \text{Im} = \text{Re}$

$\omega_G RC = 1 \Rightarrow \omega_G = \frac{1}{RC}$

Bsp.: $f_G = 10\ \text{Hz}$

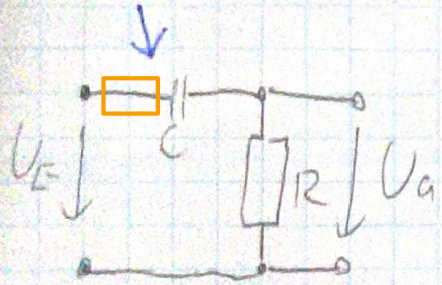
$\left(\omega_G = \frac{1}{RC} \right)$

$\omega_G = 2\pi f_G = 62{,}8\ \frac{1}{s}$ Ann. $C = 1\,\mu F$

$R = \frac{1}{\omega C} = \frac{1}{62{,}8\ \frac{1}{s} \cdot 1 \cdot 10^{-6}\ F} \doteq 19{,}9\,k\Omega$

KSV

Hochpass (lässt hohe Frequenzen heraus)



$$X_C = \frac{1}{j\omega C}$$

$$G(j\omega) = \frac{U_a}{U_e} = \frac{R}{\frac{1}{j\omega C} + R\frac{1}{R}}$$

$$= \frac{1}{\frac{1}{j\omega RC} + 1} = \frac{1}{1 + \frac{1}{j\omega RC}}$$

$R \cdot x = 1$

$x =$

$|G(j\omega)| =$

$\varphi =$

HÜ

Aufgabe:

Hochpass: f_g = Geburtsdatum · 5 kHz

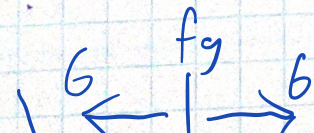
Amplitudengang

Phasengang (rechnen + zeichnen)

1) R, C

2) Tabelle

ω	$G(j\omega)$ dB	φ



5k 10k 20k 50k 100k

4te Aufgabe

Arbeitsauftrag:

$f_g = 1 \cdot 5\,kHz = 5\,kHz$

1)

$R = \frac{1}{\omega C} = \frac{1}{31415{,}93 \cdot 1\mu} = 31{,}83\,\Omega$

$C = 1\,\mu F$

$\omega = 2\pi f_g = 2\pi 5k = 31\,415{,}93\,\frac{1}{s}$

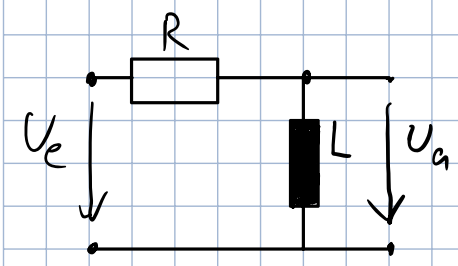
2)

ω	$\lvert G(j\omega)\rvert$ dB	φ
3141592,65	$-0{,}43 \cdot 10^{-3}$	-0,57
1256637,06	$-2{,}71 \cdot 10^{-3}$	-1,43
628318,53	$-10{,}8 \cdot 10^{-3}$	-2,86
314159,26	$-43 \cdot 10^{-3}$	-5,7
12566,371	$-263{,}3 \cdot 10^{-3}$	-14,03
6283,185	$-969{,}1 \cdot 10^{-3}$	-26,56
3141,592	-3,01	-45
1256,637	-8,60	-68,19
628,319	-14,15	-78,69
314,159	-20,04	-84,29
125,663	-27,97	-87,71
62,831	-33,98	-88,85
31,416	-40	-89,42

$$|G(j\omega)| = 20 \log \frac{1}{\sqrt{1 + \left(\frac{1}{\omega RC}\right)^2}}$$

$$\varphi = -\arctan \frac{1}{\omega RC}$$

TABELLE



1)

$$G(j\omega) = \frac{U_a}{U_e} = \frac{j\omega L}{R + j\omega L} \cdot \frac{\frac{1}{j\omega L}}{\frac{1}{j\omega L}} = \frac{1}{1 + \frac{R}{j\omega L}}$$

$$\frac{R}{j\omega L} \rightarrow 1 - j \cdot \frac{R}{\omega L}$$

$$= \frac{|Z_Z| \cdot e^{j\varphi_Z}}{|Z_N| \cdot e^{j\varphi_N}} = \frac{|Z|}{|Z|} \cdot \frac{e^{j\varphi_Z}}{e^{j\varphi_N}} = e^{j(\varphi_Z - \varphi_N)}$$

φ

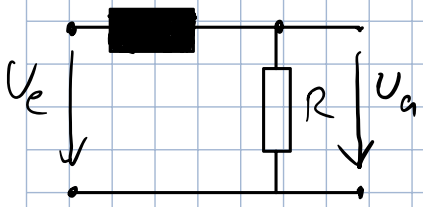
$e^{j\varphi}$

$$|G(j\omega)| = \frac{\sqrt{1^2 - 0^2}}{\sqrt{1^2 + \left(\frac{R}{\omega L}\right)^2}} = \frac{1}{\sqrt{1 + \left(\frac{R}{\omega L}\right)^2}}$$

$$\varphi = 0 - \arctan\left(\frac{-\frac{R}{\omega L}}{1}\right) = 0 - \arctan\left(-\frac{R}{\omega L}\right)$$

$k = 1$

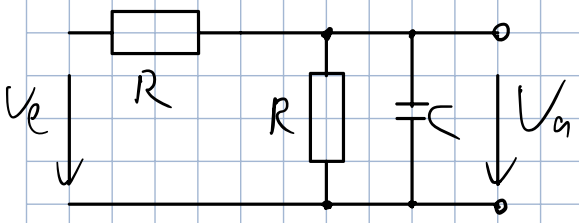
ω_G .... $Re = Im \Rightarrow \underbrace{\frac{R}{\omega L}}_{Im} = \underbrace{1}_{Re} \Rightarrow \omega_G = \frac{R}{L}$



$$G(j\omega) = \frac{U_a}{U_e} = \frac{R}{R + j\omega L} \cdot \frac{\frac{1}{R}}{\frac{1}{R}} = \frac{1}{1 + j\frac{\omega L}{R}}$$

$$|G(j\omega)| = \frac{1}{\sqrt{1^2 + \left(\frac{\omega L}{R}\right)^2}}$$

$$\varphi = 0 - \arctan\left(\frac{\omega L}{R}\right)$$



$$G(j\omega) = \frac{\frac{1}{2}}{1 + \dots}$$

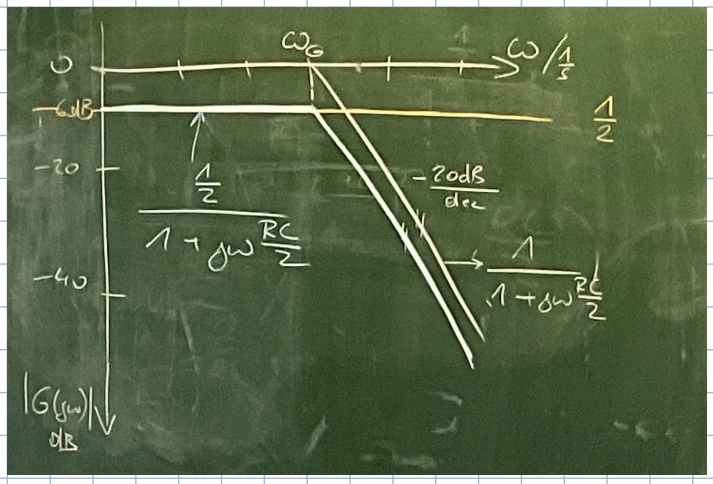
$$G(j\omega) = \frac{U_a}{U_e} = \frac{R \| X_C}{R + R \| X_C} = \frac{\frac{1}{\frac{1}{R} + j\omega C}}{R + \frac{1}{\frac{1}{R} + j\omega C}} \cdot \frac{\frac{1}{R} + j\omega C}{\frac{1}{R} + j\omega C}$$

$$= \frac{1}{\frac{R}{R} + j\omega RC + 1} = \frac{1}{2 + j\omega RC} = \frac{\frac{1}{2}}{1 + j\omega \frac{RC}{2}}$$

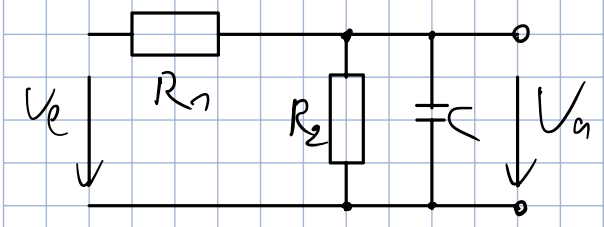
$$\varphi = \underbrace{\arctan\left(\frac{0}{1}\right)}_{=0} - \arctan\left(\frac{j\omega \frac{RC}{2}}{2}\right) \Rightarrow \varphi = -\arctan\left(\frac{\omega RC}{2}\right)$$

$$|G(j\omega)| = \frac{\frac{1}{2}}{\sqrt{1^2 + \left(\frac{\omega RC}{2}\right)^2}}$$

$$1 = \omega_G \frac{RC}{2} \Rightarrow \omega_G = \frac{1}{\frac{RC}{2}} = \frac{2}{RC}$$



Bsp.



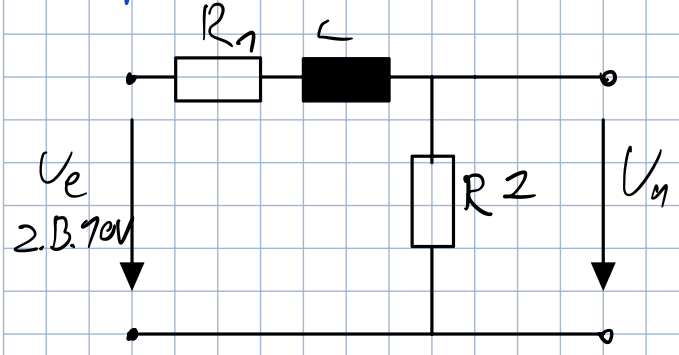
$$R_2 \| C = \frac{1}{\frac{1}{R_2} + j\omega C} = \frac{R_2}{1 + j\omega R_2 C}$$

$$G(j\omega) = \frac{U_a}{U_e} = \frac{\frac{R_2}{1+j\omega R_2 C}}{R_1 + \frac{R_2}{1+j\omega R_2 C}} \cdot \frac{1+j\omega R_2 C}{1+j\omega R_2 C} =$$

$$= \frac{R_2}{R_1 + j\omega R_2 R_1 C + R_2} = \frac{R_2}{R_1 + R_2 + j\omega R_1 R_2 C} \cdot \frac{\frac{1}{R_1+R_2}}{\frac{1}{R_1+R_2}} = \frac{\frac{R_2}{R_1+R_2}}{1 + j\omega \frac{R_1 R_2 C}{R_1+R_2}}$$

$$\Rightarrow G(j\omega) = \frac{\frac{R_2}{R_1+R_2}}{1 + j\omega \frac{R_1 R_2 C}{R_1+R_2}}$$

Bsp: 24.11.2021



$\omega \ll$ $X_L \ll$

$\omega \gg$ $X_L \gg$

$$\frac{U_{amax}}{U_e} = \frac{R_2}{R_1+R_2} \quad k$$

$$G(j\omega) = \frac{U_a}{U_e} = \frac{R_2}{R_1 + R_2 + X_L} =$$

$$U_{amax} = U_e \frac{R_2}{R_1+R_2} \quad (\omega = 0)$$

$$= \frac{R_2}{R_1 + R_2 + j\omega L} \cdot \frac{\frac{1}{R_1+R_2}}{\frac{1}{R_1+R_2}} =$$

$$= \frac{\frac{R_2}{R_1+R_2}}{1 + j\frac{\omega L}{R_1+R_2}} \quad k$$

$$|G(j\omega)| = \sqrt{\frac{R_2^2}{(R_1+R_2)^2 + (\omega L)^2}}$$

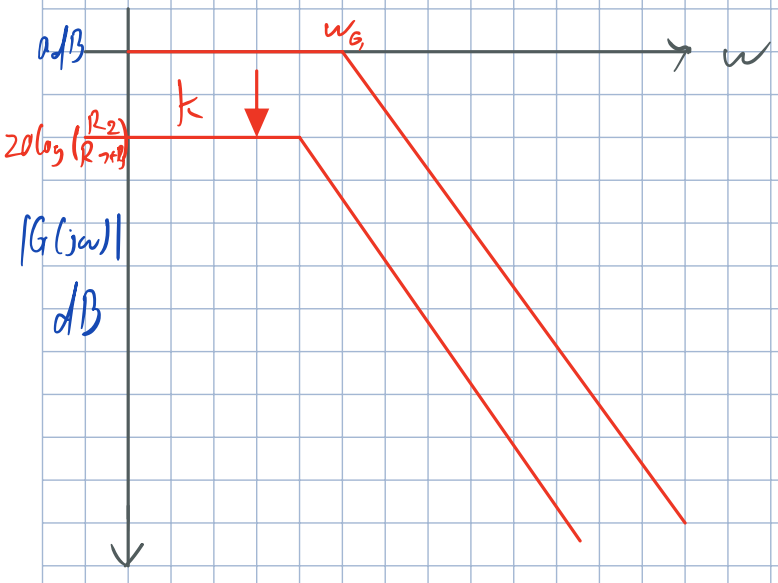
① $\varphi = -\arctan\left(\frac{\omega L}{R_1+R_2}\right)$

① ω_g: $R_1 + R_2 = \omega_g L \Rightarrow \omega_g = \frac{R_1+R_2}{L}$

② $1 = \frac{\omega_g L}{R_1+R_2} \Rightarrow \omega_g = \frac{R_1+R_2}{L}$

② $\varphi = -\arctan \frac{\omega L}{\frac{R_1 + R_2}{n}} = -\arctan \frac{\omega L}{R_1 + R_2}$

Bsp:



f_g: Geburtstag · 100 | Monat · 02 | 14 Hz ... 30 kHz · 10.2

$f_g = 1002$ Hz

R_1, R_2, L | ann.: $L = 27$ mH

R_1, R_2 errechnen

Bodediagramm / Phasengang | SIMULATION LTSPICE

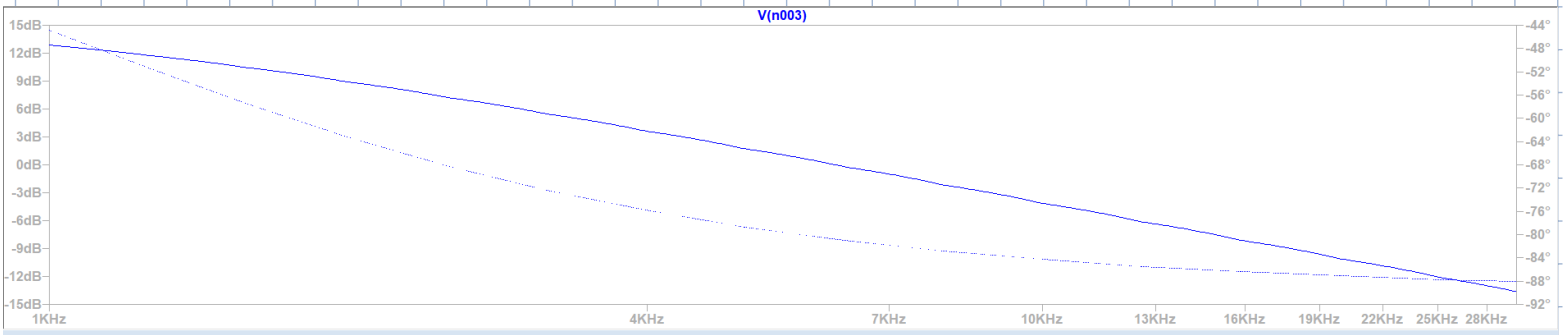
$f_g = 1 \cdot 10 + 05 = 1005$ Hz

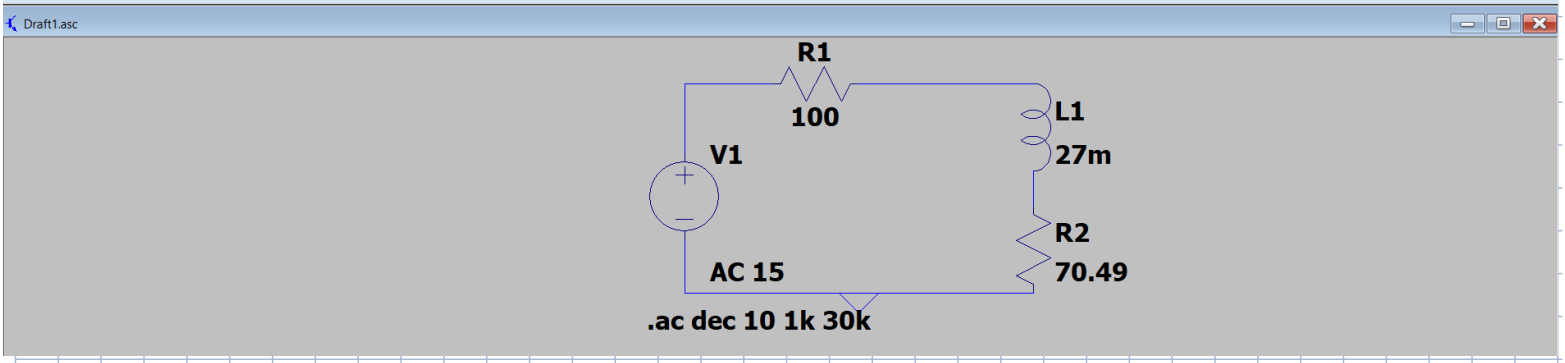
$\omega_g = 2\pi f_g = 6314,6 \frac{1}{s}$

$R_1 = 100\,\Omega$

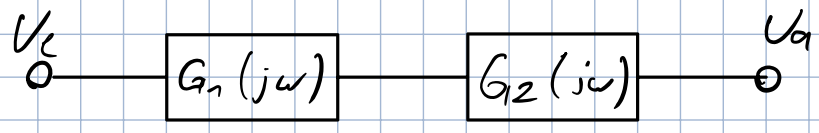
$\omega_G \cdot L = R_1 + R_2 \rightarrow R_2 = \omega_G \cdot L - R_1 = 6314,6 \frac{1}{s} \cdot 27\,\text{mH} - 100\,\Omega$

$= 70,49\,\Omega$





Zusammengesetzte Frequenzgänge



$G(j\omega) = U_e - G_1(j\omega)$

$U_a = U_{a1} \cdot G_2(j\omega)$

$U_a = U_e \cdot G_1(j\omega) \cdot G_2(j\omega) \Rightarrow \frac{U_a}{U_e} = G_1(j\omega) \cdot G_2(j\omega)$

Bsp: $G_1(j\omega) = 100$

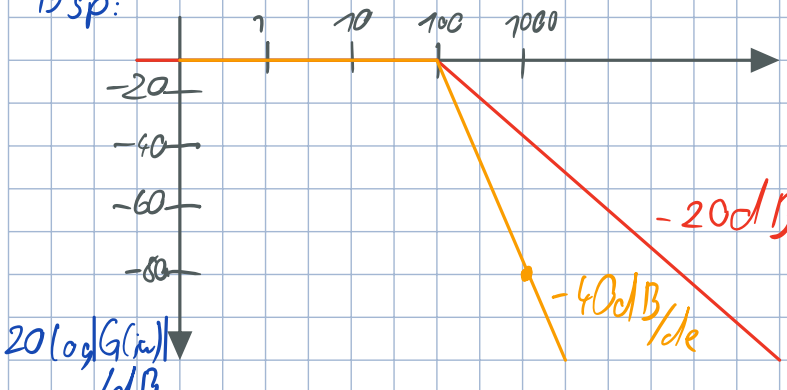
$G_2(j\omega) = 1000$

$\Rightarrow G(j\omega) = 100 \cdot 1000 = 100\,000$

$10^2 \quad 10^3 \quad 10^{2+3} = 10^5$

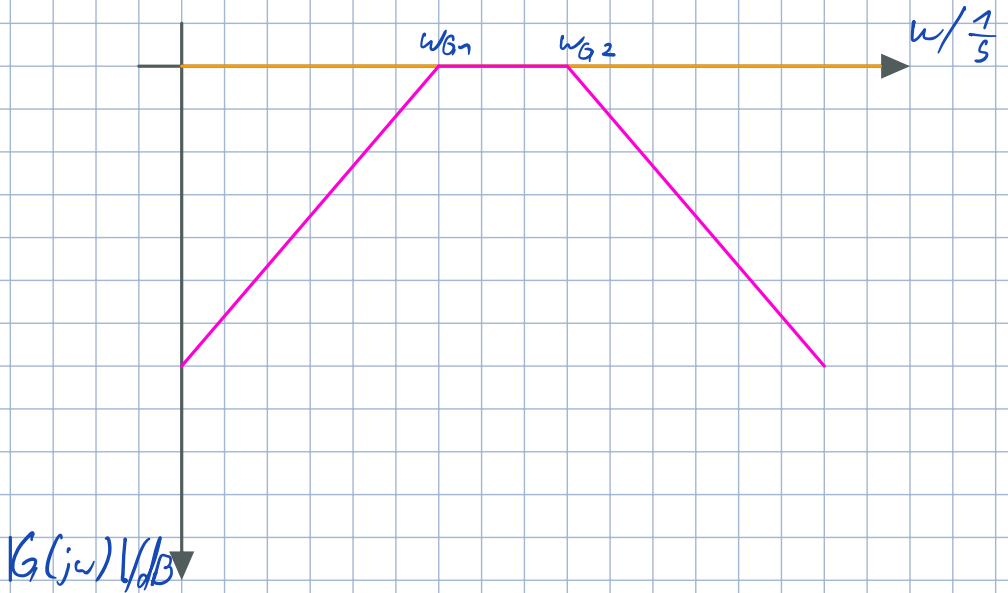
$\log|G(j\omega)| = \log|G_1(j\omega)| + \log|G_2(j\omega)|$

Bsp:



$G(j\omega) = G_1(j\omega)\, G_2(j\omega)$

Bsp: HP, TP



Bsp. phasenanhebendes System

$$G(j\omega) = \frac{1+j\omega T_1}{1+j\omega T_2} = \underbrace{(1+j\omega T_1)}_{G_1(j\omega)} \underbrace{\frac{1}{1+j\omega T_2}}_{G_2(j\omega)}$$

$T_1 = 1 \qquad T_2 = 0{,}2$

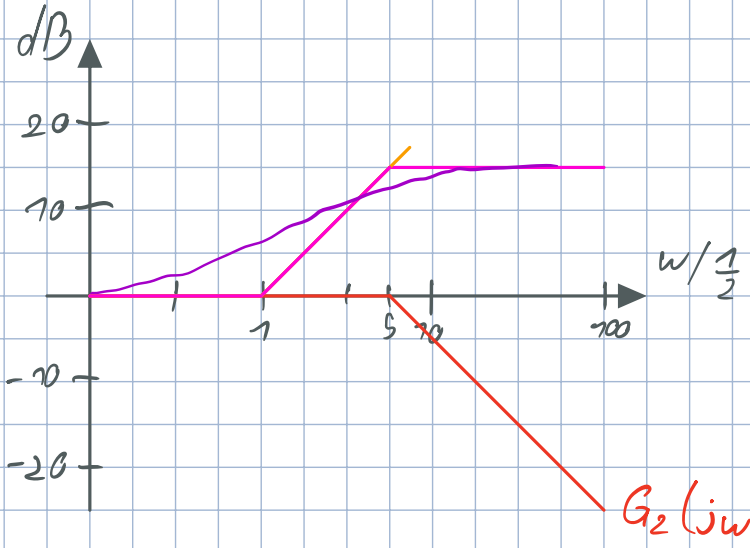
$$|G(j\omega)| = \sqrt{\frac{(1+\omega T_1)^2}{(1+\omega T_2)^2}}$$

$$\varphi = \arg(G(j\omega)) = \arctan(\omega T_1) - \arctan(\omega T_2)$$

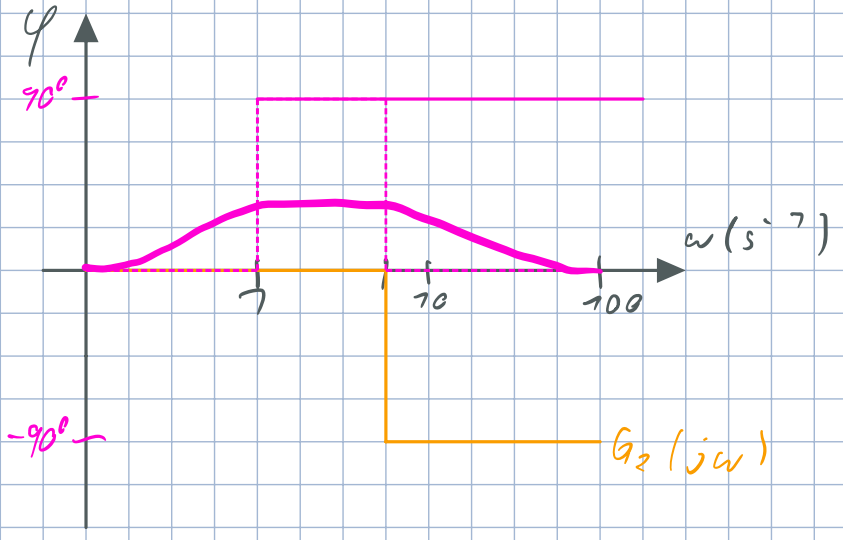
Grenzfrequenzen (Knickfrequenz)

$$\omega_{G1} = \frac{1}{T_1} = 1 \qquad \omega_{G2} = \frac{1}{0{,}2} = 5$$

Amplitudengang



Phasengang

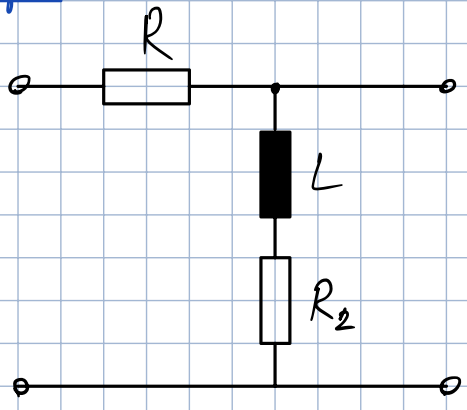


Maximum tritt im geometrischen Mittel ω auf

$$\omega_m = \frac{1}{\sqrt{T_1 \cdot T_2}} = \sqrt{5}$$

$$\varphi_{max}|_{\omega=\omega_m} = \arctan\sqrt{\frac{T_1}{T_2}} - \arctan\sqrt{\frac{T_2}{T_1}} = 47{,}8°$$

Bsp:



ges: $G(j\omega)$, ω_{a1}, ω_{a2}, k, φ

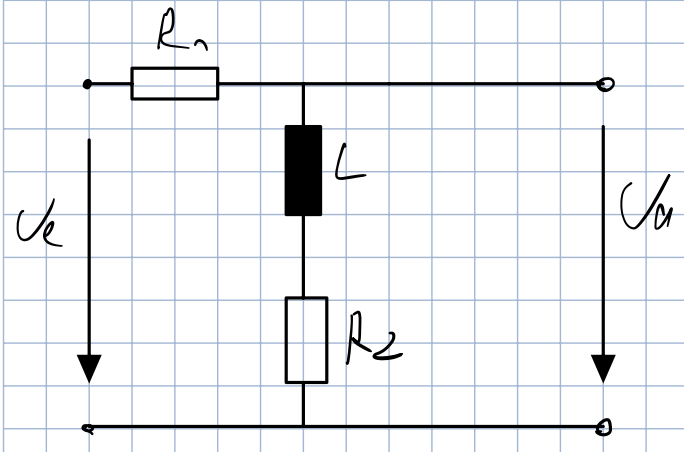
$k \cdot \frac{1+j\ldots}{1+j\ldots}$ $\quad$ k ... nicht ω-abhängig

$$k = \frac{R_2}{R_1+R_2}$$

$$G(j\omega) = \frac{U_a}{U_e} = \frac{R_2+j\omega L}{R_1+R_2+j\omega L} = \frac{R_2\left(1+\frac{j\omega L}{R_2}\right)}{R_1+R_2\left(1+\frac{j\omega L}{R_1+R_2}\right)} = \frac{R_2}{R_1+R_2} \cdot \frac{1+j\frac{\omega L}{R_2}}{1+j\frac{\omega L}{R_1+R_2}}$$

$$\varphi = \arctan\left(\frac{\frac{\omega L}{R_2}}{\frac{\omega L}{R_2}}\right)$$

Bsp.



gesucht:

- $G(j\omega)$ · Bodediagram
- $|G(j\omega)|$ · Phasengang
- φ
- f_g

$k = \frac{R_2}{R_1 + R_2}$

$$G(j\omega) = \frac{U_a}{U_e} = \frac{R_2 + j\omega L}{R_1 + R_2 + j\omega L} = \underbrace{\frac{R_2}{R_1 + R_2}}_{k} \cdot \frac{1 + j\frac{\omega L}{R_2}}{1 + j\frac{\omega L}{R_1 + R_2}} =$$

$$|G(j\omega)| = \frac{\sqrt{R_2^2 + \omega^2 L^2}}{\sqrt{(R_1 + R_2)^2 + \omega^2 L^2}}$$

$$\varphi = \arctan\left(\frac{\omega L}{R}\right) - \arctan\left(\frac{\omega L}{R_1 + R_2}\right)$$

Annahme:

$L = 100\,mH$ $\quad \omega_{g1} = 10k\frac{1}{s}$ $\quad \omega_{g2} = 100k\frac{1}{s}$

$\omega = 10, 100, 1k, 10k, 100k, 1M, 10M$

$Im_1 = Re_1 \Rightarrow R_2 = \omega L = 10k\frac{1}{s} \cdot 100\,mH = 1k\Omega$

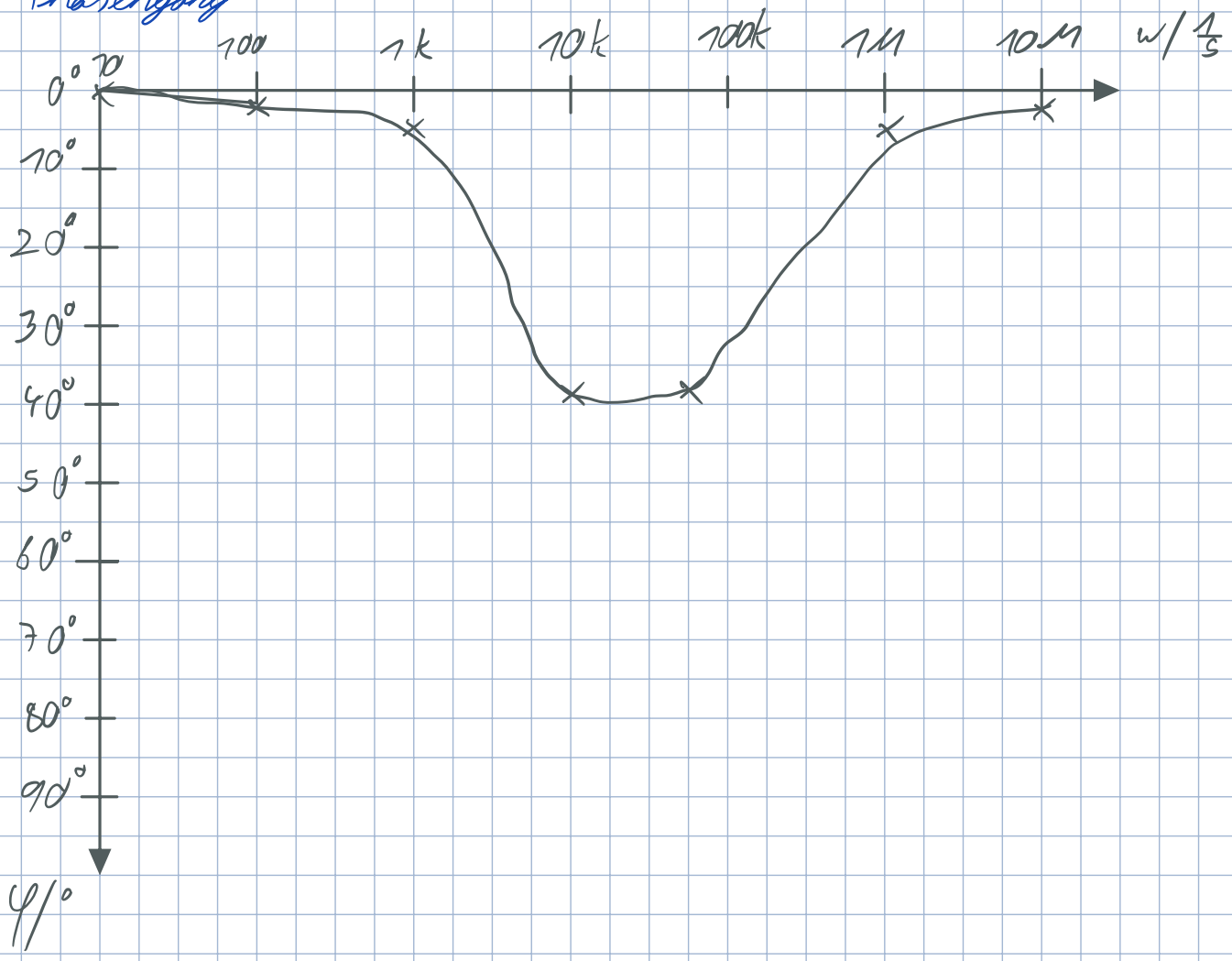
$Im_1 = Re_{1n} \Rightarrow R_1 + R_2 = \omega L = R_1 = \omega L - R_2 = 100k\frac{1}{s} \cdot 100\,mH - 1k\Omega = 9k\Omega$

$f_{g1} = \frac{\omega_{g1}}{2\pi} = \frac{10k\frac{1}{s}}{2\pi} = 1591{,}5\,Hz \approx 1{,}6\,kHz$

$f_{g2} = \frac{\omega_{g2}}{2\pi} = \frac{100k\frac{1}{s}}{2\pi} = 15915{,}5\,Hz \approx 15{,}9\,Hz$

ω	$20 \log(\lvert G(j\omega) \rvert)$	φ
10	-20 dB	0,057°
100	-19,9996 dB	0,516°
1k	-19,9572 dB	5,137°
10k	-17,03 dB	39,3°
100k	-2,967 dB	39,3°
1M	-0,043 dB	5,137°
10M	-0,00043 dB	0,517°

Phasengang



Amplitudengang