

PROBLEM SOLVING

[BACK](#)

Pólya's approach to problem-solving



image source: <http://doi.org/10.3932/ethz-a-000099441>

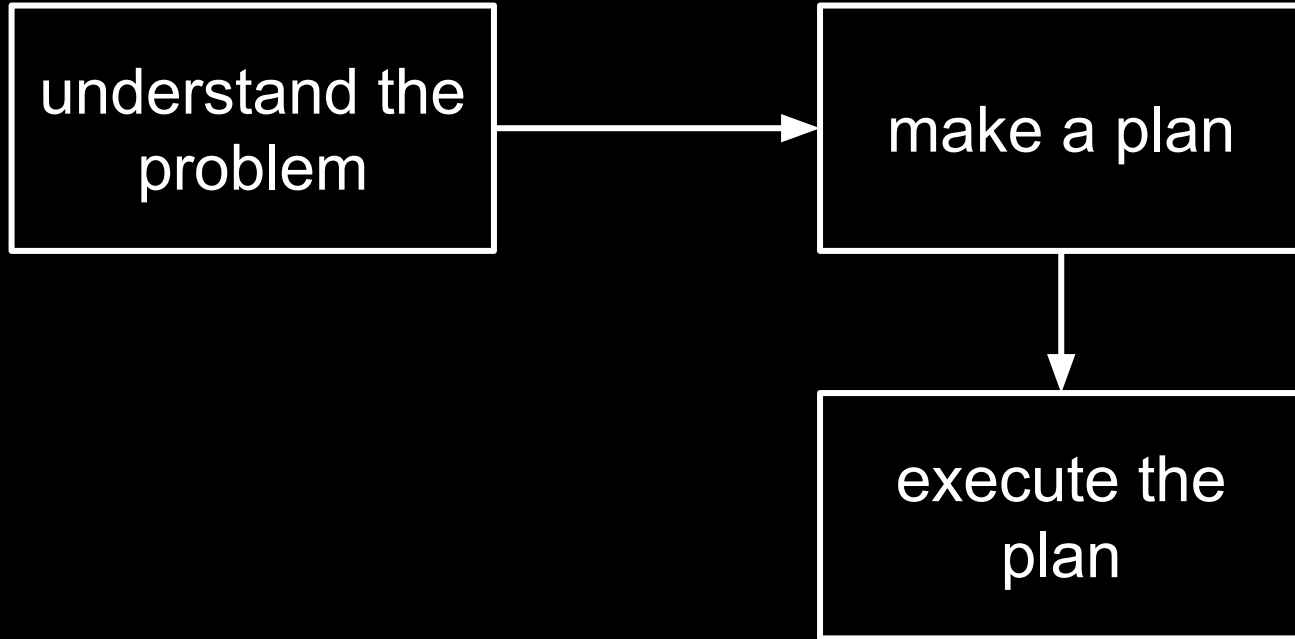
Pólya's approach to problem-solving

understand the
problem

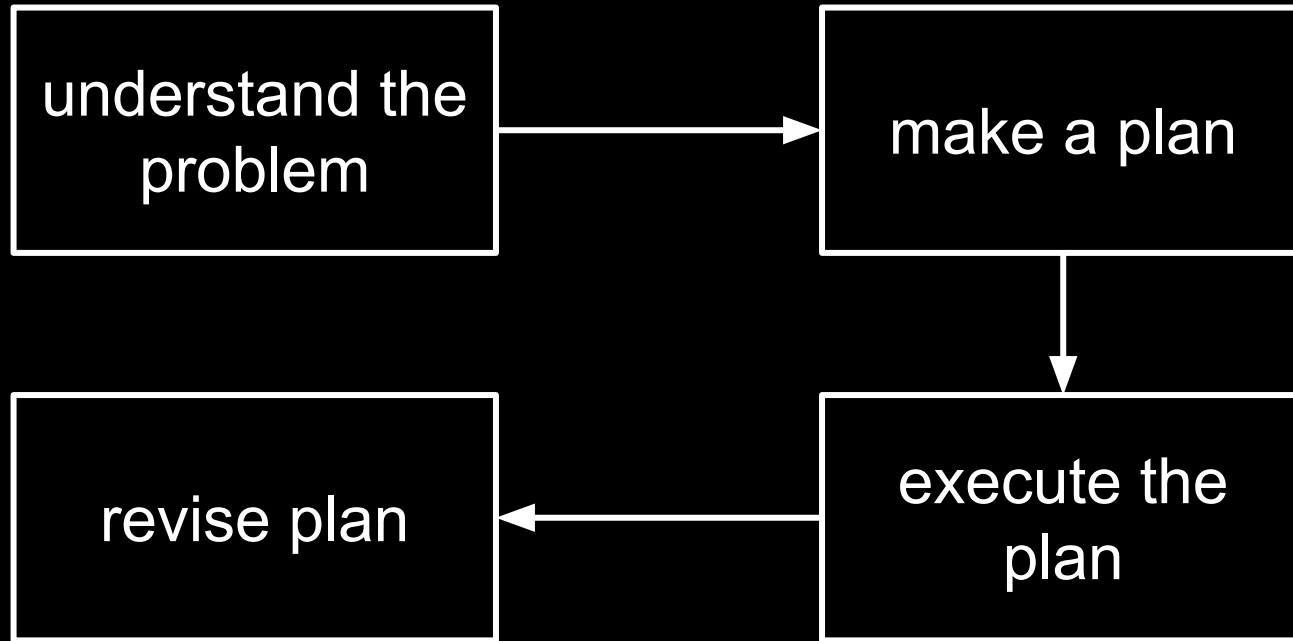
Pólya's approach to problem-solving



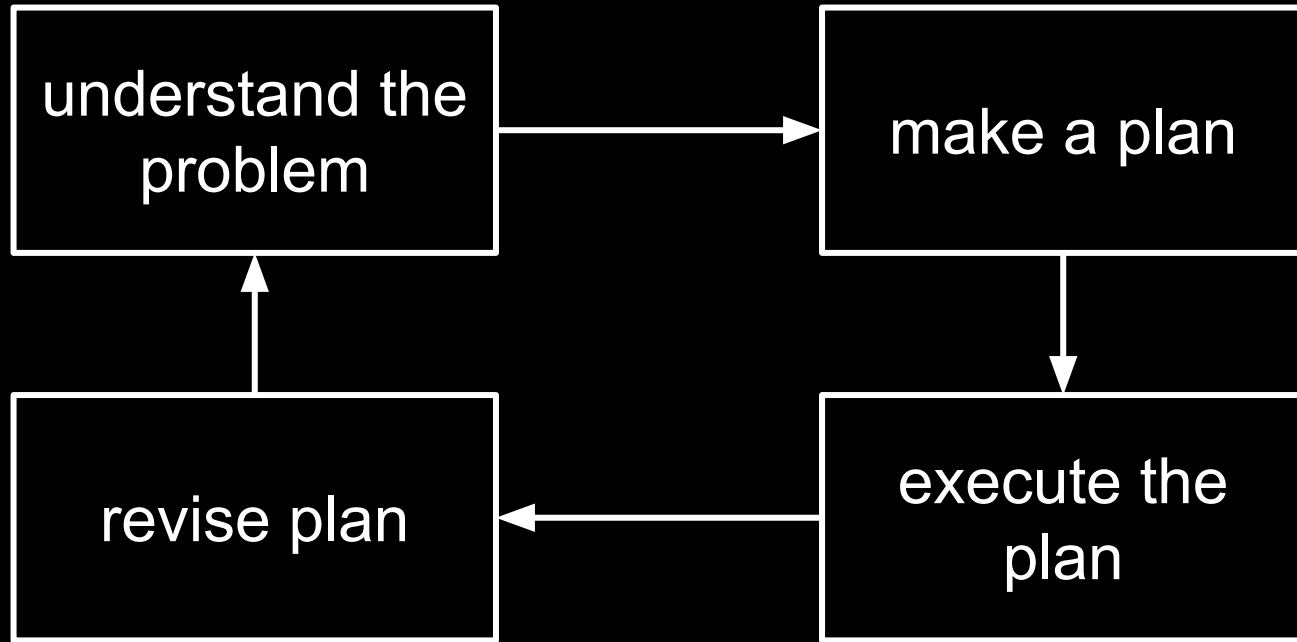
Pólya's approach to problem-solving



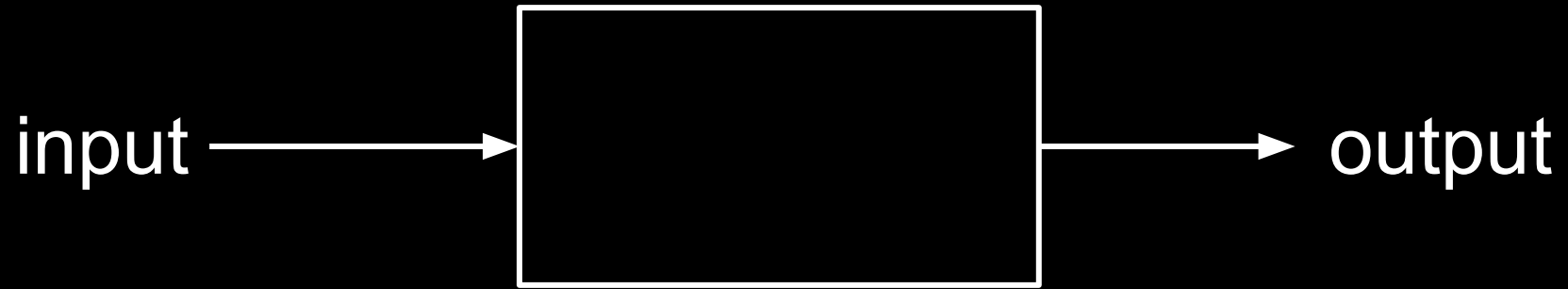
Pólya's approach to problem-solving



Pólya's approach to problem-solving



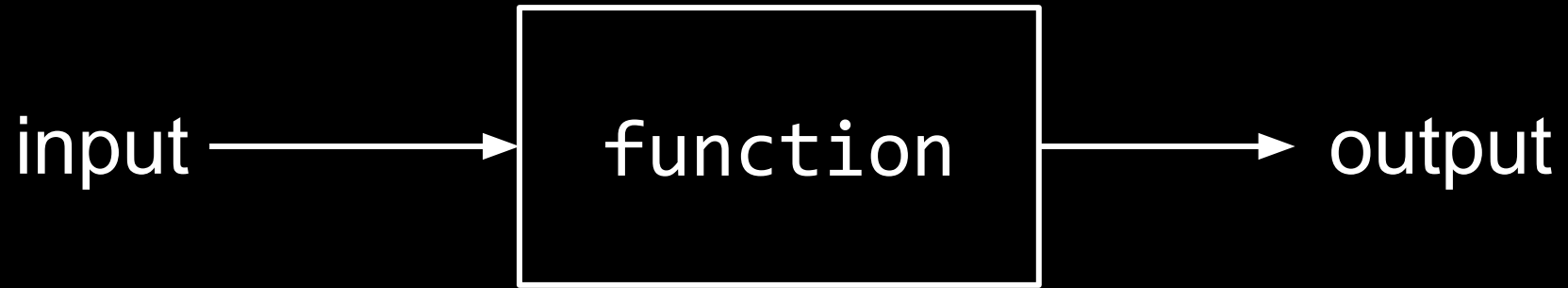
a model to represent problems



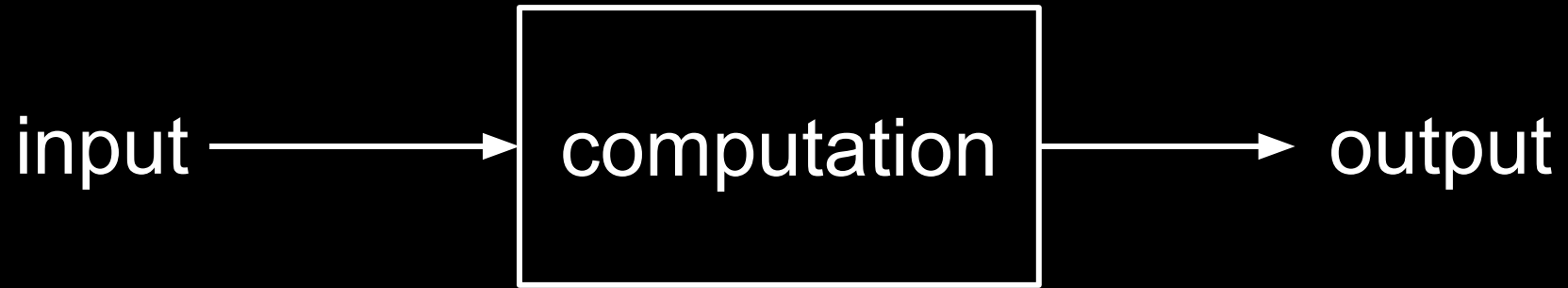
a model to represent problems



a model to represent problems

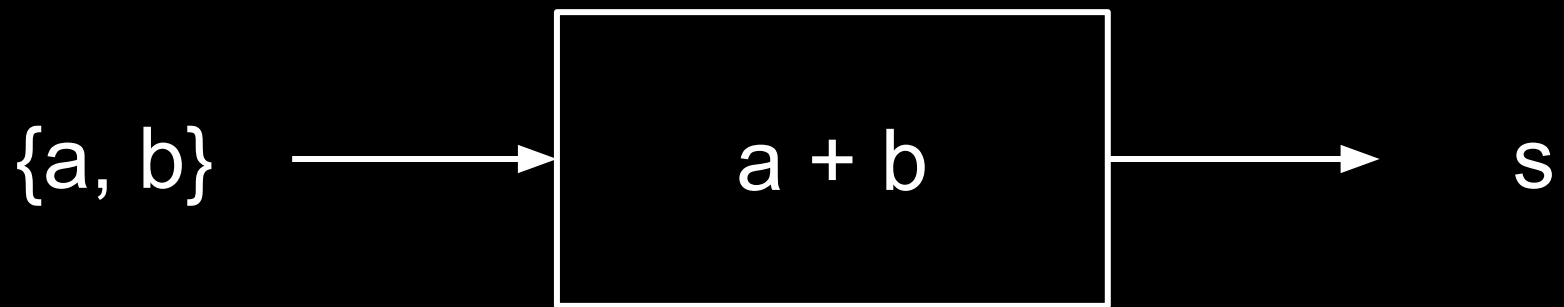


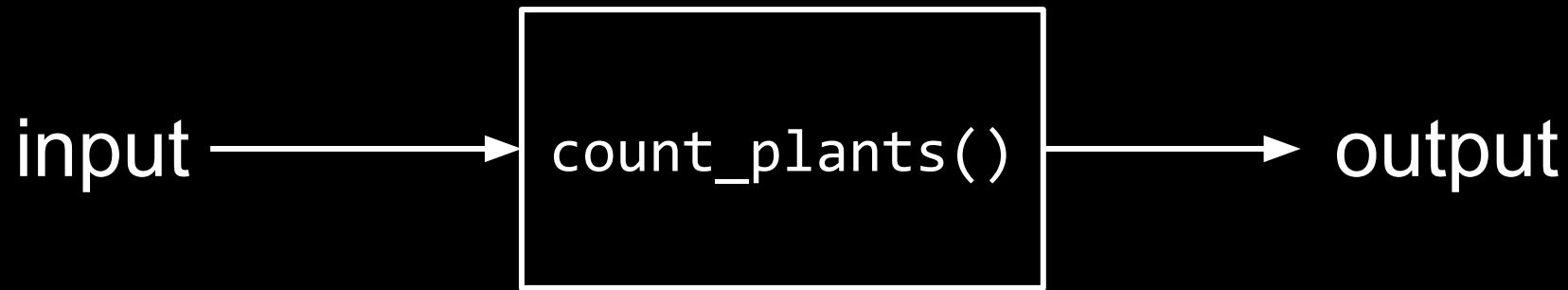
a model to represent problems

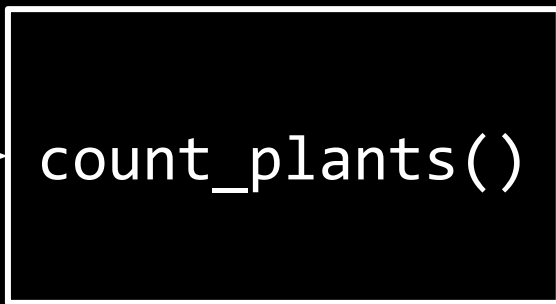


the rainbow experiment as an
input - processing - output - problem

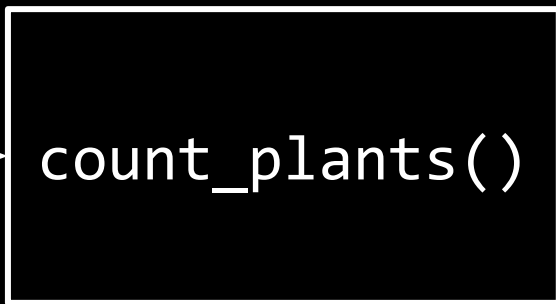






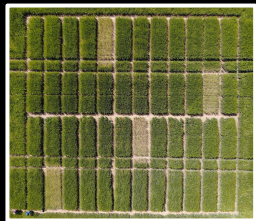


output



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processing of
information



`count_plants()`

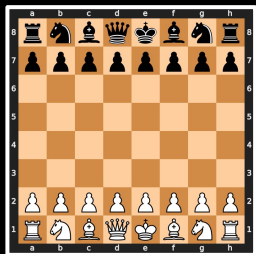
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representation of
information



next_move()

E2 → E4



1: 0R	9: 0P	57: 1R
2: 0N	10: 0P	58: 1N
3: 0B	11: 0P	59: 1B
4: 0K	12: 0P	60: 1K
5: 0Q	13: 0P	61: 1Q
6: 0B	14: 0P	62: 1B
7: 0N	15: 0P	63: 1N
8: 0R	16: 0P	64: 1R

...

representation of information



problem solving strategies

problem decomposition

large and complex problem

less complex
subproblem

less complex
subproblem

less complex subproblem

less complex
subproblem

less complex
subproblem

less complex subproblem

less complex subproblem

less complex subproblem

divide and conquer

large and complex problem of type A

smaller problem
of type A

smaller problem
of type A

smaller problem
of type A

smaller problem
of type A

even smaller problem of type A	even smaller problem of type A
even smaller problem of type A	even smaller problem of type A
even smaller problem of type A	even smaller problem of type A
even smaller problem of type A	even smaller problem of type A

sorted list +
element



search()



yes / no

is 67 a prime number?

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

linear search



2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

linear search



2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

linear search



2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

linear search



2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

linear search

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

↑

19 steps... can't we do better?

2, 3, 5, 7, 11, ~~13~~, 17, ~~19~~, ~~23~~, ~~29~~, ~~31~~, ~~37~~, ~~41~~,
~~43~~, ~~47~~, ~~53~~, ~~59~~, ~~61~~, 67, 71, 73, 79, 83, 89, 97

↑

large and complex problem

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

large and complex
problem

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

smaller
problem

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41

smaller
problem

43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

binary search

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

binary search

67 != 41



2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

binary search

67 > 41



2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, ~~41~~,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

binary search

67 > 41



2, 3, 5, 7, ~~11~~, ~~13~~, 17, 19, ~~23~~, ~~29~~, 31, 37, ~~41~~,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

binary search

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97



67 != 71

binary search

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97



67 != 71

binary search

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97



$67 < 71$

binary search

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97



67 != 59

binary search

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
~~43~~, 47, ~~53~~, 59, 61, 67, ~~71~~, ~~73~~, 79, ~~83~~, ~~89~~, 97



67 > 59

binary search

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
~~43~~, ~~47~~, ~~53~~, ~~59~~, 61, 67, ~~71~~, ~~73~~, ~~79~~, ~~83~~, ~~89~~, 97



67 = 67

binary search

2, 3, 5, 7, 11, ~~13~~, 17, 19, ~~23~~, ~~29~~, 31, 37, 41,
~~43~~, 47, ~~53~~, 59, ~~61~~, 67, ~~71~~, ~~73~~, 79, ~~83~~, ~~89~~, 97



67 = 67

3 splits → much better

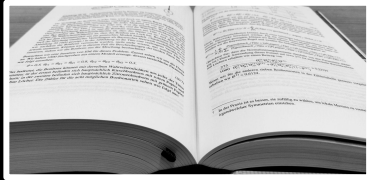
2, 3, 5, 7, 11, ~~13~~, 17, 19, ~~23~~, 29, 31, 37, 41,
~~43~~, 47, ~~53~~, 59, ~~61~~, 67, ~~71~~, ~~73~~, 79, ~~83~~, ~~89~~, 97



$$67 = 67$$



how efficient are linear and
binary search in general?



`count_words()`

word count

$\theta = 0.5$, $\theta_{T_1} = \theta_{R_1} = \theta_{I_1} = 0.8$, $\theta_{L_2} = \theta_{W_2} = \theta_{F_2} = 0.3$.

Zus bedeutet, die Bonbons können mit derselben Wahrscheinlichkeit aus jeder der Tüten ummen; in der ersten befinden sich hauptsächlich Kirschbonbons mit rotem Papier und ohne Löcher. Die Zähler für die acht möglichen Bonbonarten sehen wie folgt aus:

(20.7)

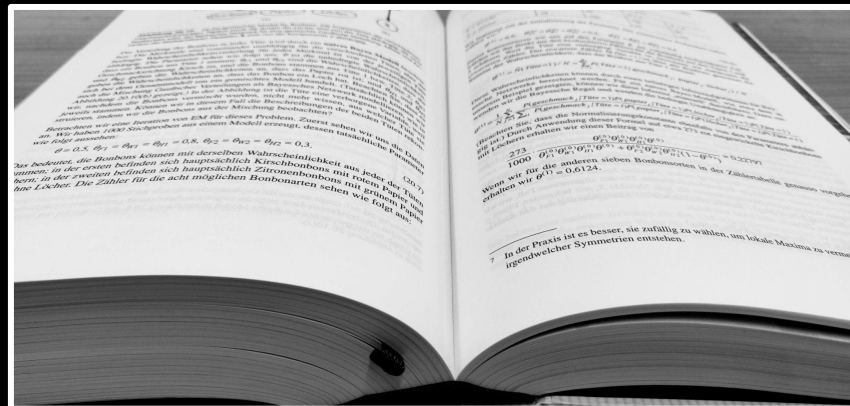
Diese Wahrscheinlichkeiten können durch einen kombinatorischen Ausdruck dargestellt werden. Ein Beispiel zeigt den Zusammenhang zwischen dem Parameter θ_i und der Anzahl der Bonbons einer bestimmten Art.

Wenn wir für die anderen sieben Bonbonsorten in der Zahlen Tabelle genauso vorgehen erhalten wir $\theta^{(1)} = 0.6124$.

In der Praxis ist es besser, sie zufällig zu wählen, um lokale Maxima zu vermeiden irgendwelcher Symmetrien entstehen.

[illegible]

strategies, anyone?

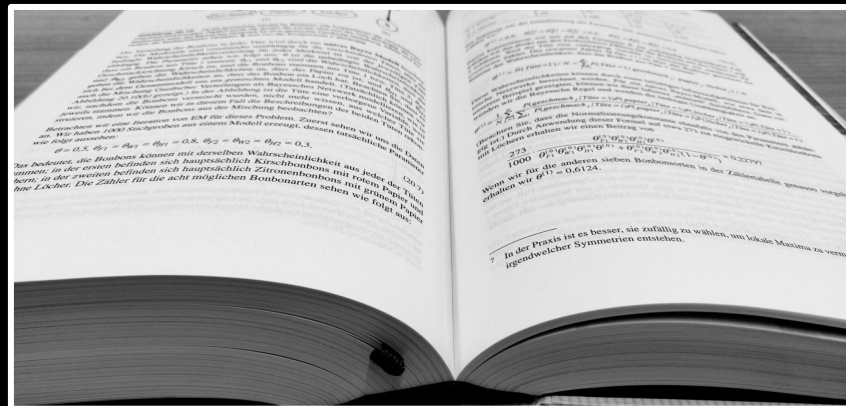


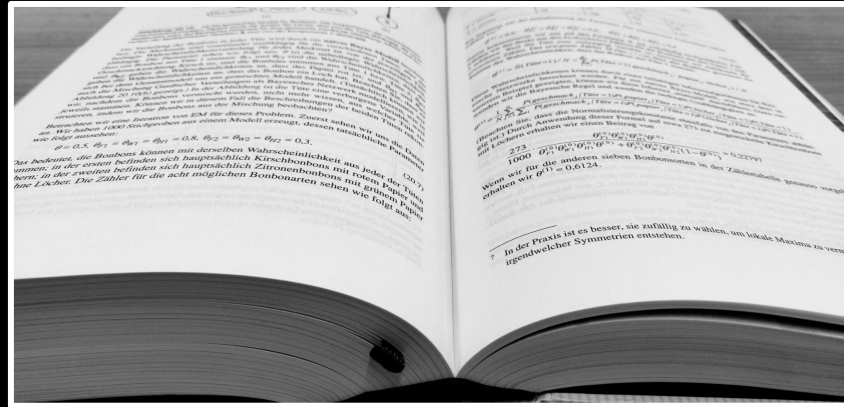
... das bedeutet, die Bonbons können mit derselben Wahrscheinlichkeit aus jeder der Urnen
kommen, in der ersten befinden sich hauptsächlich Zitronenbonbons mit einem Loch, in der
zweiten hauptsächlich Limettenbonbons mit einem Loch. Die Zähler für die acht möglichen Bonbonurten sehen wie folgt aus:

$$\theta_1 = 0.5, \theta_2 = 0.5, \theta_3 = 0.5, \theta_4 = 0.5, \theta_5 = 0.5, \theta_6 = 0.5, \theta_7 = 0.5, \theta_8 = 0.5.$$

Wenn wir für die anderen sieben Bonbonurten in der Zählertabelle gemessen werden
erhalten wir $\theta^{(1)} = 0.6124$.

In der Praxis ist es besser, sie zufällig zu wählen, um lokale Maxima zu vermeiden
irgendwelcher Symmetrien entstehen.

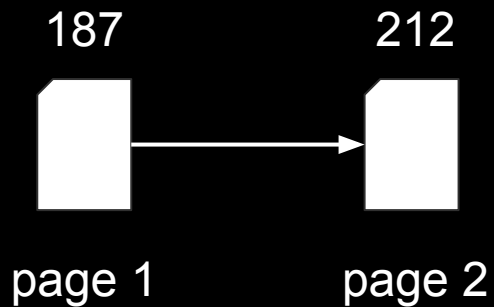
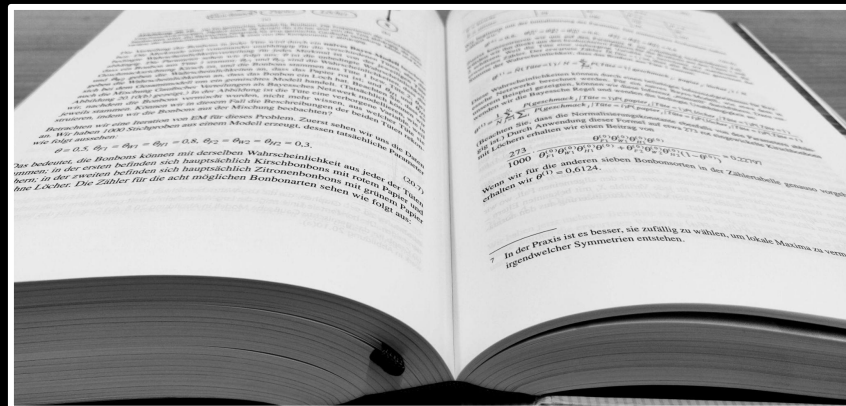


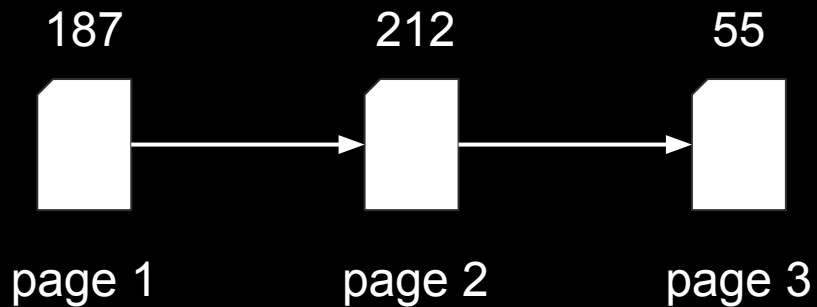
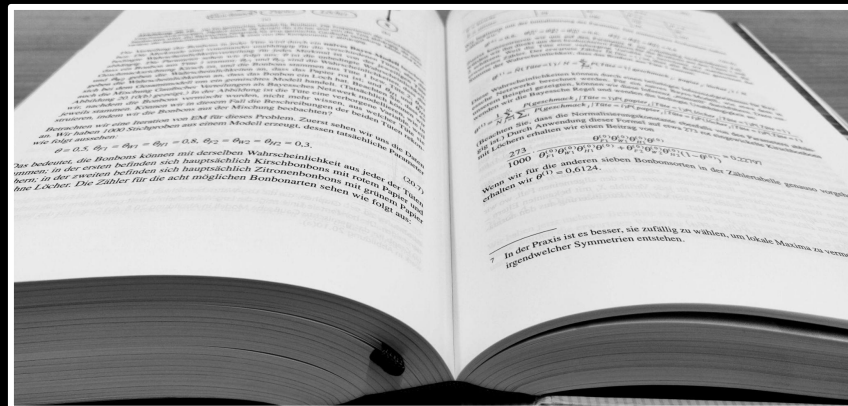


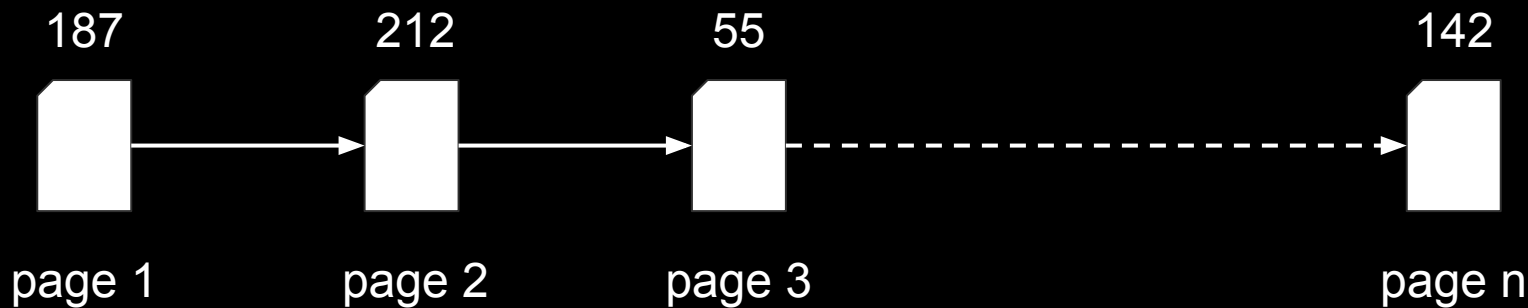
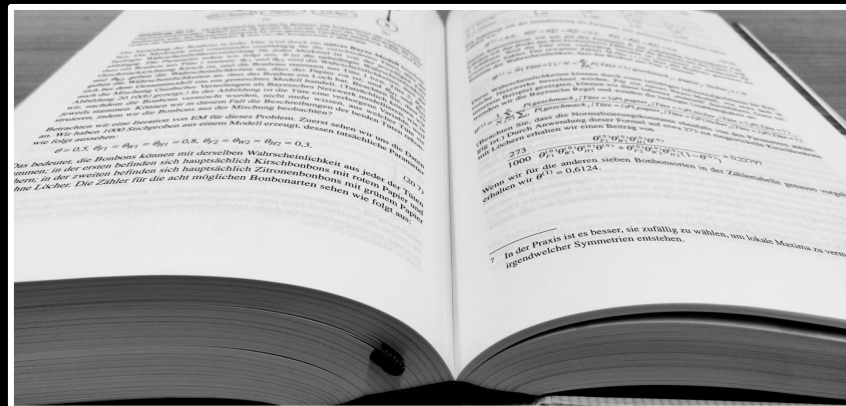
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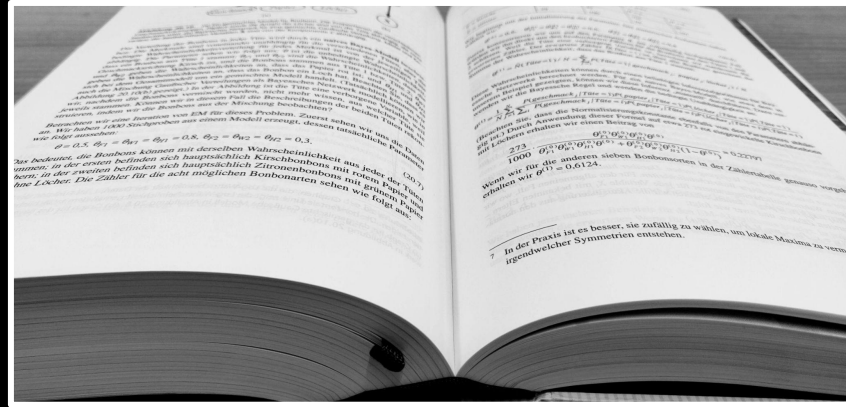


page 1









n = 1327 pages

Ø 2:23 minutes per page

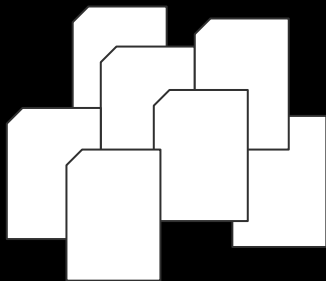
~ 52.34 hours

divide and conquer

+

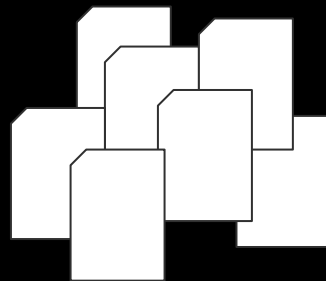
?

pages 1 - 700



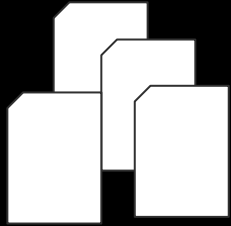
student 1

pages 701 - 1327



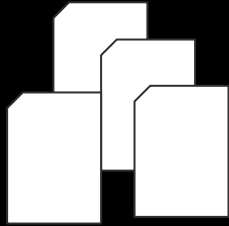
student 2

pages 1 - 350



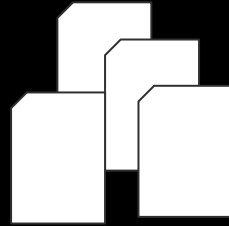
student 1

pages 351 - 700



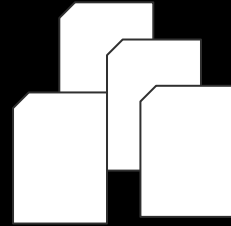
student 2

pages 701 - 1050



student 3

pages 1051- 1327



student 4

divide and conquer

+

distribution and parallelization

