INFORMATION

“Information is that which allows you to make a correct prediction with accuracy better than chance.”

Adami, Christoph. “What Is Information?” Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences, vol. 374, no. 2063, Mar. 2016, p. 20150230, https://doi.org/10.1098/rsta.2015.0230.

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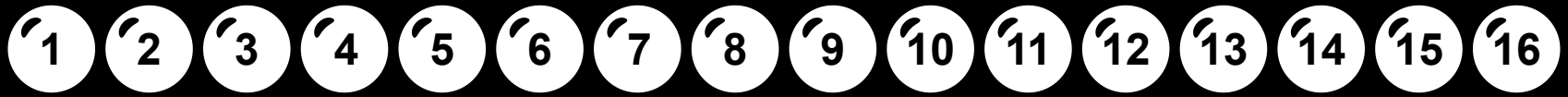
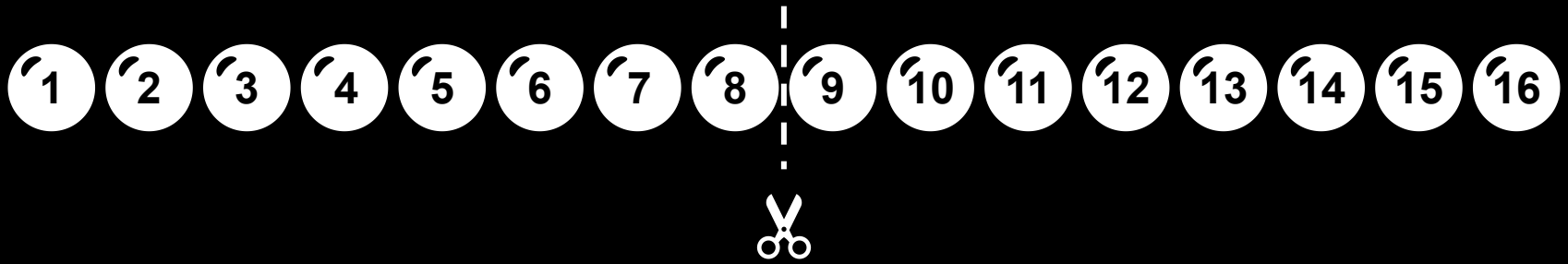
defining and measuring information

guess the number am I thinking of!

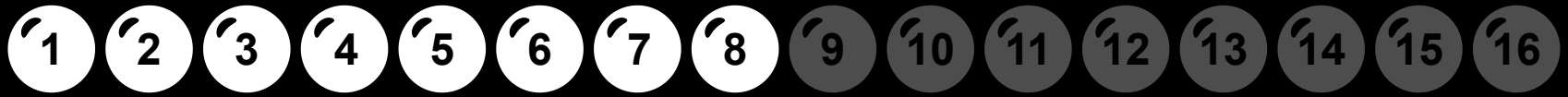
what is the most efficient approach?



is it > 8?

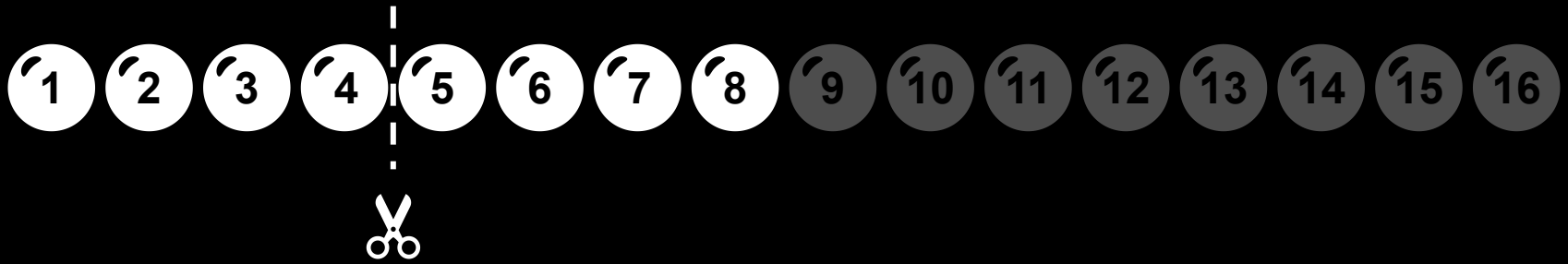


is it > 8? ✗



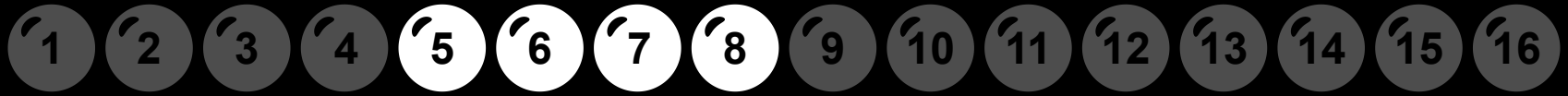
is it > 8? ✗

is it > 4?



is it > 8? ✗

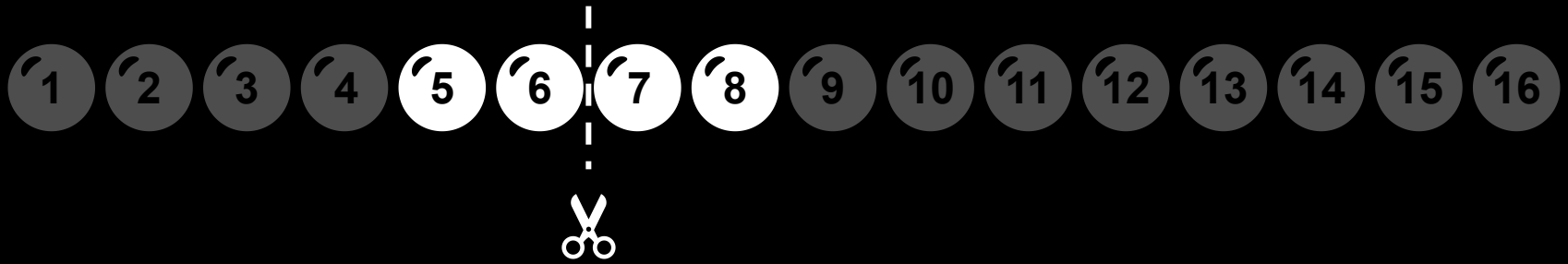
is it > 4? ✓



is it > 8? ✗

is it > 4? ✓

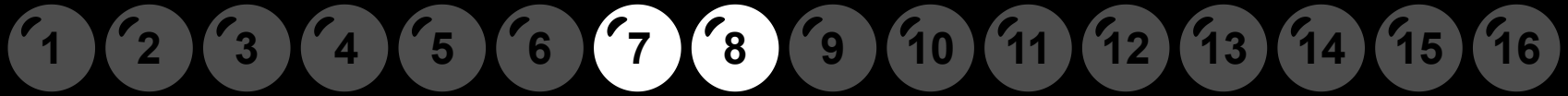
is it > 6?



is it > 8? ✗

is it > 4? ✓

is it > 6? ✓

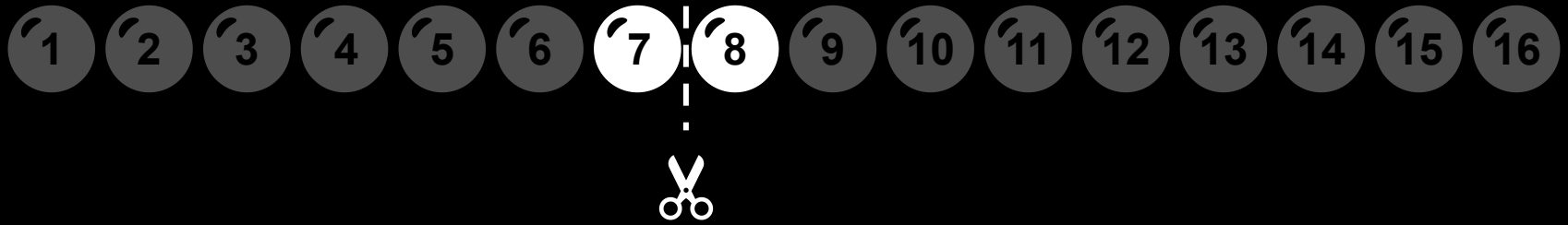


is it > 8? ✗

is it > 4? ✓

is it > 6? ✓

is it > 7?



is it > 8? ✗

is it > 4? ✓

is it > 6? ✓

is it > 7? ✗

is it > 8? ✗

is it > 4? ✓

is it > 6? ✓

is it > 7? ✗

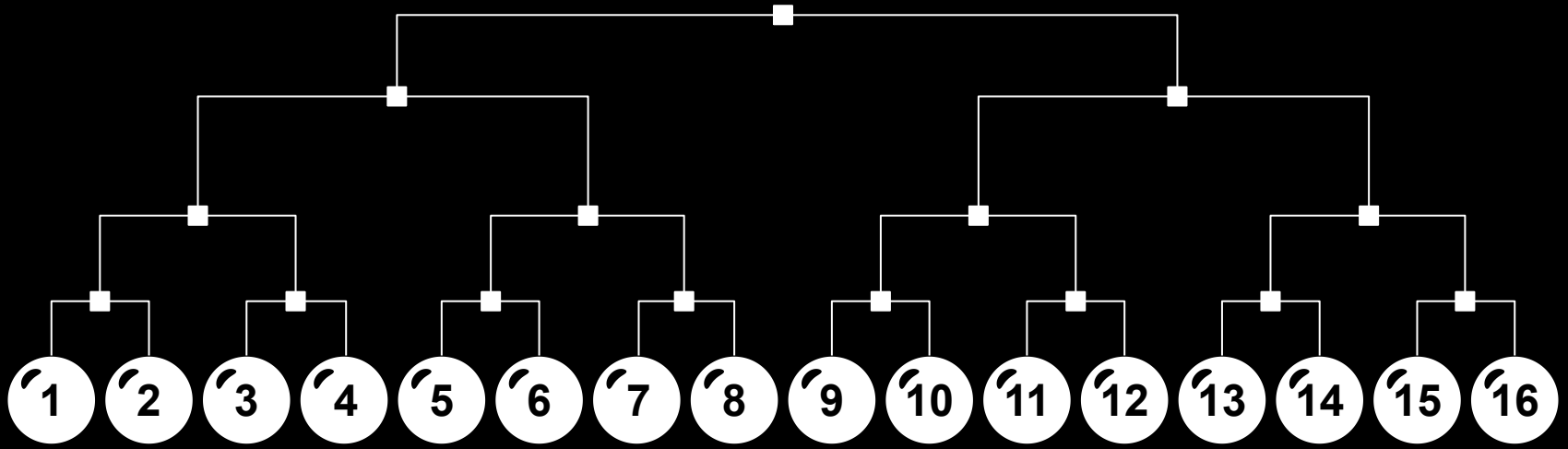
with 4 questions from 16 to 1

is it > 8? ✗ 0
is it > 4? ✓ 1
is it > 6? ✓ 1
is it > 7? ✗ 0

with 4 questions from 16 to 1

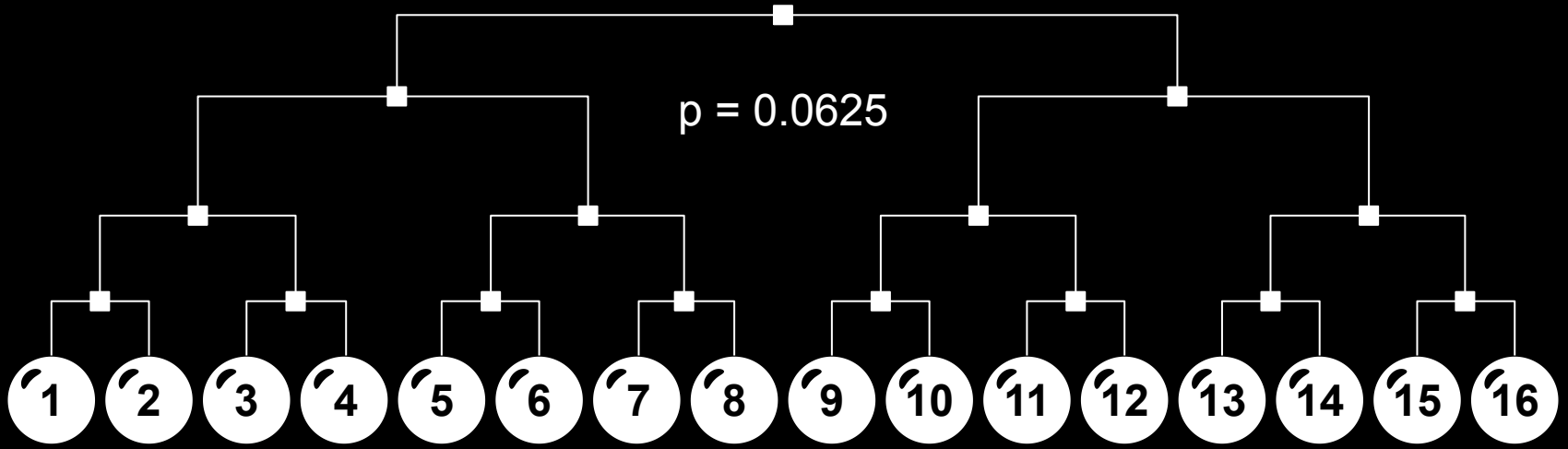
where is the information?

where is the information?



where is the information?

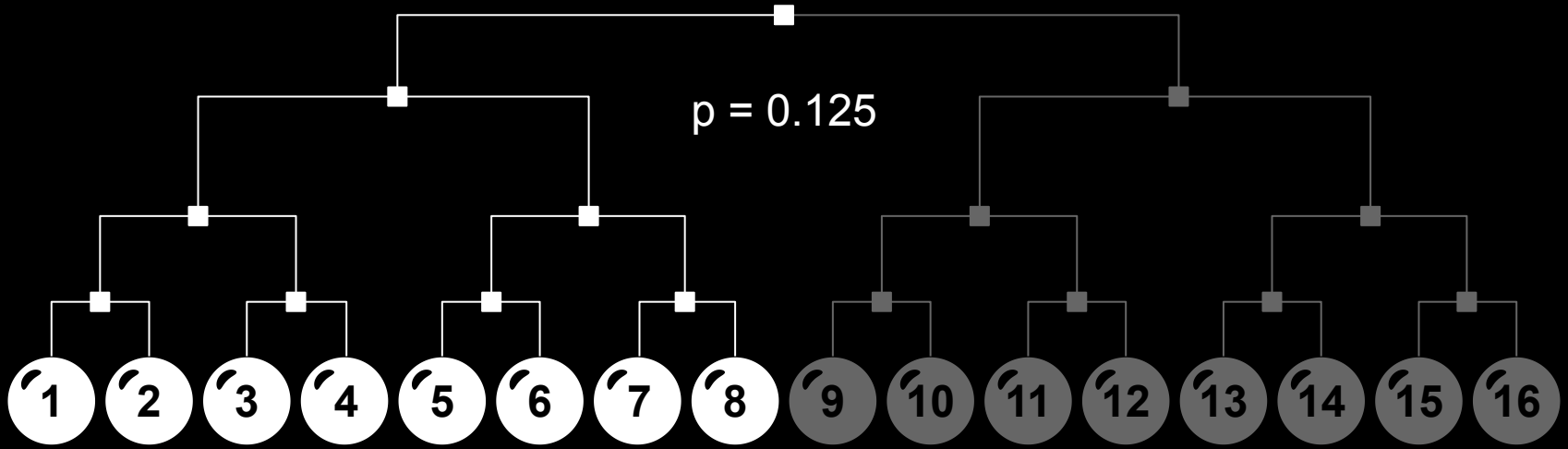
N = 16



where is the information?

~~N = 16~~

N = 8

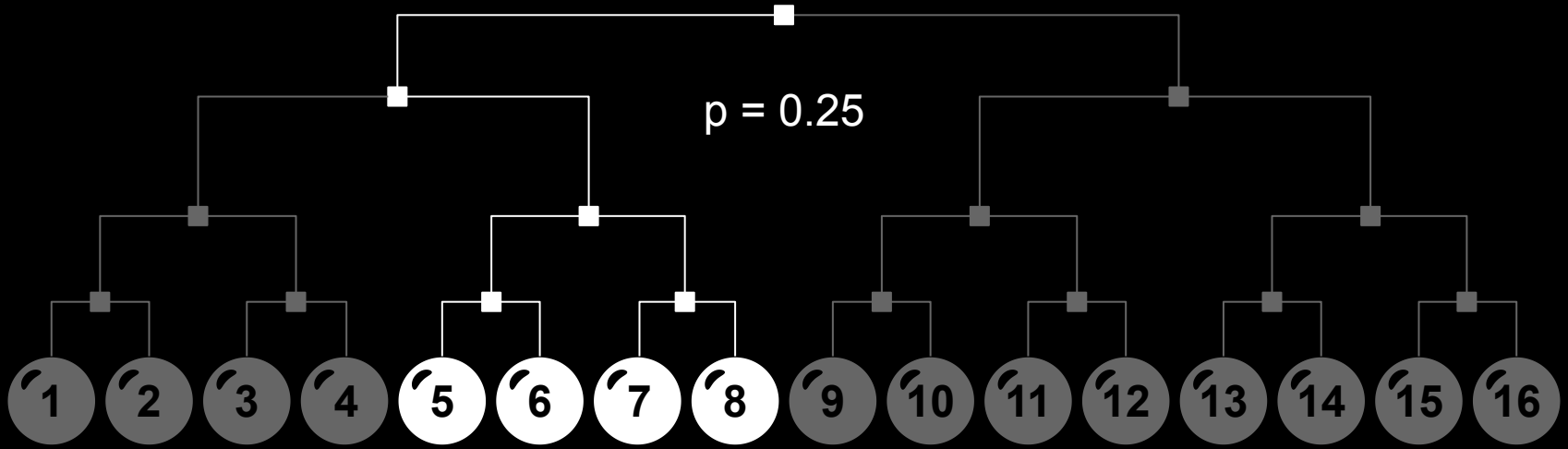


where is the information?

~~N = 16~~

~~N = 8~~

N = 4



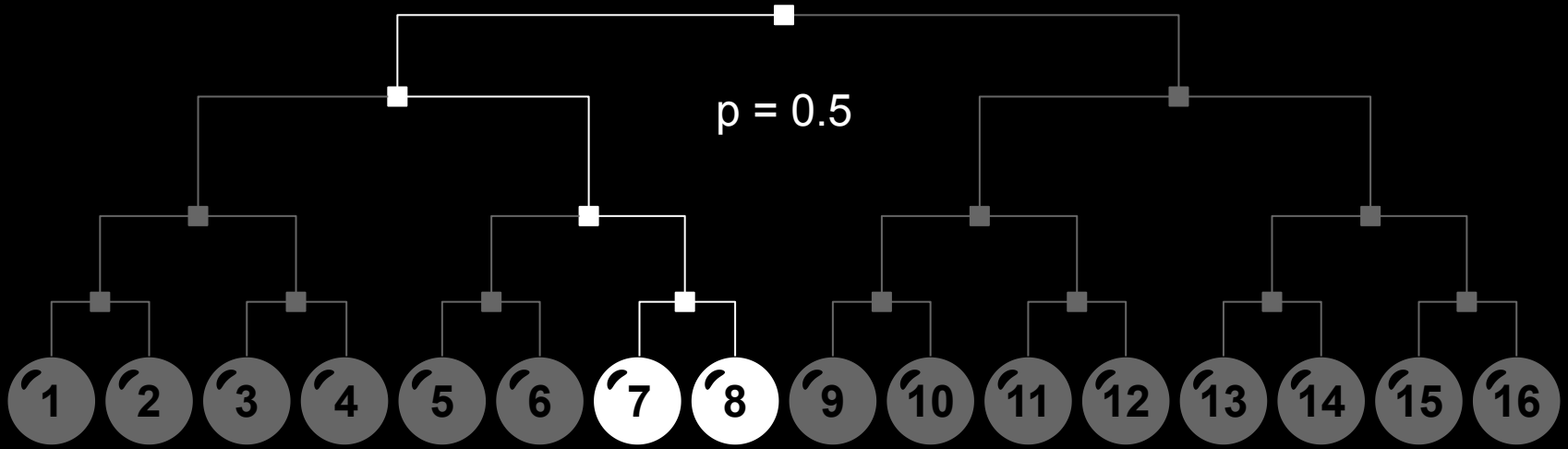
where is the information?

~~N = 16~~

~~N = 8~~

~~N = 4~~

N = 2



where is the information?

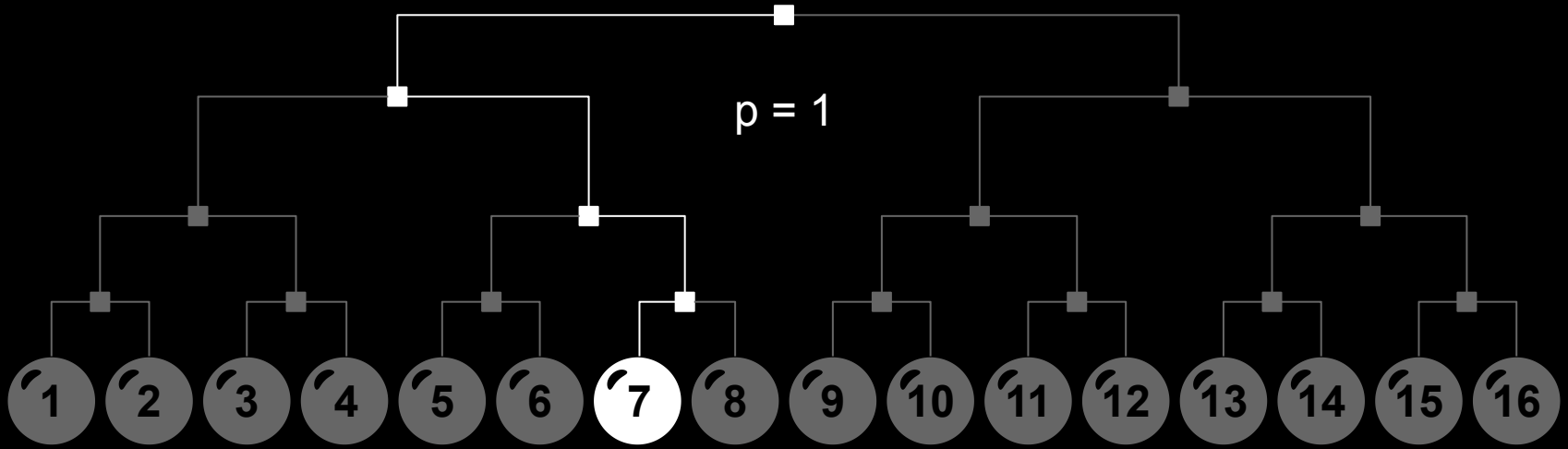
~~N = 16~~

~~N = 8~~

~~N = 4~~

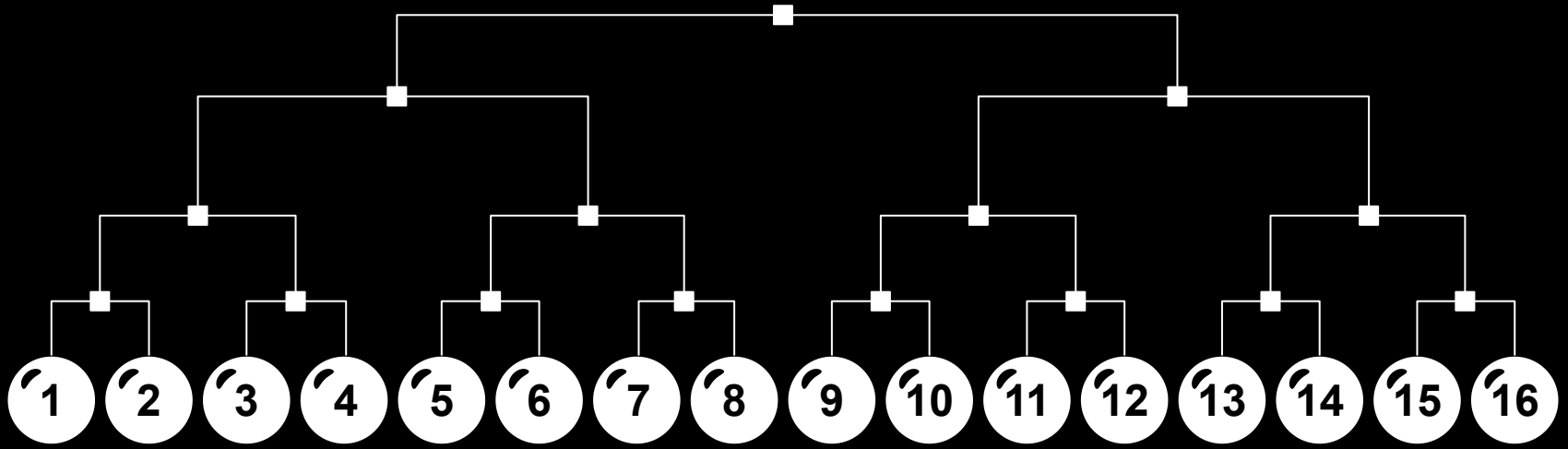
~~N = 2~~

N = 1



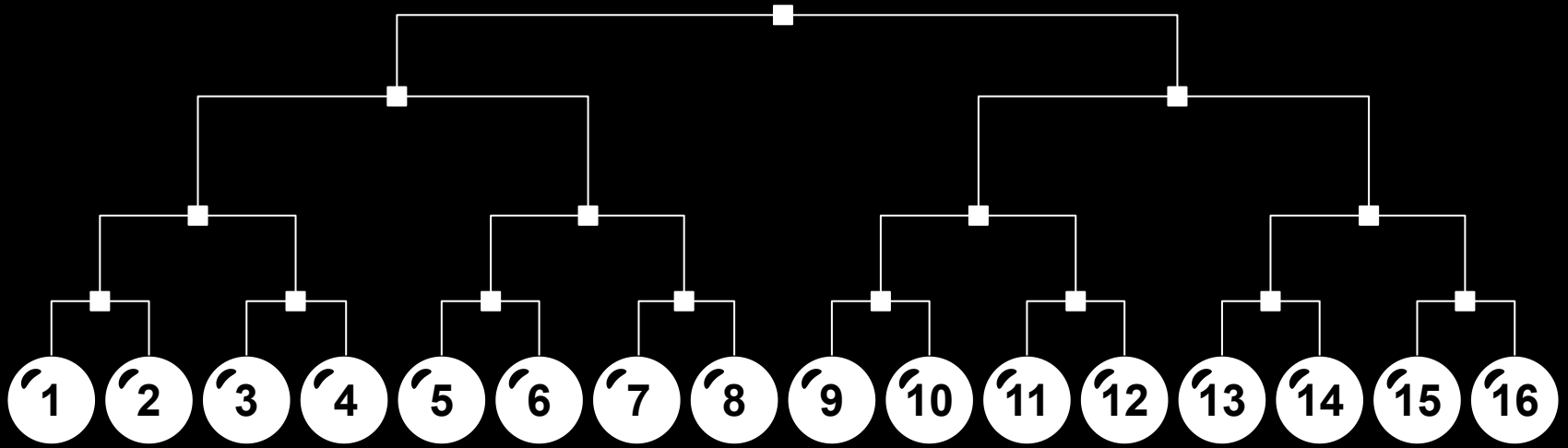
information = reduced uncertainty
uncertainty is measured with the logarithm of N

$$H = log_2(N)$$

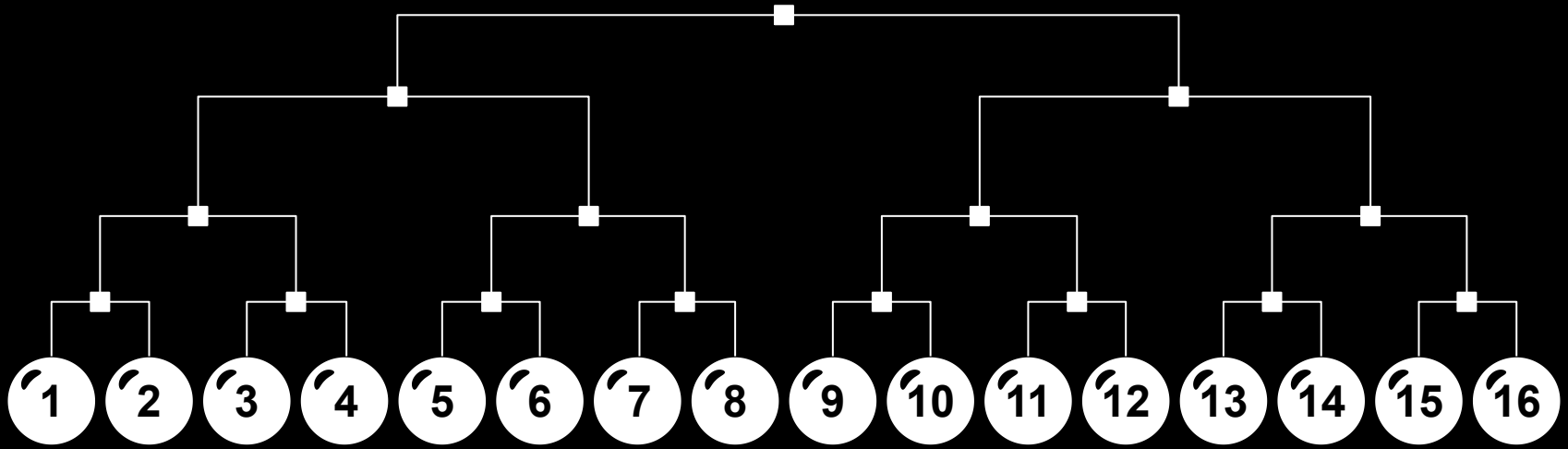


or: how often can we cut the remaining possibilities in half?

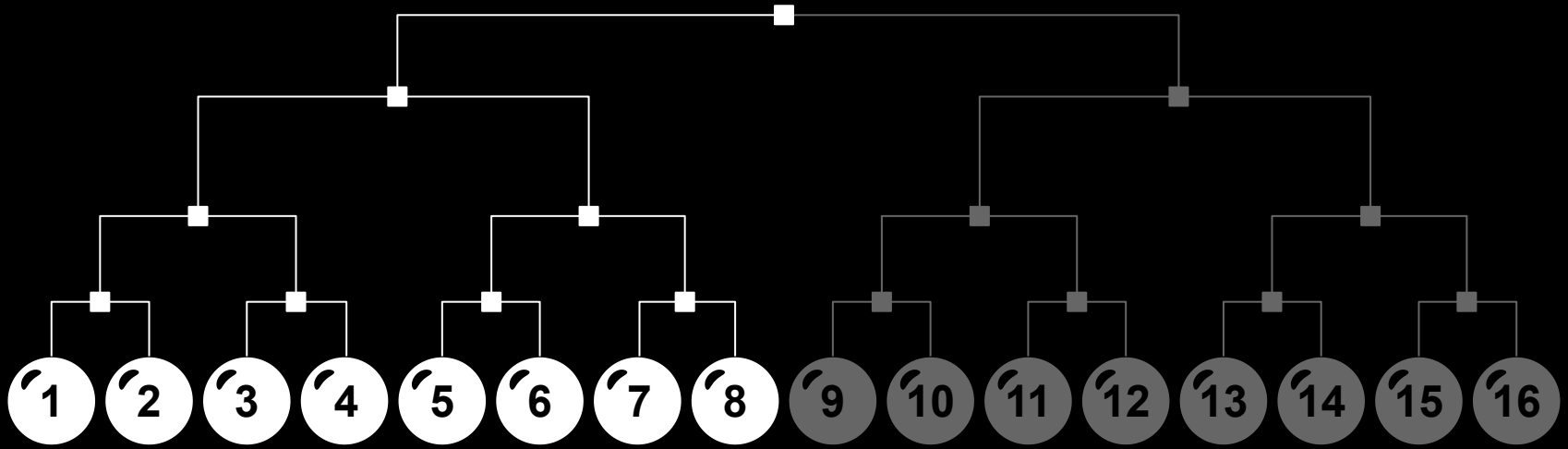
$$H = log_2(N)$$



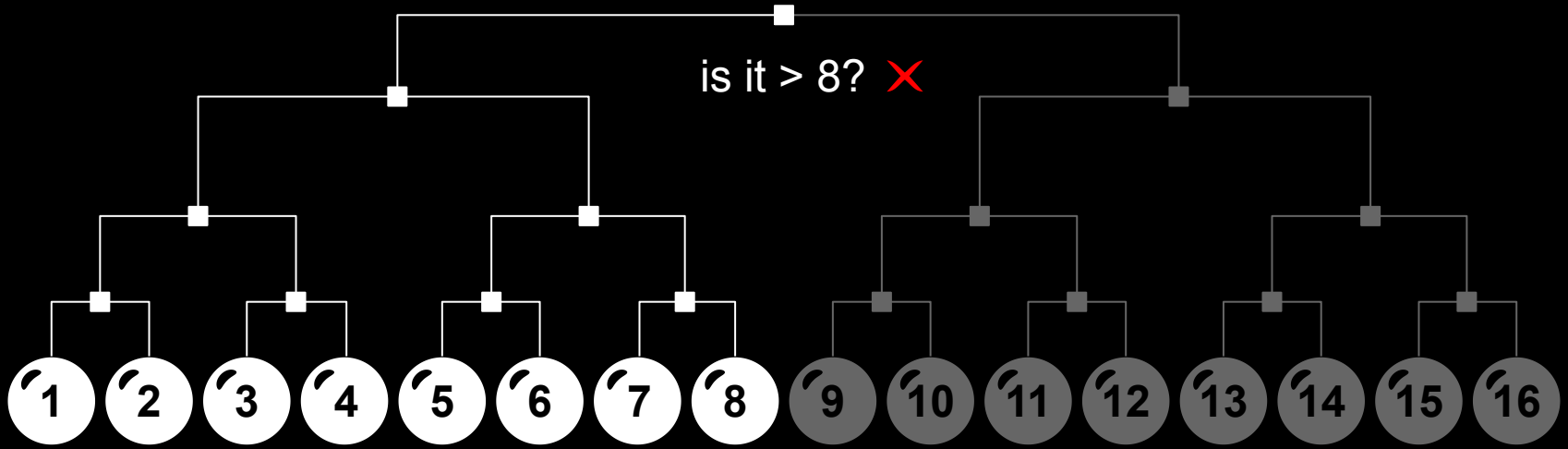
$$H_0 = log_2(16) = 4$$



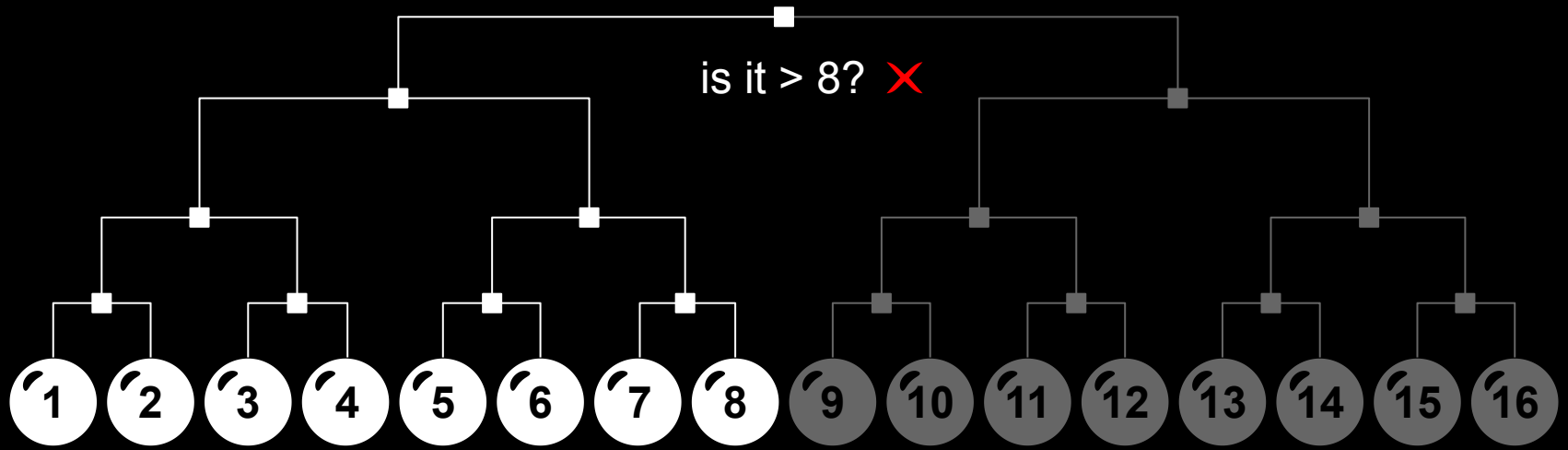
$$H_1 = log_2(8) = 3$$



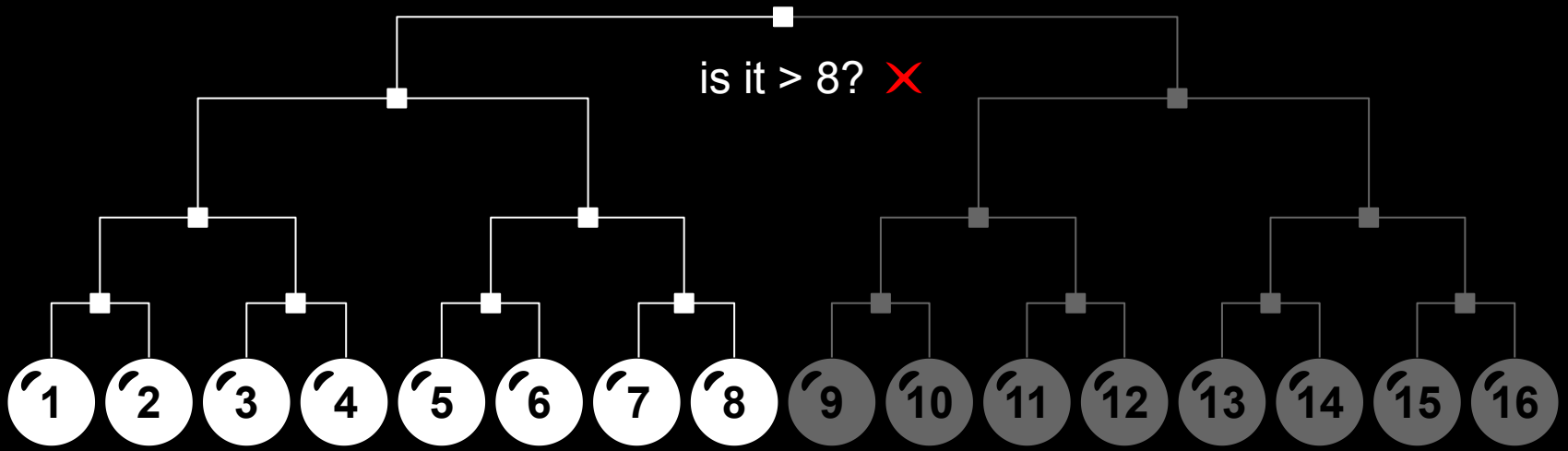
$$I = H_0 - H_1$$



$$I = log_2(16) - log_2(8)$$



$$I = 4 - 3 = 1$$



uncertainty and information are
measured in **bits**

how many yes/no questions to reduce uncertainty to zero?

$$H = 0 = log_2(1)$$

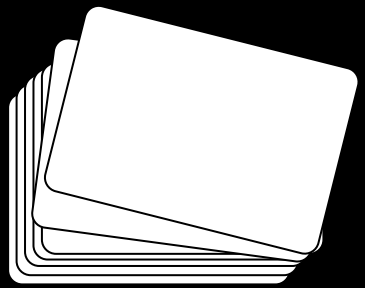
how many yes/no questions to reduce uncertainty to zero?

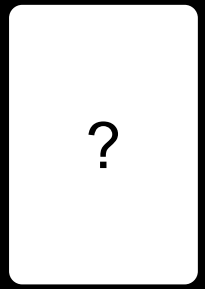
$$H = 0 = log_2(1)$$

$$H = log_2(N)$$

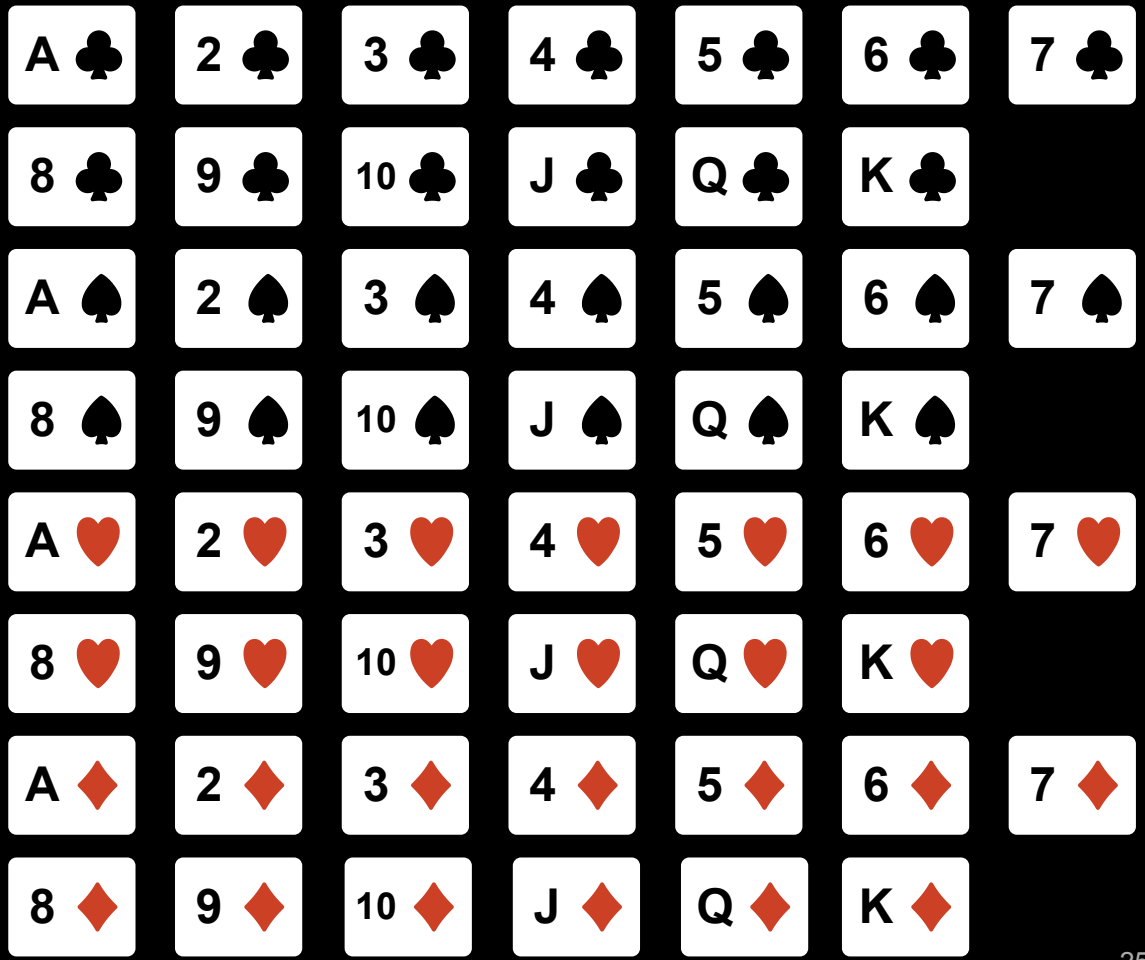
poker

which card am I holding?

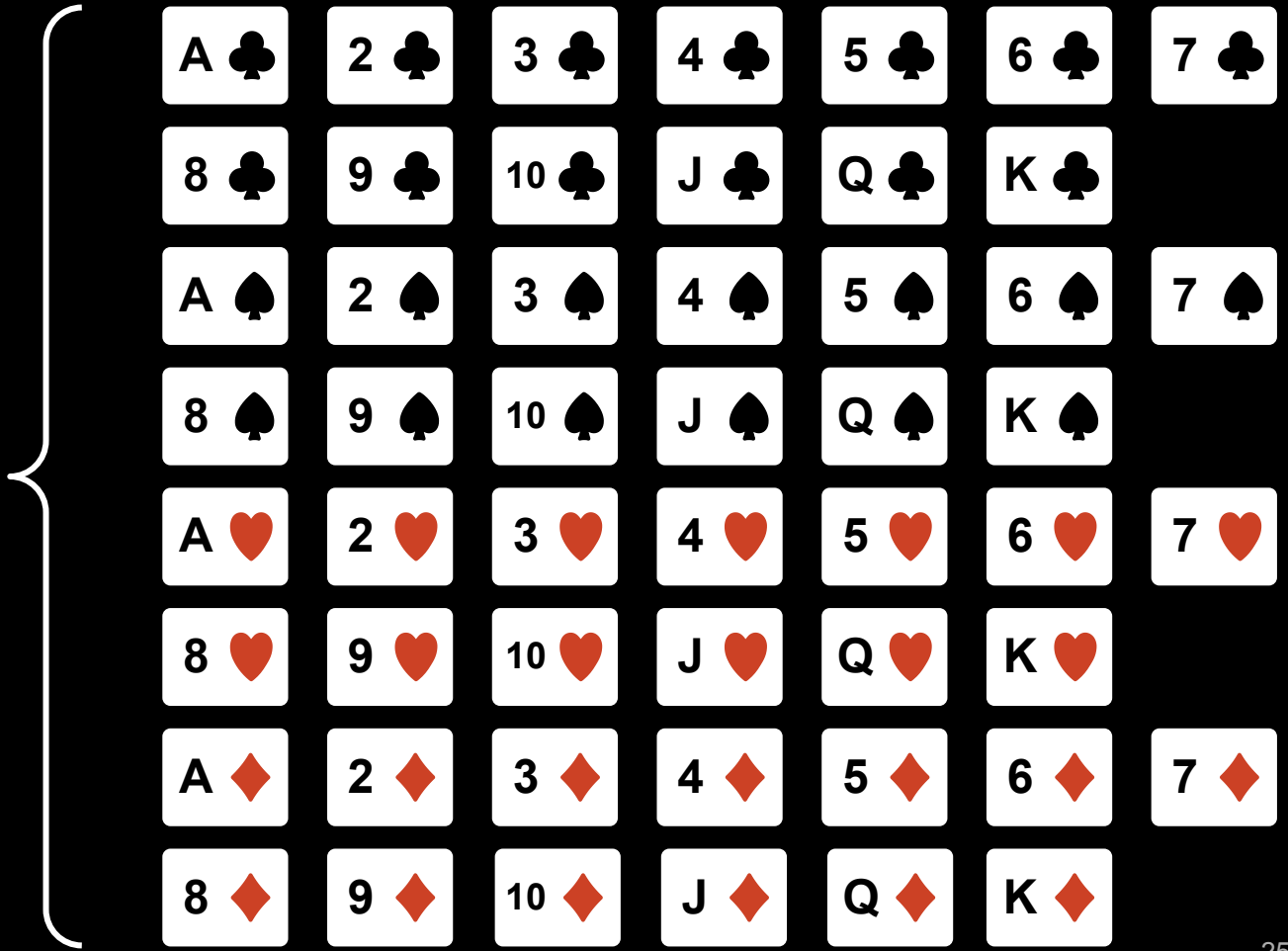




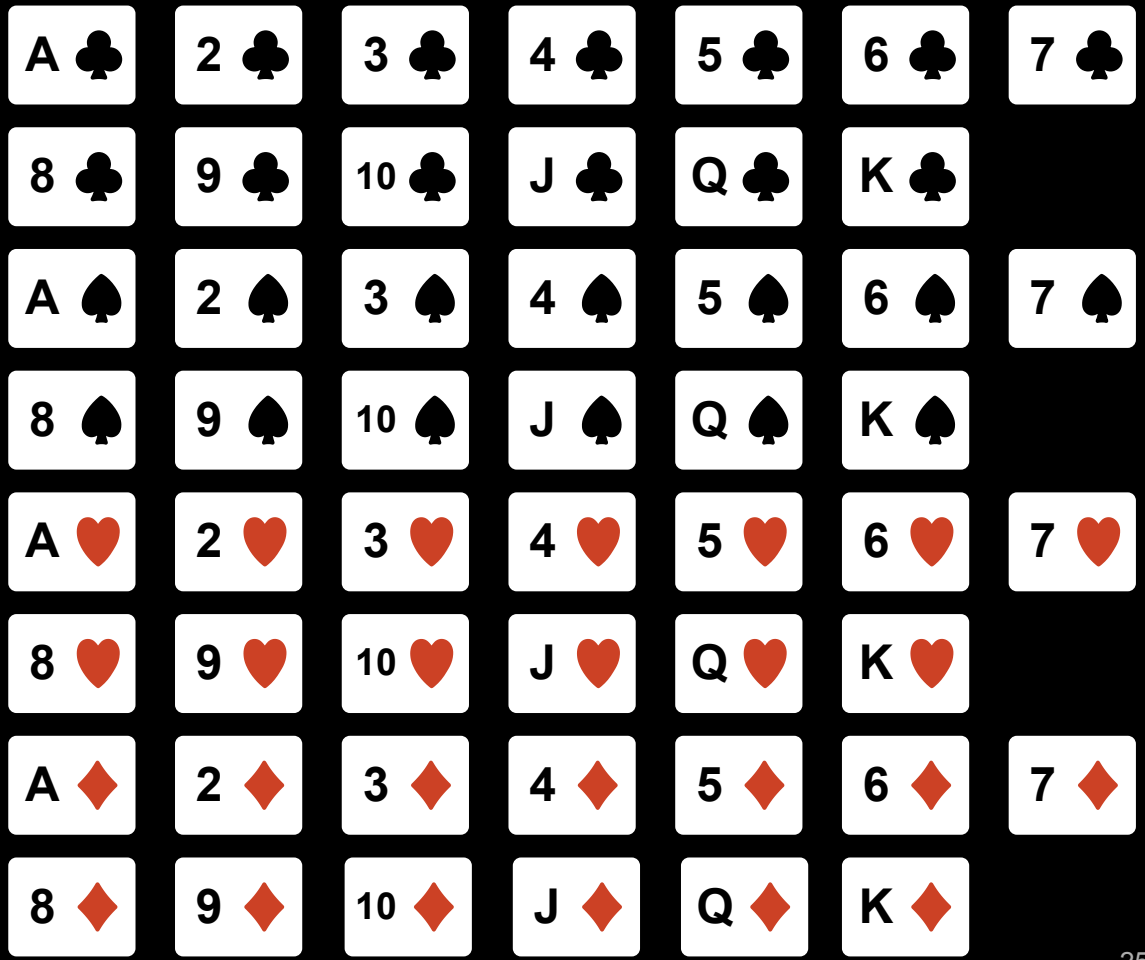
52 card poker deck


A ♣ 2 ♣ 3 ♣ 4 ♣ 5 ♣ 6 ♣ 7 ♣
8 ♣ 9 ♣ 10 ♣ J ♣ Q ♣ K ♣
A ♠ 2 ♠ 3 ♠ 4 ♠ 5 ♠ 6 ♠ 7 ♠
8 ♠ 9 ♠ 10 ♠ J ♠ Q ♠ K ♠
A ♥ 2 ♥ 3 ♥ 4 ♥ 5 ♥ 6 ♥ 7 ♥
8 ♥ 9 ♥ 10 ♥ J ♥ Q ♥ K ♥
A ♦ 2 ♦ 3 ♦ 4 ♦ 5 ♦ 6 ♦ 7 ♦
8 ♦ 9 ♦ 10 ♦ J ♦ Q ♦ K ♦

52 possible cards

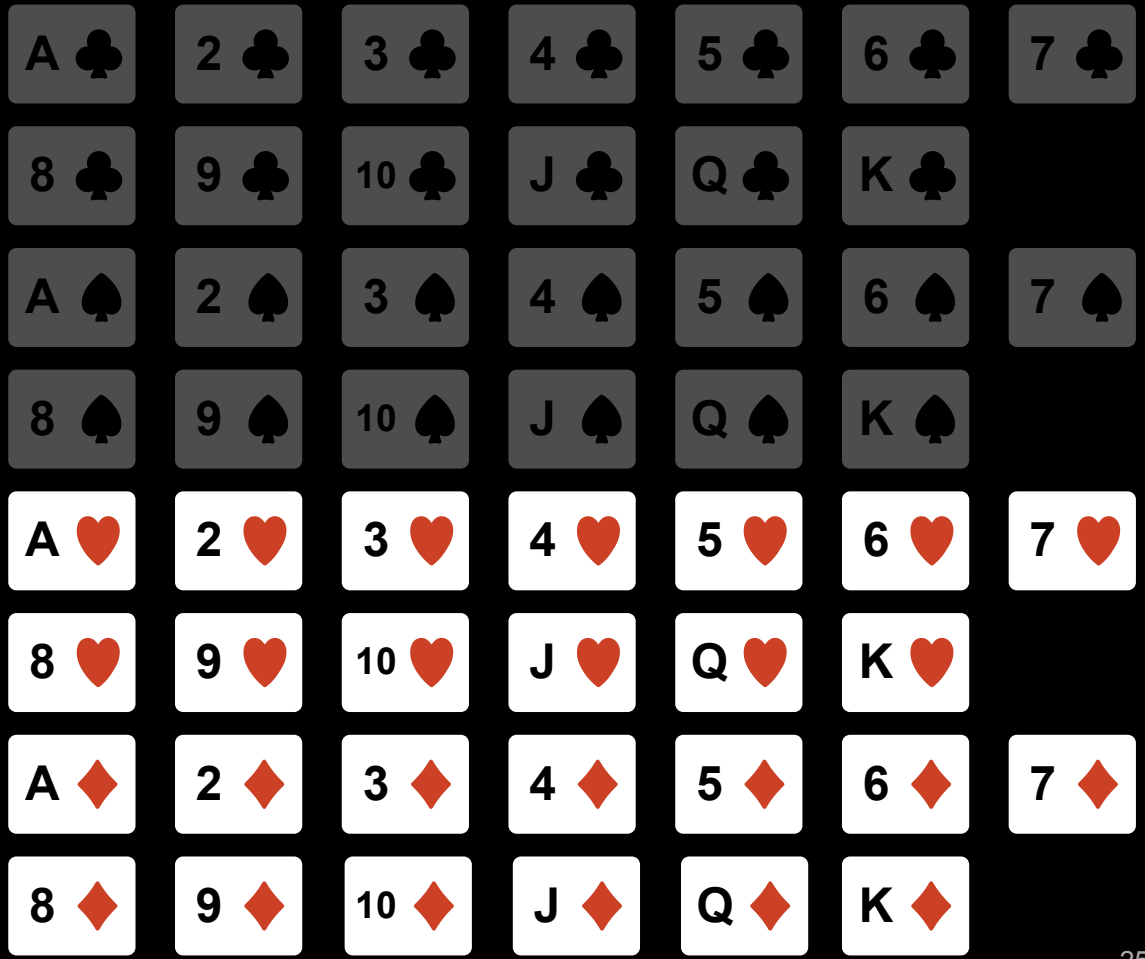


is the card black?



is the card black?

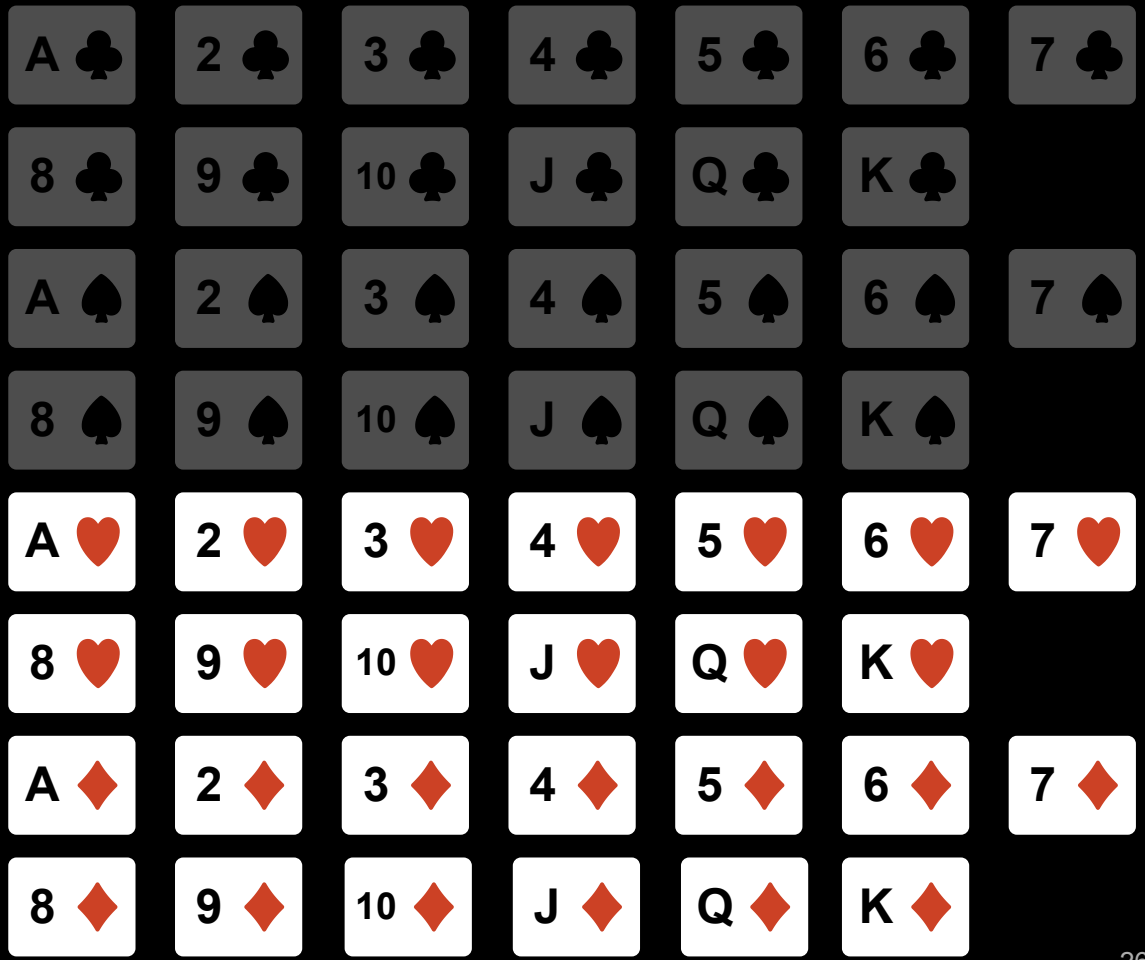
no



is the card black?

no

is it hearts?

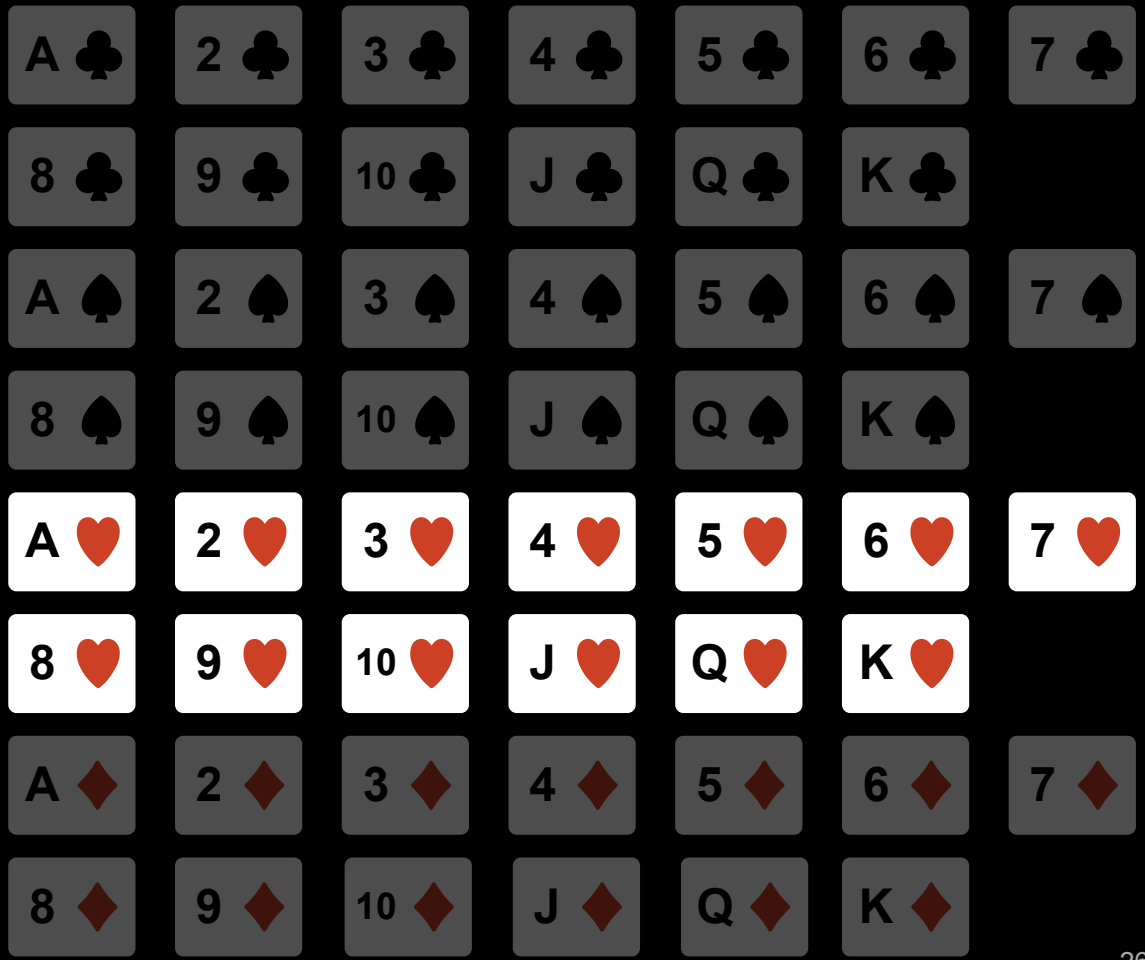


is the card black?

no

is it hearts?

yes



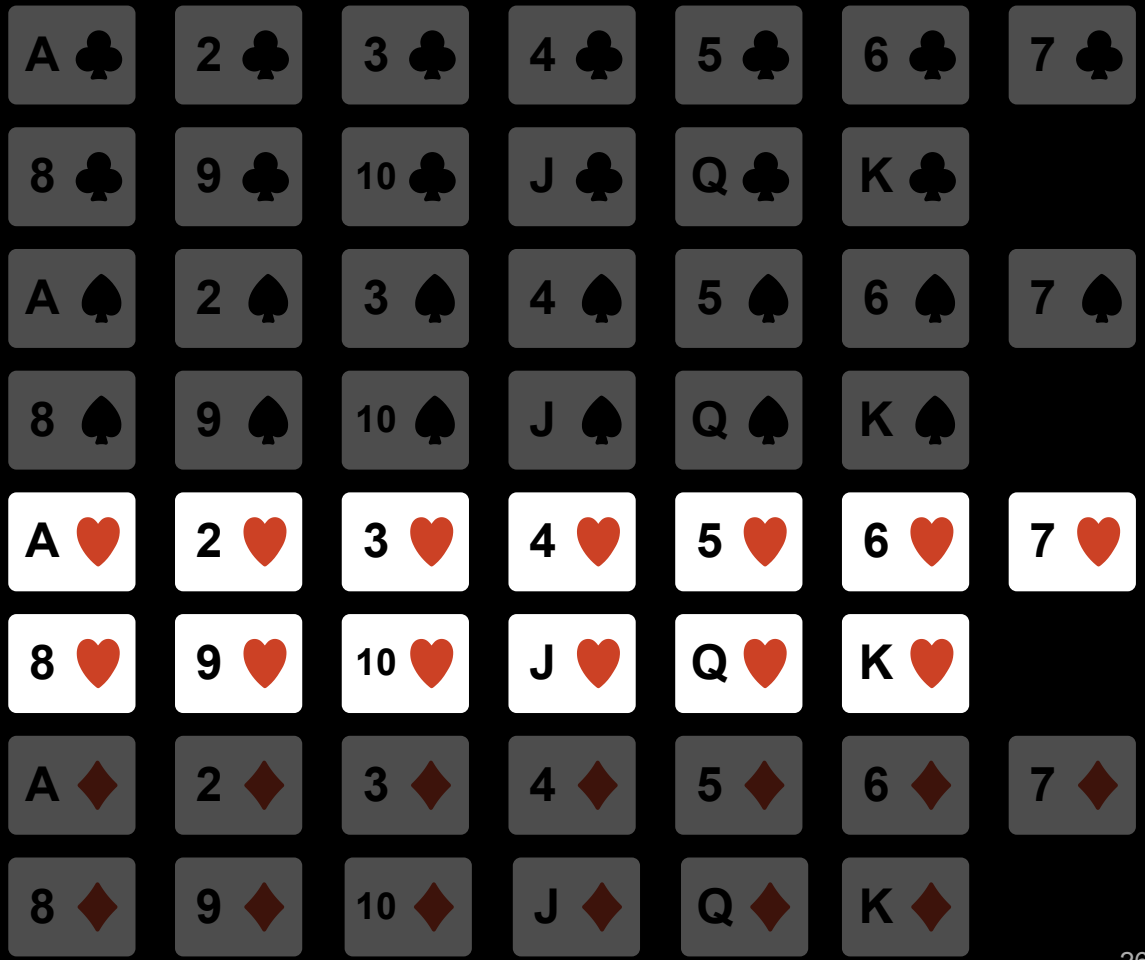
is the card black?

no

is it hearts?

yes

is it 8 or above?



is the card black?

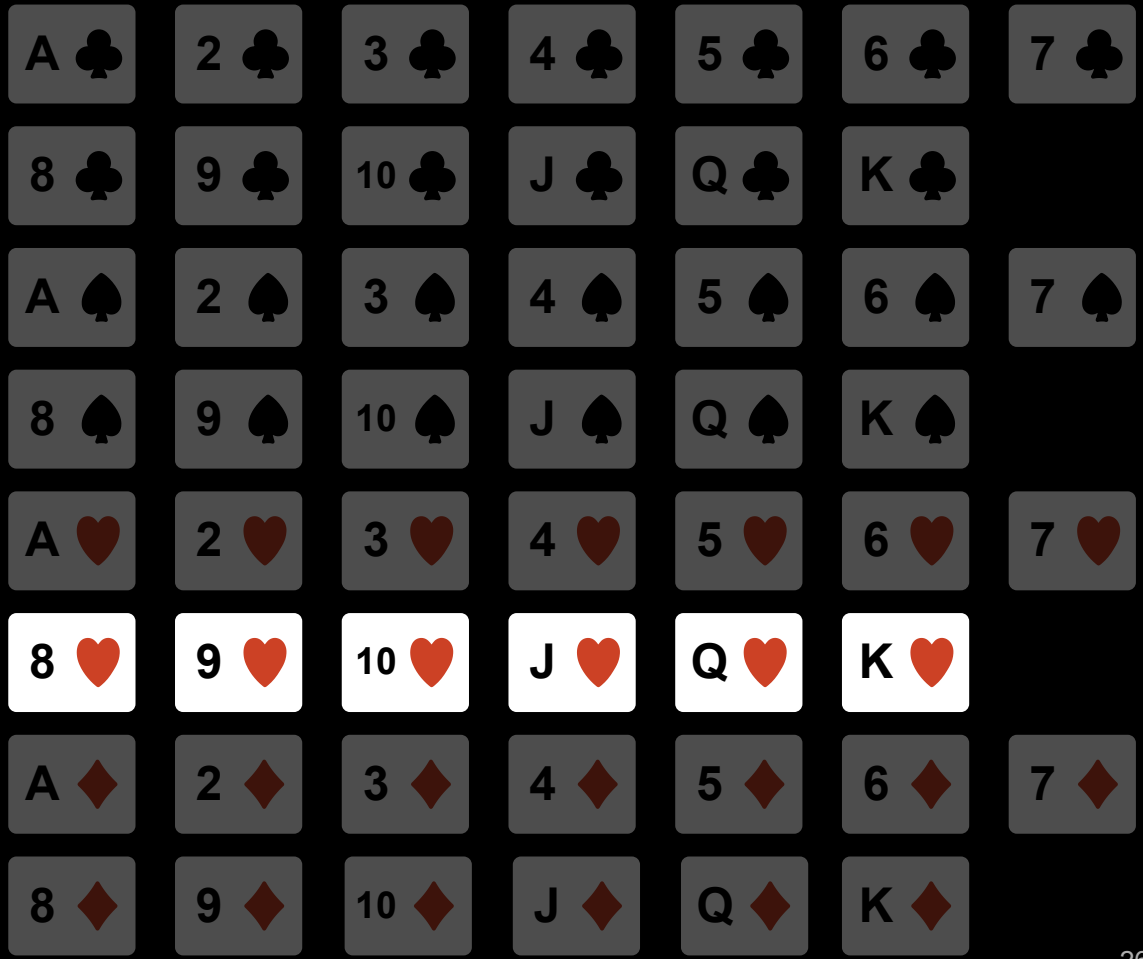
no

is it hearts?

yes

is it 8 or above?

yes



is the card black?

no

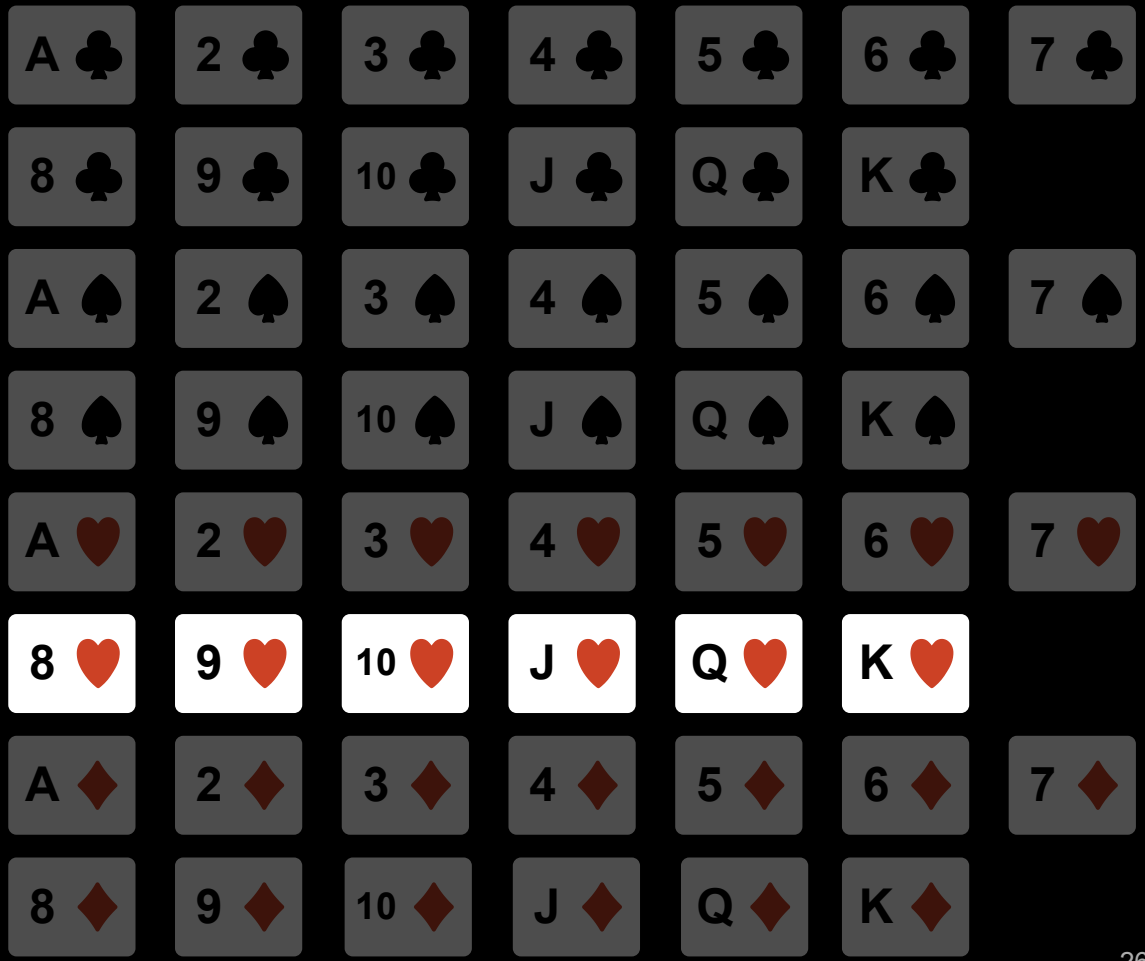
is it hearts?

yes

is it 8 or above?

yes

is it jack or above?



is the card black?

no

is it hearts?

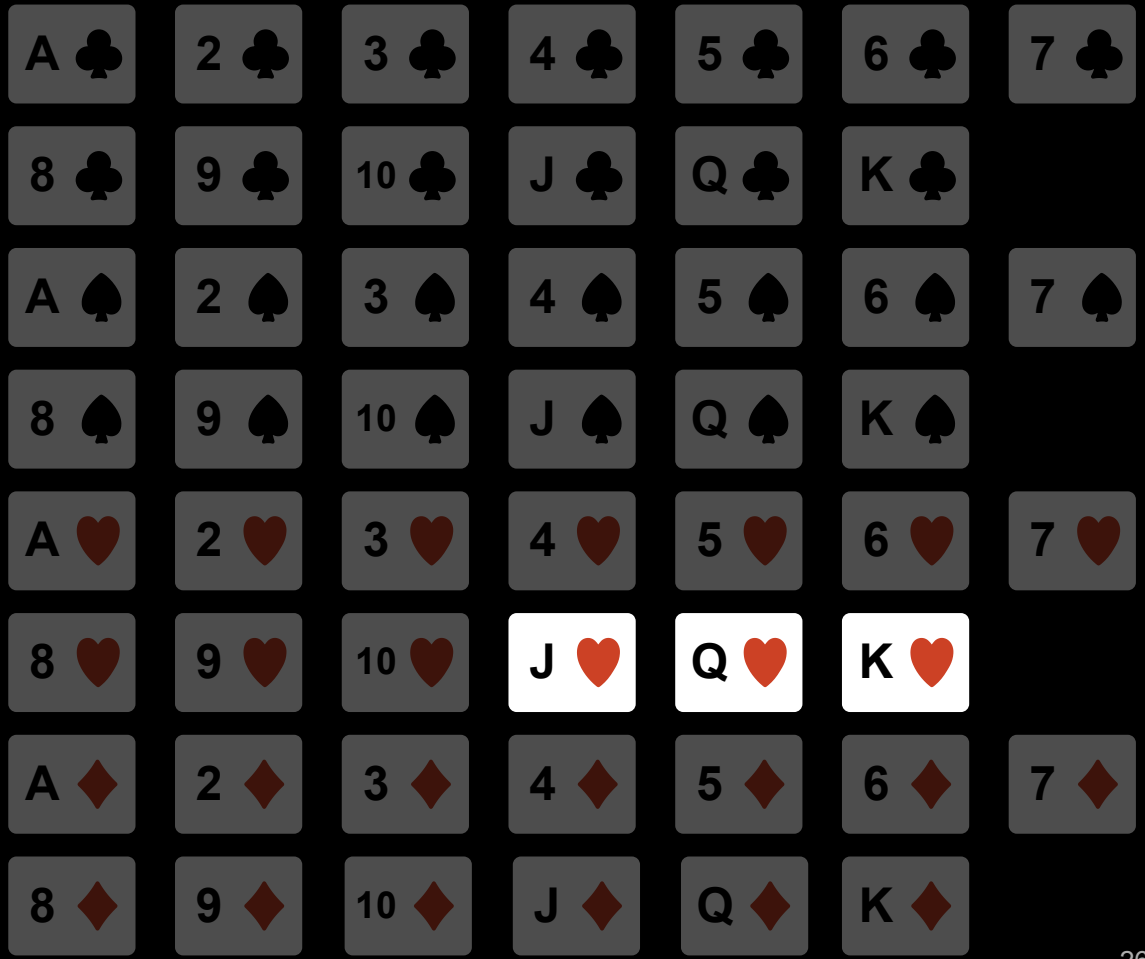
yes

is it 8 or above?

yes

is it jack or above?

yes



is the card black?

no

is it hearts?

yes

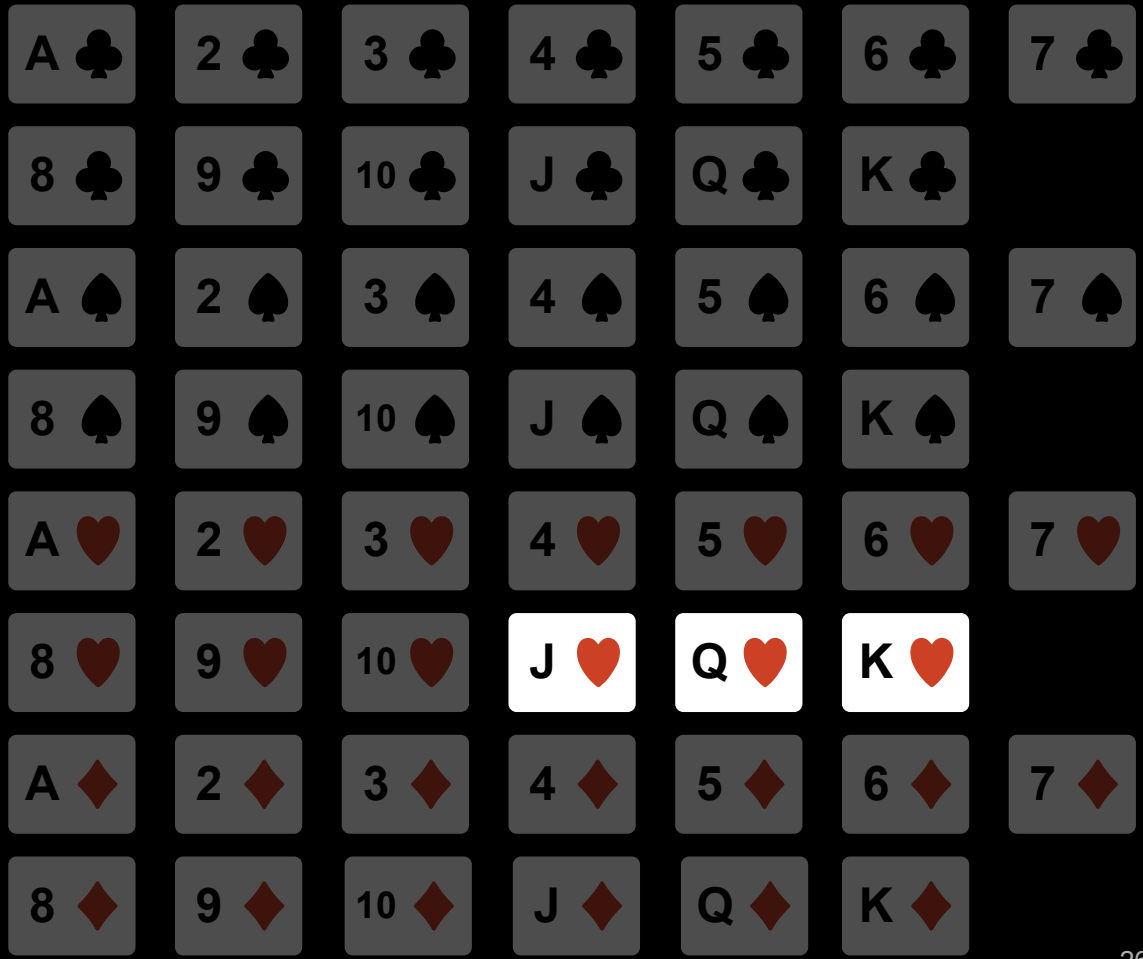
is it 8 or above?

yes

is it jack or above?

yes

is it queen or above?



is the card black?

no

is it hearts?

yes

is it 8 or above?

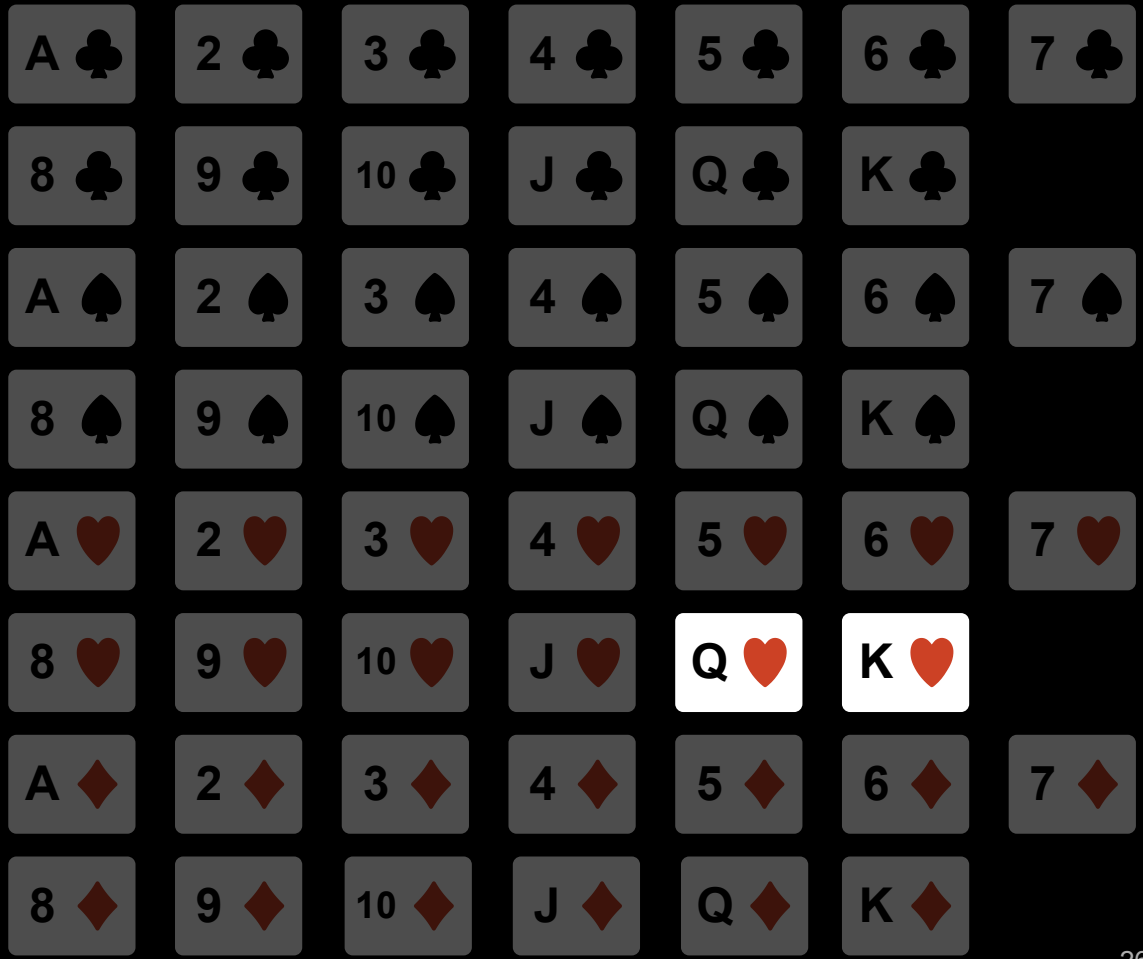
yes

is it jack or above?

yes

is it queen or above?

yes



is the card black?

no

is it hearts?

yes

is it 8 or above?

yes

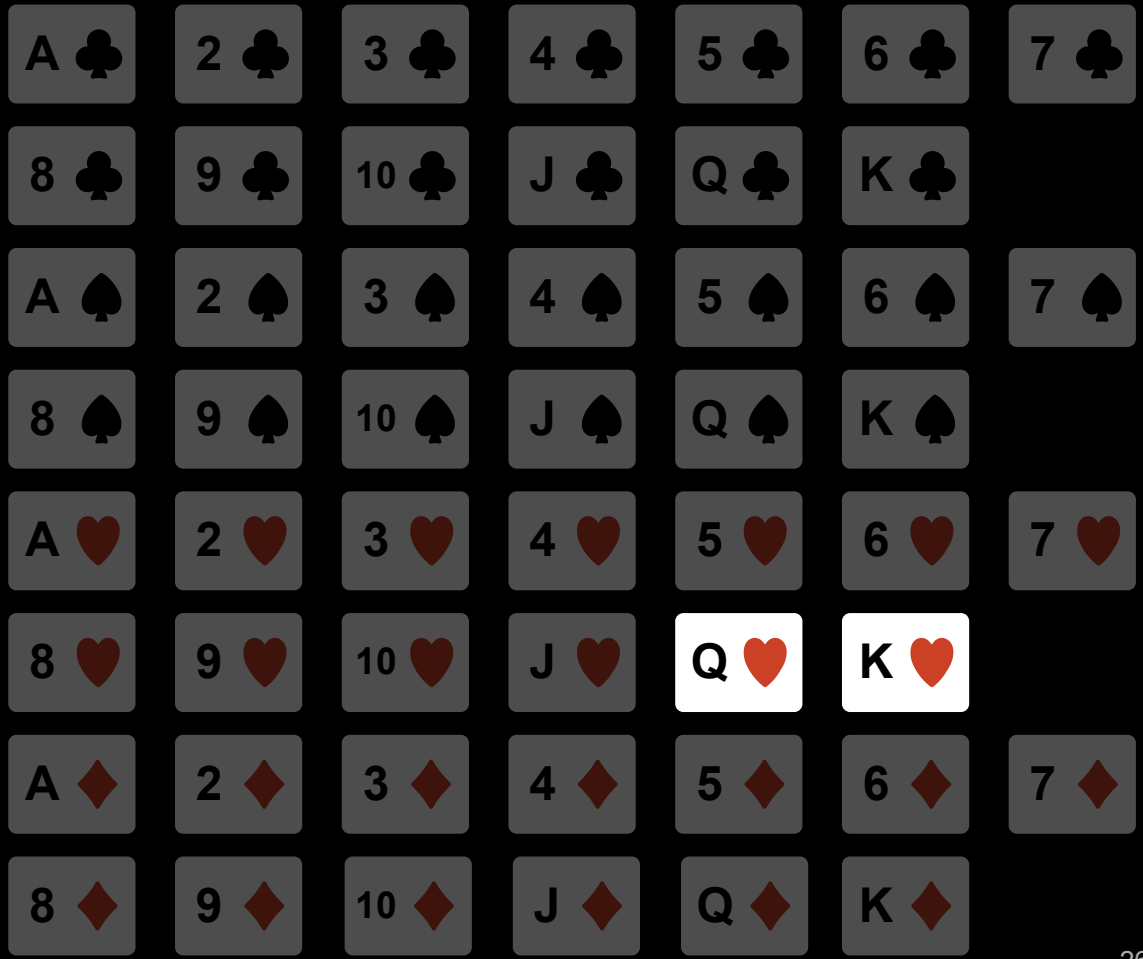
is it jack or above?

yes

is it queen or above?

yes

is it king?



is the card black?

no

is it hearts?

yes

is it 8 or above?

yes

is it jack or above?

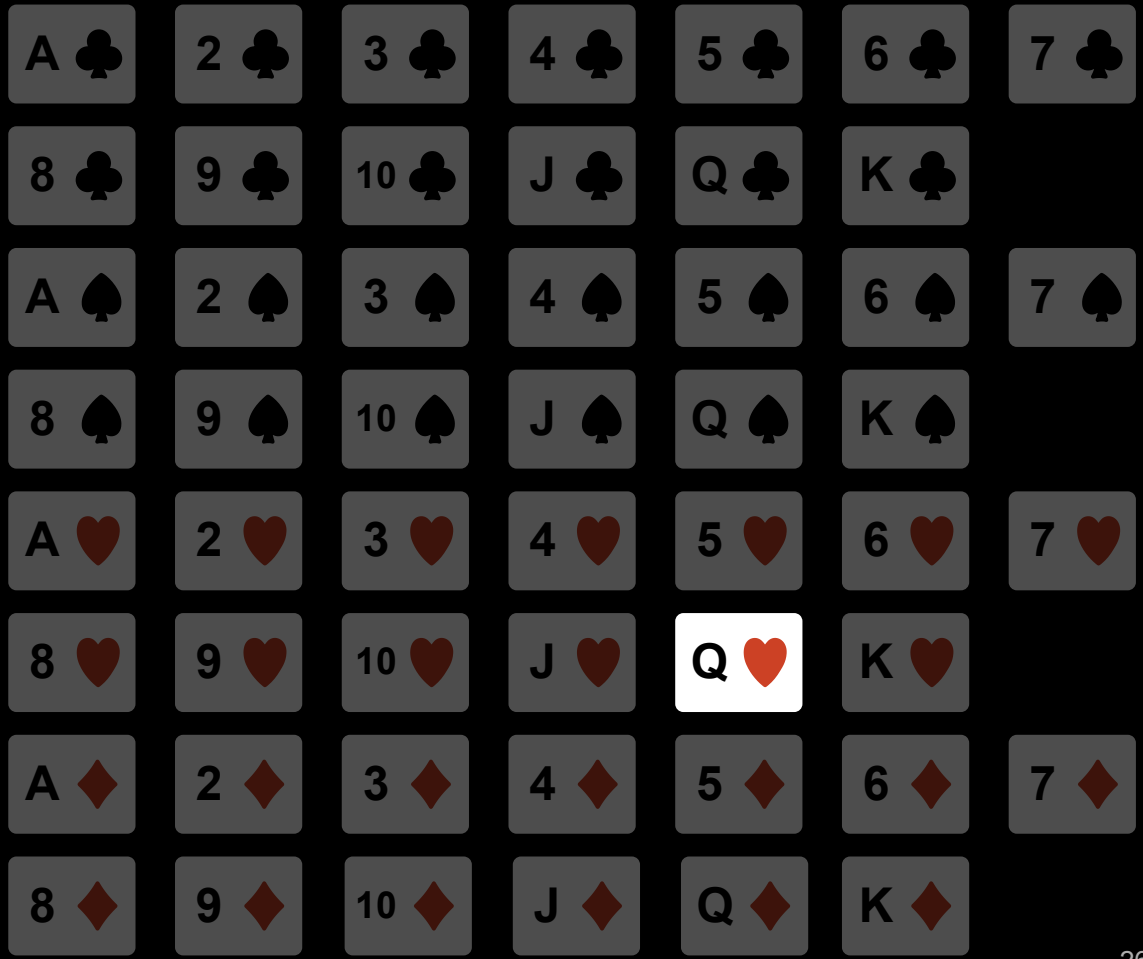
yes

is it queen or above?

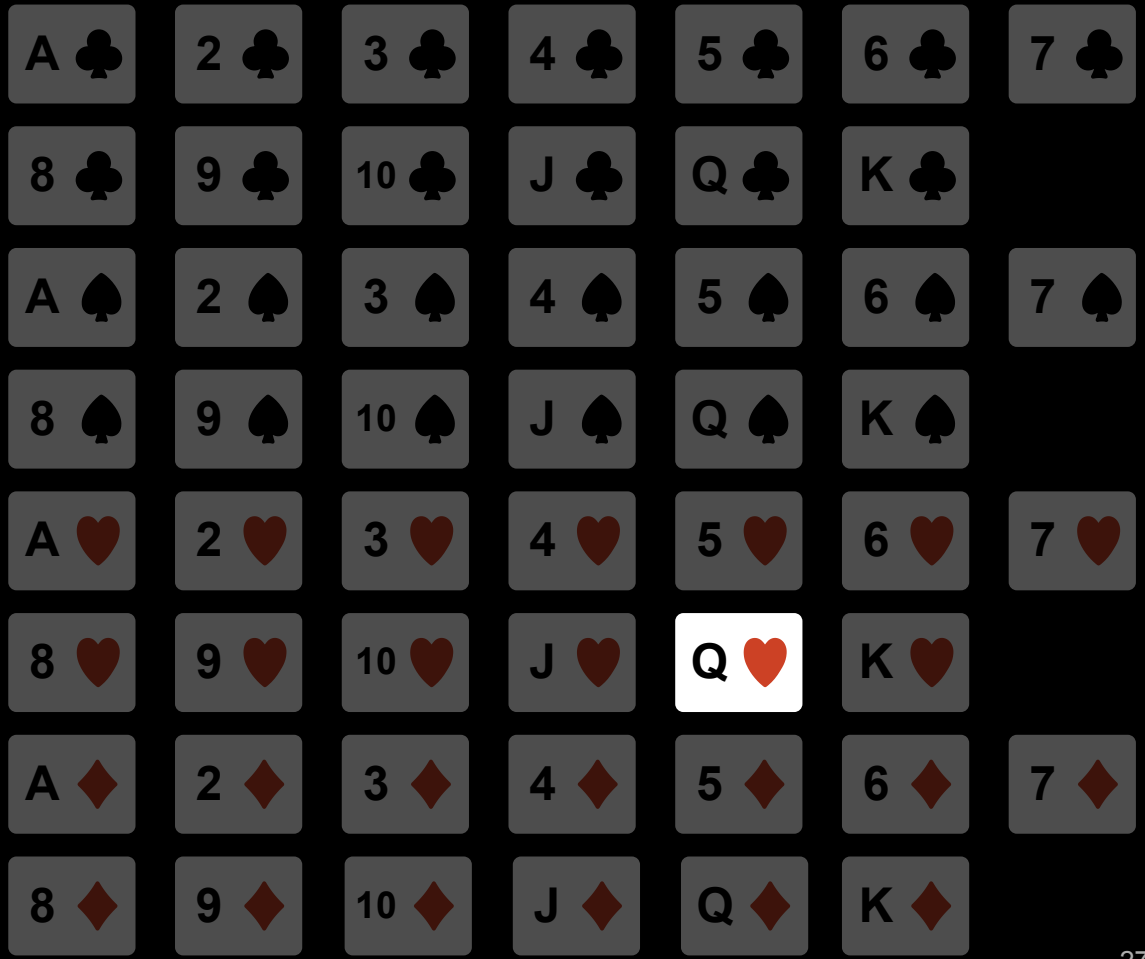
yes

is it king?

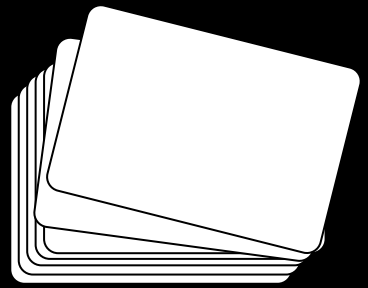
no



with 6 questions
from 52 to 1

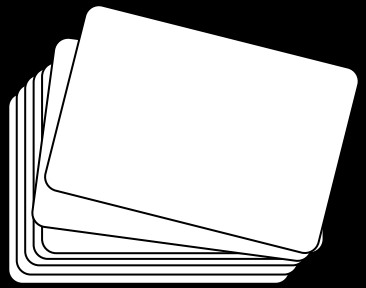


uncertainty with N = 52 possibilities?



$$H = log_2(52) \approx 5.7$$

uncertainty with N = 52 possibilities?



average # of yes/no questions

$$H = log_2(52) \approx 5.7$$

one bit of information with each answer…

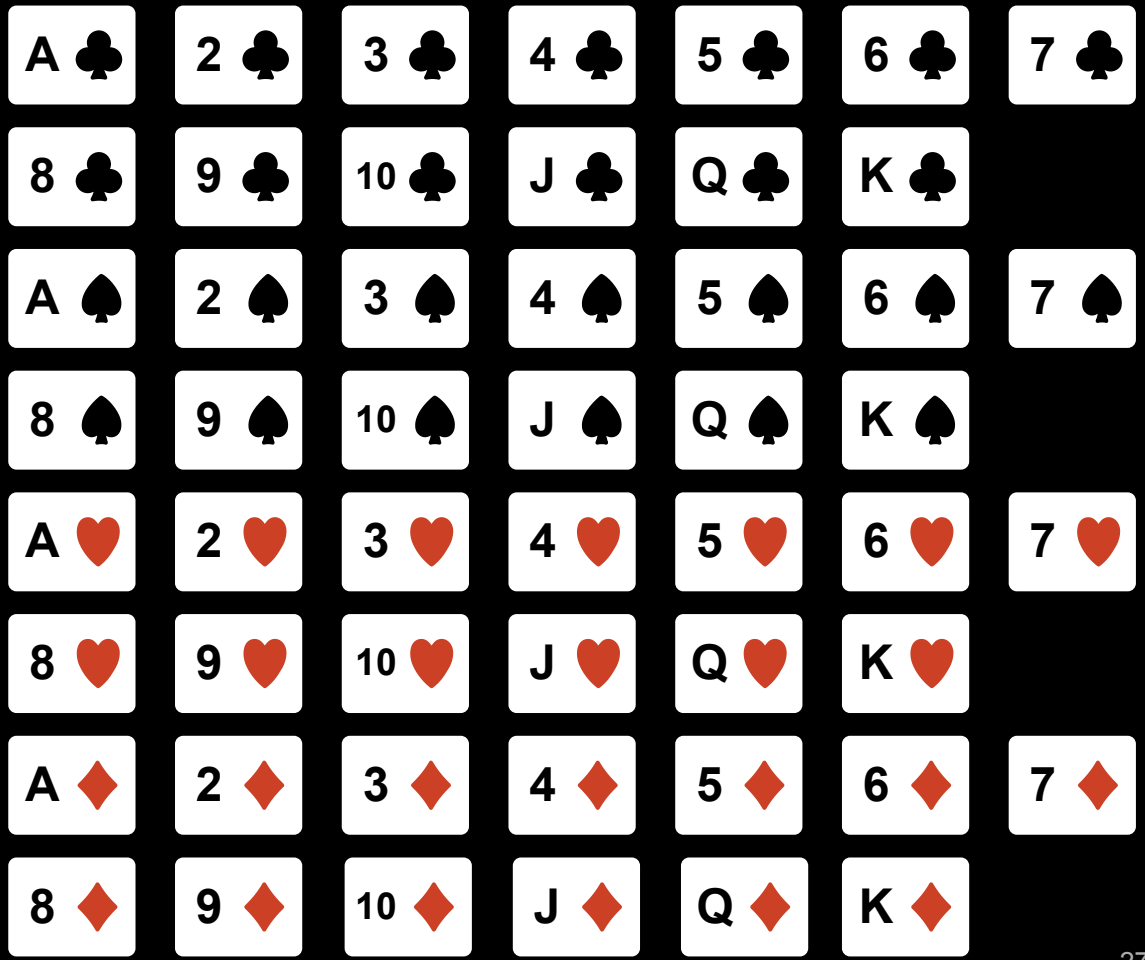
$$log_2(52) - log_2(26) = 1$$

one bit of information with each answer…

$$log_2(52) - log_2(26) = 1$$

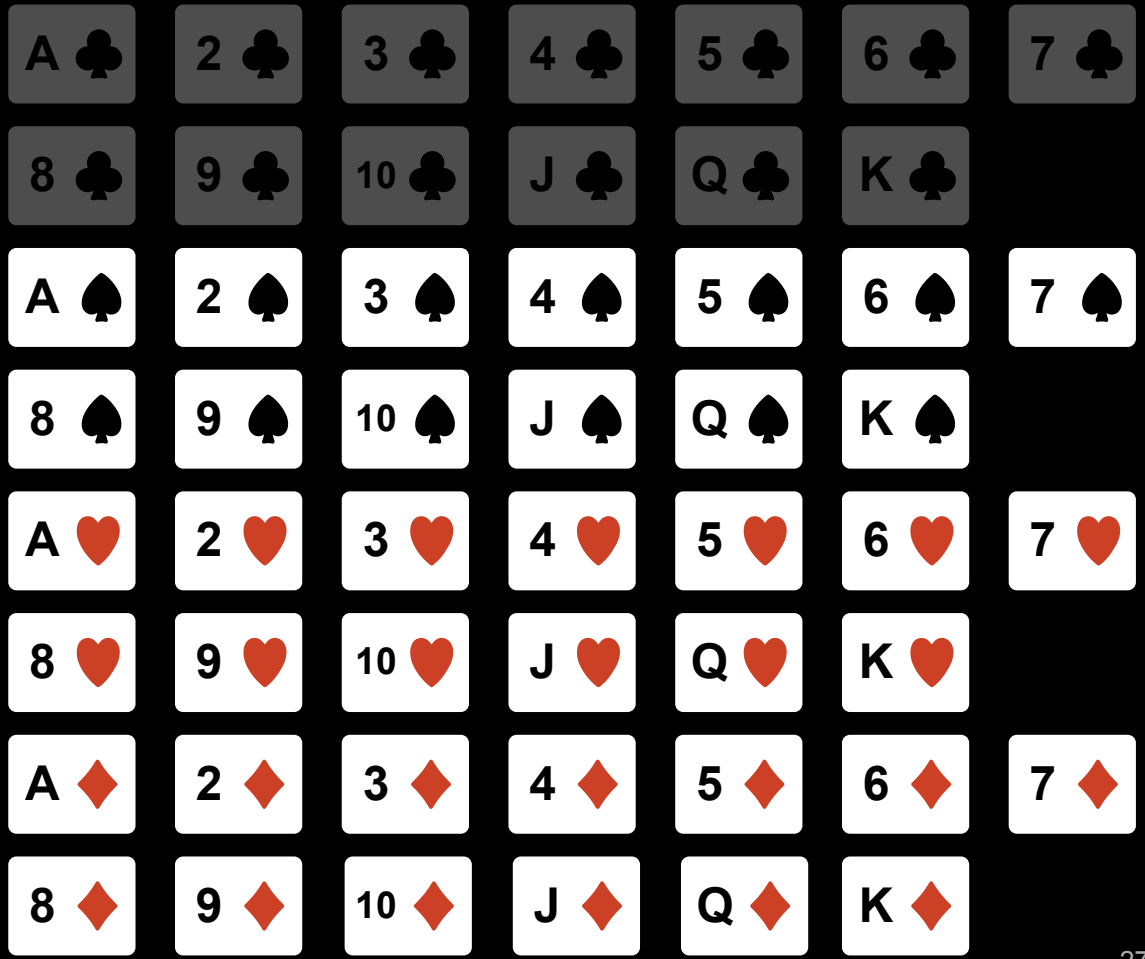
…that cuts the remaining options in half

is it a spades card?



is it a spades card?

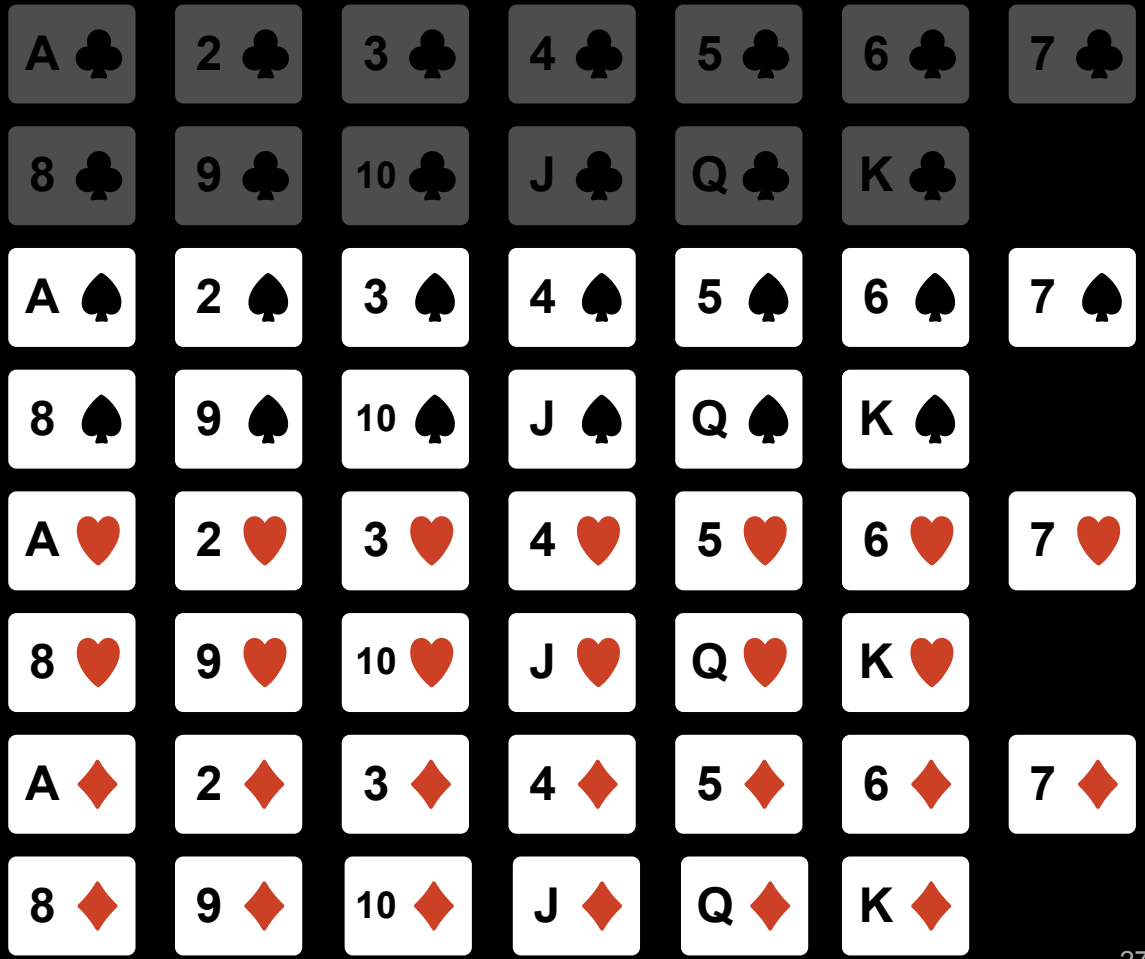
no



is it a spades card?

no

how much information?



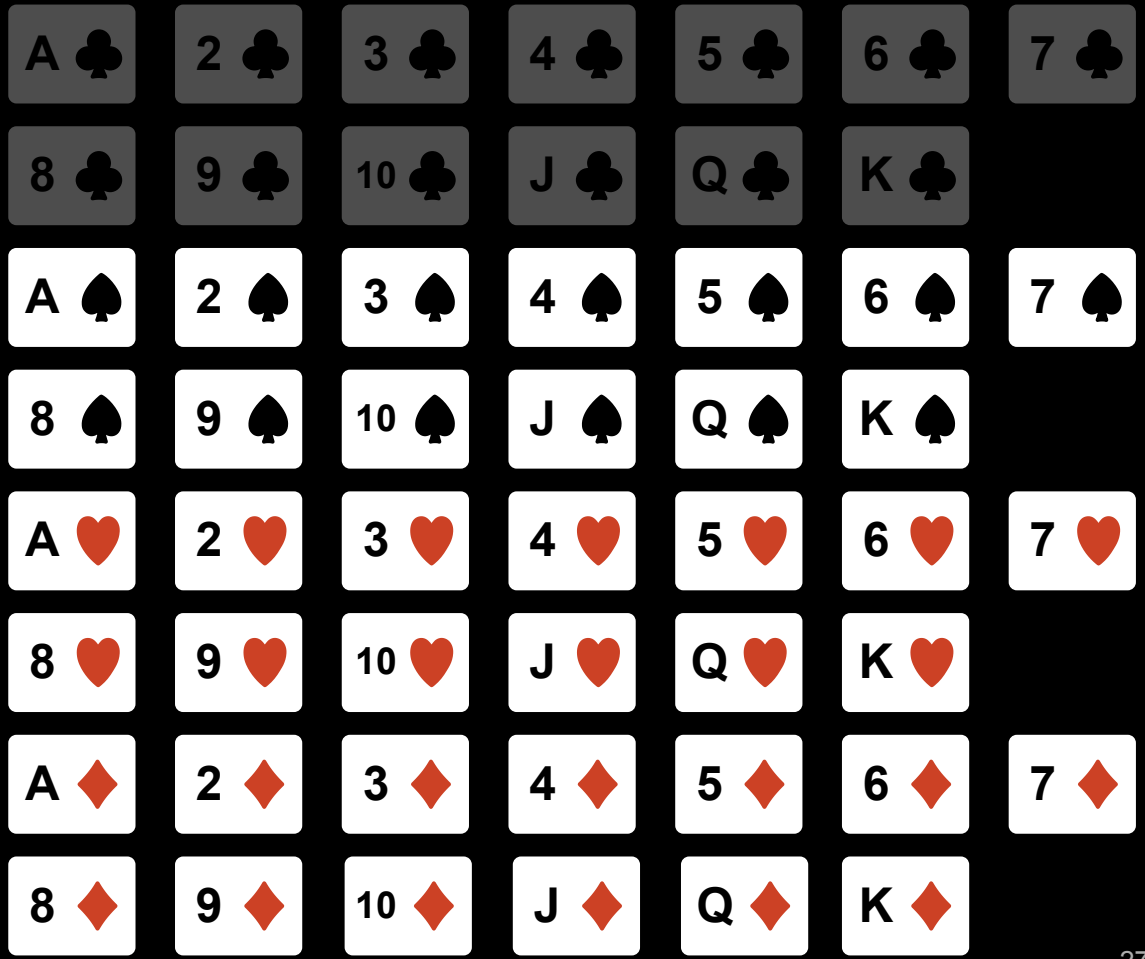
is it a spades card?

no

how much information?

$$H_0 = log_2(52) \approx 5.7$$

$$H_1 = log_2(39) \approx 5.29$$



is it a spades card?

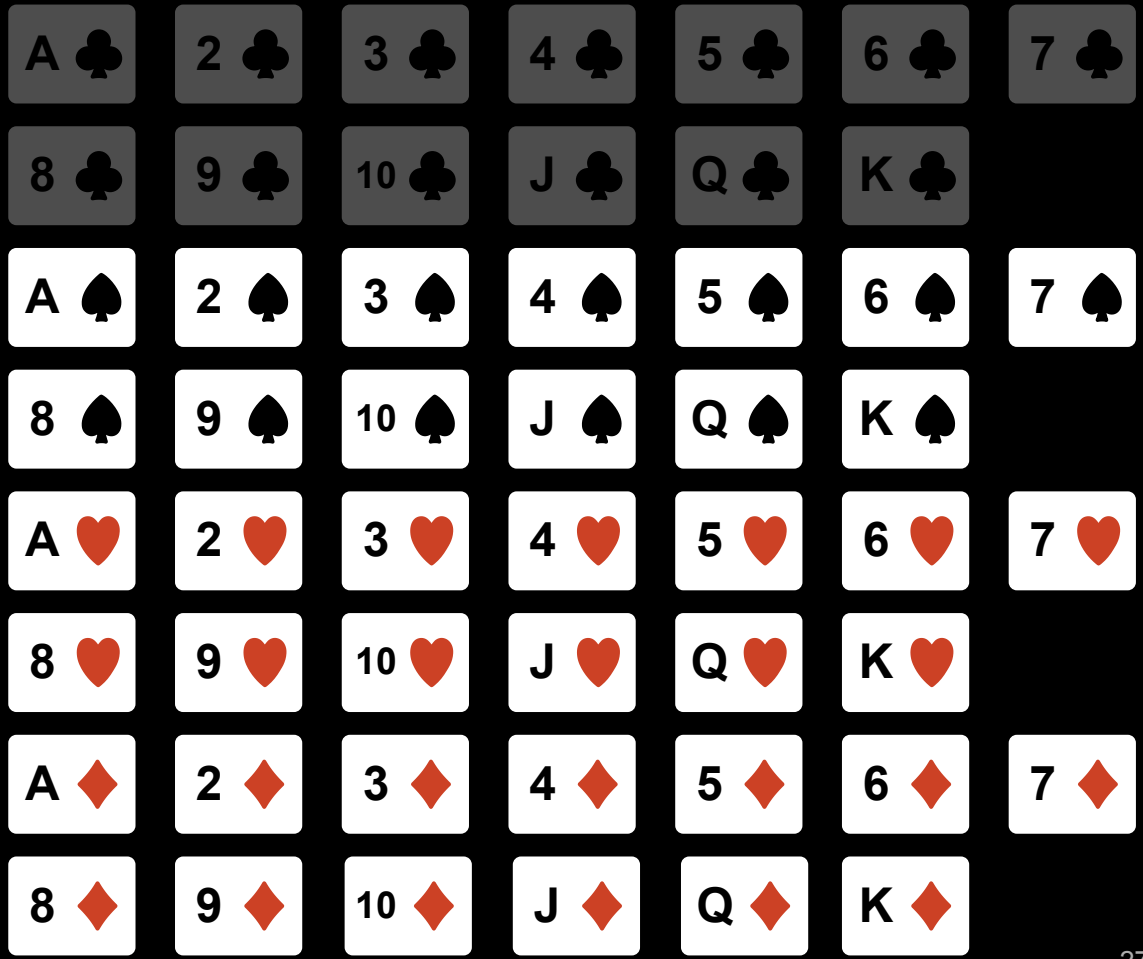
no

how much information?

$$H_0 = log_2(52) \approx 5.7$$

$$H_1 = log_2(39) \approx 5.29$$

$$H_0 - H_1 \approx 0.41$$



is it a spades card?

no

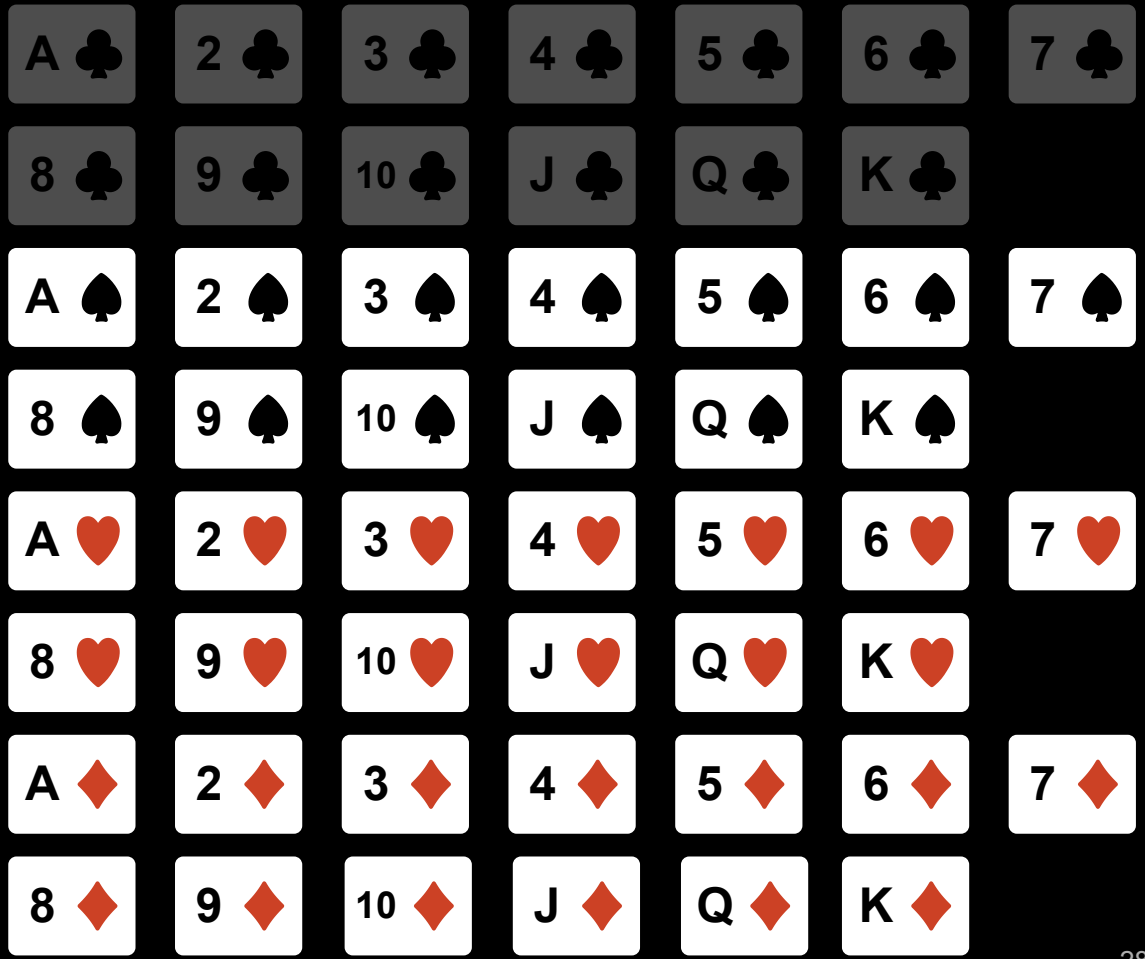
how much information?

$$H_0 = log_2(52) \approx 5.7$$

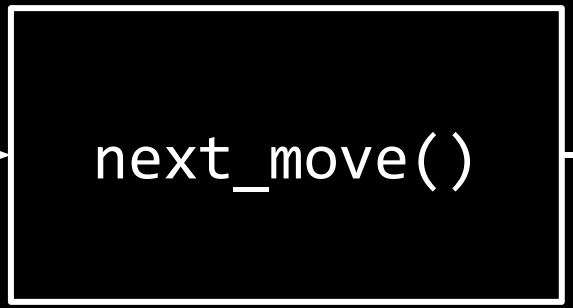
$$H_1 = log_2(39) \approx 5.29$$

$$H_0 - H_1 \approx 0.41$$

that's less than 1 bit



chess

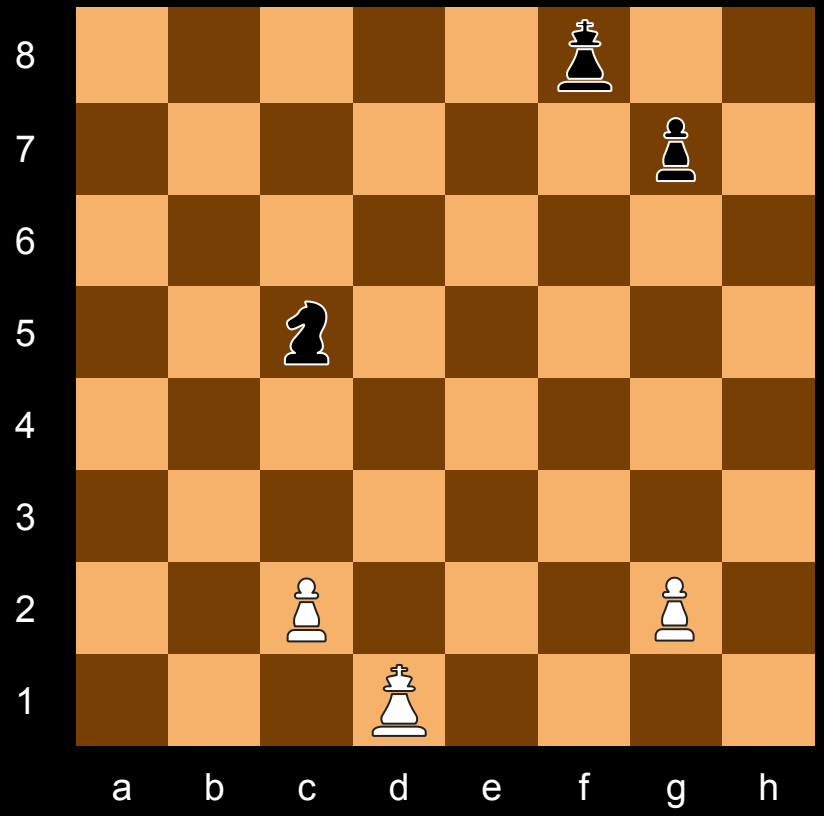


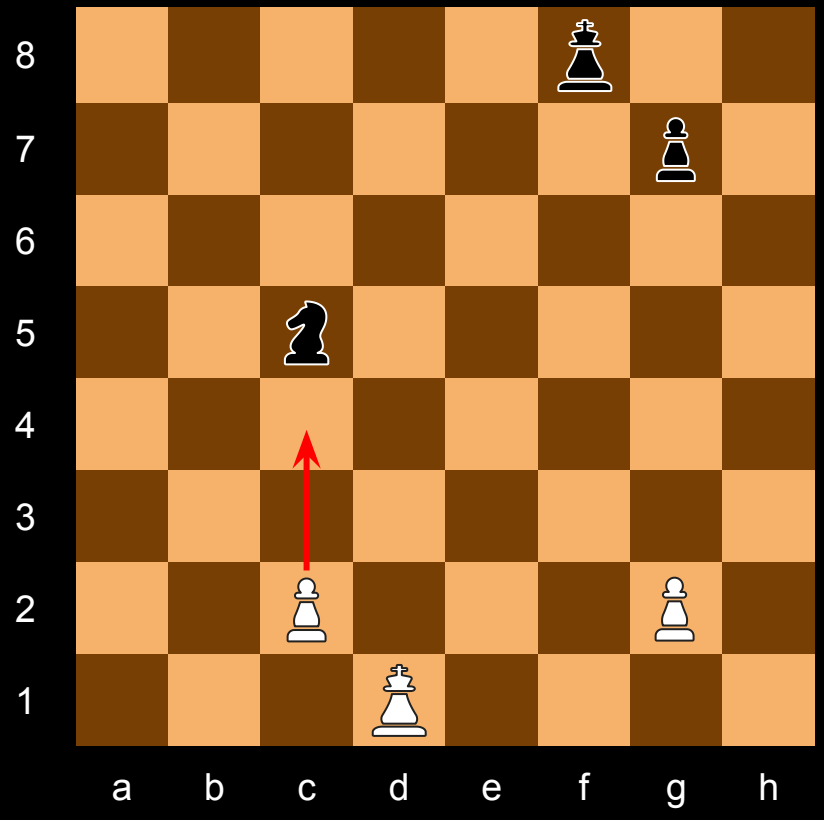
E2 → E4

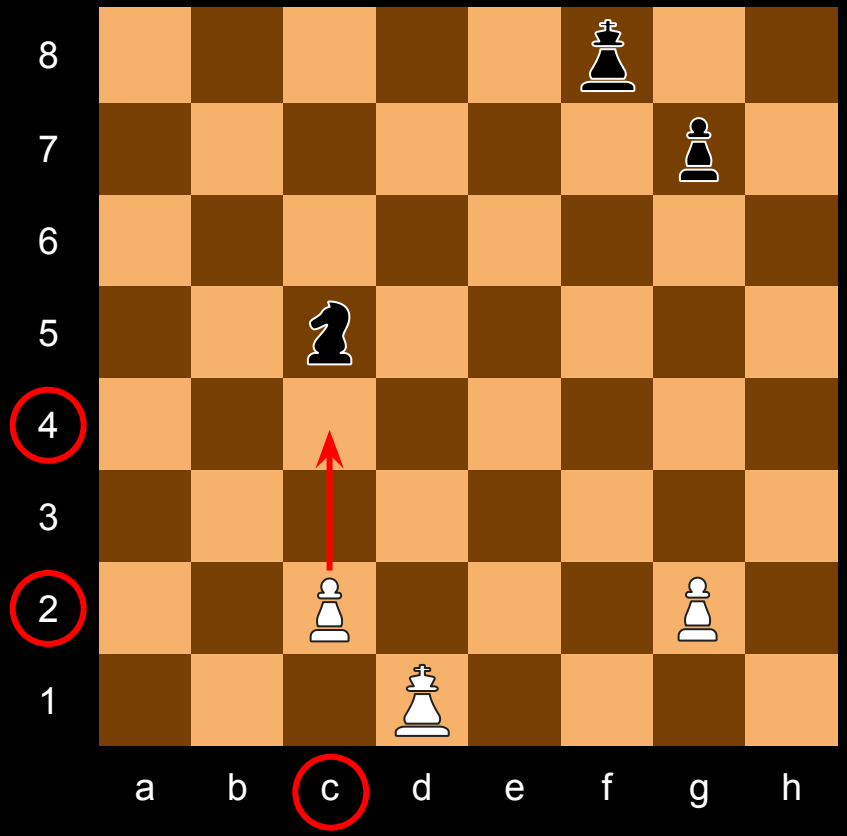
how much information is one move?

```
next_move()
```

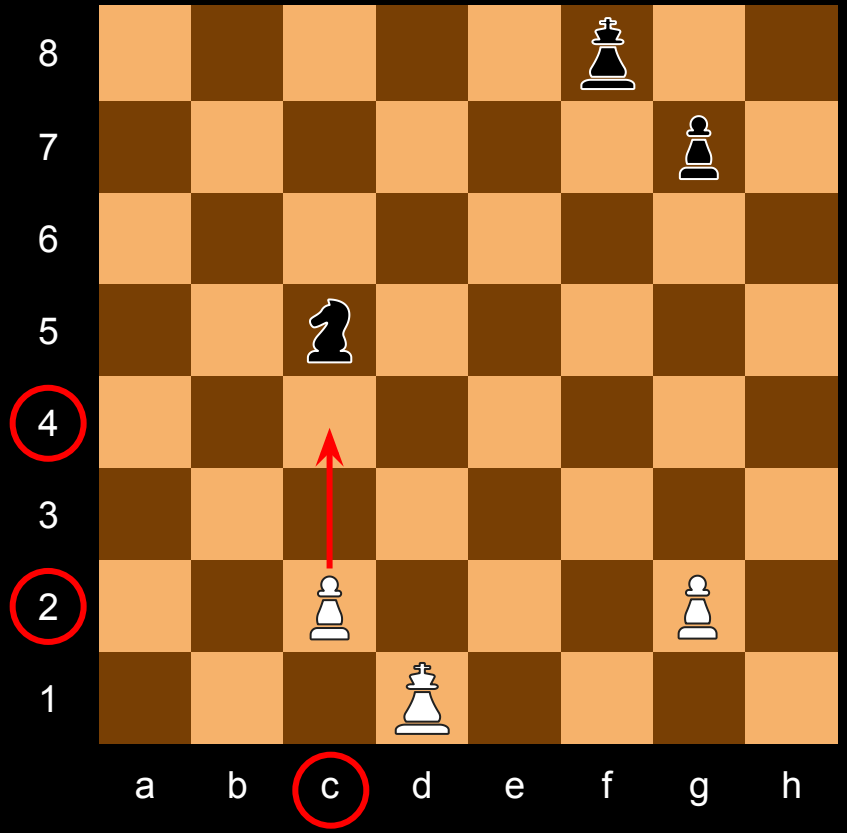
E2 → E4


8
7
6
5
4
3
2
1
a b c d e f g h

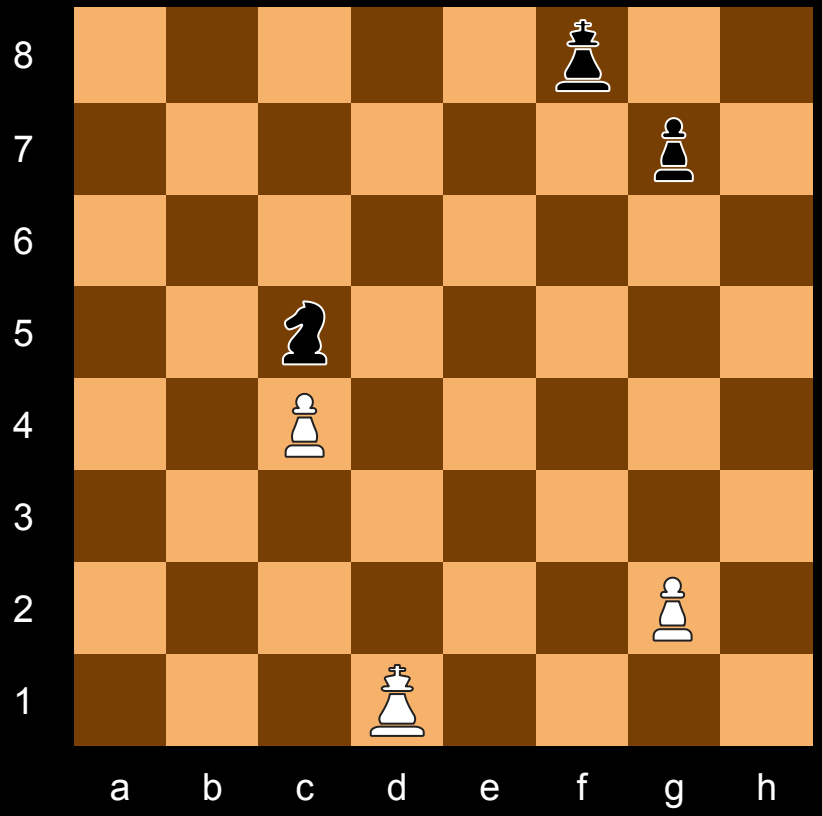

8
7
6
5
4
3
2
1
a b c d e f g h


8
7
6
5
4
3
2
1
a b c d e f g h

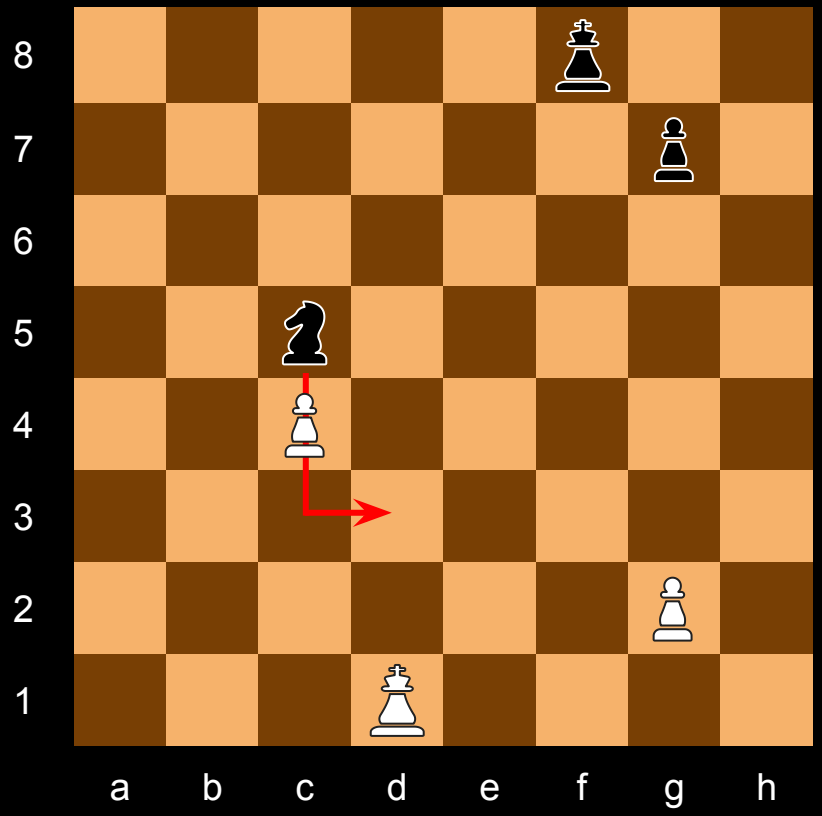
c2 → c4



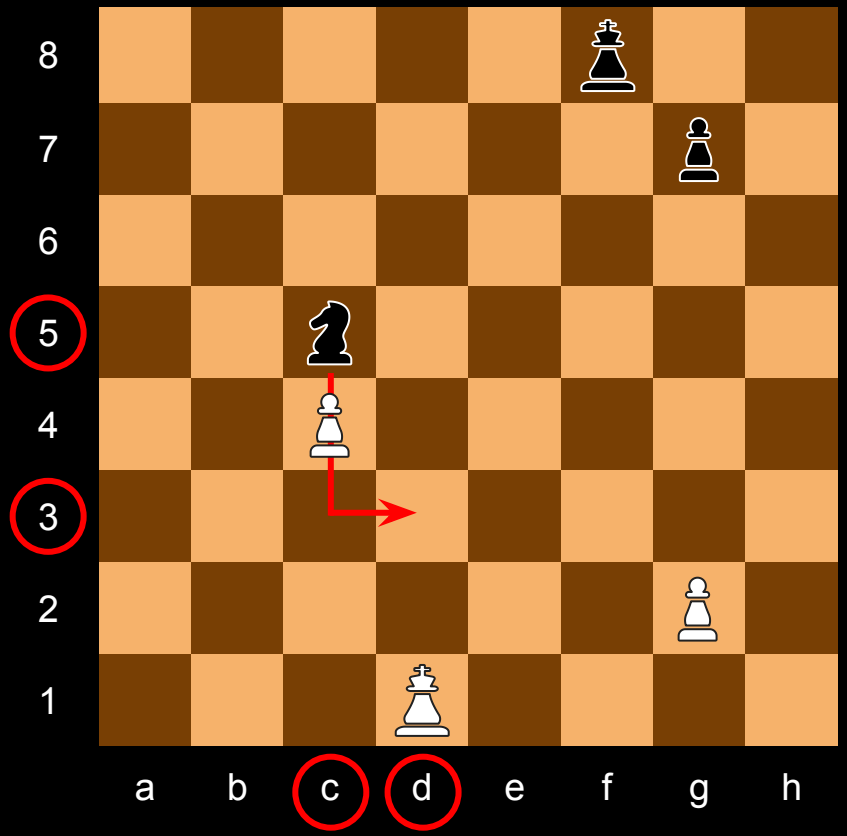
c2 → c4



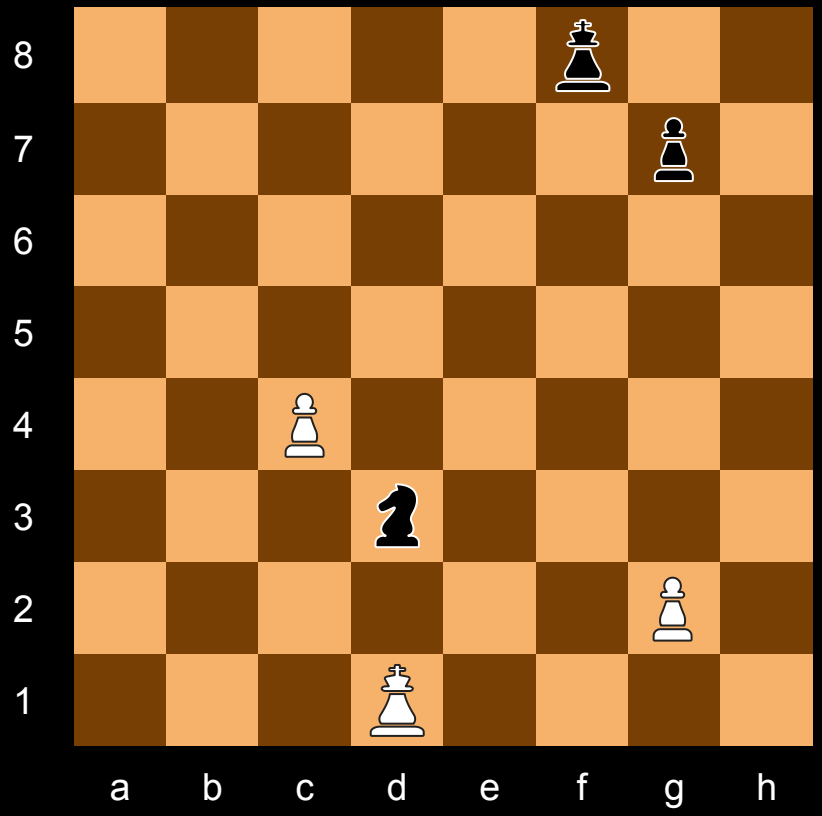
c2 → c4



c2 → c4
c5 → d3



c2 → c4
c5 → d3
…



c2 → c4

how many possibilities?

64 fields x 64 fields

how many possibilities?

64 x 64 = 4096

possible moves*

*disregarding impossible moves

64 x 64 = 4096

$$H = log_2(4096) = 12$$

64 x 64 = 4096

$$H = log_2(4096) = 12$$

one chess move is 12 bits of information

an alternative way to calculate # bits

c 2 c 4

4 digits

c 2 c 4

4 digits

8 possible symbols per digit

c 2 c 4

4 digits

8 possible symbols per digit

how many bits per digit?

c 2 c 4

4 digits

8 possible symbols per digit

how many bits per digit?

$$H_{digit} = log_2(8) = 3$$

c 2 c 4

4 digits

8 possible symbols per digit

how many bits per digit?

$$H_{digit} = log_2(8) = 3$$

$$H_{move} = log_2(8) \times 4 = 12$$

$$H_{avg} = log_2(S) \times n$$

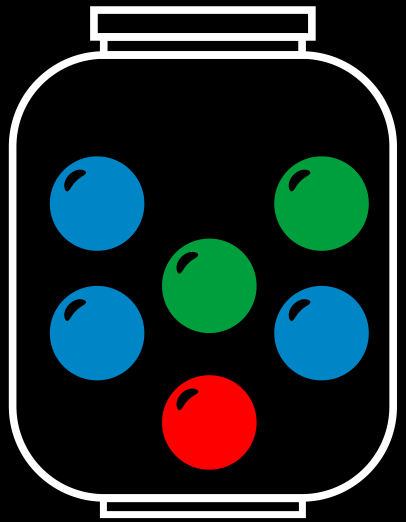
S: number of possible symbols

n: number of digits in our message

$$H_{max} = \lceil log_2(S) \rceil \times n$$

when calculating bits for storage, we must always consider the worst case

drawing balls from an urn

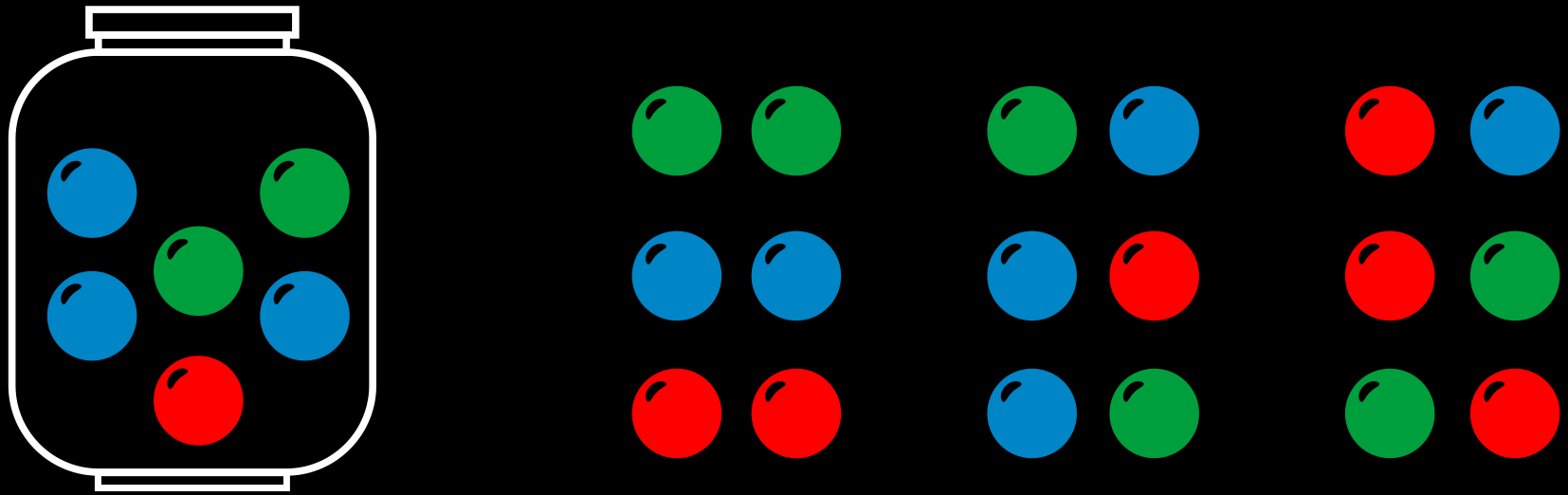


p( ) = 1/2

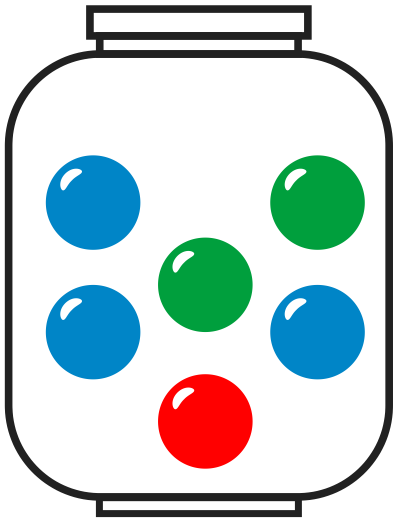
p( ) = 2/3

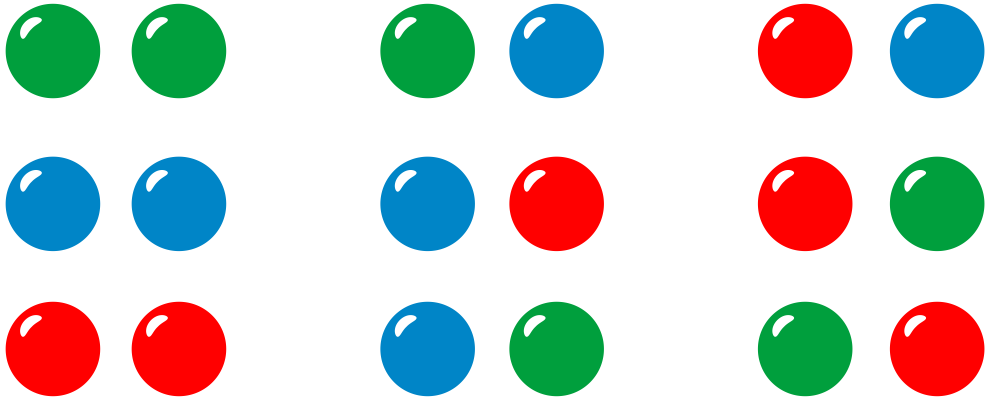
p( ) = 1/6

Example adapted from:
Pinkard, Henry, and Laura Waller. A Visual Introduction to Information Theory. arXiv:2206.07867, arXiv, 16 June 2022, https://doi.org/10.48550/arXiv.2206.07867.



Example adapted from:
Pinkard, Henry, and Laura Waller. A Visual Introduction to Information Theory. arXiv:2206.07867, arXiv, 16 June 2022, https://doi.org/10.48550/arXiv.2206.07867.

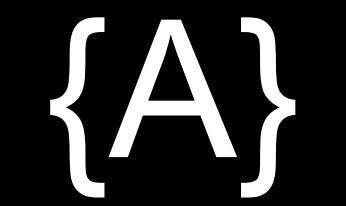




Example adapted from:
Pinkard, Henry, and Laura Waller. A Visual Introduction to Information Theory. arXiv:2206.07867, arXiv, 16 June 2022, https://doi.org/10.48550/arXiv.2206.07867.

mastermind

digits and # symbols


{A}

{A}

A A

{A, B}

_ _

{A, B}

AA, AB, BA, BB

{A, B, C}

_ _

{A, B, C}

AA, AB, BA, BB,
AC, BC, CA, CB, CC

{A, B, C, D}

_ _

{A, B, C, D}

AA, AB, BA, BB, AC, BC, CA, CB,
CC, AD, DA, BD, DB, CD, DC, DD

{A, B, C, D, E}

_ _

{A, B, C, D, E}

AA, AB, BA, BB, AC, BC, CA, CB, CC,
AD, DA, BD, DB, CD, DC, DD, AE, EA,
BE, EB, CE, EC, DE, ED, EE

with # digits n = 2

# symbols	# messages
1	1
2	4
3	9
4	16
5	25

with length n = 2

# symbols		# messages
1		1
2	f(x)	4
3	→	9
4		16
5		25

and more digits?

{A, B}

_ _ _

{A, B}

AAA, AAB, ABA, ABB,
BBB, BBA, BAA, BAB

{A, B}

_ _ _ _

{A, B}

AAAA, AAAB, AABA, AABB,
ABAA, ABAB, ABBA, ABBB,
BAAA, BAAB, BABA, BABB,
BBAA, BBAB, BBBA, BBBB

with # symbols S = 2

# digits	# messages
1	2
2	4
3	8
4	16
5	32

with # symbols S = 2

# digits	# messages
1	2
2	4
3	8
4	16
5	32

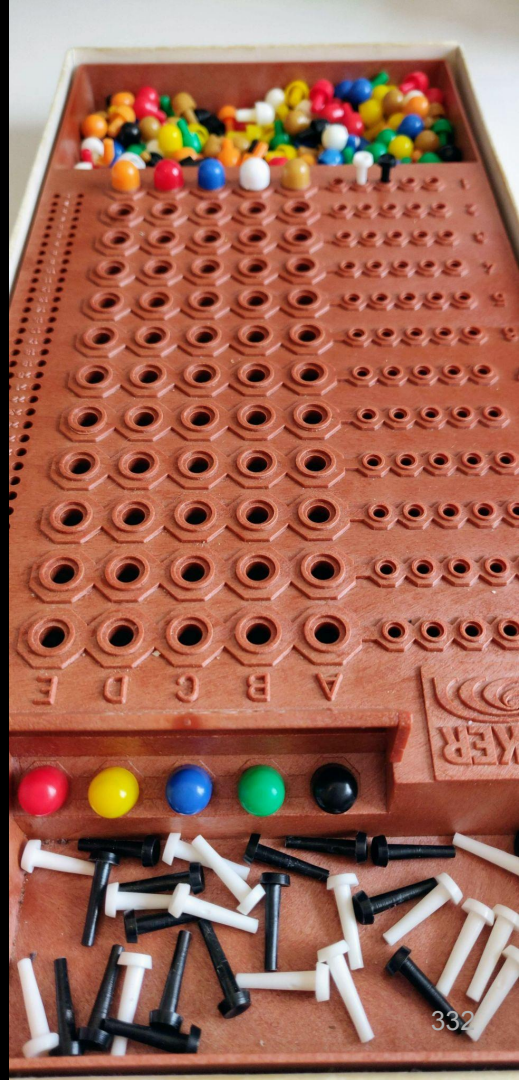
with # symbols S = 2

# digits	# messages
1	2
2	4
3	8
4	16
5	32

possible messages with *n* digits and *S* symbols

$$N = S^n$$

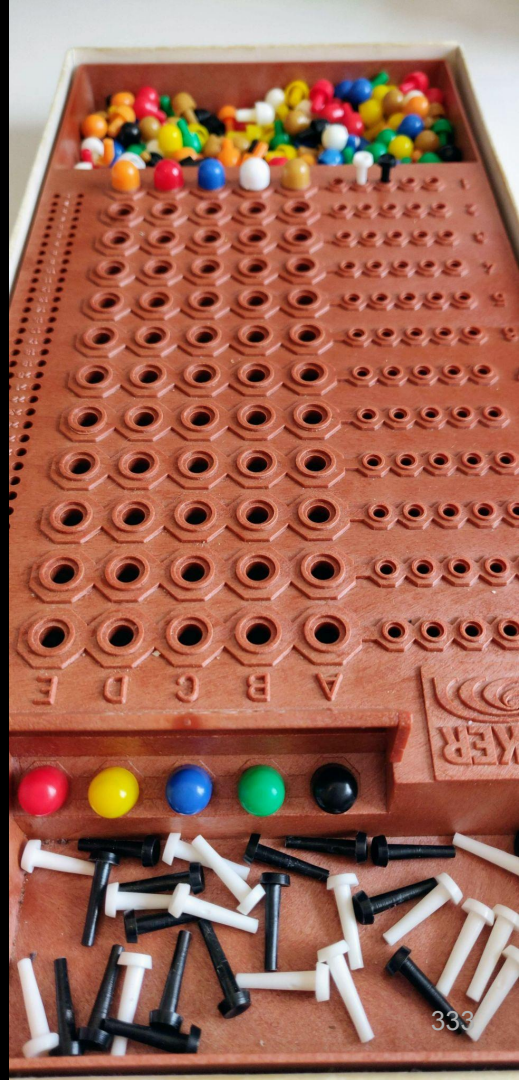
mastermind



eight colors

five slots

A B C D E



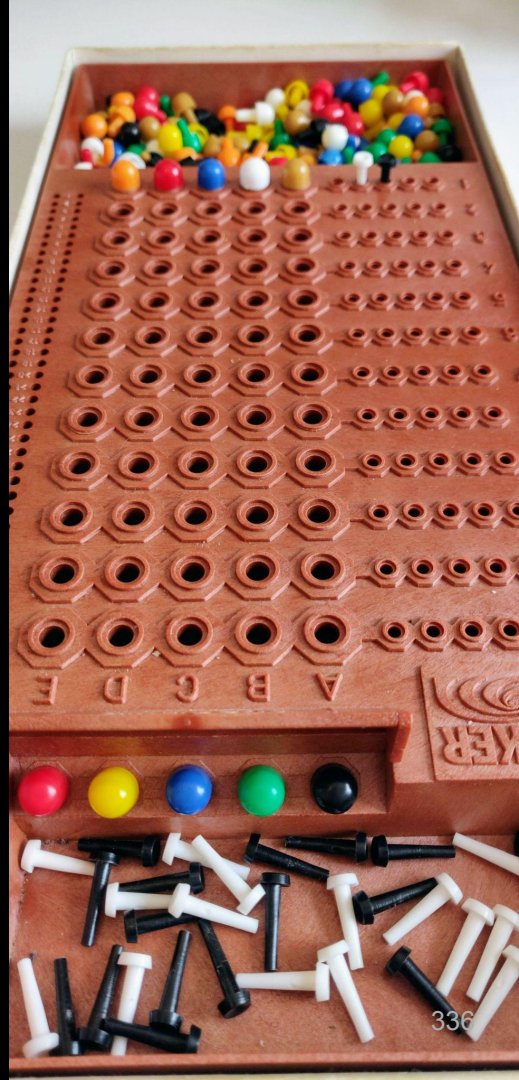
possible permutations

$$N = 5^8 = 390625$$

maximum entropy in bits

$$H_{max} = log_2\ 5^8 = 18.575$$

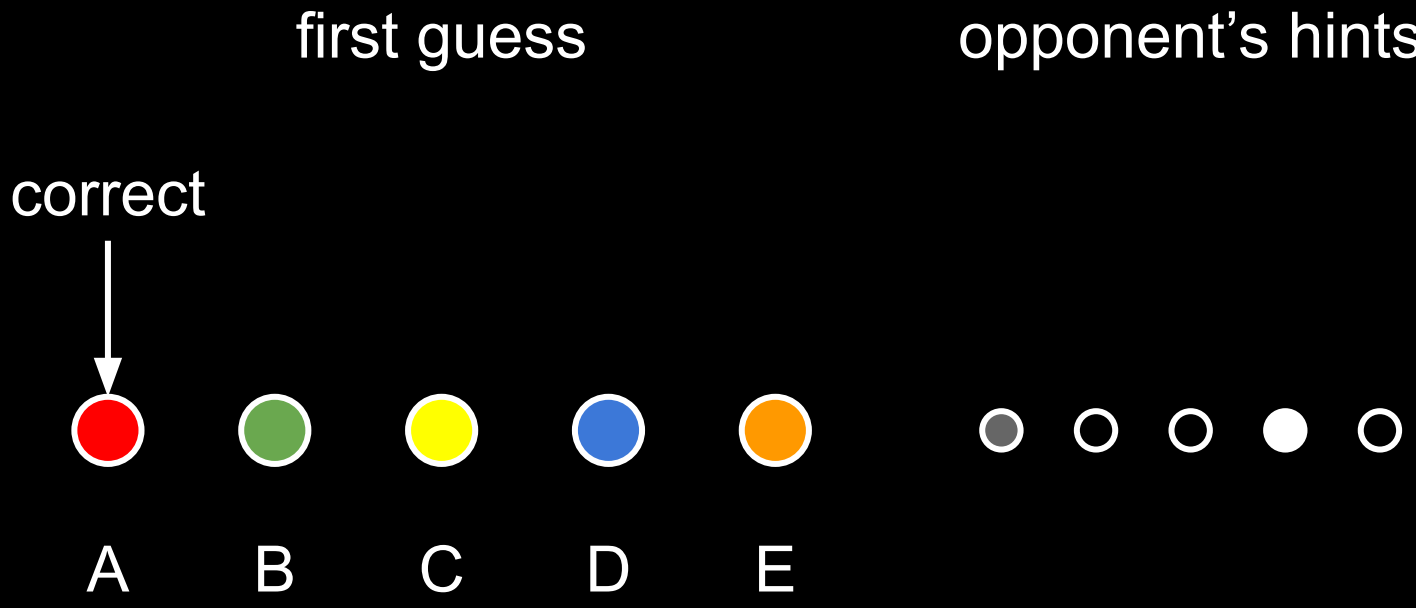
hints reduce uncertainty (entropy)



first guess

opponent’s hints

A B C D E

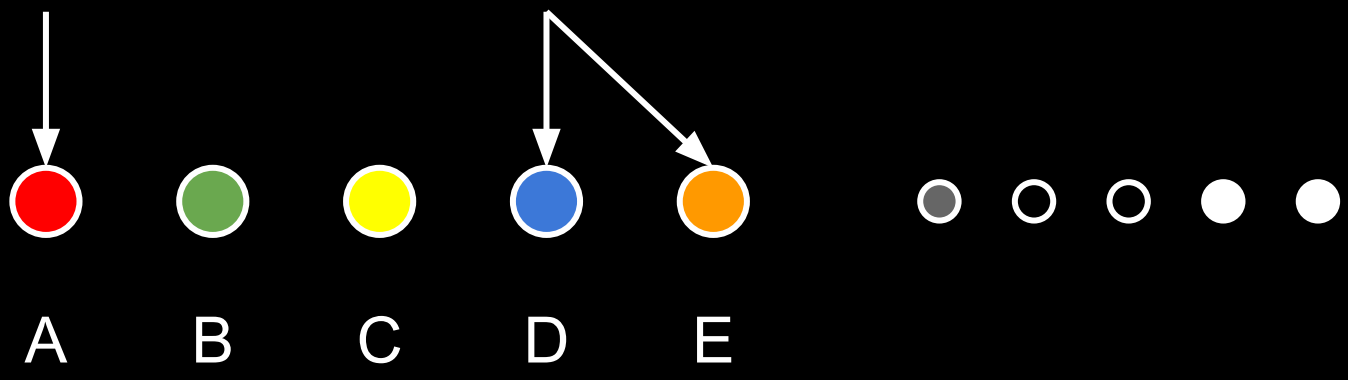

first guess
opponent’s hints
correct
A
B
C
D
E

first guess

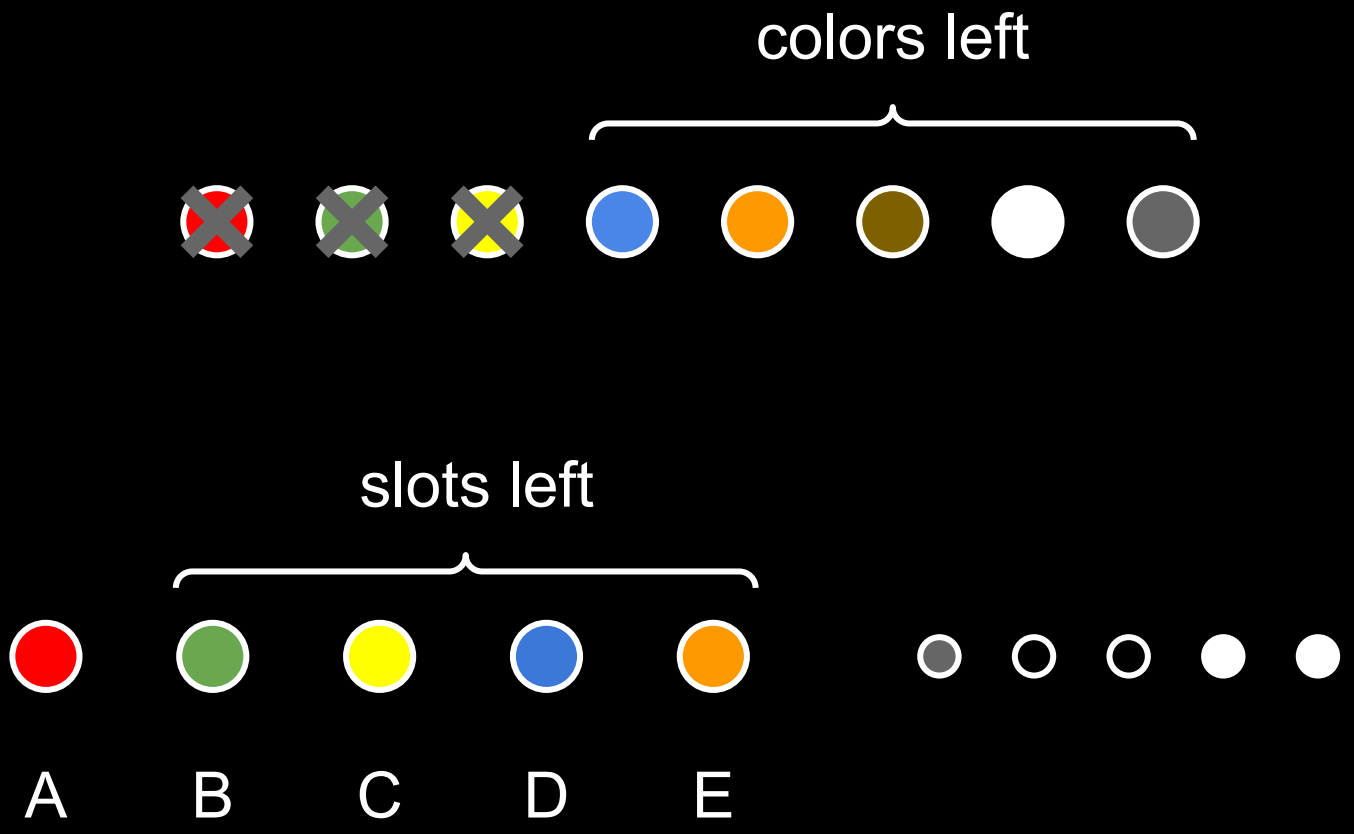
opponent’s hints

correct

wrong position



how much information did we get?


colors left
slots left
A
B
C
D
E

new possible permutations?

$$N = 4^8 = 65536$$

reduced entropy in bits?

$$H = log_2\ 4^8 = 16$$