

# INFORMATION

“Information is that which allows you to make a  
correct prediction with accuracy better than chance.”

Adami, Christoph. “What Is Information?” *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, vol. 374, no. 2063, Mar. 2016, p. 20150230, <https://doi.org/10.1098/rsta.2015.0230>.

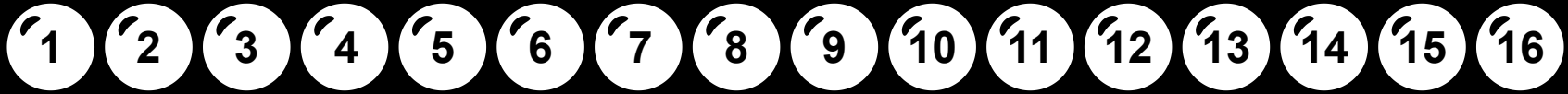
“Information is that which allows you to make a correct prediction with accuracy better than chance.”

Adami, Christoph. “What Is Information?” *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, vol. 374, no. 2063, Mar. 2016, p. 20150230, <https://doi.org/10.1098/rsta.2015.0230>.

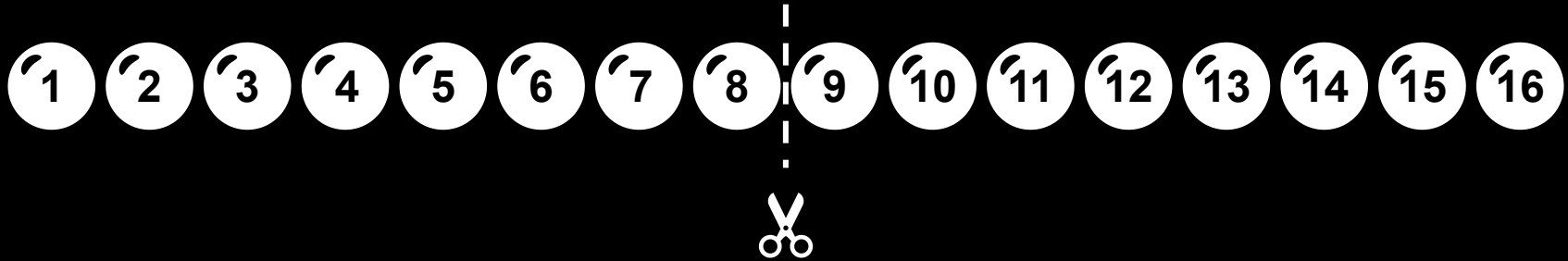
defining and measuring information

guess the number am I thinking of!

what is the most efficient approach?



is it > 8?

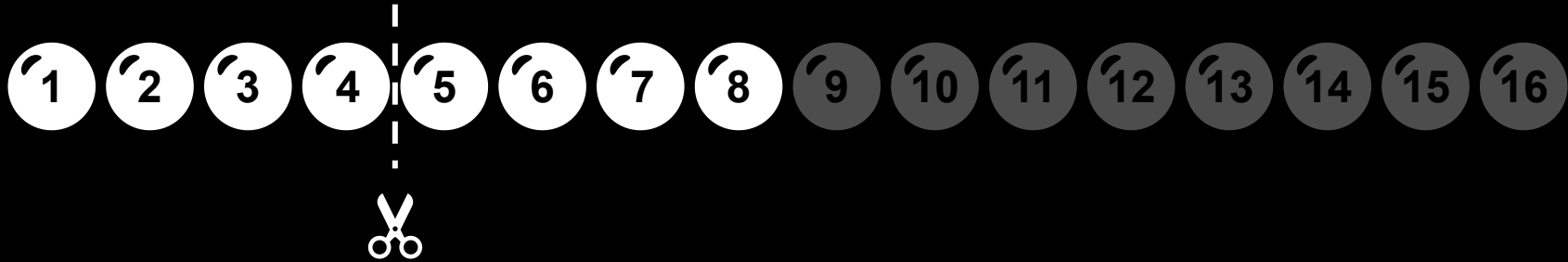


is it > 8? ✗



is it > 8? ✗

is it > 4?





is it > 8? ✗

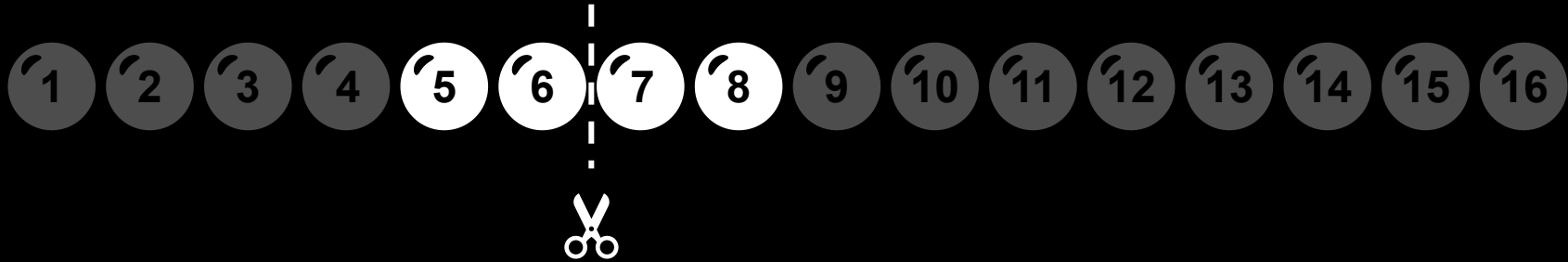
is it > 4? ✓



is it > 8? ✗

is it > 4? ✓

is it > 6?



is it > 8? ✗

is it > 4? ✓

is it > 6? ✓

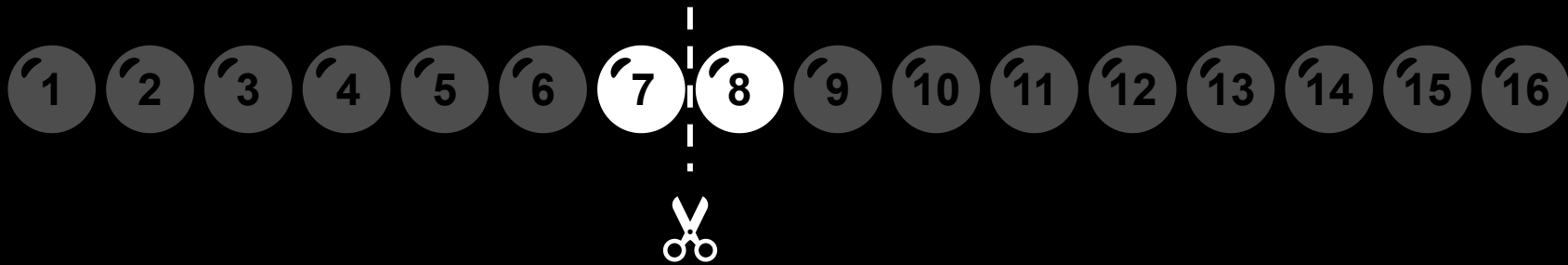


is it > 8? ✗

is it > 4? ✓

is it > 6? ✓

is it > 7?



is it > 8? ✗

is it > 4? ✓

is it > 6? ✓

is it > 7? ✗



is it > 8? ✗

is it > 4? ✓

is it > 6? ✓

is it > 7? ✗



with 4 questions from 16 to 1

is it > 8?	✗	}	0
is it > 4?	✓		1
is it > 6?	✓		1
is it > 7?	✗		0

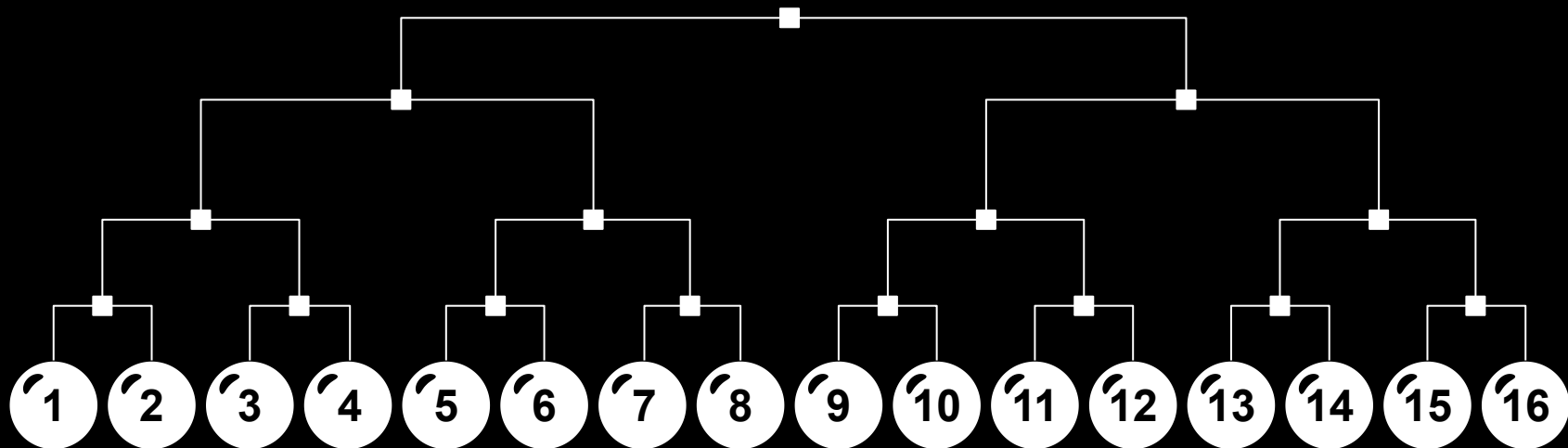


with 4 questions from 16 to 1

where is the information?

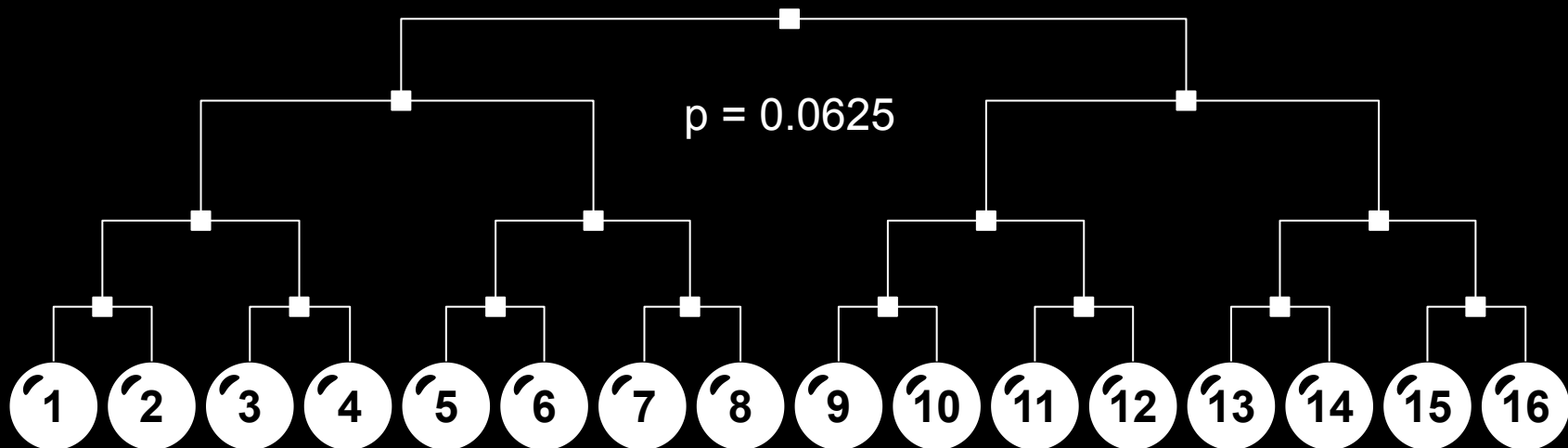


# where is the information?



# where is the information?

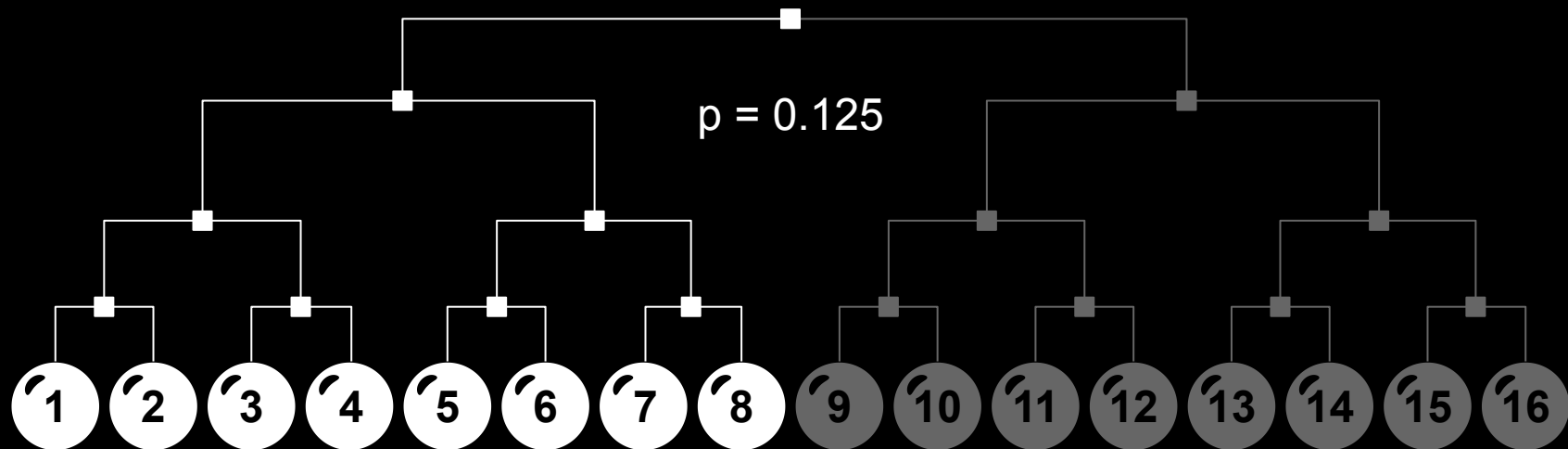
N = 16



where is the information?

~~N = 16~~

N = 8

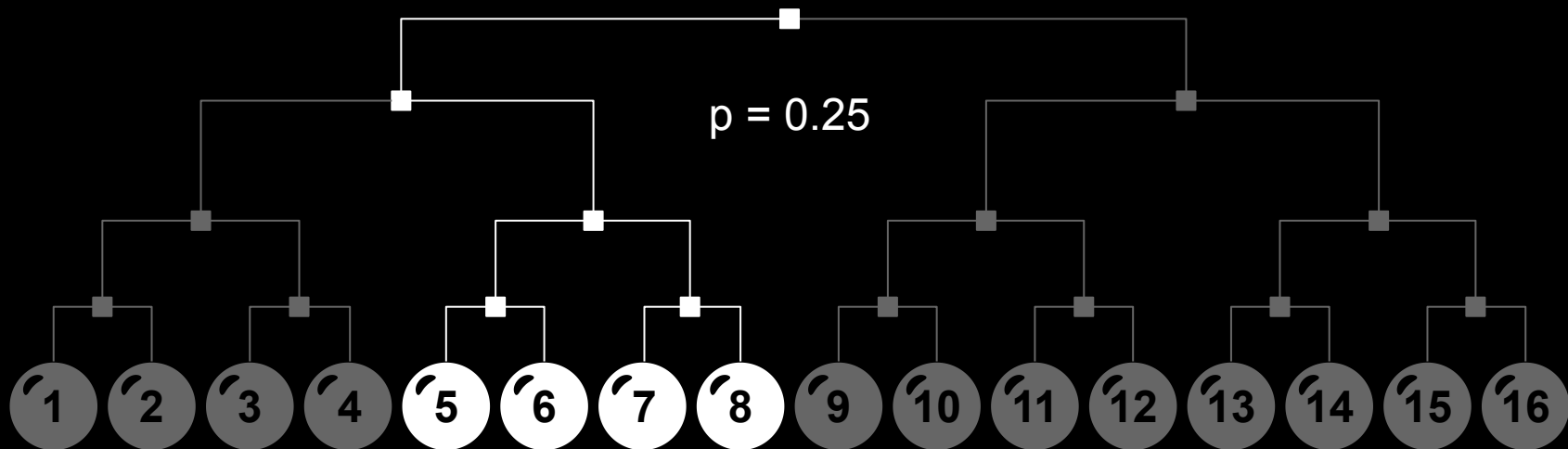


where is the information?

~~N = 16~~

~~N = 8~~

N = 4



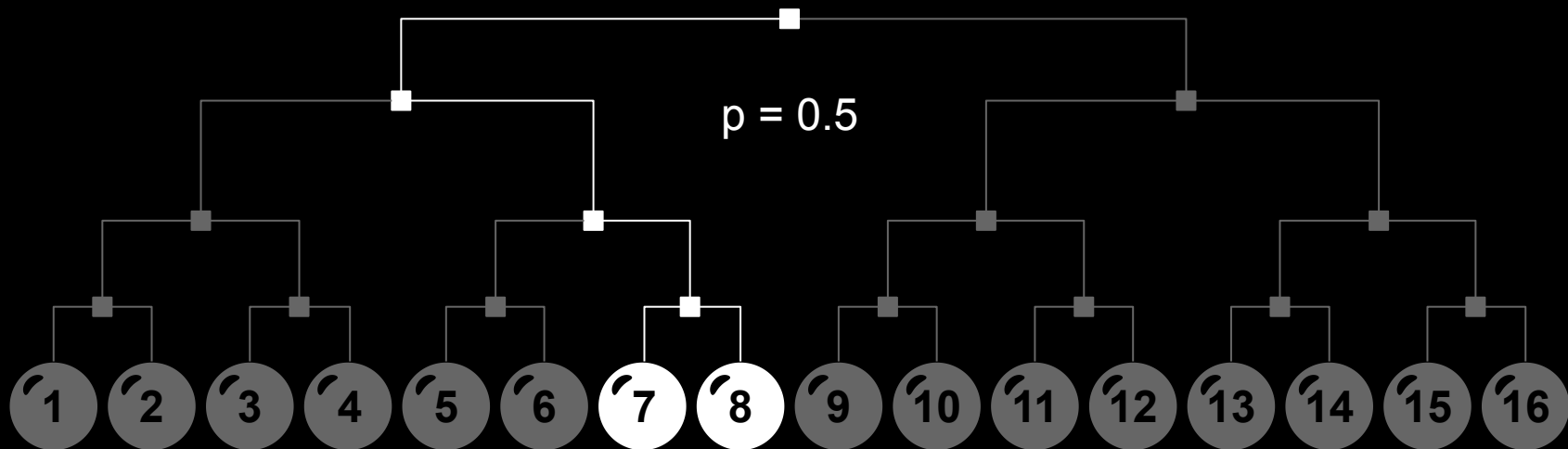
where is the information?

~~$N = 16$~~

~~$N = 8$~~

~~$N = 4$~~

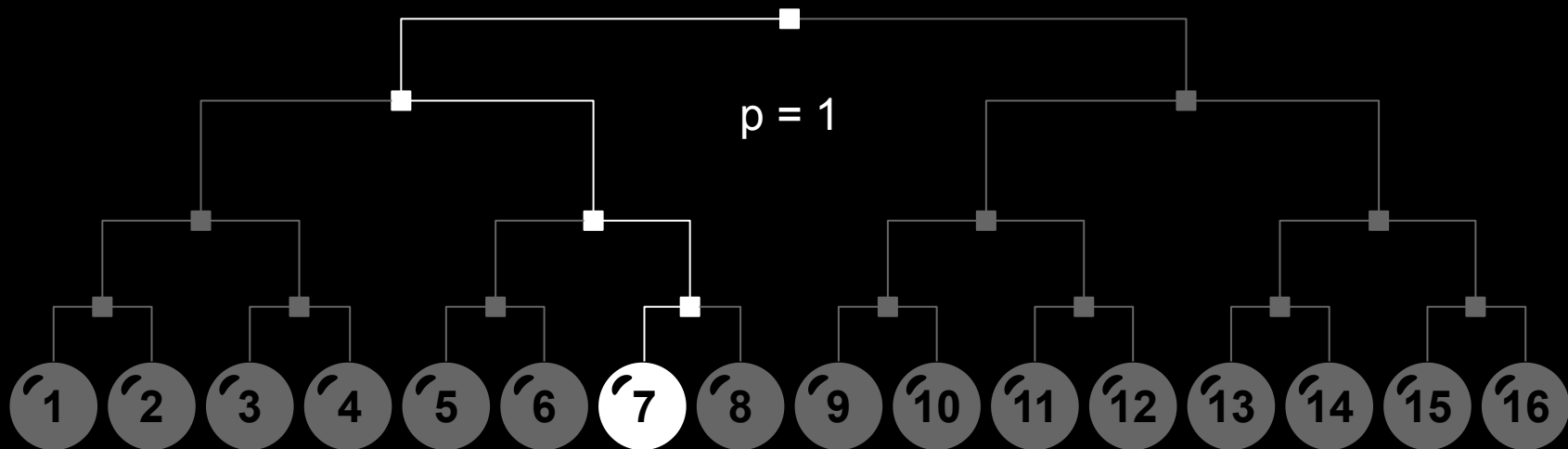
$N = 2$



# where is the information?

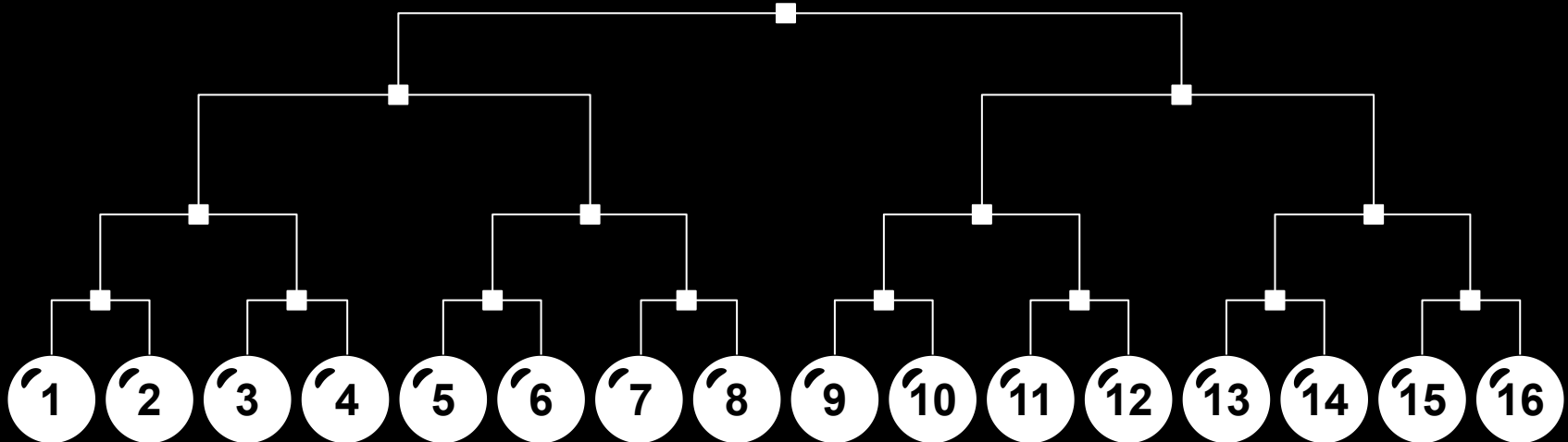
~~N = 16~~~~N=8~~~~N=4~~~~N=2~~

N = 1



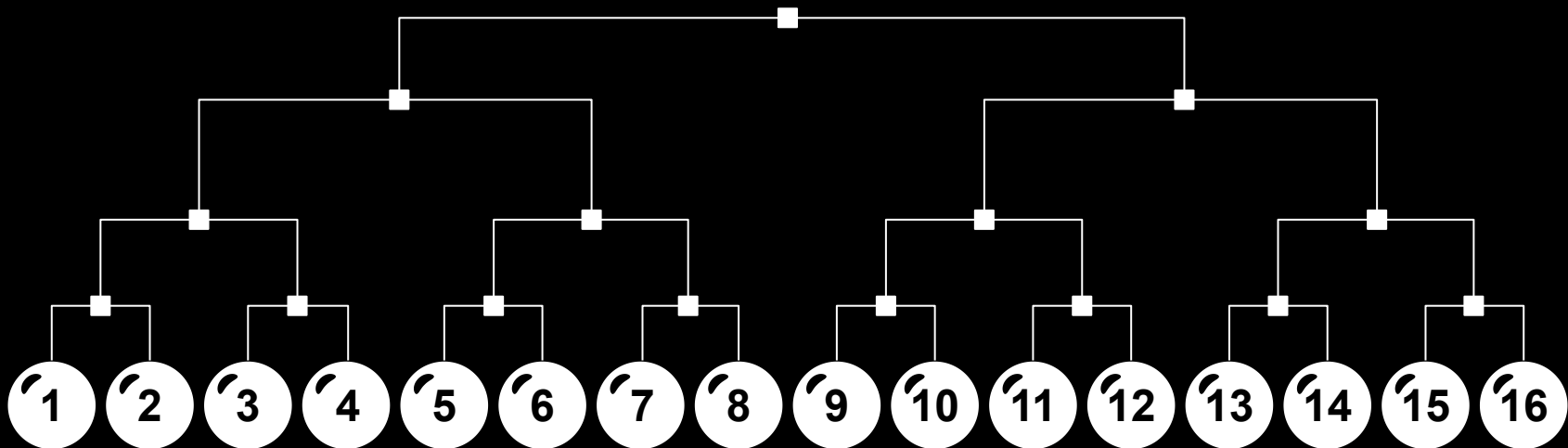
information = reduced uncertainty  
uncertainty is measured with the logarithm of N

$$H = \log_2(N)$$



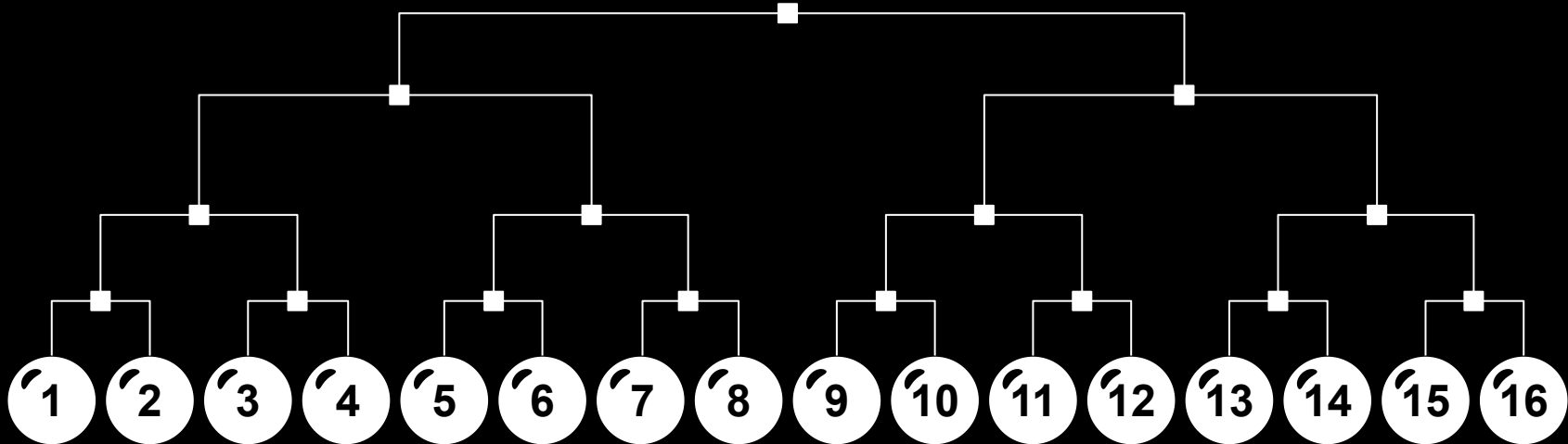
or: how often can we cut the remaining possibilities in half?

$$H = \log_2(N)$$

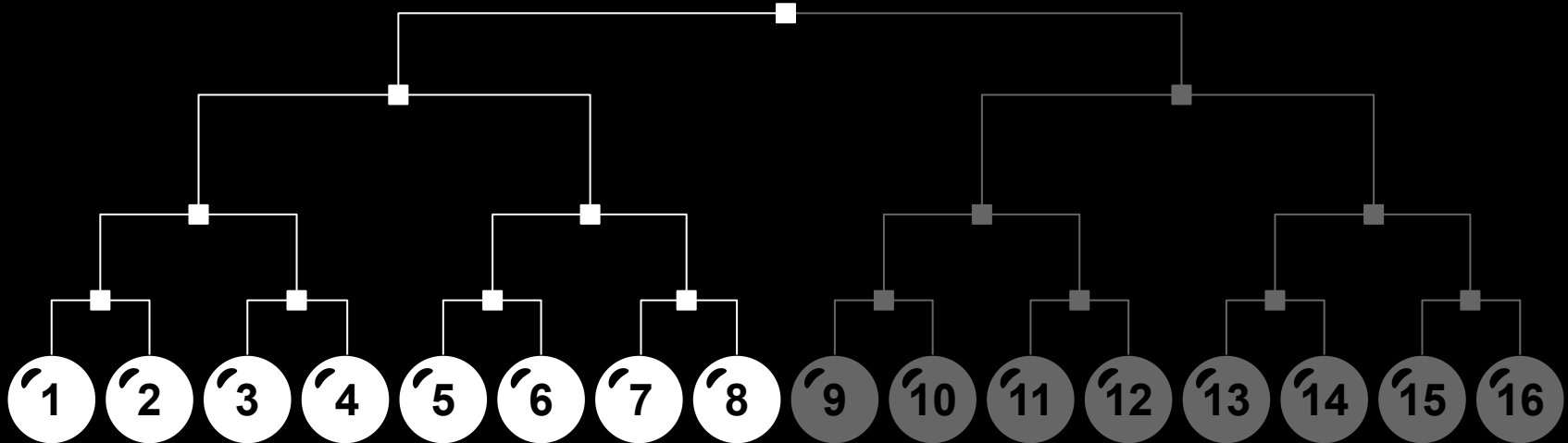




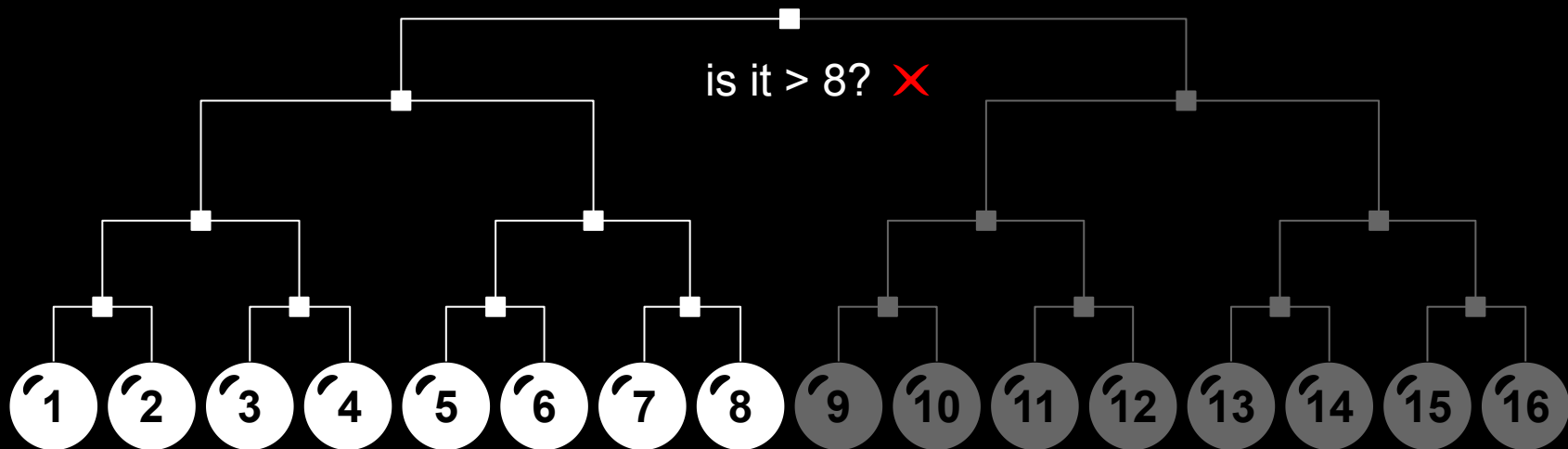
$$H_0 = \log_2(16) = 4$$



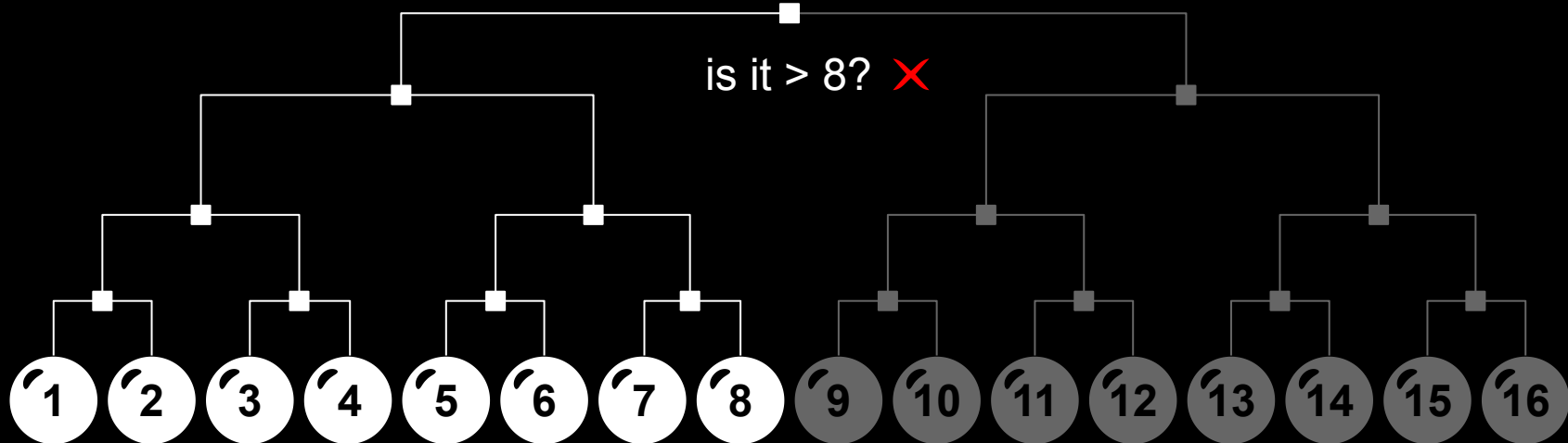
$$H_1 = \log_2(8) = 3$$



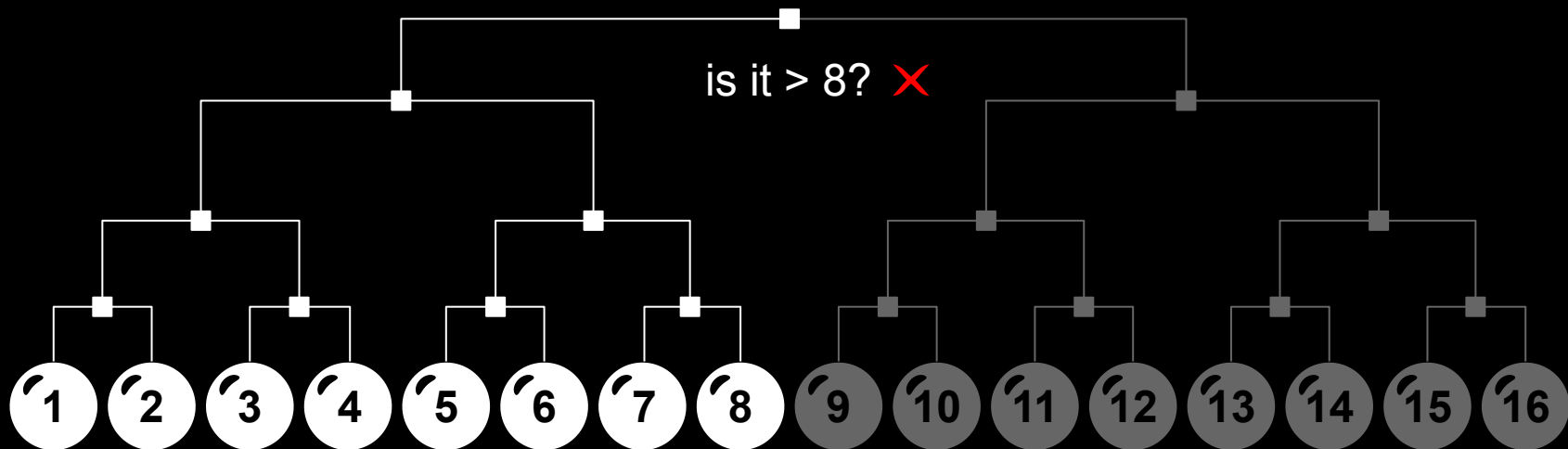
$$I = H_0 - H_1$$



$$I = \log_2(16) - \log_2(8)$$



$$I = 4 - 3 = 1$$



uncertainty and information are  
measured in **bits**

how many yes/no questions to reduce  
uncertainty to zero?

$$H = 0 = \log_2(1)$$

how many yes/no questions to reduce  
uncertainty to zero?

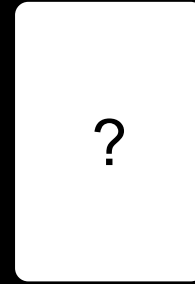
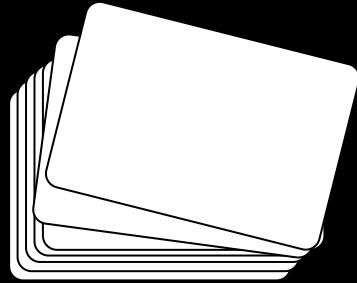
$$H = 0 = \log_2(1)$$

$$H = \log_2(N)$$























































poker

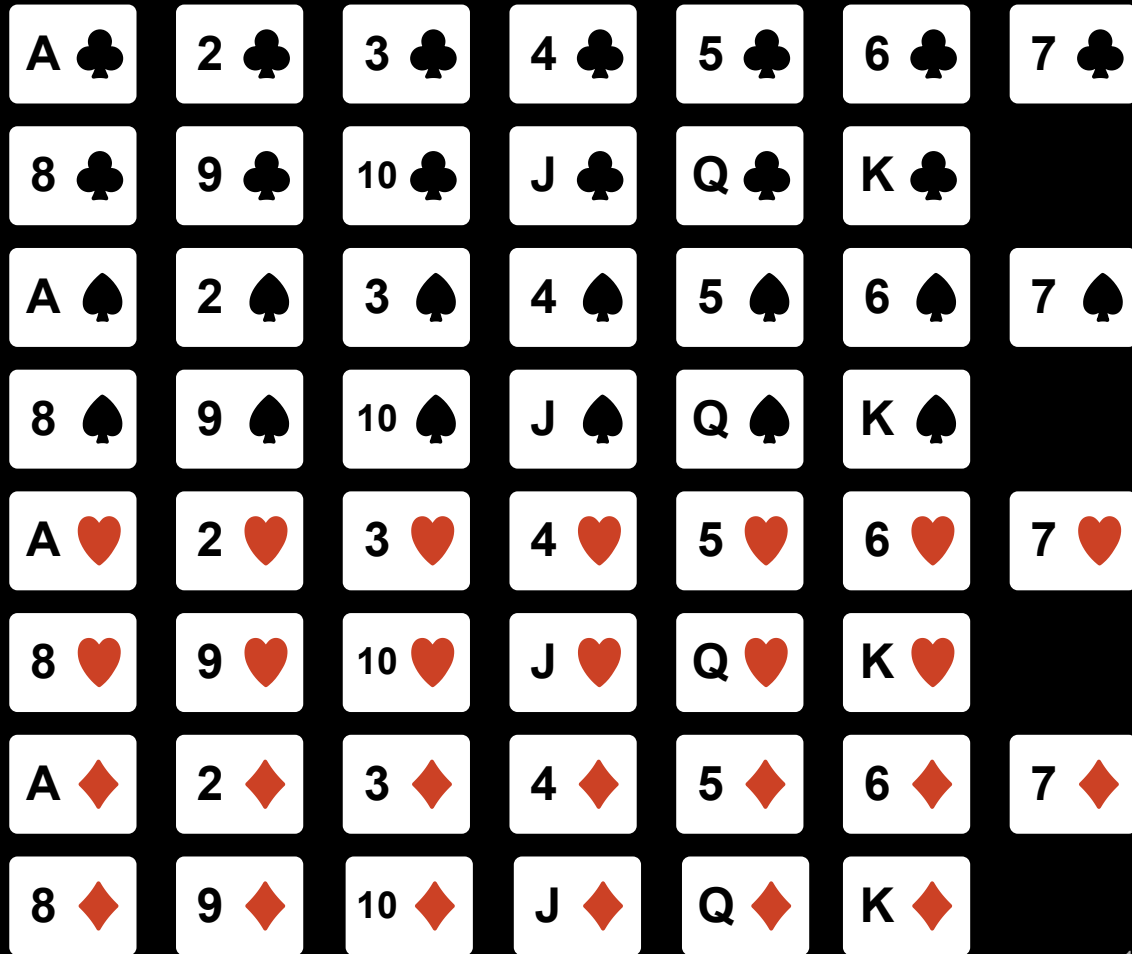
which card am I holding?



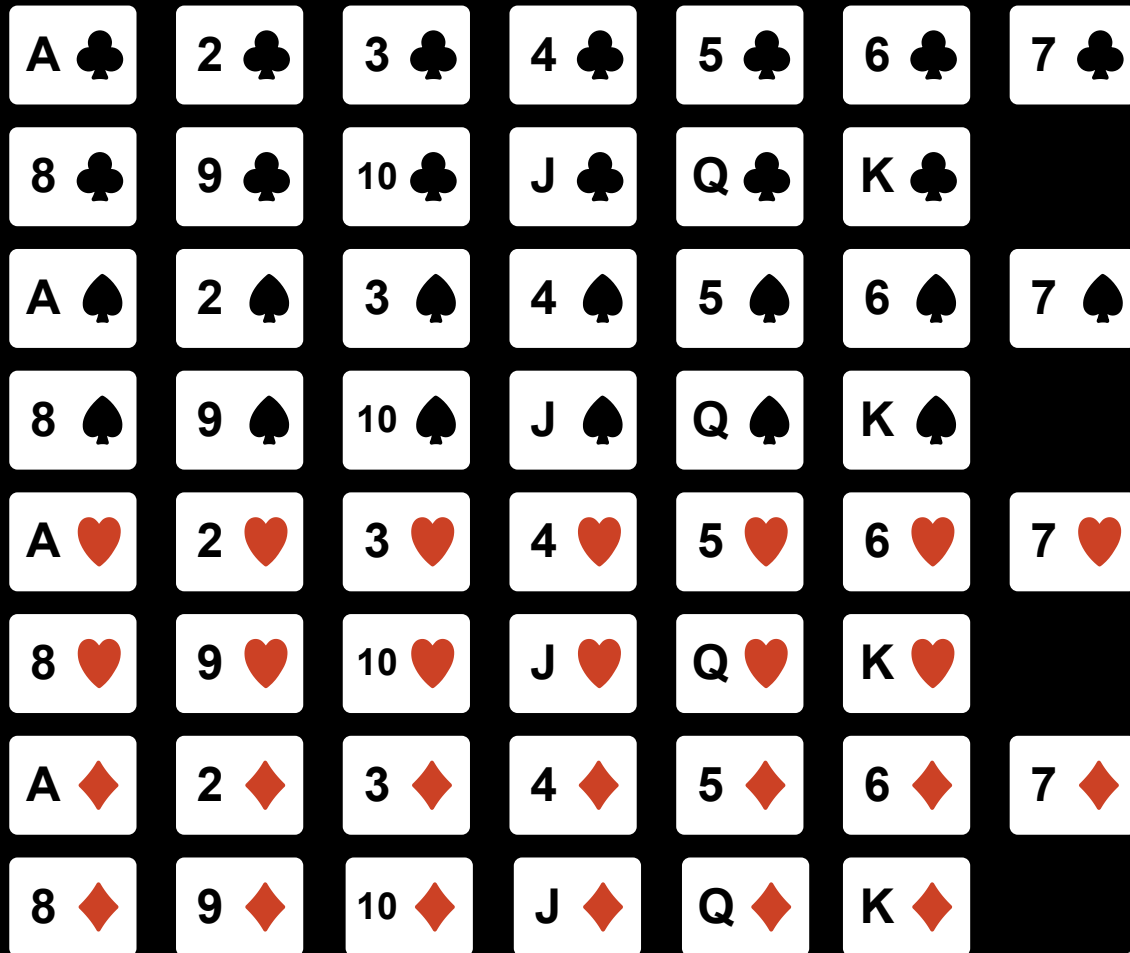
52 card poker deck

A 	2 	3 	4 	5 	6 	7 
8 	9 	10 	J 	Q 	K 	
A 	2 	3 	4 	5 	6 	7 
8 	9 	10 	J 	Q 	K 	
A 	2 	3 	4 	5 	6 	7 
8 	9 	10 	J 	Q 	K 	
A 	2 	3 	4 	5 	6 	7 
8 	9 	10 	J 	Q 	K 	

52 possible cards

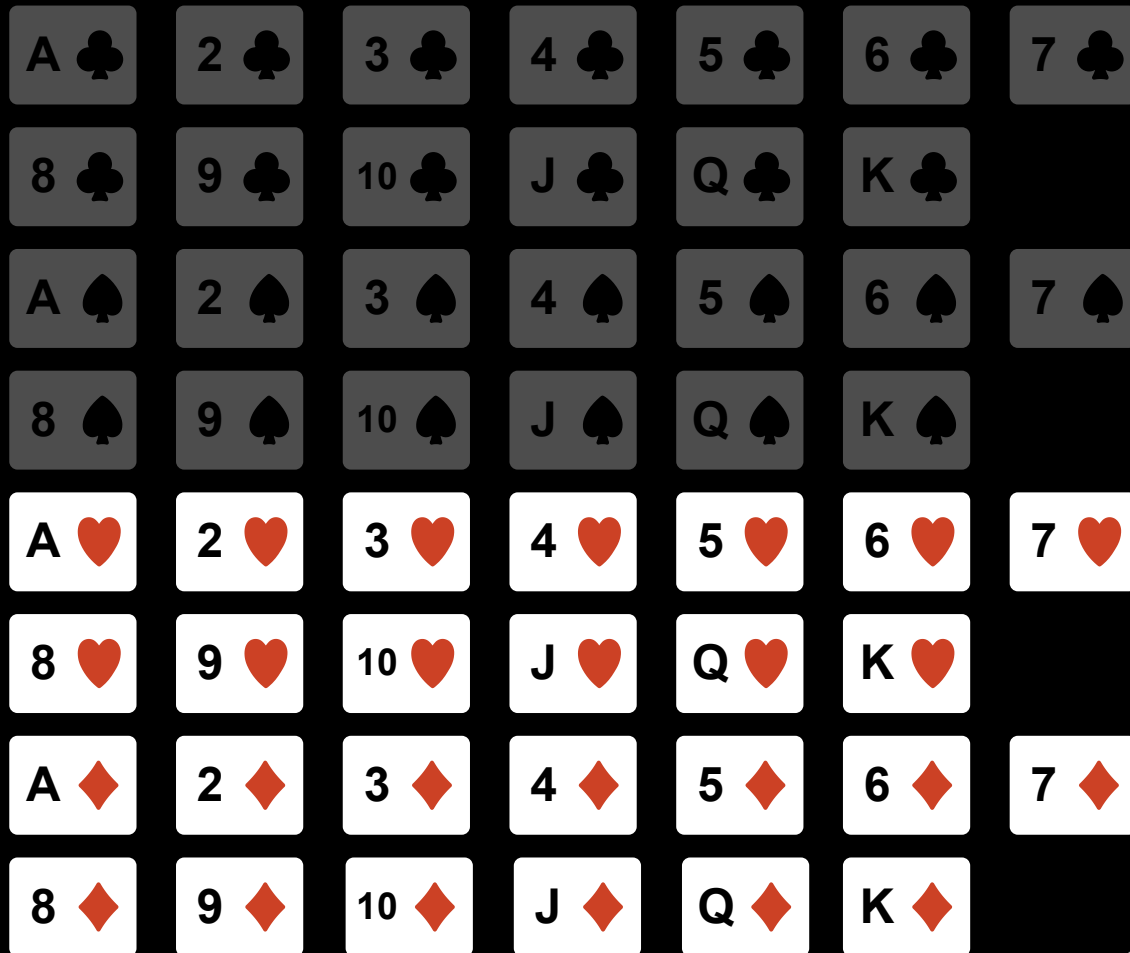


is the card black?



is the card black?

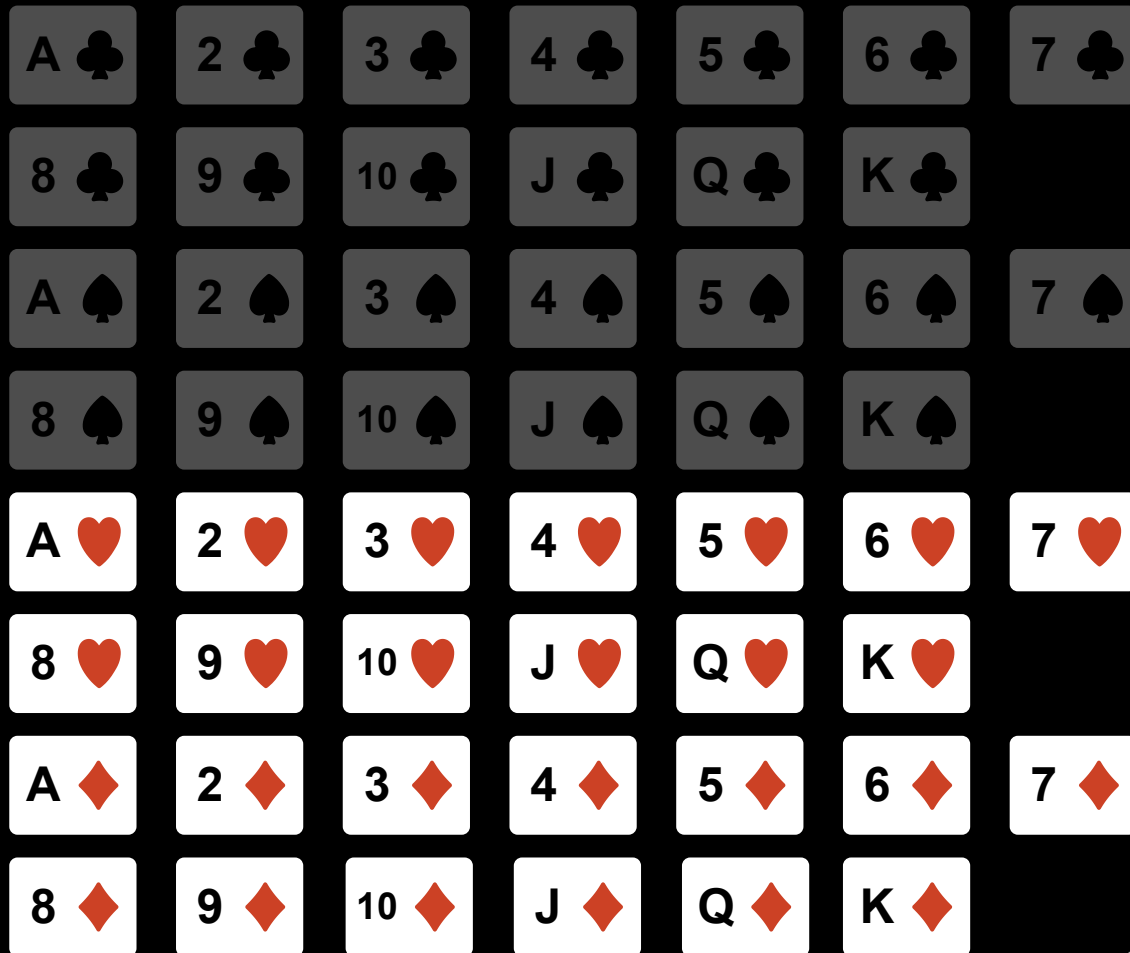
no



is the card black?

no

is it hearts?

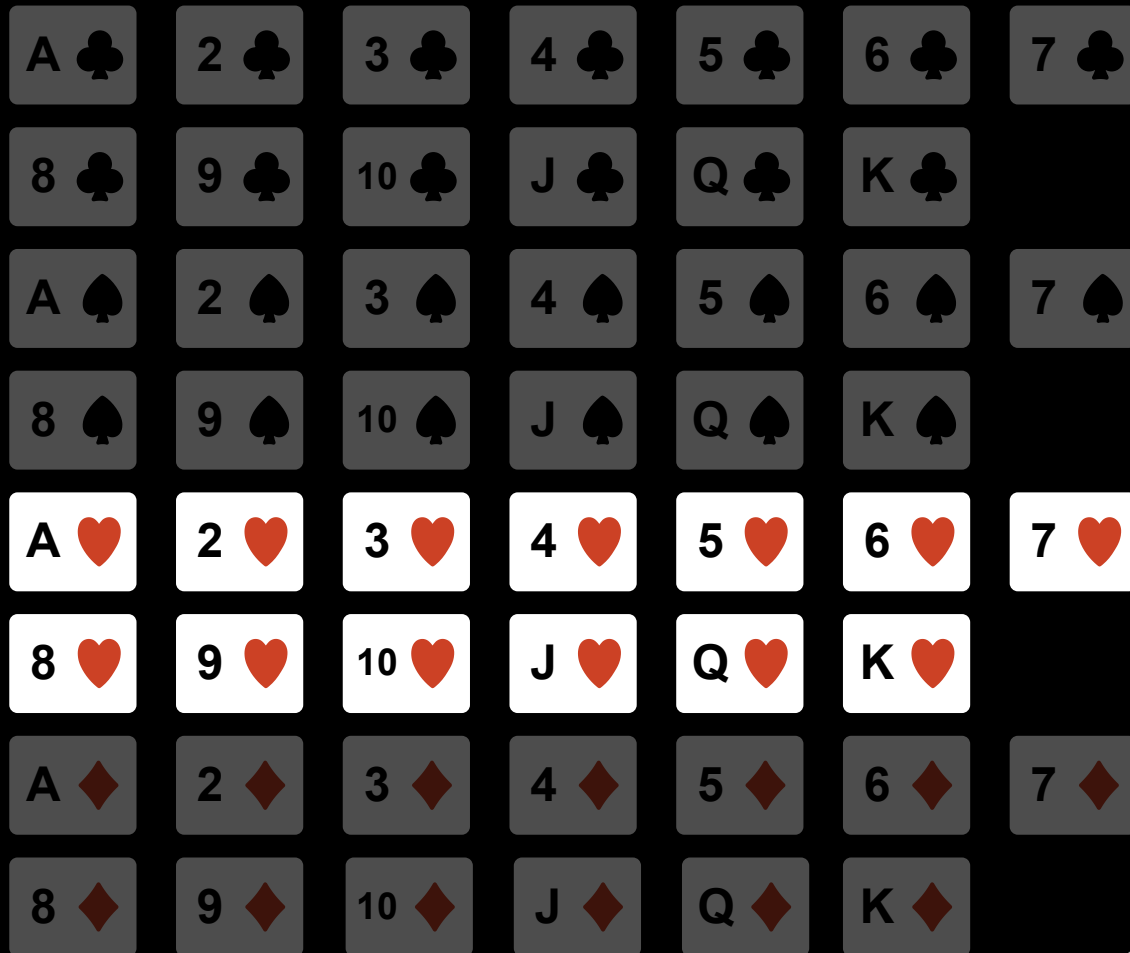


is the card black?

no

is it hearts?

yes





A horizontal row of seven playing cards, all from the Club suit. From left to right, the cards are: Ace of Clubs (A with a club symbol), 2 of Clubs (2 with a club symbol), 3 of Clubs (3 with a club symbol), 4 of Clubs (4 with a club symbol), 5 of Clubs (5 with a club symbol), 6 of Clubs (6 with a club symbol), and 7 of Clubs (7 with a club symbol). The cards are slightly overlapping and have a light gray background.

8♣ 9♣ 10♣ J♣ Q♣ K♣

A sequence of seven cards: A, 2, 3, 4, 5, 6, 7, all of the same suit (spades).

A horizontal sequence of six cards, each in a grey box. From left to right: 8 of Spades, 9 of Spades, 10 of Spades, Jack of Spades (J), Queen of Spades (Q), and King of Spades (K). All cards are of the same suit (Spades).

A horizontal sequence of seven cards: Ace of Hearts, 2 of Hearts, 3 of Hearts, 4 of Hearts, 5 of Hearts, 6 of Hearts, and 7 of Hearts.

8♥ 9♥ 10♥ J♥ Q♥ K♥

A ♦ 2 ♦ 3 ♦ 4 ♦ 5 ♦ 6 ♦ 7 ♦

8♦ 9♦ 10♦ J♦ Q♦ K♦

is the card black?

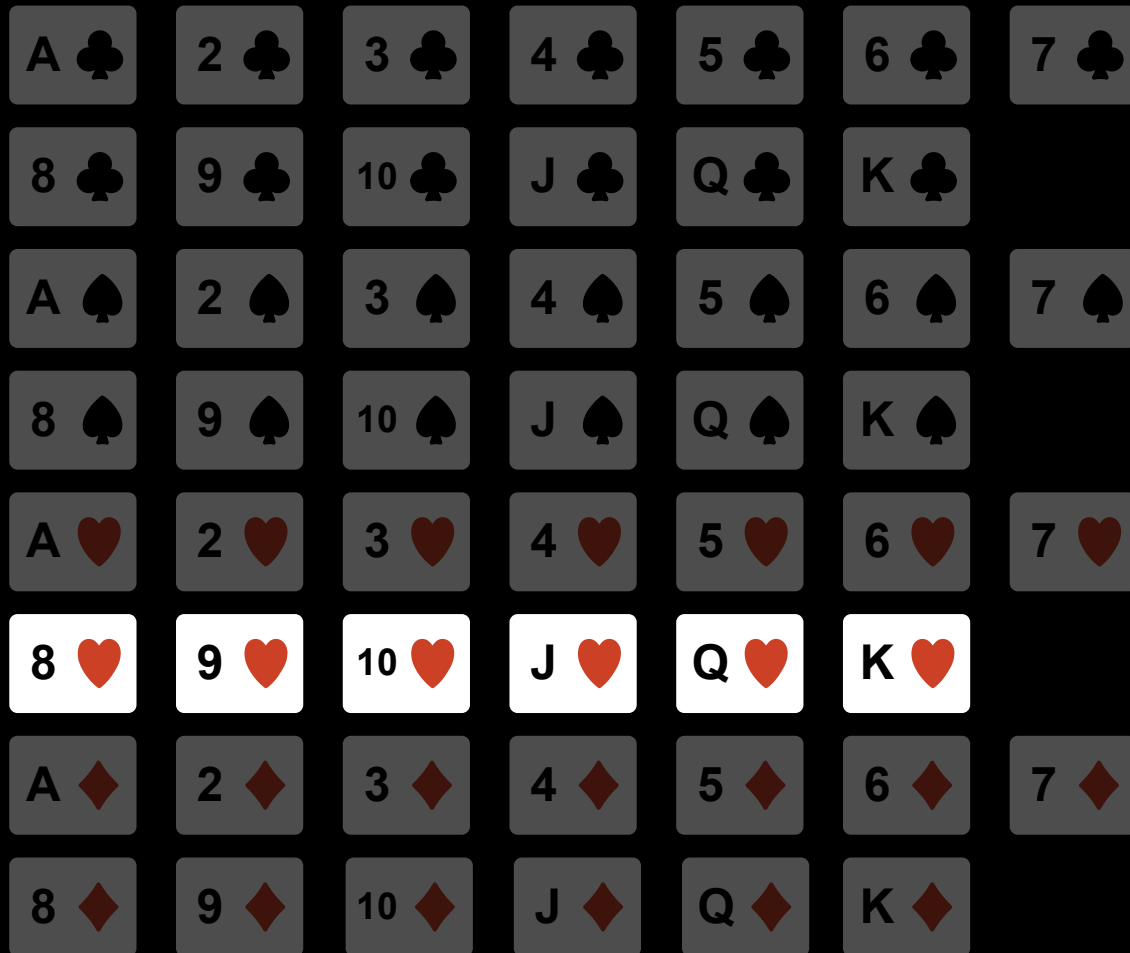
no

is it hearts?

yes

is it 8 or above?

yes



is the card black?

no

is it hearts?

yes

is it 8 or above?

yes

is it jack or above?

A ♣	2 ♣	3 ♣	4 ♣	5 ♣	6 ♣	7 ♣
8 ♣	9 ♣	10 ♣	J ♣	Q ♣	K ♣	
A ♠	2 ♠	3 ♠	4 ♠	5 ♠	6 ♠	7 ♠
8 ♠	9 ♠	10 ♠	J ♠	Q ♠	K ♠	
A ♥	2 ♥	3 ♥	4 ♥	5 ♥	6 ♥	7 ♥
8 ♥	9 ♥	10 ♥	J ♥	Q ♥	K ♥	
A ♦	2 ♦	3 ♦	4 ♦	5 ♦	6 ♦	7 ♦
8 ♦	9 ♦	10 ♦	J ♦	Q ♦	K ♦	

is the card black?

no

is it hearts?

yes

is it 8 or above?

yes

is it jack or above?

yes

A ♣	2 ♣	3 ♣	4 ♣	5 ♣	6 ♣	7 ♣
8 ♣	9 ♣	10 ♣	J ♣	Q ♣	K ♣	
A ♠	2 ♠	3 ♠	4 ♠	5 ♠	6 ♠	7 ♠
8 ♠	9 ♠	10 ♠	J ♠	Q ♠	K ♠	
A ♥	2 ♥	3 ♥	4 ♥	5 ♥	6 ♥	7 ♥
8 ♥	9 ♥	10 ♥	J ♥	Q ♥	K ♥	
A ♦	2 ♦	3 ♦	4 ♦	5 ♦	6 ♦	7 ♦
8 ♦	9 ♦	10 ♦	J ♦	Q ♦	K ♦	

is the card black?

no

is it hearts?

yes





















































is it 8 or above?

yes

is it jack or above?

yes

is it queen or above?

A 	2 	3 	4 	5 	6 	7 
8 	9 	10 	J 	Q 	K 	
A 	2 	3 	4 	5 	6 	7 
8 	9 	10 	J 	Q 	K 	
A 	2 	3 	4 	5 	6 	7 
8 	9 	10 	J 	Q 	K 	
A 	2 	3 	4 	5 	6 	7 
8 	9 	10 	J 	Q 	K 	

is the card black?

no

is it hearts?

yes

is it 8 or above?

yes

is it jack or above?

yes

is it queen or above?

yes

A ♣	2 ♣	3 ♣	4 ♣	5 ♣	6 ♣	7 ♣
8 ♣	9 ♣	10 ♣	J ♣	Q ♣	K ♣	
A ♠	2 ♠	3 ♠	4 ♠	5 ♠	6 ♠	7 ♠
8 ♠	9 ♠	10 ♠	J ♠	Q ♠	K ♠	
A ♥	2 ♥	3 ♥	4 ♥	5 ♥	6 ♥	7 ♥
8 ♥	9 ♥	10 ♥	J ♥	Q ♥	K ♥	
A ♦	2 ♦	3 ♦	4 ♦	5 ♦	6 ♦	7 ♦
8 ♦	9 ♦	10 ♦	J ♦	Q ♦	K ♦	

is the card black?

no

is it hearts?

yes

is it 8 or above?

yes





















































is it jack or above?

yes

is it queen or above?

yes

is it king?

A 	2 	3 	4 	5 	6 	7 
8 	9 	10 	J 	Q 	K 	
A 	2 	3 	4 	5 	6 	7 
8 	9 	10 	J 	Q 	K 	
A 	2 	3 	4 	5 	6 	7 
8 	9 	10 	J 	Q 	K 	
A 	2 	3 	4 	5 	6 	7 
8 	9 	10 	J 	Q 	K 	

is the card black?

no

is it hearts?

yes

is it 8 or above?

yes

is it jack or above?





















































yes

is it queen or above?

yes

is it king?

no

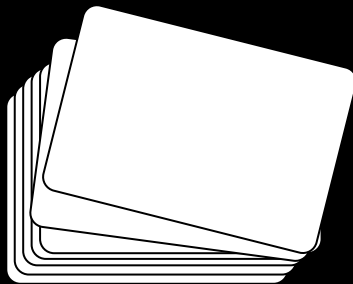
A 	2 	3 	4 	5 	6 	7 
8 	9 	10 	J 	Q 	K 	
A 	2 	3 	4 	5 	6 	7 
8 	9 	10 	J 	Q 	K 	
A 	2 	3 	4 	5 	6 	7 
8 	9 	10 	J 	Q 	K 	
A 	2 	3 	4 	5 	6 	7 
8 	9 	10 	J 	Q 	K 	



with 6 questions  
from 52 to 1

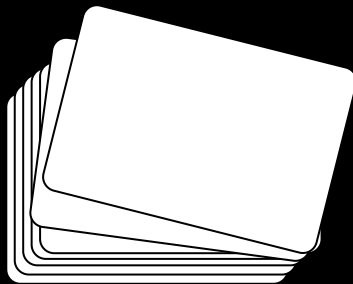
A ♣	2 ♣	3 ♣	4 ♣	5 ♣	6 ♣	7 ♣
8 ♣	9 ♣	10 ♣	J ♣	Q ♣	K ♣	
A ♠	2 ♠	3 ♠	4 ♠	5 ♠	6 ♠	7 ♠
8 ♠	9 ♠	10 ♠	J ♠	Q ♠	K ♠	
A ♥	2 ♥	3 ♥	4 ♥	5 ♥	6 ♥	7 ♥
8 ♥	9 ♥	10 ♥	J ♥	Q ♥	K ♥	
A ♦	2 ♦	3 ♦	4 ♦	5 ♦	6 ♦	7 ♦
8 ♦	9 ♦	10 ♦	J ♦	Q ♦	K ♦	

uncertainty with  $N = 52$  possibilities?

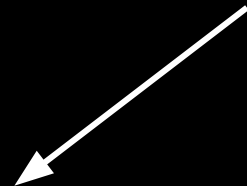


$$H = \log_2(52) \approx 5.7$$

uncertainty with  $N = 52$  possibilities?



average # of yes/no  
questions



$$H = \log_2(52) \approx 5.7$$

one bit of information with each answer...

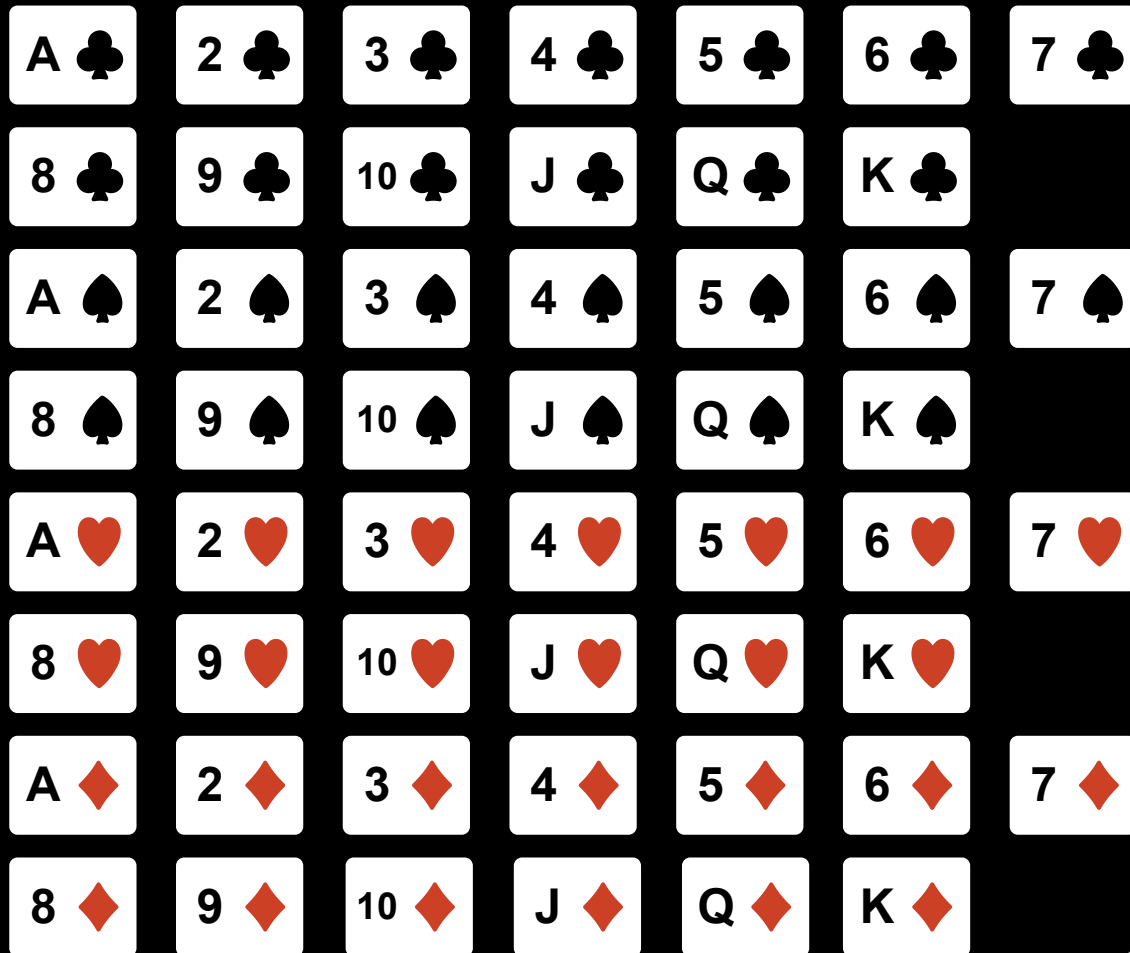
$$\log_2(52) - \log_2(26) = 1$$

one bit of information with each answer...

$$\log_2(52) - \log_2(26) = 1$$

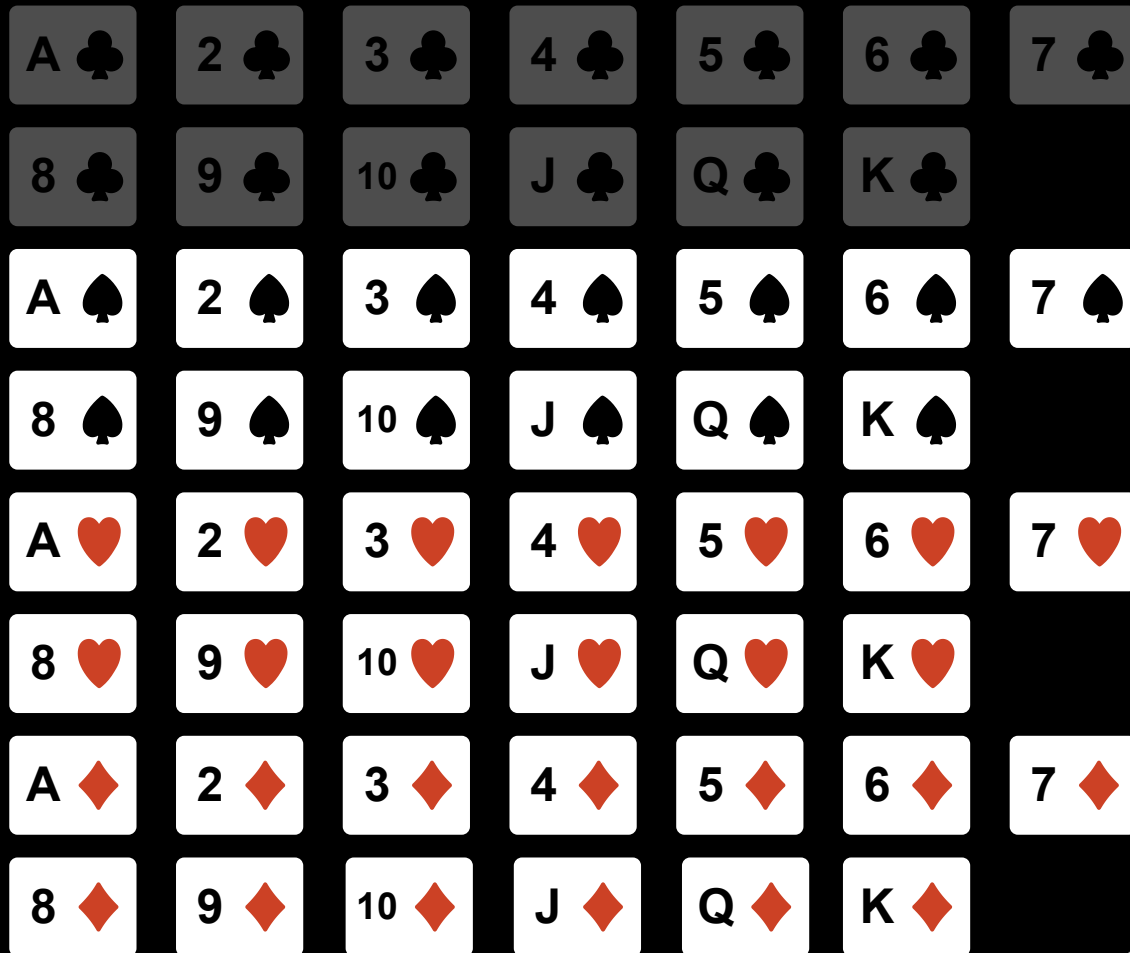
...that cuts the remaining options in half

is it a spades card?



is it a spades card?

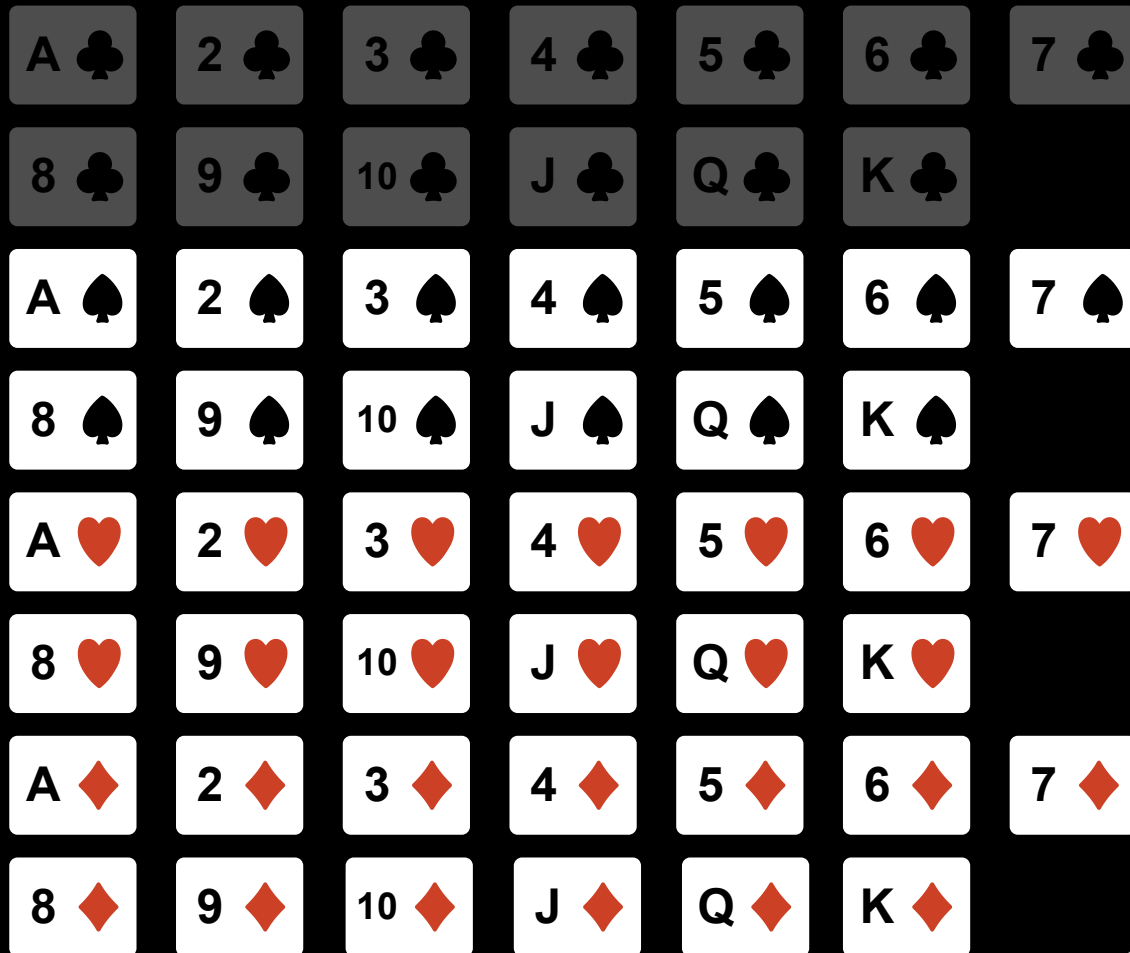
no



is it a spades card?

no

how much information?





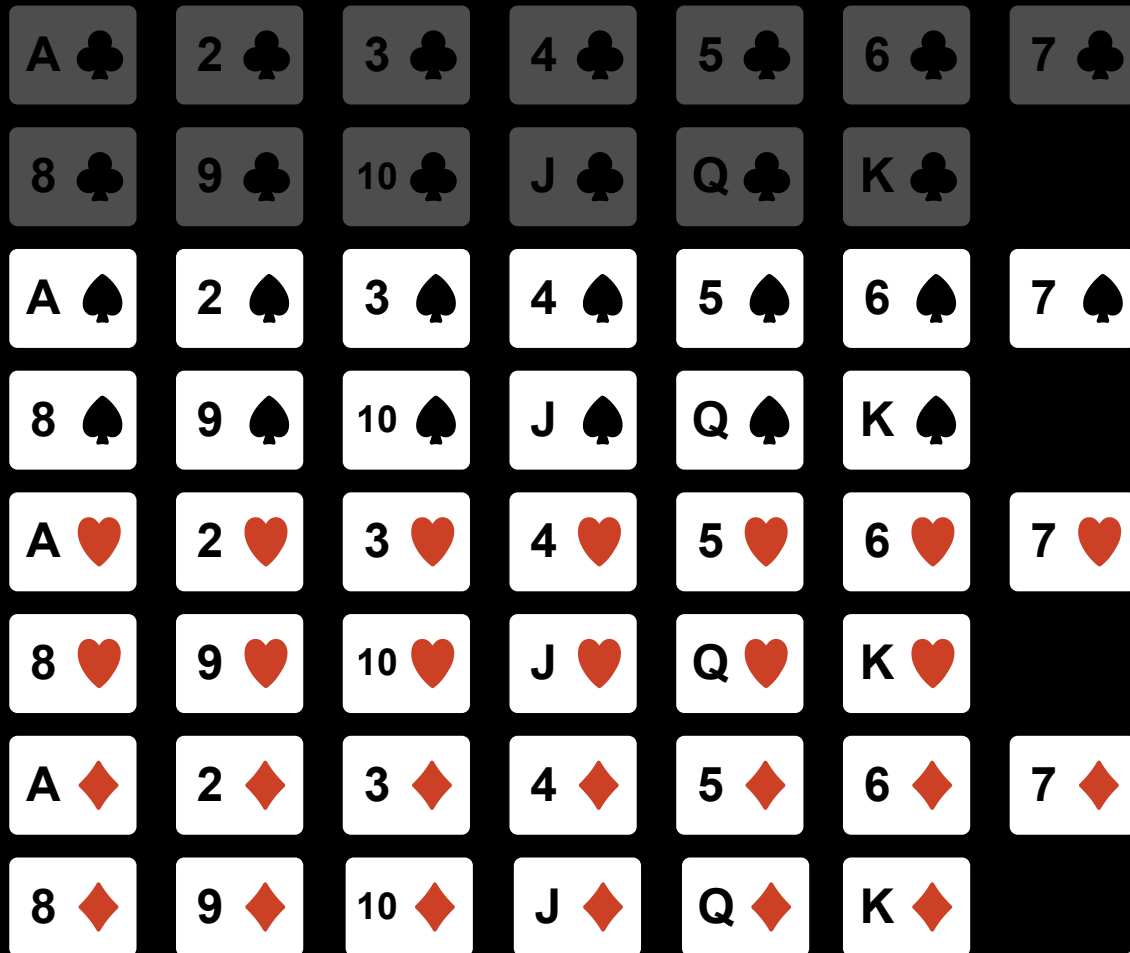
is it a spades card?

no

how much information?

$$H_0 = \log_2(52) \approx 5.7$$

$$H_1 = \log_2(39) \approx 5.29$$



is it a spades card?

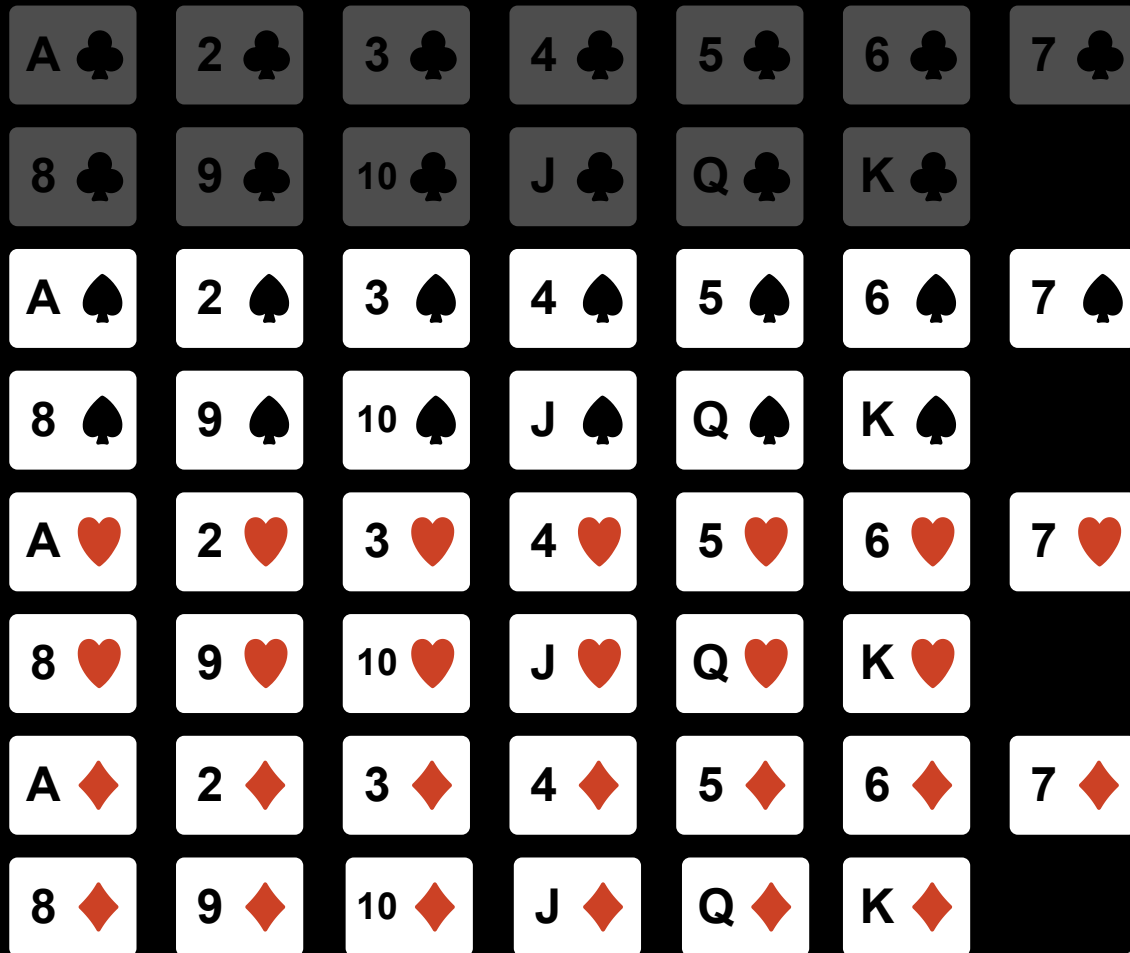
no

how much information?

$$H_0 = \log_2(52) \approx 5.7$$

$$H_1 = \log_2(39) \approx 5.29$$

$$H_0 - H_1 \approx 0.41$$



is it a spades card?

no

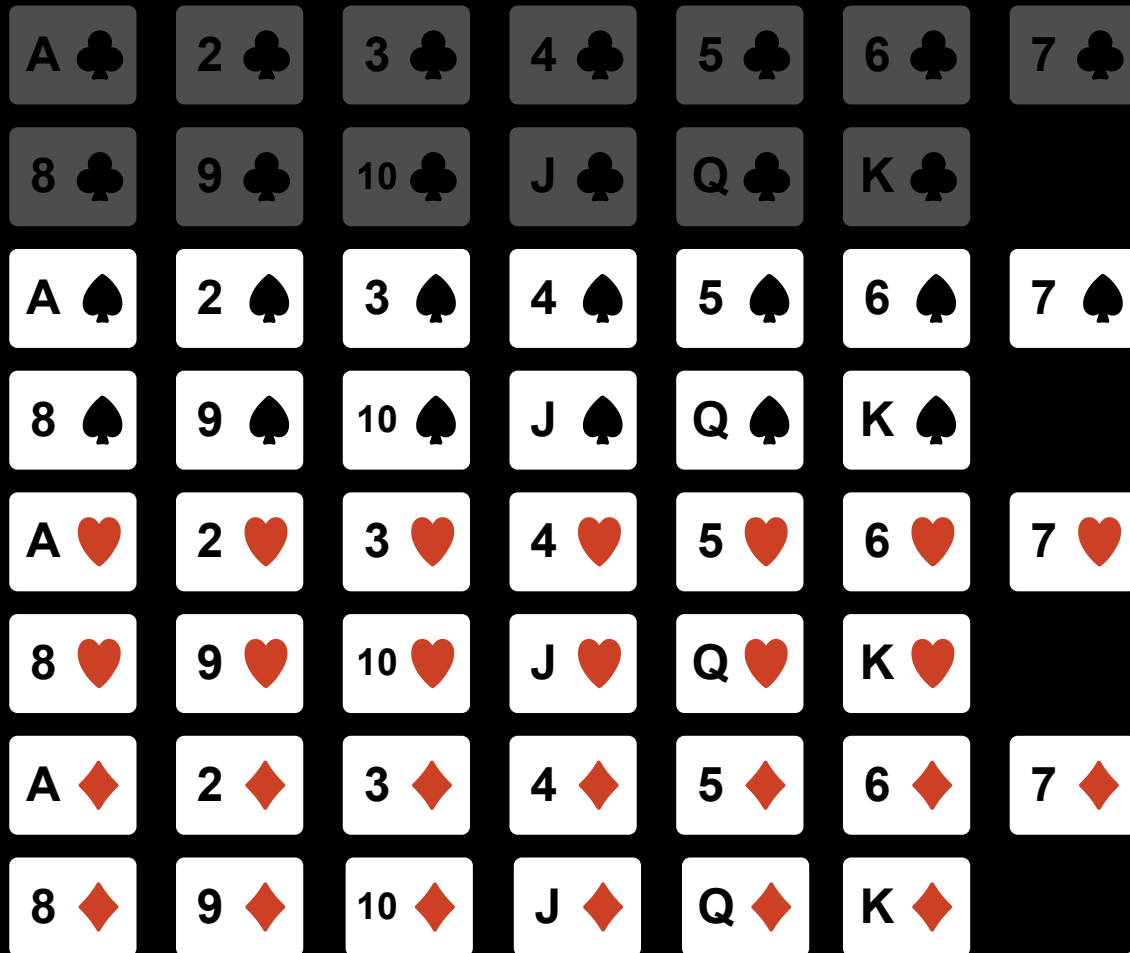
how much information?

$$H_0 = \log_2(52) \approx 5.7$$

$$H_1 = \log_2(39) \approx 5.29$$

$$H_0 - H_1 \approx 0.41$$

that's less than 1 bit



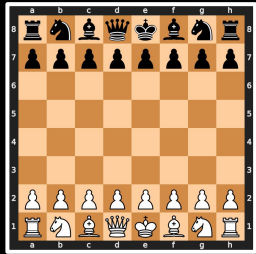
chess



next\_move()

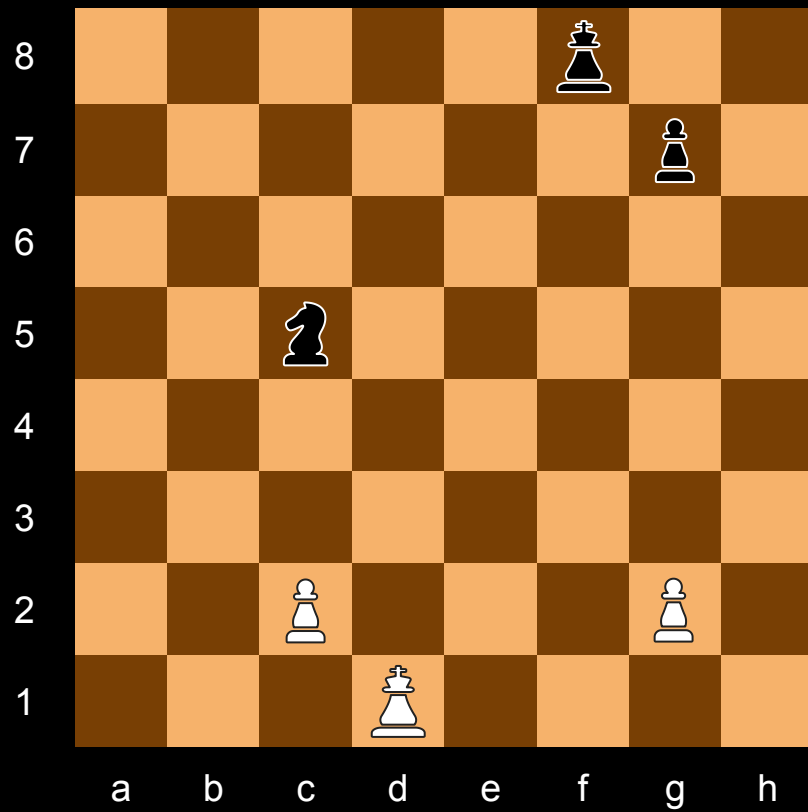
E2 → E4

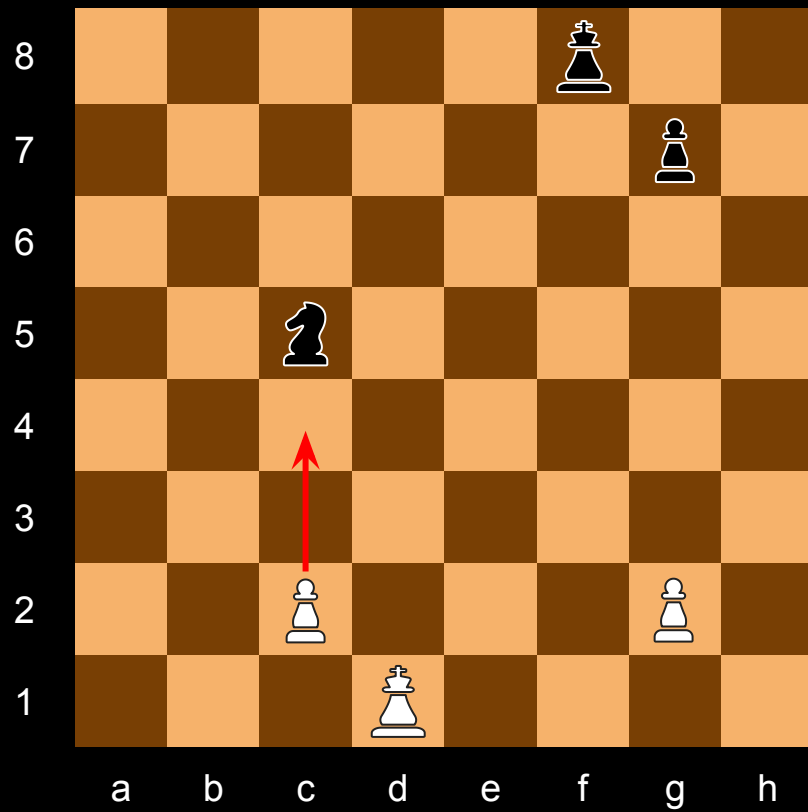
how much information is  
one move?



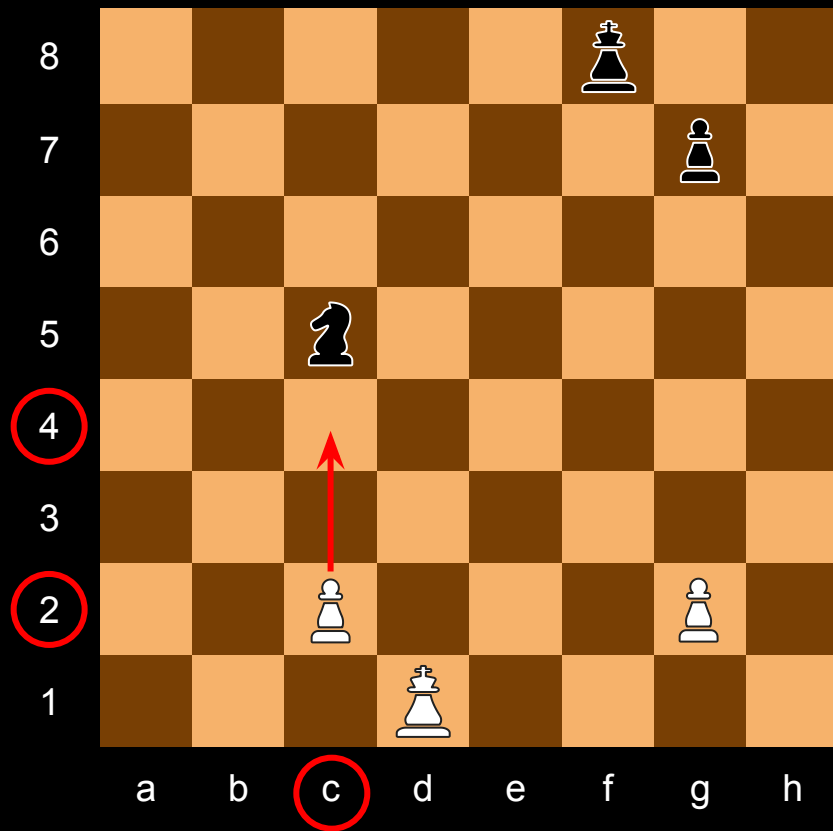
next\_move()

E2 → E4

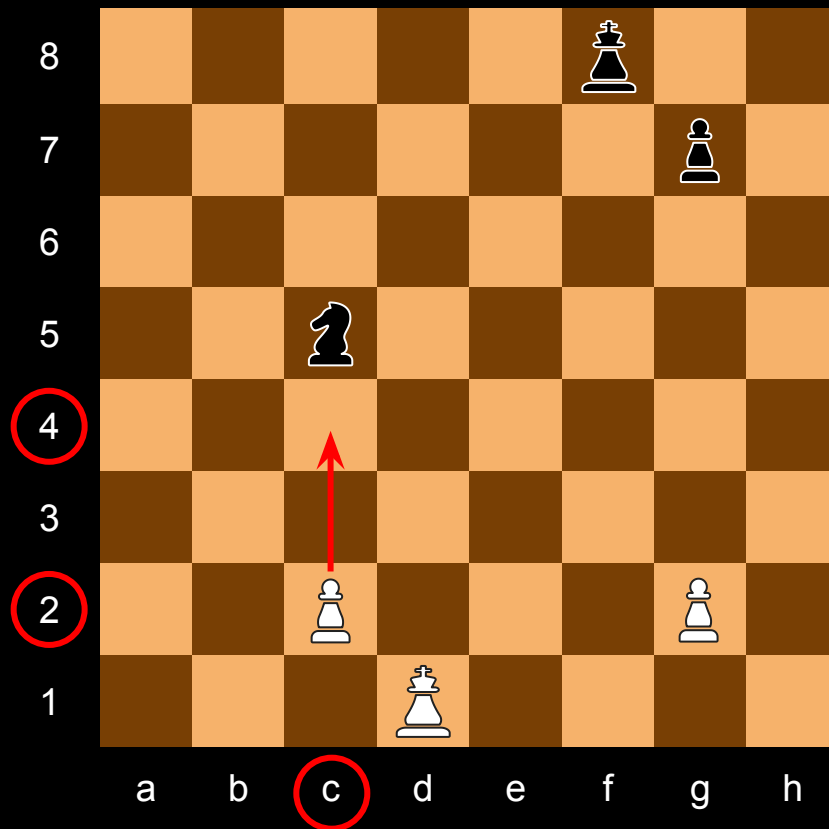




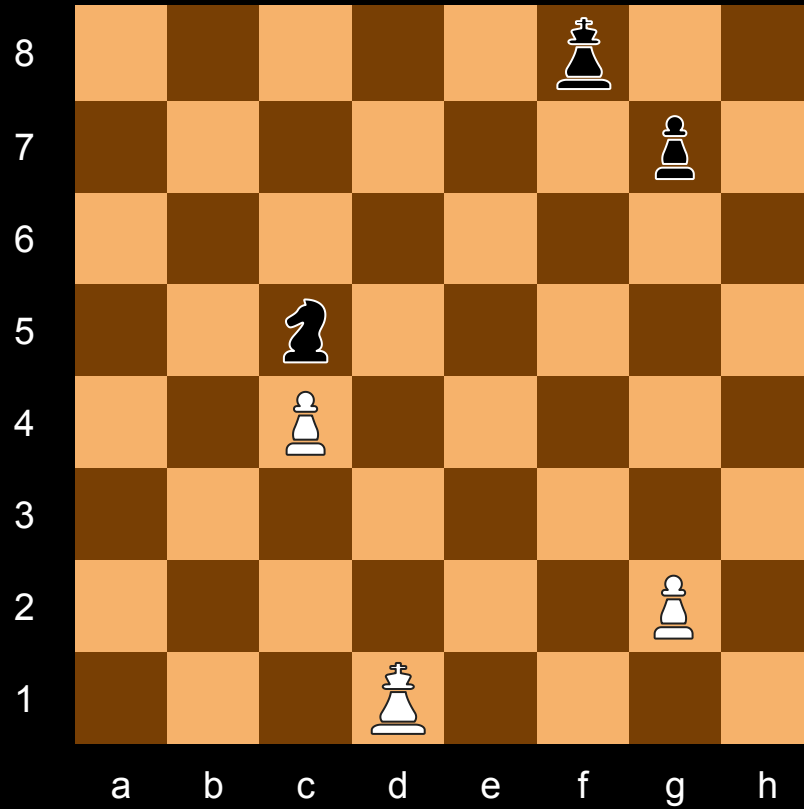




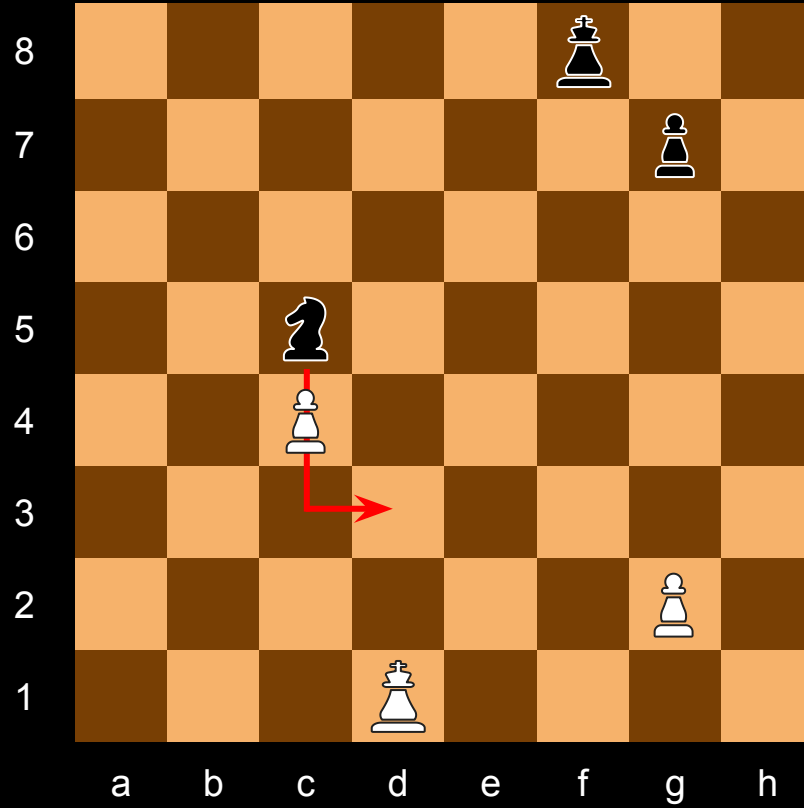
c2 → c4



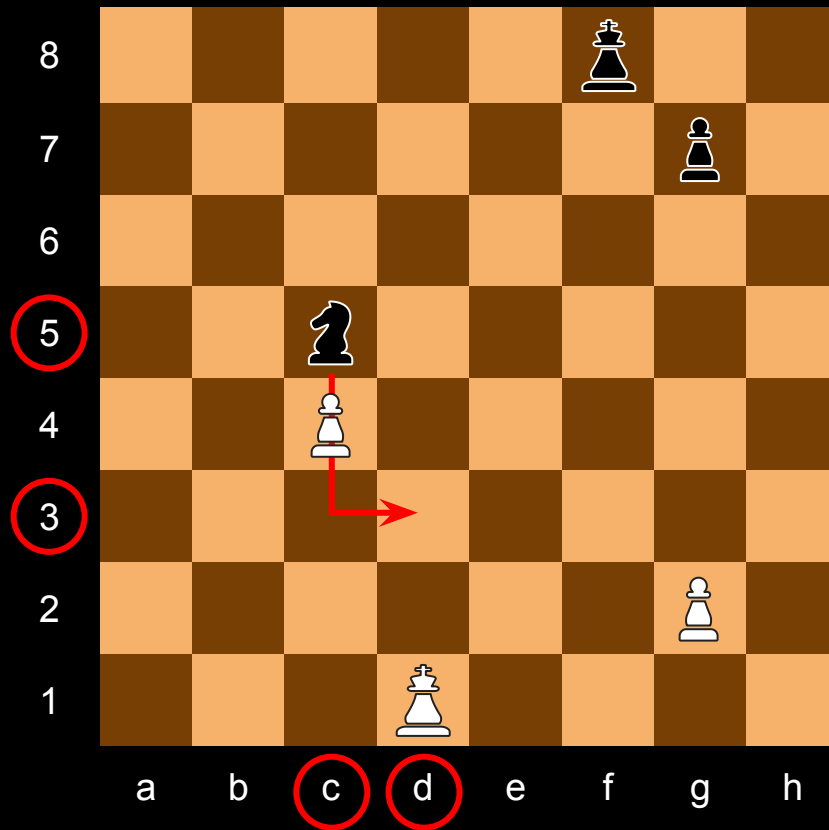
c2 → c4



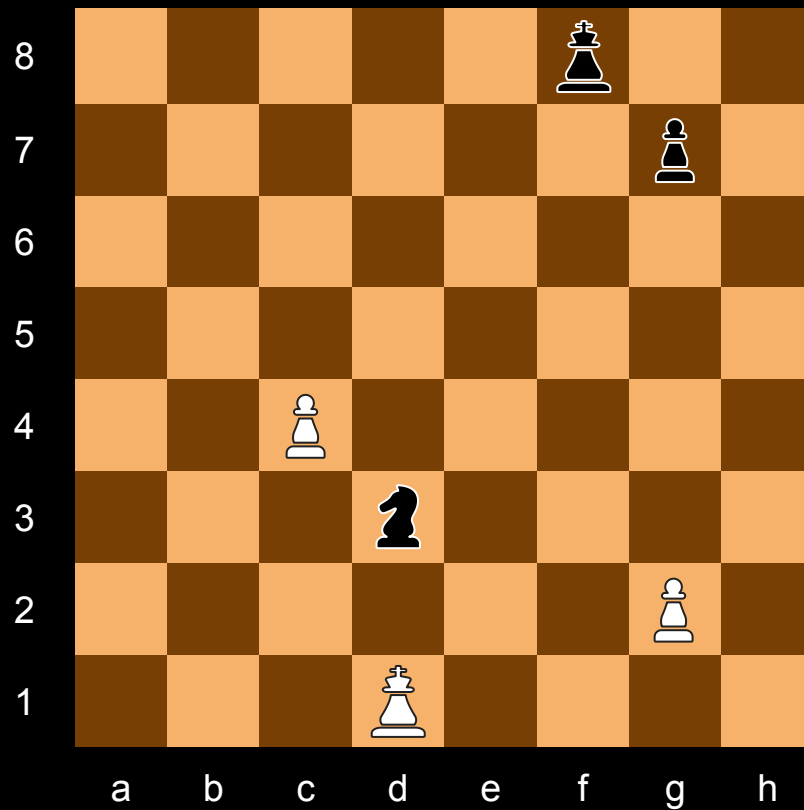
c2 → c4



c2 → c4  
c5 → d3



c2 → c4  
c5 → d3  
...



$$c_2 \rightarrow c_4$$

how many possibilities?

# 64 fields x 64 fields

how many possibilities?



$$64 \times 64 = 4096$$

possible moves\*

\*disregarding impossible moves

$$64 \times 64 = 4096$$

$$H = \log_2(4096) = 12$$

$$64 \times 64 = 4096$$

$$H = \log_2(4096) = 12$$

one chess move is 12 bits of information

an alternative way to calculate # bits

c 2 c 4

4 digits

c 2 c 4

4 digits

8 possible symbols per digit

c 2 c 4

4 digits

8 possible symbols per digit  
how many bits per digit?

c 2 c 4

4 digits

8 possible symbols per digit

how many bits per digit?

$$H_{digit} = \log_2(8) = 3$$



c 2 c 4

4 digits

8 possible symbols per digit

how many bits per digit?

$$H_{digit} = \log_2(8) = 3$$

$$H_{move} = \log_2(8) \times 4 = 12$$

$$H_{avg} = \log_2(S) \times n$$

$S$ : number of possible symbols

$n$ : number of digits in our message

$$H_{max} = \lceil \log_2(S) \rceil \times n$$

when calculating bits for storage, we must  
always consider the worst case

# digits and # symbols

$\{A\}$

— —

$\{A\}$

A A

$\{A, B\}$

— —

$\{A, B\}$ 

AA, AB, BA, BB



$\{A, B, C\}$

— —

$\{A, B, C\}$

AA, AB, BA, BB,  
AC, BC, CA, CB, CC

$\{A, B, C, D\}$

— —

$\{A, B, C, D\}$

AA, AB, BA, BB, AC, BC, CA, CB,  
CC, AD, DA, BD, DB, CD, DC, DD

{A, B, C, D, E}

— —

$\{A, B, C, D, E\}$

AA, AB, BA, BB, AC, BC, CA, CB, CC,  
AD, DA, BD, DB, CD, DC, DD, AE, EA,  
BE, EB, CE, EC, DE, ED, EE

with # digits  $n = 2$

# symbols

# messages

1

1

2

4

3

9

4

16

5

25

with length  $n = 2$

# symbols

# messages

1

1

2

4

3

9

4

16

5

25

$f(x)$





and more digits?

$\{A, B\}$

— — —

$\{A, B\}$

AAA, AAB, ABA, ABB,  
BBB, BBA, BAA, BAB

$\{A, B\}$

— — — —

$\{A, B\}$

AAAA, AAAB, AABA, AABB,  
ABAA, ABAB, ABBA, ABBB,  
BAAA, BAAB, BABA, BABB,  
BBAA, BBAB, BBBA, BBBB

with # symbols  $S = 2$

# digits

# messages

1

2

2

4

3

8

4

16

5

32

with # symbols  $S = 2$

# digits		# messages
1		2
2		4
3	$\xrightarrow{f(x)}$	8
4		16
5		32

with # symbols  $S = 2$

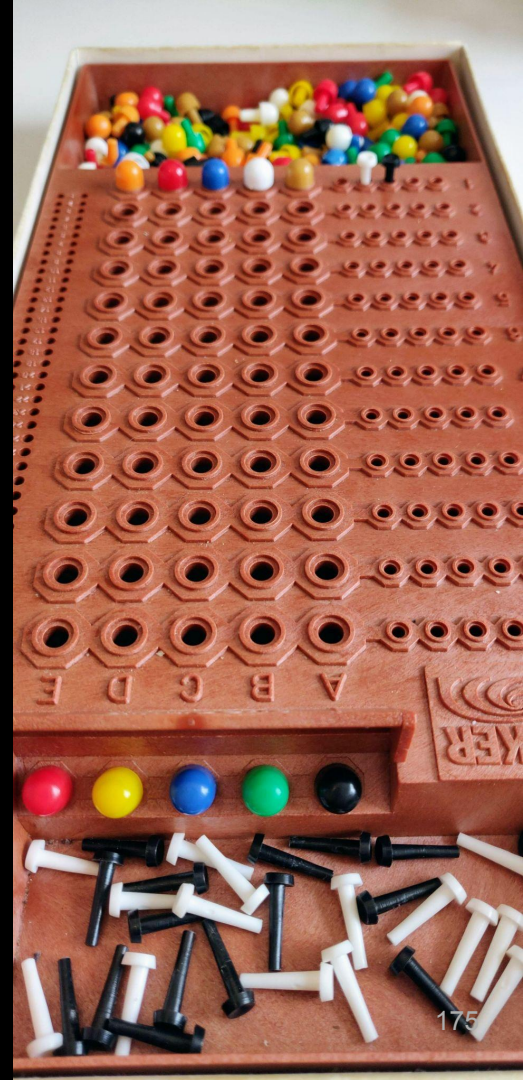
# digits		# messages
1		2
2		4
3	$\xrightarrow{f(x)}$	8
4		16
5		32



# possible messages with  $n$  digits and  $S$  symbols

$$N = S^n$$

mastermind



eight colors



five slots



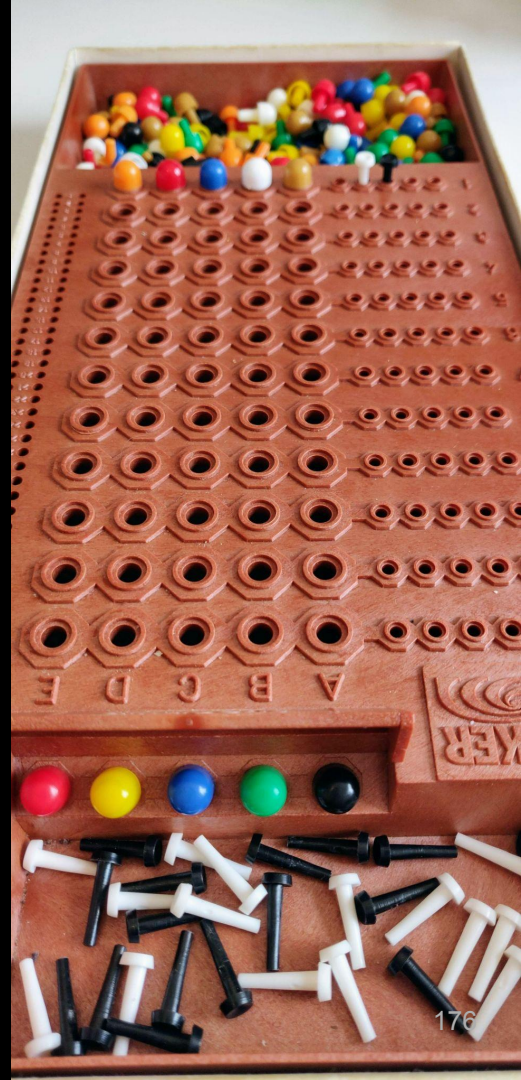
A

B

C

D

E



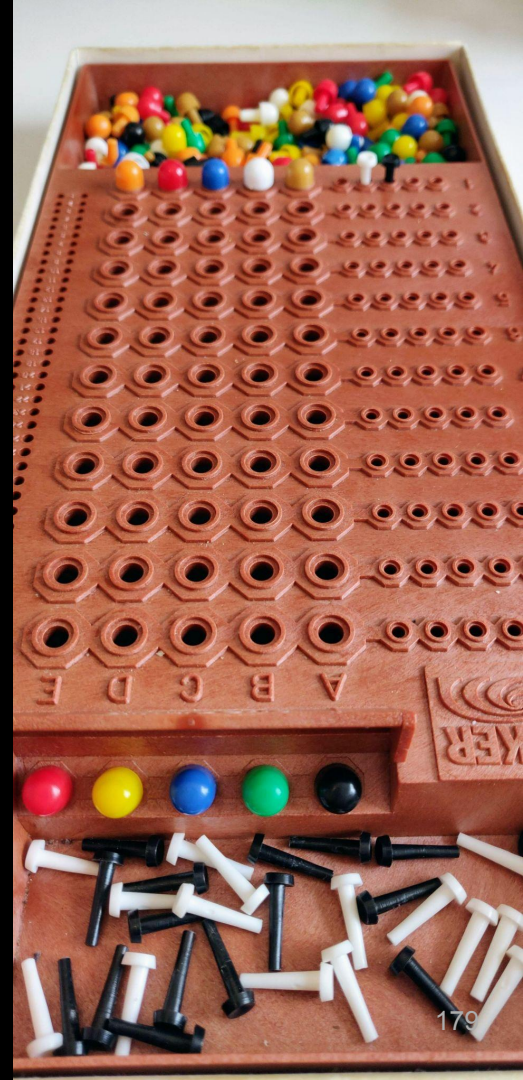
# possible permutations

$$N = 5^8 = 390625$$

# maximum entropy in bits

$$H_{max} = \log_2 5^8 = 18.575$$

hints reduce uncertainty (entropy)



first guess

opponent's hints



A

B

C

D

E

first guess

opponent's hints

correct



A

B

C

D

E



first guess

correct



A



B



C



D



E

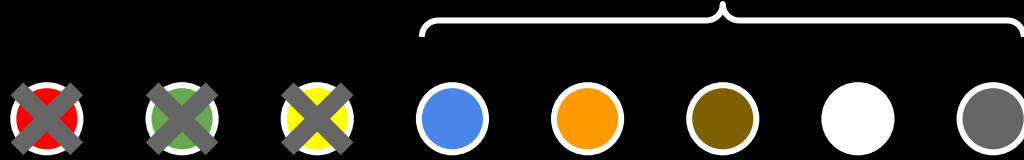
opponent's hints



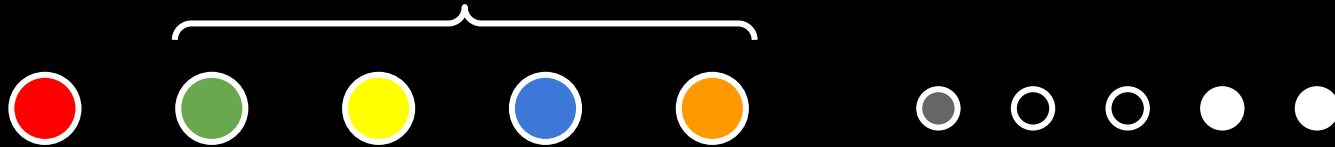
how much information did we get?



colors left



slots left



A

B

C

D

E

# new possible permutations?

$$N = 4^8 = 65536$$

# reduced entropy in bits?

$$H = \log_2 4^8 = 16$$