

0. MOTIVATION
1. PROBLEM SOLVING
2. INFORMATION
3. COUNTING
4. BITS
5. CODES
6. ALGORITHMS
7. COMPUTERS
8. ARITHMETIC
9. MEMORY
10. ANALOG VS. DIGITAL

The slides are meant as visual support for the lecture.
They are neither a documentation nor a script.

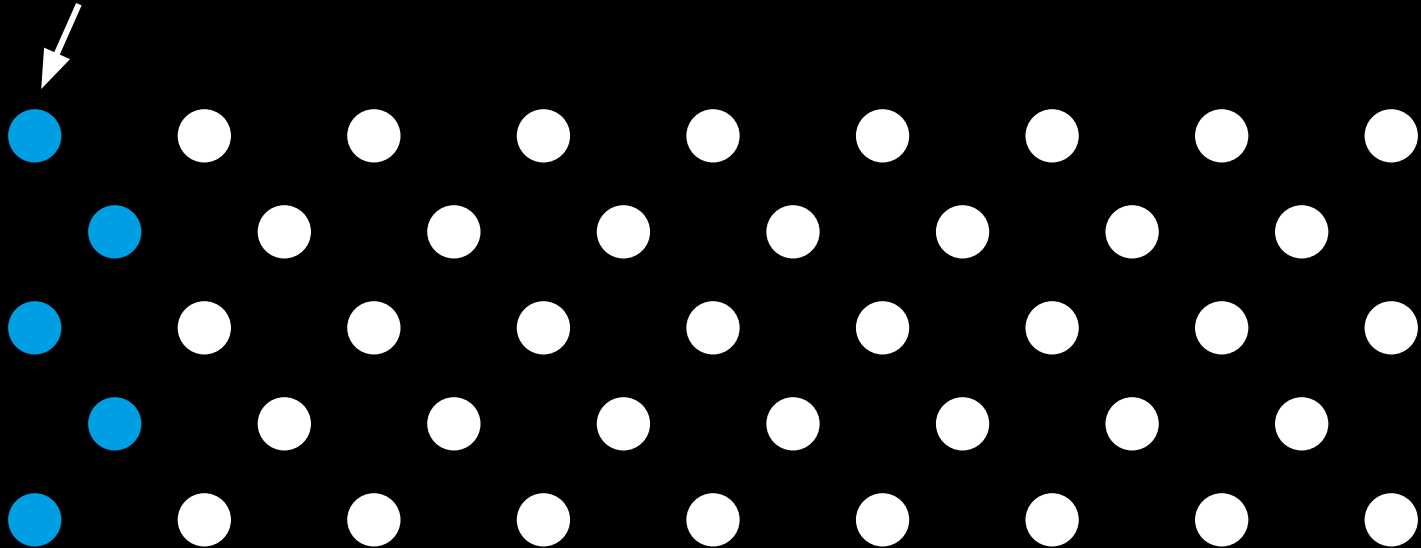
Please do not print the slides.

Comments and feedback at n.meseth@hs-osnabrueck.de

MOTIVATION

a few
experts

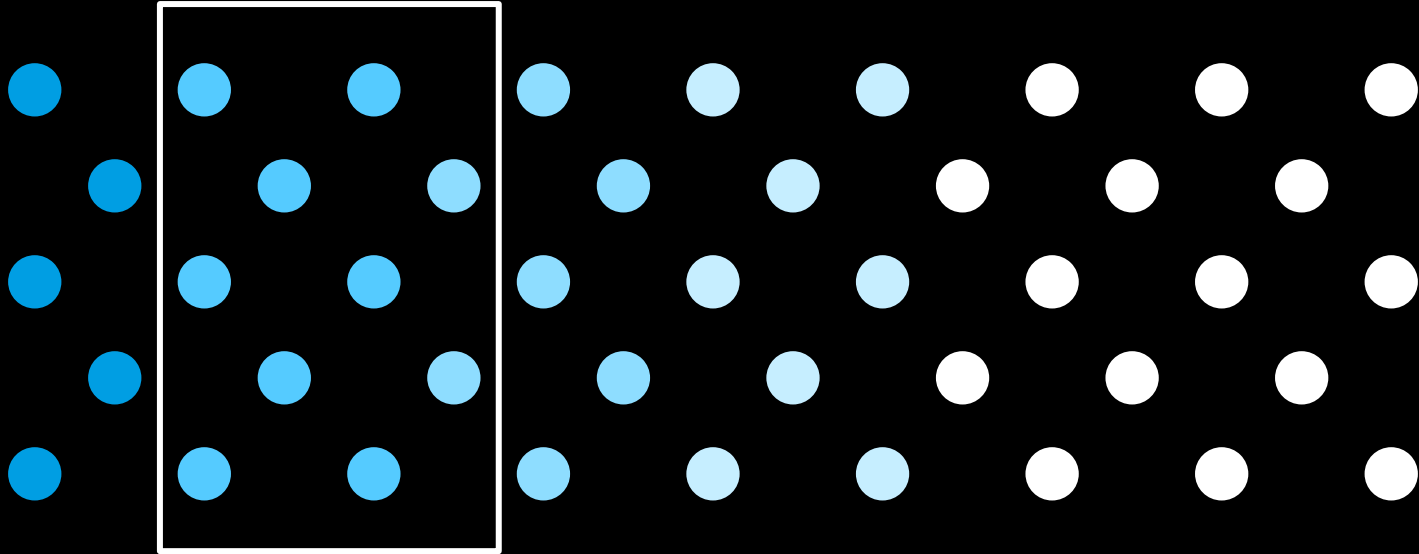
digitally uneducated
society



digitally illiterate society with a few experts

collective understanding

you?



society with a distributed and high degree of digital education

representation

representation

processing

representation

processing

programming

representation

processing

programming

digital fundamentals

data analysis

representation

processing

programming

digital fundamentals

data analysis

artificial
intelligence

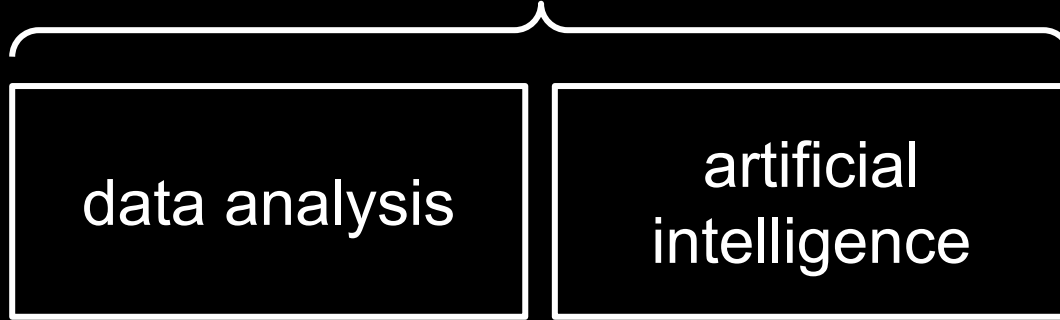
representation

processing

programming

digital fundamentals

digital applications



digital fundamentals

digital applications

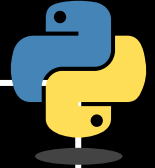
data analysis

artificial
intelligence

representation

processing

programming



digital fundamentals

digital applications

data analysis

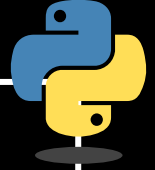
artificial
intelligence



representation

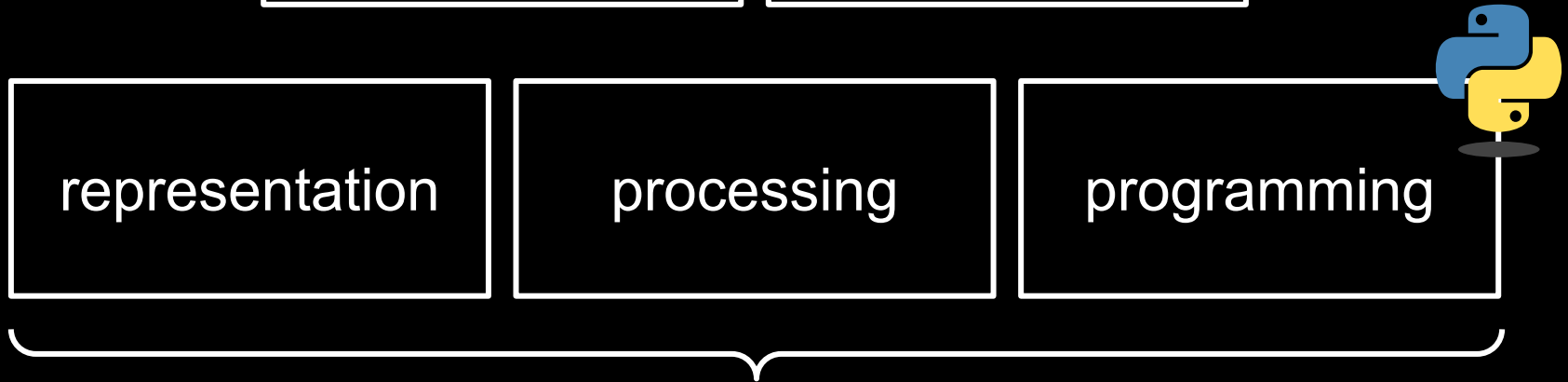
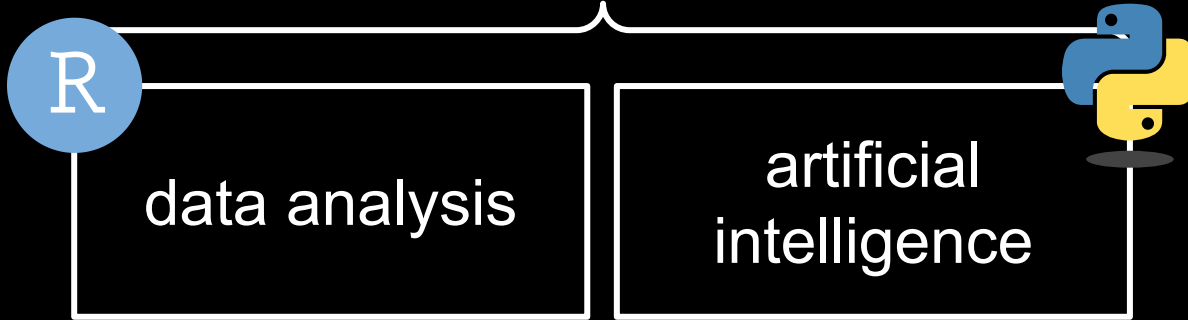
processing

programming



digital fundamentals

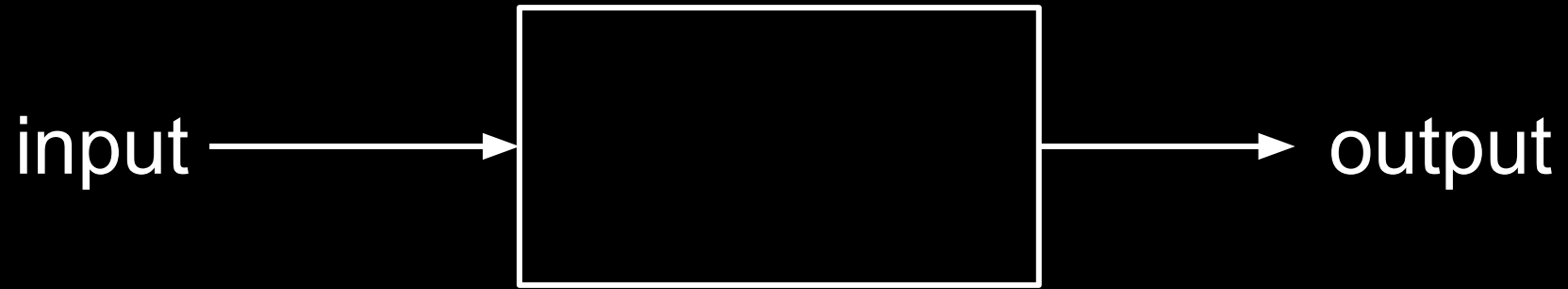
digital applications



digital fundamentals

PROBLEM SOLVING

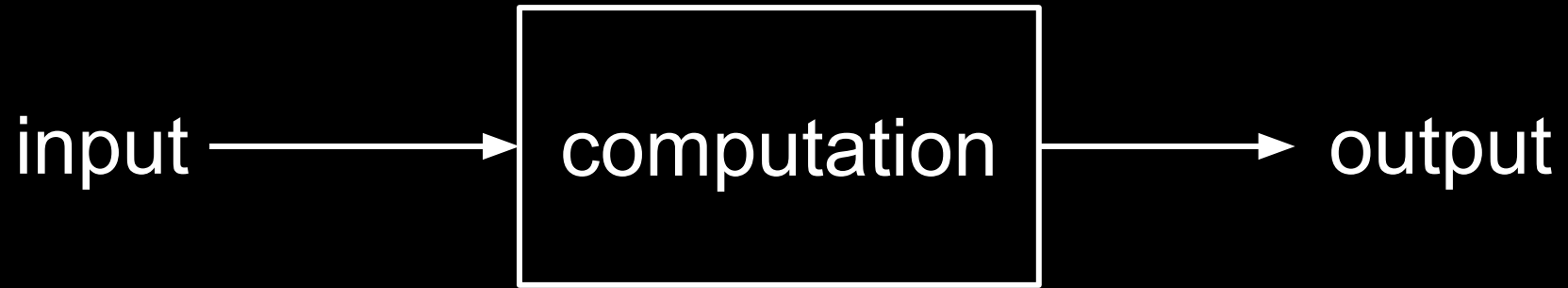
a model for solving problems

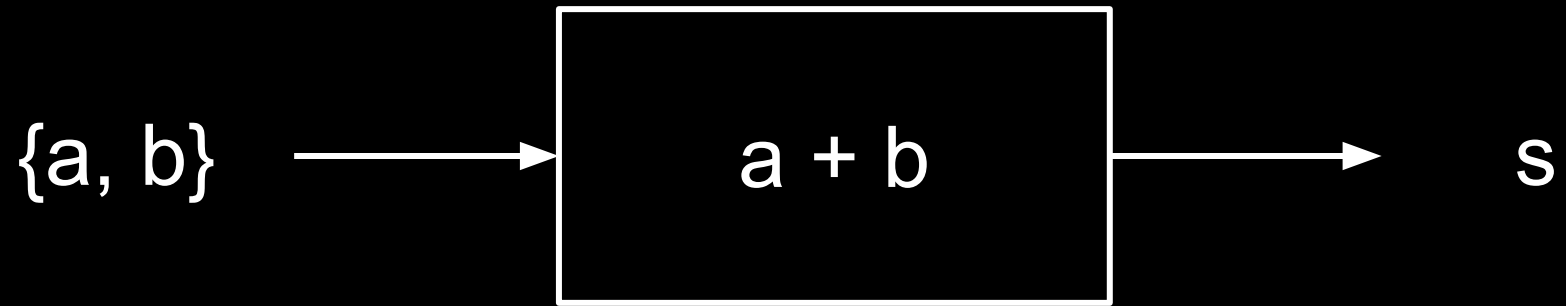


a model for solving problems

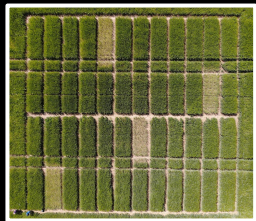


a model for solving problems







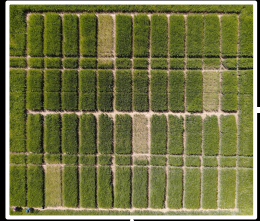


output



42

processing of
information



`count_plants()`

42

representation of
information





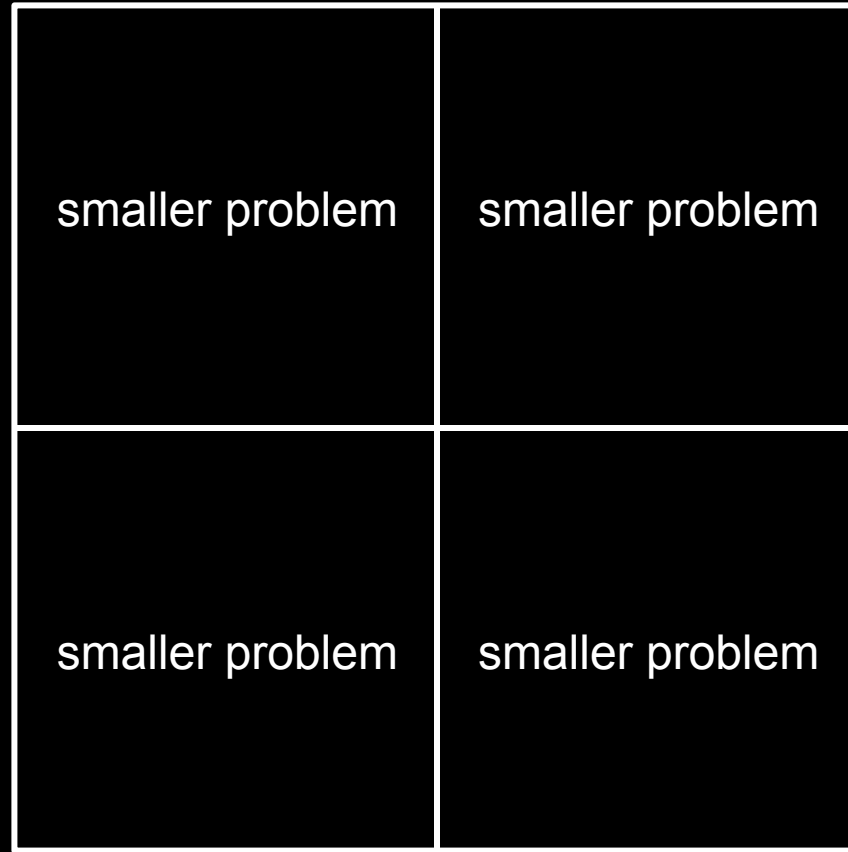
next_move()

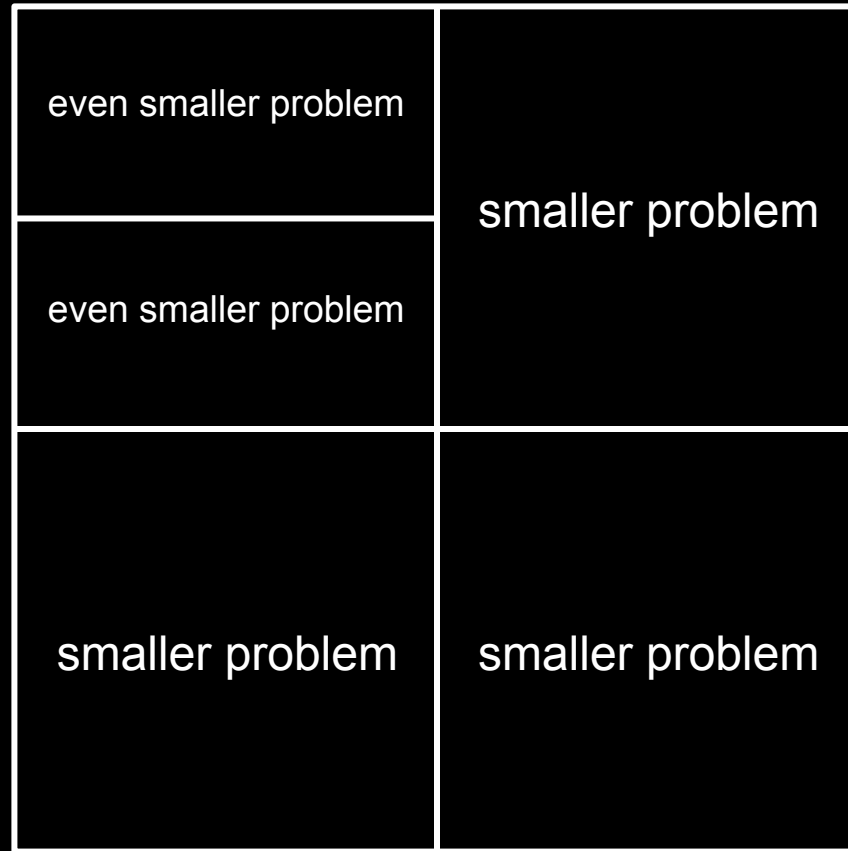
E2 → E4

problem solving strategies

divide and conquer

large and complex problem





sorted list +
element



search()



yes / no

is 67 a prime number?

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

linear search



2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

linear search



2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

linear search



2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

linear search



2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

linear search

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
~~43~~, 47, ~~53~~, 59, ~~61~~, 67, 71, 73, 79, 83, 89, 97
↑

19 steps... can't we do better?

2, 3, 5, 7, 11, ~~13~~, 17, ~~19~~, ~~23~~, ~~29~~, ~~31~~, ~~37~~, ~~41~~,
~~43~~, ~~47~~, ~~53~~, ~~59~~, ~~61~~, 67, 71, 73, 79, 83, 89, 97

↑

large and complex problem

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

large and complex
problem

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

smaller
problem

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41

smaller
problem

43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

binary search

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

binary search

67 != 41



2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, ~~41~~,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

binary search

67 > 41



2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, ~~41~~,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

binary search

67 > 41



2, 3, 5, 7, ~~11~~, ~~13~~, 17, 19, ~~23~~, ~~29~~, ~~31~~, ~~37~~, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

binary search

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97



67 != 71

binary search

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97



67 != 71

binary search

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97



$67 < 71$

binary search

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97



67 != 59

binary search

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
~~43~~, 47, ~~53~~, 59, 61, 67, ~~71~~, ~~73~~, 79, ~~83~~, ~~89~~, 97



67 > 59

binary search

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
~~43~~, ~~47~~, ~~53~~, ~~59~~, 61, 67, ~~71~~, ~~73~~, ~~79~~, ~~83~~, ~~89~~, 97



67 = 67

binary search

2, 3, 5, 7, 11, ~~13~~, 17, 19, ~~23~~, ~~29~~, 31, 37, 41,
~~43~~, 47, ~~53~~, 59, ~~61~~, 67, ~~71~~, ~~73~~, 79, ~~83~~, ~~89~~, 97



67 = 67

3 splits → much better

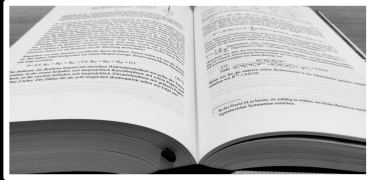
2, 3, 5, 7, 11, ~~13~~, 17, 19, ~~23~~, ~~29~~, 31, 37, 41,
~~43~~, 47, ~~53~~, 59, ~~61~~, 67, ~~71~~, ~~73~~, 79, ~~83~~, ~~89~~, 97



67 = 67



how efficient are linear and
binary search in general?



`count_words()`

word count

Die Mischung der Bonbons in jeder Tüte wird durch ein wahres Bayes-Modell beschrieben. Die Wahrscheinlichkeit, dass eine Tüte einen bestimmten Geschmack hat, ist gegeben durch die Wahrscheinlichkeit, dass sie aus einem bestimmten Packen stammt, multipliziert mit der Wahrscheinlichkeit, dass sie aus diesem Packen stammt.

Die Parameter des Modells sind:

- θ_i : Die Wahrscheinlichkeit, dass eine Tüte den Geschmack i hat.
- ϕ_j : Die Wahrscheinlichkeit, dass eine Tüte aus dem Packen j stammt.

Das Problem besteht darin, die Parameter θ_i und ϕ_j zu schätzen, wenn wir nur die Beobachtung eines Geschmacks pro Tüte haben. Eine naive Schätzung würde bedeuten, dass das Papier rot ist, weil es in einer roten Tüte war, was offensichtlich falsch ist.

Eine bessere Schätzung erhält man, indem man die Bonbons aus allen Tüten mischt und die relative Häufigkeit jedes Geschmacks berechnet. Dies ist äquivalent zur Maximum-Likelihood-Schätzung (MLE) für das Mischungsmodell.

Betrachten wir eine Iteration von EM für dieses Problem. Zuerst sehen wir uns die Daten an. Wir haben 1000 Stichproben aus einem Modell erzeugt, dessen tatsächliche Parameter $\theta = 0.5, \phi_1 = \phi_{W1} = \phi_{H1} = 0.8, \phi_2 = \phi_{W2} = \phi_{H2} = 0.3$.

Dies bedeutet, dass die Bonbons können mit derselben Wahrscheinlichkeit aus jeder der Tüten stammen; in der ersten befinden sich hauptsächlich Kirschbonbons mit rotem Papier und in der zweiten befinden sich hauptsächlich Zitronenbonbons mit grünem Papier ohne Löcher. Die Zähler für die acht möglichen Bonbonarten sehen wie folgt aus:

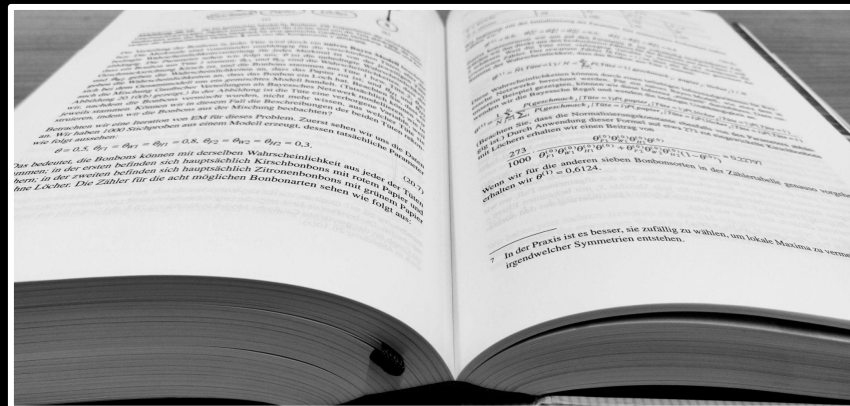
Bonbonart	Zähler
Kirsch (rot)	792
Zitrone (grün)	208
Kirsch (grün)	192
Zitrone (rot)	208

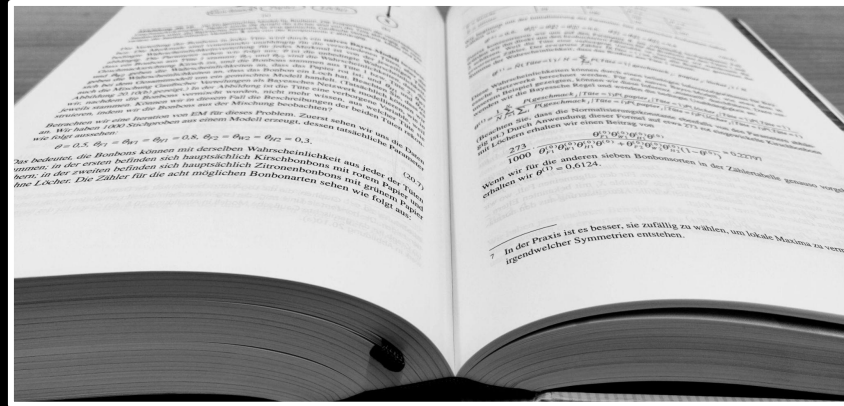
(20.3)

Wenn wir die anderen sieben Bonbonsorten in der Zählertabelle genauso vorgehen erhalten wir $\phi^{(1)} = 0.6124$.

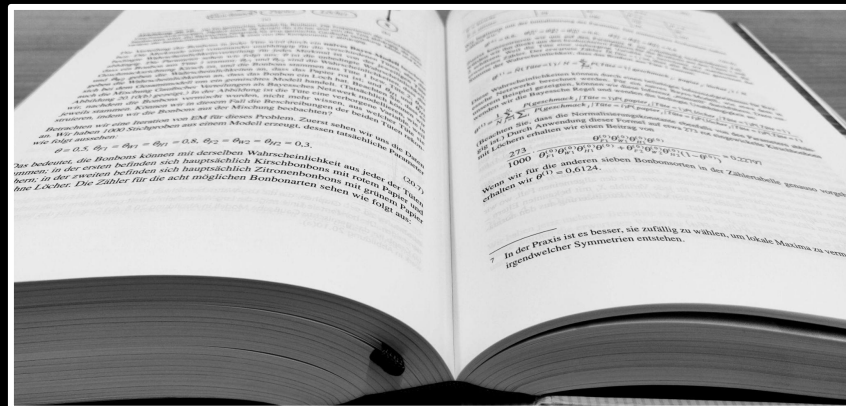
In der Praxis ist es besser, sie zufällig zu wählen, um lokale Maxima zu vermeiden, irgendwelcher Symmetrien entstehen.

strategies, anyone?





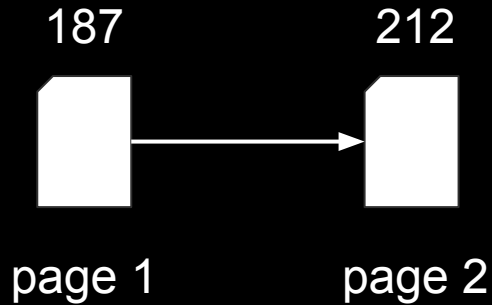
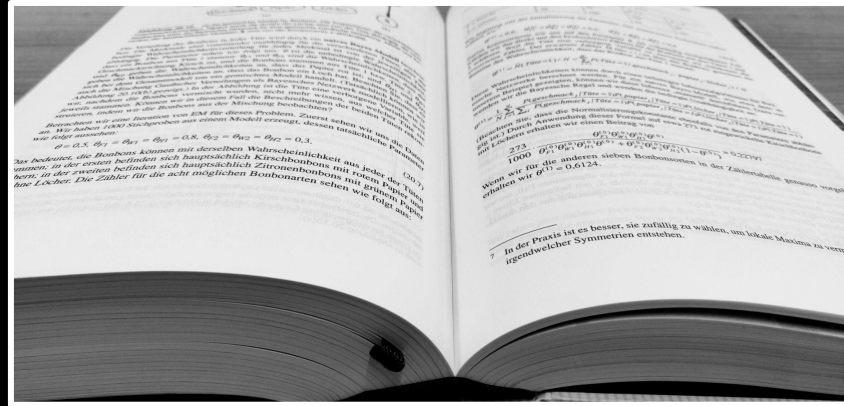
page 1

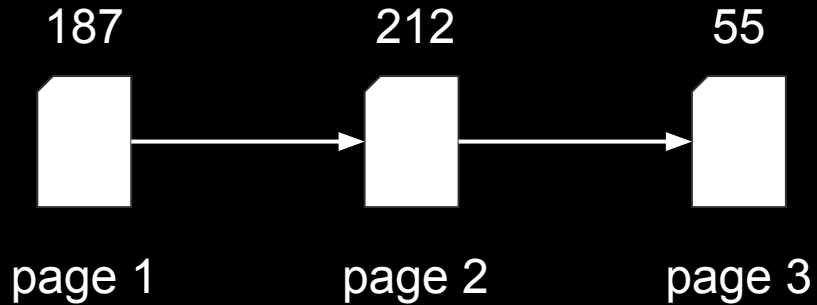


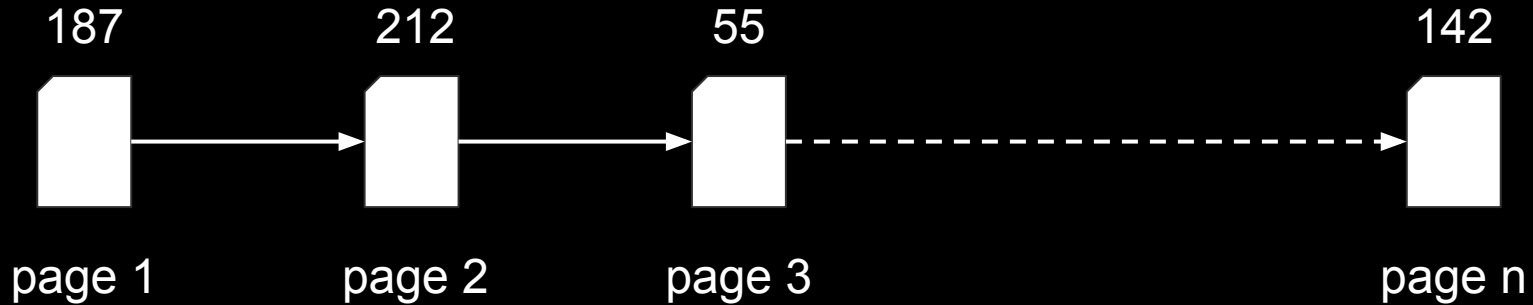
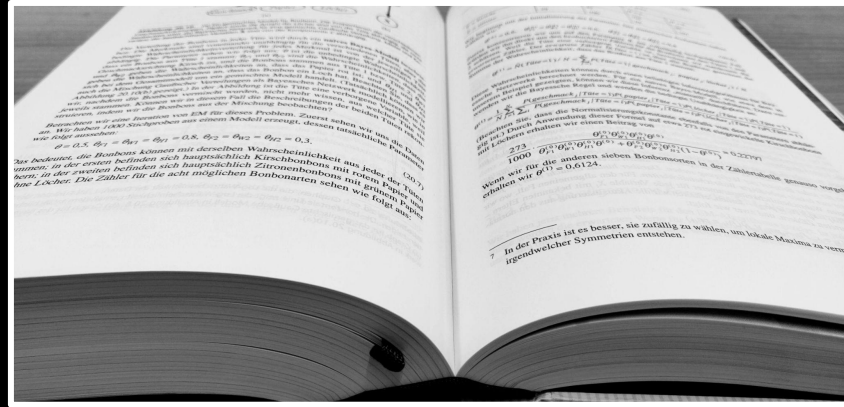
187



page 1









$n = 1327$ pages

\varnothing 2:23 minutes per page

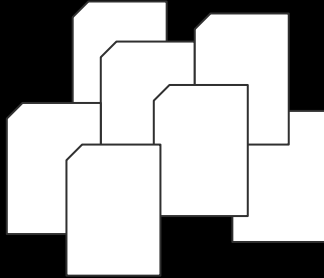
~ 52.34 hours

divide and conquer

+

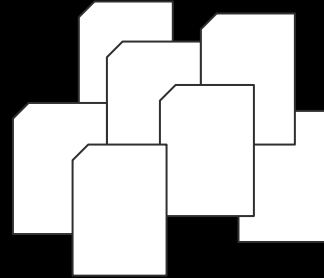
?

pages 1 - 700



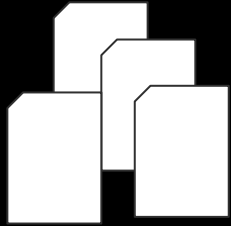
student 1

pages 701 - 1327



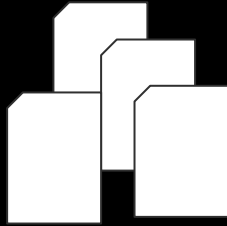
student 2

pages 1 - 350



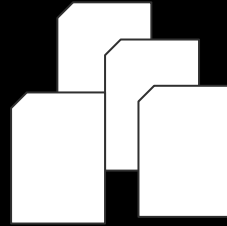
student 1

pages 351 - 700



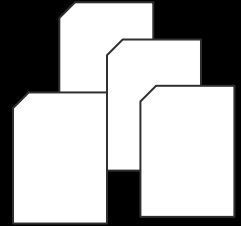
student 2

pages 701 - 1050



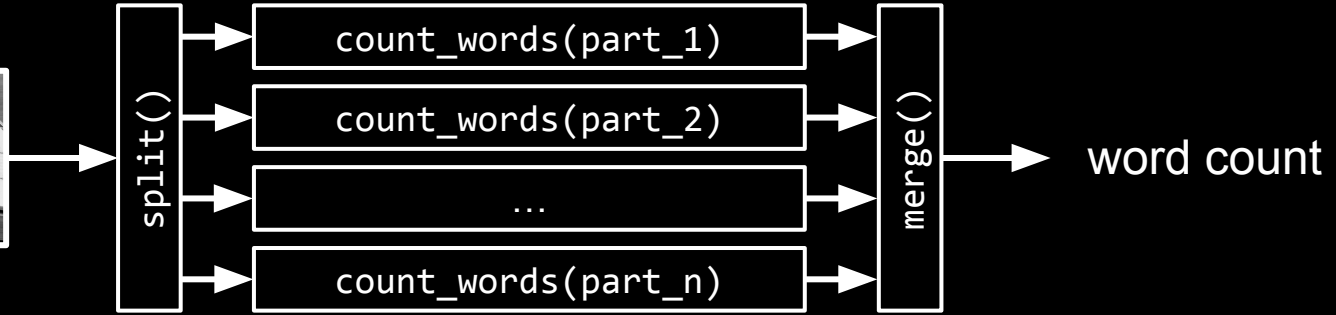
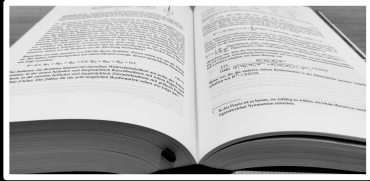
student 3

pages 1051- 1327



student 4

divide and conquer
+
distribution and parallelization



INFORMATION

“Information is that which allows you to make a
correct prediction with accuracy better than chance.”

Adami, Christoph. “What Is Information?” *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, vol. 374, no. 2063, Mar. 2016, p. 20150230, <https://doi.org/10.1098/rsta.2015.0230>.

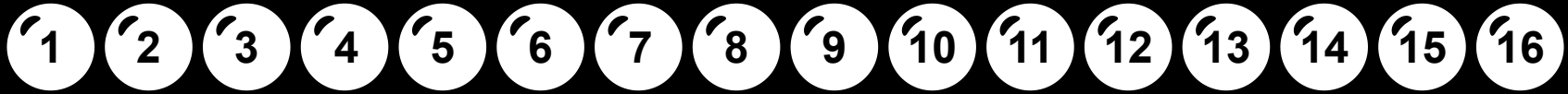
“Information is that which allows you to make a correct prediction with accuracy better than chance.”

Adami, Christoph. “What Is Information?” *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, vol. 374, no. 2063, Mar. 2016, p. 20150230, <https://doi.org/10.1098/rsta.2015.0230>.

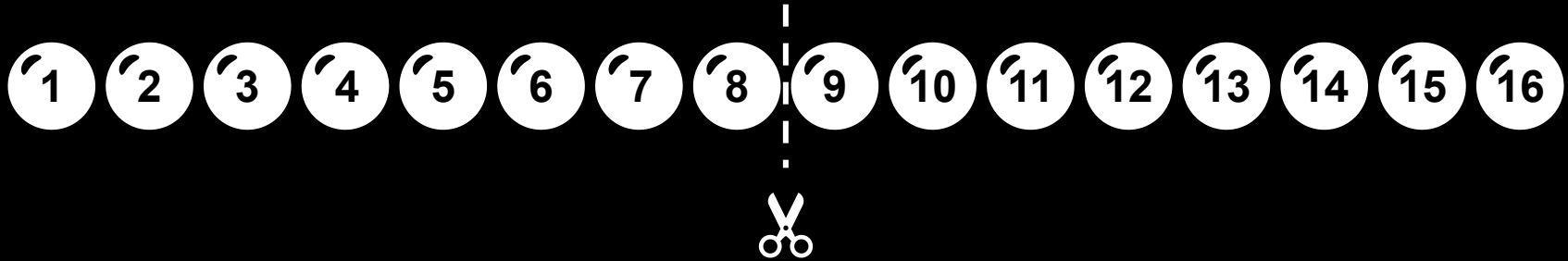
defining and measuring information

guess the number am I thinking of!

what is the most efficient approach?



is it > 8?

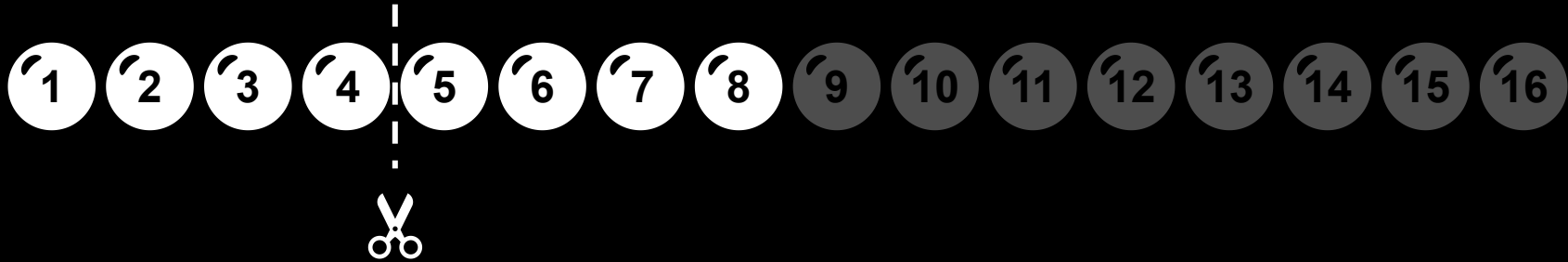


is it > 8? ✗



is it > 8? ✗

is it > 4?



is it > 8? ✗

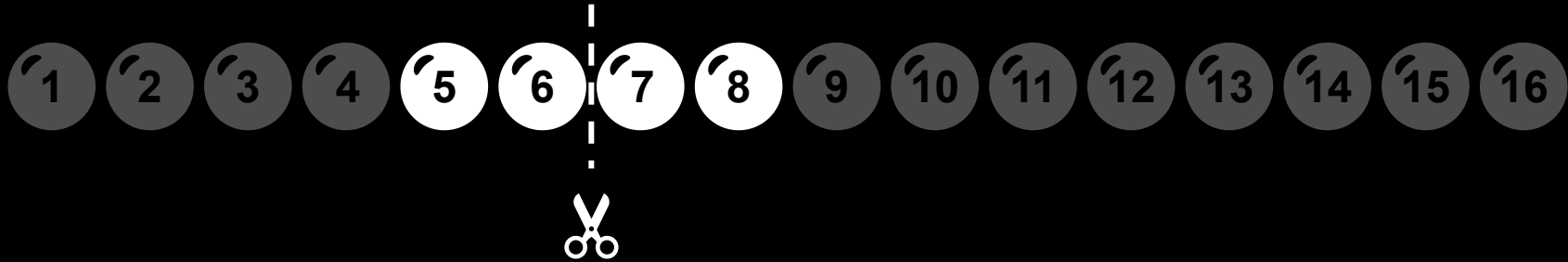
is it > 4? ✓



is it > 8? ✗

is it > 4? ✓

is it > 6?



is it > 8? ✗

is it > 4? ✓

is it > 6? ✓

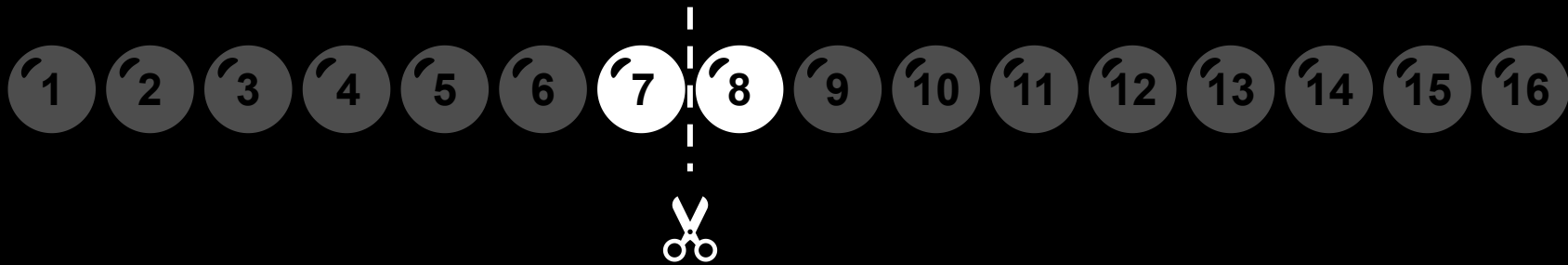


is it > 8? ✗

is it > 4? ✓

is it > 6? ✓

is it > 7?



is it > 8? ✗

is it > 4? ✓

is it > 6? ✓

is it > 7? ✗



is it > 8? ✗

is it > 4? ✓

is it > 6? ✓

is it > 7? ✗



with 4 questions from 16 to 1

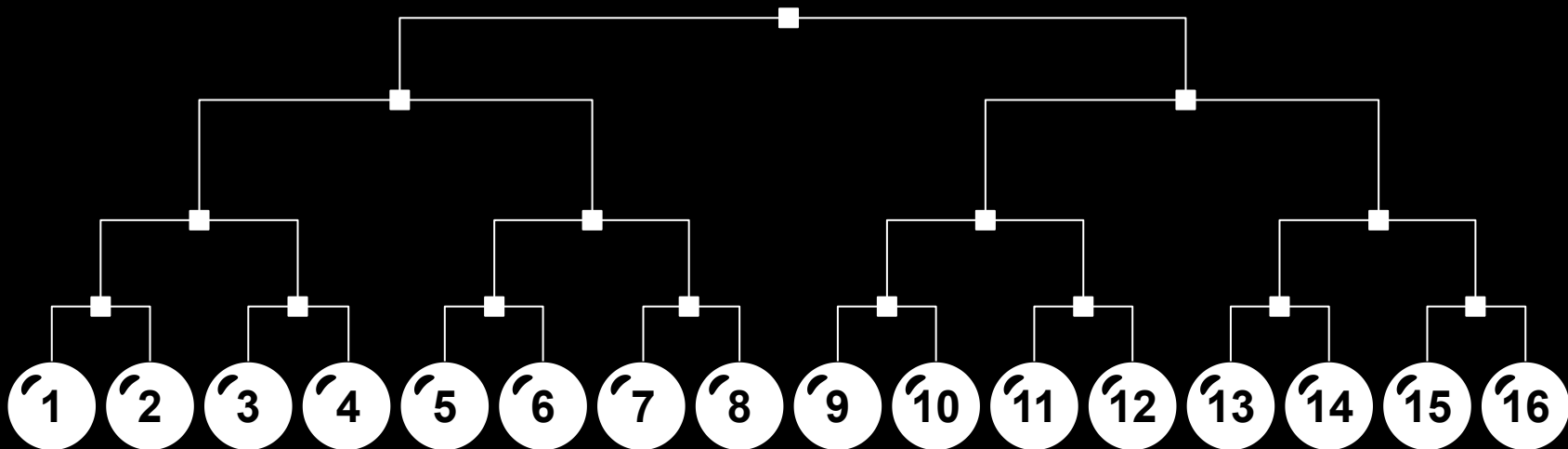
is it > 8?	✗	}	0
is it > 4?	✓		1
is it > 6?	✓		1
is it > 7?	✗		0



with 4 questions from 16 to 1

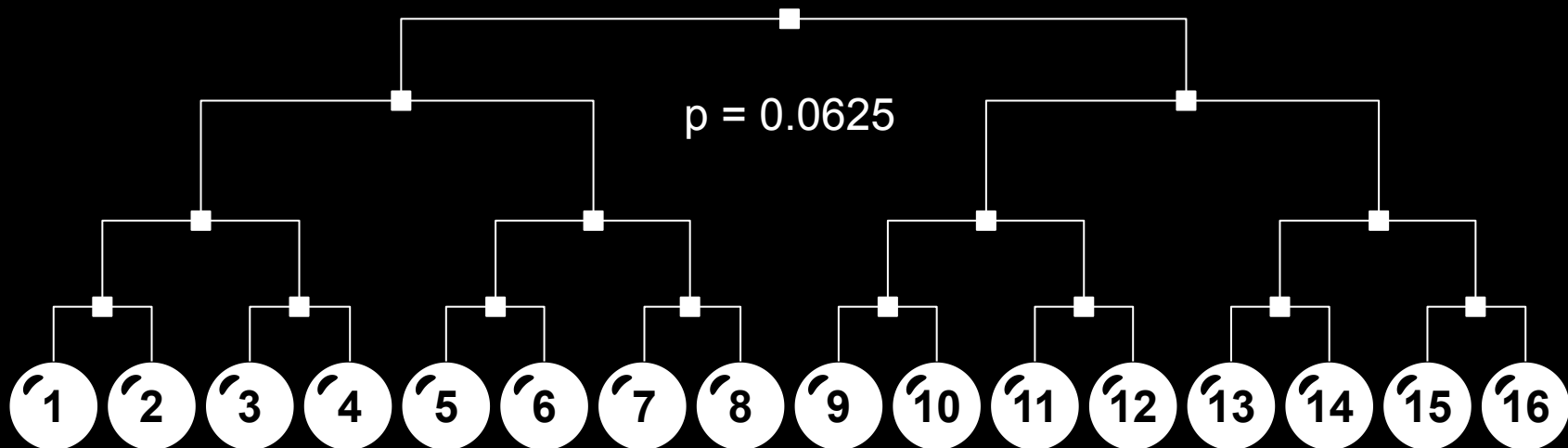
where is the information?

where is the information?



where is the information?

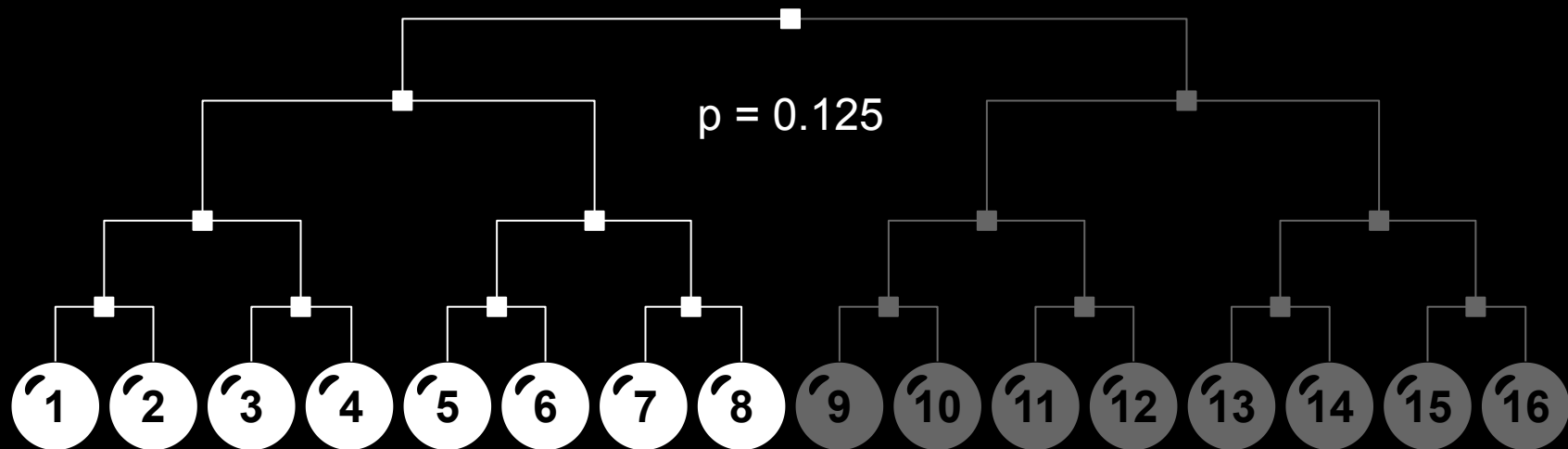
$N = 16$



where is the information?

~~N = 16~~

N = 8

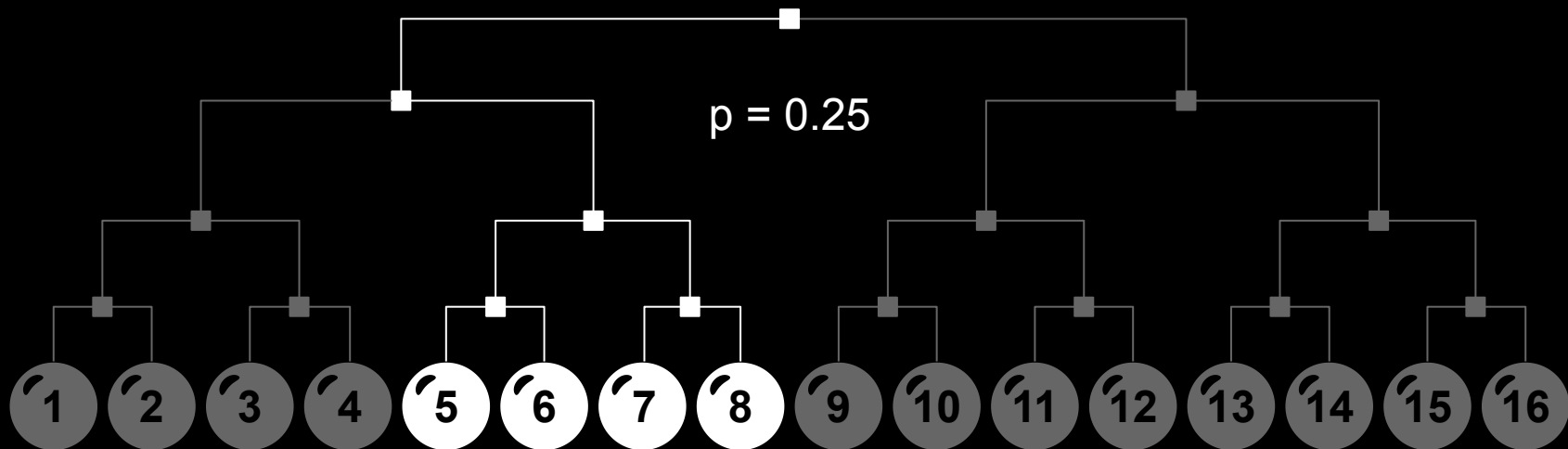


where is the information?

~~N = 16~~

~~N = 8~~

N = 4



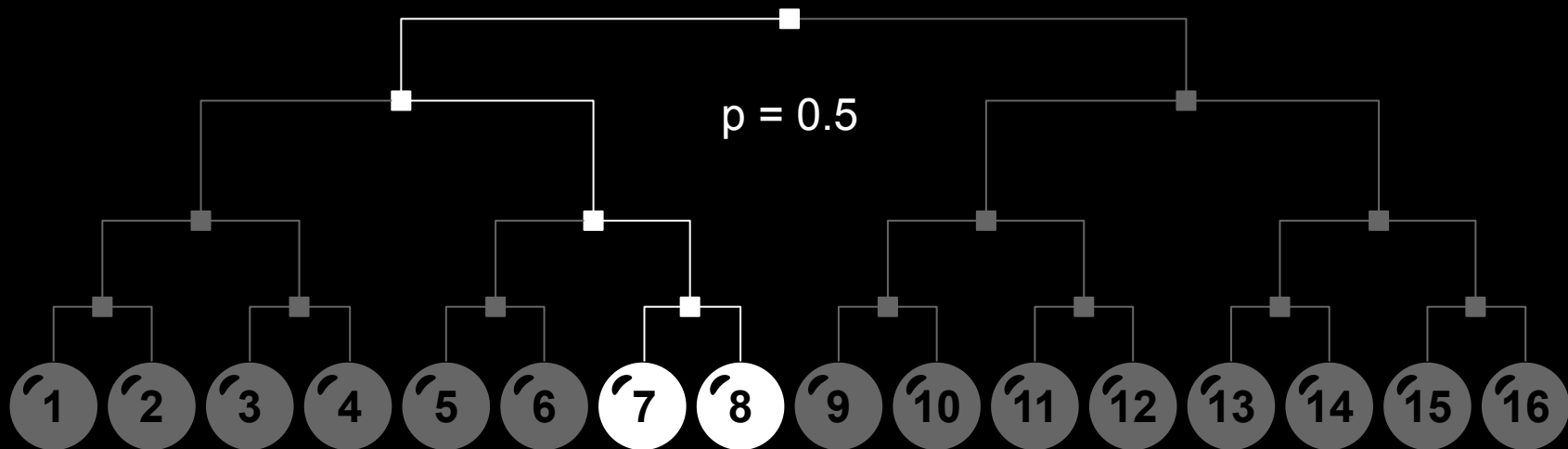
where is the information?

~~$N = 16$~~

~~$N = 8$~~

~~$N = 4$~~

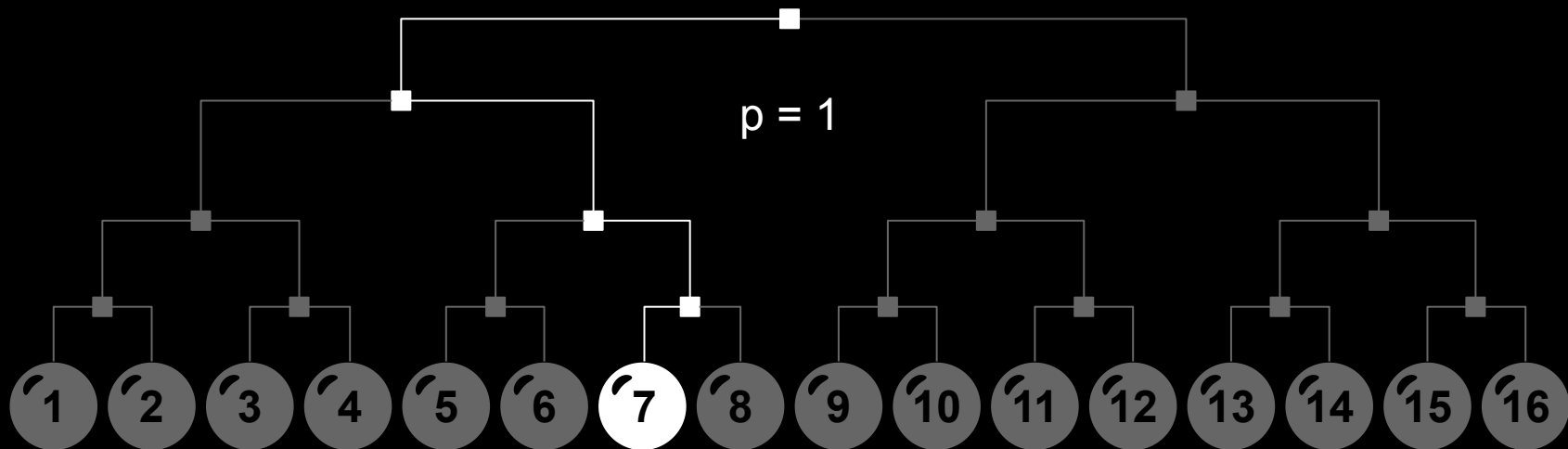
$N = 2$



where is the information?

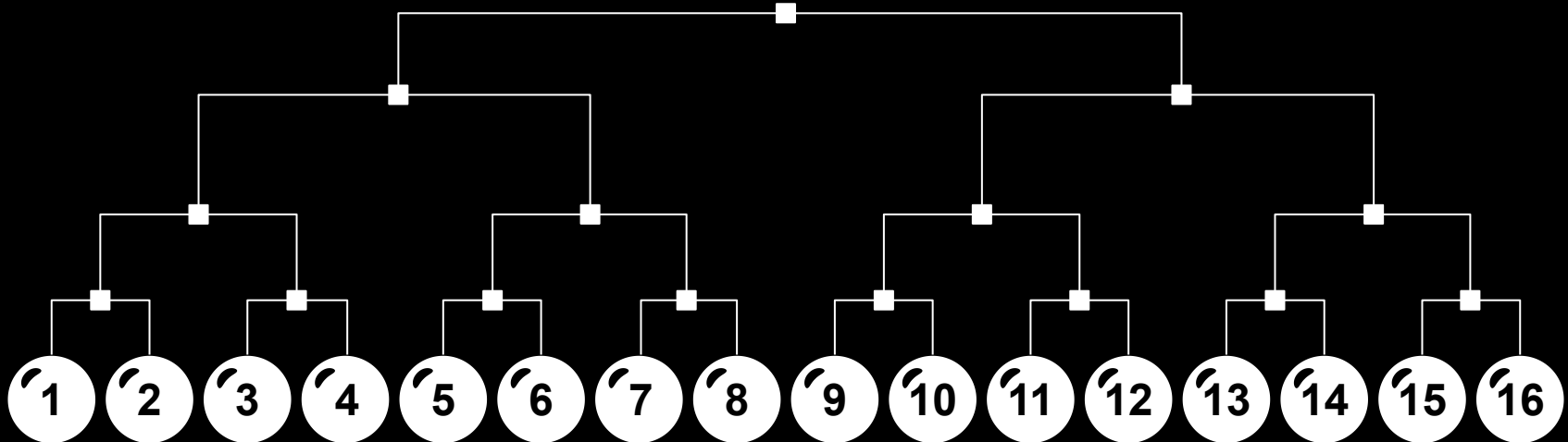
~~N = 16~~~~N=8~~~~N=4~~~~N=2~~

$N = 1$



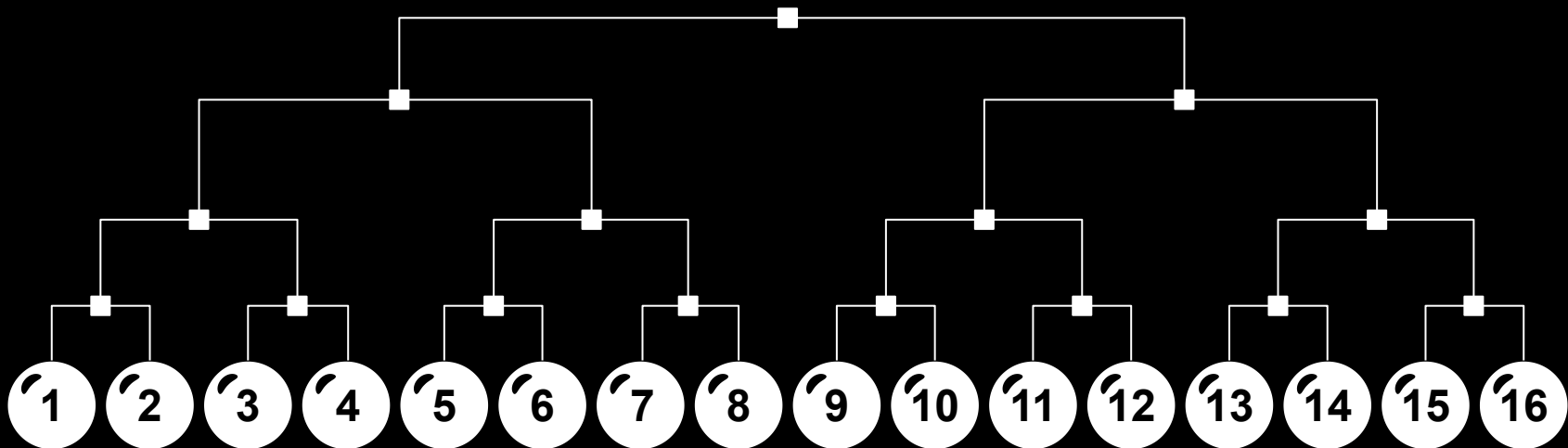
information = reduced uncertainty
uncertainty is measured with the logarithm of N

$$H = \log_2(N)$$

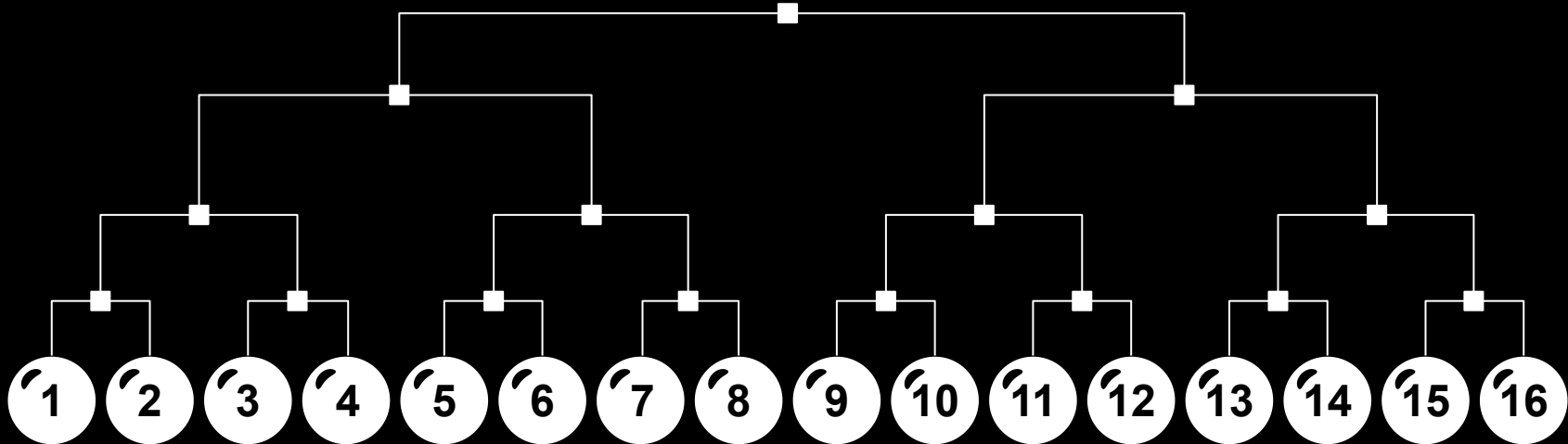


or: how often can we cut the remaining possibilities in half?

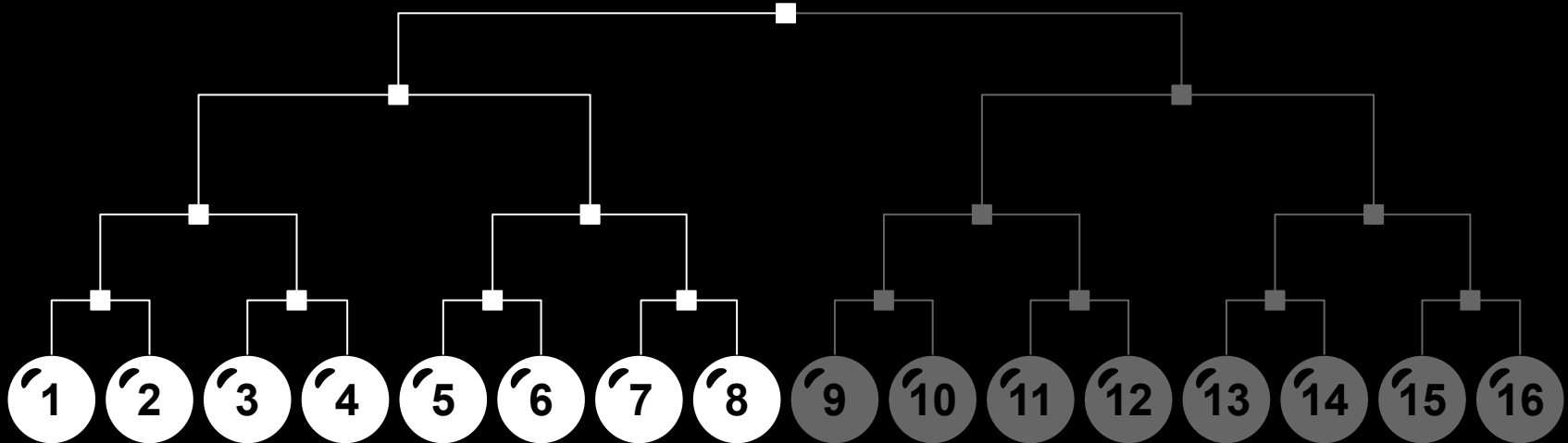
$$H = \log_2(N)$$



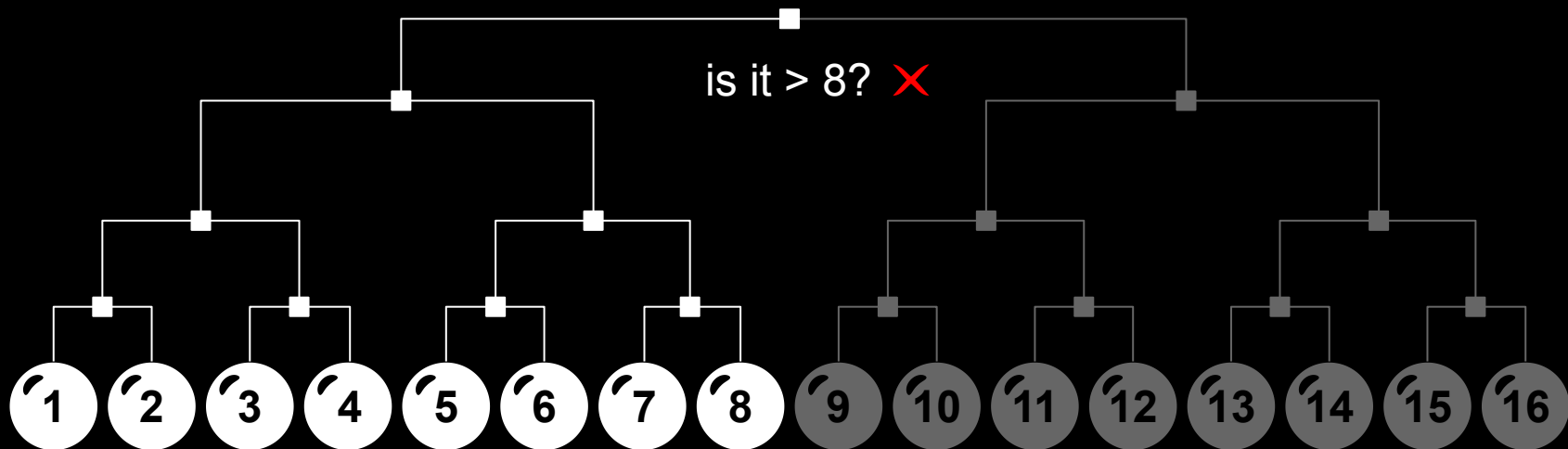
$$H_0 = \log_2(16) = 4$$



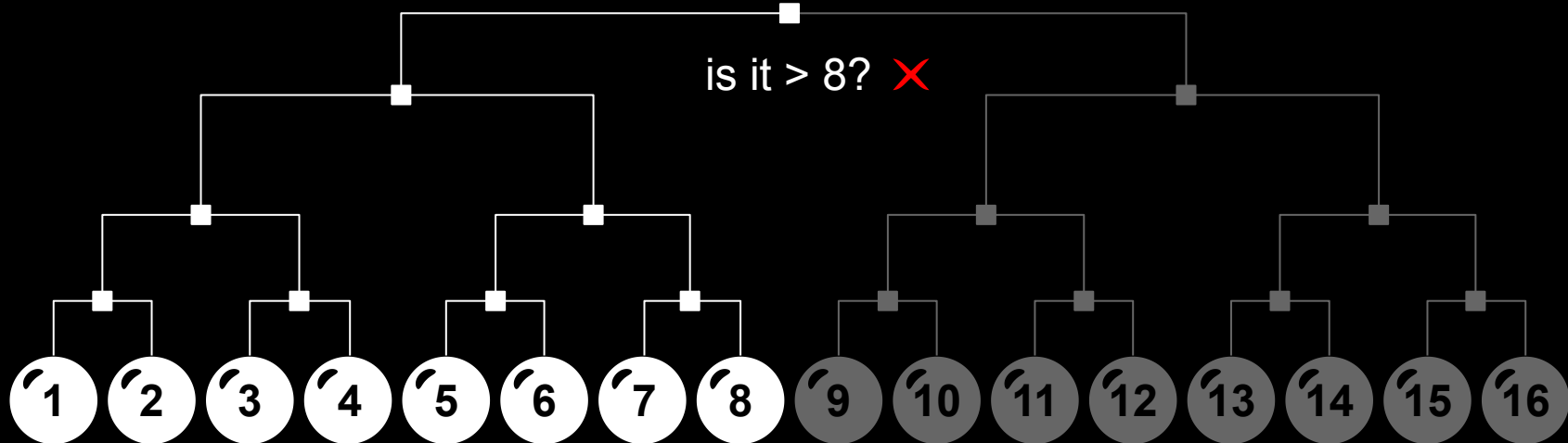
$$H_1 = \log_2(8) = 3$$



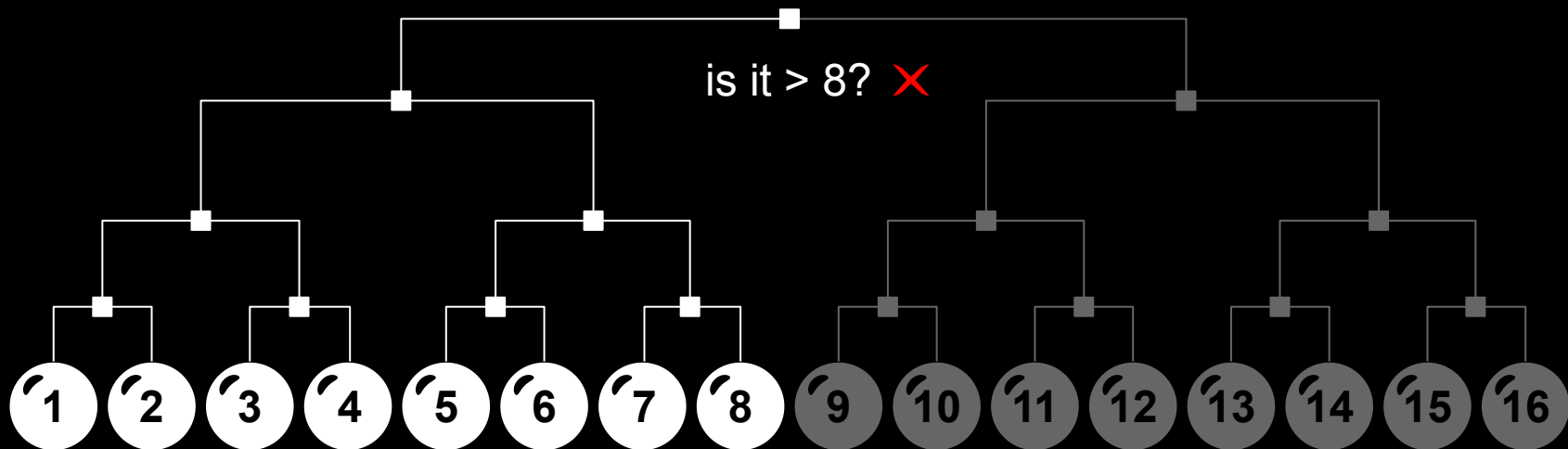
$$I = H_0 - H_1$$



$$I = \log_2(16) - \log_2(8)$$



$$I = 4 - 3 = 1$$



uncertainty and information are
measured in **bits**

how many yes/no questions to reduce
uncertainty to zero?

$$H = 0 = \log_2(1)$$

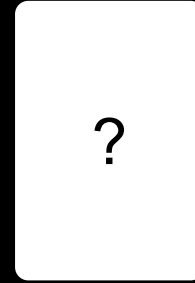
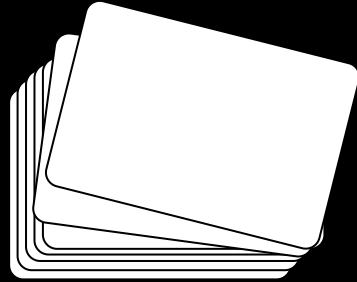
how many yes/no questions to reduce
uncertainty to zero?

$$H = 0 = \log_2(1)$$





















































$$H = \log_2(N)$$

poker

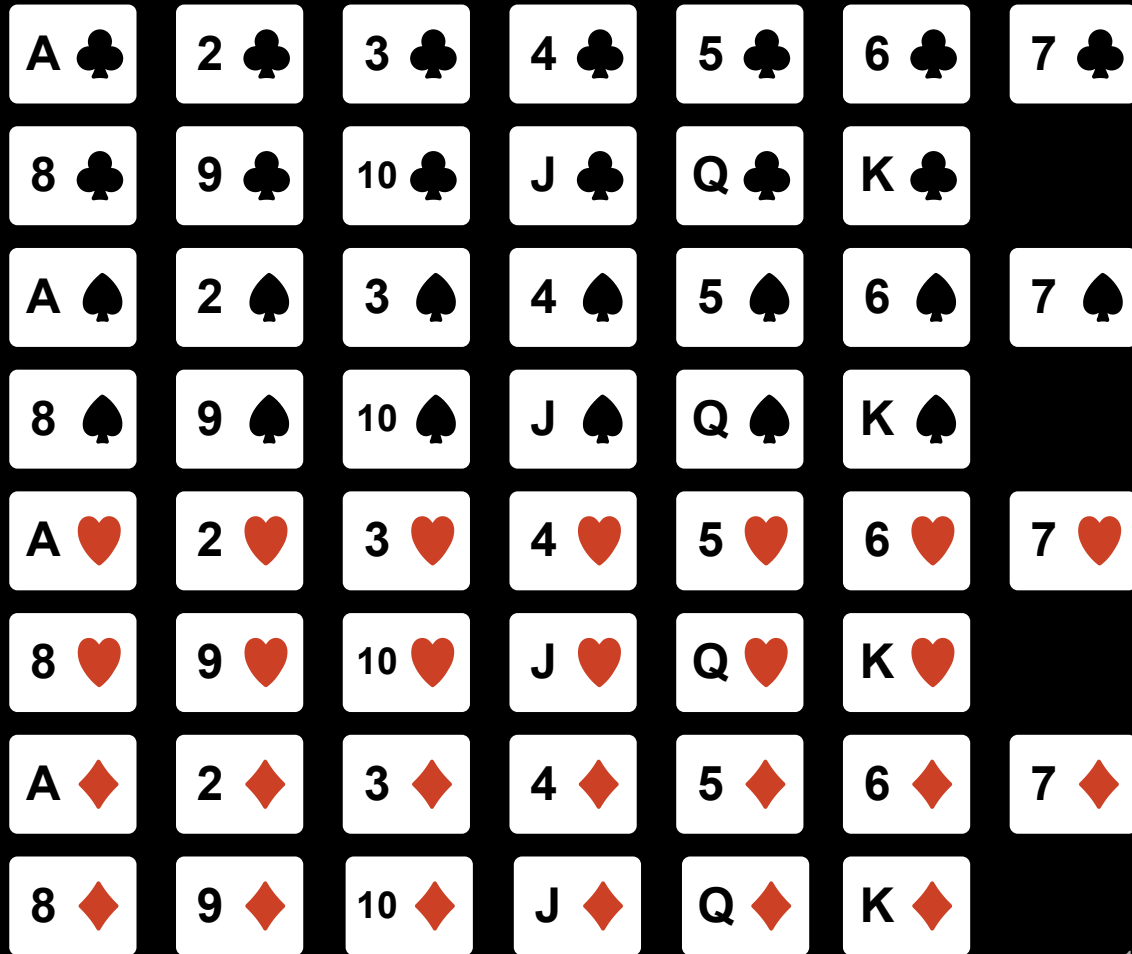
which card am I holding?



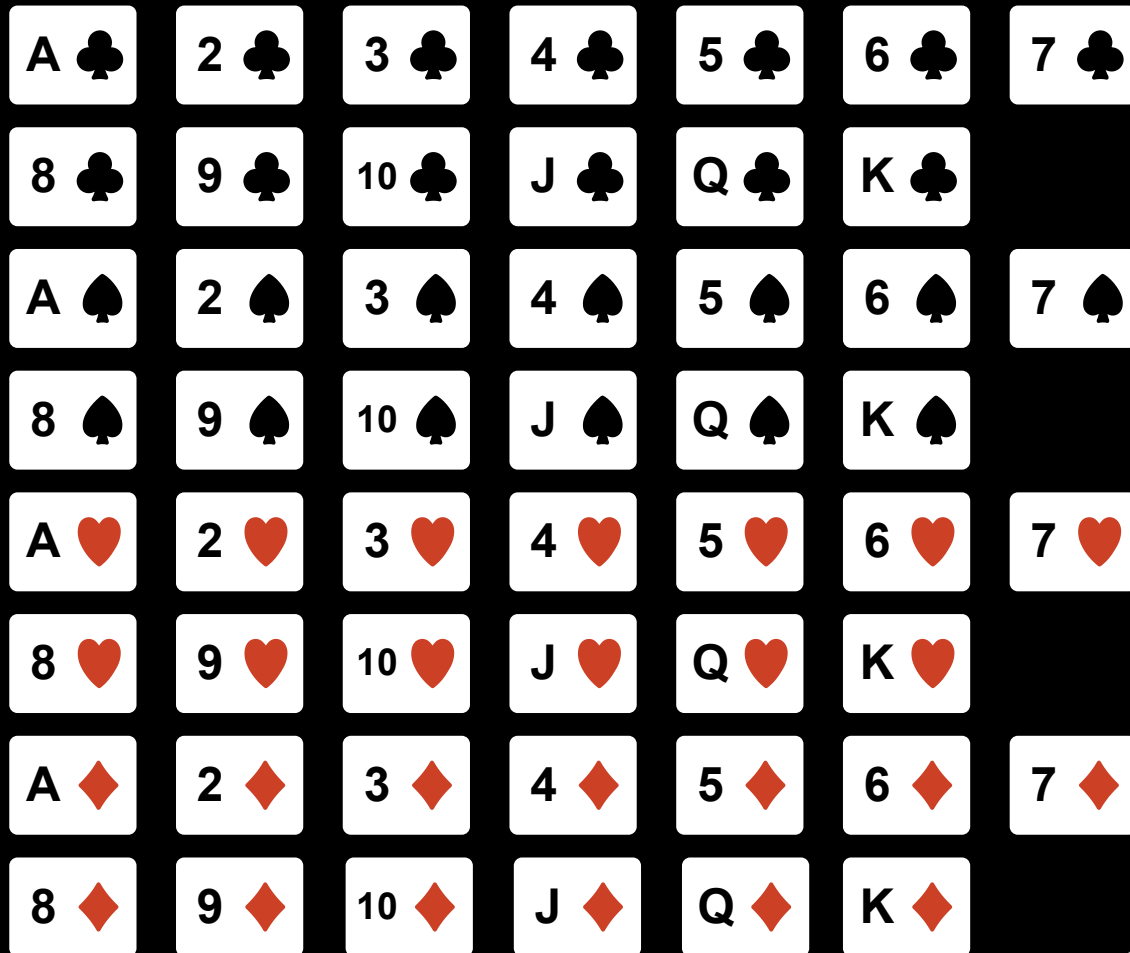
52 card poker deck

A 	2 	3 	4 	5 	6 	7 
8 	9 	10 	J 	Q 	K 	
A 	2 	3 	4 	5 	6 	7 
8 	9 	10 	J 	Q 	K 	
A 	2 	3 	4 	5 	6 	7 
8 	9 	10 	J 	Q 	K 	
A 	2 	3 	4 	5 	6 	7 
8 	9 	10 	J 	Q 	K 	

52 possible cards

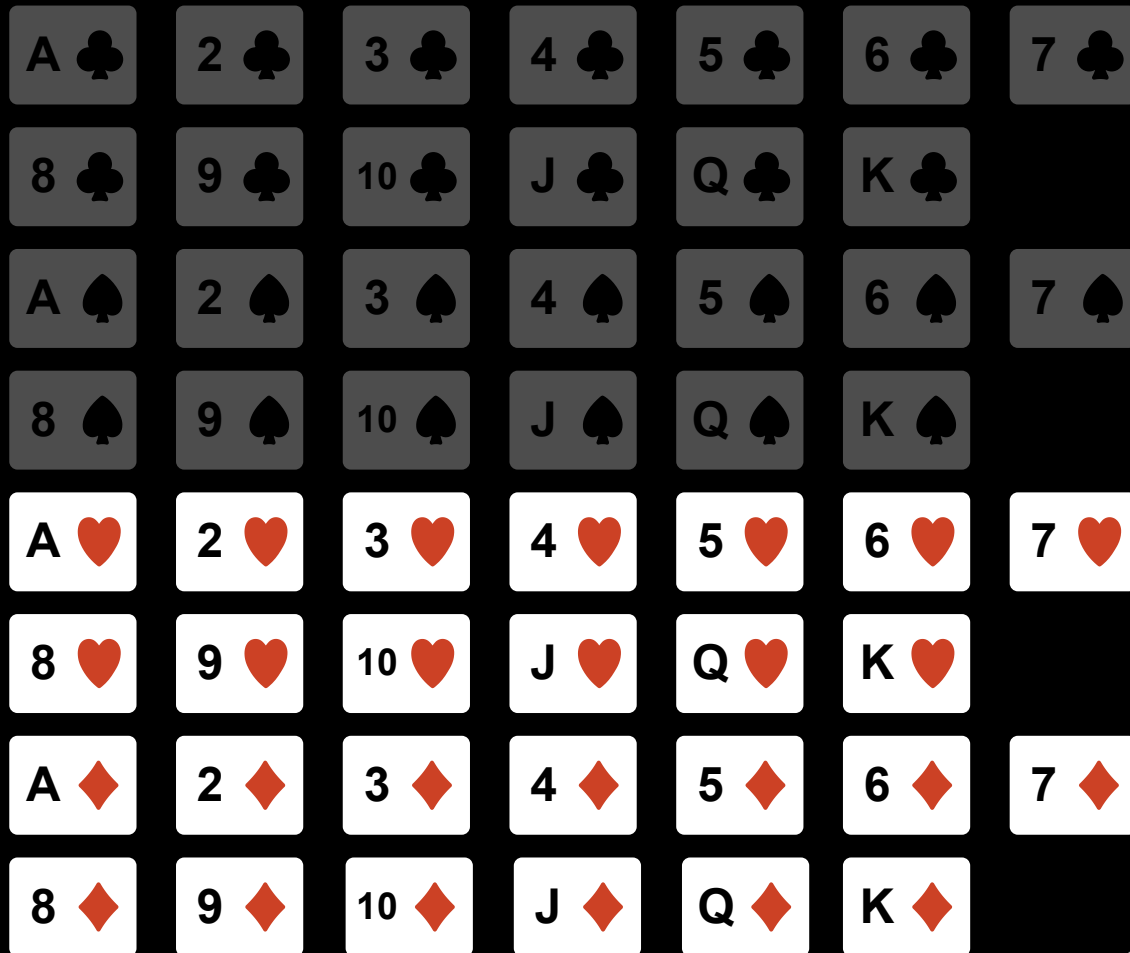


is the card black?



is the card black?

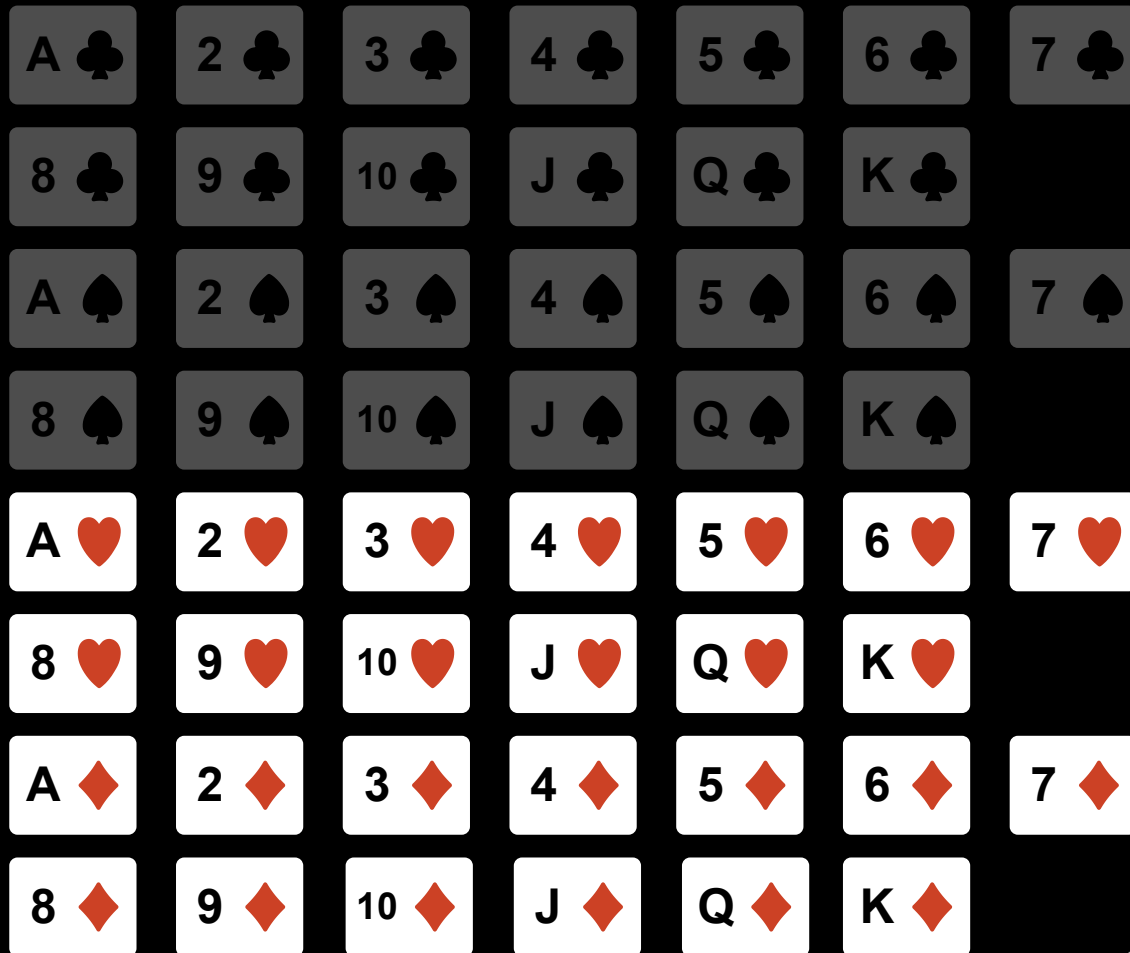
no



is the card black?

no

is it hearts?

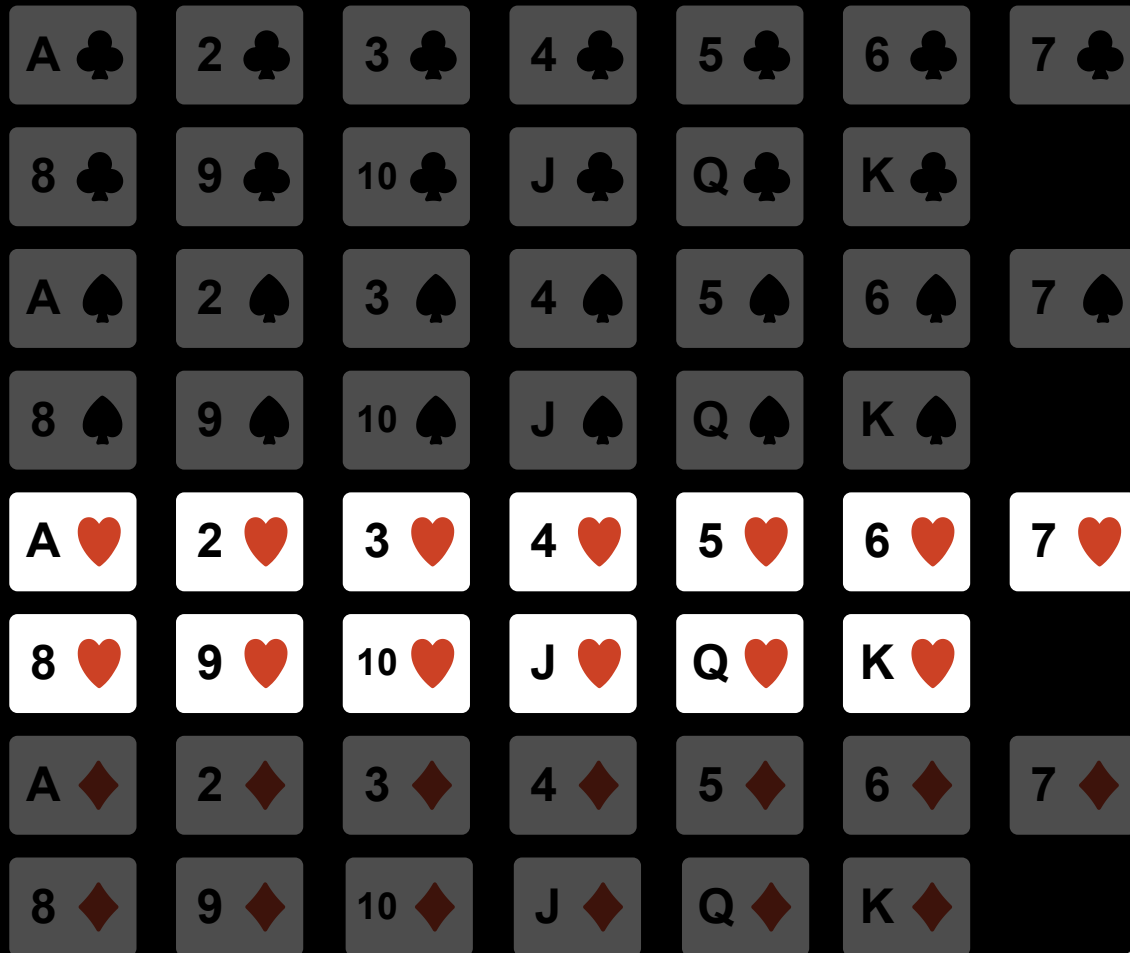


is the card black?

no

is it hearts?

yes



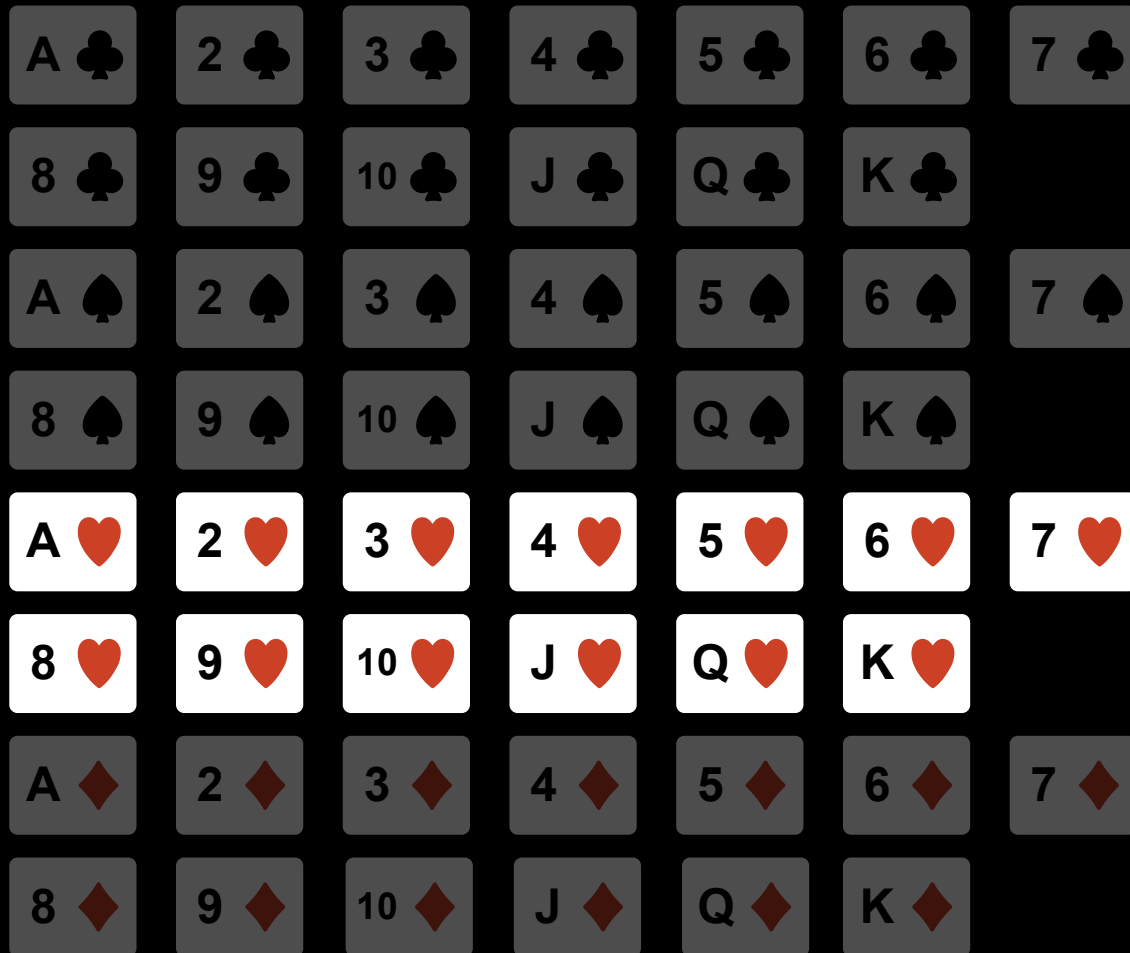
is the card black?

no

is it hearts?

yes

is it 8 or above?



is the card black?

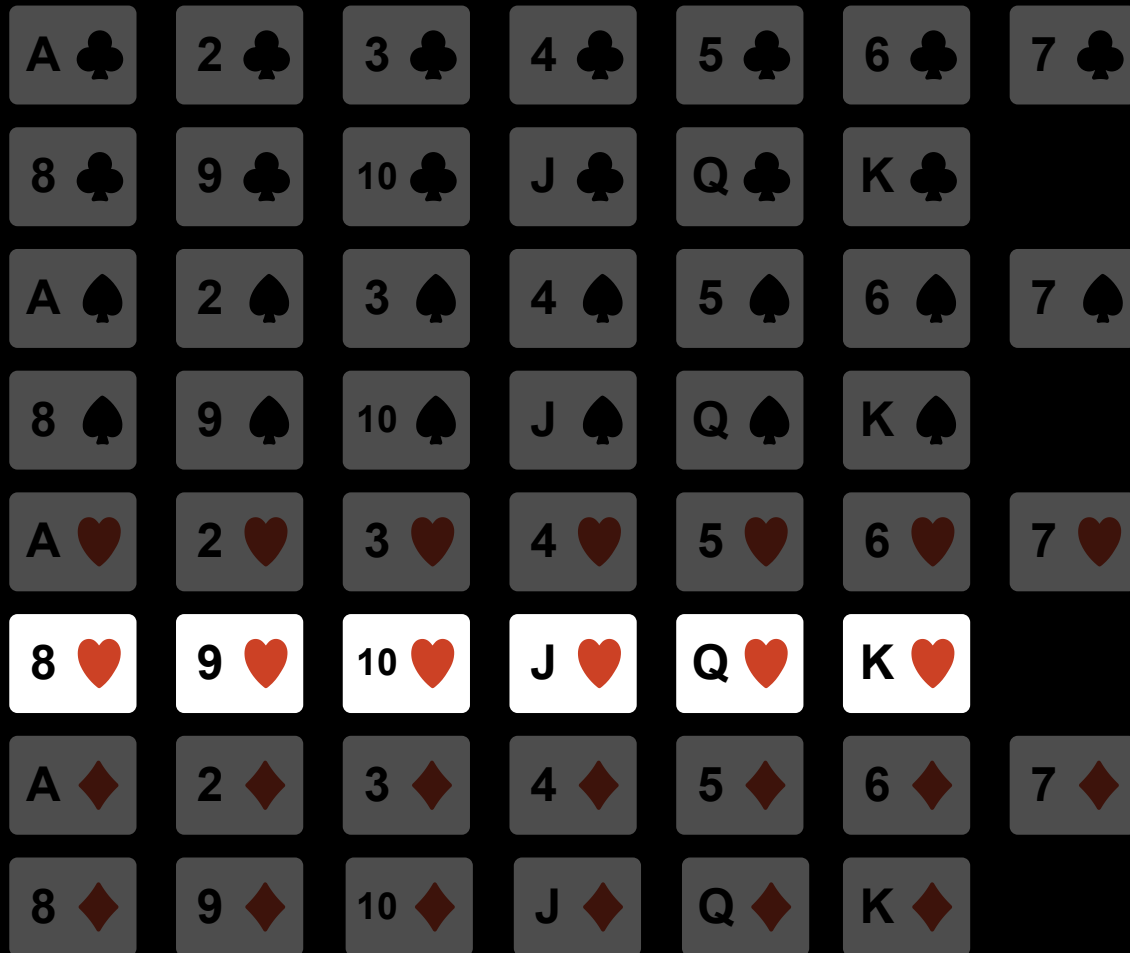
no

is it hearts?

yes

is it 8 or above?

yes



is the card black?

no

is it hearts?

yes

is it 8 or above?

yes

is it jack or above?

A ♣	2 ♣	3 ♣	4 ♣	5 ♣	6 ♣	7 ♣
8 ♣	9 ♣	10 ♣	J ♣	Q ♣	K ♣	
A ♠	2 ♠	3 ♠	4 ♠	5 ♠	6 ♠	7 ♠
8 ♠	9 ♠	10 ♠	J ♠	Q ♠	K ♠	
A ♥	2 ♥	3 ♥	4 ♥	5 ♥	6 ♥	7 ♥
8 ♥	9 ♥	10 ♥	J ♥	Q ♥	K ♥	
A ♦	2 ♦	3 ♦	4 ♦	5 ♦	6 ♦	7 ♦
8 ♦	9 ♦	10 ♦	J ♦	Q ♦	K ♦	

is the card black?

no

is it hearts?

yes

is it 8 or above?

yes

is it jack or above?

yes

A ♣	2 ♣	3 ♣	4 ♣	5 ♣	6 ♣	7 ♣
8 ♣	9 ♣	10 ♣	J ♣	Q ♣	K ♣	
A ♠	2 ♠	3 ♠	4 ♠	5 ♠	6 ♠	7 ♠
8 ♠	9 ♠	10 ♠	J ♠	Q ♠	K ♠	
A ♥	2 ♥	3 ♥	4 ♥	5 ♥	6 ♥	7 ♥
8 ♥	9 ♥	10 ♥	J ♥	Q ♥	K ♥	
A ♦	2 ♦	3 ♦	4 ♦	5 ♦	6 ♦	7 ♦
8 ♦	9 ♦	10 ♦	J ♦	Q ♦	K ♦	

is the card black?

no

is it hearts?

yes

is it 8 or above?

yes

is it jack or above?

yes

is it queen or above?

A ♣	2 ♣	3 ♣	4 ♣	5 ♣	6 ♣	7 ♣
8 ♣	9 ♣	10 ♣	J ♣	Q ♣	K ♣	
A ♠	2 ♠	3 ♠	4 ♠	5 ♠	6 ♠	7 ♠
8 ♠	9 ♠	10 ♠	J ♠	Q ♠	K ♠	
A ♥	2 ♥	3 ♥	4 ♥	5 ♥	6 ♥	7 ♥
8 ♥	9 ♥	10 ♥	J ♥	Q ♥	K ♥	
A ♦	2 ♦	3 ♦	4 ♦	5 ♦	6 ♦	7 ♦
8 ♦	9 ♦	10 ♦	J ♦	Q ♦	K ♦	

is the card black?

no

is it hearts?

yes

is it 8 or above?

yes

is it jack or above?

yes

is it queen or above?

yes

A ♣	2 ♣	3 ♣	4 ♣	5 ♣	6 ♣	7 ♣
8 ♣	9 ♣	10 ♣	J ♣	Q ♣	K ♣	
A ♠	2 ♠	3 ♠	4 ♠	5 ♠	6 ♠	7 ♠
8 ♠	9 ♠	10 ♠	J ♠	Q ♠	K ♠	
A ♥	2 ♥	3 ♥	4 ♥	5 ♥	6 ♥	7 ♥
8 ♥	9 ♥	10 ♥	J ♥	Q ♥	K ♥	
A ♦	2 ♦	3 ♦	4 ♦	5 ♦	6 ♦	7 ♦
8 ♦	9 ♦	10 ♦	J ♦	Q ♦	K ♦	

is the card black?

no

is it hearts?

yes

is it 8 or above?

yes





















































is it jack or above?

yes

is it queen or above?

yes

is it king?

A 	2 	3 	4 	5 	6 	7 
8 	9 	10 	J 	Q 	K 	
A 	2 	3 	4 	5 	6 	7 
8 	9 	10 	J 	Q 	K 	
A 	2 	3 	4 	5 	6 	7 
8 	9 	10 	J 	Q 	K 	
A 	2 	3 	4 	5 	6 	7 
8 	9 	10 	J 	Q 	K 	

is the card black?

no

is it hearts?

yes

is it 8 or above?

yes

is it jack or above?





















































yes

is it queen or above?

yes

is it king?

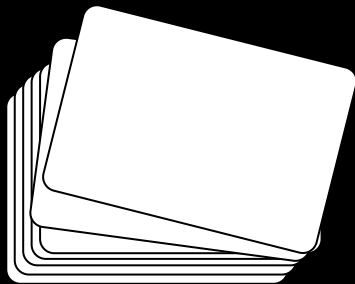
no

A 	2 	3 	4 	5 	6 	7 
8 	9 	10 	J 	Q 	K 	
A 	2 	3 	4 	5 	6 	7 
8 	9 	10 	J 	Q 	K 	
A 	2 	3 	4 	5 	6 	7 
8 	9 	10 	J 	Q 	K 	
A 	2 	3 	4 	5 	6 	7 
8 	9 	10 	J 	Q 	K 	

with 6 questions
from 52 to 1

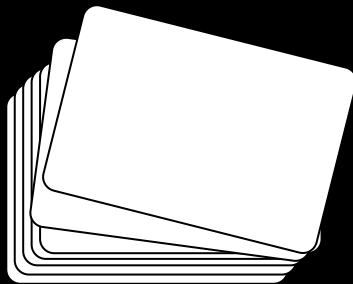
A ♣	2 ♣	3 ♣	4 ♣	5 ♣	6 ♣	7 ♣
8 ♣	9 ♣	10 ♣	J ♣	Q ♣	K ♣	
A ♠	2 ♠	3 ♠	4 ♠	5 ♠	6 ♠	7 ♠
8 ♠	9 ♠	10 ♠	J ♠	Q ♠	K ♠	
A ♥	2 ♥	3 ♥	4 ♥	5 ♥	6 ♥	7 ♥
8 ♥	9 ♥	10 ♥	J ♥	Q ♥	K ♥	
A ♦	2 ♦	3 ♦	4 ♦	5 ♦	6 ♦	7 ♦
8 ♦	9 ♦	10 ♦	J ♦	Q ♦	K ♦	

uncertainty with $N = 52$ possibilities?

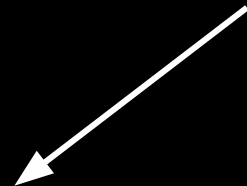


$$H = \log_2(52) \approx 5.7$$

uncertainty with $N = 52$ possibilities?



average # of yes/no
questions



$$H = \log_2(52) \approx 5.7$$

one bit of information with each answer...

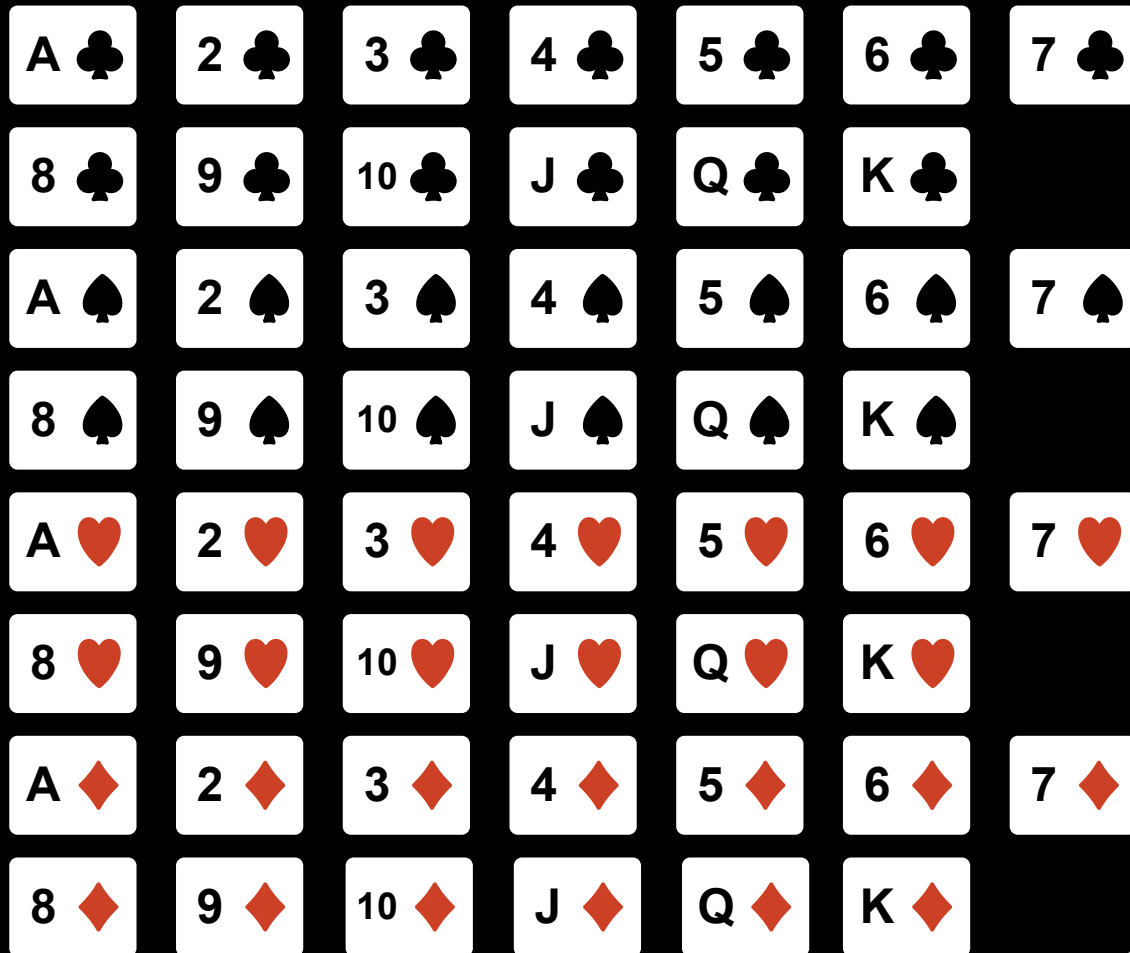
$$\log_2(52) - \log_2(26) = 1$$

one bit of information with each answer...

$$\log_2(52) - \log_2(26) = 1$$

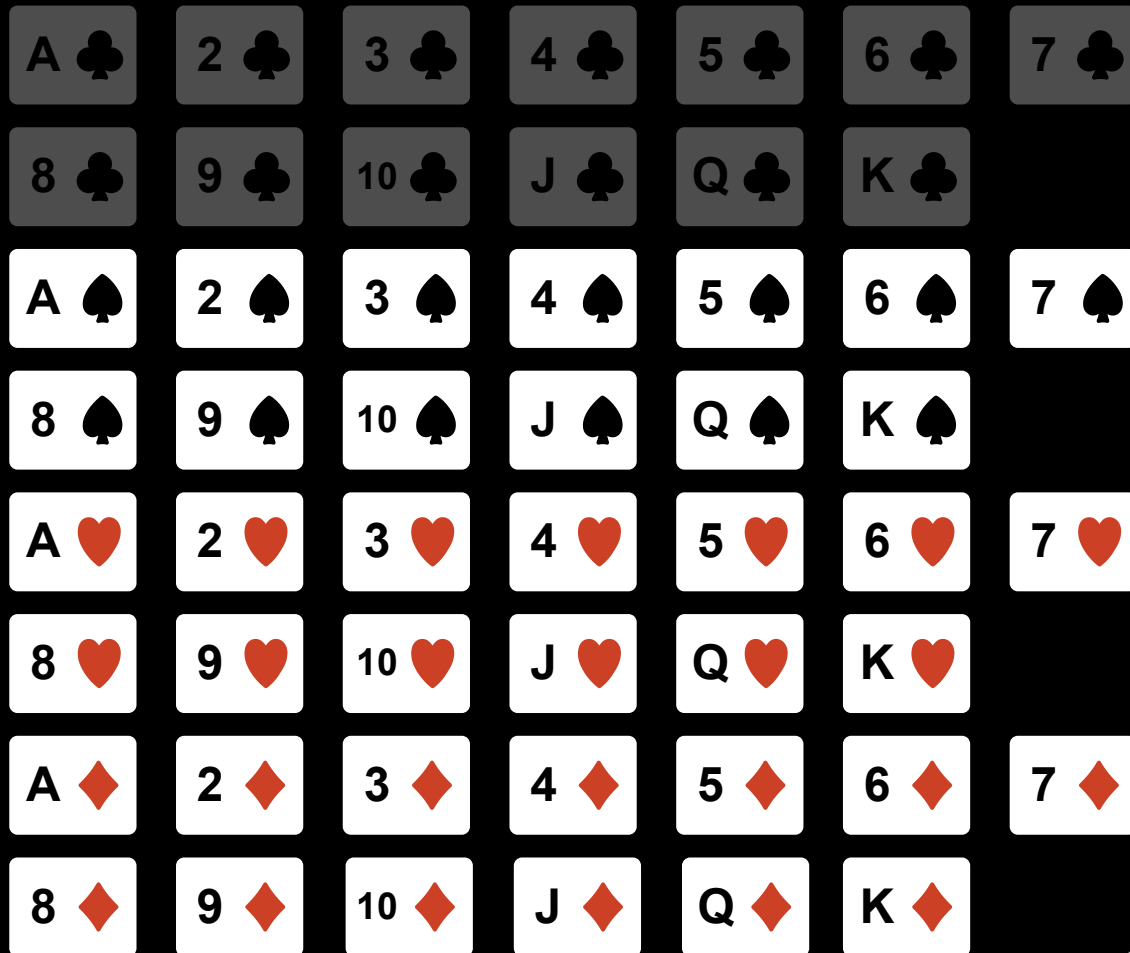
...that cuts the remaining options in half

is it a spades card?



is it a spades card?

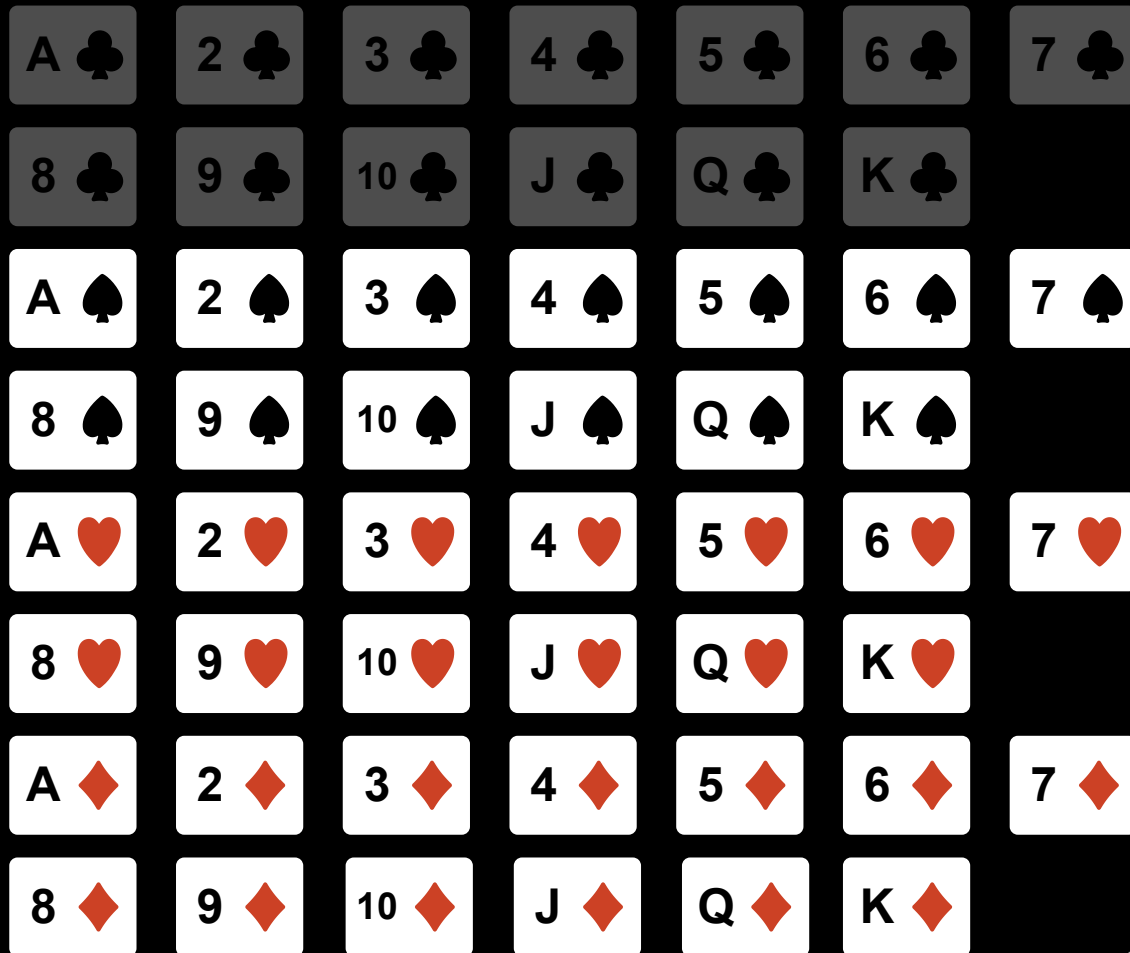
no



is it a spades card?

no

how much information?



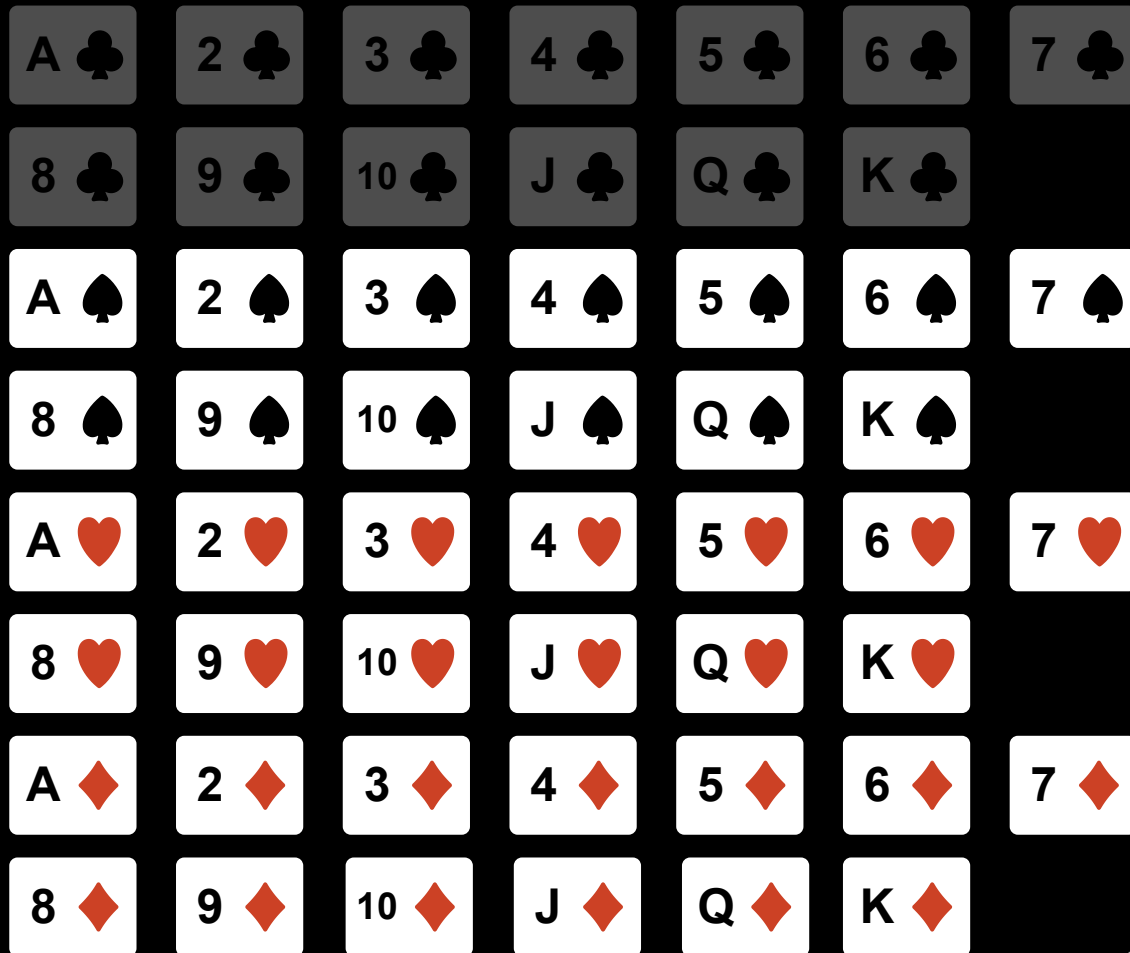
is it a spades card?

no

how much information?

$$H_0 = \log_2(52) \approx 5.7$$

$$H_1 = \log_2(39) \approx 5.29$$



is it a spades card?

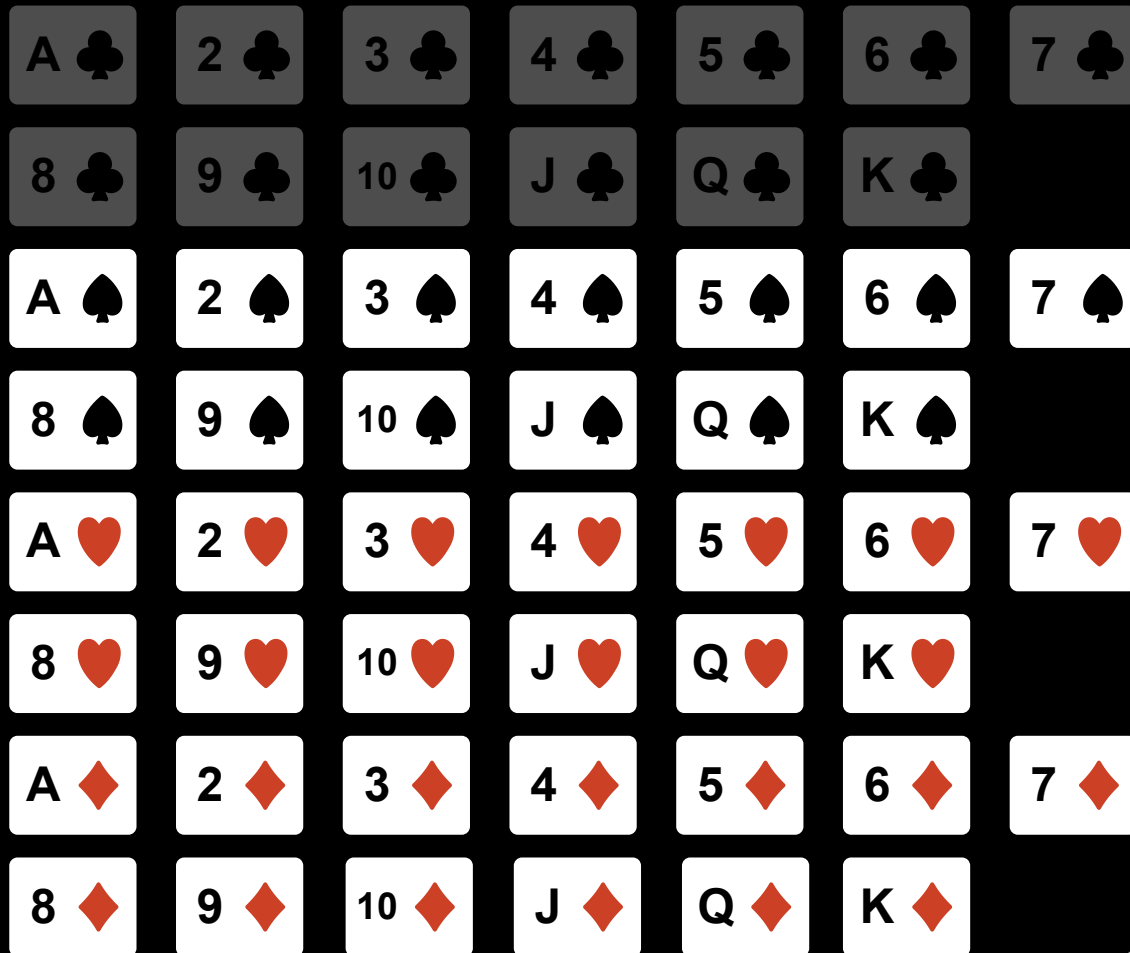
no

how much information?

$$H_0 = \log_2(52) \approx 5.7$$

$$H_1 = \log_2(39) \approx 5.29$$

$$H_0 - H_1 \approx 0.41$$



is it a spades card?

no

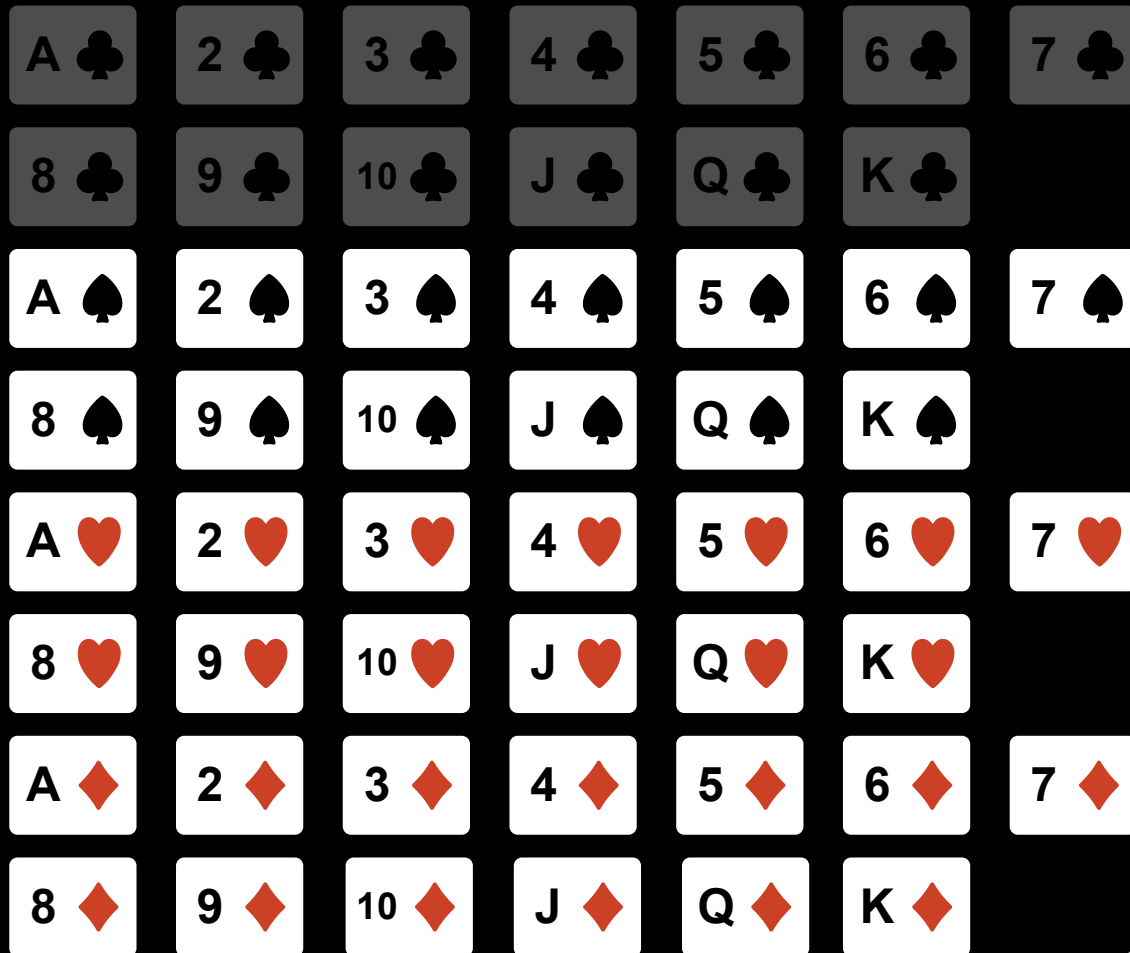
how much information?

$$H_0 = \log_2(52) \approx 5.7$$

$$H_1 = \log_2(39) \approx 5.29$$

$$H_0 - H_1 \approx 0.41$$

that's less than 1 bit



chess



next_move()

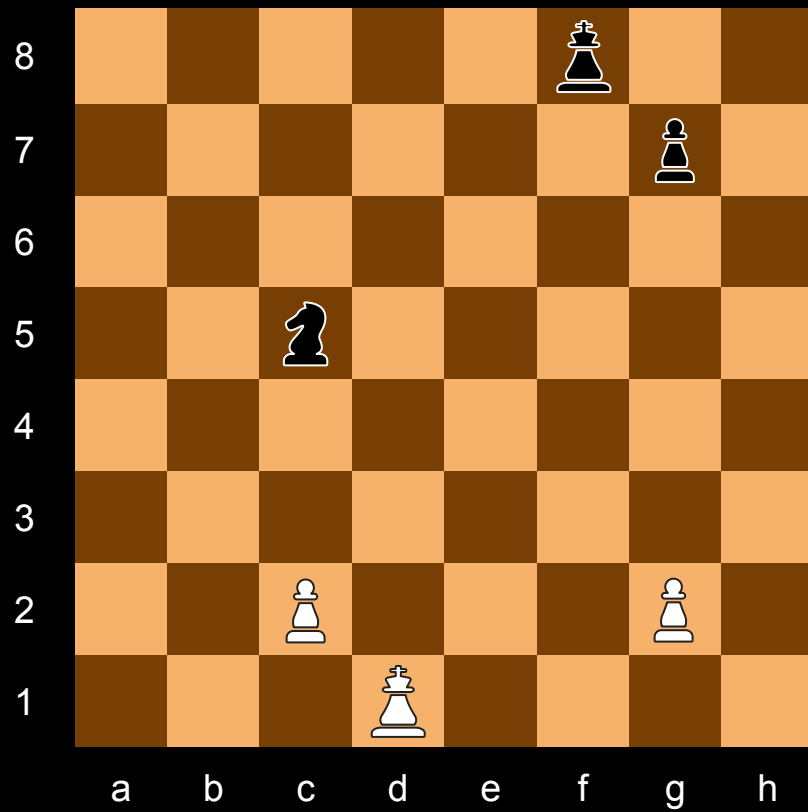
E2 → E4

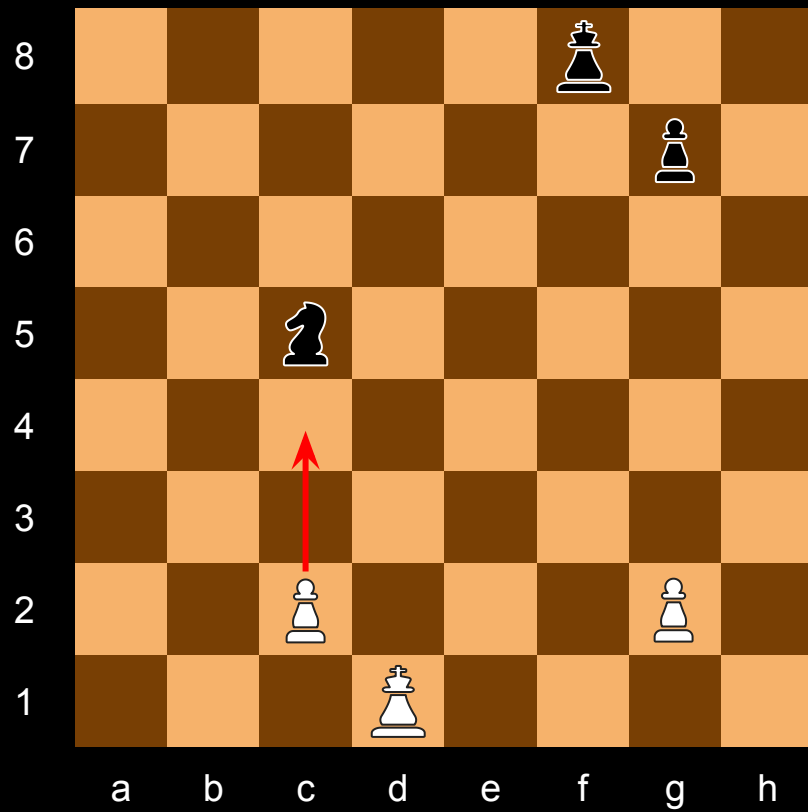
how much information is
one move?

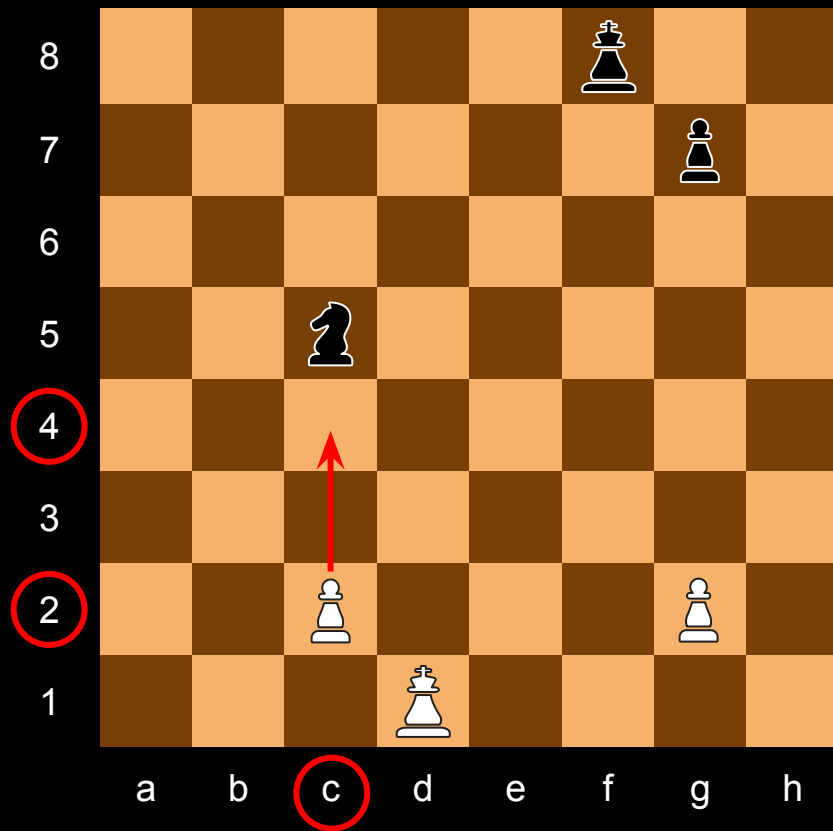


next_move()

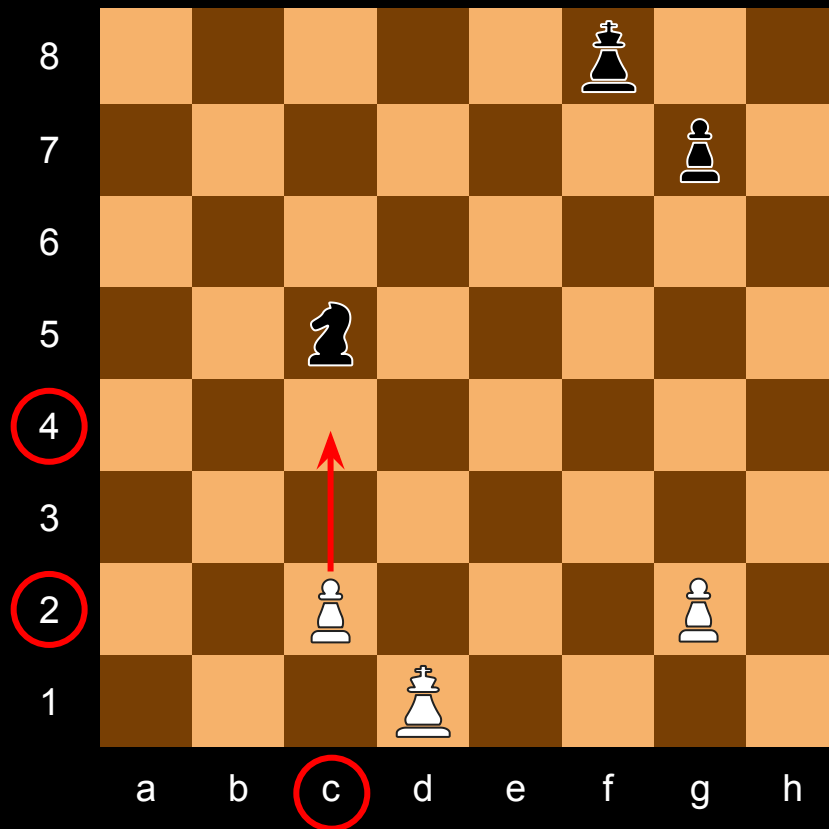
E2 → E4



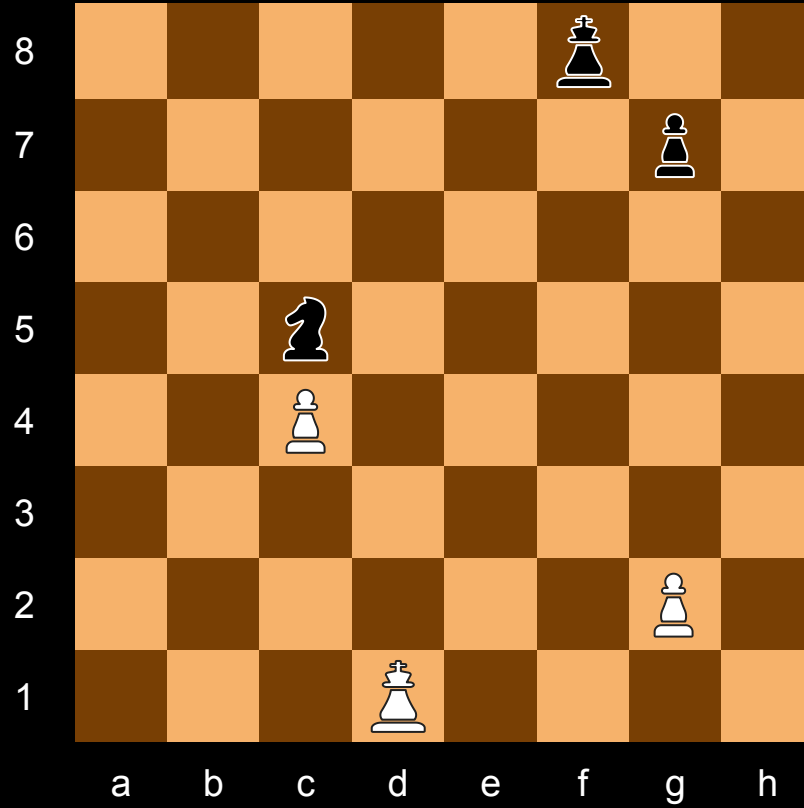




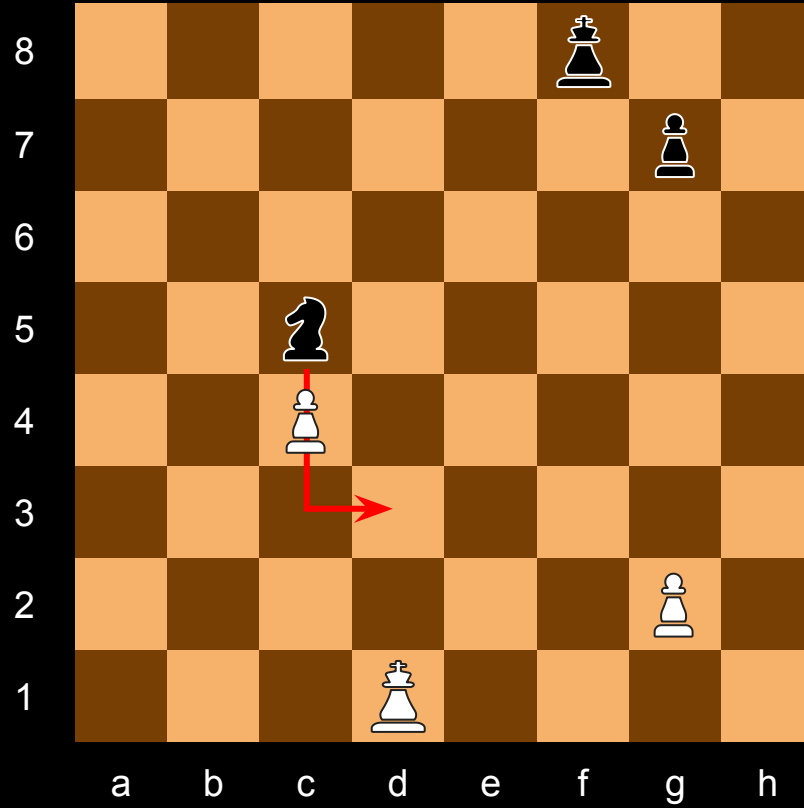
c2 → c4



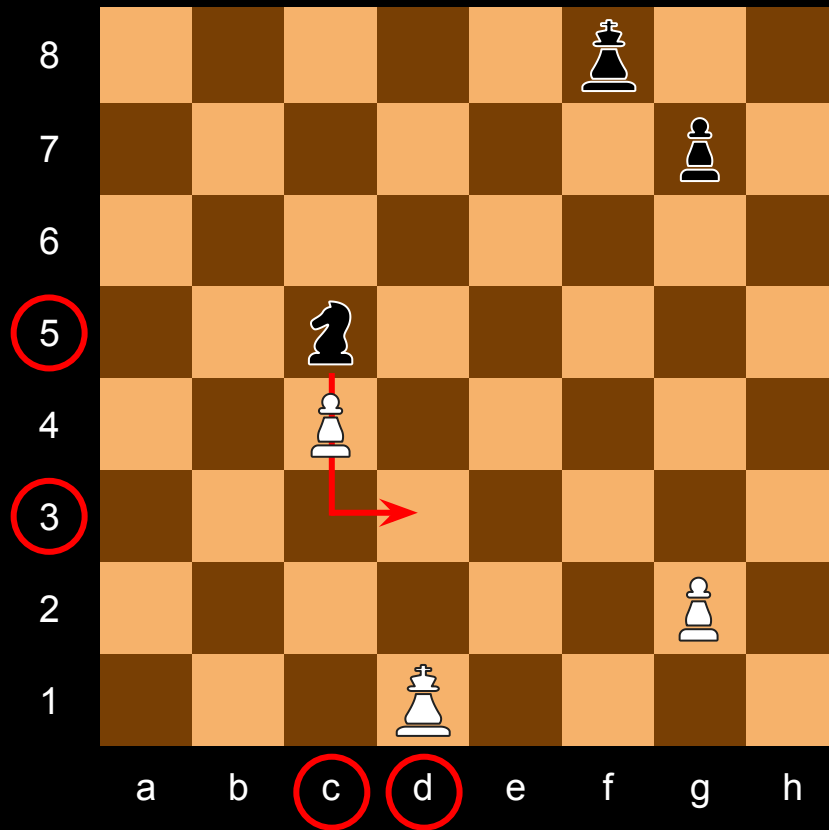
c2 → c4



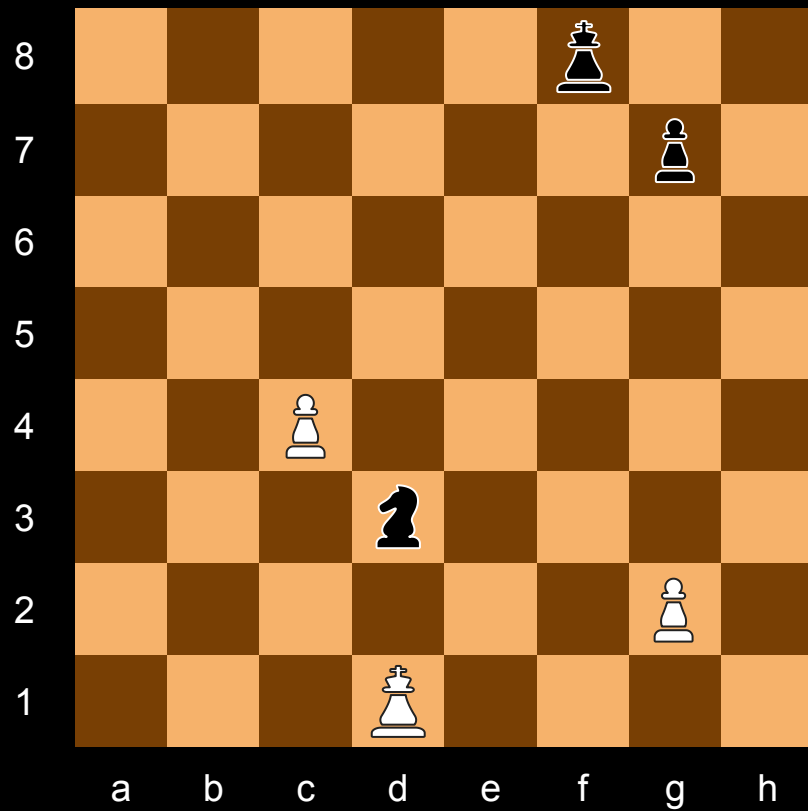
c2 → c4



c2 → c4
c5 → d3



c2 → c4
c5 → d3
...



$$c_2 \rightarrow c_4$$

how many possibilities?

64 fields x 64 fields

how many possibilities?

$$64 \times 64 = 4096$$

possible moves*

*disregarding impossible moves

$$64 \times 64 = 4096$$

$$H = \log_2(4096) = 12$$

$$64 \times 64 = 4096$$

$$H = \log_2(4096) = 12$$

one chess move is 12 bits of information

an alternative way to calculate # bits

c 2 c 4

4 digits

c 2 c 4

4 digits

8 possible symbols per digit

c 2 c 4

4 digits

8 possible symbols per digit

how many bits per digit?

c 2 c 4

4 digits

8 possible symbols per digit

how many bits per digit?

$$H_{digit} = \log_2(8) = 3$$

c 2 c 4

4 digits

8 possible symbols per digit

how many bits per digit?

$$H_{digit} = \log_2(8) = 3$$

$$H_{move} = \log_2(8) \times 4 = 12$$

$$H_{avg} = \log_2(S) \times n$$

S : number of possible symbols

n : number of digits in our message

$$H_{max} = \lceil \log_2(S) \rceil \times n$$

when calculating bits for storage, we must
always consider the worst case

digits and # symbols

$\{A\}$

— —

$\{A\}$

A A

$\{A, B\}$

— —

$\{A, B\}$

AA, AB, BA, BB

$\{A, B, C\}$

— —

$\{A, B, C\}$

AA, AB, BA, BB,
AC, BC, CA, CB, CC

$\{A, B, C, D\}$

— —

$\{A, B, C, D\}$

AA, AB, BA, BB, AC, BC, CA, CB,
CC, AD, DA, BD, DB, CD, DC, DD

{A, B, C, D, E}

— —

$\{A, B, C, D, E\}$

AA, AB, BA, BB, AC, BC, CA, CB, CC,
AD, DA, BD, DB, CD, DC, DD, AE, EA,
BE, EB, CE, EC, DE, ED, EE

with # digits $n = 2$

symbols

messages

1

1

2

4

3

9

4

16

5

25

with length $n = 2$

symbols

messages

1

1

2

4

3

9

4

16

5

25

$f(x)$



and more digits?

$\{A, B\}$

— — —

$\{A, B\}$

AAA, AAB, ABA, ABB,
BBB, BBA, BAA, BAB

$\{A, B\}$

— — — —

$\{A, B\}$

AAAA, AAAB, AABA, AABB,
ABAA, ABAB, ABBA, ABBB,
BAAA, BAAB, BABA, BABB,
BBAA, BBAB, BBBA, BBBB

with # symbols $S = 2$

digits

messages

1

2

2

4

3

8

4

16

5

32

with # symbols $S = 2$

# digits		# messages
1		2
2		4
3	$\xrightarrow{f(x)}$	8
4		16
5		32

with # symbols $S = 2$

# digits		# messages
1		2
2		4
3	$\xrightarrow{f(x)}$	8
4		16
5		32

possible messages with n digits and S symbols

$$N = S^n$$