Interpolation

- let w_s be the interpolation weight on sample s.
- let $\text{interpolate}(x_i)_s = w_s$ be the interpolation method
- let $y(x_i)_j = Y_j^s w_s$ be the interpolated value

1. Bilinear

naive form:

$$I(x_0, x_1) = (x_0 x_1, x_0(1 - x_1), (1 - x_0)x_1, (1 - x_0)(1 - x_1))$$

tensor form:

$$I(x_i)_s = A_s^{ii} x_i x_i + B_s^i x_i + C_s$$

- $A_s^{ii} = \begin{pmatrix} Q^{ii} & -Q^{ii} & -Q^{ii} & Q^{ii} \end{pmatrix}$
  - $Q^{ii} = \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix}$
- $B_s^i = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ -1 & -1 \end{pmatrix}$
- $C_s = \begin{pmatrix} 0 & 0 & 0 & 1 \end{pmatrix}$

or more compact:

$$I(x_i)_s = A_s^{i \times i + i + 1} \left(x_{i \times i}^2 \oplus x_i \oplus 1 \right) = A_s^{i'} \ \text{poly}(x, 2)_{i'}$$

- $$A = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & -\frac{1}{2} & 0 & 1 & 0 & 0 \\ 0 & -\frac{1}{2} & -\frac{1}{2} & 0 & 0 & 1 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & -1 & -1 & 1 \end{pmatrix}$$

1.1. Inverse Interpolation

write the equation as:

$$y_j = Y_j^s \ \mathrm{interpolate}(x_i)_s = Y_j^s A_s^{ii} x_i x_i + Y_j^s B_s^i x_i + Y_j^s C_s$$

let:

$$f(x_i)_j = {A'}_j^{ii} x_i x_i + {B'}_j^i x_i + C'_j = 0$$

the derivative:

$$\frac{\partial f(x_i)_j}{\partial x_i} = 2{A'}_j^{ii} x_i + {B'}_j^i$$

then we can solve x_i by newton method.

we can also write it as:

$$f'(x_i)_i = x_i + \left({A'}_j^i x_i + {B'}_j^i\right)^{-1} C'_j = 0$$

the derivative:

$$\frac{\partial f'(x_i)}{\partial x_i} = I_i - \left({A'}_i^j x^i + {B'}_i^j\right)^{-1} {A'}_j^i \left({A'}_i^j x^i + {B'}_i^j\right)^{-1} C'_j$$

this is called modified-newton method.