LECTURE 16

In this lecture, we introduce t-structures on stable ∞-categories.

1. Definitions

1.1. Let C be a stable ∞-category. A t-structure on C is just a t-structure on the triangulated category $h\mathsf{C}$.

Definition 1.2. *Let C be a stable ∞-category. A **t-structure** on C is defined to be a pair of full sub-∞-categories $(\mathsf{C}^{\leq 0}, \mathsf{C}^{\geq 0})$ of C such that when we write*

$$\mathsf{C}^{\leq n} := \mathsf{C}^{\leq 0}[-n], \ \mathsf{C}^{\geq n} := \mathsf{C}^{\geq 0}[-n]$$

we have:

- (0) *Both $\mathsf{C}^{\leq 0}$ and $\mathsf{C}^{\geq 0}$ are stable under isomorphisms in C.*
- (1) *For $X \in \mathsf{C}^{\leq 0}$ and $Y \in \mathsf{C}^{\geq 1}$, we have $\mathsf{Maps}_{\mathsf{C}}(X, Y) \simeq \{0\}$.*
- (2) *We have inclusions $\mathsf{C}^{\leq -1} \subseteq \mathsf{C}^{\leq 0}$ and $\mathsf{C}^{\geq 1} \subseteq \mathsf{C}^{\geq 0}$.*
- (3) *For any $X \in \mathsf{C}$, there exists a fiber-cofiber sequence $X' \to X \to X''$ such that $X' \in \mathsf{C}^{\leq 0}$ and $X'' \in \mathsf{C}^{\geq 1}$.*

Warning 1.3. *We use the **cohomological convention**. To compare with notations in the homological convention, let*

$$\mathsf{C}_{\leq n} := \mathsf{C}^{\geq -n}, \ \mathsf{C}_{\geq n} := \mathsf{C}^{\leq -n}$$

Exercise 1.4. *The assignment*

$$(\mathsf{C}^{\leq 0}, \mathsf{C}^{\geq 0}) \mapsto (h\mathsf{C}^{\leq 0}, h\mathsf{C}^{\geq 0})$$

gives a bijection between t-structures on C with t-structures on the triangulated category $h\mathsf{C}$.

Exercise 1.5. *The assignment*

$$(\mathsf{C}^{\leq 0}, \mathsf{C}^{\geq 0}) \mapsto ((\mathsf{C}^{\geq 0})^{\mathsf{op}}, (\mathsf{C}^{\leq 0})^{\mathsf{op}})$$

gives a bijection between t-structures on C with t-structures on C^{op}.

Exercise 1.6. *Show that for $m < n$, $\mathsf{C}^{\leq m} \cap \mathsf{C}^{\geq n} \simeq \{0\}$.*

Lemma 1.7. *Let $n \geq 0$. For any $X \in \mathsf{C}^{\leq 0}$ and $Y \in \mathsf{C}^{\geq -n}$, the mapping space $\mathsf{Maps}_{\mathsf{C}}(X, Y)$ is a homotopy n-type.*

Sketch. First note that the connected components of $\mathsf{Maps}_{\mathsf{C}}(X, Y)$ are weakly homotopy equivalent to each other because it is the loop space of $\mathsf{Maps}_{\mathsf{C}}(X, \Sigma Y)$. Hence we only need to show $\pi_m \mathsf{Maps}_{\mathsf{C}}(X, Y) \simeq 0$ for $m > n$ and the base point $0 \in \mathsf{Maps}_{\mathsf{C}}(X, Y)$. This follows from observation that $\Omega^{n+1}\mathsf{Maps}_{\mathsf{C}}(X, Y) \simeq \mathsf{Maps}_{\mathsf{C}}(X, \Omega^{n+1}Y)$ is weakly contractible because $\Omega^{n+1}Y \in \mathsf{C}^{\geq 1}$. $\square$

Definition 1.8. *Let* C *be a stable* ∞-*category equipped with a t-structure. The* **heart** *of this t-structure is defined to be*

$$\mathsf{C}^{\heartsuit} := \mathsf{C}^{\leq 0} \cap \mathsf{C}^{\geq 0}.$$

Theorem 1.9. *The* ∞-*category* $\mathsf{C}^{\heartsuit}$ *is an ordinary abelian category. Moreover,* $0 \to X'' \to X \to X' \to 0$ *is a short exact sequence in* $\mathsf{C}^{\heartsuit}$ *iff* $X'' \to X \to X'$ *is a fiber-cofiber sequence of* C *contained in* $\mathsf{C}^{\heartsuit}$.

Sketch. By Lemma 1.7, $\mathsf{C}^{\heartsuit}$ is an ordinary category. It follows it can be identified with $\mathsf{hC}^{\leq 0} \cap \mathsf{hC}^{\geq 0}$, which is the heart of the triangulated category hC. Now the claims follow from the corresponding well-known claims for triangulated categories. $\square$

Warning 1.10. *A t-structure is not determined by its heart. Most information in a stable* ∞-*category cannot be recovered from the heart. For instance, for* $X, Y \in \mathsf{C}^{\heartsuit}$, *the groups* $\mathsf{Ext}^{\bullet}_{\mathsf{C}}(X, Y)$ *and* $\mathsf{Ext}^{\bullet}_{\mathsf{C}^{\heartsuit}}(X, Y)$ *are not isomorphic in general.*

Definition 1.11. *Let* C *and* D *be stable* ∞-*categories equipped with t-structures. For an exact functor* $F : \mathsf{C} \to \mathsf{D}$, *we say*
- *the functor* F *is* **left t-exact** *if it sends* $\mathsf{C}^{\geq 0}$ *into* $\mathsf{D}^{\geq 0}$
- *the functor* F *is* **right t-exact** *if it sends* $\mathsf{C}^{\leq 0}$ *into* $\mathsf{D}^{\leq 0}$
- *the functor* F *is* **t-exact** *if it is both left t-exact and right t-exact.*

Exercise 1.12. *Show that the left adjoint of a left t-exact functor is right t-exact.*

Exercise 1.13. *Construct an exact functor* $F : \mathsf{C} \to \mathsf{D}$ *that preserves the hearts but is not t-exact.*

2. Cohomologies

Proposition-Definition 2.1. *Let* C *be a stable* ∞-*category equipped with a t-structure. For any* n, *the inclusion functor* $\mathsf{C}^{\leq n} \to \mathsf{C}$ *admits a right adjoint* $\tau^{\leq n}$, *and the inclusion functor* $\mathsf{C}^{\geq n} \to \mathsf{C}$ *admits a left adjoint* $\tau^{\geq n}$. *These functors are called the* **truncation functors** *for the t-structure.*

Remark 2.2. *One can memorize the above handedness as* $\mathsf{C}^{\leq 0} \rightleftarrows \mathsf{C} \rightleftarrows \mathsf{C}^{\geq 1}$.

Exercise 2.3. *Prove the above proposition by verifying the following claim. Let* $X \in \mathsf{C}$ *and* $X' \to X \to X''$ *be any fiber-cofiber sequence with* $X' \in \mathsf{C}^{\leq 0}$ *and* $X'' \in \mathsf{C}^{\geq 1}$, *then*

(1) *For any* $Y \in \mathsf{C}^{\leq 0}$, *the morphism* $X' \to X$ *induces equivalences*

$$\mathsf{Maps}_{\mathsf{C}}(Y, X') \xrightarrow{\simeq} \mathsf{Maps}_{\mathsf{C}}(Y, X).$$

(2) *For any* $Z \in \mathsf{C}^{\geq 1}$, *the morphism* $X \to X''$ *induces equivalences*

$$\mathsf{Maps}_{\mathsf{C}}(X, Z) \xrightarrow{\simeq} \mathsf{Maps}_{\mathsf{C}}(X'', Z).$$

Deduce that $X' \simeq \tau^{\leq 0} X$ *and* $X'' \simeq \tau^{\geq 1} X$.

Exercise 2.4. *We have Cartesian squares*

In particular, $\mathsf{C}^{\leq 0}$ *and* $\mathsf{C}^{\geq 0}$ *determine each other.*

Exercise 2.5. *The full sub-∞-category $\mathsf{C}^{\leq 0} \subseteq \mathsf{C}$ is stable under colimits, while $\mathsf{C}^{\leq 0} \subseteq \mathsf{C}$ is stable under limits.*

Exercise 2.6. *A limit in $\mathsf{C}^{\leq 0}$ is isomorphic to the $\tau^{\leq 0}$-truncation of the corresponding limit in C. Dually, a colimit in $\mathsf{C}^{\geq 0}$ is isomorphic to the $\tau^{\geq 0}$-truncation of the corresponding colimit in C.*

2.7. The following exercises say that for a fiber-cofiber sequence $X' \to X \to X''$, *amplitude estimations* of two terms give an estimation for the third one.

Exercise 2.8. *Let $X' \to X \to X''$ be a fiber-cofiber sequence.*
  (1) *If $X', X'' \in \mathsf{C}^{\leq 0}$, then $X \in \mathsf{C}^{\leq 0}$.*
  (2) *If $X \in \mathsf{C}^{\leq 0}$ and $X'' \in \mathsf{C}^{\leq -1}$, then $X' \in \mathsf{C}^{\leq 0}$.*
  (3) *If $X \in \mathsf{C}^{\leq 0}$ and $X' \in \mathsf{C}^{\leq 1}$, then $X'' \in \mathsf{C}^{\leq 0}$.*
In particular, the full sub-∞-categories $\mathsf{C}^{\leq 0} \subseteq \mathsf{C} \supseteq \mathsf{C}^{\geq 0}$ are stable under extensions.

Exercise 2.9. *Give examples to show the above estimations are optimal.*

Exercise 2.10. *The truncation functors $\tau_{\leq \bullet}$ and $\tau_{\geq \bullet}$ commute with each other.*

Remark 2.11. *The precise meaning of the above exercise consists of the following. For m and n, we have*

  • $\tau_{\geq m} \circ \tau_{\geq n} \simeq \tau_{\geq \max\{m,n\}}, \quad \tau_{\leq m} \circ \tau_{\leq n} \simeq \tau_{\leq \max\{m,n\}}.$
  • *The commutative square*

$$
\begin{array}{ccc}
\mathsf{C}^{\leq m} \cap \mathsf{C}^{\geq n} & \xrightarrow{\subseteq} & \mathsf{C}^{\leq m} \\
\downarrow{\scriptstyle \subseteq} & & \downarrow{\scriptstyle \subseteq} \\
\mathsf{C}^{\geq n} & \xrightarrow{\subseteq} & \mathsf{C}
\end{array}
$$

  is left adjointable along the horizontal direction, and the induced commutative square

$$
\begin{array}{ccc}
\mathsf{C}^{\leq m} \cap \mathsf{C}^{\geq n} & \xleftarrow{\tau^{\geq n}} & \mathsf{C}^{\leq m} \\
\downarrow{\scriptstyle \subseteq} & & \downarrow{\scriptstyle \subseteq} \\
\mathsf{C}^{\geq n} & \xleftarrow{\tau^{\geq n}} & \mathsf{C}
\end{array}
$$

  is right adjointable along the vertical direction, i.e., induces a commutative square

$$
\begin{array}{ccc}
\mathsf{C}^{\leq m} \cap \mathsf{C}^{\geq n} & \xleftarrow{\tau^{\geq n}} & \mathsf{C}^{\leq m} \\
\uparrow{\scriptstyle \tau^{\leq m}} & & \uparrow{\scriptstyle \tau^{\leq m}} \\
\mathsf{C}^{\geq n} & \xleftarrow{\tau^{\geq n}} & \mathsf{C}
\end{array}
$$

Definition 2.12. *Let C be a stable ∞-category equipped with a t-structure. Consider the functor*

$$
\mathsf{H}^n : \mathsf{C} \xrightarrow{\tau^{\leq n} \circ \tau^{\geq n}} \mathsf{C}^{\leq n} \cap \mathsf{C}^{\geq n} \xrightarrow{[-n]} \mathsf{C}^{\heartsuit}.
$$

*For $X \in \mathsf{C}$, we call $\mathsf{H}^n(X) \in \mathsf{C}^{\heartsuit}$ the n-**th cohomology of** X.*

Warning 2.13. *It may happen that $\mathsf{H}^n(X) \simeq 0$ while X is not isomorphic to 0. For instance, $\mathsf{C}^{\leq 0} := \mathsf{C}$ and $\mathsf{C}^{\geq 0} := \{0\}$ give a t-structure with $\mathsf{C}^{\heartsuit} \simeq \{0\}$. There are lots of interesting examples in*

- non-regular *algebraic geometry*
- infinite type *algebraic geometry*
- infinite dimensional *representation theory*
- *...*

2.14.   The following result follows from the corresponding well-known result for triangulated categories.

Proposition 2.15. *Let* C *be a stable* ∞-*category equipped with a t-structure. For any fiber-cofiber sequence* $X' \to X \to X''$, *we have a long exact sequence in* $\mathsf{C}^\heartsuit$

$$\cdots \to \mathsf{H}^n(X') \to \mathsf{H}^n(X) \to \mathsf{H}^n(X'') \xrightarrow{\delta} \mathsf{H}^{n+1}(X') \to \cdots,$$

where the connecting morphism δ *is induced by the morphism* $X'' \to X'[1]$.

2.16.   The following result relates t-exact functors with exact functors between the hearts.

Exercise 2.17. *Let* $F : \mathsf{C} \to \mathsf{D}$ *be a left t-exact functor. Show that the composition*

$$\mathsf{H}^0 F : \mathsf{C}^\heartsuit \to \mathsf{C}^{\geq 0} \xrightarrow{F} \mathsf{D}^{\geq 0} \xrightarrow{\tau^{\leq 0}} \mathsf{D}^\heartsuit$$

is left exact.

Warning 2.18. *As in Warning 1.10, even a t-exact functor* F *cannot be recovered from* $\mathsf{H}^0 F$.

Remark 2.19. *Next time, we will define various versions of* derived ∞-categories *of an abelian categories* A, *which are equipped with t-structures whose hearts can be identified with* A. *By definition, all information about these derived* ∞-*categories can be recovered from* A.

Under certain assumptions, a left/right t-exact functor F *out of these derived* ∞-*categories can be identified with the right/left derived functor of* $\mathsf{H}^0 F$.

3. Bounded, separated and complete

Definition 3.1. *Let* C *be a stable* ∞-*category equipped with a t-structure. For an object* $X \in \mathsf{C}$,

- *we say* X *is* **connective** *if* $X \in \mathsf{C}^{\leq 0}$;
- *we say* X *is* **coconnective** *if* $X \in \mathsf{C}^{\geq 0}$;
- *we say* X *is* n-**connective** *if* $X \in \mathsf{C}^{\leq -n}$;
- *we say* X *is* n-**coconnective** *if* $X \in \mathsf{C}^{\geq -n}$;
- *we say* X *is* **eventually connective**, *or* **right bounded** *if*

$$X \in \mathsf{C}^- := \bigcup \mathsf{C}^{\leq n}$$

- *we say* X *is* **eventually coconnective**, *or* **left bounded** *if*

$$X \in \mathsf{C}^+ := \bigcup \mathsf{C}^{\geq n}$$

- *we say* X *is* **bounded** *if* X *is both left bounded and right bounded:*

$$X \in \mathsf{C}^\mathsf{b} := \mathsf{C}^+ \bigcap \mathsf{C}^-.$$

Warning 3.2. *As in Warning 2.13, the above properties cannot be tested via the cohomologies.*

Exercise 3.3. *Show that if* X *is left bounded, then* $X \in \mathsf{C}^{\geq 0}$ *iff* $\mathsf{H}^i(X) \simeq 0$ *for* $i < 0$.

Definition 3.4. *Let* C *be a stable* ∞-*category equipped with a t-structure.*

- *We say* C *is* **right bounded** *if* $C \simeq C^-$.
- *We say* C *is* **left bounded** *if* $C \simeq C^+$.
- *We say* C *is* **bounded** *if* $C \simeq C^b$.
- *We say* C *is* **right separated** *if*

$$C^{\geq \infty} := \bigcap_n C^{\geq n} \simeq \{0\}.$$

- *We say* C *is* **left separated** *if*

$$C^{\leq -\infty} := \bigcap_n C^{\leq n} \simeq \{0\}.$$

Definition 3.5. *Let* C *be a stable* ∞-*category equipped with a t-structure. The* **left completion** $\widehat{C}$ *of* C *is defined to be the limit of the following diagram*

$$\cdots \xrightarrow{\tau} C^{\geq -2} \xrightarrow{\tau} C^{\geq -1} \xrightarrow{\tau} C^{\geq 0} \xrightarrow{\tau} \cdots.$$

We say C *is* **left complete** *if the functor* $C \to \widehat{C}$ *is an equivalence.*

Remark 3.6. *Dually, the* **right completion** *is defined to be the limit of*

$$\cdots \xrightarrow{\tau} C^{\leq 2} \xrightarrow{\tau} C^{\leq 1} \xrightarrow{\tau} C^{\leq 0} \xrightarrow{\tau} \cdots.$$

Most unbounded t-structures appearing naturally are right complete, hence people do not give a notation to the right completion. Note however that the right completion can be identified with $(\widehat{C^{op}})^{op}$.

Exercise 3.7. *Show that the left completion* $\widehat{C}$ *is stable and the functor* $C \to \widehat{C}$ *is exact.*

Exercise 3.8. *Show that the left completion* $\widehat{C}$ *admits an essential unique t-structure such that* $C^{\geq 0} \xrightarrow{\simeq} (\widehat{C})^{\geq 0}$.

Exercise 3.9. *Show that* C *is left separated iff the functor* $C \to \widehat{C}$ *detects zero objects. In particular,* C *is left separated if it is left complete.*

Warning 3.10. *A left separated t-structure may not be left complete. For example, any left bounded t-structure is left separated, but is almost never left complete.*

Exercise 3.11. *Suppose* C *admits countable products and* $C^{\leq 0} \subseteq C$ *is stable under countable products[1]. Then* C *is left separated iff it is left complete.*

Exercise 3.12. *Show that* C *is left complete iff it is* **Postnikov complete***. The latter means*

- *Every object* $X \in C$ *is the limit of its* **Postnikov tower***:*

$$X \simeq \lim \left[\cdots \to \tau^{\geq n-1} X \to \tau^{\geq n} X \to \tau^{\geq n+1} X \to \cdots \right]$$

- *Any* **Postnikov tower** *in* C *converges. In other words, any collection* $X_n \in C^{\geq n}$ *equipped with isomorphisms* $\tau^{\geq n} X_{n-1} \xrightarrow{\simeq} X_n$ *is the Postnikov tower of*

$$X := \lim \left[\cdots \to X_{n-1} \to X_n \to X_{n+1} \to \cdots \right] \in C$$

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[1]These conditions are often referred as: taking countable products is right t-exact.

4. t-STRUCTURES ON PRESENTABLE STABLE ∞-CATEGORIES

Proposition-Definition 4.1 (HA.1.4.4.13)**.** *Let* C *be a presentable stable* ∞-*category equipped with a t-structure. The following conditions are equivalent:*

- *The* ∞-*category* $\mathsf{C}^{\leq 0}$ *is presentable.*
- *The* ∞-*category* $\mathsf{C}^{\leq 0}$ *is accessible.*
- *The* ∞-*category* $\mathsf{C}^{\geq 0}$ *is presentable.*
- *The* ∞-*category* $\mathsf{C}^{\geq 0}$ *is accessible.*
- *The composition* $\mathsf{C} \xrightarrow{\tau_{\leq 0}} \mathsf{C}^{\leq 0} \to \mathsf{C}$ *is accessible.*
- *The composition* $\mathsf{C} \xrightarrow{\tau_{\geq 0}} \mathsf{C}^{\geq 0} \to \mathsf{C}$ *is accessible.*

We say a t-structure on C *is* ***accessible*** *if it satisfies the above conditions.*

Proposition 4.2 (HA.1.4.4.11)**.** *Let* C *be a presentable stable* ∞-*category. A full sub-*∞-*category* $\mathsf{C}' \subseteq \mathsf{C}$ *determines an accessible t-structure with* $\mathsf{C}^{\leq 0} \simeq \mathsf{C}'$ *iff*

- C' *is accessible*
- $\mathsf{C}' \subseteq \mathsf{C}$ *is closed under colimits and extensions.*

Remark 4.3. *In practice, one can apply the above proposition to the smallest full sub-*∞-*category* C' *generated by a* small *collection of objects under small colimits and extensions. Such* C' *is always accessible.*

Definition 4.4. *Let* C *be a presentable stable* ∞-*category. We say a t-structure on* C *is* ***compatible with filtered colimits*** *if taking filtered colimits is t-exact.*

Exercise 4.5. *A t-structure on* C *is compatible with filtered colimits iff taking small coproducts is t-exact.*

Definition 4.6. *Let* C *be a presentable stable* ∞-*category. We say a t-structure on* C *is* ***compactly generated*** *if* $\mathsf{C}^{\leq 0}$ *is compactly generated.*

Exercise 4.7. *Let* C *be a presentable stable* ∞-*category equipped with a compactly generated t-structure.*

(1) *Show that the t-structure is right complete and compatible with filtered colimits.*
(2) *Show that* C *is compactly generated, and compact objects in* C *are exactly given by* $X[n]$ *such that* X *is compact object in* $\mathsf{C}^{\leq 0}$.

Exercise 4.8. *Let* C_0 *be a small stable* ∞-*category equipped with a bounded t-structure. Show that* $\mathsf{C} := \mathsf{Ind}(\mathsf{C}_0)$ *has a compactly generated t-structure with* $\mathsf{C}^{\leq 0} := \mathsf{Ind}(\mathsf{C}_0^{\leq 0})$.

Warning 4.9. *Compact objects in* C *may not be stable under truncations. Hence not all compactly generated t-structures come from ind-completion. There are lots of interesting examples in the settings listed in Warning 2.13.*

Remark 4.10. *In practice, most t-structures on presentable stable* ∞-*categories are right complete and compatible with filtered colimits, or even compactly generated.*

5. t-STRUCTURE ON Sptr

Proposition-Construction 5.1 (HA.1.4.3.4)**.** *Let* C *be a pointed presentable* ∞-*category. We have an accessible t-structure on* $\mathsf{Sptr}(\mathsf{C})$ *given by the following.*

- *Let* $\mathsf{Sptr}(\mathsf{C})^{\geq 1}$ *be the full sub-*∞-*category of objects* X *such that* $\Omega^{\infty}(X) \simeq 0$.

- Let $\mathsf{Sptr}(\mathsf{C})^{\leq 0}$ be the full sub-∞-category generated by the essential image of the functor Σ^{∞} under extensions and small colimits.

Exercise 5.2. *Show that* $\mathsf{Sptr}(\mathsf{Sptr}(\mathsf{C})^{\leq 0}) \to \mathsf{Sptr}(\mathsf{C})$ *is a t-exact equivalence.*

Proposition 5.3 (HA.1.4.3.6)**.** *Let* Sptr *be equipped with the above t-structure. Then*

(1) *This t-structure is compactly generated[2] and left complete.*
(2) *The Eilenberg–Maclane spectrum functor* $A \mapsto \mathbb{H}A$ *gives an equivalence* $\mathsf{Ab} \xrightarrow{\simeq} \mathsf{Sptr}^{\heartsuit}$.

Exercise 5.4. *Show that the cohomology functor* $\mathsf{H}^i : \mathsf{Sptr} \to \mathsf{Sptr}^{\heartsuit}$ *can be identified with* π_{-i} *via the equivalence* $\mathsf{Ab} \xrightarrow{\simeq} \mathsf{Sptr}^{\heartsuit}$.

Warning 5.5. *Let* A_1 *and* A_2 *be abelian groups. In general, the extension groups* $\mathsf{Ext}^i_{\mathsf{Sptr}}(\mathbb{H}A_1, \mathbb{H}A_2)$ *and* $\mathsf{Ext}^i(A_1, A_2)$ *are not isomorphic. For instance, the graded ring* $\mathsf{Ext}^{\bullet}_{\mathsf{Sptr}}(\mathbb{H}\mathbb{F}_q, \mathbb{H}\mathbb{F}_q)$ *is the **mod p Steenrod algebra**.*

5.6.   Next time, we will construct a natural homomorphism

$$\mathsf{Ext}^i(A_1, A_2) \to \mathsf{Ext}^i_{\mathsf{Sptr}}(\mathbb{H}A_1, \mathbb{H}A_2)$$

which should be viewed as derived direct images along $\mathsf{Spec}\,\mathbb{Z} \to \mathsf{Spec}\,\mathbb{S}$.

APPENDIX A. PRESTABLE ∞-CATEGORIES

A.1.   Let C be a stable ∞-category equipped with a t-structure. Sometimes it is more convenient to study $\mathsf{C}^{\leq 0}$ rather then C. It is possible to find several axioms that characterize ∞-categories of the form $\mathsf{C}^{\leq 0}$. Such ∞-categories are called **prestable ∞-categories**.

A.2. **Suggested readings.** SAG.C.

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[2]Compact generation is not proved in HA, but it follows from the fact that Spc_* is compactly generated.