Numeric and Optimize Method

1. Matrix Decomposition/ Factorization

reference:

- wiki

1.1. LU

reference:

- wiki

fomulation:

$$A = LU \tag{1}$$

- L: lower triangular
- U: upper triangular

1.1.1. Partial Piviting

+fomulation:

$$PA = LU \tag{2}$$

- P: permutation that reorder rows

feature:

- numerically stable

1.1.2. Full Piviting

+fomulation:

$$PAQ = LU \quad <3>$$

- Q: permutation that reorder columns

1.1.3. LDU

+fomulation:

$$A = LDU \quad <4>$$

- D: diagonal
- L,U: +unitraingular

1.2. Cholesky

formulation:

$$A = UU^T \quad <5>$$

- A: symmetric, positive (semi-)defined
- U: upper traingular, (semi-)positive diagonal entries

1.2.1. LDL/LDLT

+formulation:

$$A = LDL^T \quad <6>$$

- L: lower unitraingular
- D: diagonal

feature:

- +square-root-free

1.3. QR

formulation:

$$A = QR \tag{7}$$

- Q: orthogonal
- R: upper traingular

compute:

1.3.1. Gram-Schmidt Process

feature:

- low numeric stability
- easy implementation

1.3.2. Householder Reflections

feature

- better numeric stability than Gram-Schmidt Process
- bandwidth heavy
- not parallelizable

1.3.3. Givens Rotations

feature:

- sparse
- parallelizable

2. Equaltion

2.1. Linear

formulation:

$$A\boldsymbol{x} + \boldsymbol{b} = 0 \tag{8}$$

transform:
if rank(A) < $\boldsymbol{b}$

$$A^{\top}A\boldsymbol{x} + A^{\top}\boldsymbol{b} = 0 \tag{9}$$

2.1.1. Conjugate Gradient

reference:

- wiki
- cornell.edu

3. Optimization

3.1. Quadral

formulation:

$$\min_{x} \| A\boldsymbol{x} - \boldsymbol{b} \|_2 \tag{10}$$

3.1.1. Newton

3.1.2. Quasi-Newton

reference:

- wiki

3.1.2.1. BFGS

reference:

- wiki

3.1.2.1.1. L-BFGS

reference:

- wiki

3.1.2.2. Compact Representation

reference:

- wiki

4. Constraint

4.1. Single Linear Equal

formulation:

$$\begin{gathered} \boldsymbol{x} = N\boldsymbol{\lambda}, \\ \text{rank}(N) < \text{rank}(\boldsymbol{x}) \end{gathered} \tag{11}$$

4.1.1. Qualdral Optimization (Linear Least Squares)

reference:

- wiki

transform:

$$\begin{gathered} \text{<10>, <11>} \Rightarrow \\ \min_{\boldsymbol{\lambda}} \| AN\boldsymbol{\lambda} - \boldsymbol{b} \|_2 \end{gathered} \tag{12}$$

$$\min_{\boldsymbol{x}'} \| A'\boldsymbol{x}' - \boldsymbol{b} \|_2 \quad <13>$$

$$2A^{\top}(A'\boldsymbol{x}' - \boldsymbol{b}) = 0 \quad <14>$$

4.1.2. Rank $n-1$ formulation:

$$\boldsymbol{n}\boldsymbol{x} + m = 0 \quad <15>$$

4.1.2.1. Qualdral Optimization transform:

$$<10>, <15> \Rightarrow \quad \lambda\boldsymbol{n} + 2A^{\top}(A\boldsymbol{x} - \boldsymbol{b}) = 0 \quad <16>$$

$$\lambda\boldsymbol{n} + 2A^{\top}A\boldsymbol{x} - 2A^{\top}\boldsymbol{b} = 0 \quad <17>$$

$$<15>, <17> \Leftrightarrow \quad \mathrm{diag}(2A^{\top}A, \boldsymbol{n})(\boldsymbol{x} \oplus \lambda) - 2A^{\top}\boldsymbol{b} = 0 \quad <18>$$

$$A'\boldsymbol{x}' + \boldsymbol{b}' = 0 \quad <19>$$