Numeric and Optimize Method

1. Tensor Perspective of Matrix

index invisible formulation:

- matrix:$M_{\text{col row}}$
  - $\left(M_{ij}\right)^{T} = M_{ji}$
  - $\left(M_{ij}\right)^{-1} = M_{ji}$
- vector:V_{any}

that neighbor indices from different symbols will go contraction.

2. Matrix Decomposition/ Factorization

reference:

- wiki

2.1. LU

reference:

- wiki

fomulation:

$$A = LU$$

- L: lower triangular
- U: upper triangular

2.1.1. Partial Piviting

+fomulation:

$$PA = LU$$

- P: permutation that reorder rows

feature:

- numerically stable

2.1.2. Full Piviting

+fomulation:

$$PAQ = LU$$

- Q: permutation that reorder columns

2.1.3. LDU

+fomulation:

$$A = LDU$$

- D: diagonal
- L,U: +unitraingular

2.2. Cholesky

formulation:

$$A = UU^T$$

- A: symmetric, positive (semi-)defined
- U: upper traingular, (semi-)positive diagonal entries

2.2.1. LDL/LDLT

+formulation:

$$A = LDL^T$$

- L: lower unitraingular
- D: diagonal

feature:

- +square-root-free

2.3. QR

formulation:

$$A = QR$$

- Q: orthogonal
- R: upper traingular

compute:

2.3.1. Gram-Schmidt Process

feature:

- low numeric stability
- easy implementation

2.3.2. Householder Reflections

feature

- better numeric stability than Gram-Schmidt Process
- bandwidth heavy
- not parallelizable

2.3.3. Givens Rotations

feature:

- sparse
- parallelizable