# LECTURE 3

In this lecture, we give a quick review of the theory of algebraic groups. This theory is analogous to that of complex Lie groups, but the techniques are more algebraic and some proofs are subtler[1]. Standard textbooks include: [B], [H] and [Sp]. See [M] for a modern treatment of this theory, which is also my favourite.

## 1. Algebraic Groups

**Definition 1.1.** An **algebraic group** over $k$ is a finite type $k$-scheme $G$ equipped with a group structure, i.e., a multiplication map $m : G \times G \to G$ subject to axioms similar to those for an abstract group.

Homomorphisms between algebraic groups are defined in the obvious way. Let $\mathsf{Grp}_k$ be the category of algebraic groups.

*Remark* 1.2. As in the study of abstract groups, the unit and the inversion maps are determined by the multiplication map. We denote them respectively by:

$$e : \mathsf{pt} \to G,\ \sigma : G \to G,$$

where $\mathsf{pt} := \mathsf{Spec}(k)$.

**Construction 1.3.** *Let $G$ be an algebraic group. For any commutative $k$-algebra $A$, write*

$$G(A) := \mathsf{Hom}(\mathsf{Spec}(A), G)$$

*be the set of maps $\mathsf{Spec}(A) \to G$ between $k$-schemes. The group structure on $G$ induces a group structure on $G(A)$.*

*Note that for $A \to B$, we have a homomorphism $G(A) \to G(B)$. Hence we obtain a functor*

$$G(-) : \mathsf{CAlg}_k \to \mathsf{Grp}$$

*from the category of commutative $k$-algebras to the category of (abstract) groups. By the Yoneda lemma, the algebraic group $G$ is determined by this functor.*

**Example 1.4.** The **additive group** $\mathbb{G}_\mathsf{a}$ is defined such that $\mathbb{G}_\mathsf{a}(A) = A$, viewed as a commutative group under addition. The underlying $k$-scheme is the affine line $\mathbb{A}^1$.

**Example 1.5.** The **multplicative group** $\mathsf{GL}_1 = \mathbb{G}_\mathsf{m}$ is defined such that $\mathbb{G}_\mathsf{m}(A) = A^\times$, i.e. the subset of unit elements in $A$, viewed as a commutative group under multiplication. The underlying $k$-scheme is $\mathbb{A}^1 \smallsetminus 0$, i.e., the affine line with the origin removed.

**Example 1.6.** One can define algebraic groups $G := \mathsf{GL}_n$, $\mathsf{SL}_n$, $\mathsf{SO}_n$, etc. such that $G(A)$ is the group of matrices of the corresponding type with coefficients in $A$.

**Example 1.7.** One can define the algebraic group $\mathsf{PGL}_n$ such that $\mathcal{O}_{\mathsf{PGL}_n}$ is a subring of $\mathcal{O}_{\mathsf{GL}_n}$[2].

---

*Date*: Mar 11, 2024.

[1]Especially if one allows positive-characteristic or non-algebraic-closed base field $k$.

[2]Warning: $\mathsf{PGL}_n(A) \neq \mathsf{GL}_n(A)/\mathsf{GL}_1(A)$ for general $A$. In fact, viewed as functors in $A$, the LHS is the sheafification of the RHS in the fpqc topology.

*Remark* 1.8. Not every functor $\mathsf{CAlg}_k \to \mathsf{Grp}$ comes from an algebraic group. For example, any $k$-vector space $V$ defines a functor

$$\mathsf{GL}_V(-) : \mathsf{CAlg}_k \to \mathsf{Grp}$$

that sends $A$ to the group of $A$-linear automorphisms of $A \otimes V$. This functor is not represented by an algebraic group unless $V$ is finite-dimensional.

## 2. Hopf algebras

From now on, we assume $G$ is affine[3][4].

**Construction 2.1.** *An affine algebraic group $G$ is determined by its ring of functions $\mathcal{O}_G$. The maps $m : G \times G \to G$ and $e : \mathsf{pt} \to G$ correspond to homomorphisms between algebras*

$$\Delta : \mathcal{O}_G \to \mathcal{O}_G \otimes \mathcal{O}_G,\ \epsilon : \mathcal{O}_G \to k,$$

*which are called the* ***comultiplication*** *and* ***counit*** *maps of $\mathcal{O}_G$. Together with the usual multiplication and unit maps of $\mathcal{O}_G$, we obtain a* ***bialgebra*** *$(\mathcal{O}_G, \cdot, \Delta)$.*

*Note that this bialgebra is commutative but not cocommutative unless $G$ is so.*

*The inverse map $\sigma : G \to G$ corresponds to a homomorphism $S : \mathcal{O}_G \to \mathcal{O}_G$, which is called the* ***antipode*** *of $\mathcal{O}_G$. This makes $\mathcal{O}_G$ into a commutative* ***Hopf algebra****.*

*Remark* 2.2. A Hopf algebra is a bialgebra $A$ equipped with an antipode map $S : A \to A$ subject to a certain axiom. For our purposes, it is less useful to memorize this axiom than to imagine it amounts to say "$\mathsf{Spec}(A)$"[5] has an inversion map.

**Example 2.3.** For $G = \mathbb{G}_{\mathsf{a}}$, we have $\mathcal{O}_G = k[t]$ and

$$\Delta(t) = t \otimes 1 + 1 \otimes t,\ \epsilon(f) = f(0),\ S(f)(t) = f(-t).$$

**Example 2.4.** For $G = \mathbb{G}_{\mathsf{m}}$, we have $\mathcal{O}_G = k[t, t^{-1}]$ and

$$\Delta(t) = t \otimes t,\ \epsilon(f) = f(1),\ S(f)(t) = f(t^{-1}).$$

**Example 2.5.** The universal enveloping algebra $U(\mathfrak{g})$ of any Lie algebra is a Hopf algebra. The comultiplication $\Delta : U(\mathfrak{g}) \to U(\mathfrak{g}) \otimes U(\mathfrak{g})$ is determined by $\Delta(x) = x \otimes 1 + 1 \otimes x$, $x \in \mathfrak{g} \subset U(\mathfrak{g})$ and its compatibility with the multiplication. Similarly, the antipode is determined by $S(x) = -x$, $x \in \mathfrak{g}$.

The Hopf algebra $U(\mathfrak{g})$ is cocommutative but not commutative unless $\mathfrak{g}$ is abelian.

*Remark* 2.6. Using the Hopf algebra structure on $U(\mathfrak{g})$, the tensor product structure in $\mathfrak{g}\mathsf{-mod}$ can be defined as follows. Let $V_1$ and $V_2$ be left $U(\mathfrak{g})$-modules. Their tensor product $V_1 \otimes V_2$ is naturally a $U(\mathfrak{g}) \otimes U(\mathfrak{g})$-module. Restricting along $\Delta$, we can view $V_1 \otimes V_2$ as a $U(\mathfrak{g})$-module.

*Remark* 2.7. There are also interesting Hopf algebras that are neither commutative nor cocommutative. For example, *quantum algebras* are such gadgets. See [L] for a standard textbook.

---

[3] Any affine algebraic group over field of characteristic 0 is smooth.

[4] Projective algebraic groups, a.k.a., abelian varieties, are also important and play a central role in modern mathematics.

[5] Note however that this does not make sense if $A$ is not commutative.

## 3. Tangent spaces

As in the theory of Lie groups, the tangent space of an algebraic group at its unit is a Lie algebra. To desribe this, let us review the definition of tangent spaces in algebraic geometry.

**Definition 3.1.** Let $X$ be any $k$-scheme and $x \in X$ be a $k$-point, i.e., a map $x : \mathsf{Spec}(k) \to X$. The **tangent space** of $X$ at $x$, denoted by $T_xX$, is the *set* of dotted arrows making the following diagram commute:

$$\begin{tikzcd} \mathsf{Spec}(k) \arrow[r, "x"] \arrow[d] & X \\ \mathsf{Spec}(k[\epsilon]/\epsilon^2). \arrow[ur, dotted] & \end{tikzcd}$$

Here the vertical map is given by the homomorphism $k[\epsilon]/\epsilon^2 \to k$, $\epsilon \mapsto 0$.

Elements in $T_xX$ are called **tangent vectors** of $X$ at $x$.

Tangent vectors are related to *derivations*. Let us review its definition in the algebraic setting.

**Definition 3.2.** Let $A$ be a $k$-algebra and $M$ be an $A$-module. A $k$**-derivation** of $A$ into $M$ is a $k$-linear map $D : A \to M$ satisfying the **Lebniz rule**

$$D(f \cdot g) = f \cdot D(g) + g \cdot D(f).$$

Let $\mathsf{Der}_k(A, M)$ be the set of such $k$-derivations. This is naturally an $A$-module.

**Construction 3.3.** *Let $X = \mathsf{Spec}(A)$ be an affine scheme and $x : \mathsf{Spec}(k) \to X$ be given by a homomorphism $\phi : A \to k$. View $k$ as an $A$-module via this homomorphism and denote it by $k_x$. For any $D \in \mathsf{Der}_k(A, k_x)$, the map*

$$A \to k[\epsilon]/\epsilon^2,\ f \mapsto \phi(f) + D(f)\epsilon$$

*is a homomorphism and thereby gives a map $\mathsf{Spec}(k[\epsilon]/\epsilon^2) \to X$, which is an element in $T_xX$. It is easy to see this gives a bijection*

$$\mathsf{Der}_k(A, k_x) \simeq T_xX.$$

*In particular, we obtain an $A$-module structure on $T_xX$. Note that the action of $A$ factors through $A \twoheadrightarrow k_x$.*

**Construction 3.4.** *Let $f : X \to Y$ be a morphism between $k$-schemes. Let $x \in X$ and $y := f(x) \in Y$ be $k$-points. There is an obvious $k$-linear map*

$$df : T_xX \to T_yY$$

*given by composing with $f$. We call it the* ***differential*** *of $f$.*

We have the following obvious result:

**Lemma 3.5.** *Let $X$ and $Y$ be $k$-schemes. Let $x \in X$ and $y \in Y$ be $k$-points. For $\partial_1 \in T_xX$ and $\partial_2 \in T_yY$, write $\partial_1 \oplus \partial_2 \in T_{(x,y)}(X \times Y)$ for the map*

$$\mathsf{Spec}(k[\epsilon]/\epsilon^2) \xrightarrow{(\partial_1, \partial_2)} X \times Y.$$

*Then the map*

$$T_xX \times T_yY \to T_{(x,y)}(X \times Y),\ (\partial_1, \partial_2) \mapsto \partial_1 \oplus \partial_2 \tag{3.1}$$

*induces an isomorphism $T_xX \oplus T_yY = T_{(x,y)}(X \times Y)$.*

## 4. Lie Algebras and Algebraic Groups

**Notation 4.1.** *Let $G$ be an algebraic group. Define*

$$\mathsf{Lie}(G) \coloneqq T_e G.$$

**Example 4.2.** For $G = \mathsf{GL}_n$, we have $\mathsf{Lie}(\mathsf{GL}_n) \simeq \mathfrak{gl}_n$. In particular, $\mathsf{Lie}(\mathsf{GL}_n)$ is naturally a Lie algebra. We have similar results for other classical subgroups of $\mathsf{GL}_n$.

We state the following result without proof:

**Theorem 4.3.** *There is a canonical functor $\mathsf{Grp}_k \to \mathsf{Lie}_k$ sending $G$ to $\mathsf{Lie}(G)$ equipped with a natural Lie bracket, such that for $G = \mathsf{GL}_n$, the Lie bracket on $\mathsf{Lie}(\mathsf{GL}_n)$ is given by that on $\mathfrak{gl}_n$.*

*Remark* 4.4. The above functor $\mathsf{Grp}_k \to \mathsf{Lie}_k$ is unique if stated properly. See [M, Theorem 10.23].

**Warning 4.5.** *It is not true that every Lie algebra can be obtained from algebraic groups. See [Bou, I, §5, Exercise 6] for a counterexample.*

*Remark* 4.6. Consider the multiplication map $m : G \times G \to G$ and its differential $dm : \mathsf{Lie}(G \times G) \to \mathsf{Lie}(G)$. One can show that the composition

$$\mathsf{Lie}(G) \oplus \mathsf{Lie}(G) \simeq \mathsf{Lie}(G \times G) \xrightarrow{dm} \mathsf{Lie}(G)$$

sends $(\partial_1, \partial_2)$ to $\partial_1 + \partial_2$. This is the algebraic analogue of the formula $\exp(tu) \cdot \exp(tv) = \exp(t(u+v)) + O(t^2)$, $u, v \in \mathsf{Lie}(G)$ that appears in the study of Lie groups.

*Remark* 4.7. Let $G$ be an affine algebraic group and $\mathfrak{g} \coloneqq \mathsf{Lie}(G)$ be its Lie algebra. The Hopf algebras $\mathcal{O}_G$ and $U(\mathfrak{g})$ are related as follows.

Consider the dual $U(\mathfrak{g})^* \coloneqq \mathsf{Hom}_k(U(\mathfrak{g}), k)$ as a topological vector space[6]. The cocommutative Hopf algebra structure on $U(\mathfrak{g})$ induces a commutative topological Hopf algebra structure on it[7].

On the other hand, consider the maximal ideal $\mathfrak{m} \subset \mathcal{O}_G$ corresponding to the unit point $e : \mathsf{pt} \to G$. Let $\hat{\mathcal{O}}_G$ be the $\mathfrak{m}$-adic completion of $\mathcal{O}_G$. The commutative Hopf algebra structure on $\mathcal{O}_G$ induces a commutative topological Hopf algebra structure on it.

We have

$$U(\mathfrak{g})^* \simeq \hat{\mathcal{O}}_G$$

as commutative topological Hopf algebras.

**Example 4.8.** Note that the Lie algebras of $\mathbb{G}_{\mathsf{a}}$ and $\mathbb{G}_{\mathsf{m}}$ are isomorphic, it follows that $\hat{\mathcal{O}}_{\mathbb{G}_{\mathsf{a}}} \simeq k[[t]]$ and $\hat{\mathcal{O}}_{\mathbb{G}_{\mathsf{m}}} = k[[t-1]]$ are isomorphic. Up to a scaler, this isomorphism is given by

$$k[[t-1]] \to k[[t]],\ f \mapsto f(\exp(t)).$$

Note that given a power series $a_0 + a_1(t-1) + a_2(t-1)^2 + \cdots$, the series $a_0 + a_1(\exp(t) - 1) + a_2(\exp(t)-1)^2 + \cdots$ indeed converges in the $t$-adic topology.

---

[6]For a vector space $V$ equipped with the discrete topology, the dual $V^*$ is equipped with the weakest topology such that for any finite-dimensional subspace $V_0 \subset V$, the map $V^* \to V_0^*$ is continuous. Here $V_0^*$ is equipped with the discrete topology. Equivalently, we can define $V^*$ as an object in the pro-category $\mathsf{Pro}(\mathsf{Vect}_{k,\mathsf{fd}})$ of finite dimensional vector spaces.

[7]Here we must use the *complete tensor product* $U(\mathfrak{g})^* \hat{\otimes} U(\mathfrak{g})^*$ instead of the usual tensor product. By design, this is the dual of $U(\mathfrak{g}) \otimes U(\mathfrak{g})$.

## 5. Representations of Algebraic Groups

In this section, $G$ is an affine algebraic group.

**Definition 5.1.** A **representation of** $G$, or equivalently a $G$**-module** is a $k$-vector space $V$ equipped with a natural transformation $G(-) \to \mathsf{GL}_V(-)$ as functors $\mathsf{CAlg}_k \to \mathsf{Grp}$.

Let $V$ and $W$ be $G$-modules, a $G$**-linear map** $\phi : V \to W$ is a $k$-linear map such that for any $A \in \mathsf{CAlg}_k$, the following diagram commutes

$$\begin{tikzcd} G(A) \arrow[r] \arrow[d] & \mathsf{GL}_V(A) \arrow[d, "\phi\circ-"] \\ \mathsf{GL}_W(A) \arrow[r, "-\circ\phi"] & \mathsf{Hom}_A(A \otimes V, A \otimes W). \end{tikzcd}$$

Let $\mathsf{Rep}(G)$ be the category of $G$-modules.

**Example 5.2.** Any $V$ can be equipped with a trivial $G$-module structure such that the homomorphisms $G(A) \to \mathsf{GL}_V(A)$ are trivial.

**Proposition 5.3.** *The category* $\mathsf{Rep}(G)$ *is an abelian category and the forgetful functor* $\mathsf{Rep}(G) \to \mathsf{Vect}_k$ *is exact.*

*Remark* 5.4. If $V$ is finite-dimensional, then $\mathsf{GL}_V$ is represented by an algebraic group. A $G$-module structure on $V$ is just a homomorphism $G \to \mathsf{GL}_V$ between algebraic groups.

**Warning 5.5.** *Evaluate at* $k \in \mathsf{CAlg}_k$*, we obtain a group homomorphism* $G(k) \to \mathsf{GL}_V(k) = \mathsf{GL}(V)$*. But a* $G$*-module structure contains more information than such a homomorphism. This can be seen from the following exercise. (Note that for* $k = \mathbb{C}$*, the* abstract *groups* $\mathbb{G}_\mathsf{a}(\mathbb{C}) = \mathbb{C}$ *and* $\mathbb{G}_\mathsf{m}(\mathbb{C}) = \mathbb{C}^\times$ *are isomorphic via the exponential map.)*

*Exercise* 5.6. This is Homework 1, Problem 4.

(1) Find all maps between $k$-schemes $\mathbb{A}^1 \to \mathbb{A}^1 \smallsetminus 0$.
(2) Find all 1-dimensional representations of the additive group $\mathbb{G}_\mathsf{a}$.
(3) Find all maps between $k$-schemes $\mathbb{A}^1 \smallsetminus 0 \to \mathbb{A}^1 \smallsetminus 0$.
(4) Find all 1-dimensional representations of the multiplicative group $\mathbb{G}_\mathsf{m}$.

**Proposition 5.7.** *Any* $G$*-action on a vector space* $V$ *is locally finite, i.e.,* $V$ *is the union of its finite-dimensional subrepresentations.*

**Proposition 5.8.** *There is a canonical equivalence*

$$\mathsf{Rep}(G) \simeq \mathcal{O}_G\text{–}\mathsf{comod}$$

*from the category of* $G$*-modules to the category of* $\mathcal{O}_G$*-comodules. This equivalence is compatible with the forgetful functors to* $\mathsf{Vect}_k$*.*

*Remark* 5.9. The functor $\mathsf{Rep}(G) \simeq \mathcal{O}_G\text{–}\mathsf{comod}$ is constructed as follows. Let $V \in \mathsf{Rep}(G)$. Consider $A := \mathcal{O}_G$ and the homomorphism $G(A) \to \mathsf{GL}_V(A)$. The identity map $G \to G$ can be written as $\mathsf{Spec}(A) \to G$ which corresponds to an element in $G(A)$[8]. Consider the image of this element in $\mathsf{GL}_V(A)$, which is a $A$-linear map $A \otimes V \to A \otimes V$. This is the same as a $k$-linear map $V \to A \otimes V$, i.e., a $k$-linear map

$$V \to \mathcal{O}_G \otimes V.$$

One can verify this defines a $\mathcal{O}_G$-comodule structure on $V$.

[8] Warning: this is *not* the unit element of this group.

**Construction 5.10.** *There is a forgetful functor*

$$\mathsf{Rep}(G) \to \mathsf{Lie}(G)\text{–}\mathsf{mod}$$

*that can be constructed in the following equivalent ways:*

- *Let we first suppose $V \in \mathsf{Rep}(G)$ is finite-dimensional. Then we have a homomorphism between algebraic groups $G \to \mathsf{GL}_V$ which induces a homomorphism between their Lie algebras $\mathsf{Lie}(G) \to \mathsf{Lie}(\mathsf{GL}_V) = \mathfrak{gl}(V)$, i.e., a $\mathsf{Lie}(G)$-module structure on $V$.*

  *When $V$ is infinite-dimensional, the functor $\mathsf{GL}_V(-)$ is no longer represented by an algebraic group, but there is a formal method to define its Lie algebra and make the above construction work.*
- *Any $V \in \mathsf{Rep}(G)$ has an $\mathcal{O}_G$-comodule structure. By co-restricting along the map $\mathcal{O}_G \to \hat{\mathcal{O}}_G$, we obtain a (continuous) comodule structure for $\hat{\mathcal{O}}_G \simeq U(\mathfrak{g})^*$. Passing to duality, we obtain a module structure for $U(\mathfrak{g})$.*

**Lemma 5.11.** *The adjoint representation $\mathsf{Lie}(G) \in \mathsf{Lie}(G)\text{–}\mathsf{mod}$ has a canonical lift to an object in $\mathsf{Rep}(G)$. We call it the **adjoint action of** $G$ **on** $\mathsf{Lie}(G)$.*

*Remark* 5.12. On the level of $k$-points, the conjugate action of $G(k)$ on $G$ induces an (abstract) action of $G(k)$ on $\mathsf{Lie}(G)$. This can be generalized to an action of $G(A)$ on $A \otimes \mathsf{Lie}(G)$ for any $A \in \mathsf{CAlg}_k$ and thereby obtain the desired $G$-module structure on $\mathsf{Lie}(G)$.

**Proposition 5.13.** *If $G$ is connected, then the functor $\mathsf{Rep}(G) \to \mathsf{Lie}(G)\text{–}\mathsf{mod}$ is fully faithful. In particular, the $G$-invariance and $\mathsf{Lie}(G)$-invariance for a $G$-module are the same.*

**Definition 5.14.** If $G$ is connected, we say an object $V \in \mathsf{Lie}(G)\text{–}\mathsf{mod}$ is **$G$-integrable** if it is contained in the essential image of the above functor.

**Warning 5.15.** *The similar claim for derived categories is false. In other words, extensions of $G$-integrable modules are not necessarily $G$-integrable. The multiplicative group $\mathbb{G}_{\mathsf{m}}$ is a counterexample.*

## 6. Semisimple Algebraic Groups

**Theorem 6.1.** *Any semisimple Lie algebra $\mathfrak{g}$ can be realized as the Lie algebra of an algebraic group. In the category of connected algebraic groups $G$ with $\mathsf{Lie}(G) \simeq \mathfrak{g}$, there is a final object $G_{\mathsf{ad}}$ and an initial object $G_{\mathsf{sc}}$. Moreover, the homomorphisms*

$$G_{\mathsf{sc}} \to G \to G_{\mathsf{ad}}$$

*are* isogenies*, i.e., are surjective and have finite kernels.*

**Example 6.2.** For $\mathfrak{g} = \mathfrak{sl}_n$, we have $G_{\mathsf{sc}} = \mathsf{SL}_n$ and $G_{\mathsf{ad}} = \mathsf{PGL}_n$.

**Definition 6.3.** We say $G$ is **semisimple**[9] if it is connected and its Lie algebra is semisimple. For a semisimple algebraic group $G$, we say it is **of adjoint type** (resp. **simply connected**) if it is of the form $G_{\mathsf{ad}}$ (resp. $G_{\mathsf{sc}}$).

**Theorem 6.4.** *If $G_{\mathsf{sc}}$ is a simply-connected semisimple algebraic group, then any finite-dimensional $\mathfrak{g}$-module is $G_{\mathsf{sc}}$-integrable, i.e.,*

$$\mathsf{Rep}(G_{\mathsf{sc}})_{\mathsf{fd}} = \mathfrak{g}\text{–}\mathsf{mod}_{\mathsf{fd}}.$$

**Warning 6.5.** *The similar claim for infinite-dimensional representation is false. This can be seen from the following exercise.*

[9] This is an ad hoc definition that only is only correct under our assumptions on $k$.

*Exercise* 6.6. This is Homework 1, Problem 5. Let $G$ be any semisimple algebraic group with Lie algbra $\mathfrak{g}$. Prove: any Verma module of $\mathfrak{g}$ is not $G$-integrable.

**Theorem 6.7.** *Let $G$ be any semisimple algebraic group with Lie algebra $\mathfrak{g}$. Then the abelian categories $\mathsf{Rep}(G)$ and $\mathfrak{g}\text{–}\mathsf{mod}_{\mathsf{fd}}$ are semisimple*[10]*. Simple objects in $\mathsf{Rep}(G)$ are finite-dimensional, and an object $V \in \mathsf{Rep}(G)$ is simple iff it is an simple object in $\mathfrak{g}\text{–}\mathsf{mod}$.*

## References


[B] Borel, Armand. Linear algebraic groups. Vol. 126. Springer Science & Business Media, 2012.

[Bou] Bourbaki, Nicolas. Groupes et Algebres de Lie: Elements de Mathematique. Hermann, 1968.

[H] Humphreys, James E. Linear algebraic groups. Vol. 21. Springer Science & Business Media, 2012.

[L] Lusztig, George. Introduction to quantum groups. Springer Science & Business Media, 2010.

[M] Milne, James S. Algebraic groups: the theory of group schemes of finite type over a field. Vol. 170. Cambridge University Press, 2017.

[Sp] Springer, Tonny Albert. "Linear algebraic groups." In Algebraic Geometry IV: Linear Algebraic Groups Invariant Theory, pp. 1-121. Berlin, Heidelberg: Springer Berlin Heidelberg, 1998.


[10] I.e., any object can be written as a direct sum of simple objects.