0. MOTIVATION
1. PROBLEM SOLVING
2. INFORMATION
3. COUNTING
4. BITS
5. CODES
6. ALGORITHMS
7. COMPUTERS
8. ARITHMETIC
9. MEMORY
10. ANALOG VS. DIGITAL

The slides are meant as visual support for the lecture.
They are neither a documentation nor a script.

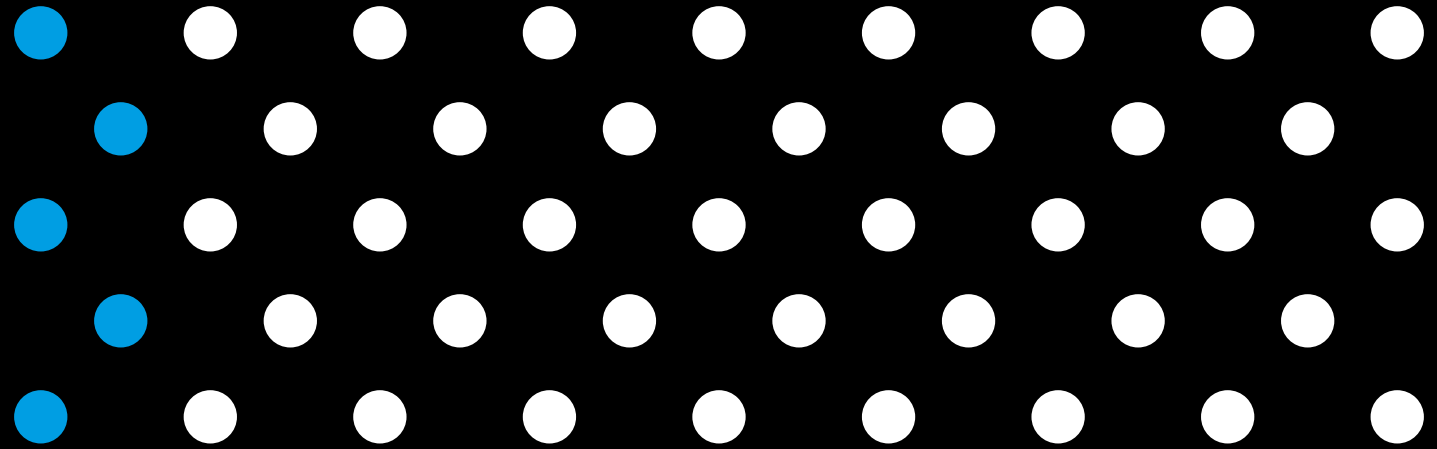
Please do not print the slides.

Comments and feedback at n.meseth@hs-osnabrueck.de

MOTIVATION

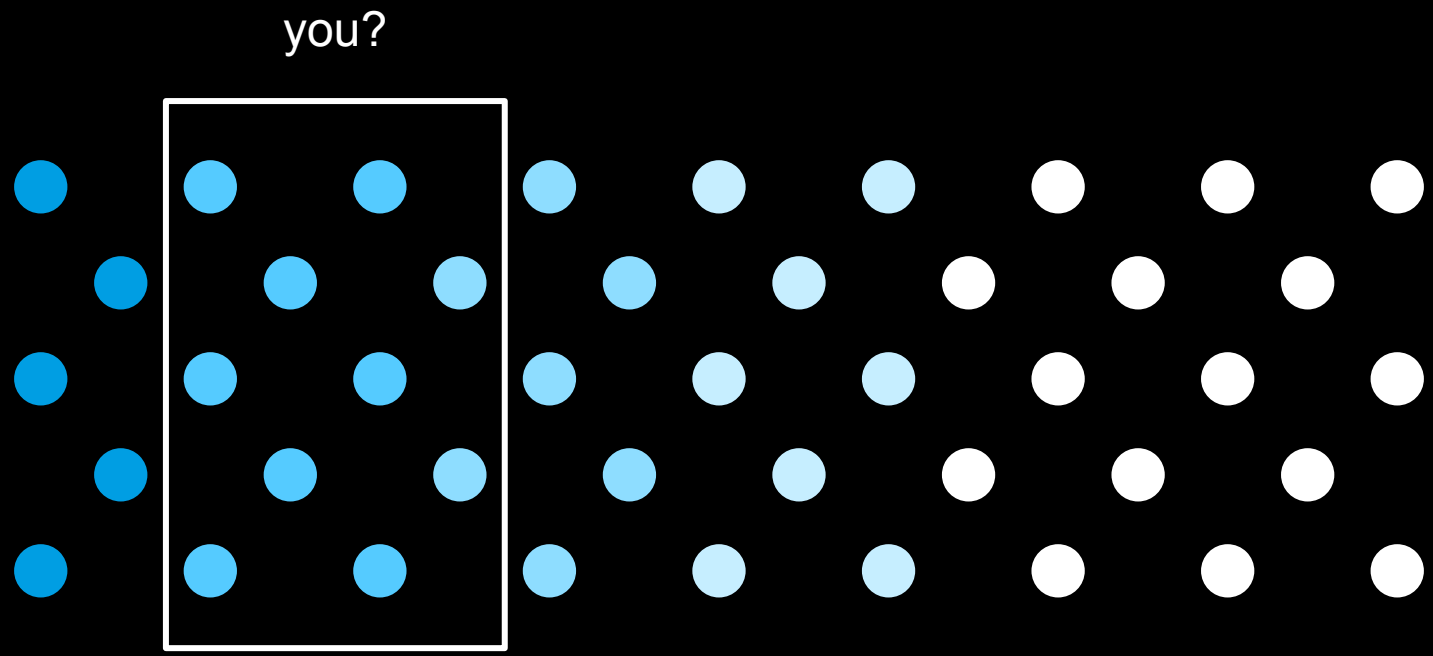
a few experts

digitally uneducated society



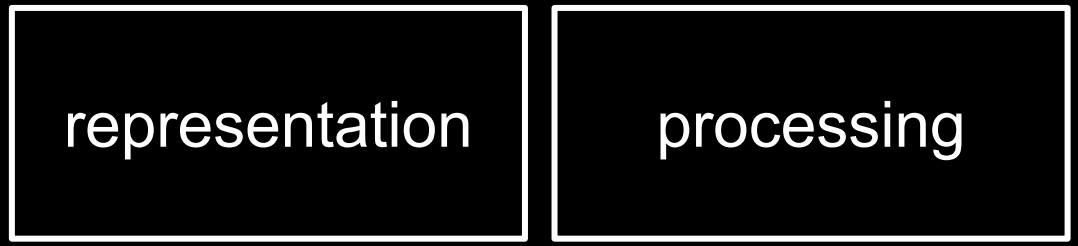
digitally illiterate society with a few experts

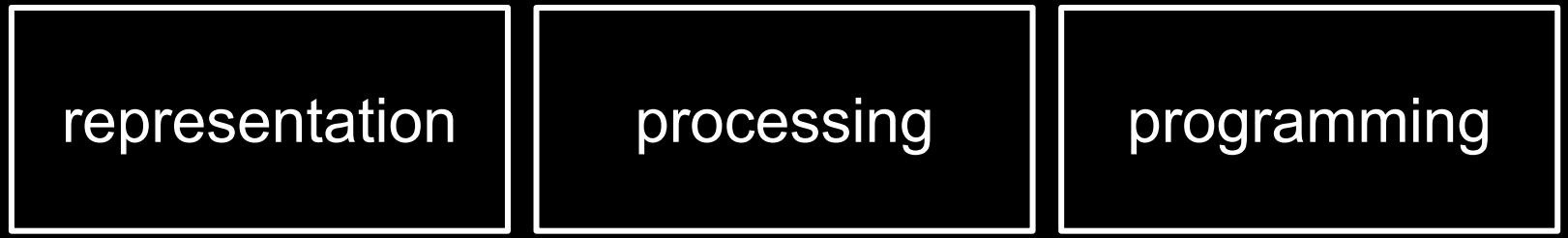
collective understanding

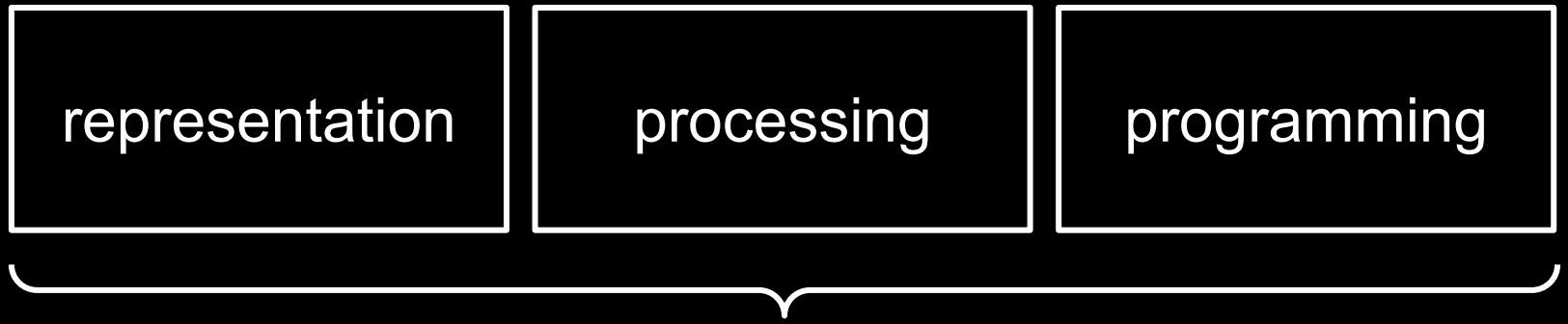


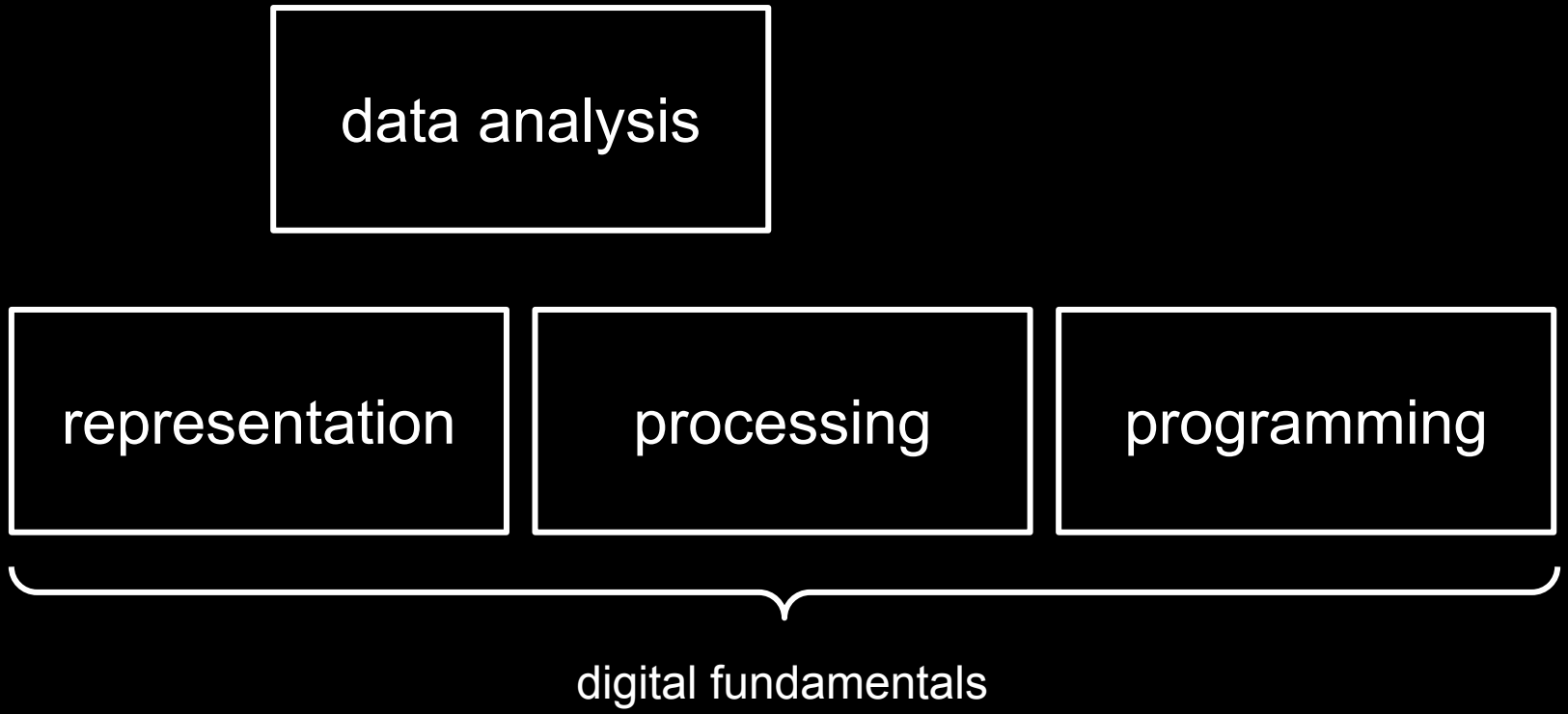
society with a distributed and high degree of digital education

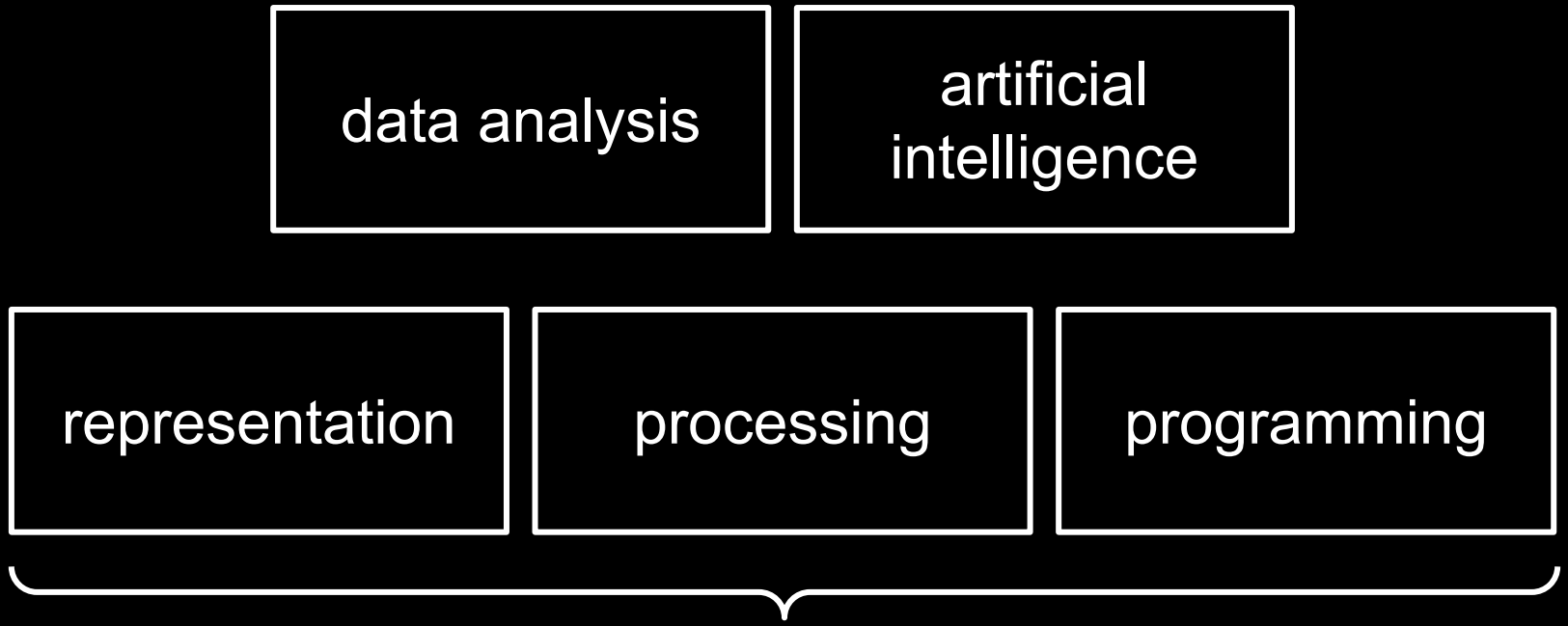
representation

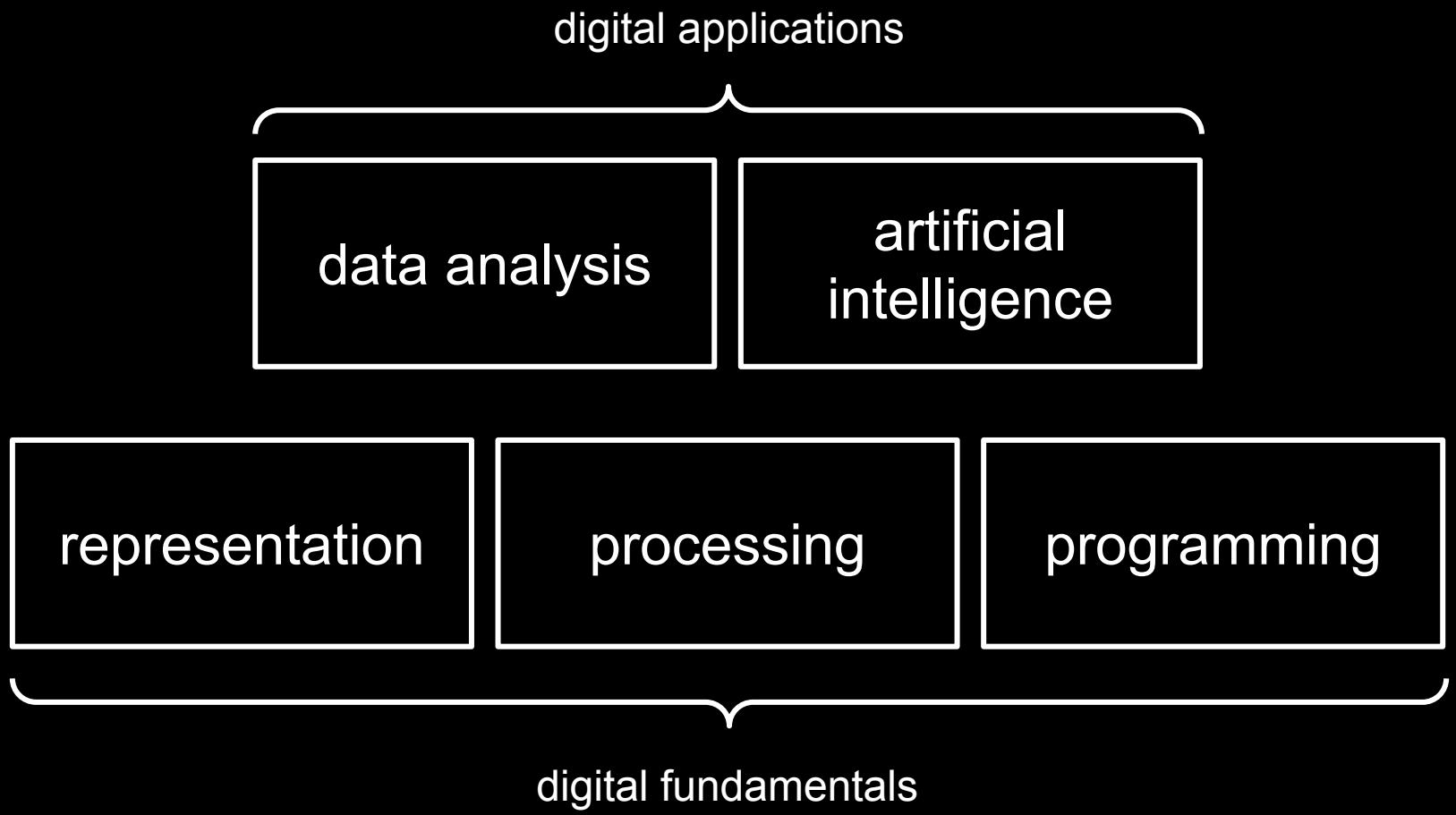

representation
processing


representation
processing
programming


representation
processing
programming
digital fundamentals


data analysis
representation
processing
programming
digital fundamentals

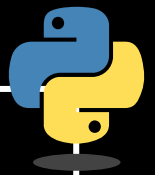

data analysis
artificial intelligence
representation
processing
programming
digital fundamentals


digital applications
data analysis
artificial intelligence
representation
processing
programming
digital fundamentals

digital applications

data analysis

artificial intelligence

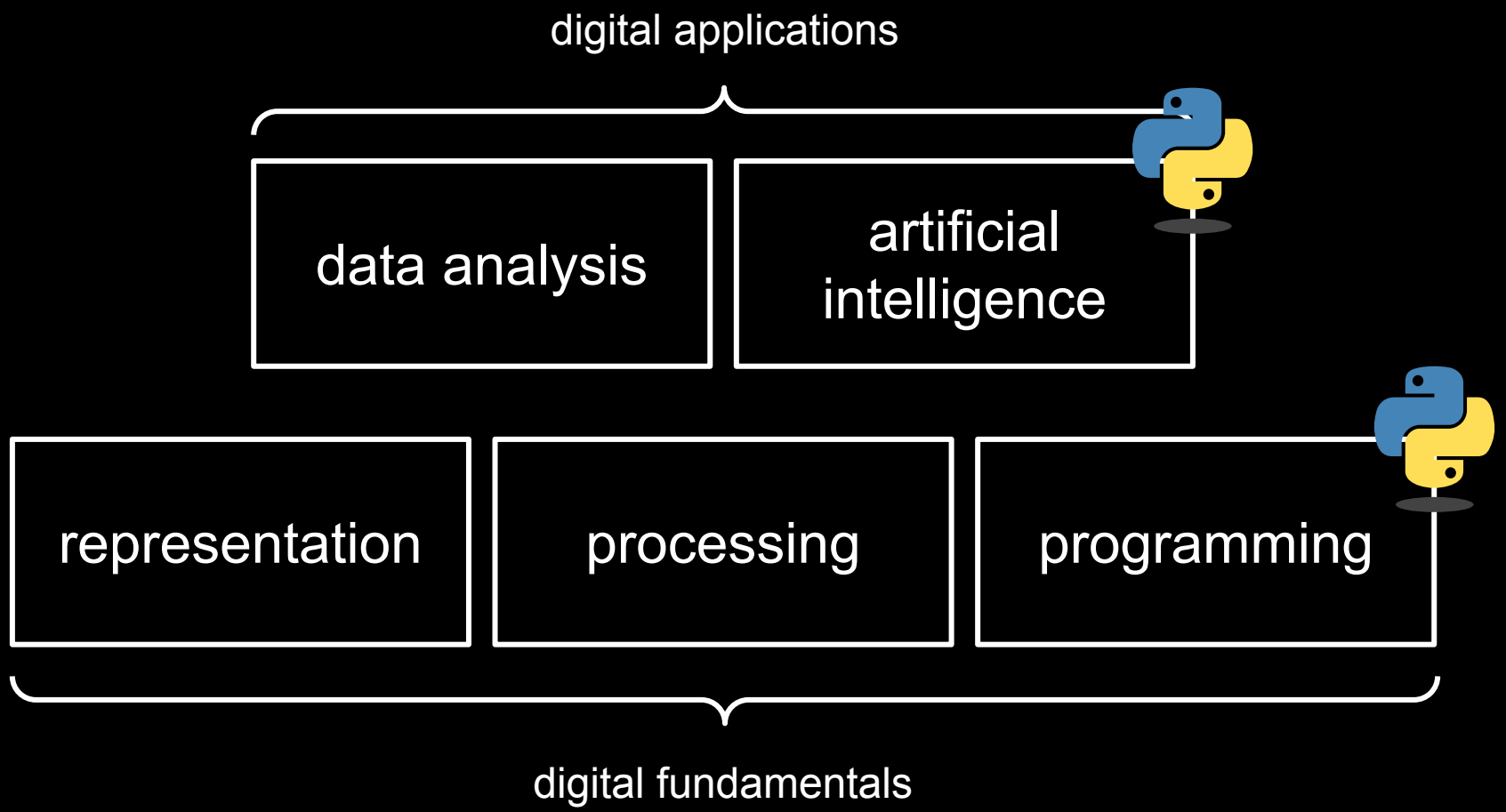


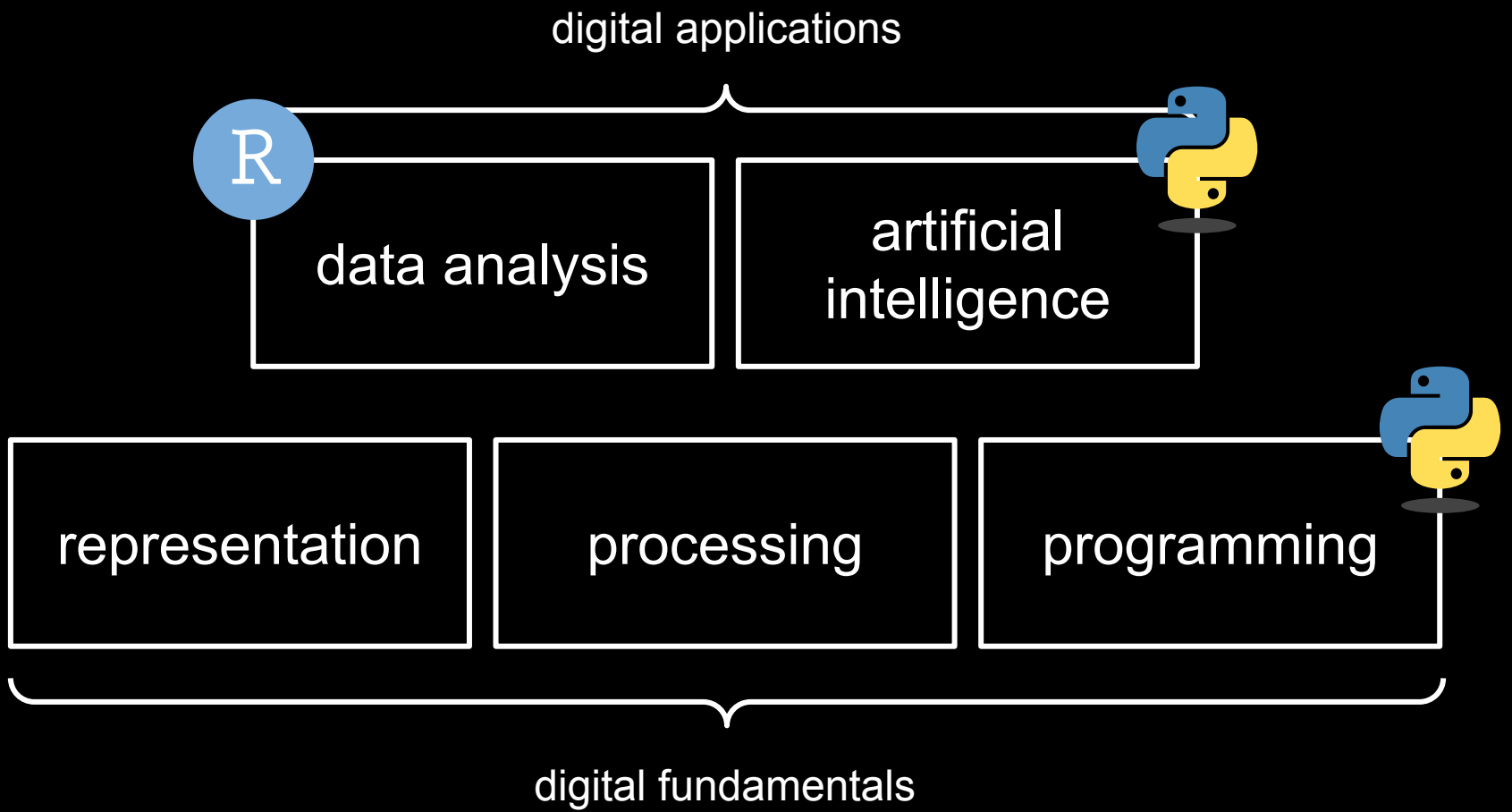
representation

processing

programming

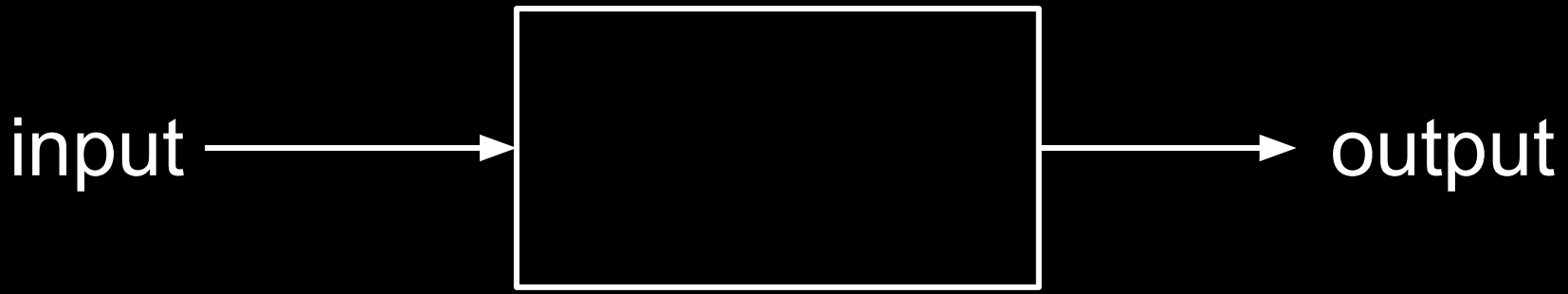
digital fundamentals


digital applications
data analysis
artificial intelligence
representation
processing
programming
digital fundamentals

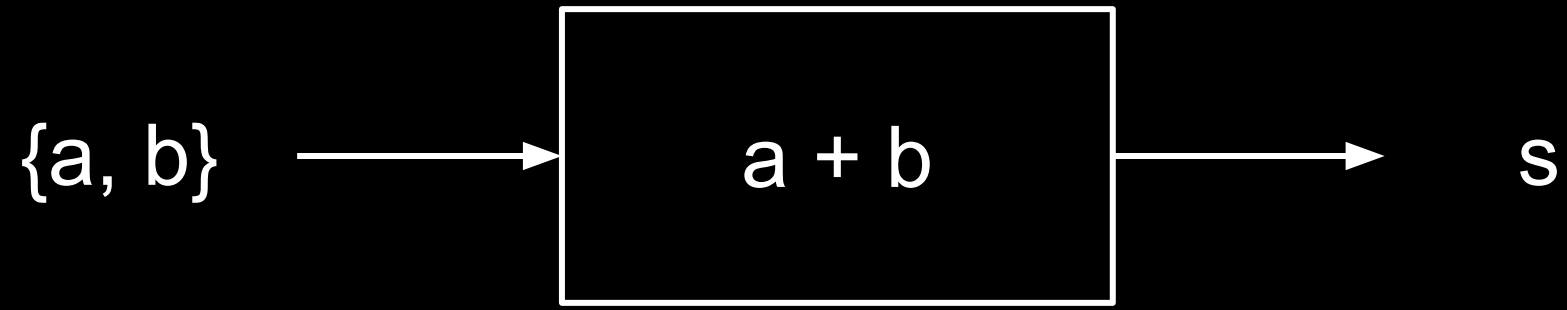

digital applications
R
data analysis
artificial
intelligence
representation
processing
programming
digital fundamentals

PROBLEM SOLVING

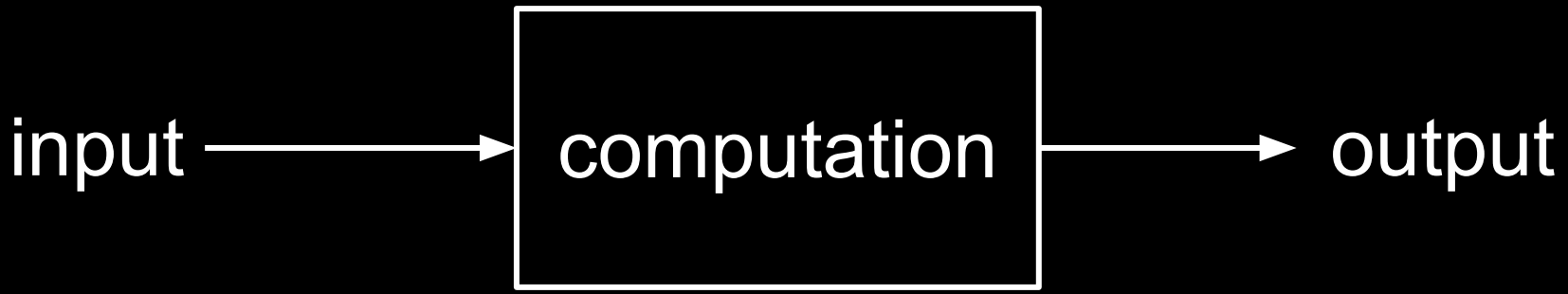
a model for solving problems

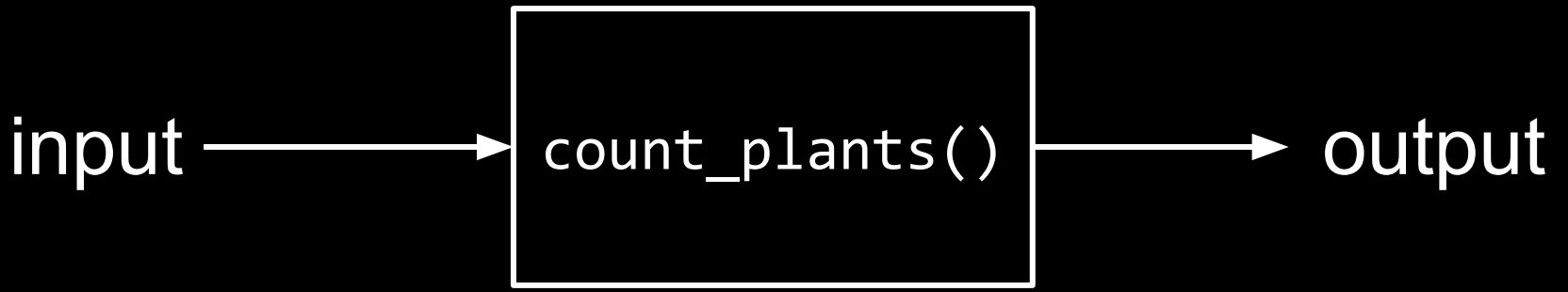


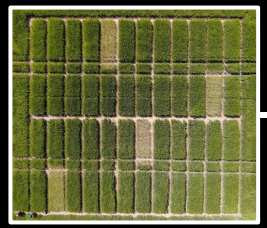
a model for solving problems


{a, b}
a + b
s

a model for solving problems

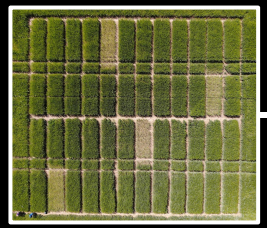



input
count_plants()
output



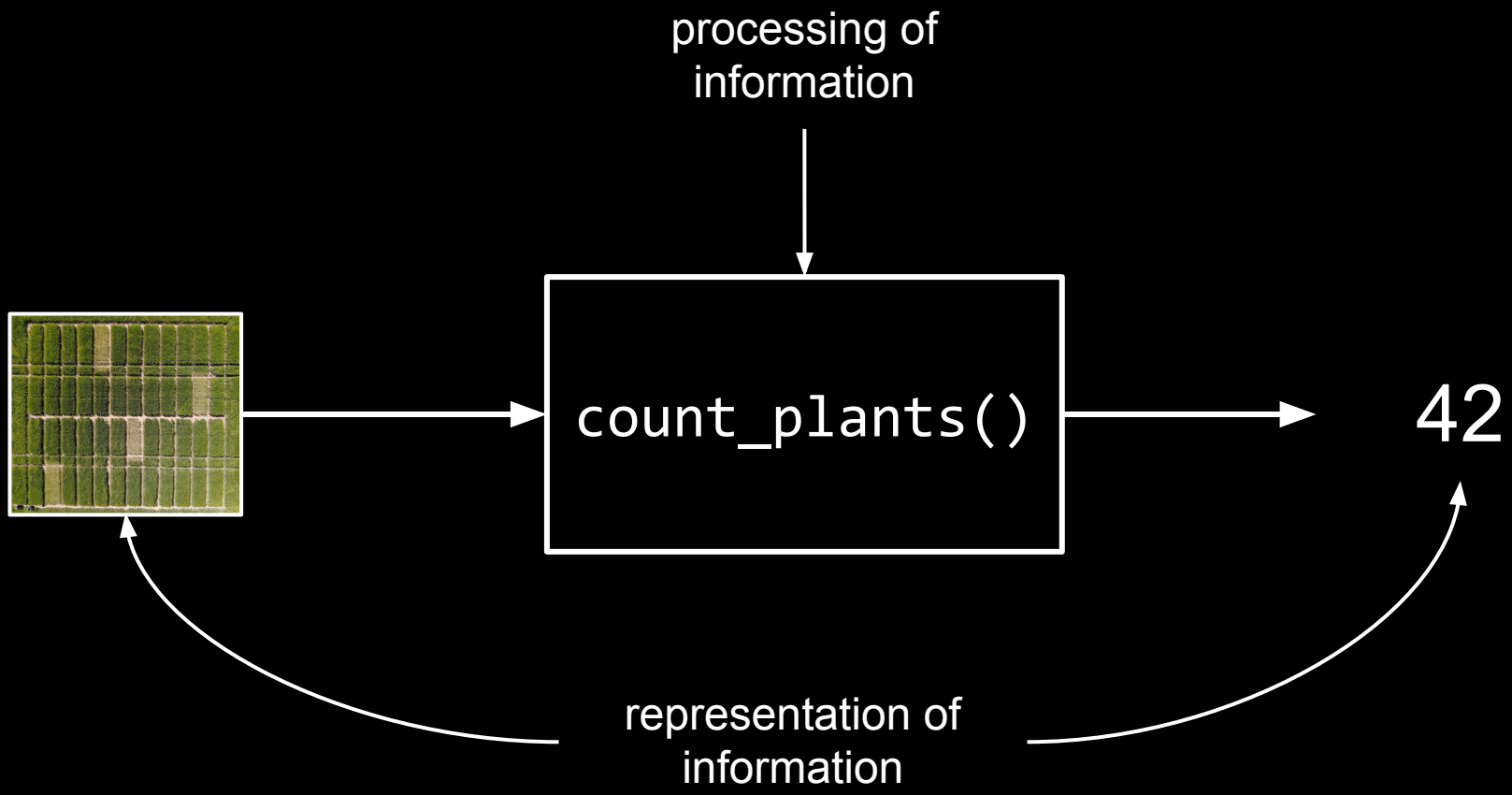
```
count_plants()
```

output

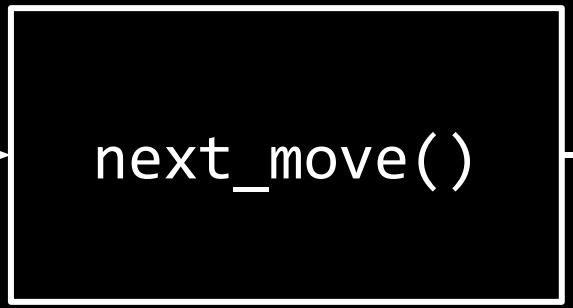


```
count_plants()
```

42


processing of
information
count_plants()
42
representation of
information

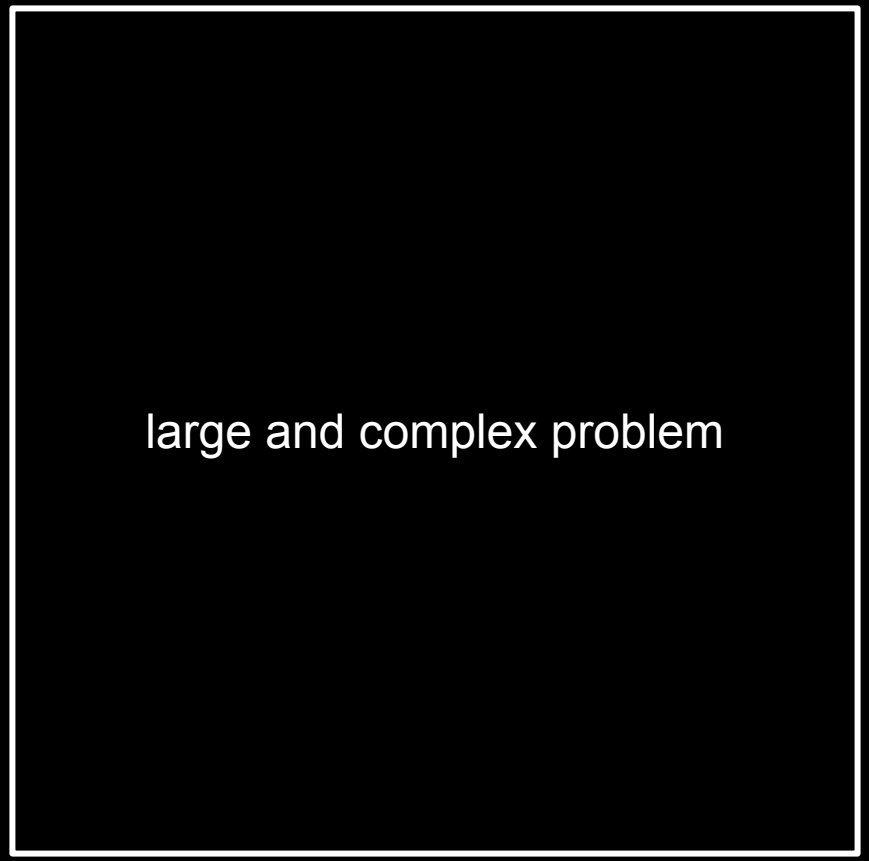

input
ChatGPT
output

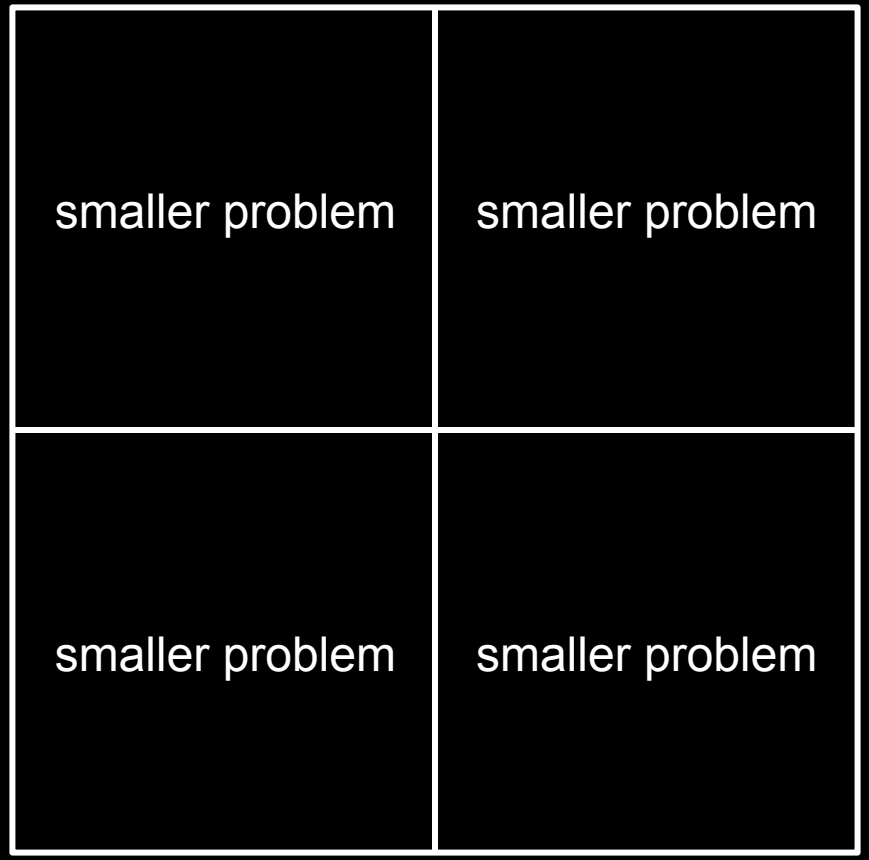


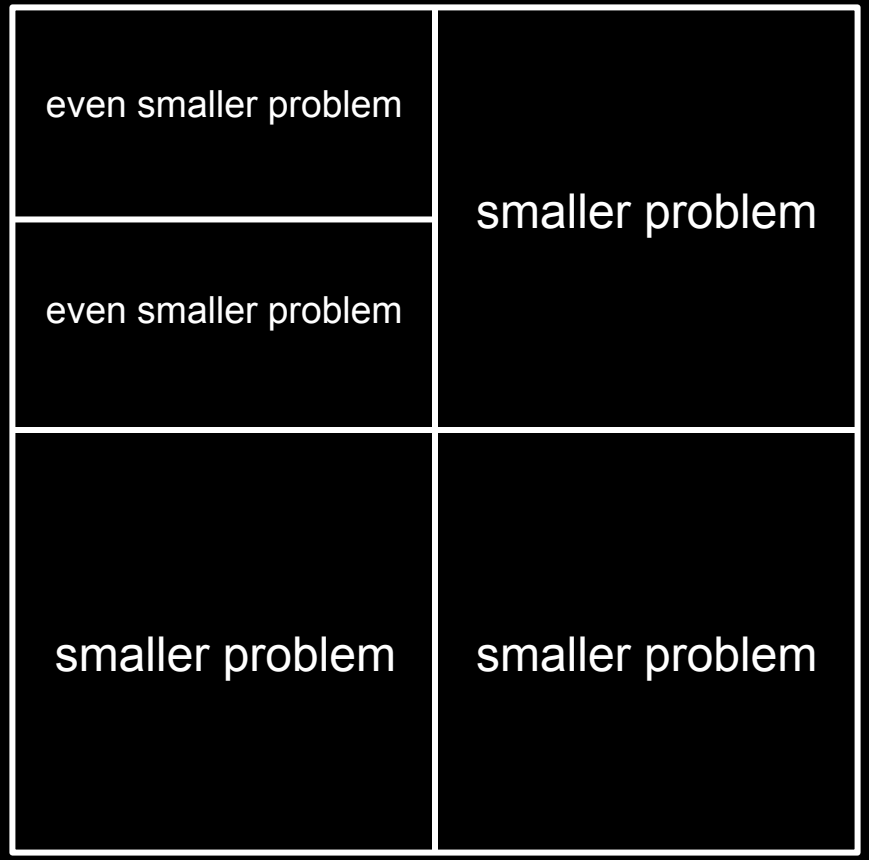
E2 → E4

problem solving strategies

divide and conquer


large and complex problem


smaller problem
smaller problem
smaller problem
smaller problem


even smaller problem
even smaller problem
smaller problem
smaller problem
smaller problem


sorted list +
element
search()
yes / no

is 67 a prime number?

```
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97
```

linear search

~~2,~~ 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

linear search

```
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97
```

linear search

~~2,~~ ~~3,~~ 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

linear search

~~2~~, ~~3~~, ~~5~~, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

linear search

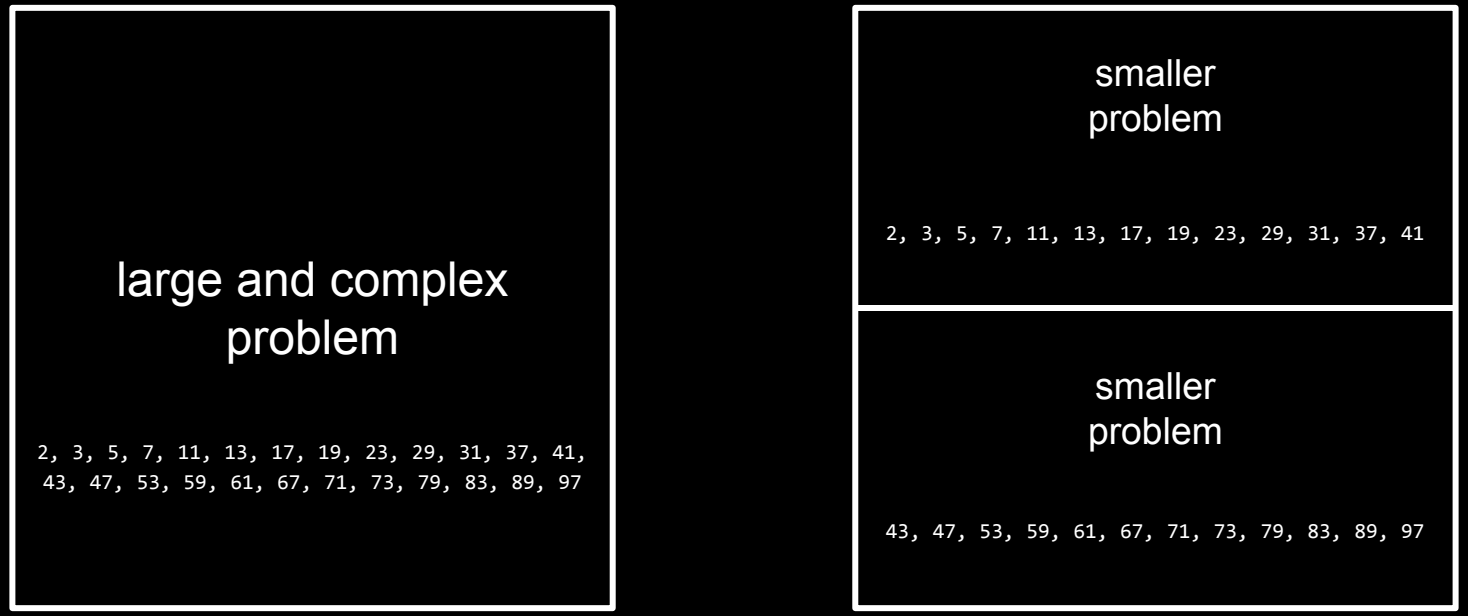
~~2~~, ~~3~~, ~~5~~, ~~7~~, ~~11~~, ~~13~~, ~~17~~, ~~19~~, ~~23~~, ~~29~~, ~~31~~, ~~37~~, ~~41~~,
~~43~~, ~~47~~, ~~53~~, ~~59~~, ~~61~~, 67, 71, 73, 79, 83, 89, 97

19 steps… can’t we do better?

~~2~~, ~~3~~, ~~5~~, ~~7~~, ~~11~~, ~~13~~, ~~17~~, ~~19~~, ~~23~~, ~~29~~, ~~31~~, ~~37~~, ~~41~~,
~~43~~, ~~47~~, ~~53~~, ~~59~~, ~~61~~, 67, 71, 73, 79, 83, 89, 97

large and complex problem

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97


large and complex problem
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97
smaller problem
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41
smaller problem
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

binary search

```
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97
```

binary search

67 != 41

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, ~~41~~,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

binary search

67 > 41

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, ~~41~~,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

binary search

67 > 41

~~2~~, ~~3~~, ~~5~~, ~~7~~, ~~11~~, ~~13~~, ~~17~~, ~~19~~, ~~23~~, ~~29~~, ~~31~~, ~~37~~, ~~41~~,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

binary search

~~2~~, ~~3~~, ~~5~~, ~~7~~, ~~11~~, ~~13~~, ~~17~~, ~~19~~, ~~23~~, ~~29~~, ~~31~~, ~~37~~, ~~41~~,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

67 != 71

binary search

~~2~~, ~~3~~, ~~5~~, ~~7~~, ~~11~~, ~~13~~, ~~17~~, ~~19~~, ~~23~~, ~~29~~, ~~31~~, ~~37~~, ~~41~~,
43, 47, 53, 59, 61, 67, ~~71~~, 73, 79, 83, 89, 97

67 != 71

binary search

~~2~~, ~~3~~, ~~5~~, ~~7~~, ~~11~~, ~~13~~, ~~17~~, ~~19~~, ~~23~~, ~~29~~, ~~31~~, ~~37~~, ~~41~~,
43, 47, 53, 59, 61, 67, ~~71~~, ~~73~~, ~~79~~, ~~83~~, ~~89~~, ~~97~~

67 < 71

binary search

~~2~~, ~~3~~, ~~5~~, ~~7~~, ~~11~~, ~~13~~, ~~17~~, ~~19~~, ~~23~~, ~~29~~, ~~31~~, ~~37~~, ~~41~~,
43, 47, 53, ~~59~~, 61, 67, ~~71~~, ~~73~~, ~~79~~, ~~83~~, ~~89~~, ~~97~~

67 != 59

binary search

~~2~~, ~~3~~, ~~5~~, ~~7~~, ~~11~~, ~~13~~, ~~17~~, ~~19~~, ~~23~~, ~~29~~, ~~31~~, ~~37~~, ~~41~~,
~~43~~, ~~47~~, ~~53~~, ~~59~~, 61, 67, ~~71~~, ~~73~~, ~~79~~, ~~83~~, ~~89~~, ~~97~~

67 > 59

binary search

~~2~~, ~~3~~, ~~5~~, ~~7~~, ~~11~~, ~~13~~, ~~17~~, ~~19~~, ~~23~~, ~~29~~, ~~31~~, ~~37~~, ~~41~~,
~~43~~, ~~47~~, ~~53~~, ~~59~~, 61, 67, ~~71~~, ~~73~~, ~~79~~, ~~83~~, ~~89~~, ~~97~~

67 = 67

binary search

~~2~~, ~~3~~, ~~5~~, ~~7~~, ~~11~~, ~~13~~, ~~17~~, ~~19~~, ~~23~~, ~~29~~, ~~31~~, ~~37~~, ~~41~~,
~~43~~, ~~47~~, ~~53~~, ~~59~~, ~~61~~, 67, ~~71~~, ~~73~~, ~~79~~, ~~83~~, ~~89~~, ~~97~~

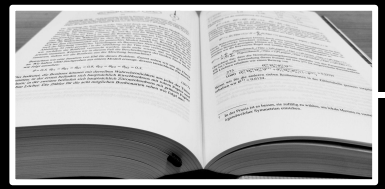
67 = 67

3 splits → much better

~~2~~, ~~3~~, ~~5~~, ~~7~~, ~~11~~, ~~13~~, ~~17~~, ~~19~~, ~~23~~, ~~29~~, ~~31~~, ~~37~~, ~~41~~,
~~43~~, ~~47~~, ~~53~~, ~~59~~, ~~61~~, 67, ~~71~~, ~~73~~, ~~79~~, ~~83~~, ~~89~~, ~~97~~

67 = 67

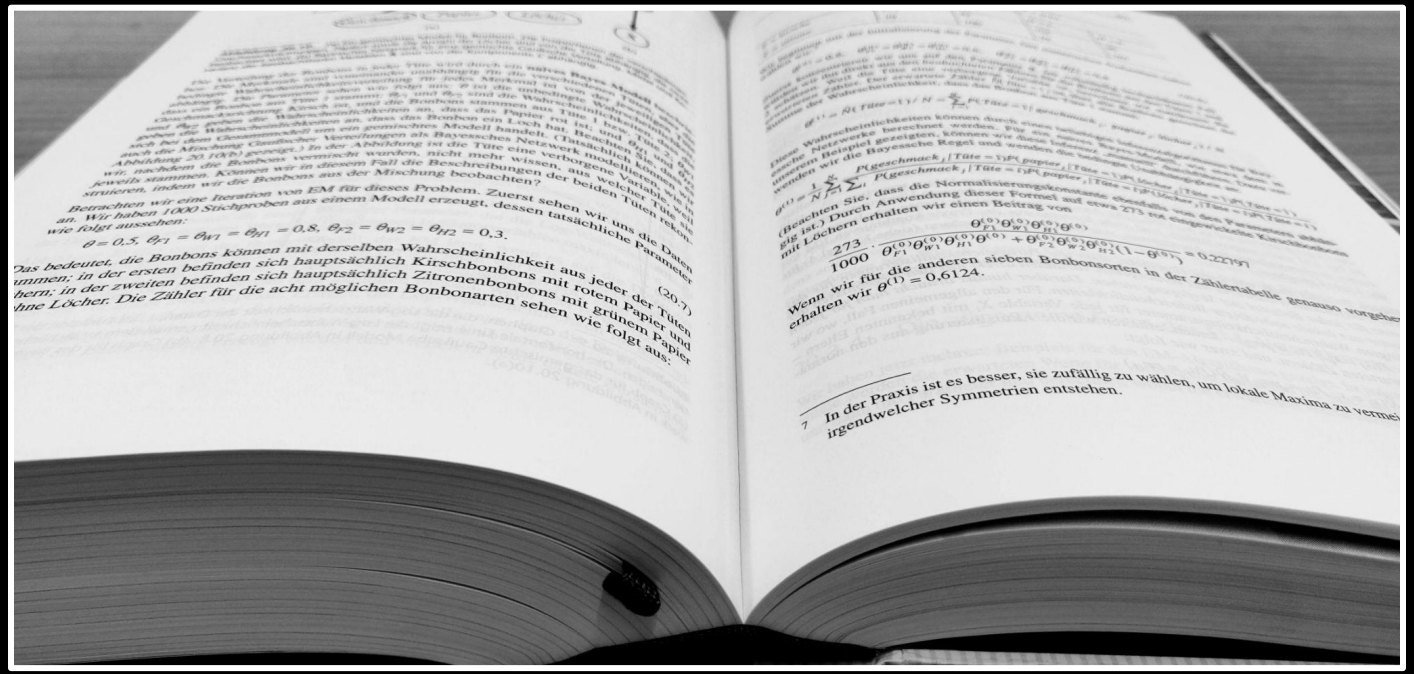
how efficient are linear and binary search in general?



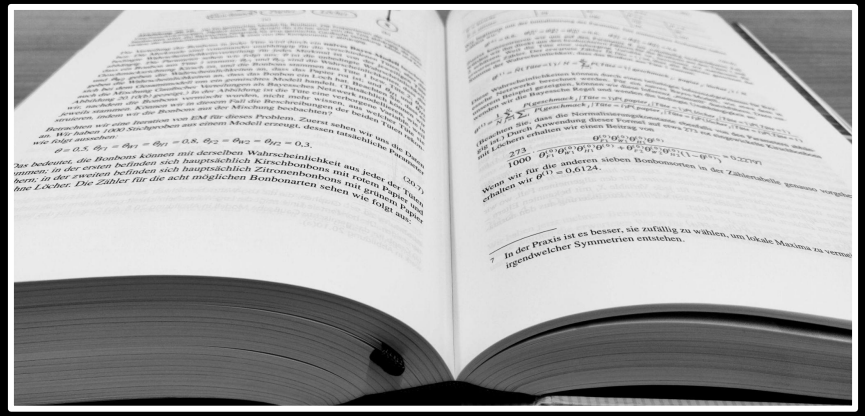
`count_words()`

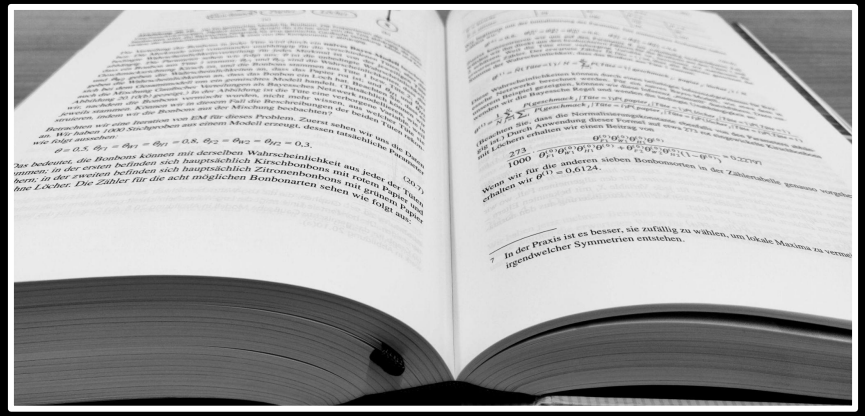
word count

how many words are in the book?

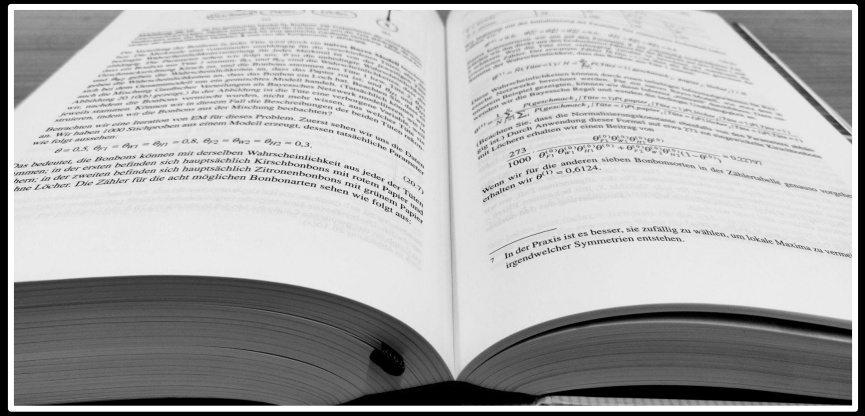


strategies, anyone?



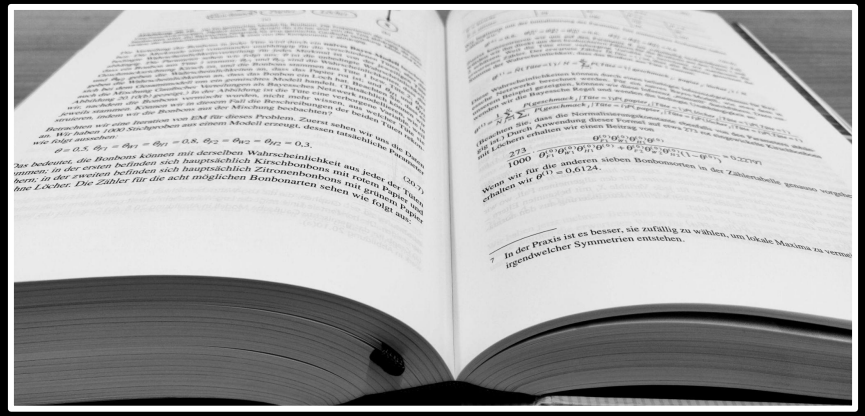


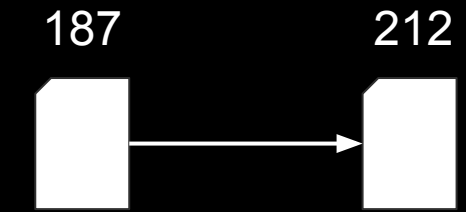
page 1



187

page 1

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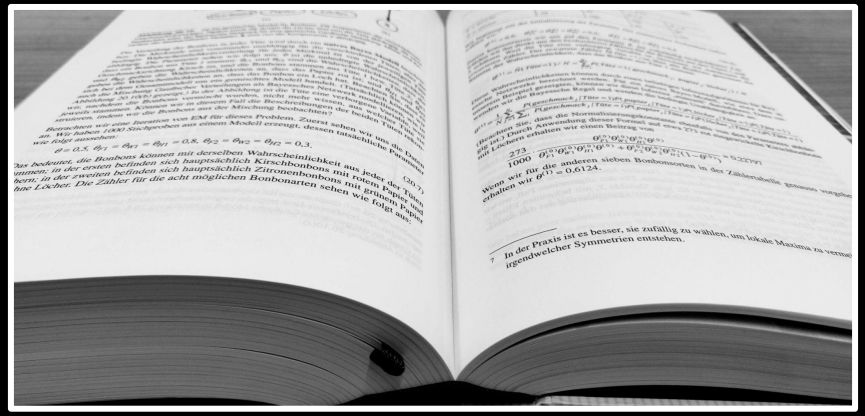
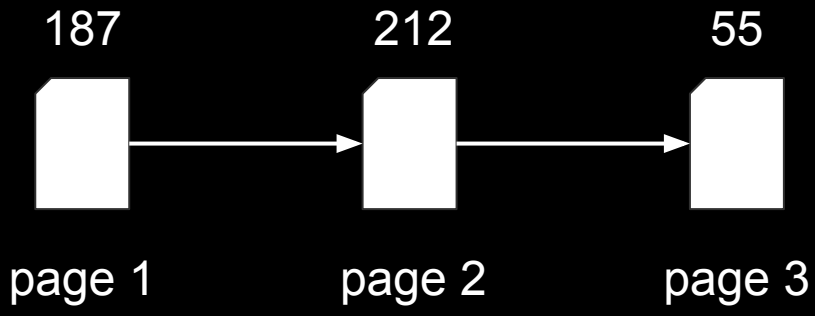
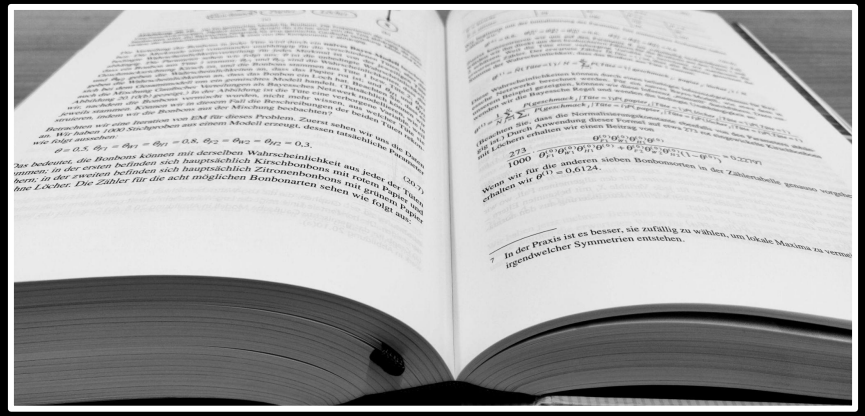
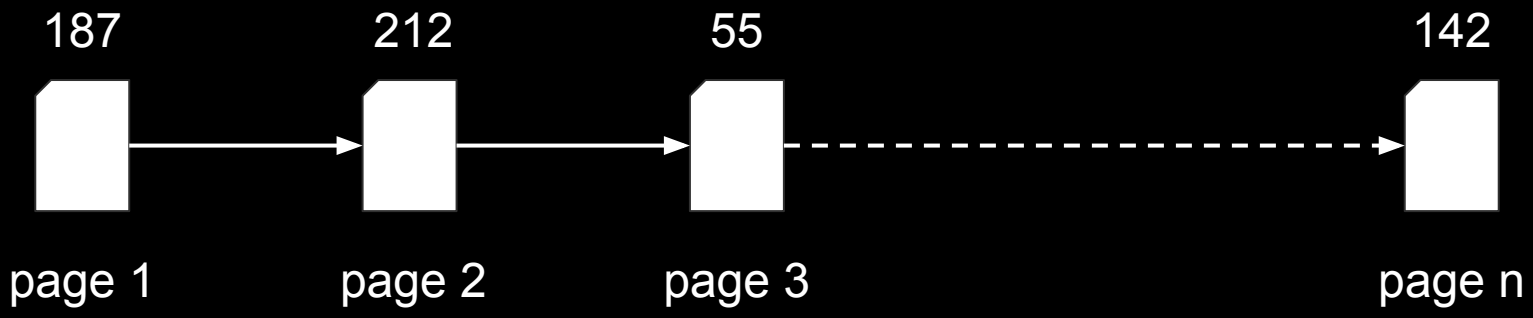
212
page 1
page 2




187
212
55
page 1
page 2
page 3




187
212
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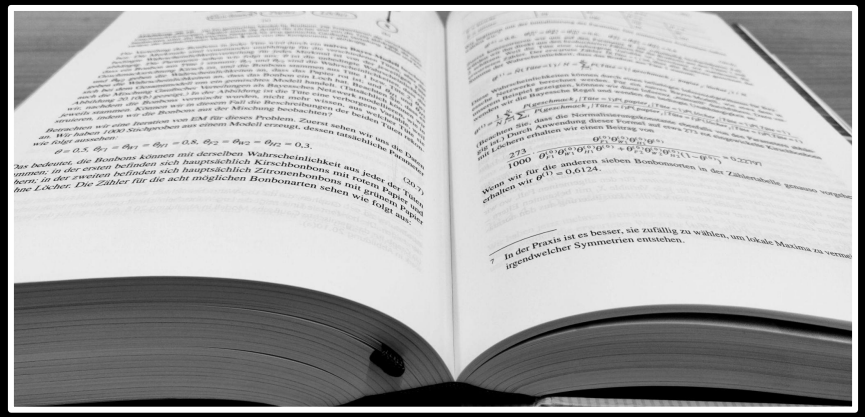
page 1
page 2
page 3
page n



n = 1327 pages

Ø 2:23 minutes per page

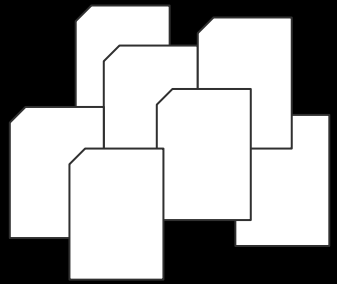
~ 52.34 hours

divide and conquer

+

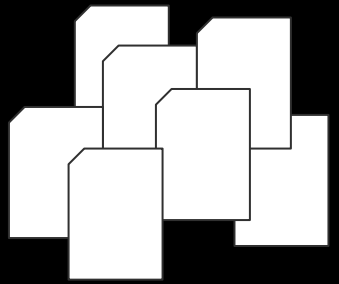
?

pages 1 - 700



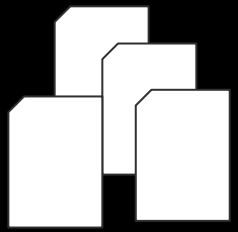
student 1

pages 701 - 1327



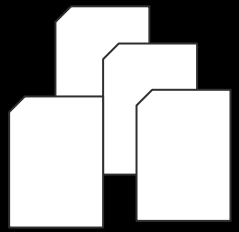
student 2

pages 1 - 350



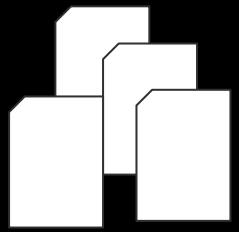
student 1

pages 351 - 700



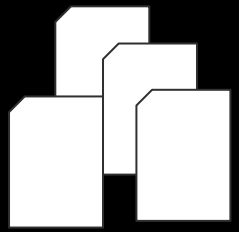
student 2

pages 701 - 1050



student 3

pages 1051- 1327

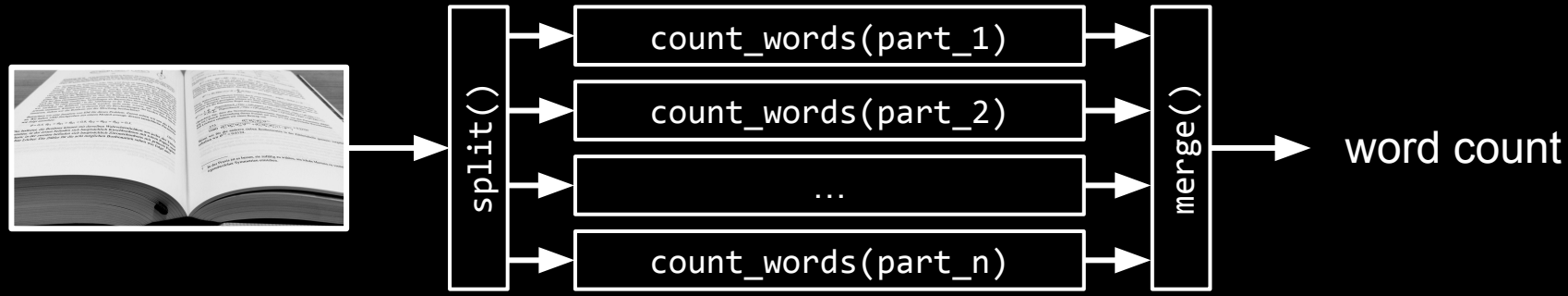


student 4

divide and conquer

+

distribution and parallelization


split()
count_words(part_1)
count_words(part_2)
...
count_words(part_n)
merge()
word count

INFORMATION

“Information is that which allows you to make a correct prediction with accuracy better than chance.”

Adami, Christoph. “What Is Information?” Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences, vol. 374, no. 2063, Mar. 2016, p. 20150230, https://doi.org/10.1098/rsta.2015.0230.

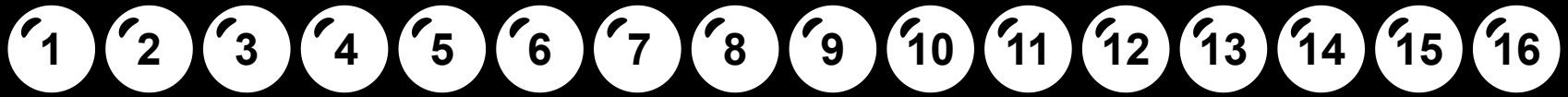
“Information is that which allows you to make a correct prediction with accuracy better than chance.”

Adami, Christoph. “What Is Information?” Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences, vol. 374, no. 2063, Mar. 2016, p. 20150230, https://doi.org/10.1098/rsta.2015.0230.

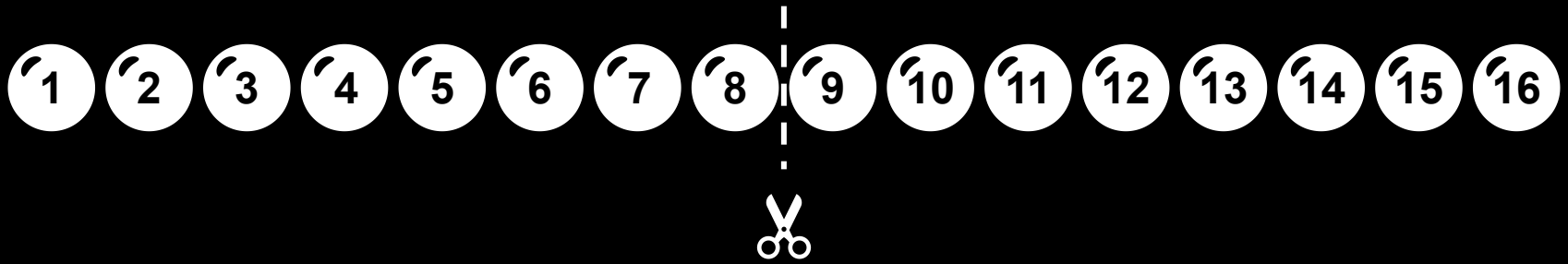
defining and measuring information

guess the number am I thinking of!

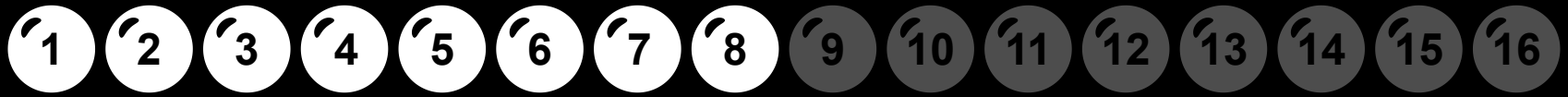
what is the most efficient approach?



is it > 8?

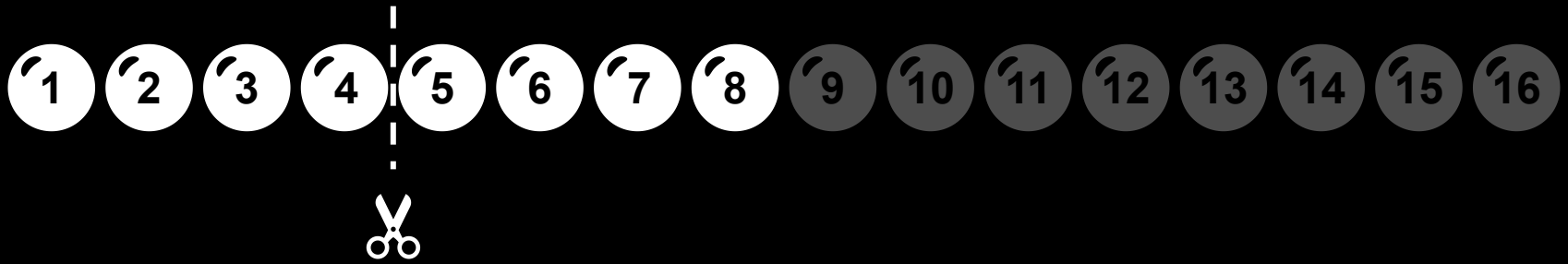


is it > 8? ✗



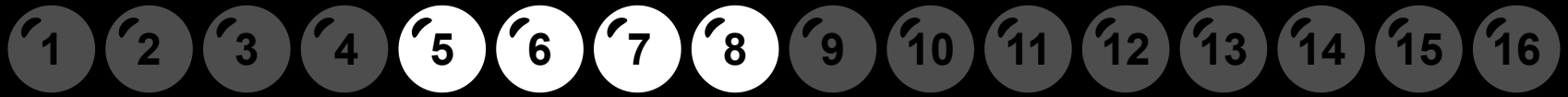
is it > 8? ✗

is it > 4?



is it > 8? ✗

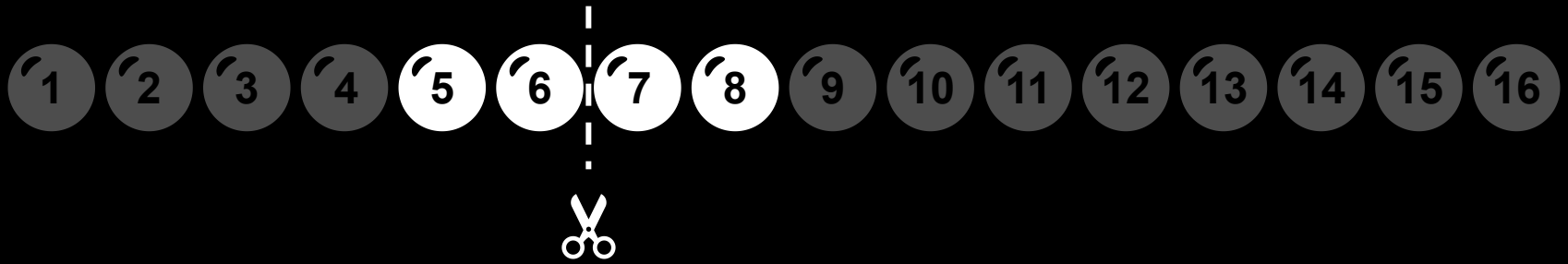
is it > 4? ✓



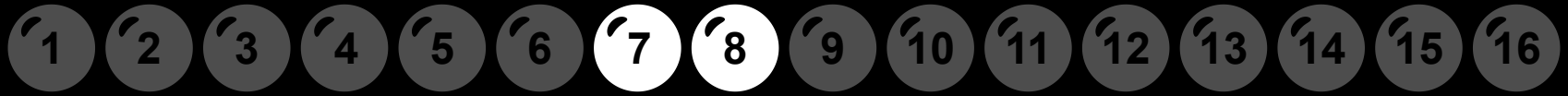
is it > 8? ✗

is it > 4? ✓

is it > 6?



is it > 8? ✗
is it > 4? ✓
is it > 6? ✓

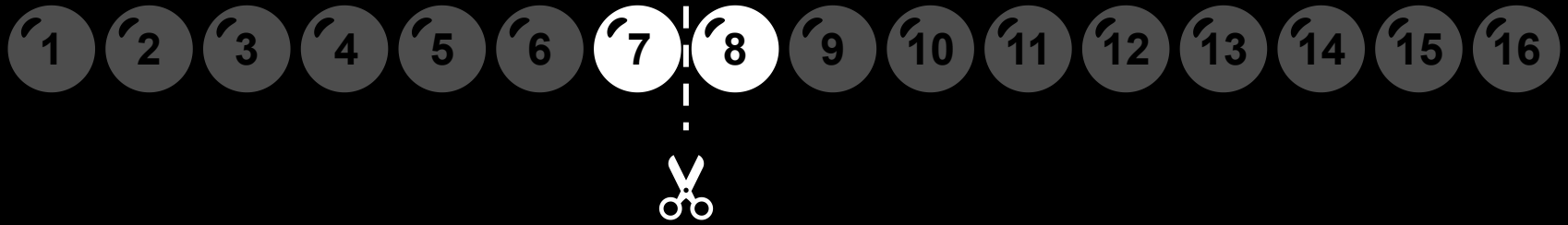


is it > 8? ✗

is it > 4? ✓

is it > 6? ✓

is it > 7?



is it > 8? ✗
is it > 4? ✓
is it > 6? ✓
is it > 7? ✗

is it > 8? ✗
is it > 4? ✓
is it > 6? ✓
is it > 7? ✗

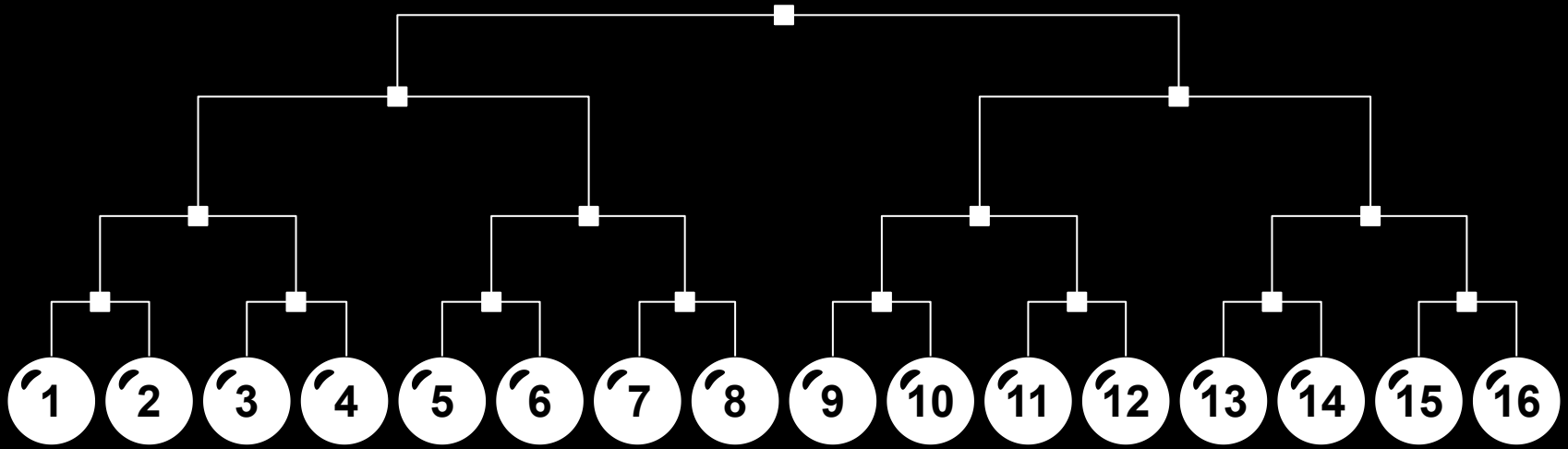
with 4 questions from 16 to 1

is it > 8? ✗ 0
is it > 4? ✓ 1
is it > 6? ✓ 1
is it > 7? ✗ 0

with 4 questions from 16 to 1

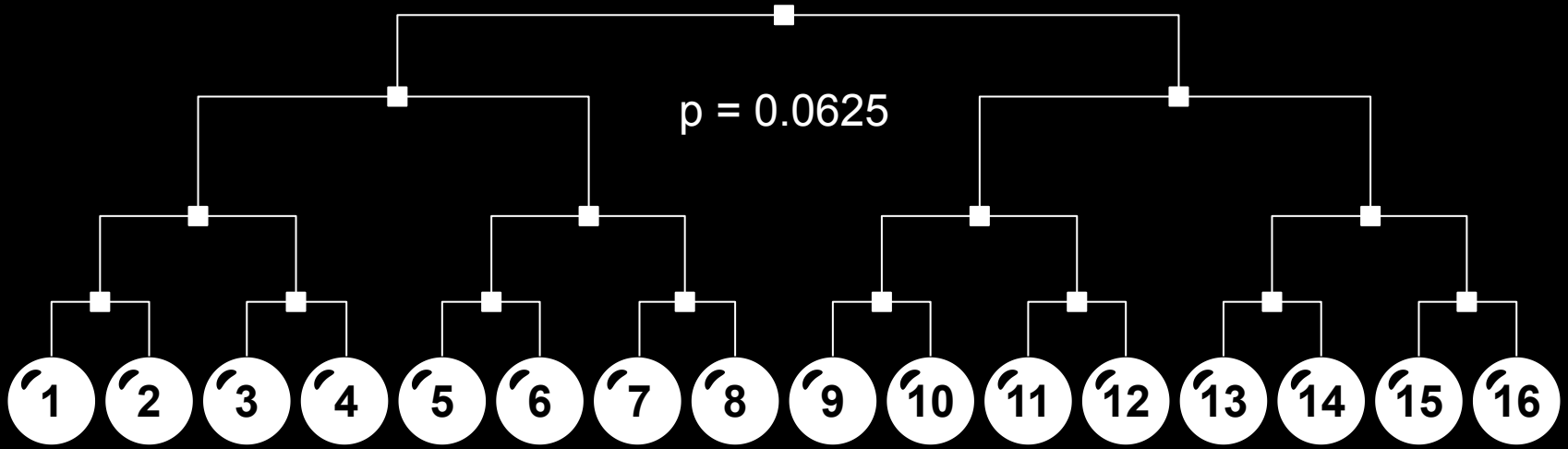
where is the information?

where is the information?



where is the information?

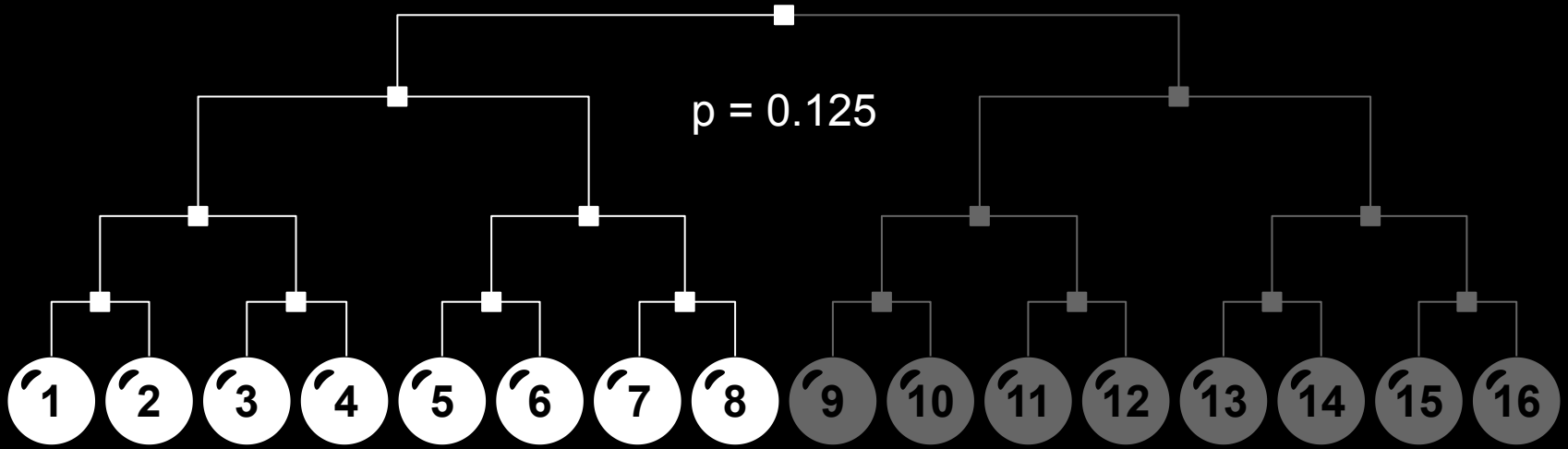
N = 16



where is the information?

~~N = 16~~

N = 8

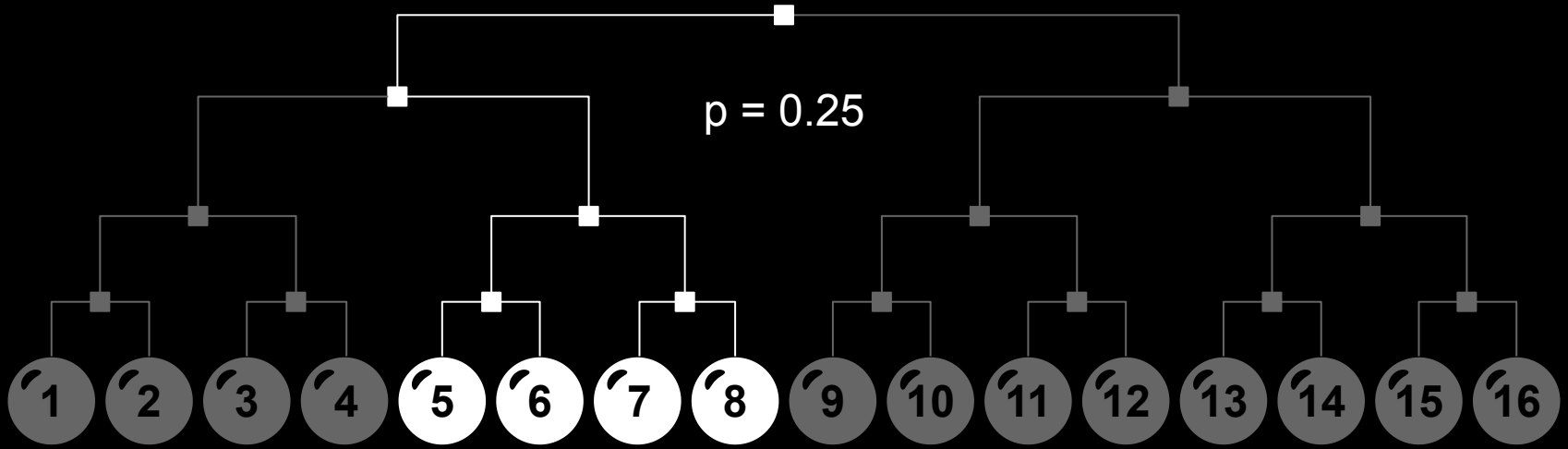


where is the information?

~~N = 16~~

~~N = 8~~

N = 4



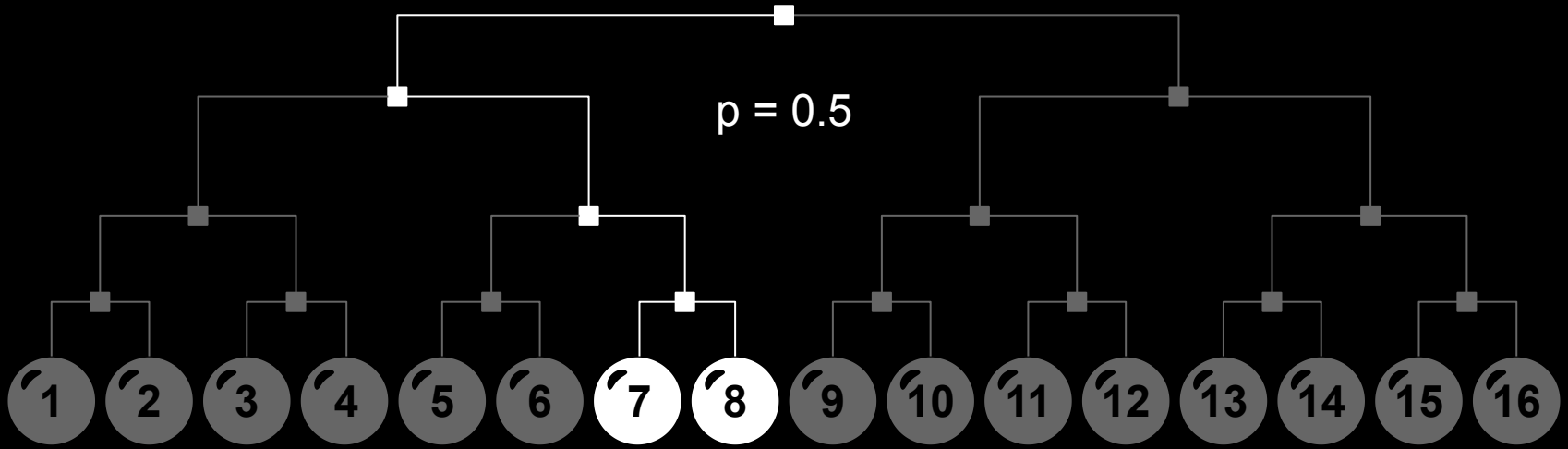
where is the information?

~~N = 16~~

~~N = 8~~

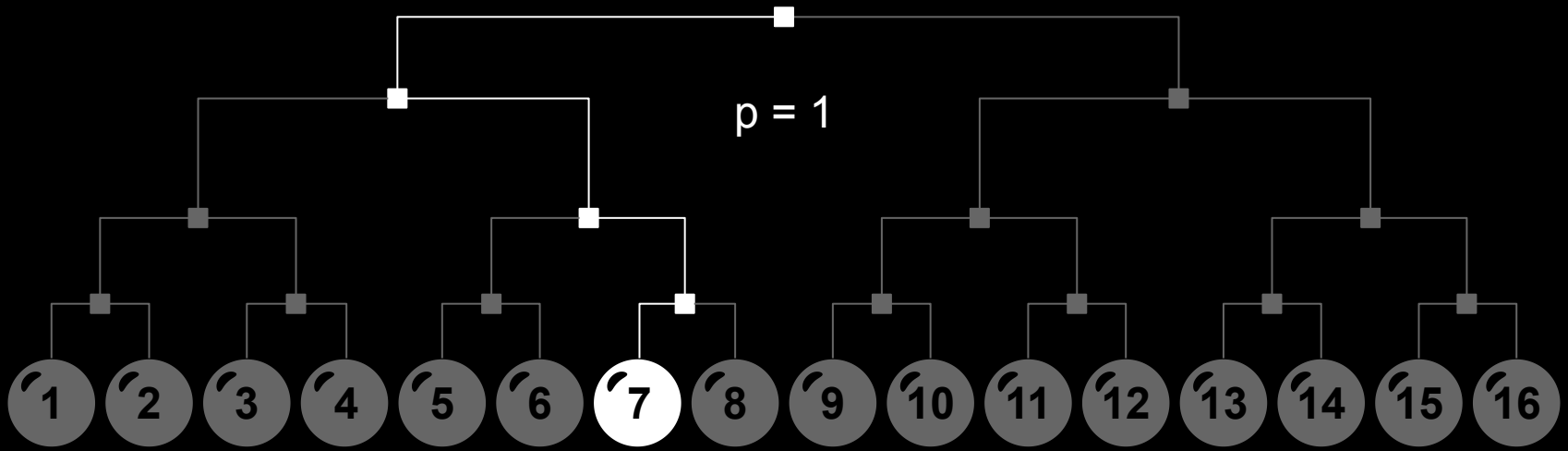
~~N = 4~~

N = 2



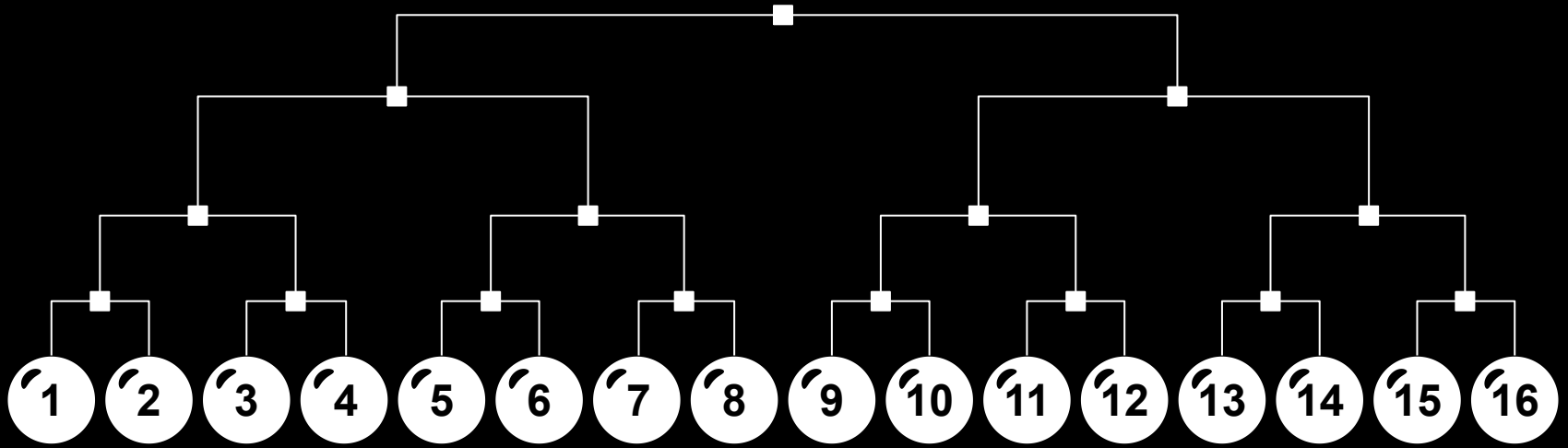
where is the information?

~~N = 16~~
~~N = 8~~
~~N = 4~~
~~N = 2~~
N = 1



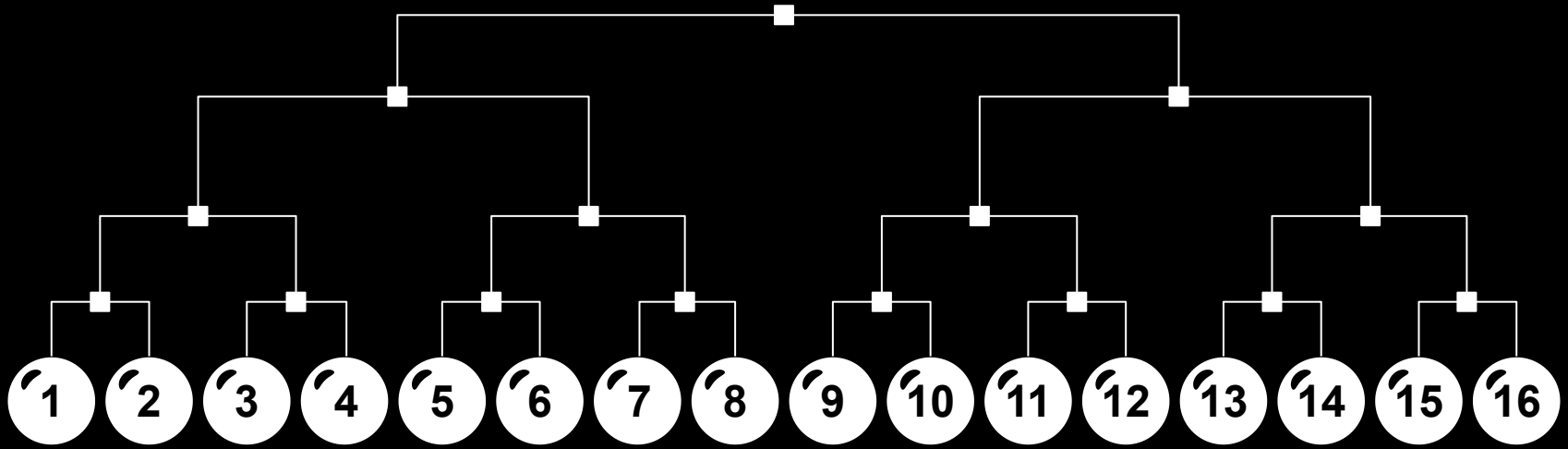
information = reduced uncertainty
uncertainty is measured with the logarithm of N

$$H = log_2(N)$$

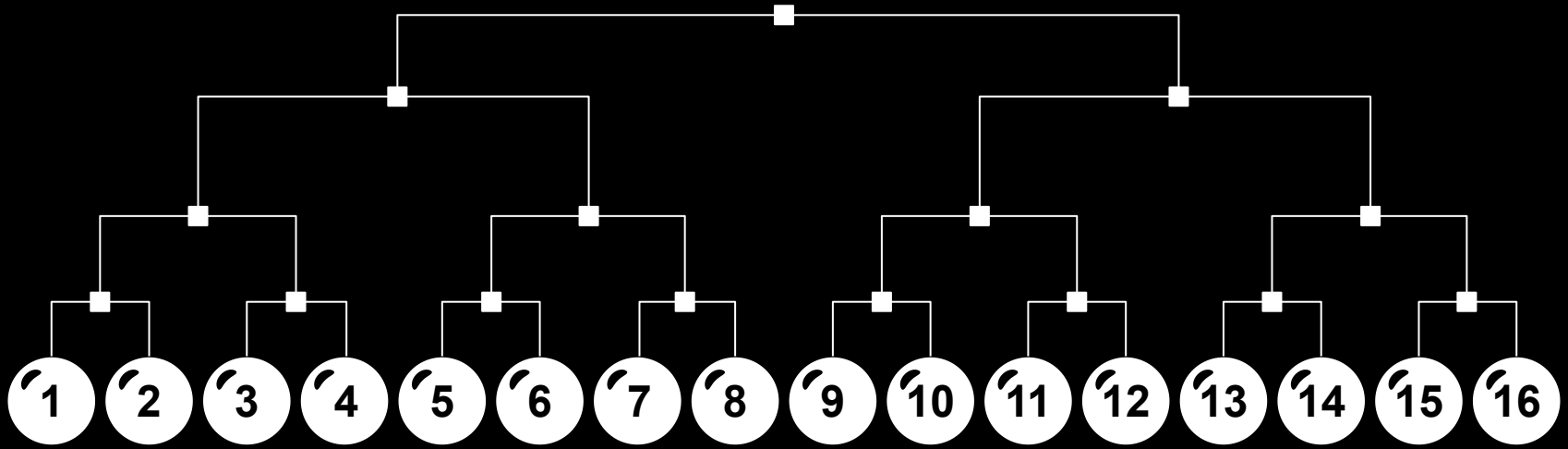


or: how often can we cut the remaining possibilities in half?

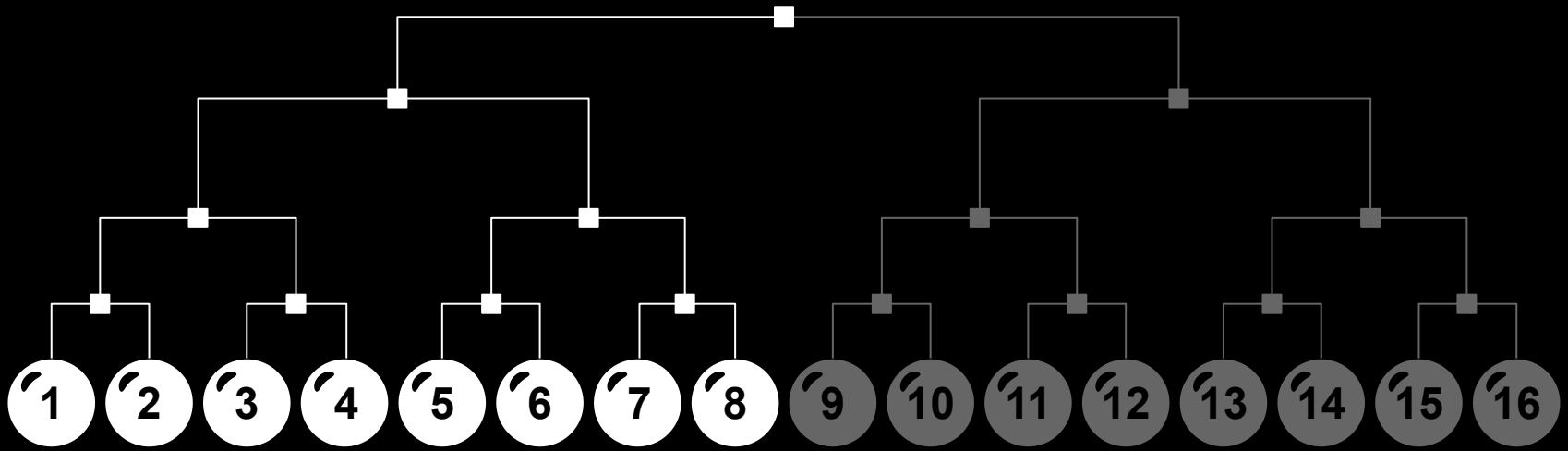
$$H = log_2(N)$$



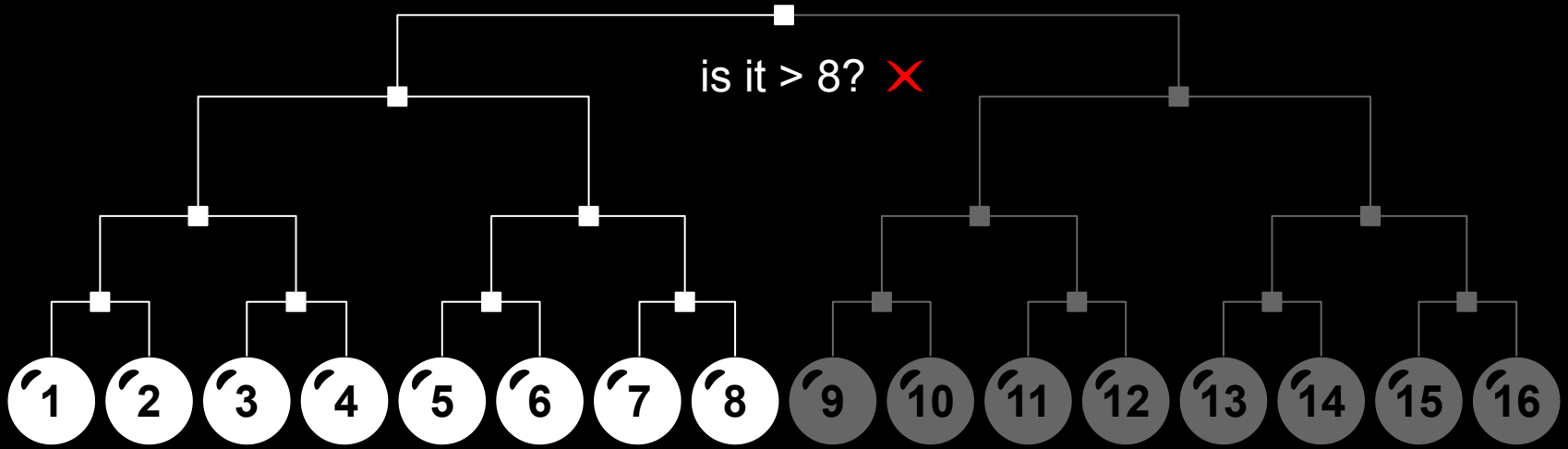
$$H_0 = log_2(16) = 4$$



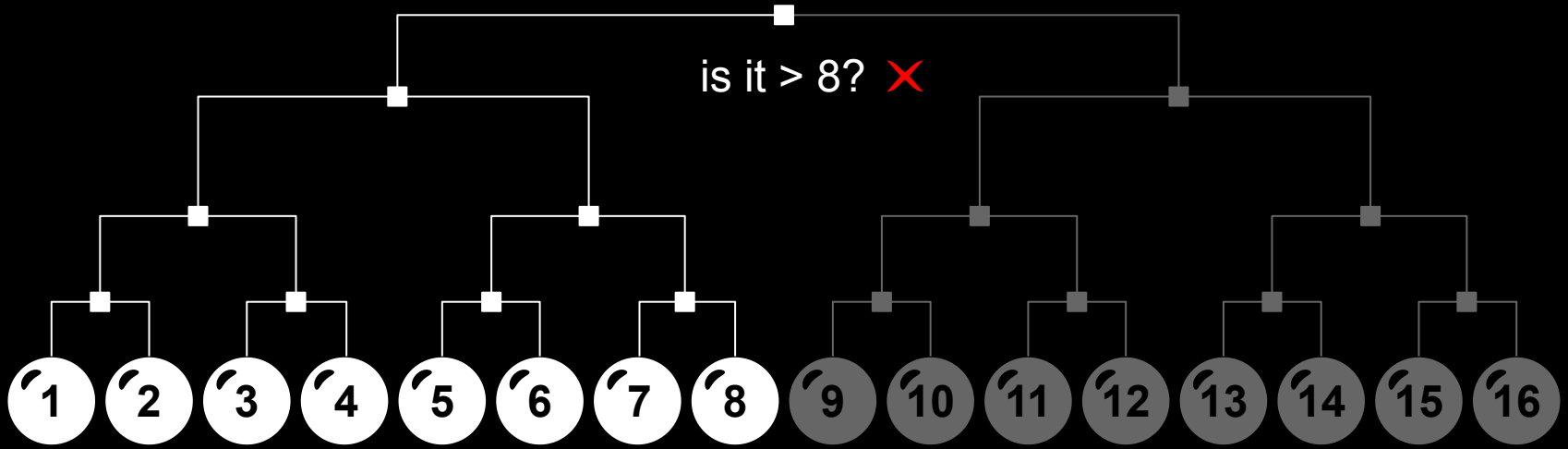
$$H_1 = log_2(8) = 3$$



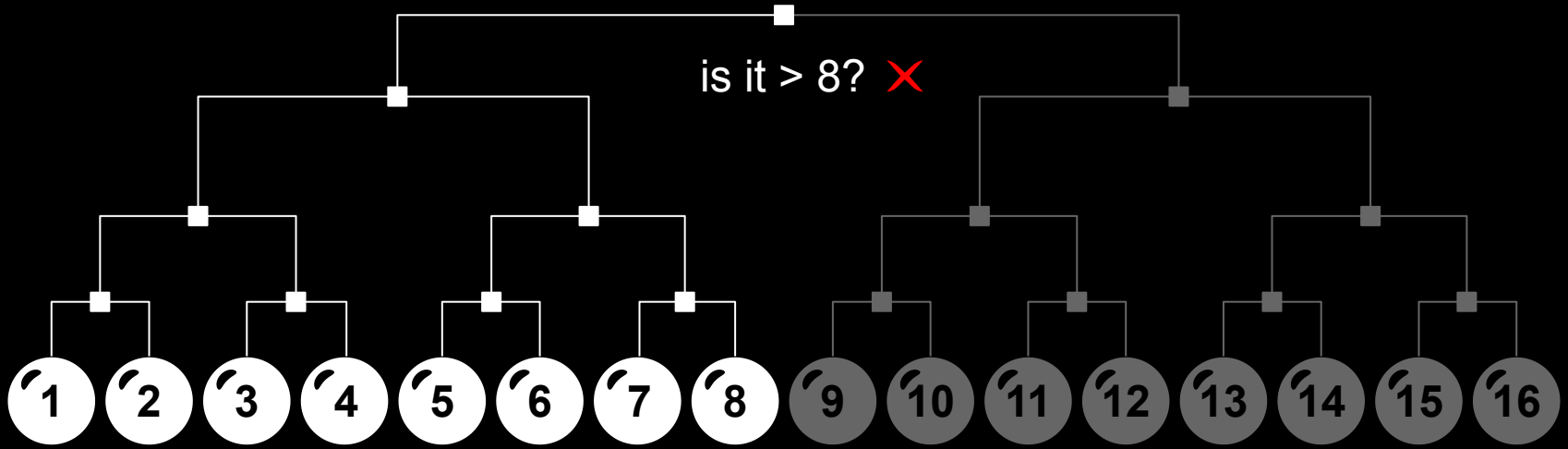
$$I = H_0 - H_1$$



$$I = log_2(16) - log_2(8)$$



$$I = 4 - 3 = 1$$



uncertainty and information are
measured in **bits**

how many yes/no questions to reduce uncertainty to zero?

$$H = 0 = log_2(1)$$

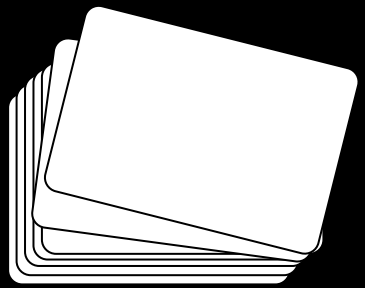
how many yes/no questions to reduce uncertainty to zero?

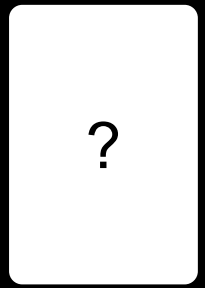
$$H = 0 = log_2(1)$$

$$H = log_2(N)$$

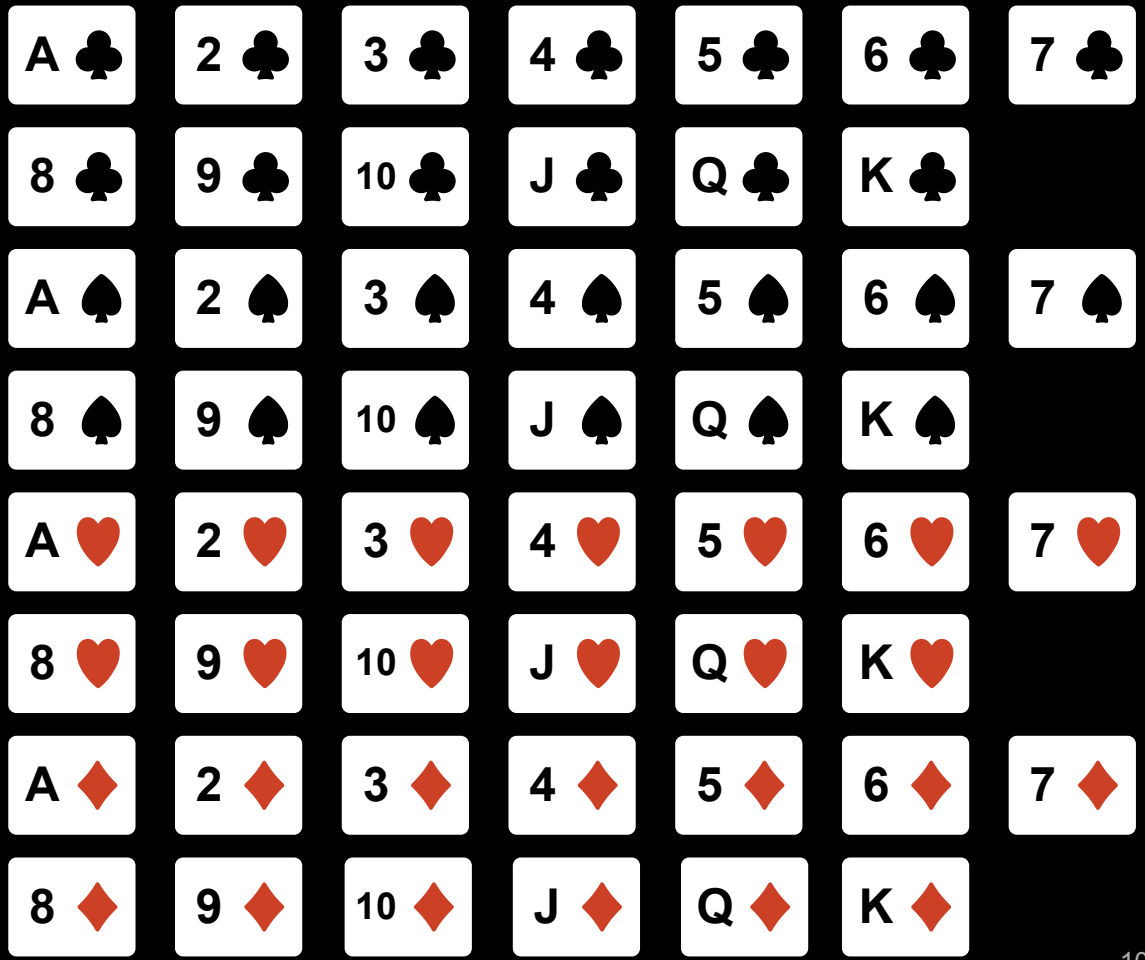
poker

which card am I holding?

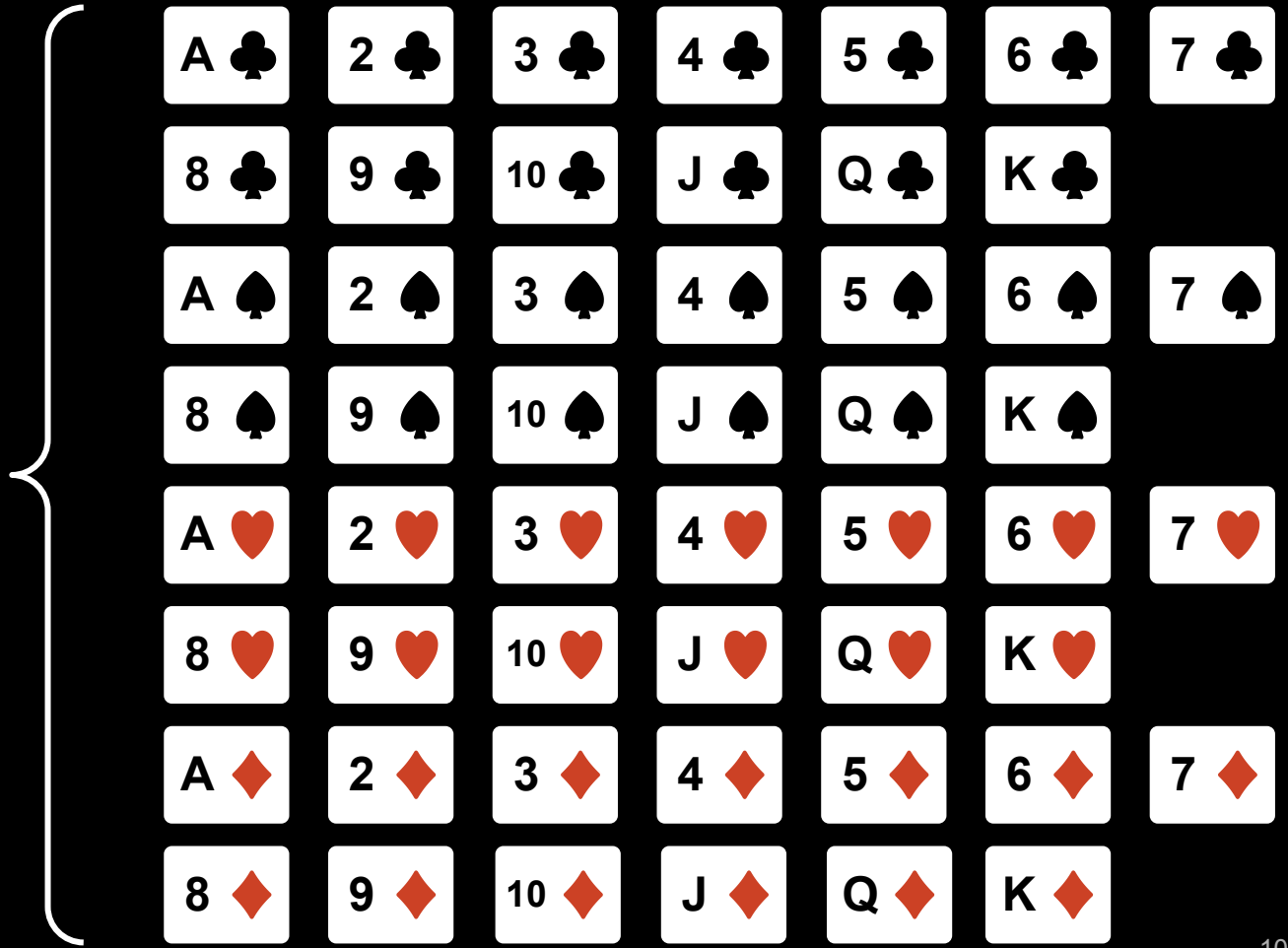




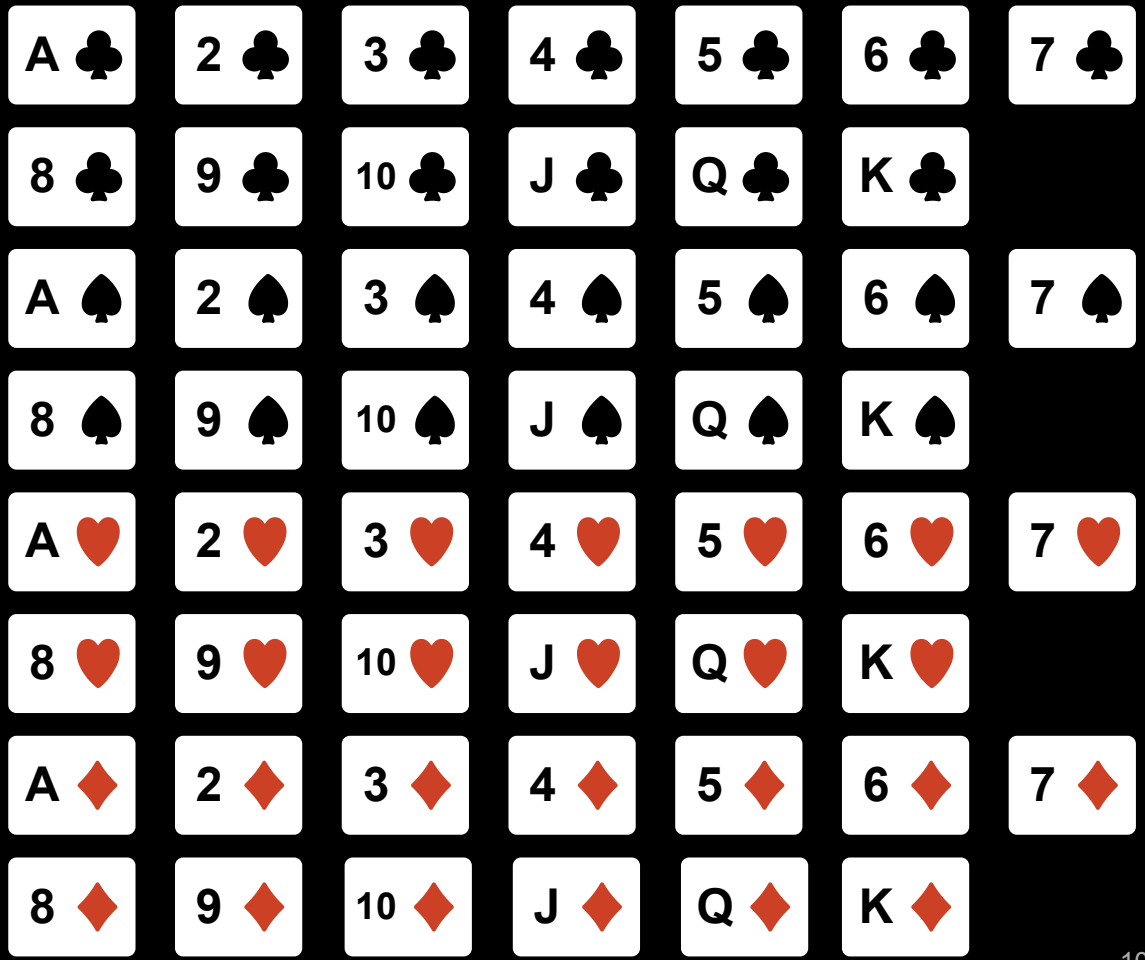
52 card poker deck


A ♣ 2 ♣ 3 ♣ 4 ♣ 5 ♣ 6 ♣ 7 ♣
8 ♣ 9 ♣ 10 ♣ J ♣ Q ♣ K ♣
A ♠ 2 ♠ 3 ♠ 4 ♠ 5 ♠ 6 ♠ 7 ♠
8 ♠ 9 ♠ 10 ♠ J ♠ Q ♠ K ♠
A ♥ 2 ♥ 3 ♥ 4 ♥ 5 ♥ 6 ♥ 7 ♥
8 ♥ 9 ♥ 10 ♥ J ♥ Q ♥ K ♥
A ♦ 2 ♦ 3 ♦ 4 ♦ 5 ♦ 6 ♦ 7 ♦
8 ♦ 9 ♦ 10 ♦ J ♦ Q ♦ K ♦

52 possible cards

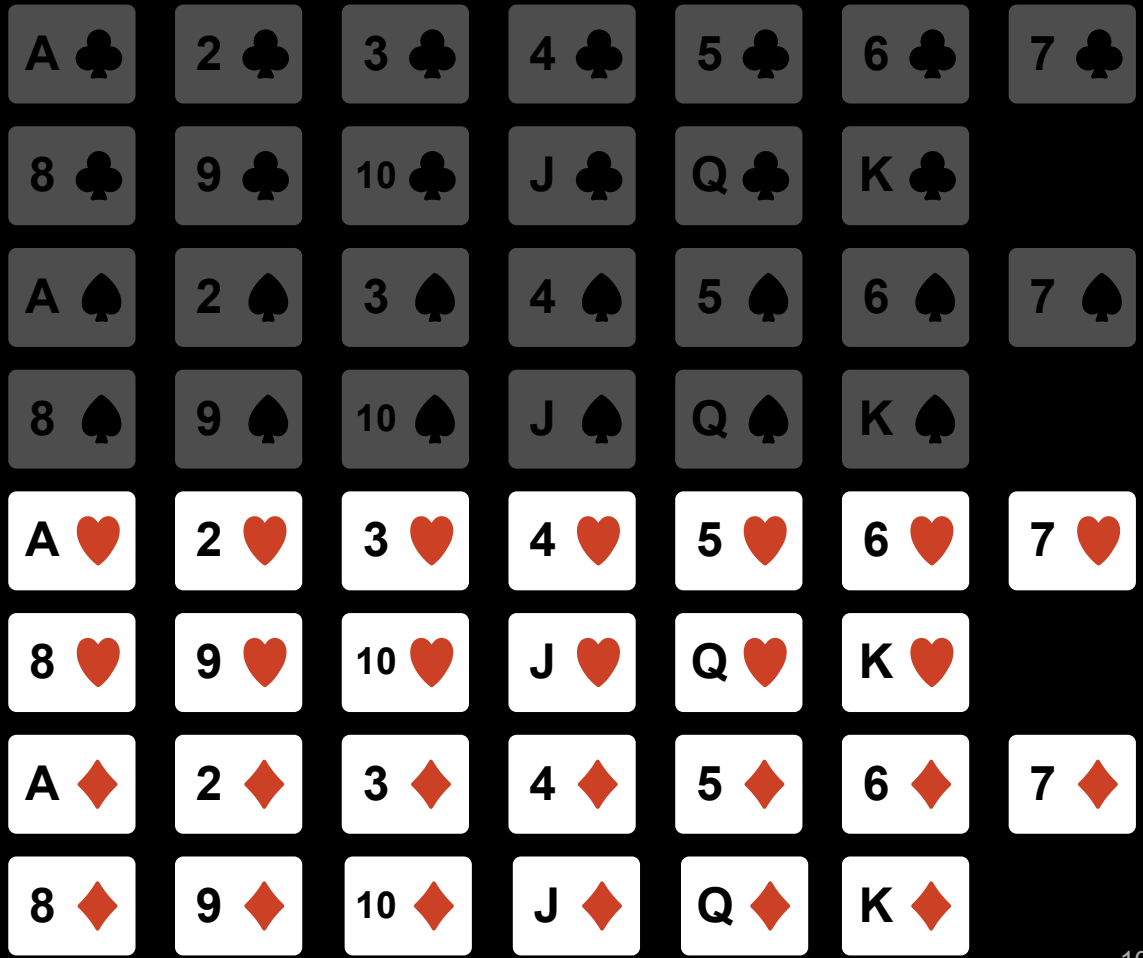


is the card black?



is the card black?

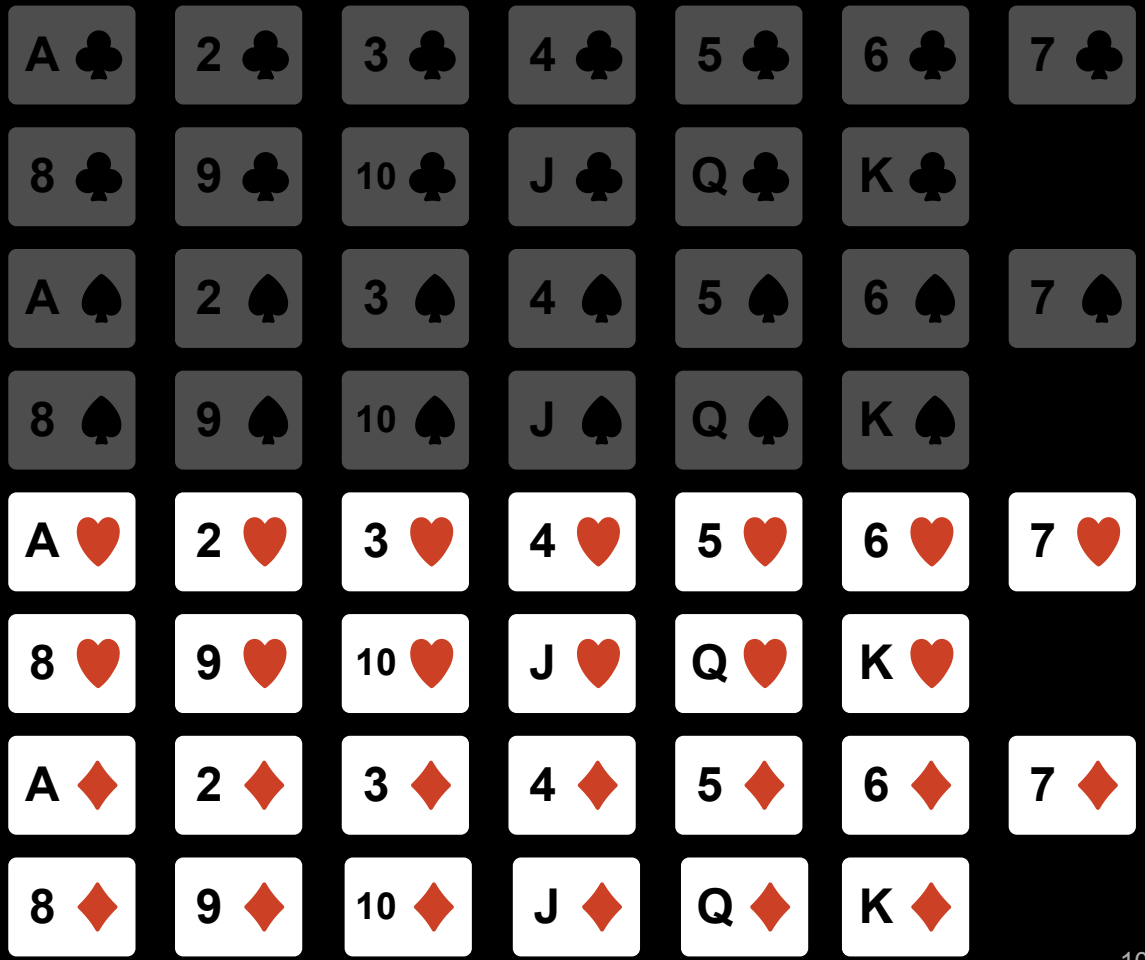
no



is the card black?

no

is it hearts?

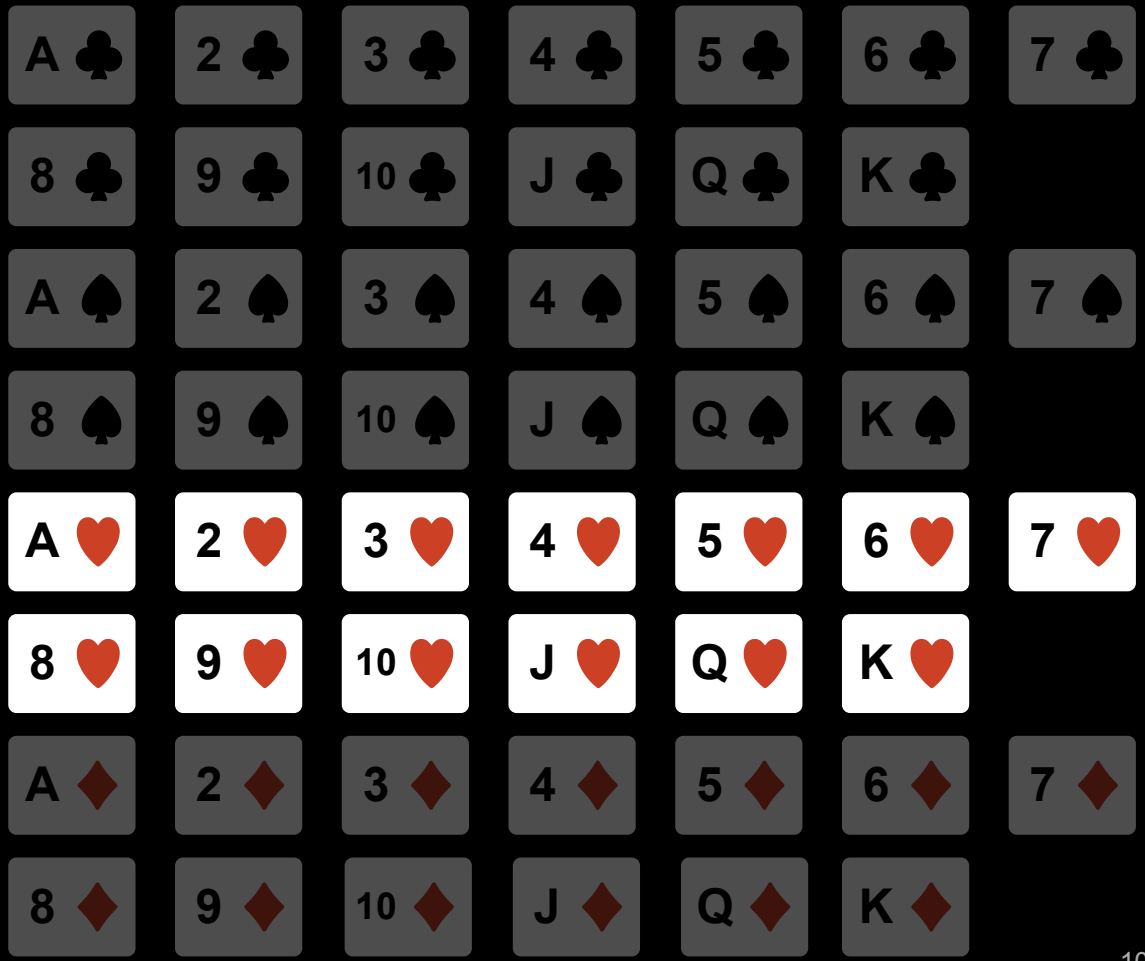


is the card black?

no

is it hearts?

yes



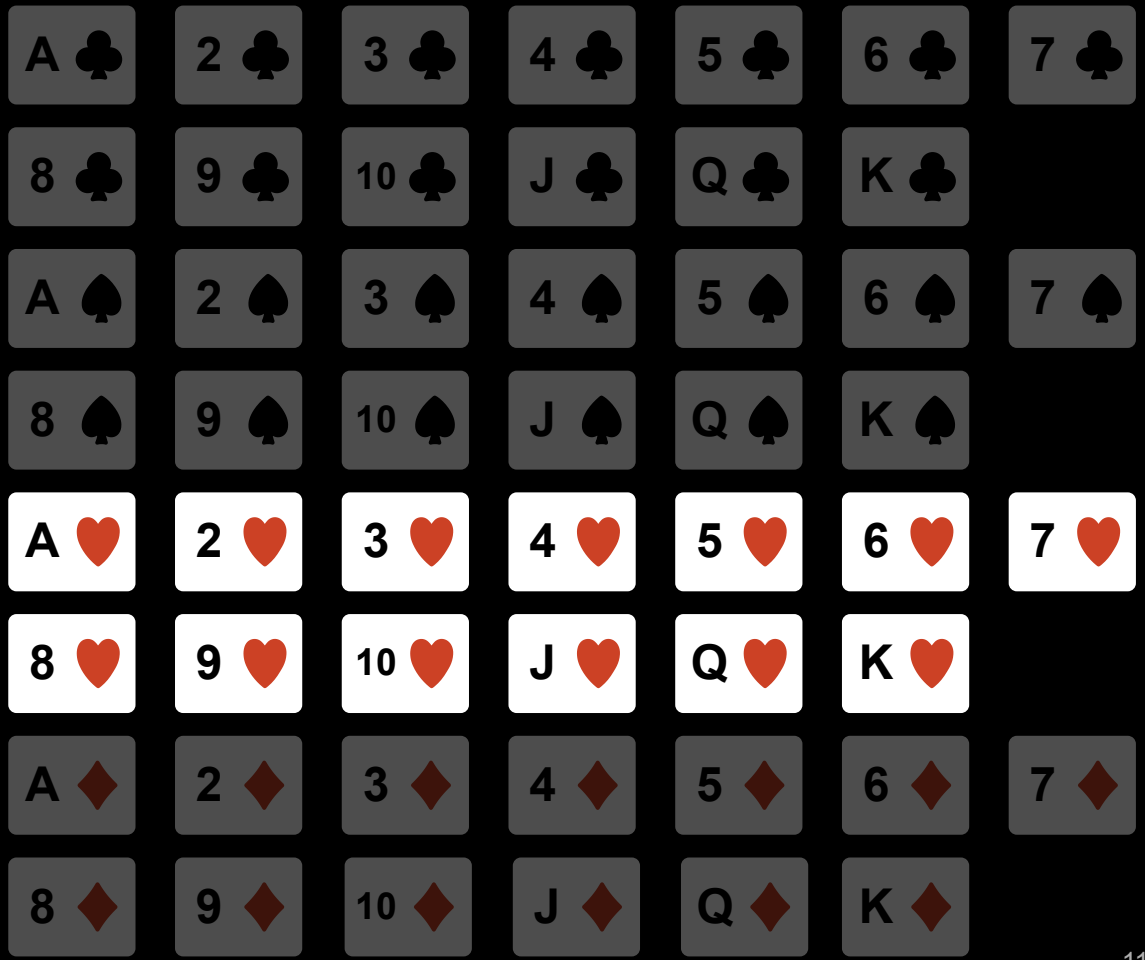
is the card black?

no

is it hearts?

yes

is it 8 or above?



is the card black?

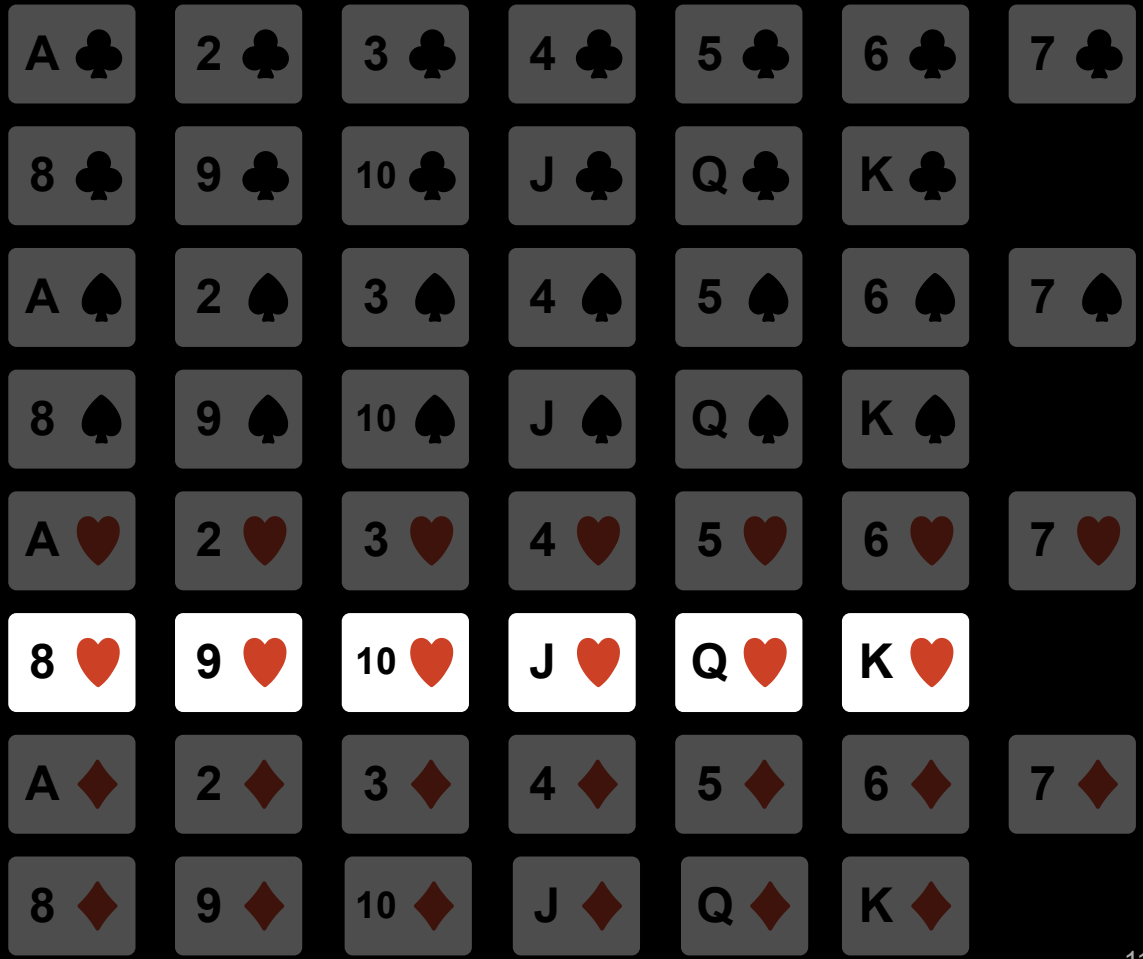
no

is it hearts?

yes

is it 8 or above?

yes



is the card black?

no

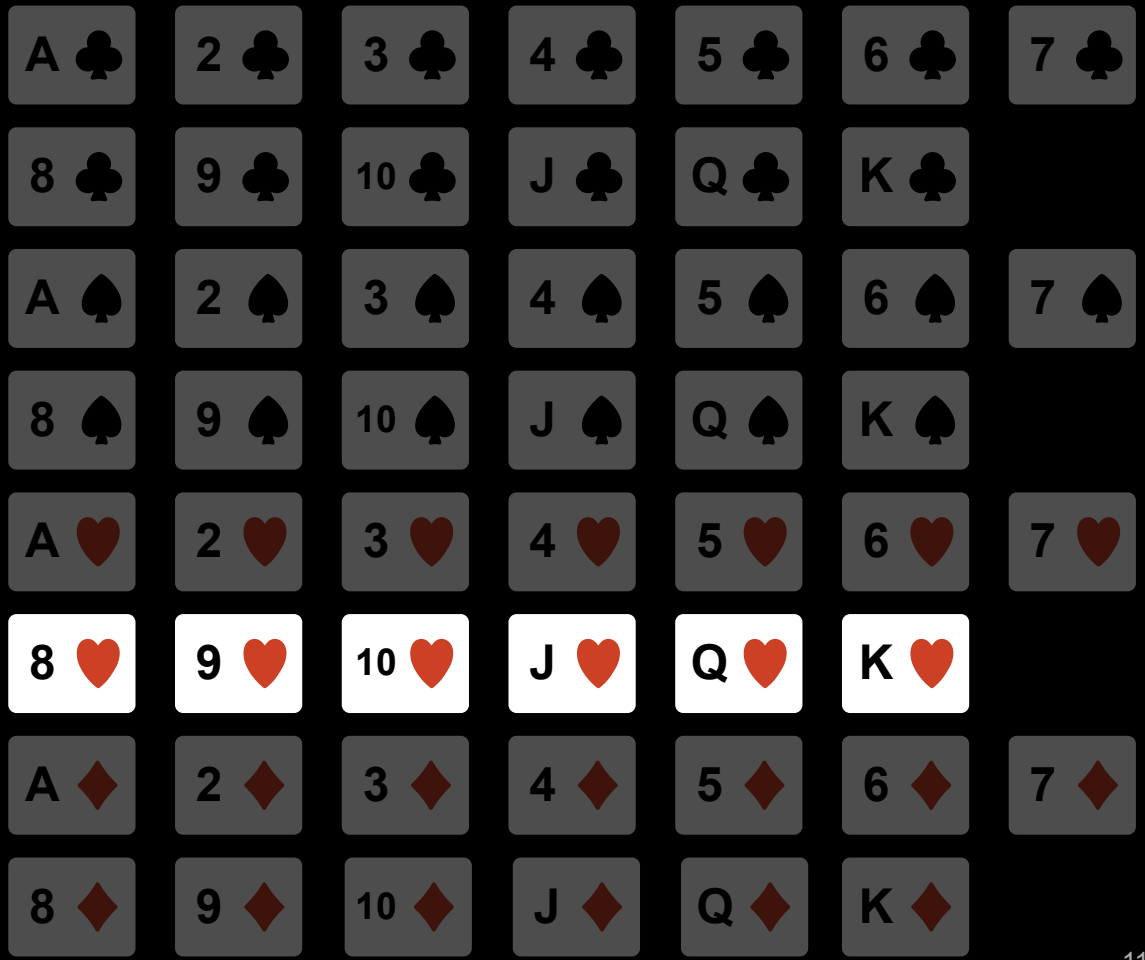
is it hearts?

yes

is it 8 or above?

yes

is it jack or above?



is the card black?

no

is it hearts?

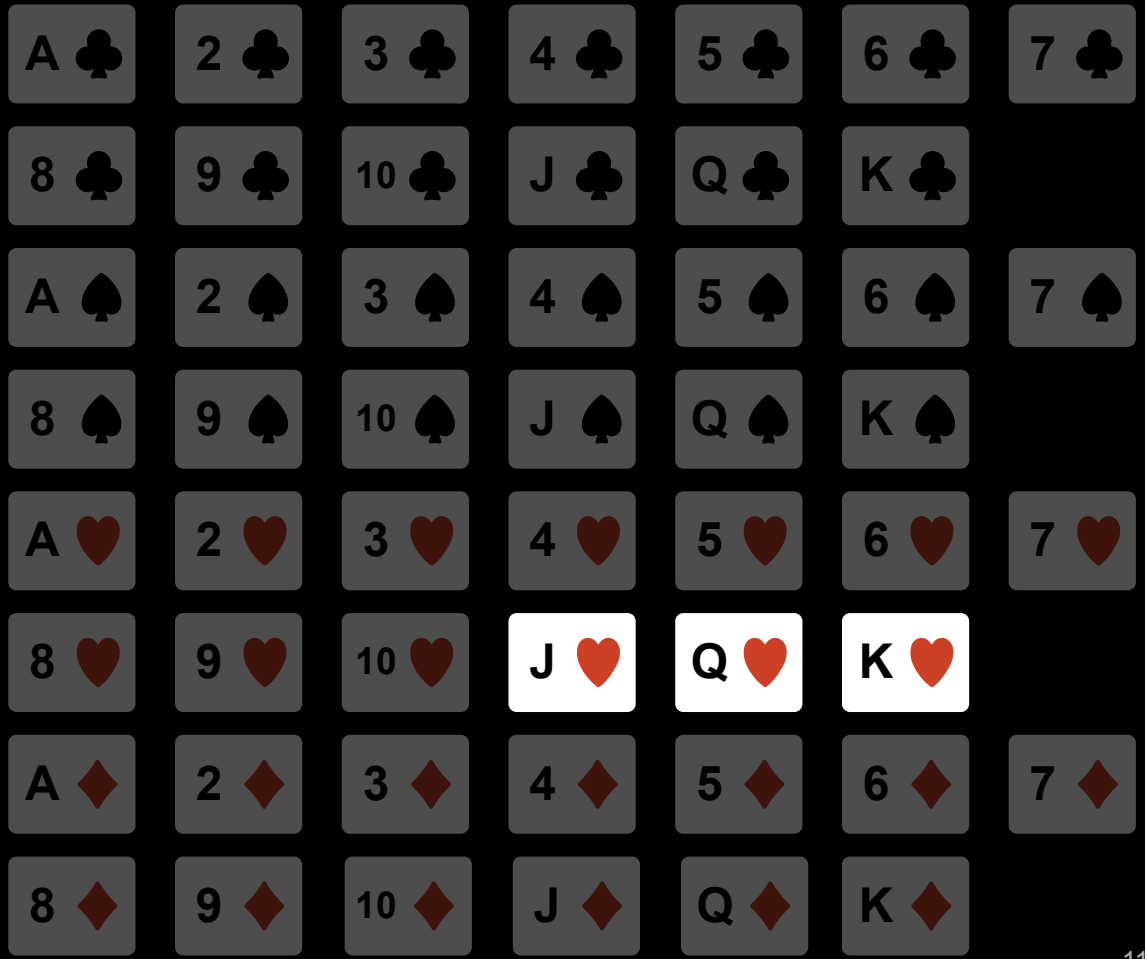
yes

is it 8 or above?

yes

is it jack or above?

yes



is the card black?

no

is it hearts?

yes

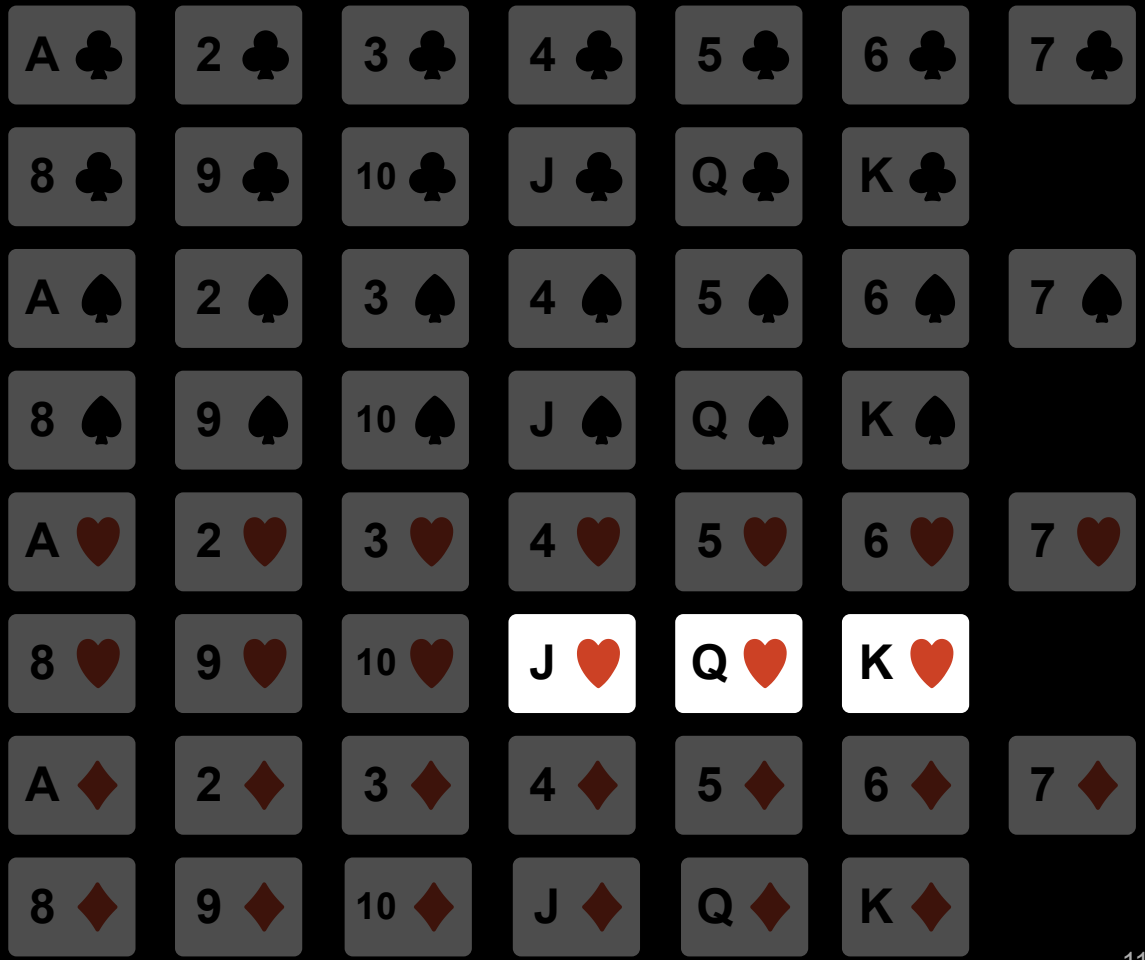
is it 8 or above?

yes

is it jack or above?

yes

is it queen or above?



is the card black?

no

is it hearts?

yes

is it 8 or above?

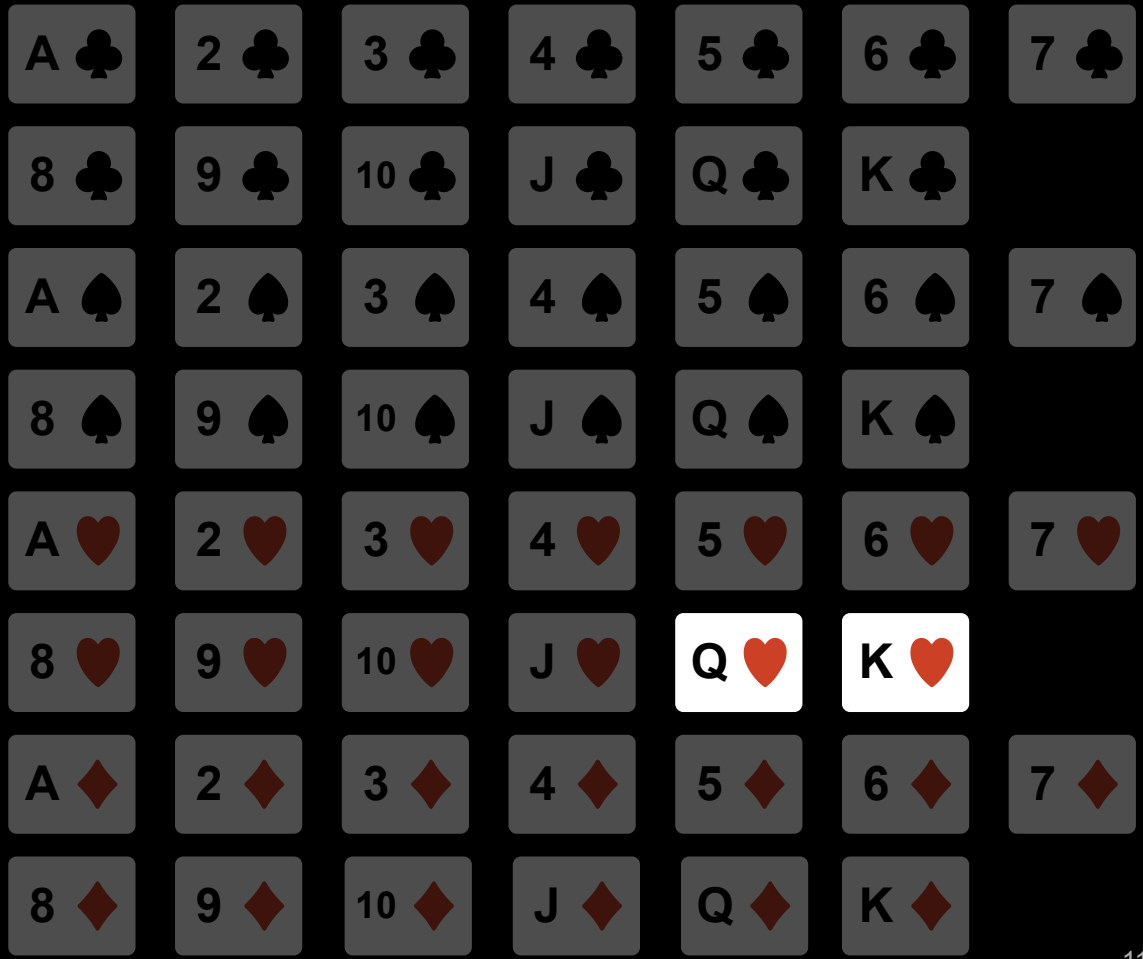
yes

is it jack or above?

yes

is it queen or above?

yes



is the card black?

no

is it hearts?

yes

is it 8 or above?

yes

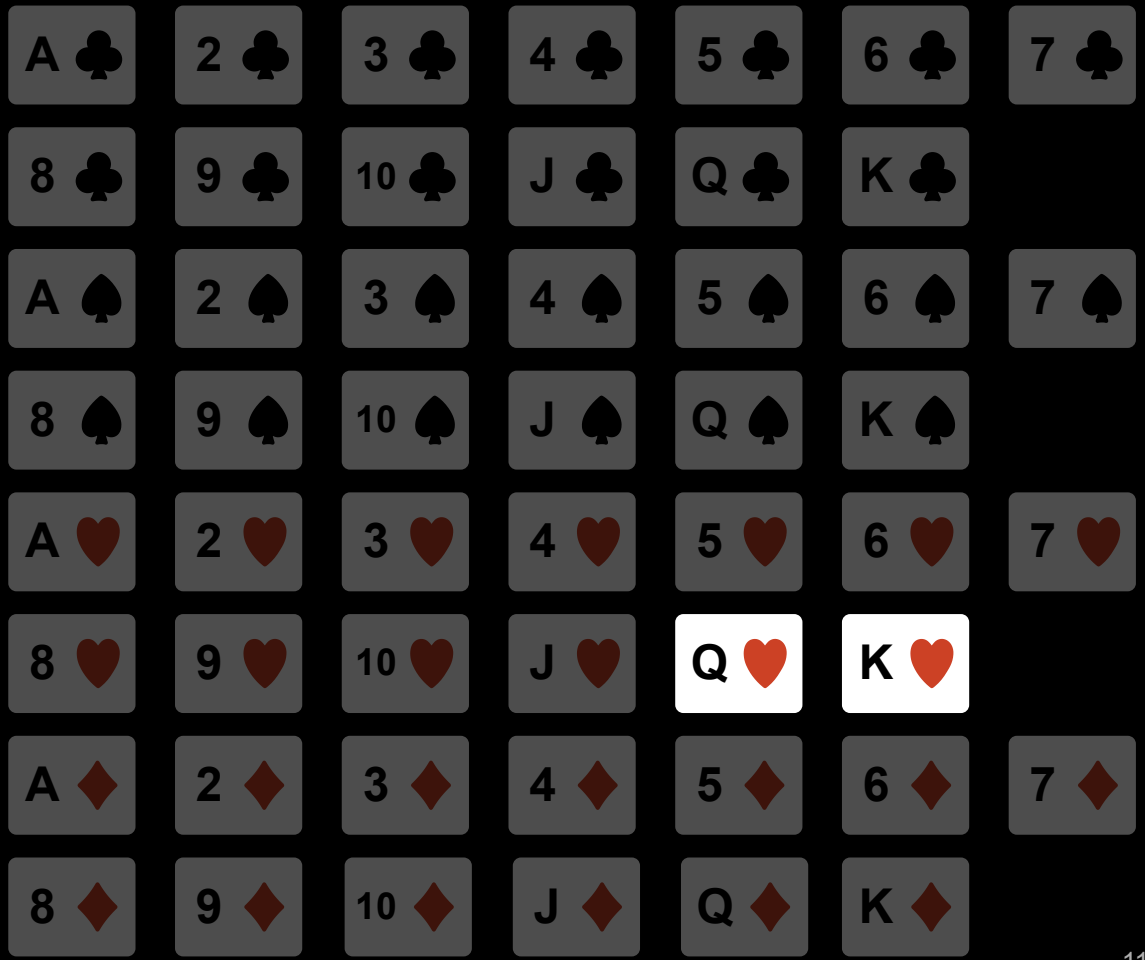
is it jack or above?

yes

is it queen or above?

yes

is it king?



is the card black?

no

is it hearts?

yes

is it 8 or above?

yes

is it jack or above?

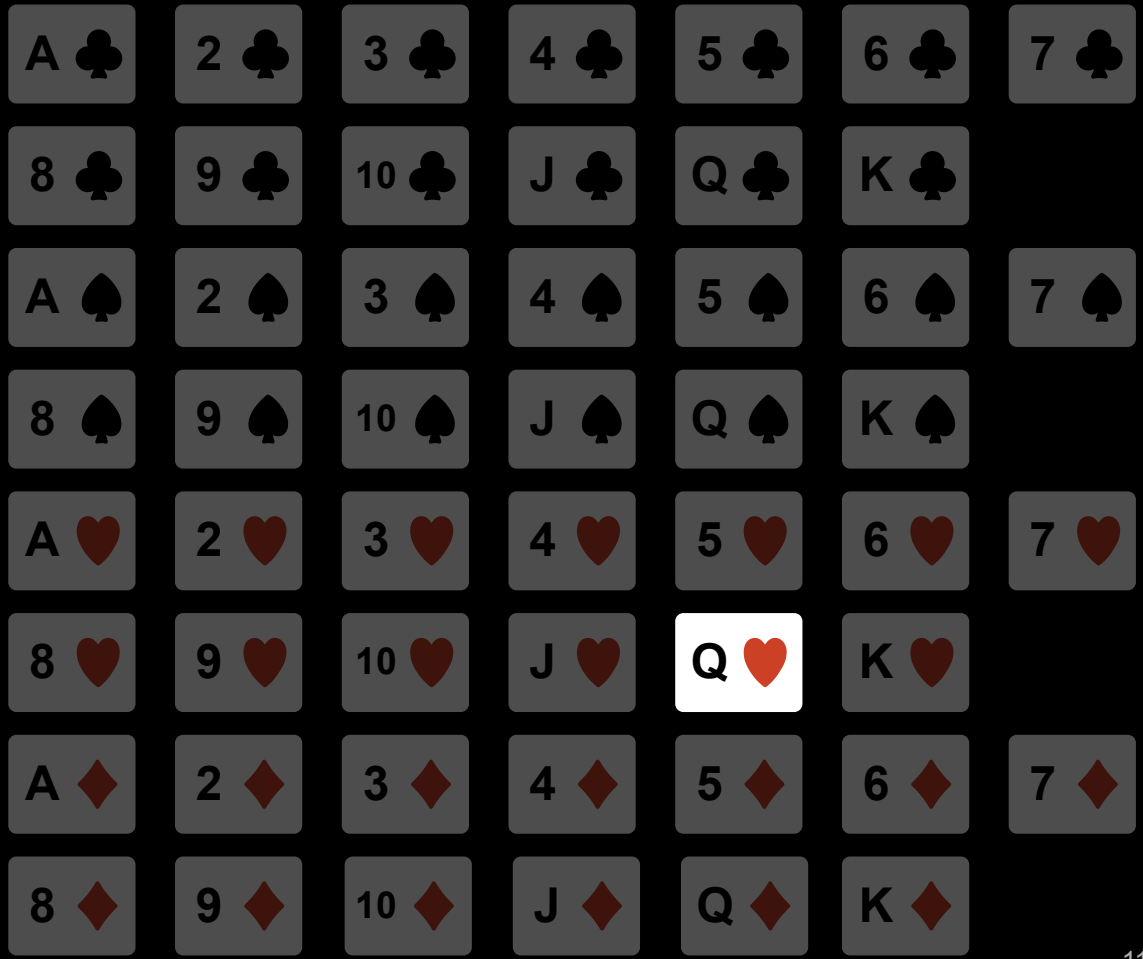
yes

is it queen or above?

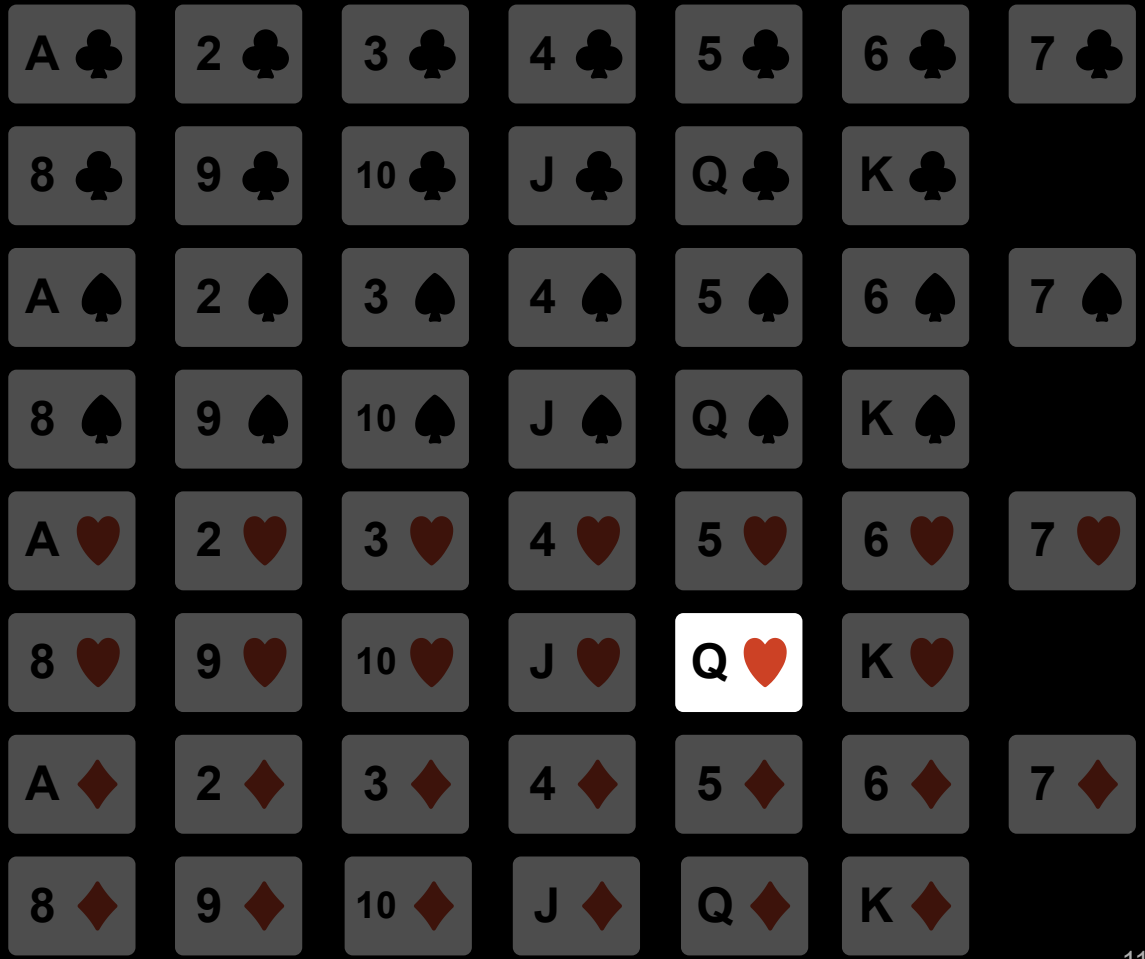
yes

is it king?

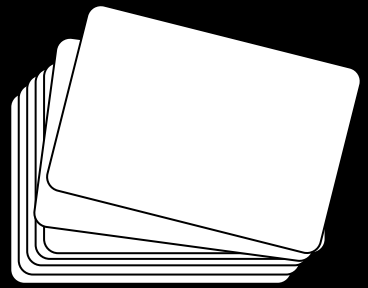
no



with 6 questions
from 52 to 1

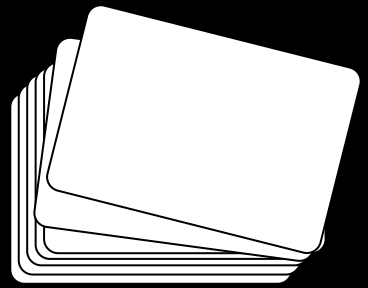


uncertainty with N = 52 possibilities?



$$H = log_2(52) \approx 5.7$$

uncertainty with N = 52 possibilities?



average # of yes/no questions

$$H = log_2(52) \approx 5.7$$

one bit of information with each answer…

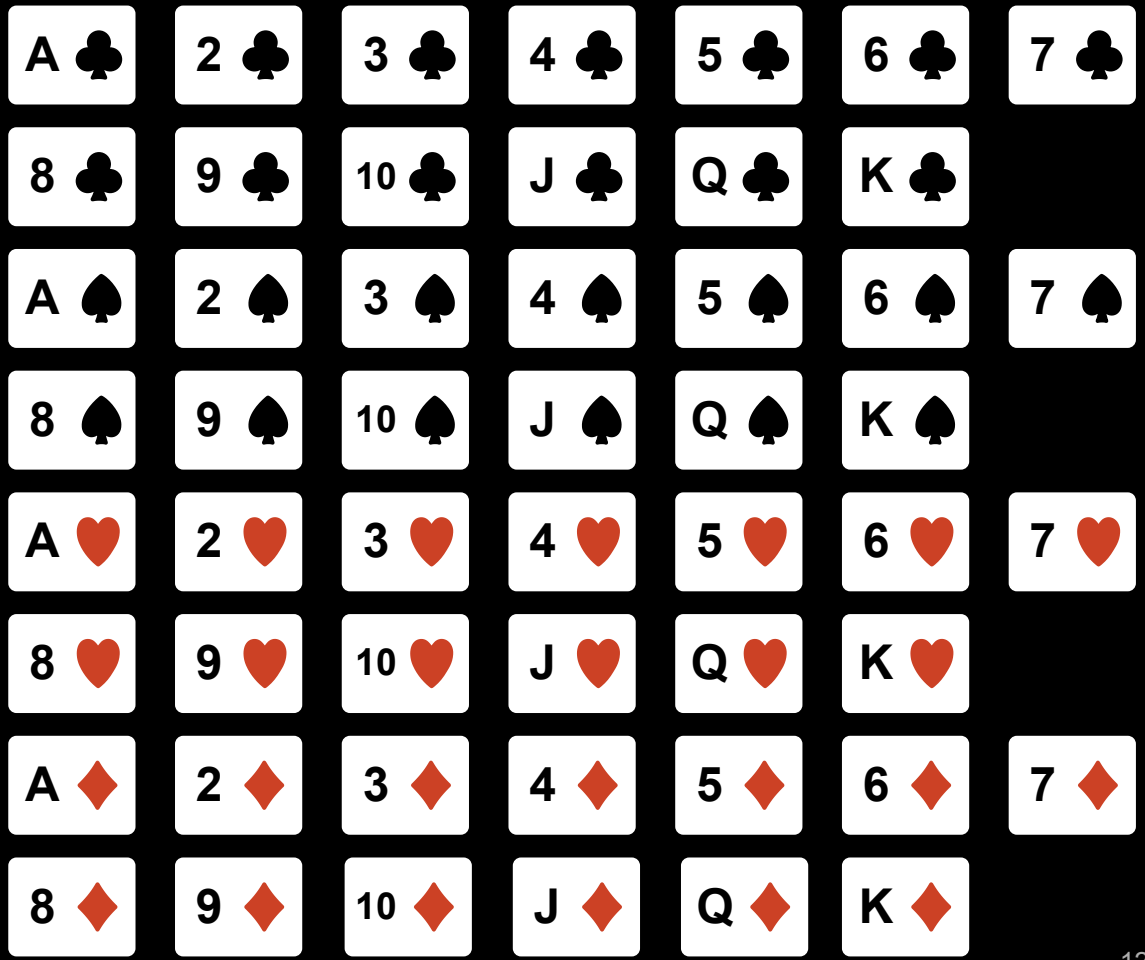
$$log_2(52) - log_2(26) = 1$$

one bit of information with each answer…

$$log_2(52) - log_2(26) = 1$$

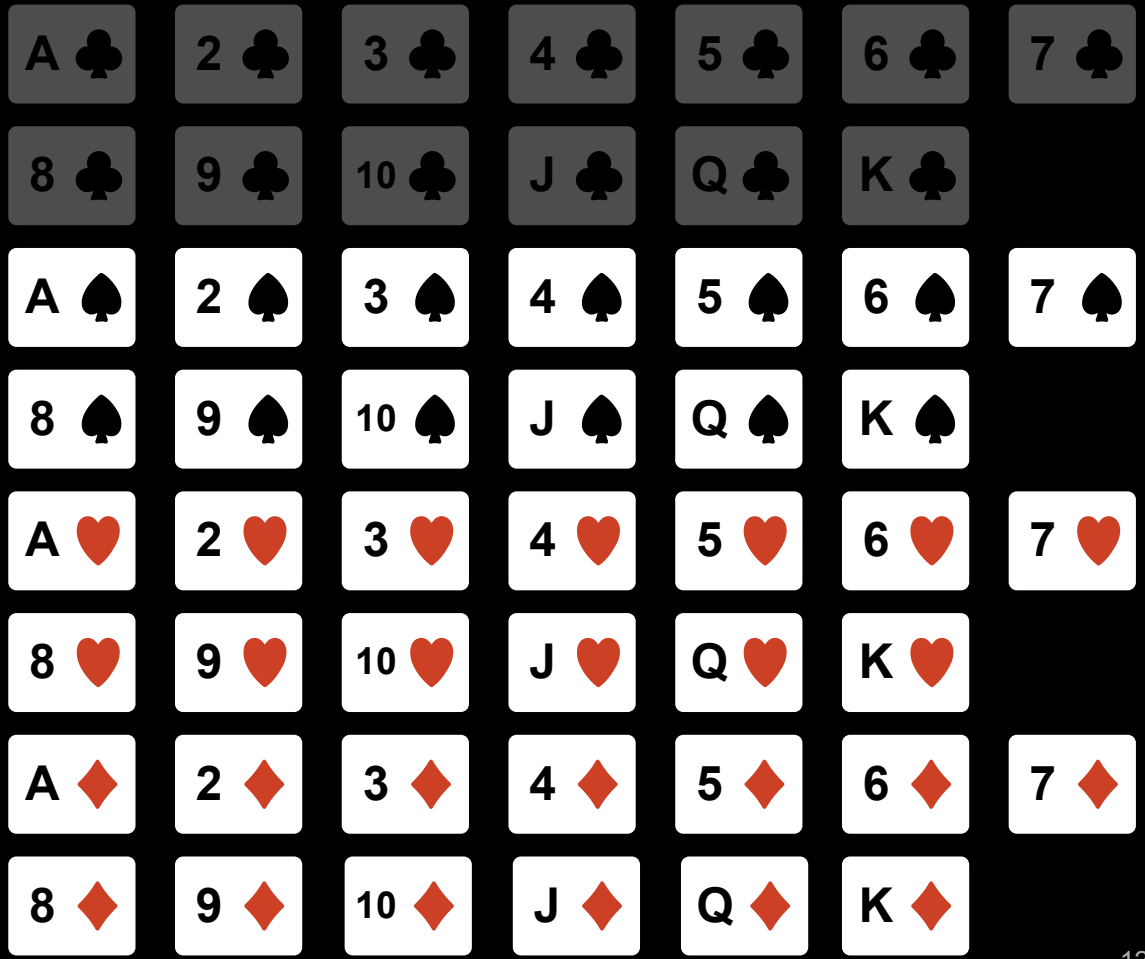
…that cuts the remaining options in half

is it a spades card?



is it a spades card?

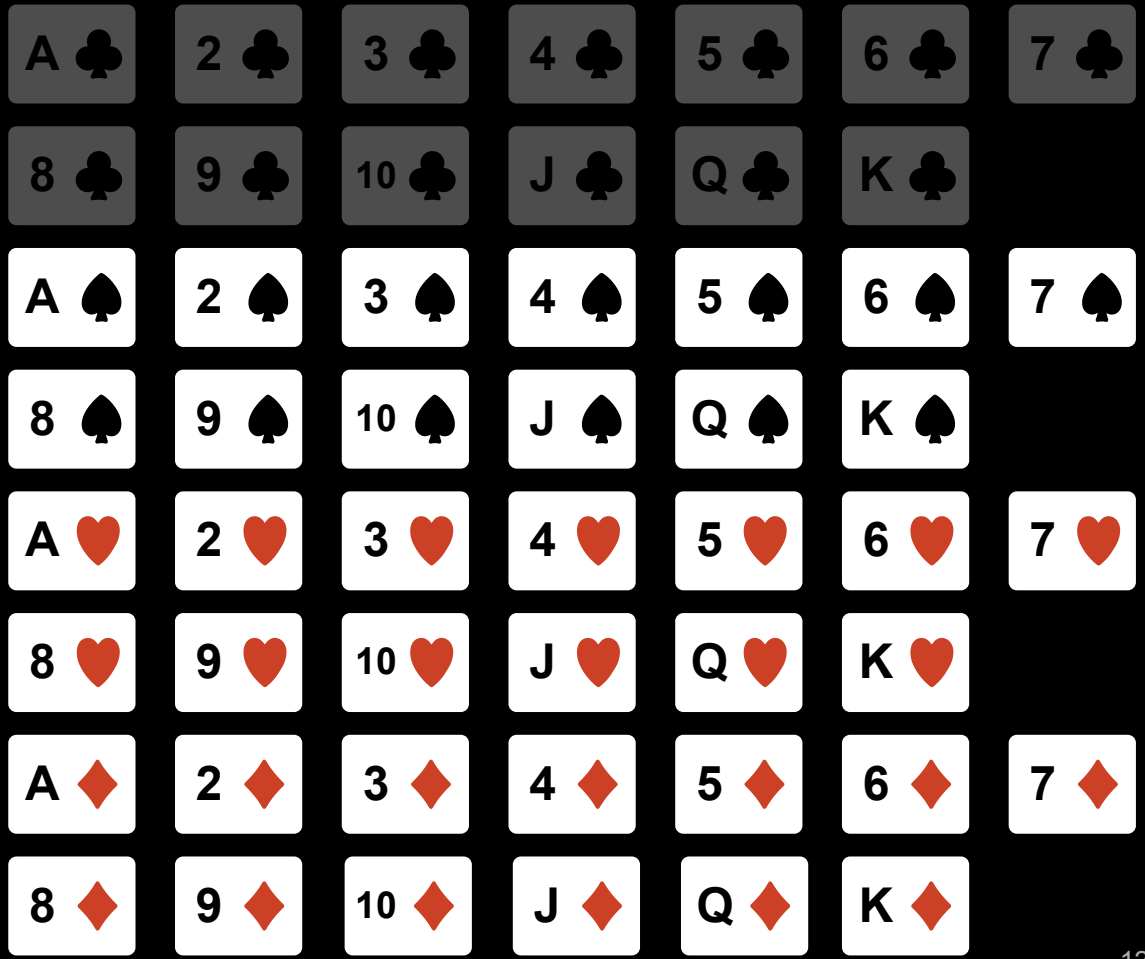
no



is it a spades card?

no

how much information?



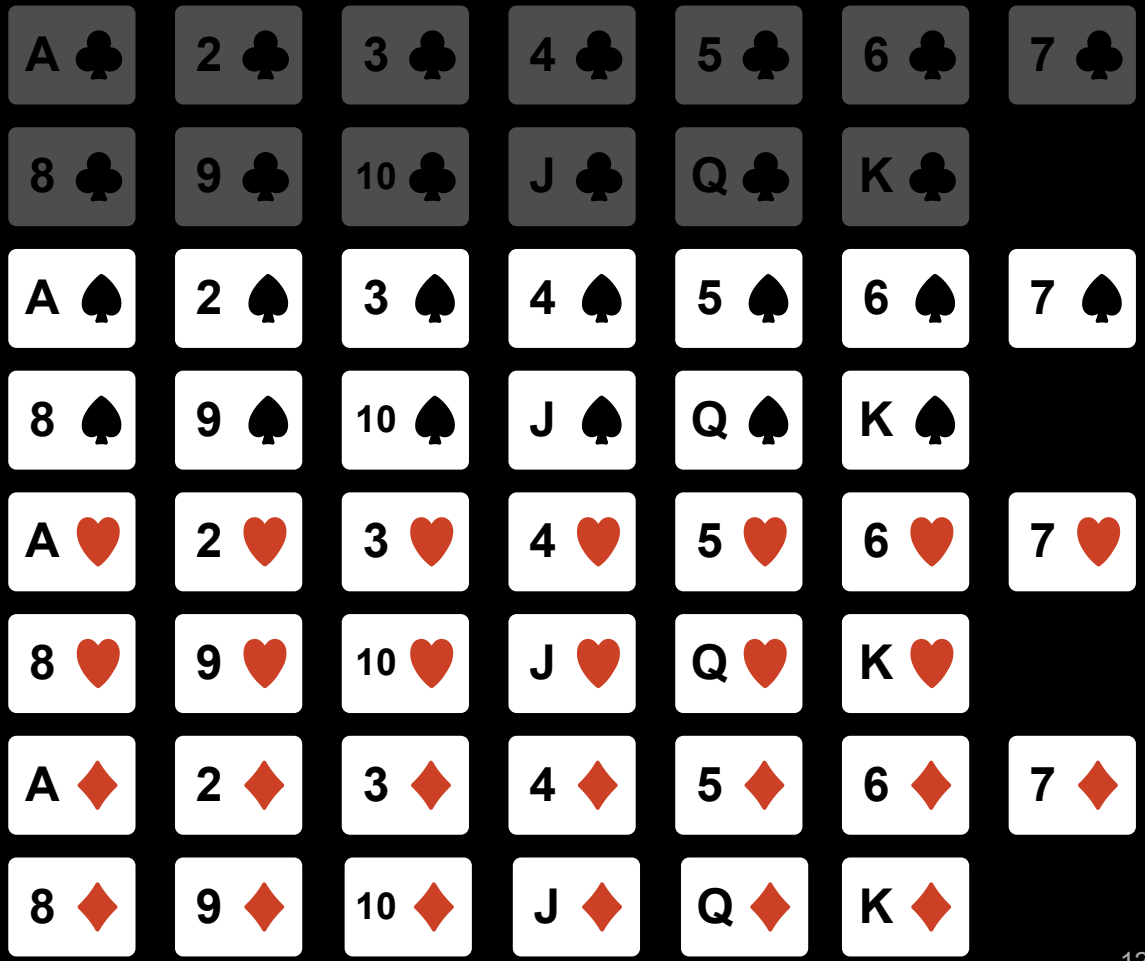
is it a spades card?

no

how much information?

$$H_0 = log_2(52) \approx 5.7$$

$$H_1 = log_2(39) \approx 5.29$$



is it a spades card?

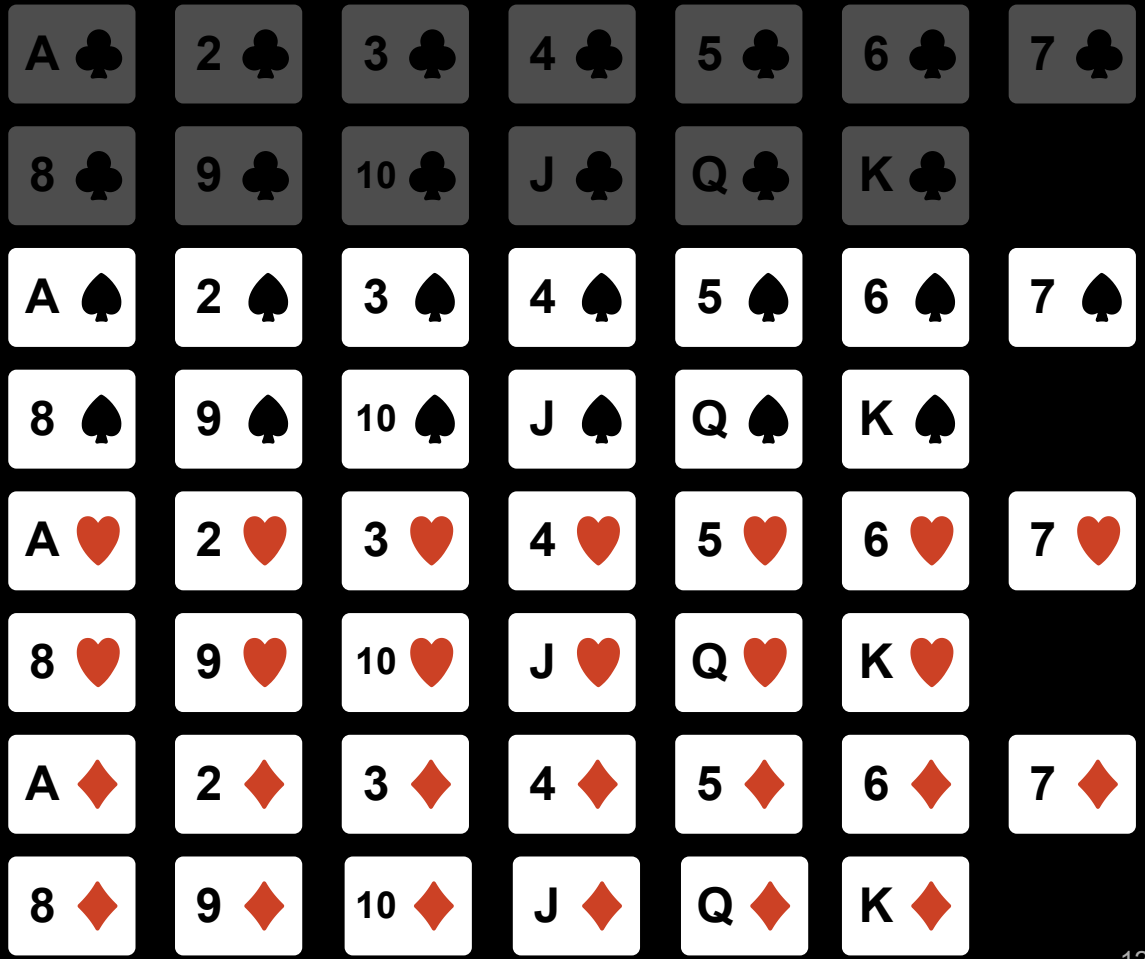
no

how much information?

$$H_0 = log_2(52) \approx 5.7$$

$$H_1 = log_2(39) \approx 5.29$$

$$H_0 - H_1 \approx 0.41$$



is it a spades card?

no

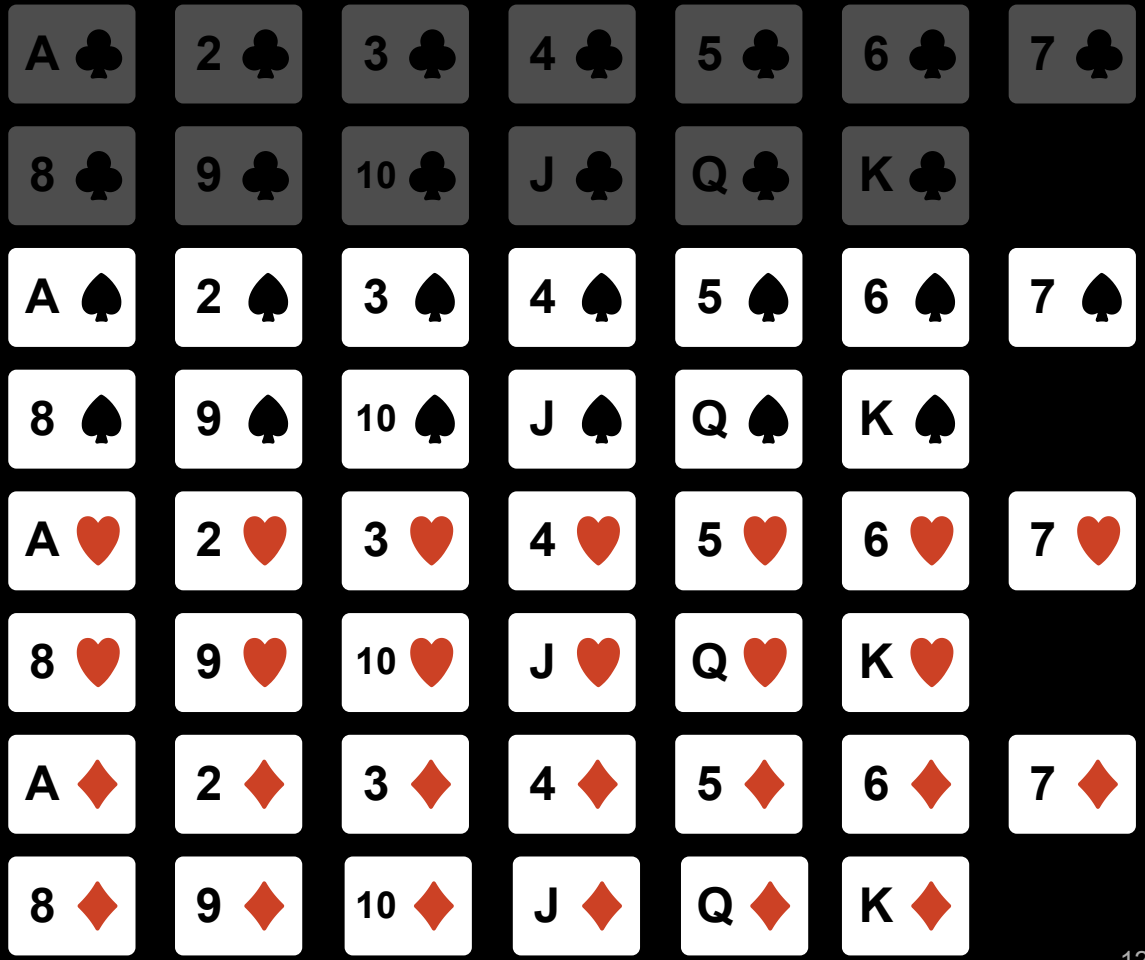
how much information?

$$H_0 = log_2(52) \approx 5.7$$

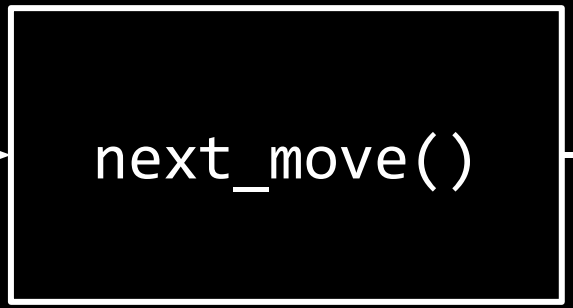
$$H_1 = log_2(39) \approx 5.29$$

$$H_0 - H_1 \approx 0.41$$

that's less than 1 bit

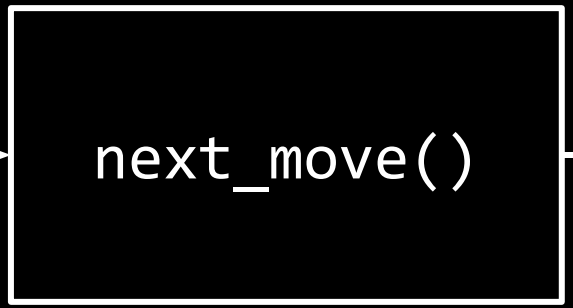


chess

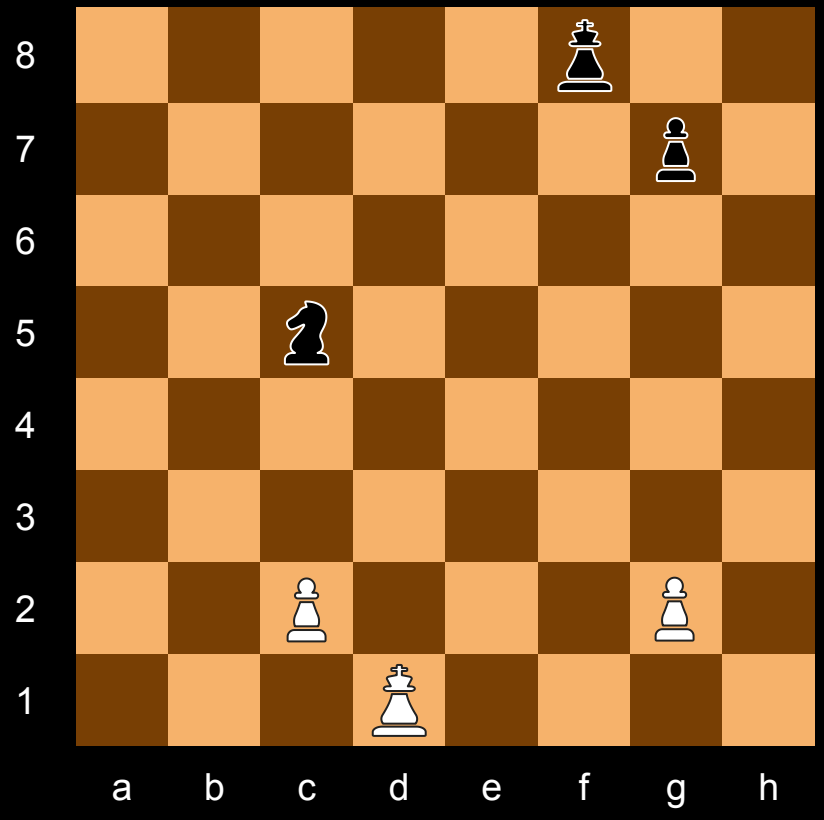


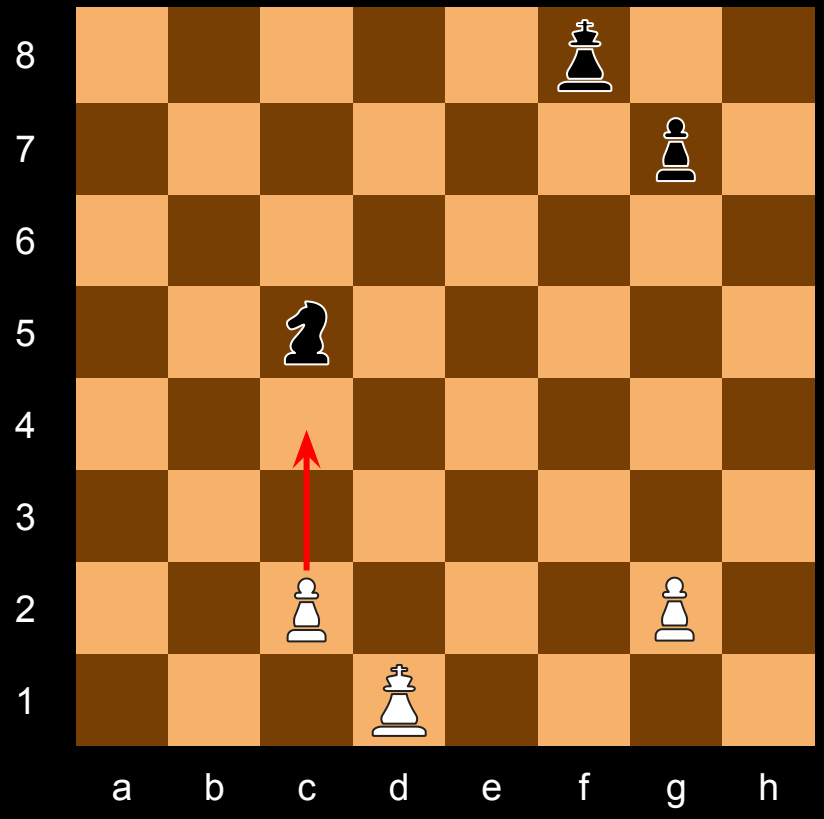
E2 → E4

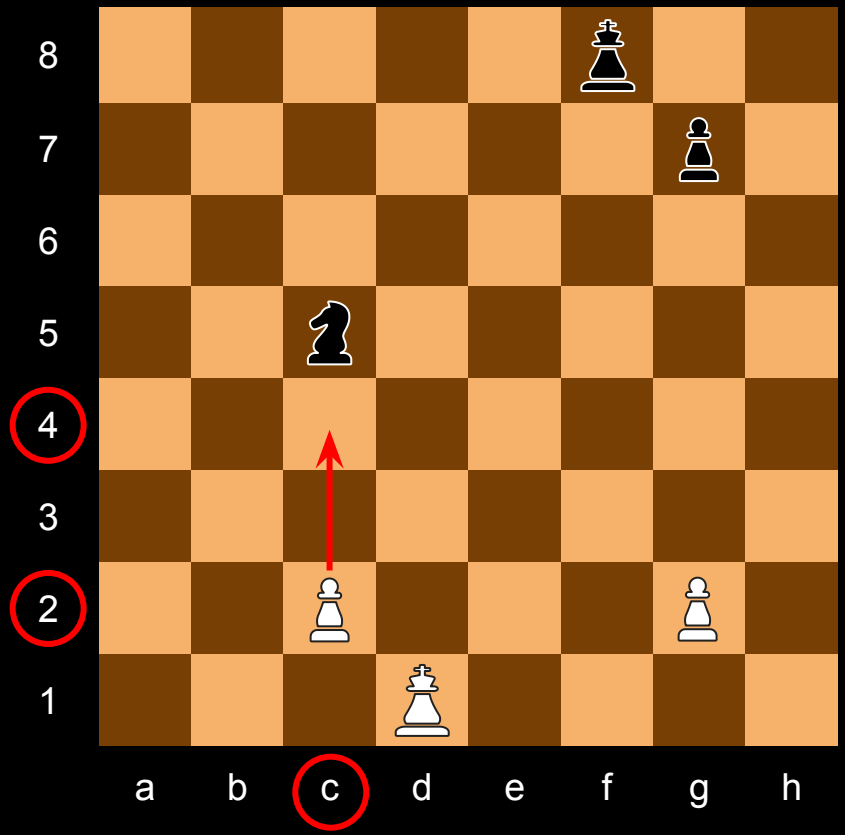
how much information is one move?



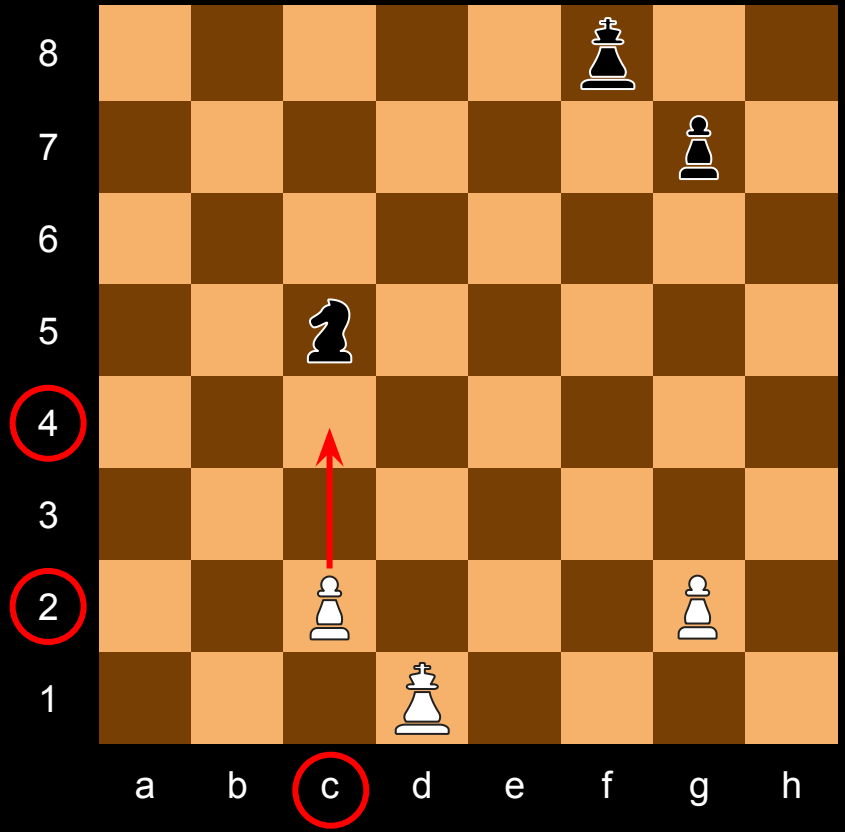
E2 → E4


8
7
6
5
4
3
2
1
a b c d e f g h

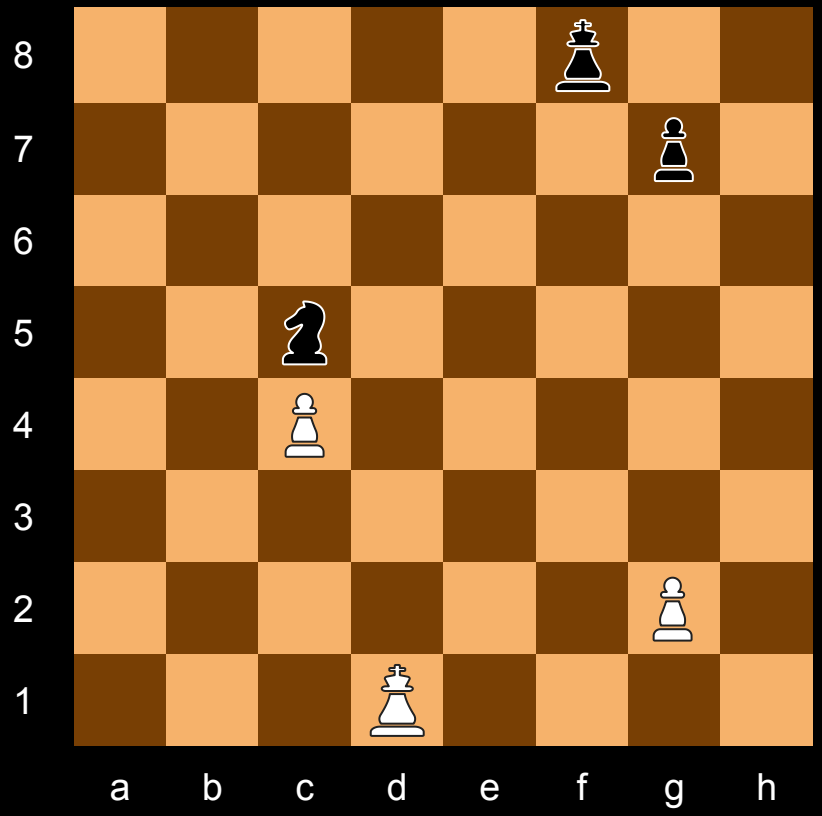

8
7
6
5
4
3
2
1
a b c d e f g h


8
7
6
5
4
3
2
1
a b c d e f g h

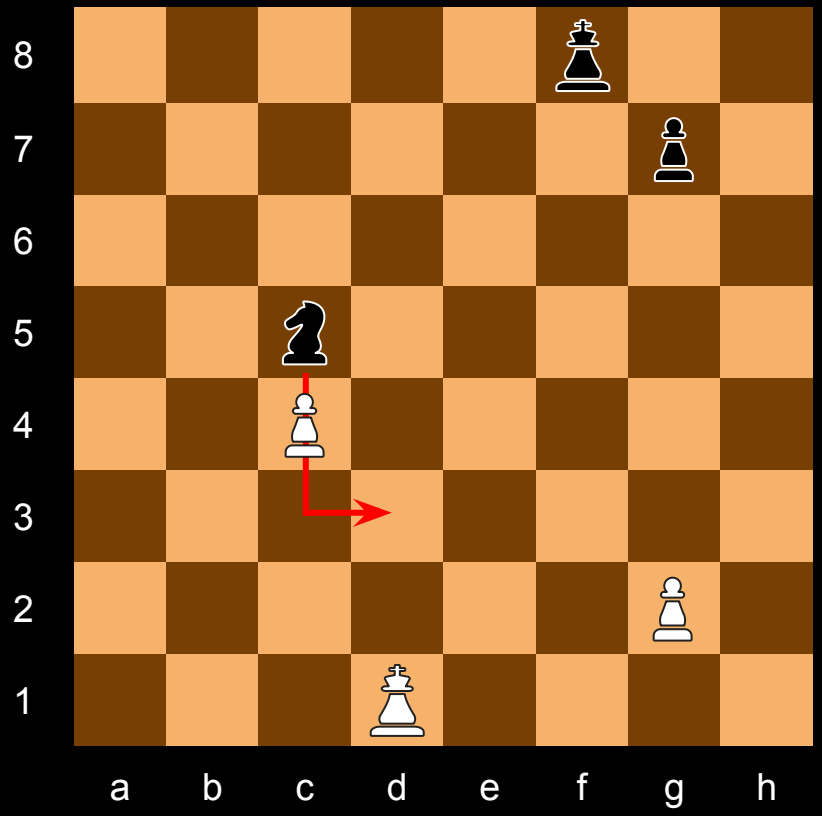
c2 → c4



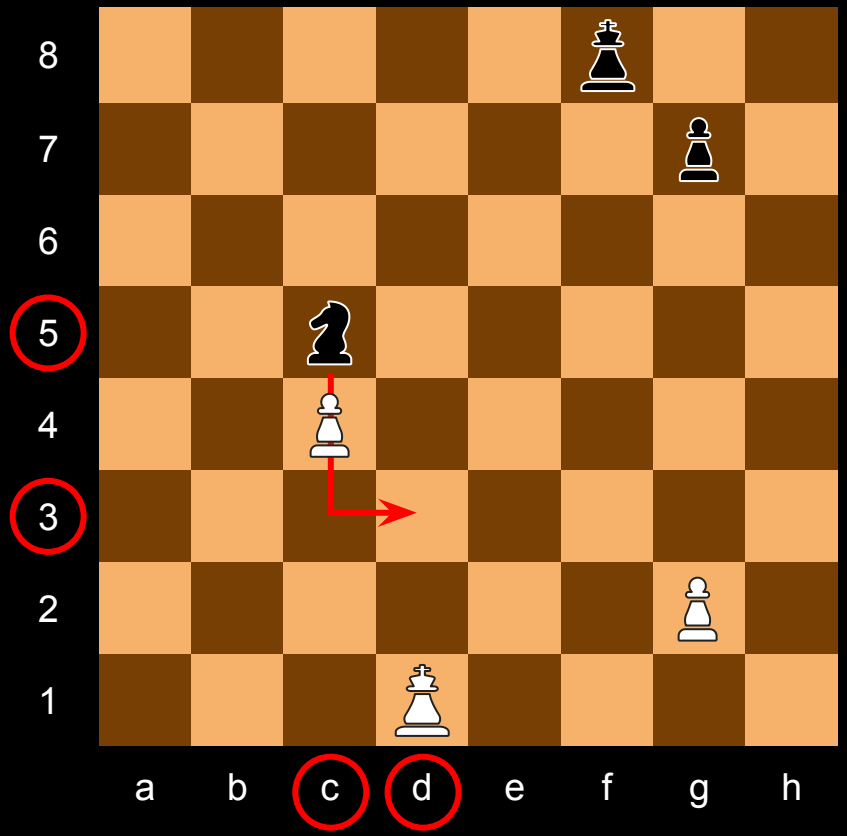
c2 → c4



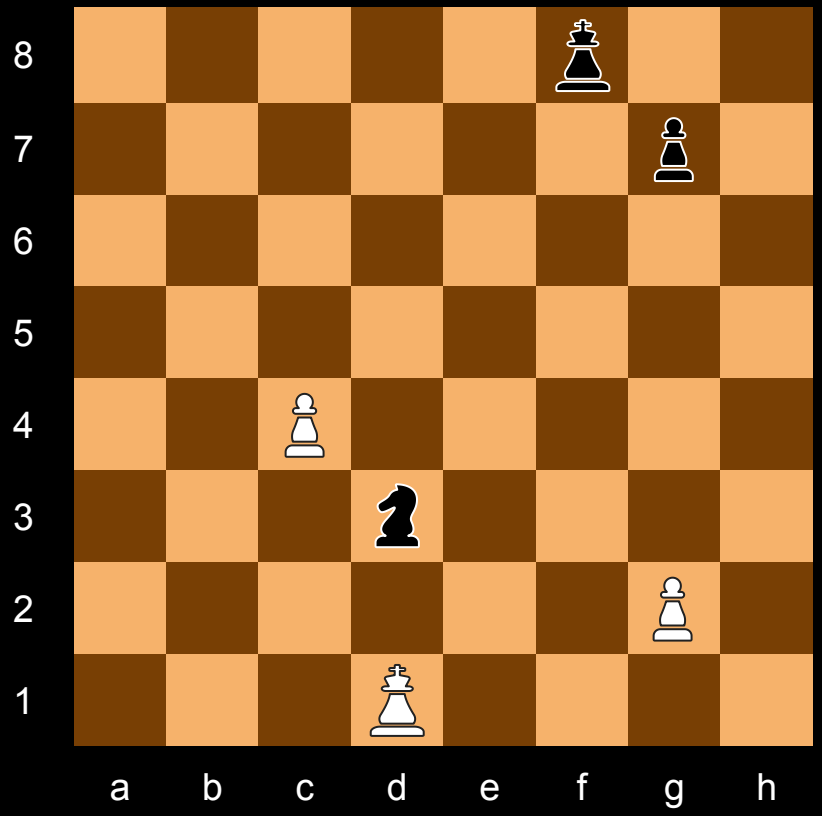
c2 → c4



c2 → c4
c5 → d3



c2 → c4
c5 → d3
…



c2 → c4

how many possibilities?

64 fields x 64 fields

how many possibilities?

64 x 64 = 4096

possible moves*

*disregarding impossible moves

64 x 64 = 4096

$$H = log_2(4096) = 12$$

64 x 64 = 4096

$$H = log_2(4096) = 12$$

one chess move is 12 bits of information

an alternative way to calculate # bits

c 2 c 4

4 digits

c 2 c 4

4 digits

8 possible symbols per digit

c 2 c 4

4 digits

8 possible symbols per digit

how many bits per digit?

c 2 c 4

4 digits

8 possible symbols per digit

how many bits per digit?

$$H_{digit} = log_2(8) = 3$$

c 2 c 4

4 digits

8 possible symbols per digit

how many bits per digit?

$$H_{digit} = log_2(8) = 3$$

$$H_{move} = log_2(8) \times 4 = 12$$

$$H_{avg} = log_2(S) \times n$$

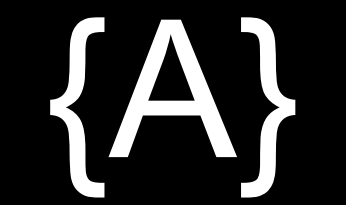
S: number of possible symbols

n: number of digits in our message

$$H_{max} = \lceil log_2(S) \rceil \times n$$

when calculating bits for storage, we must always consider the worst case

digits and # symbols


{A}

{A}

A A

{A, B}

_ _

{A, B}

AA, AB, BA, BB

{A, B, C}

_ _

{A, B, C}

AA, AB, BA, BB,
AC, BC, CA, CB, CC

{A, B, C, D}

_ _

{A, B, C, D}

AA, AB, BA, BB, AC, BC, CA, CB,
CC, AD, DA, BD, DB, CD, DC, DD

{A, B, C, D, E}

_ _

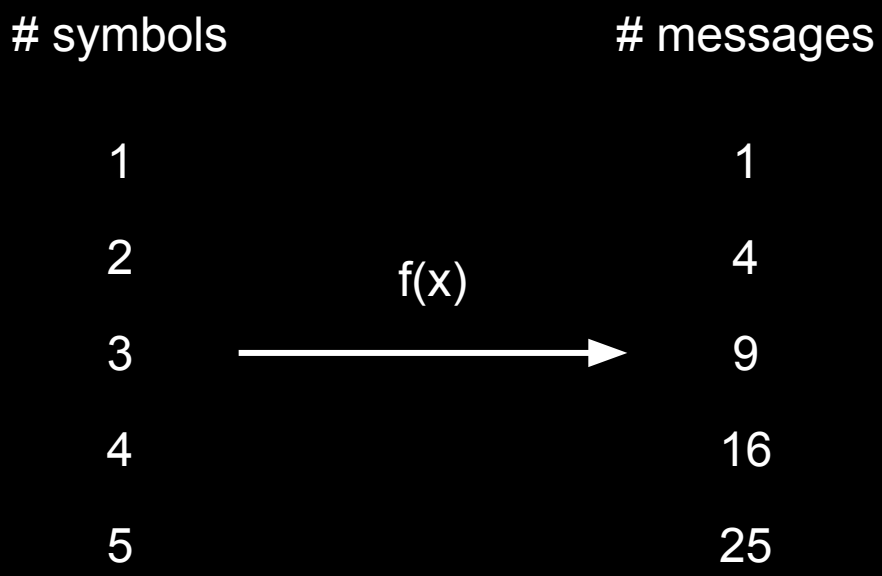
{A, B, C, D, E}

AA, AB, BA, BB, AC, BC, CA, CB, CC,
AD, DA, BD, DB, CD, DC, DD, AE, EA,
BE, EB, CE, EC, DE, ED, EE

with # digits n = 2

# symbols	# messages
1	1
2	4
3	9
4	16
5	25

with length n = 2



and more digits?

{A, B}

_ _ _

{A, B}

AAA, AAB, ABA, ABB,
BBB, BBA, BAA, BAB

{A, B}

_ _ _ _

{A, B}

AAAA, AAAB, AABA, AABB,
ABAA, ABAB, ABBA, ABBB,
BAAA, BAAB, BABA, BABB,
BBAA, BBAB, BBBA, BBBB

with # symbols S = 2

# digits	# messages
1	2
2	4
3	8
4	16
5	32

with # symbols S = 2

# digits		# messages
1		2
2	f(x)	4
3	→	8
4		16
5		32

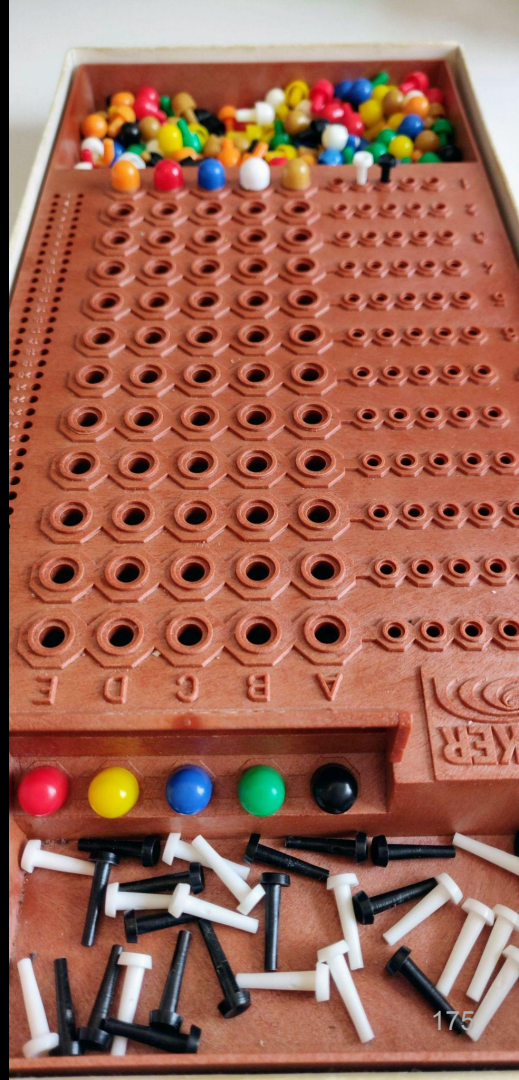
with # symbols S = 2

# digits		# messages
1		2
2	f(x)	4
3	→	8
4		16
5		32

possible messages with *n* digits and *S* symbols

$$N = S^n$$

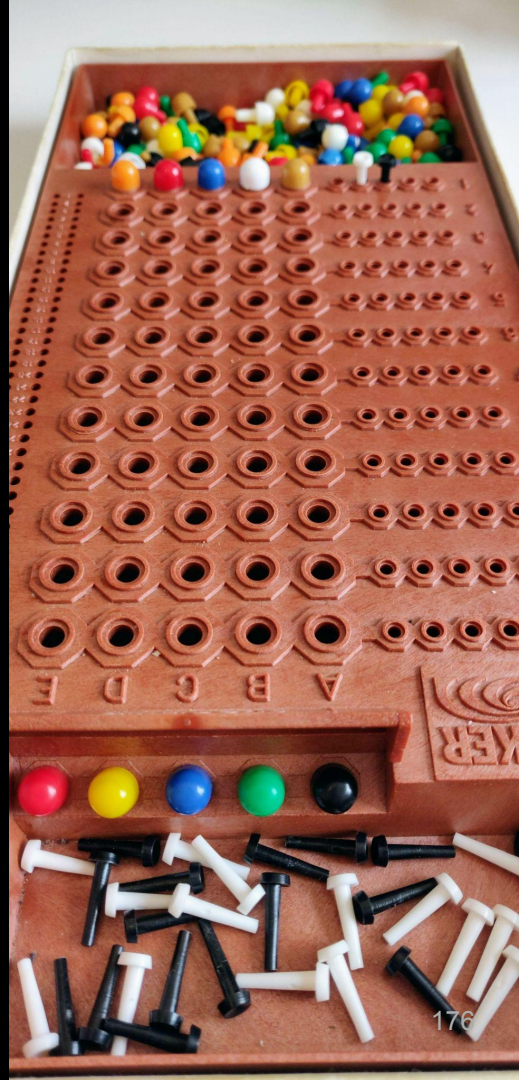
mastermind



eight colors

five slots

A B C D E



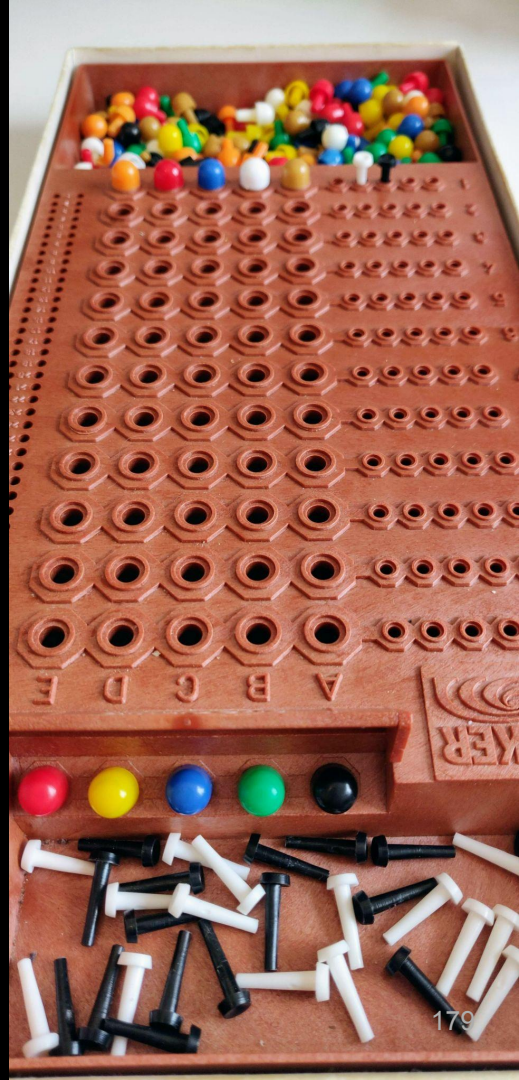
possible permutations

$$N = 5^8 = 390625$$

maximum entropy in bits

$$H_{max} = log_2\ 5^8 = 18.575$$

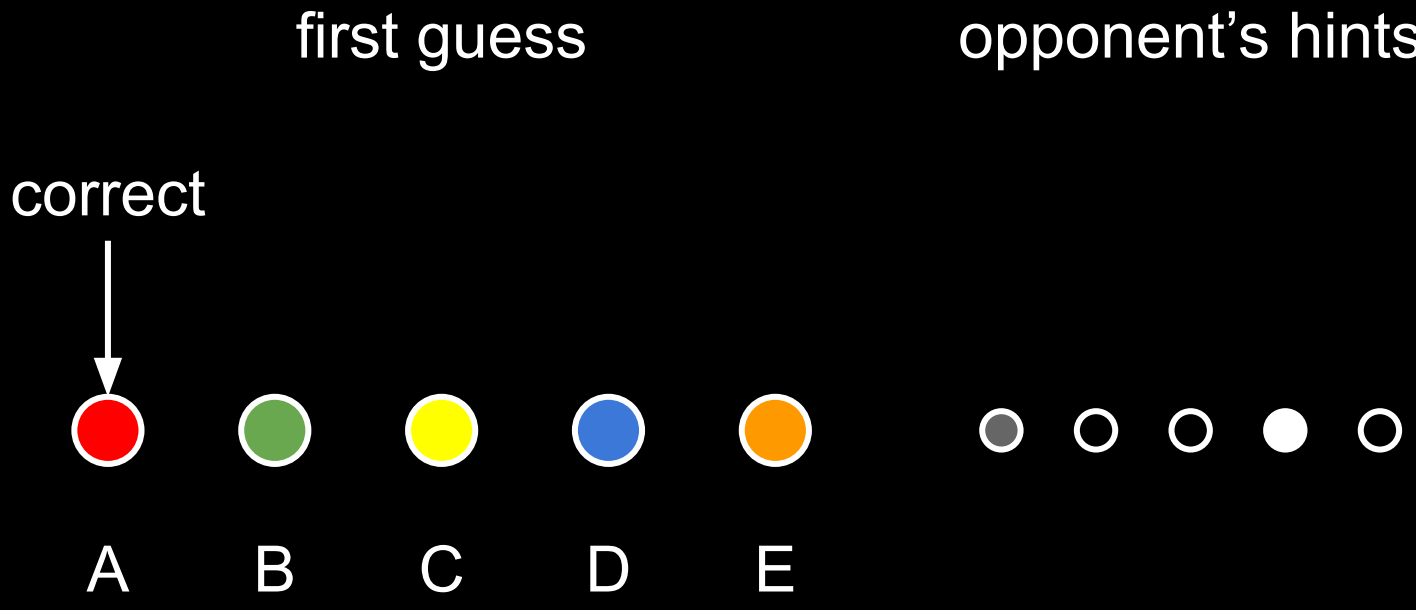
hints reduce uncertainty (entropy)



first guess

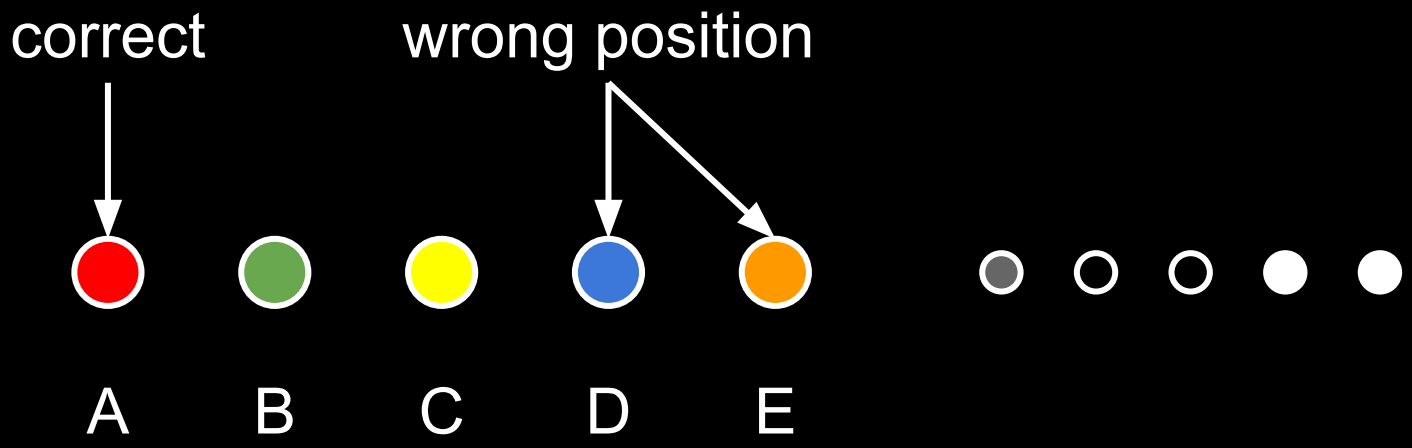
opponent’s hints

A B C D E


first guess
opponent’s hints
correct
A
B
C
D
E

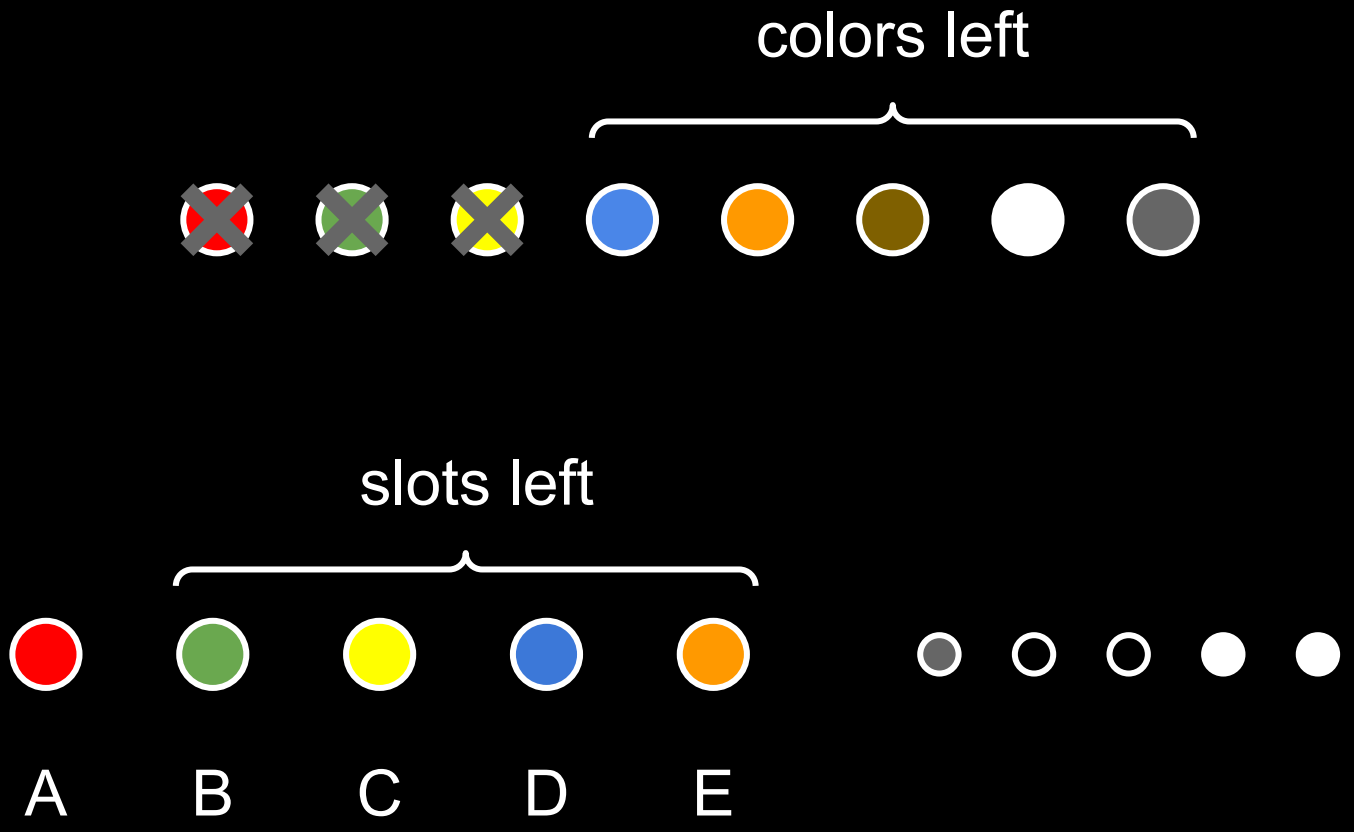
first guess

opponent’s hints



how much information did we get?

A B C D E


colors left
slots left
A
B
C
D
E

new possible permutations?

$$N = 4^8 = 65536$$

reduced entropy in bits?

$$H = log_2\ 4^8 = 16$$

COUNTING

1 2 3

1	2	3
10^2	10^1	10^0

1	2	3
10^2	10^1	10^0

$= 1 \times 10^2 + 2 \times 10^1 + 3 \times 10^0$

$= 1 \times 100 + 2 \times 10 + 3 \times 1$

$= 123$

4	1	2	3
?	10^2	10^1	10^0

4	1	2	3
?	10^2	10^1	10^0

$$= 4 \times 10^3 + 1 \times 10^2 + 2 \times 10^1 + 3 \times 10^0$$

4	1	2	3
?	10^2	10^1	10^0

$= 4 \times 10^3 + 1 \times 10^2 + 2 \times 10^1 + 3 \times 10^0$

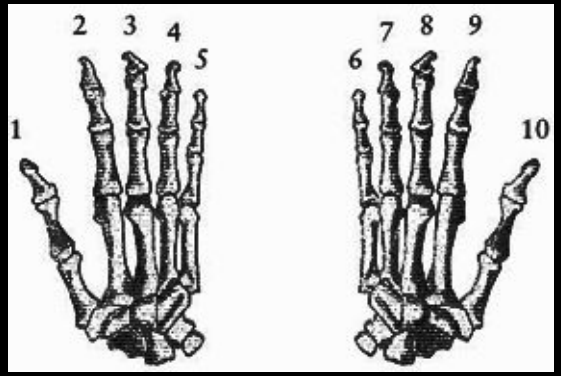
$= 4 \times 1000 + 1 \times 100 + 2 \times 10 + 3 \times 1$

4	1	2	3
?	10^2	10^1	10^0

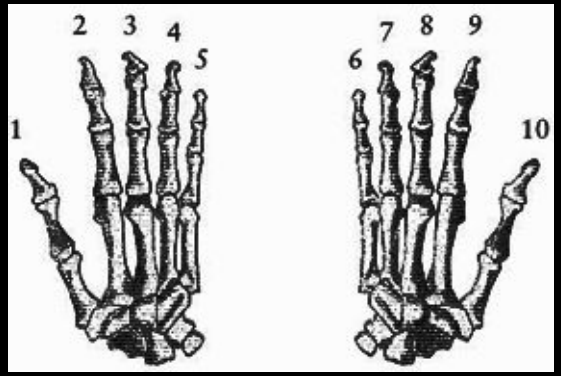
$= 4 \times 10^3 + 1 \times 10^2 + 2 \times 10^1 + 3 \times 10^0$

$= 4 \times 1000 + 1 \times 100 + 2 \times 10 + 3 \times 1$

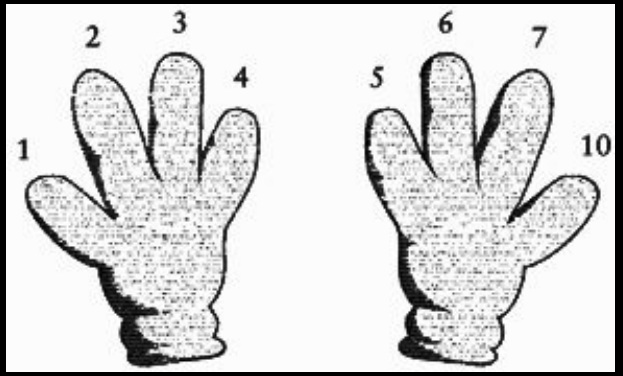
$= 4123$



human hand



human hand



cartoon character’s hand

1 2 3 (octal)

1	2	3	(octal)
8^2	8^1	8^0	

1	2	3
8^2	8^1	8^0

(octal)

$$= 1 \times 8^2 + 2 \times 8^1 + 3 \times 8^0$$

1	2	3
8^2	8^1	8^0

(octal)

$$= 1 \times 8^2 + 2 \times 8^1 + 3 \times 8^0$$

$$= 1 \times 64 + 2 \times 8 + 3 \times 1$$

1	2	3
8^2	8^1	8^0

(octal)

$= 1 \times 8^2 + 2 \times 8^1 + 3 \times 8^0$

$= 1 \times 64 + 2 \times 8 + 3 \times 1$

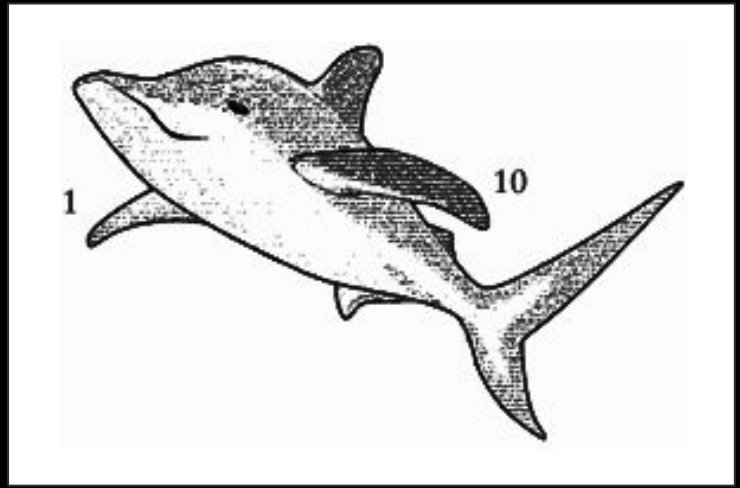
$= 83$ (decimal)


decimal
octal
8
?


decimal
octal
?
7


decimal
octal
16
?


decimal
octal
?
100



what now?

0, 1, …

0, 1, 10, …

0, 1, 10, 11, …

0, 1, 10, 11, 100, …

0, 1, 10, 11, 100, 101, …

0, 1, 10, 11, 100, 101, 110

1 1 0 (binary)

1	1	0	(binary)
2^2	2^1	2^0	

$$1 \quad 1 \quad 0 \quad \text{(binary)}$$

$$2^2 \qquad 2^1 \qquad 2^0$$

$$= 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$$

1 1 0 (binary)

2^2 2^1 2^0

$= 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$

$= 1 \times 4 + 1 \times 2 + 0 \times 1$

1	1	0
2^2	2^1	2^0

(binary)

$= 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$

$= 1 \times 4 + 1 \times 2 + 0 \times 1$

$= 6$ (decimal)

2 3 4 5 6

0, 1, 10, 11, 100, 101, 110

place value systems

$$N = d_n * R^{n-1} + \ldots + d_1 * R^1 + d_0 * R^0$$

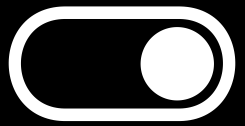
$d \in \{ 0, 1, \ldots R-1 \}$

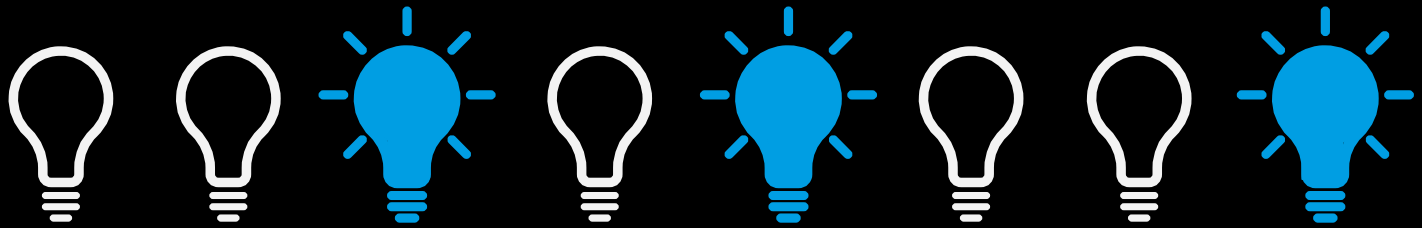
n = number of digits

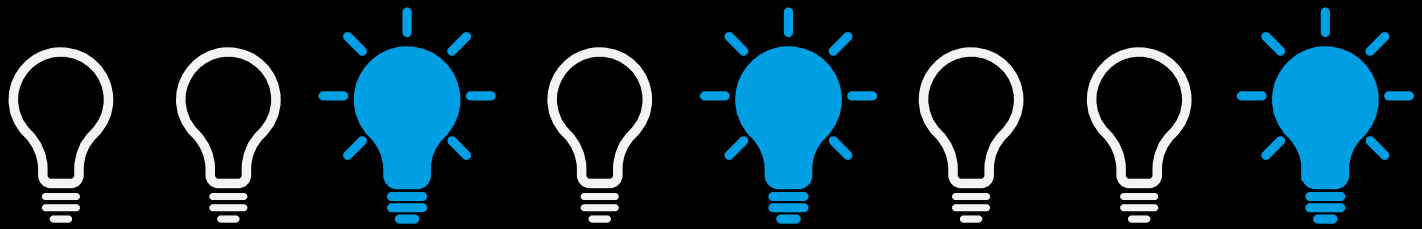
$$R \geq 2$$

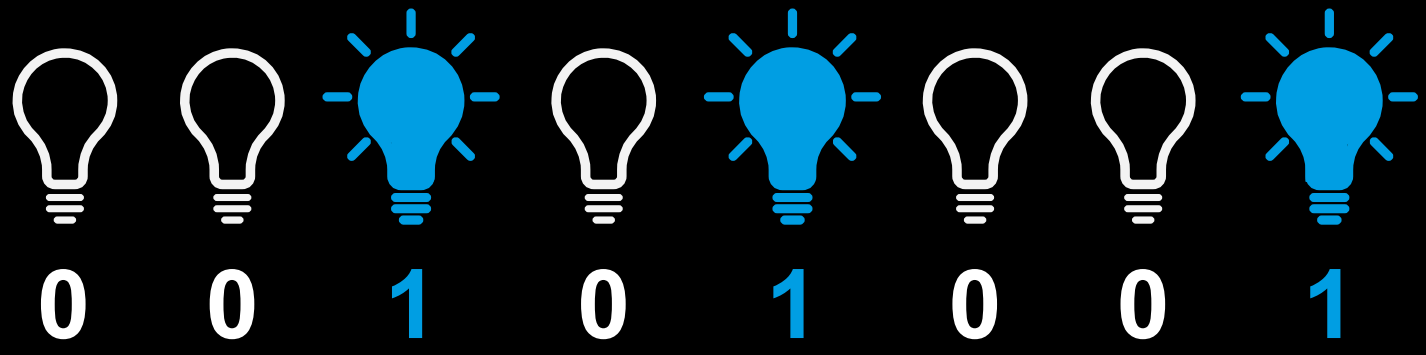
BITS

why do computers think binary?

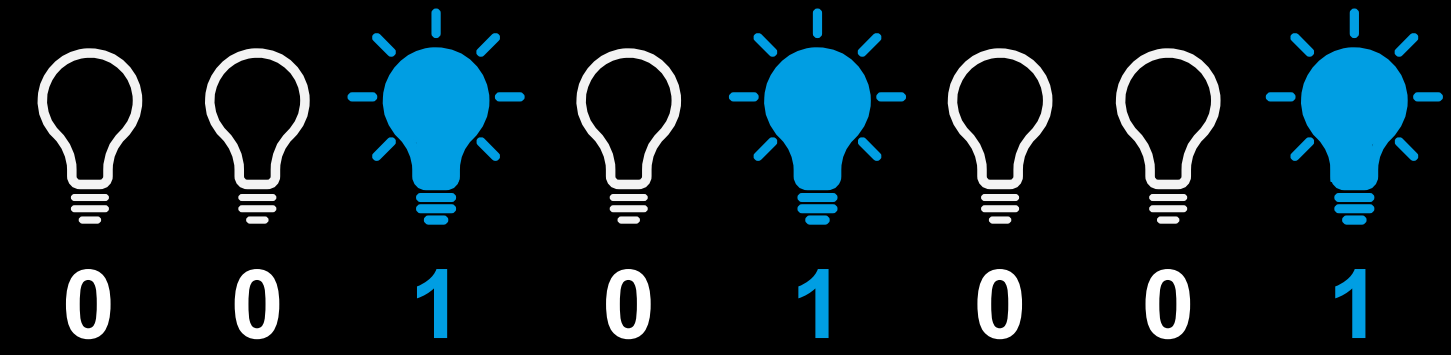





0 0 1 0 1 0 0 1

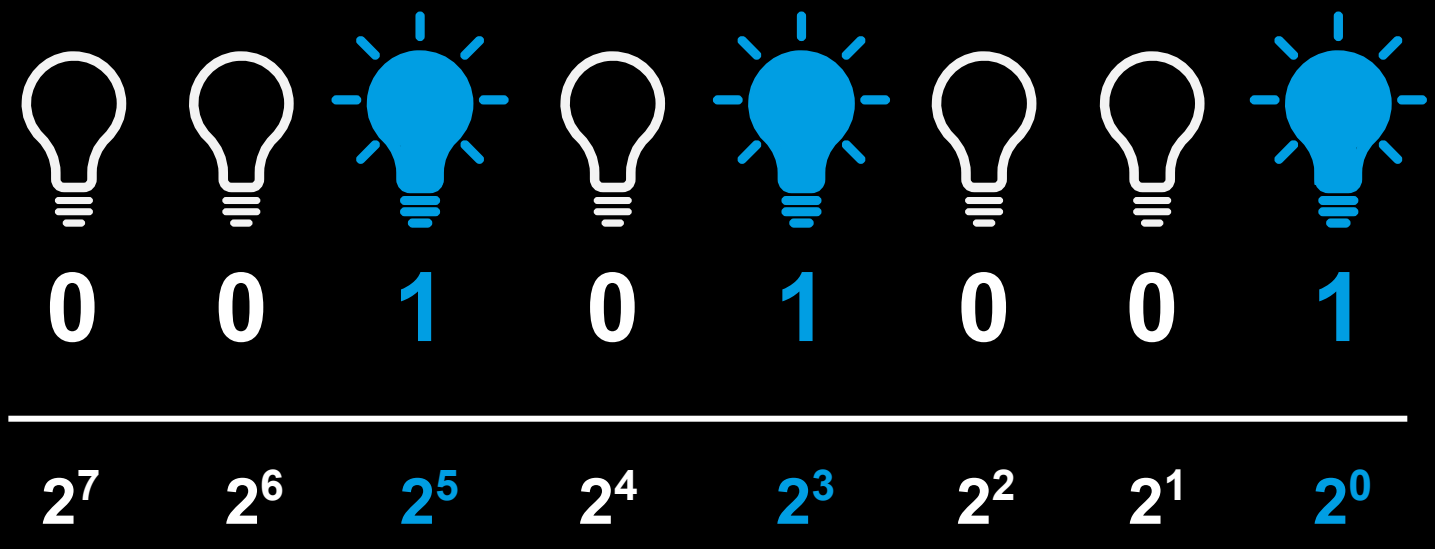


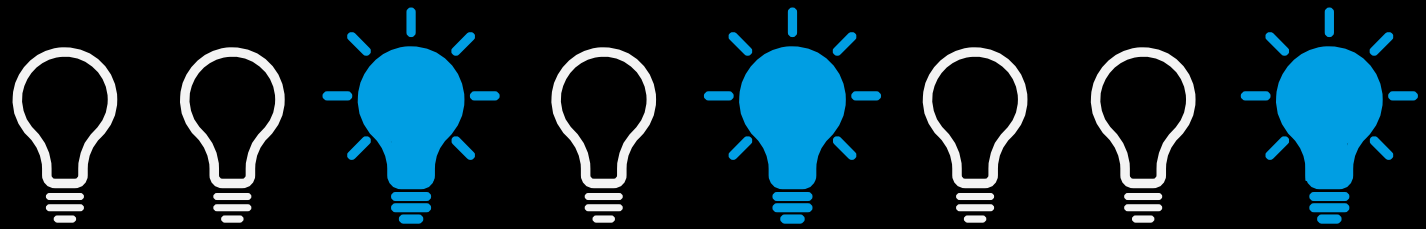
a **bit** (binary digit)



a **bit** (binary digit)

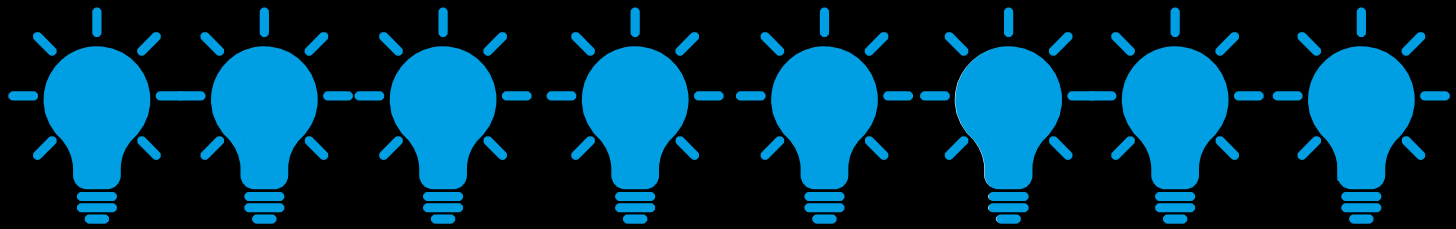
a **byte** (8 bits)


0 0 1 0 1 0 0 1
2^7 2^6 2^5 2^4 2^3 2^2 2^1 2^0



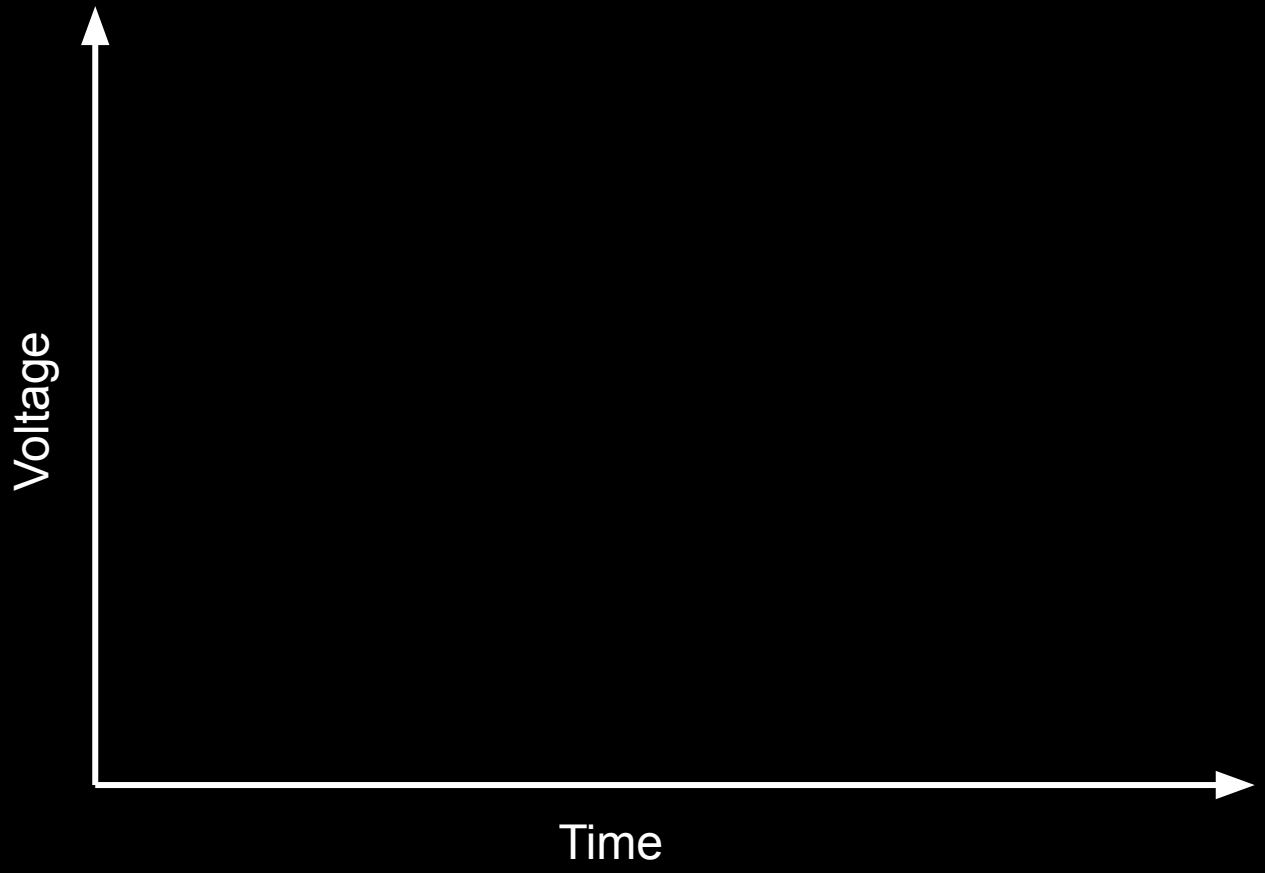
0	0	1	0	1	0	0	1

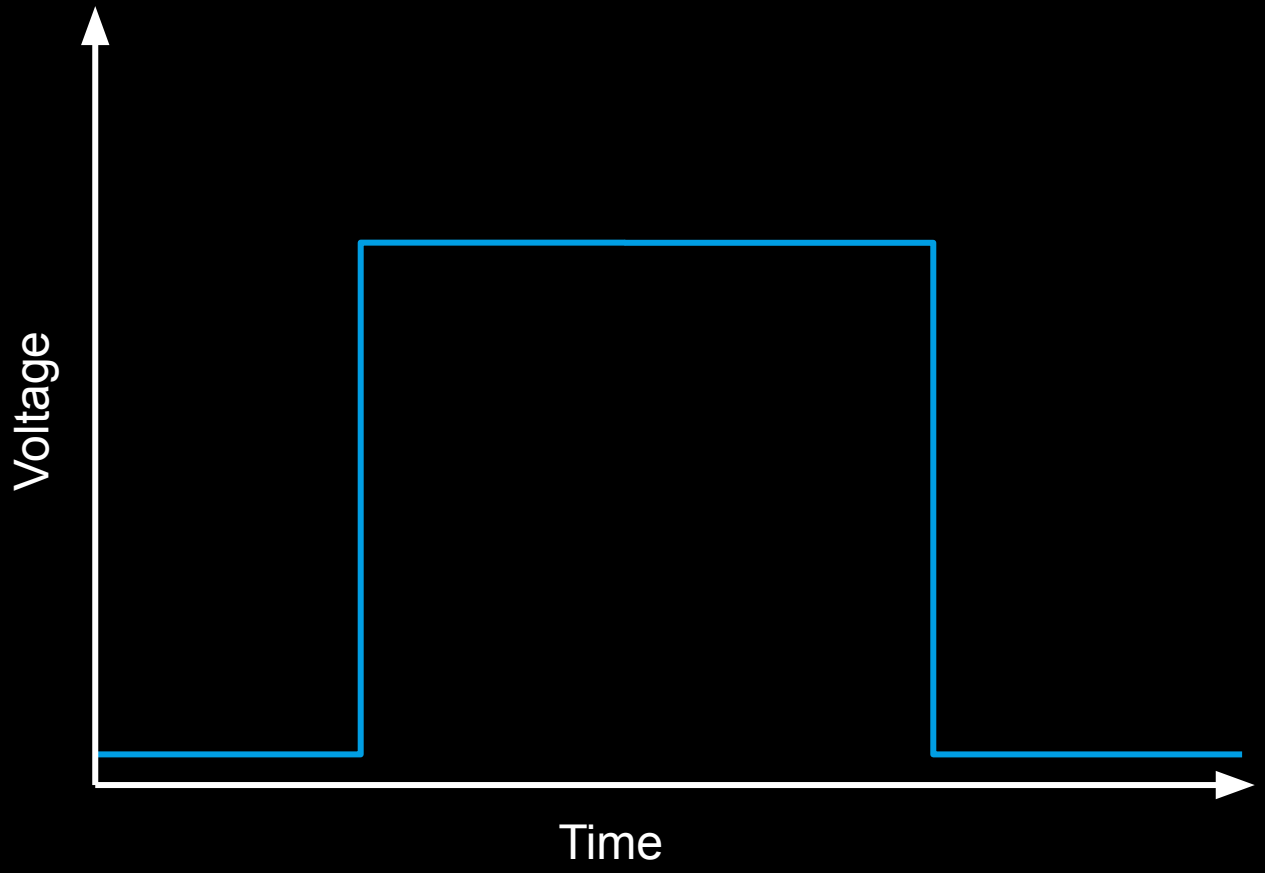
2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
128	64	32	16	8	4	2	1

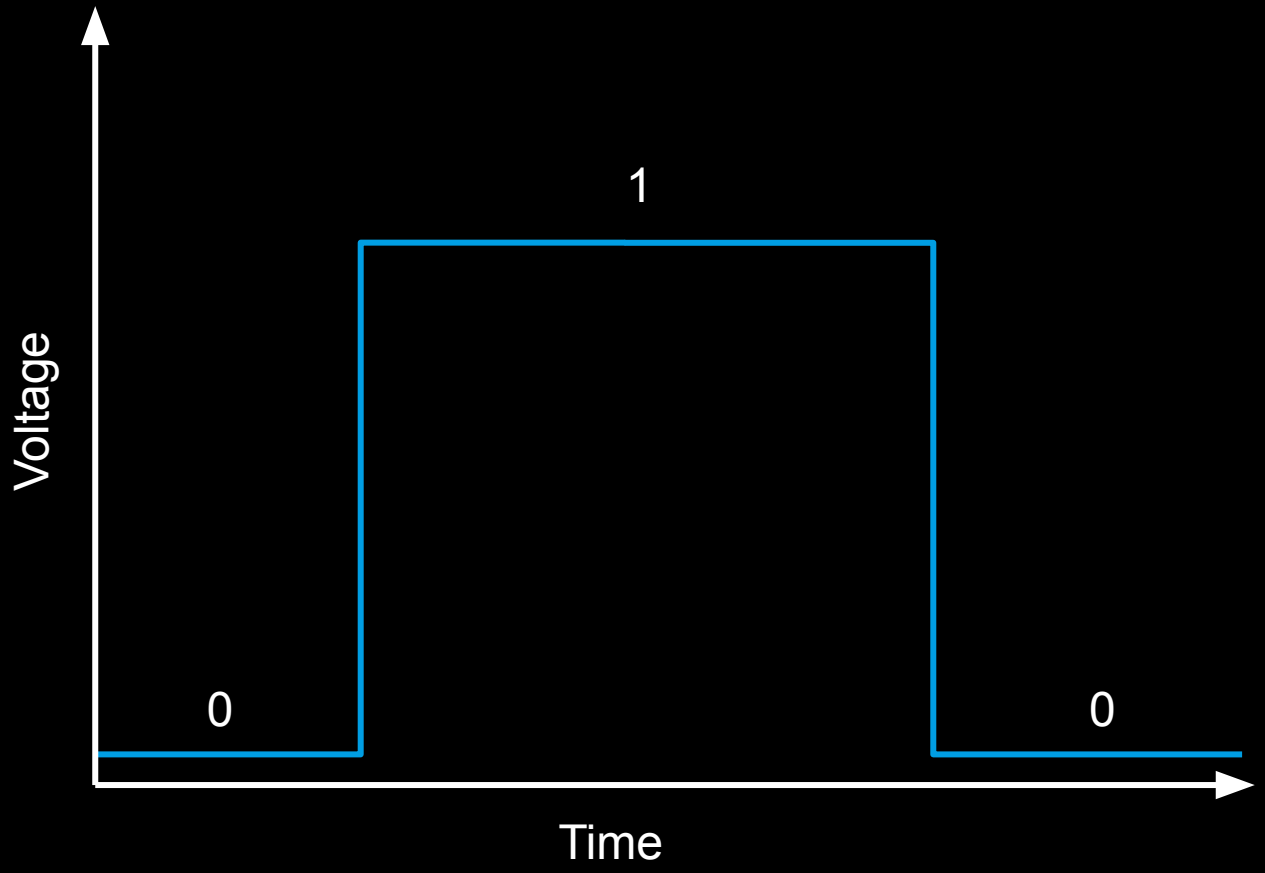


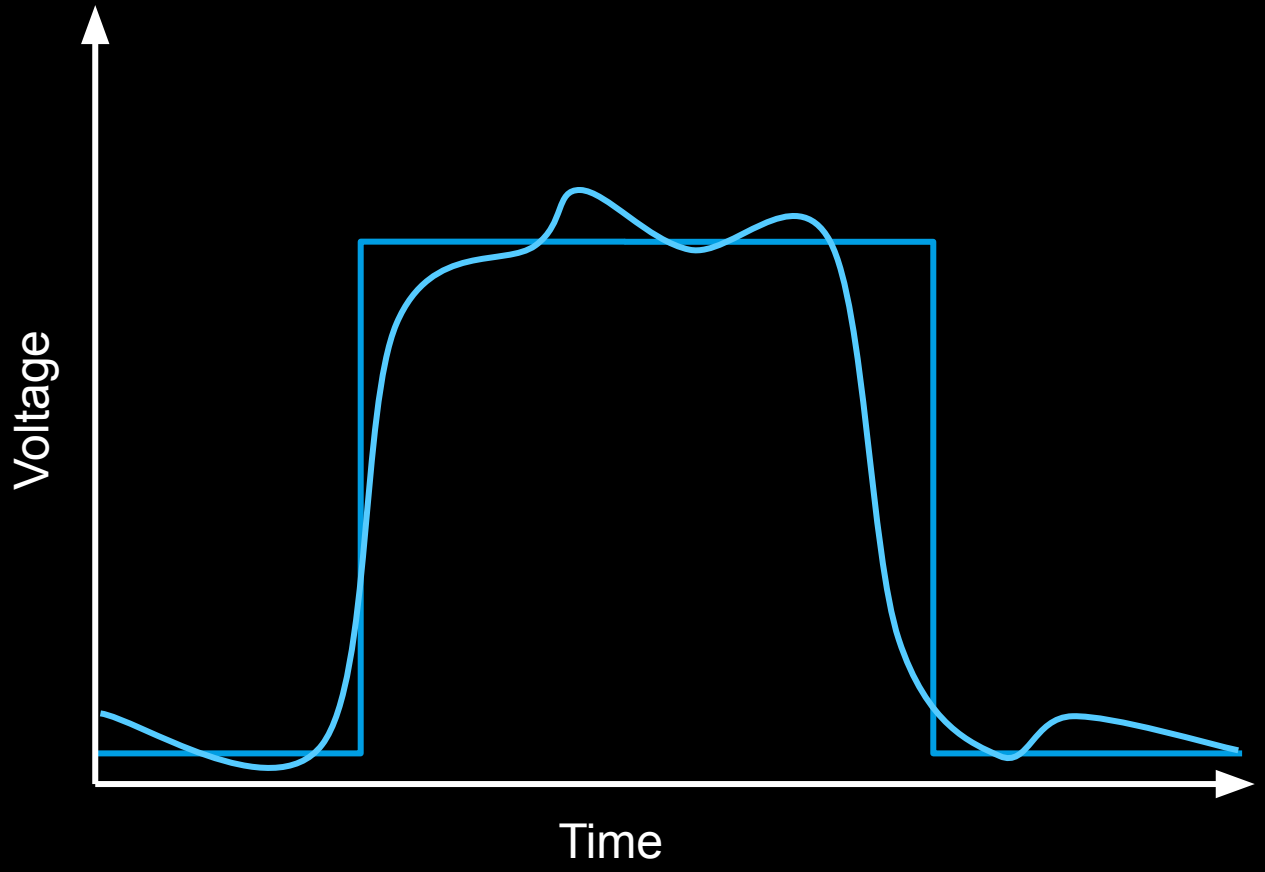
what can we store in one byte?

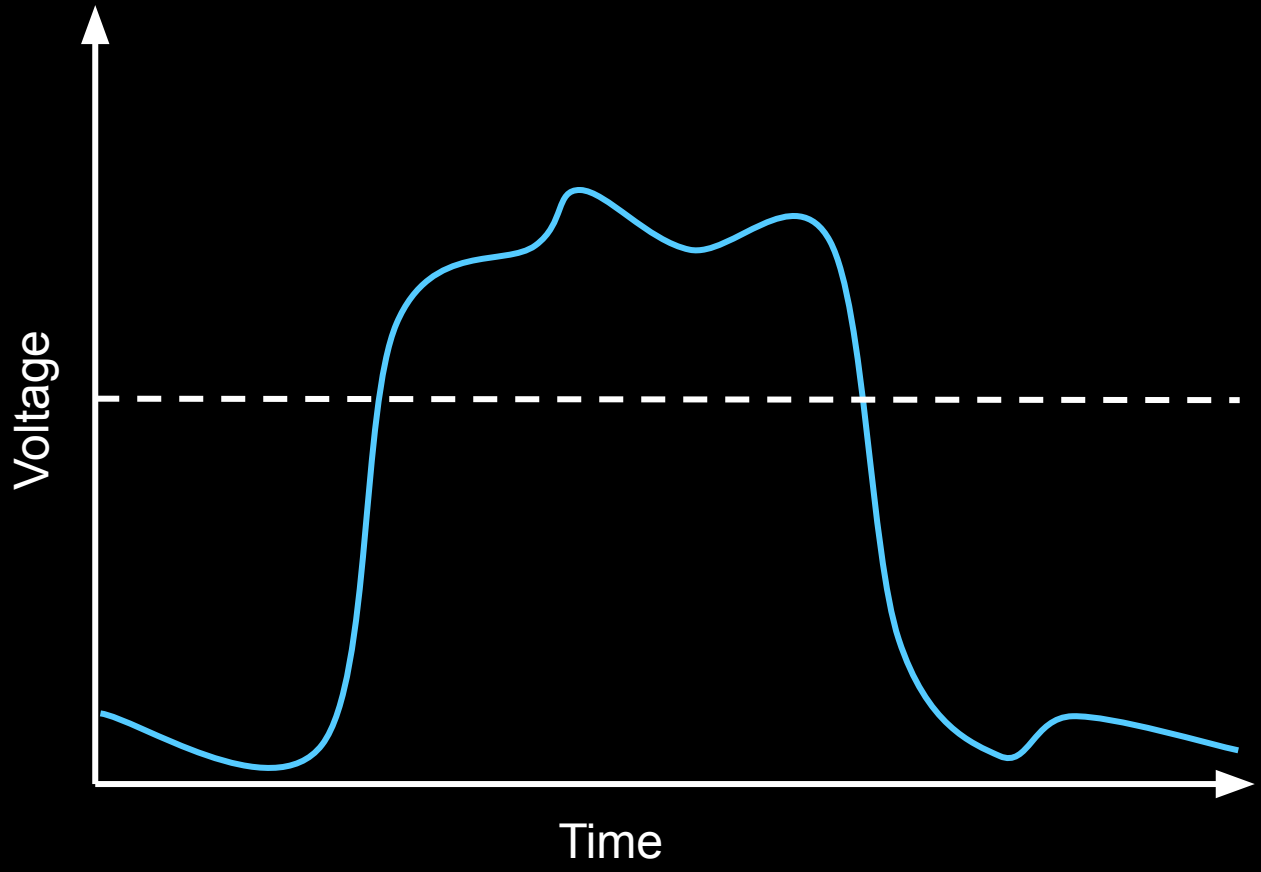
are we stuck with binary?

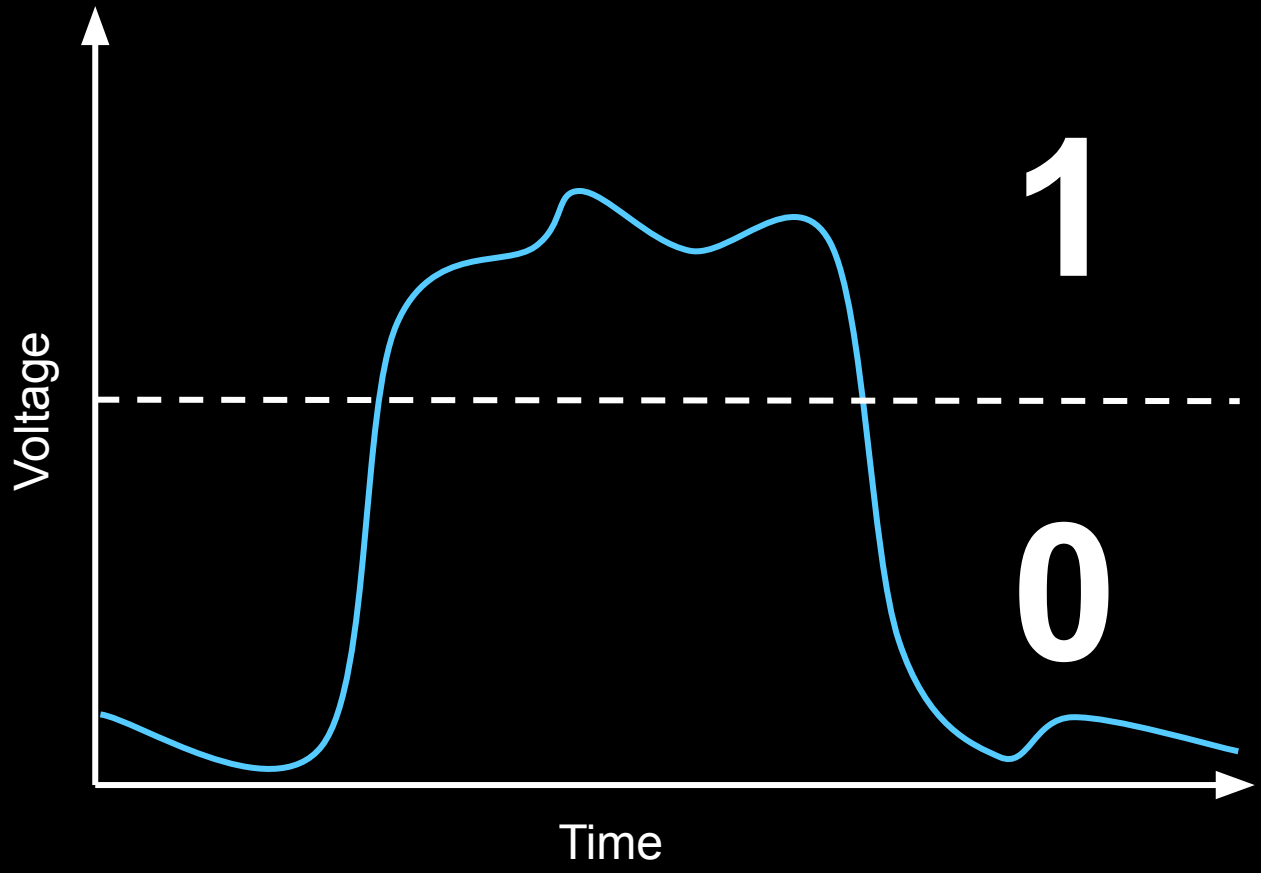

Voltage
Time

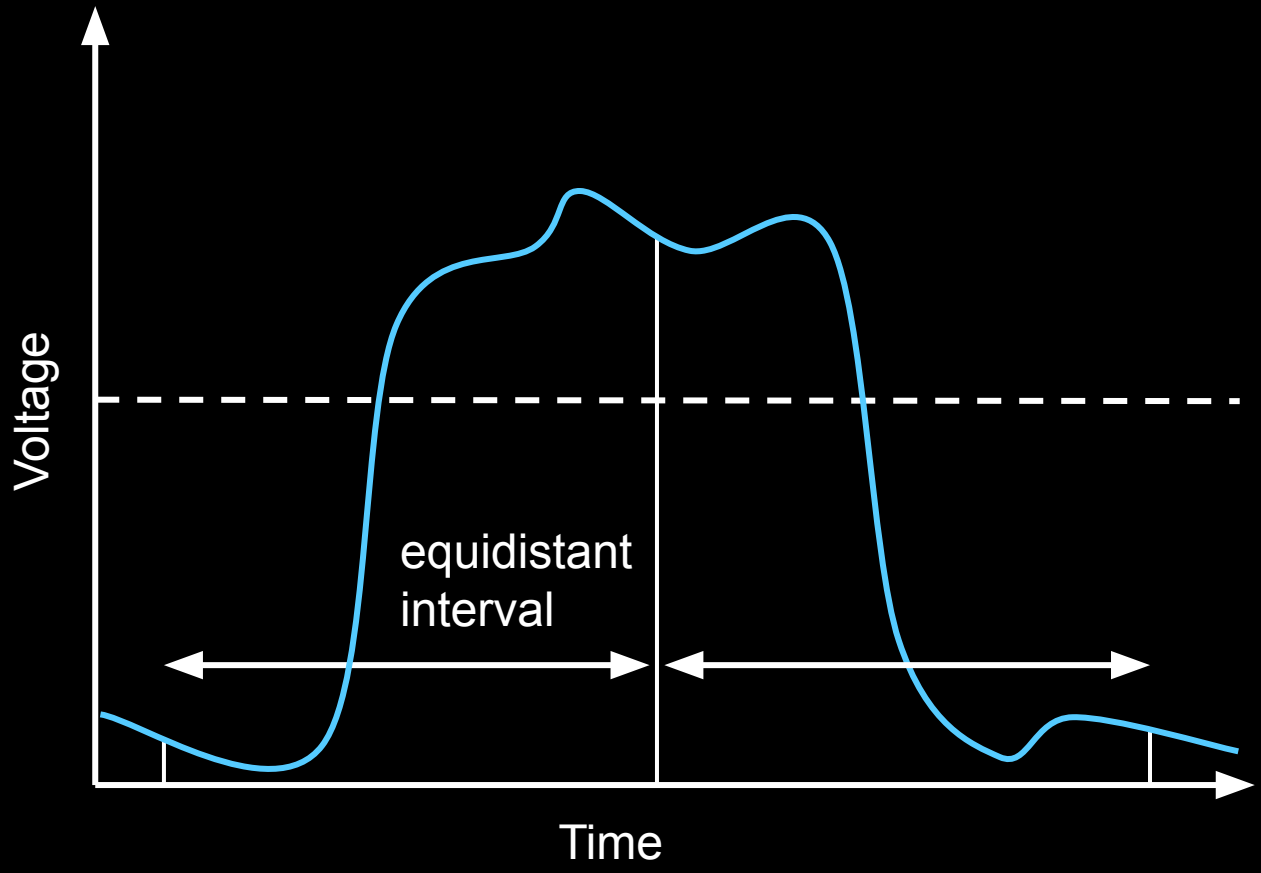

Voltage
Time

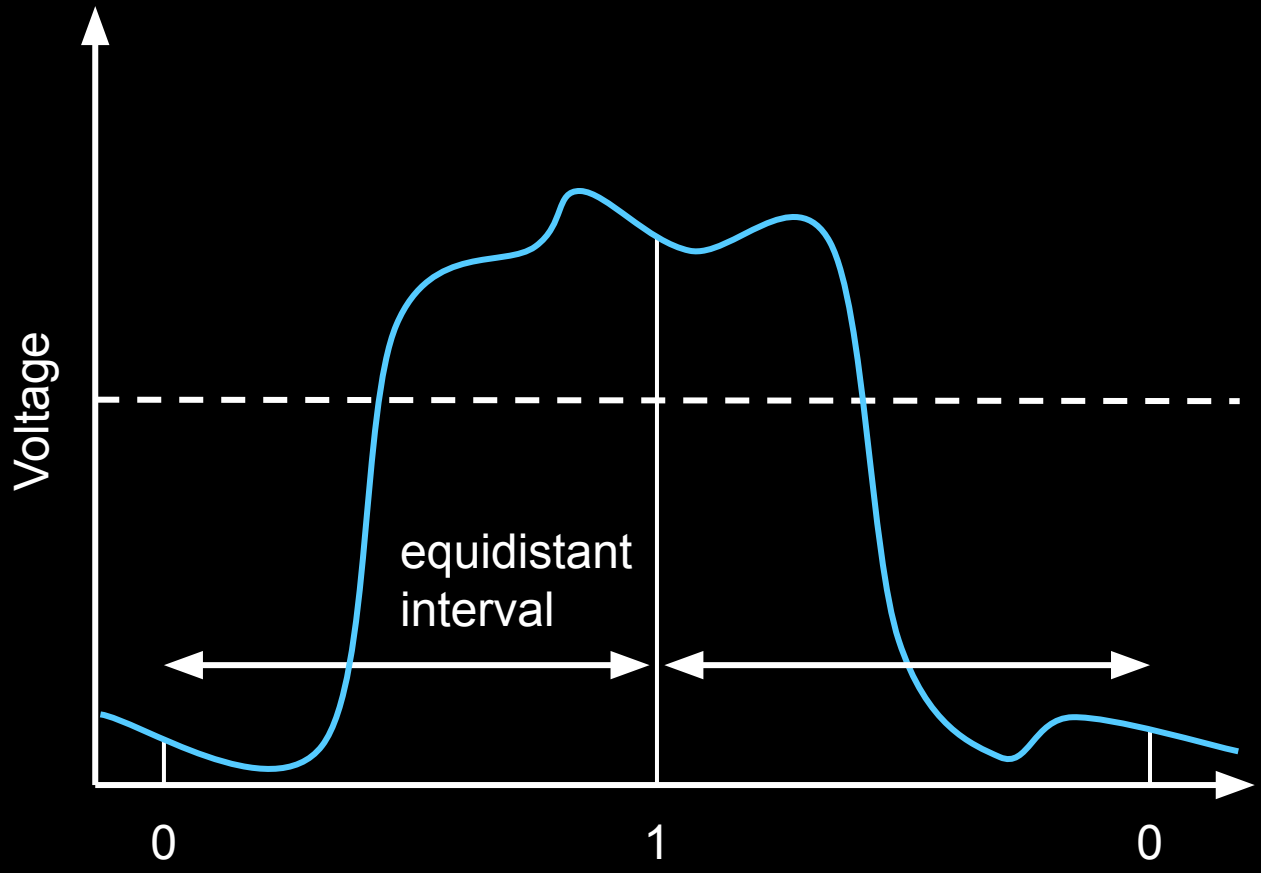

1
Voltage
0
0
Time

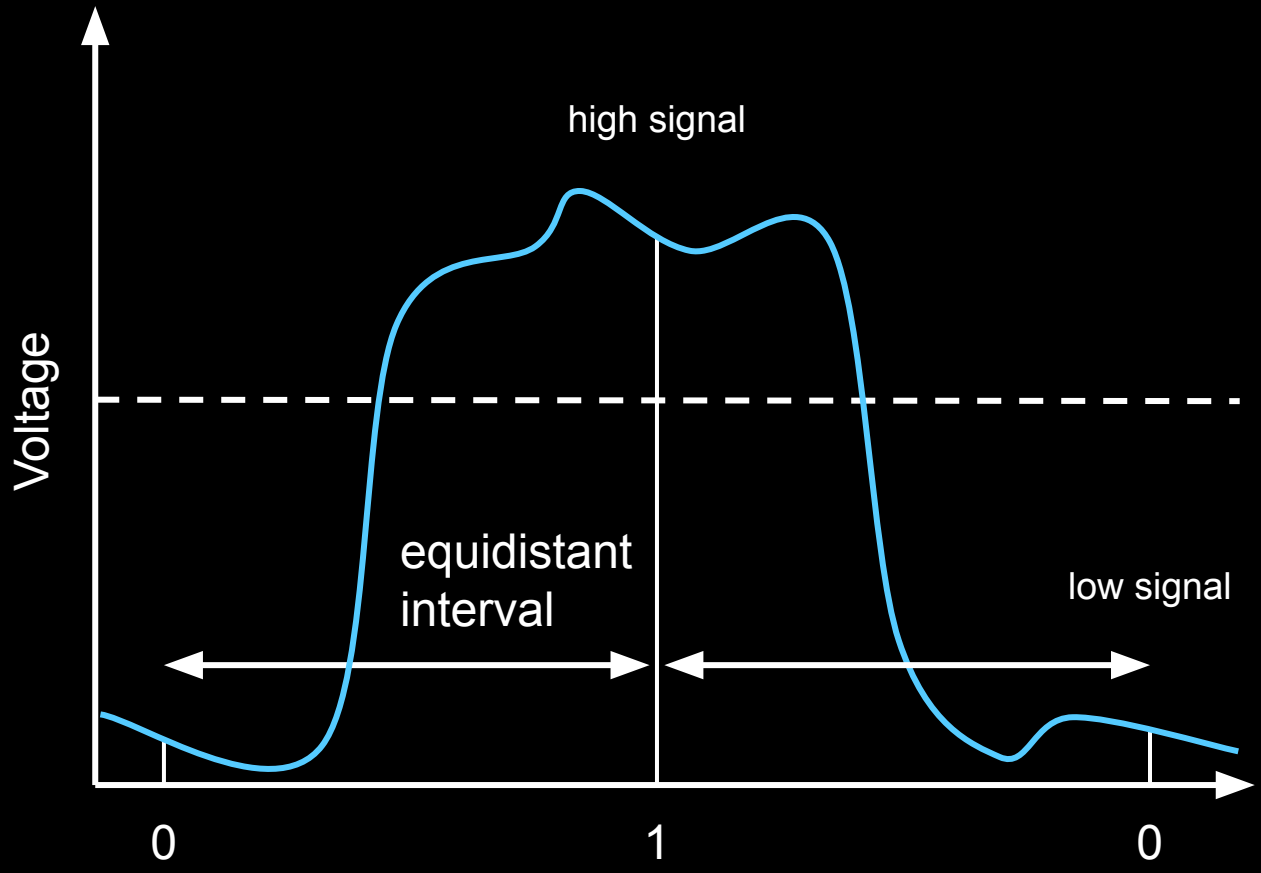

Voltage
Time


Voltage
Time


Voltage
Time
1
0

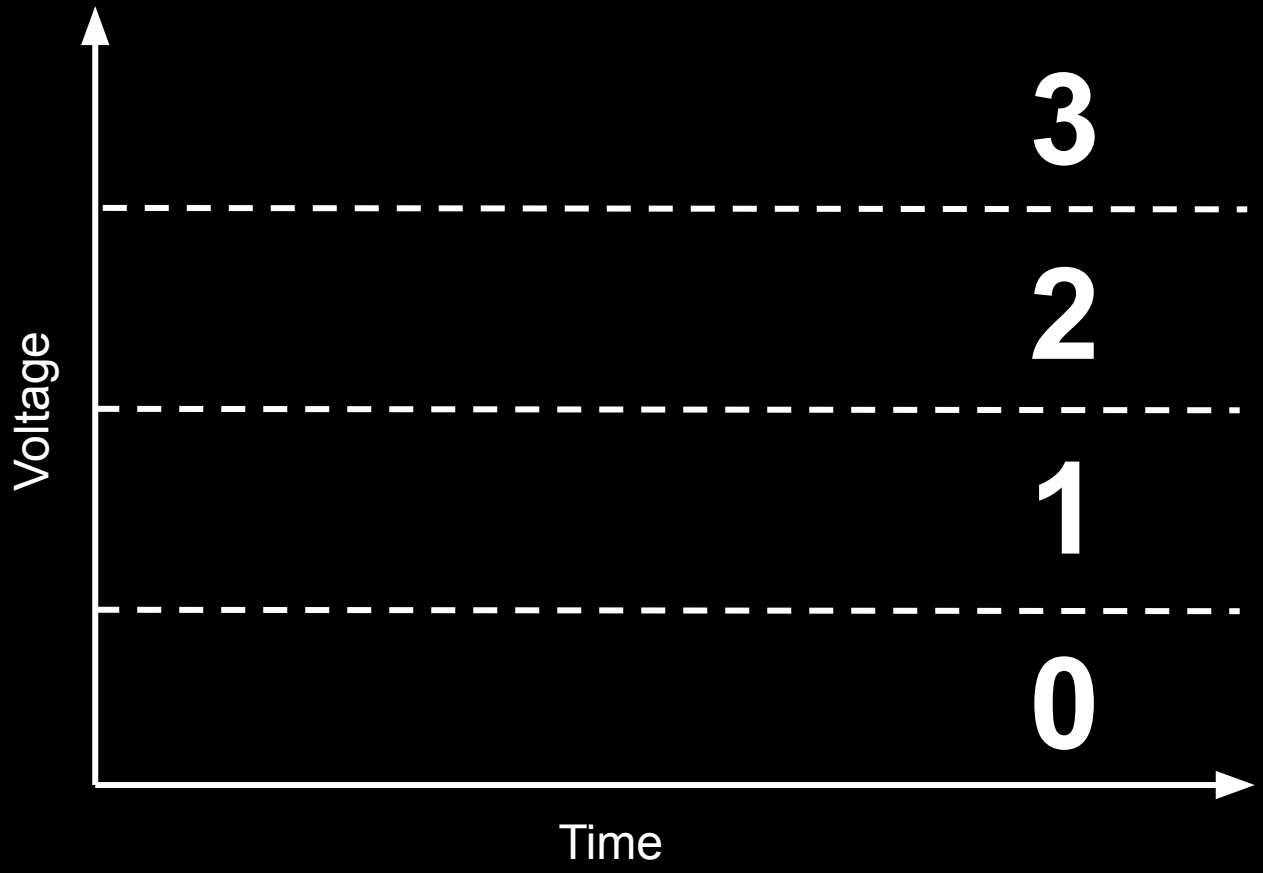

Voltage
equidistant
interval
Time

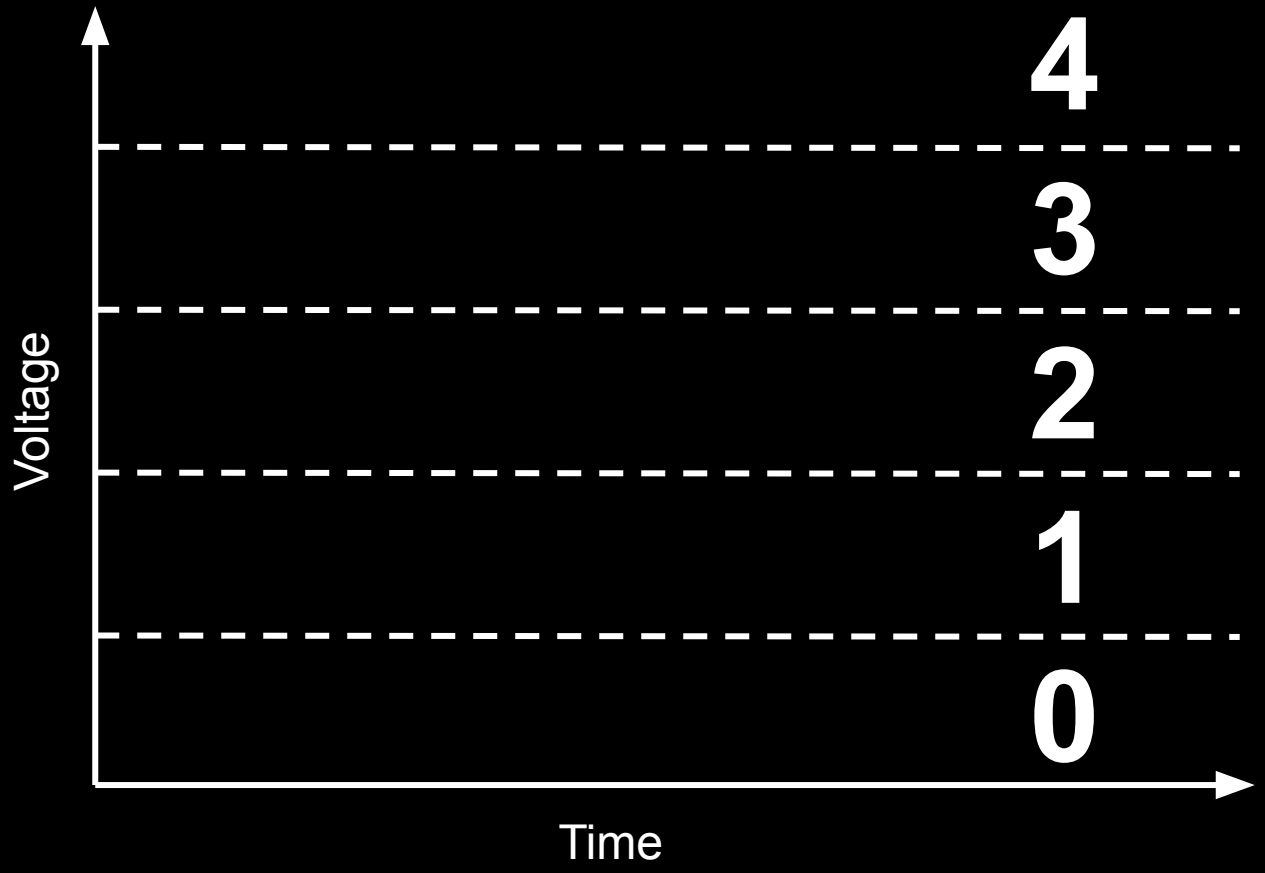

Voltage
equidistant
interval
0
1
0

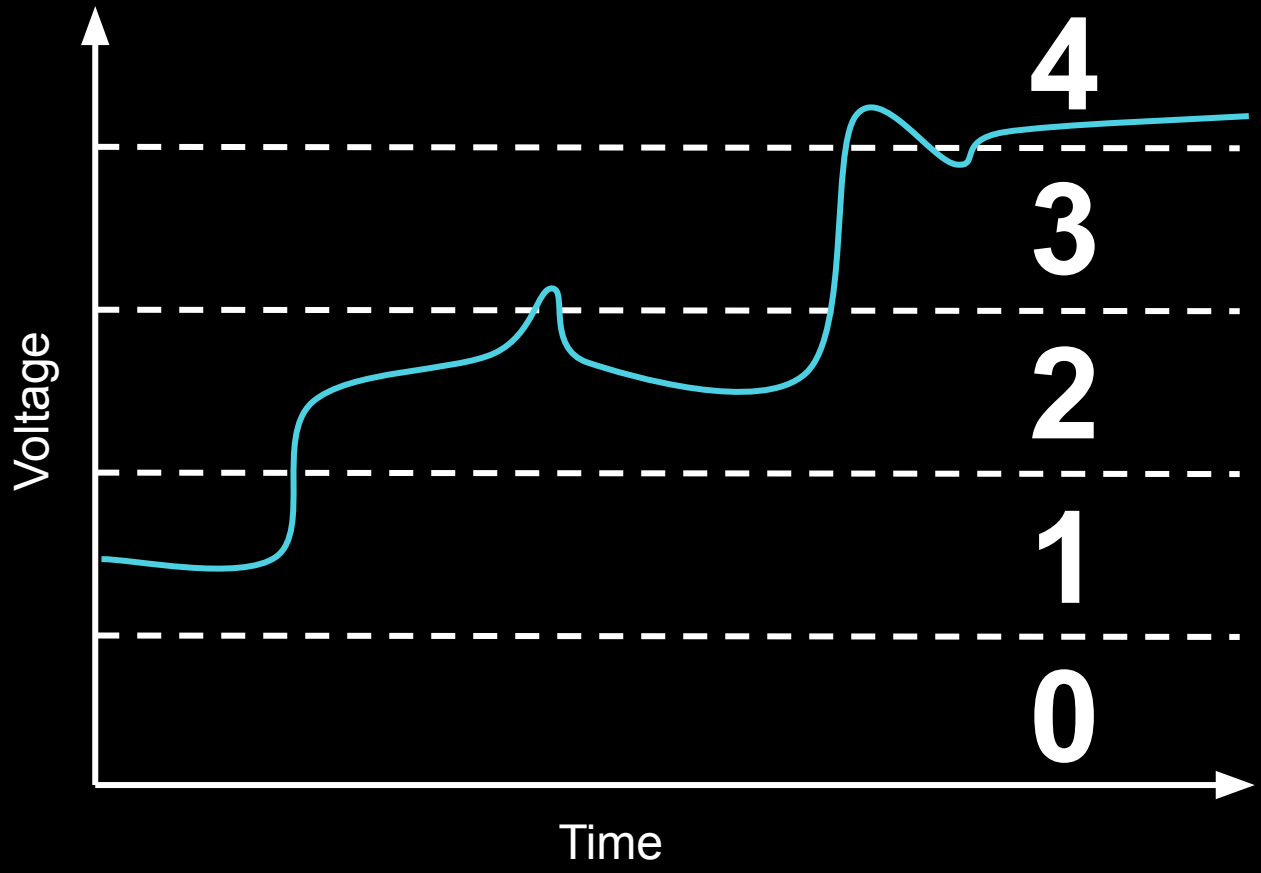

high signal
Voltage
equidistant
interval
low signal
0
1
0

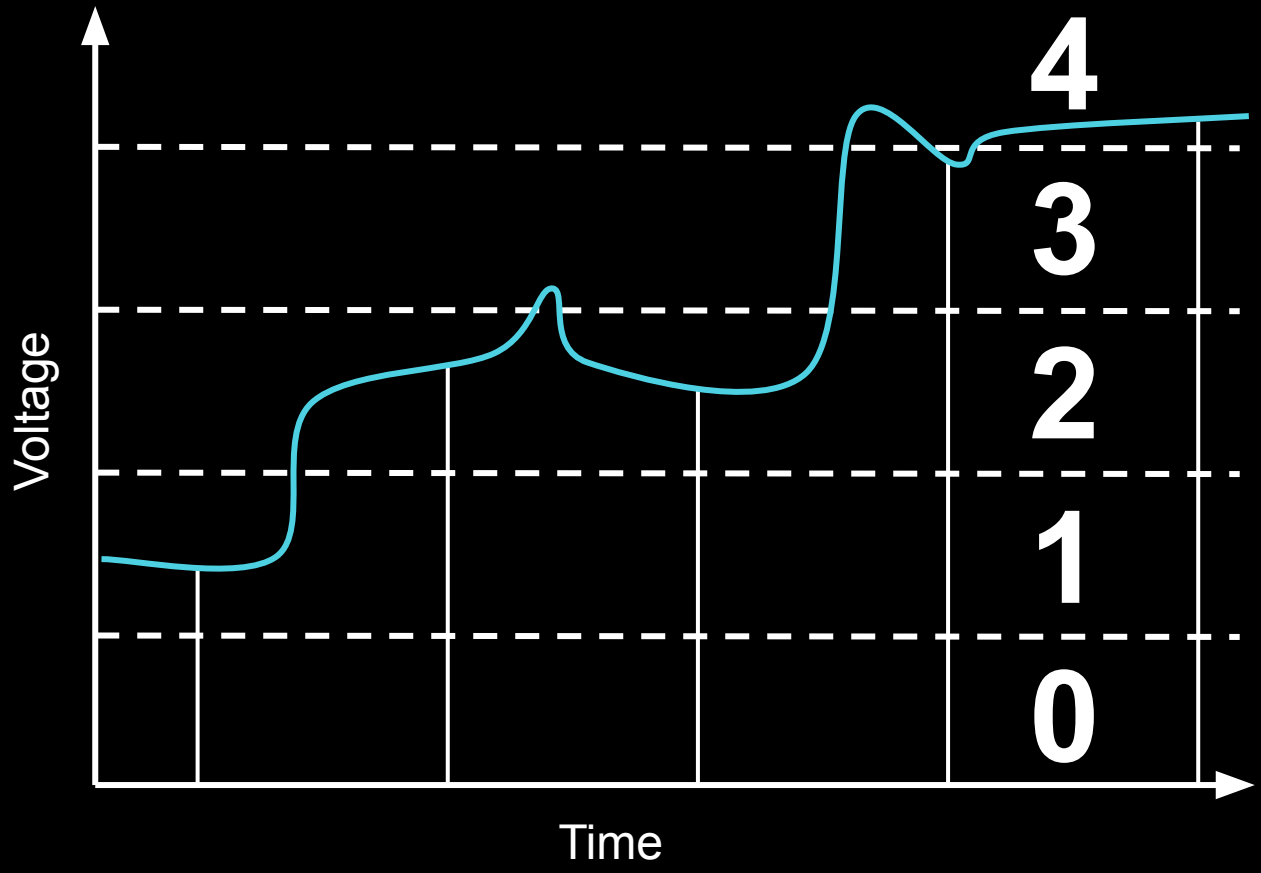
what about ternary?

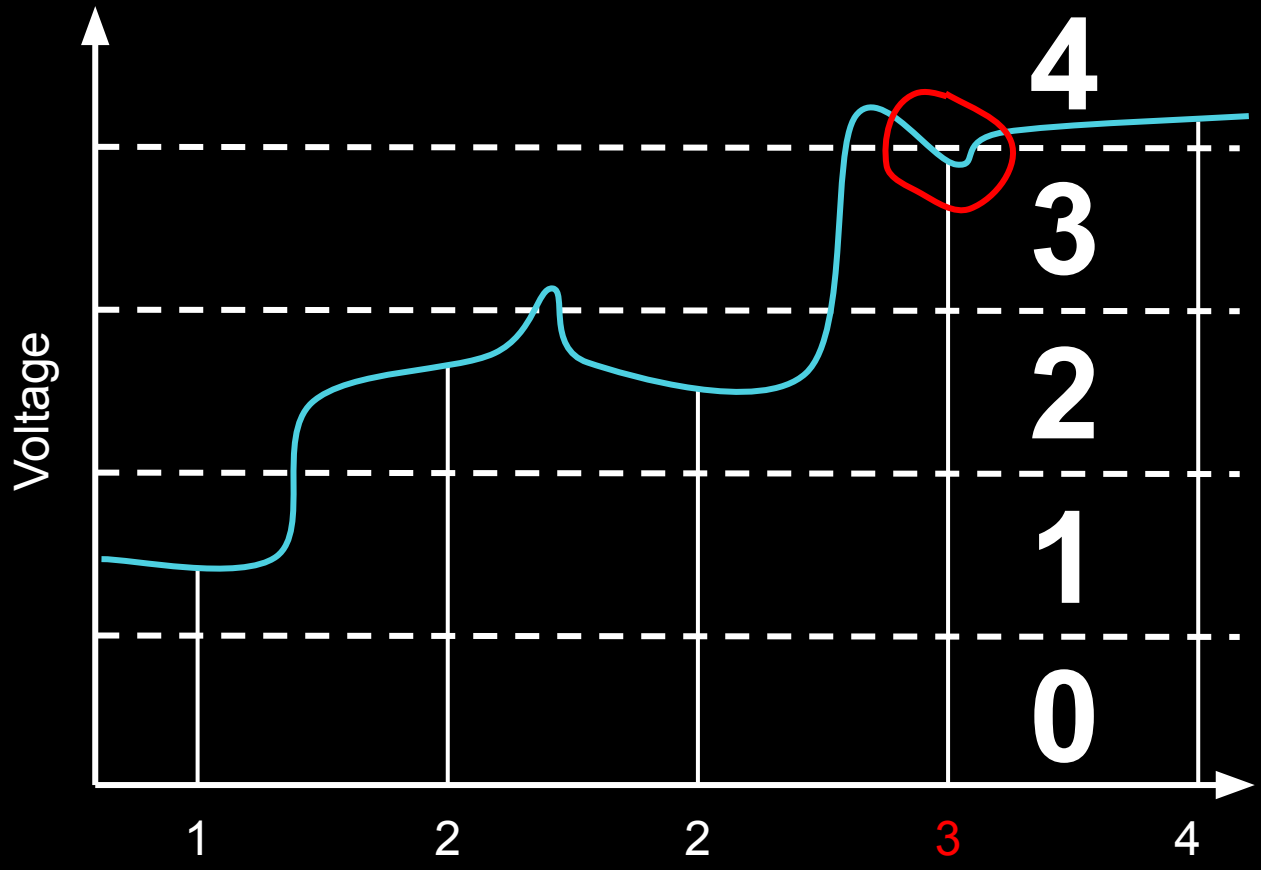

2
1
0
Voltage
Time


3
2
1
0
Voltage
Time


4
3
2
1
0
Voltage
Time


4
3
2
1
0
Voltage
Time


4
3
2
1
0
Voltage
Time


Voltage
4
3
2
1
0
1
2
2
3
4

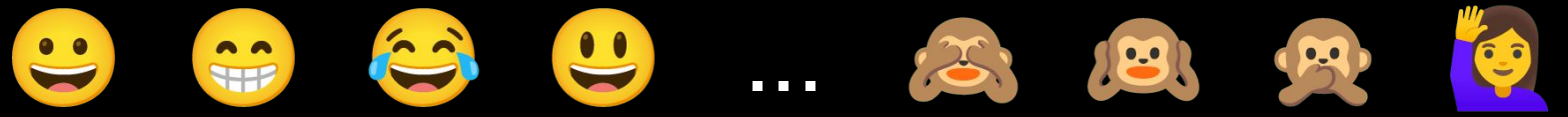
CODES

representing text

A	B	C	D	...	a	b	c	d
65	66	67	68		97	98	99	100

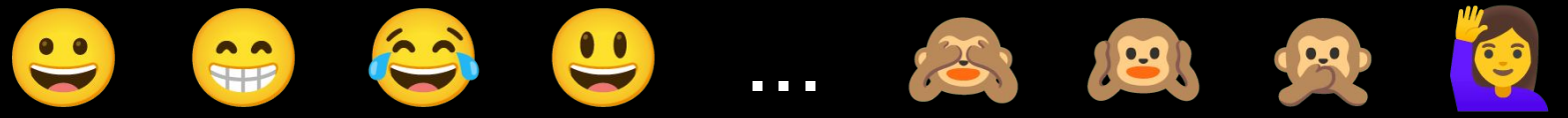
ASCII Code

A	B	C	D	...	a	b	c	d
65	66	67	68		97	98	99	100



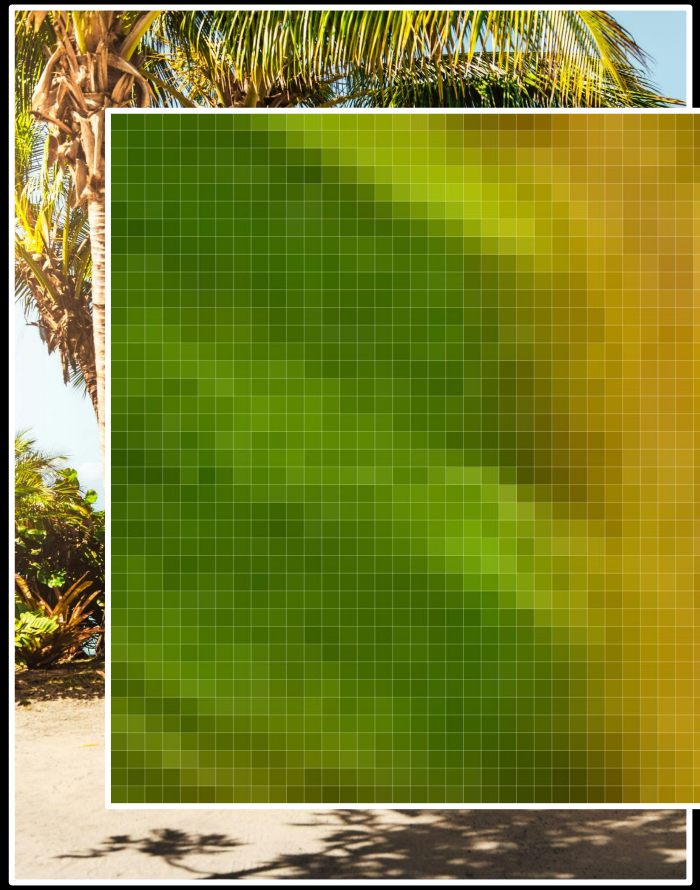
1F600 1F601 1F602 1F603 ... 1F648 1F649 1F64A 1F64B

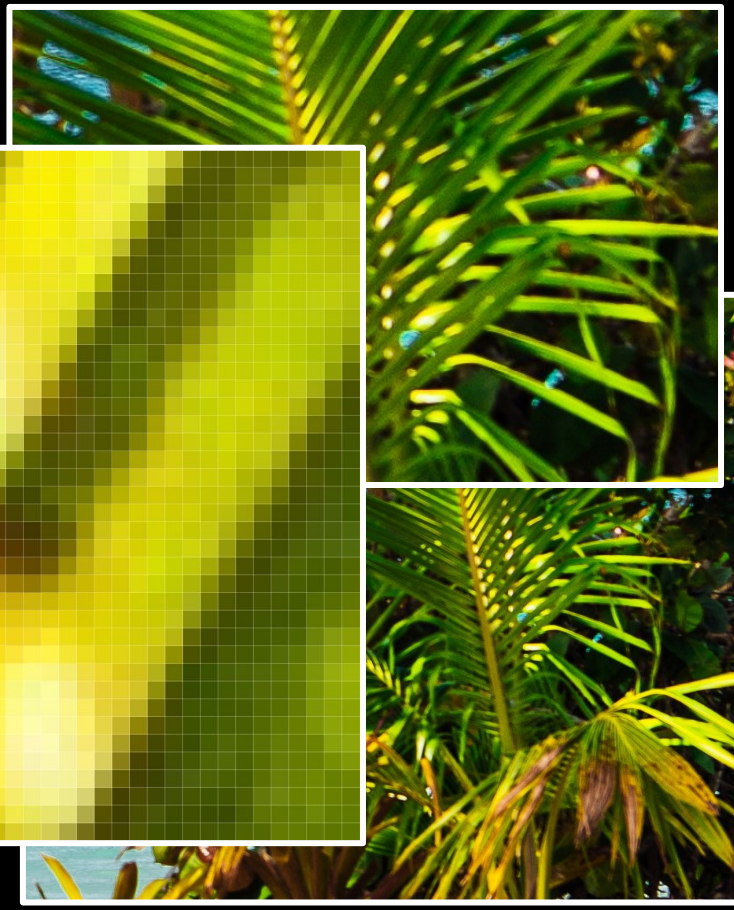
Unicode

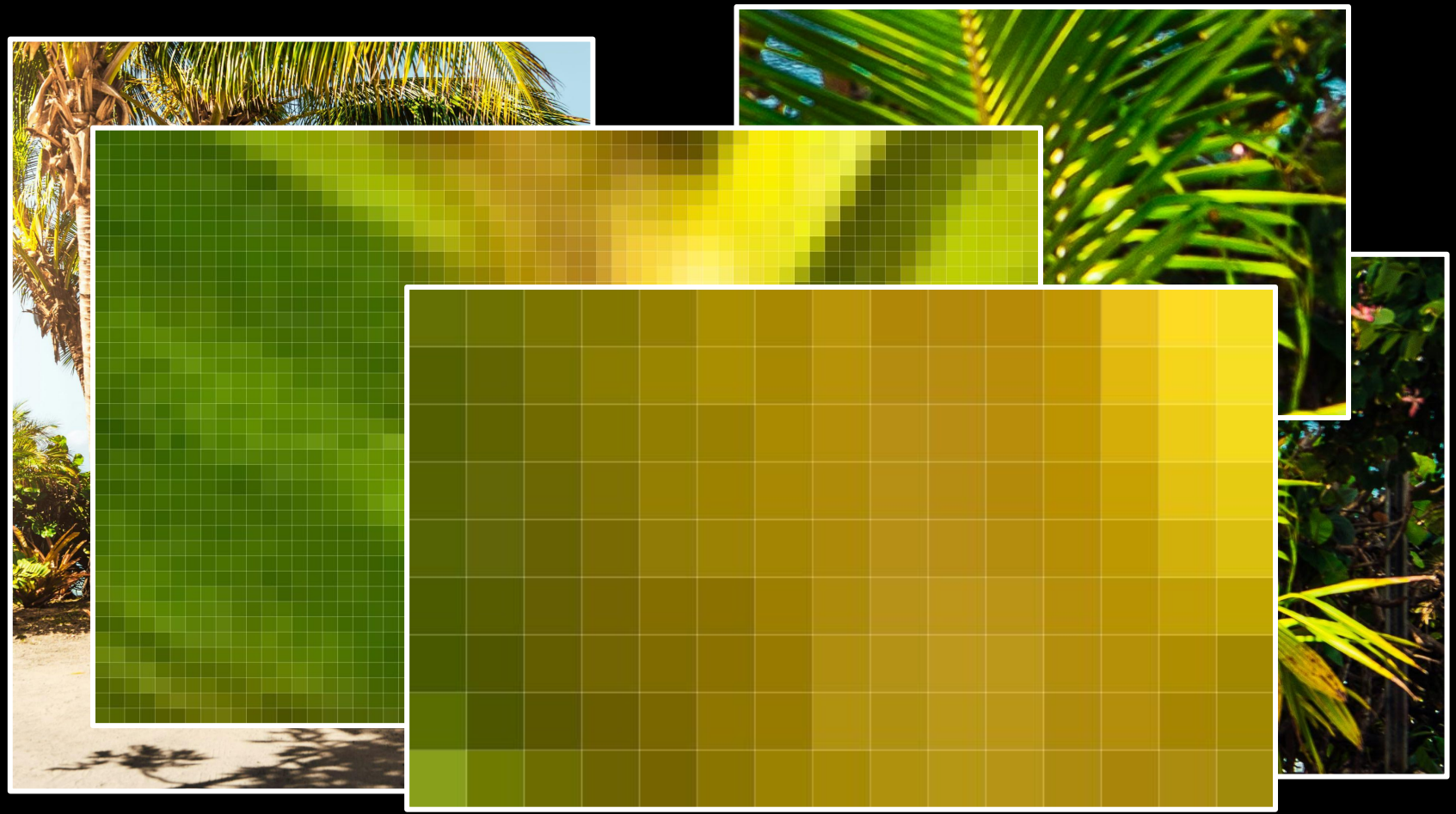


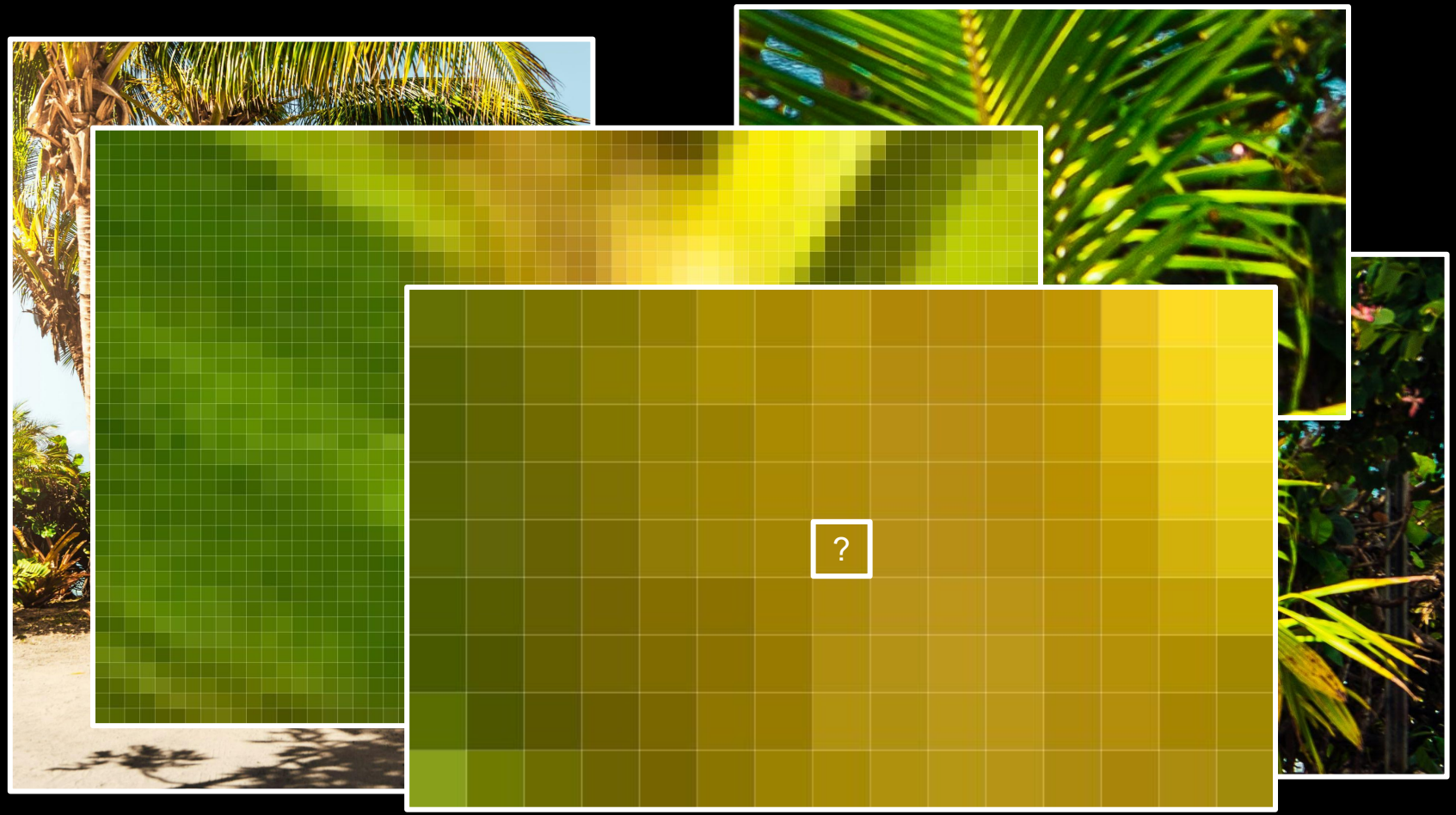
1F600 1F601 1F602 1F603 ... 1F648 1F649 1F64A 1F64B

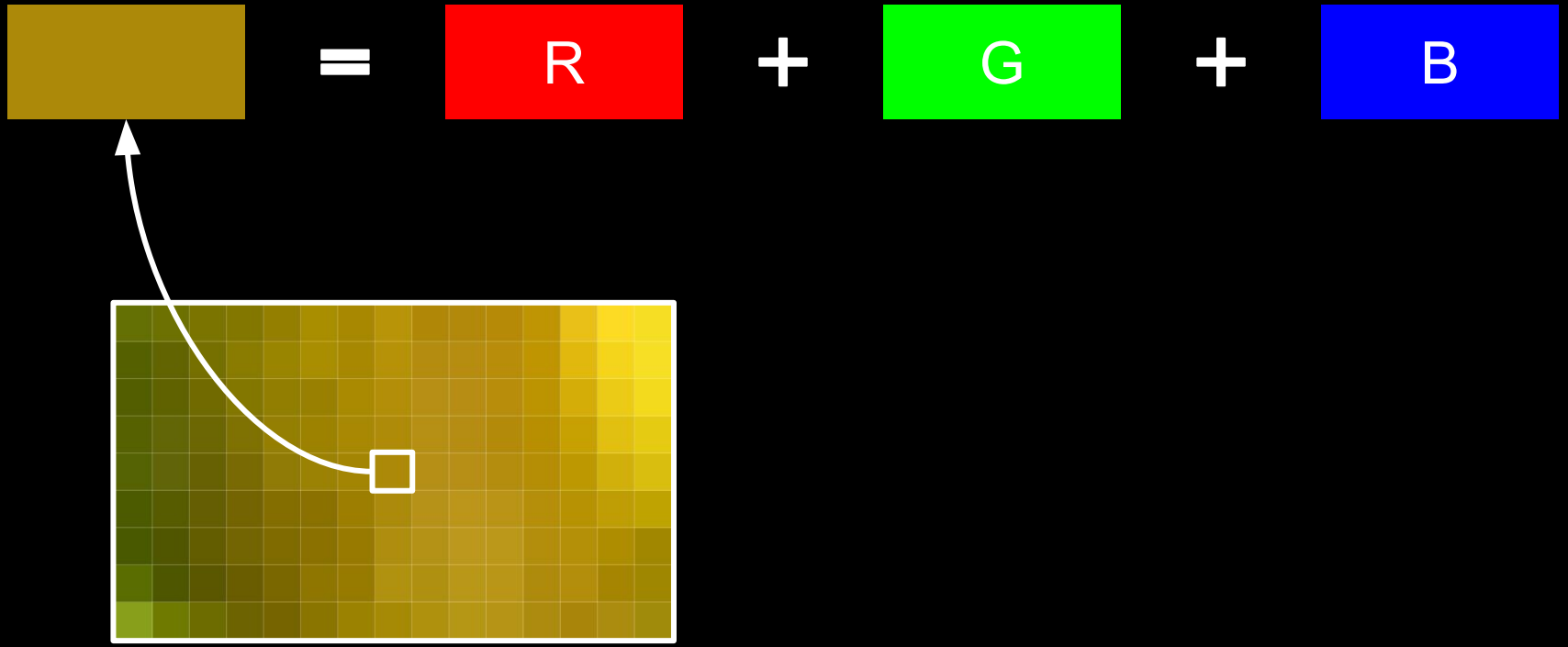
representing images

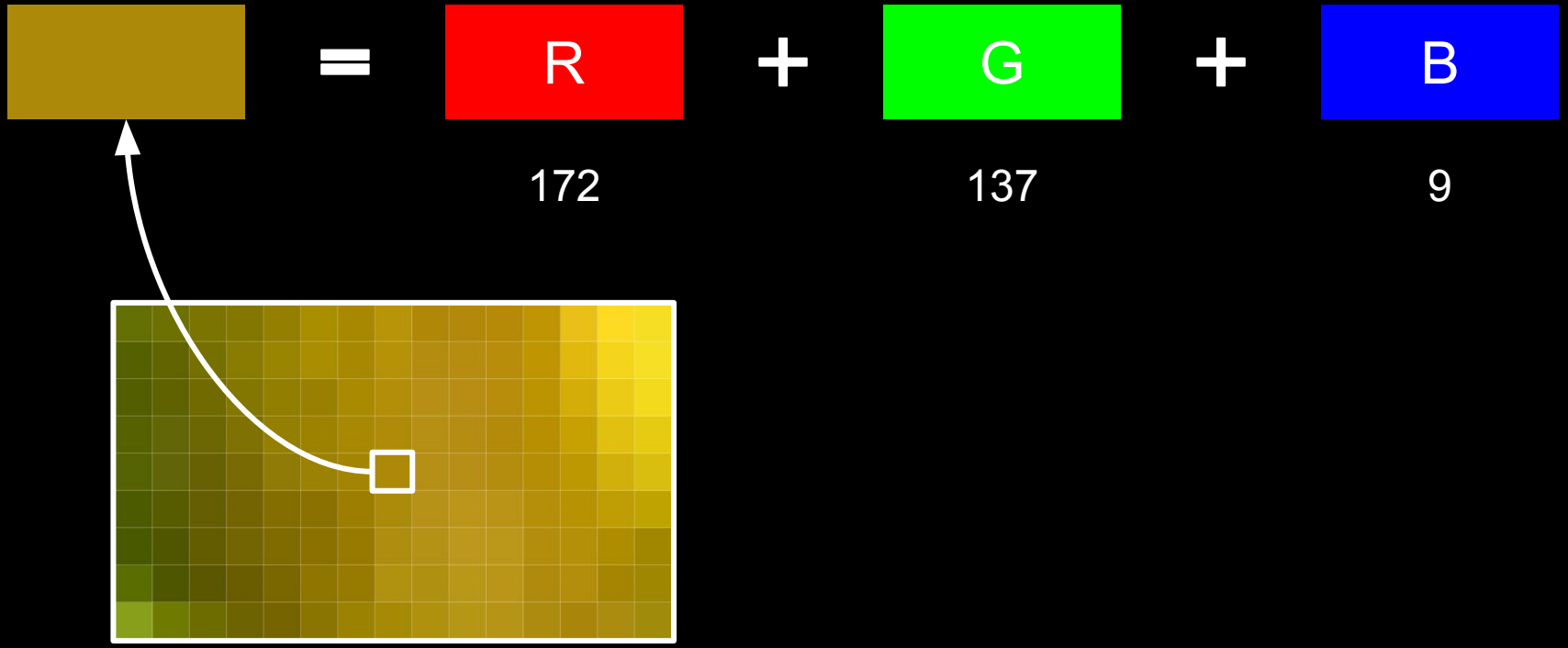


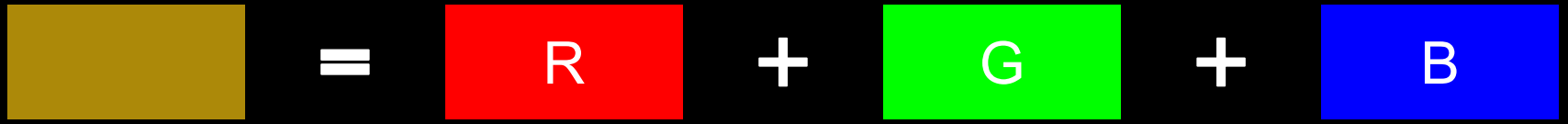




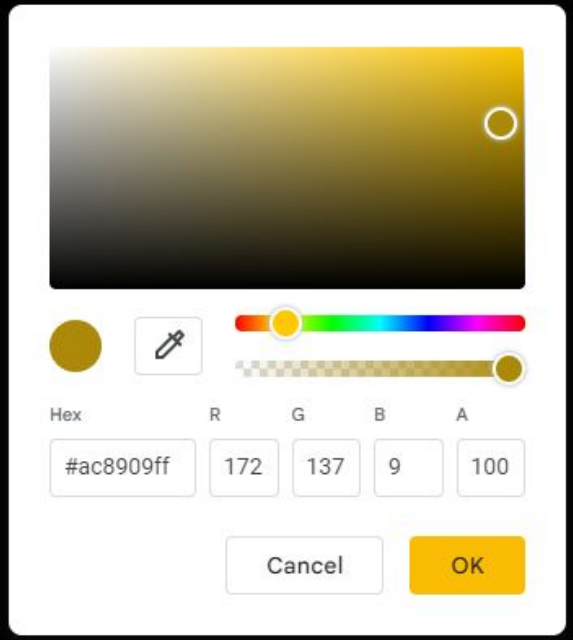



=
R
+
G
+
B


=
R
+
G
+
B
172
137
9



#AC8909


Hex
R
G
B
A
#ac8909ff
172
137
9
100
Cancel
OK

	172	137	9
#AC8909	AC	89	09

=

R

+

G

+

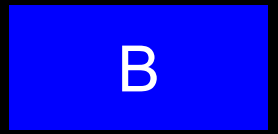
B

	172	137	9
#AC8909	AC	89	09
	10101100	01011001	00001001

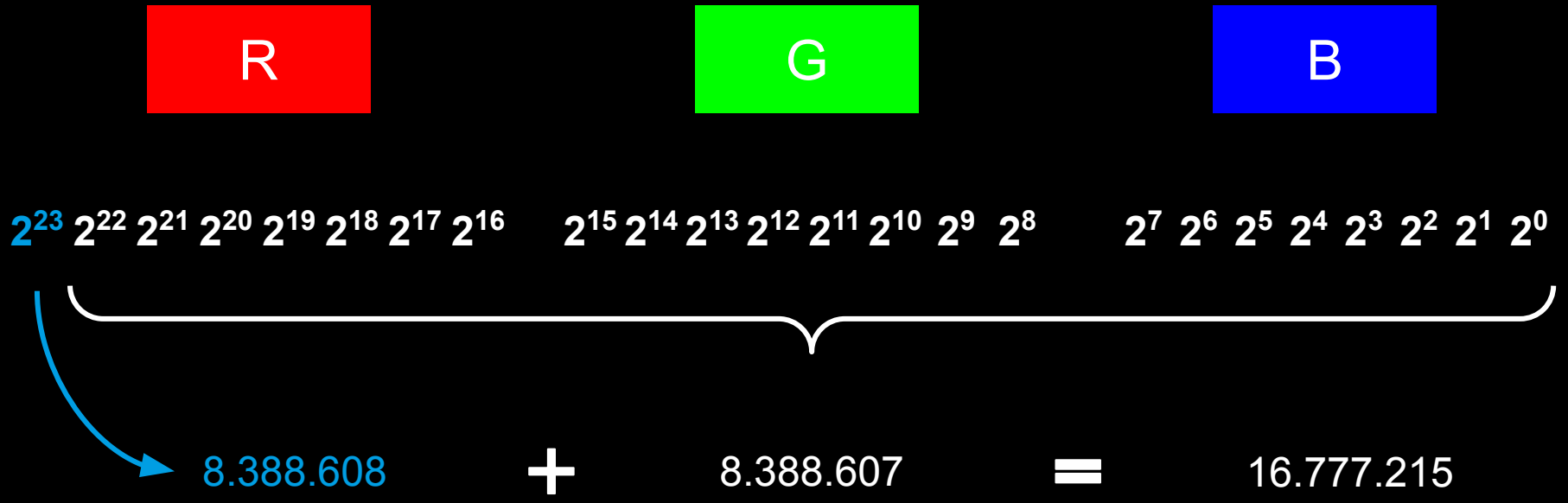
possible colors?

R

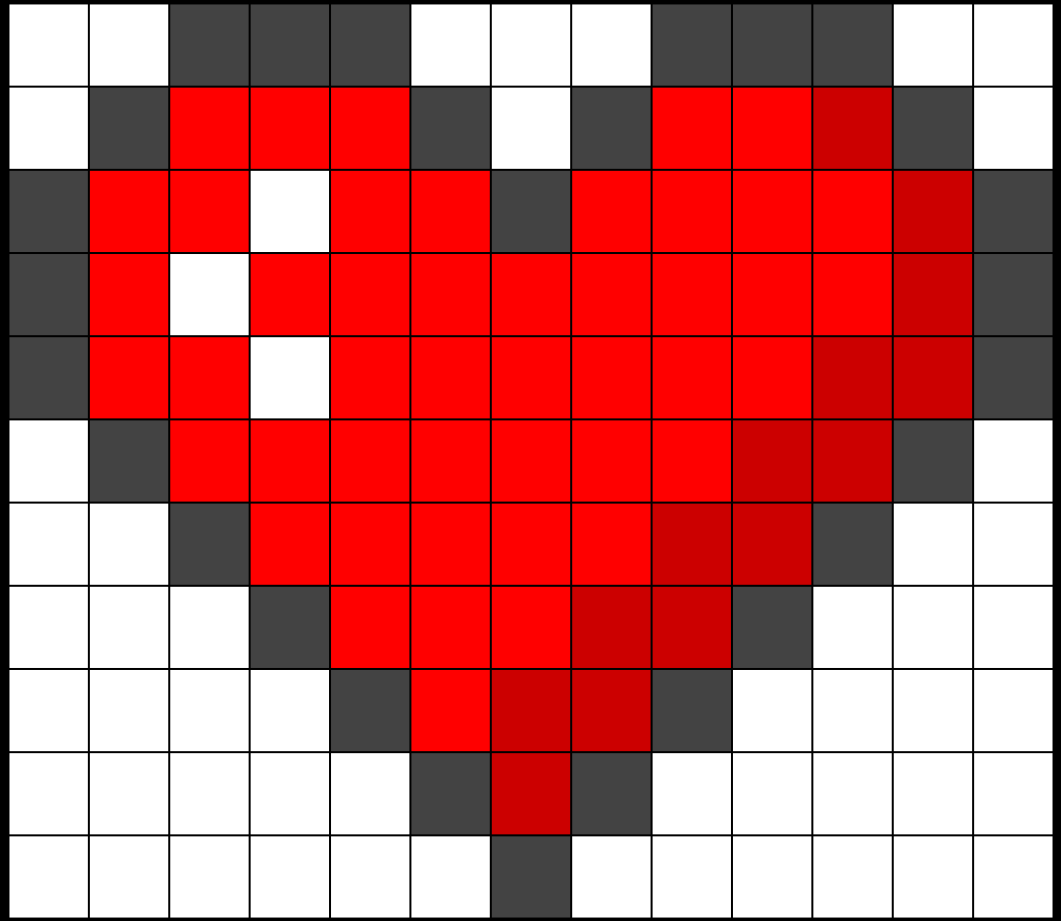
2^7 2^6 2^5 2^4 2^3 2^2 2^1 2^0



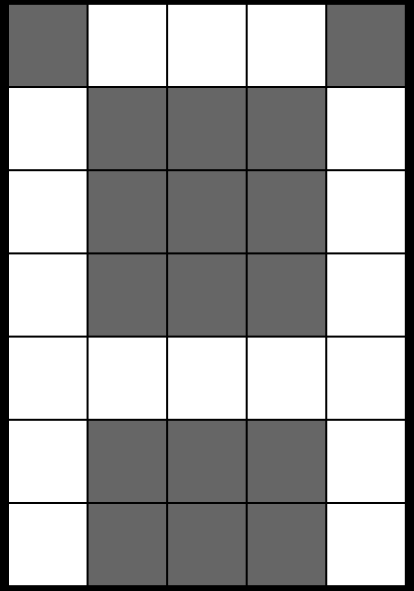
2^{23} 2^{22} 2^{21} 2^{20} 2^{19} 2^{18} 2^{17} 2^{16} 2^{15} 2^{14} 2^{13} 2^{12} 2^{11} 2^{10} 2^{9} 2^{8} 2^{7} 2^{6} 2^{5} 2^{4} 2^{3} 2^{2} 2^{1} 2^{0}

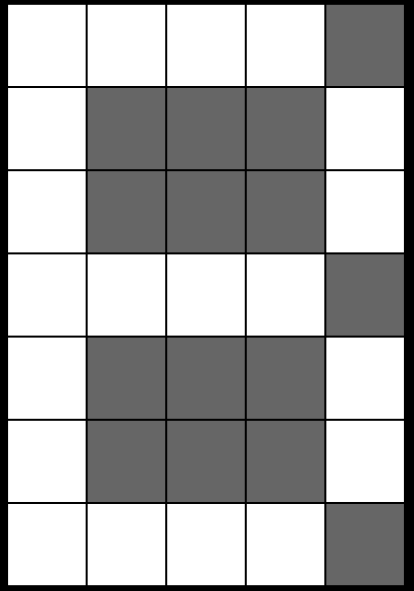

R
G
B
2^{23} 2^{22} 2^{21} 2^{20} 2^{19} 2^{18} 2^{17} 2^{16} 2^{15} 2^{14} 2^{13} 2^{12} 2^{11} 2^{10} 2^{9} 2^{8} 2^{7} 2^{6} 2^{5} 2^{4} 2^{3} 2^{2} 2^{1} 2^{0}
8.388.608
+
8.388.607
=
16.777.215

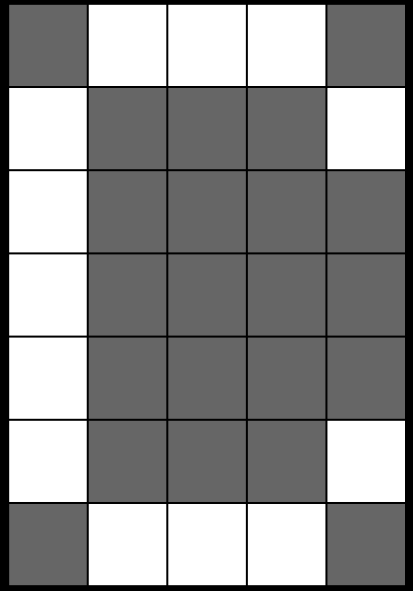

R
G
B
2^{23} 2^{22} 2^{21} 2^{20} 2^{19} 2^{18} 2^{17} 2^{16}
2^{15} 2^{14} 2^{13} 2^{12} 2^{11} 2^{10} 2^{9} 2^{8}
2^{7} 2^{6} 2^{5} 2^{4} 2^{3} 2^{2} 2^{1} 2^{0}
256
×
256
×
256

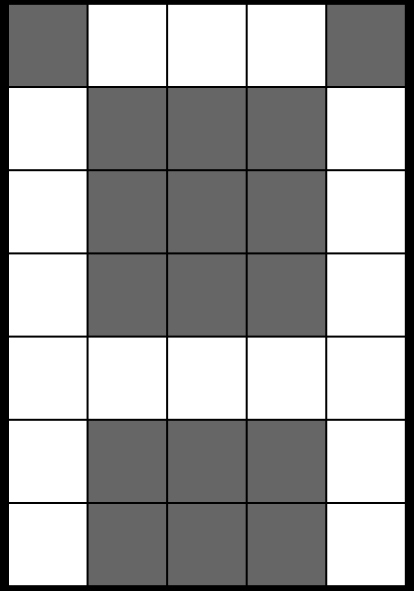


compression?

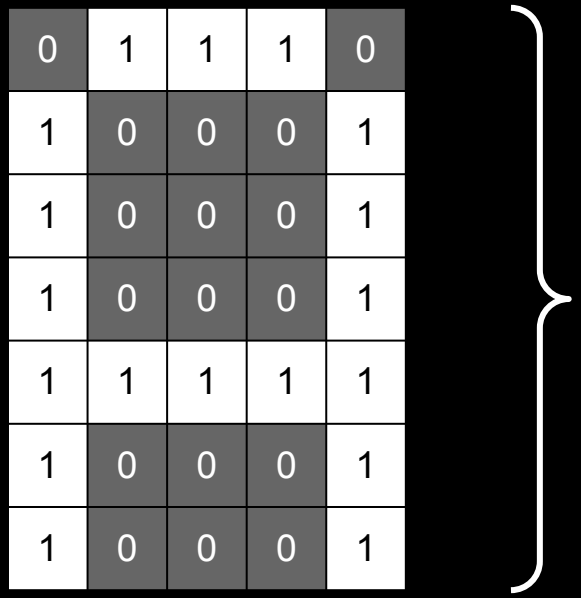








0	1	1	1	0
1	0	0	0	1
1	0	0	0	1
1	0	0	0	1
1	1	1	1	1
1	0	0	0	1
1	0	0	0	1



0 1 1 1 0 1 0 0 0 1 1 0 0 0 1 1 0 0
0 1 1 1 1 1 1 1 0 0 0 1 1 0 0 0 1

5

7

0	1	1	1	0
1	0	0	0	1
1	0	0	0	1
1	0	0	0	1
1	1	1	1	1
1	0	0	0	1
1	0	0	0	1

0 1 1 1 0 1 0 0 0 1 1 0 0 0 1 1 0 0
0 1 1 1 1 1 1 1 0 0 0 1 1 0 0 0 1

bitmap

vector graphics

grayscale

representing sound

telephone

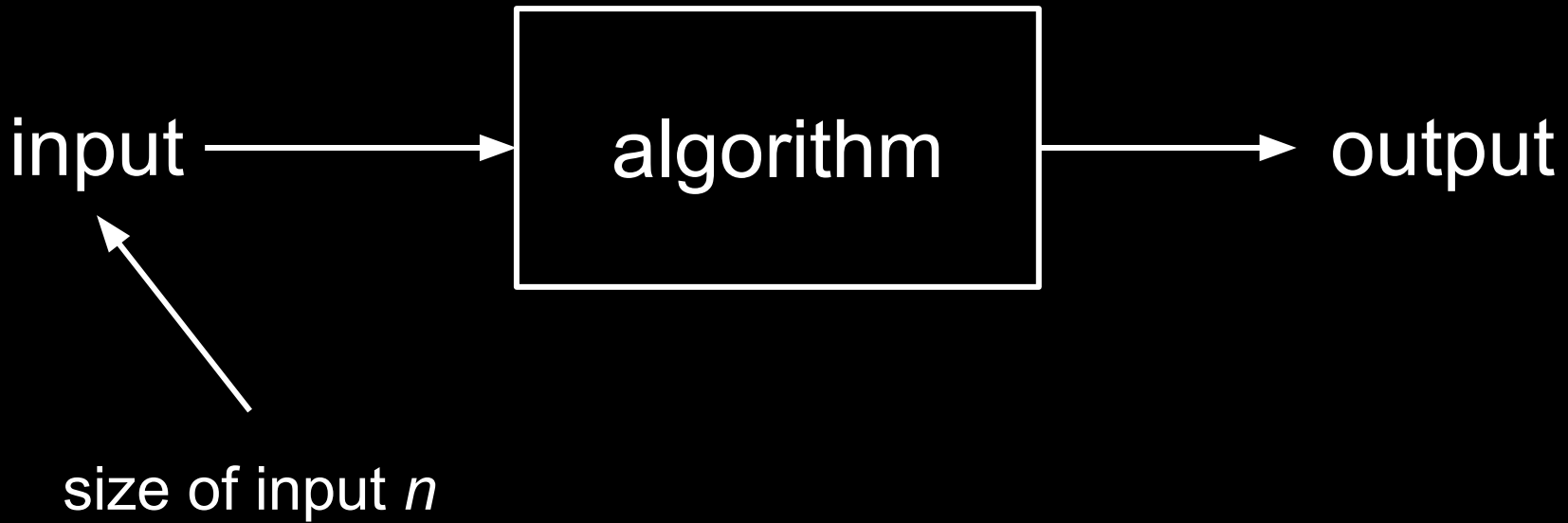
music

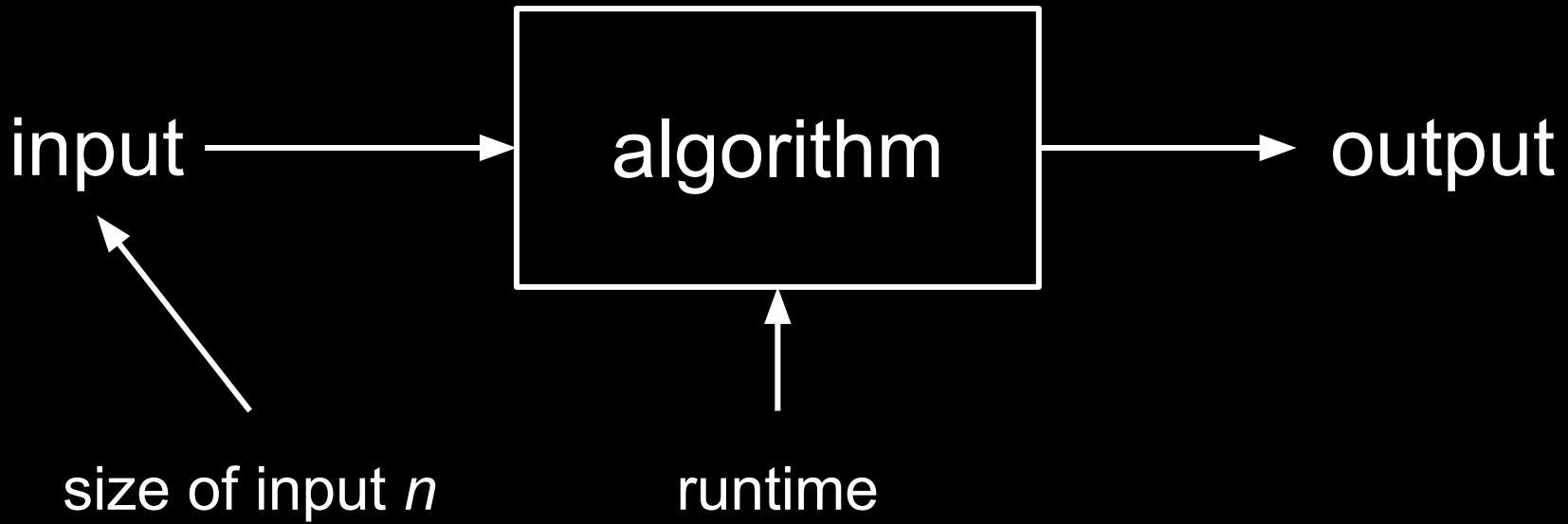
musical instrument digital interface (MIDI)

ALGORITHMS

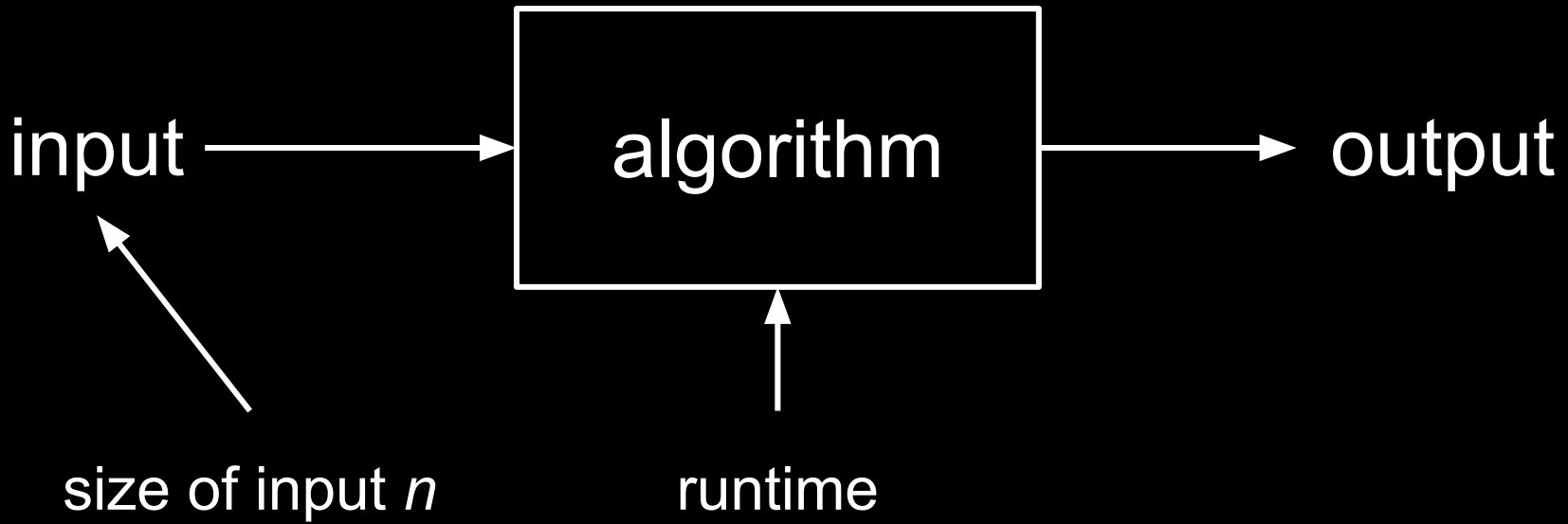
complexity


input
algorithm
output


input
algorithm
output
size of input *n*


input
algorithm
output
size of input *n*
runtime

O(*n*)



%%algorithms_size_runtime%%

COMPUTERS

ARITHMETIC

MEMORY

ANALOG VS. DIGITAL

which is better?