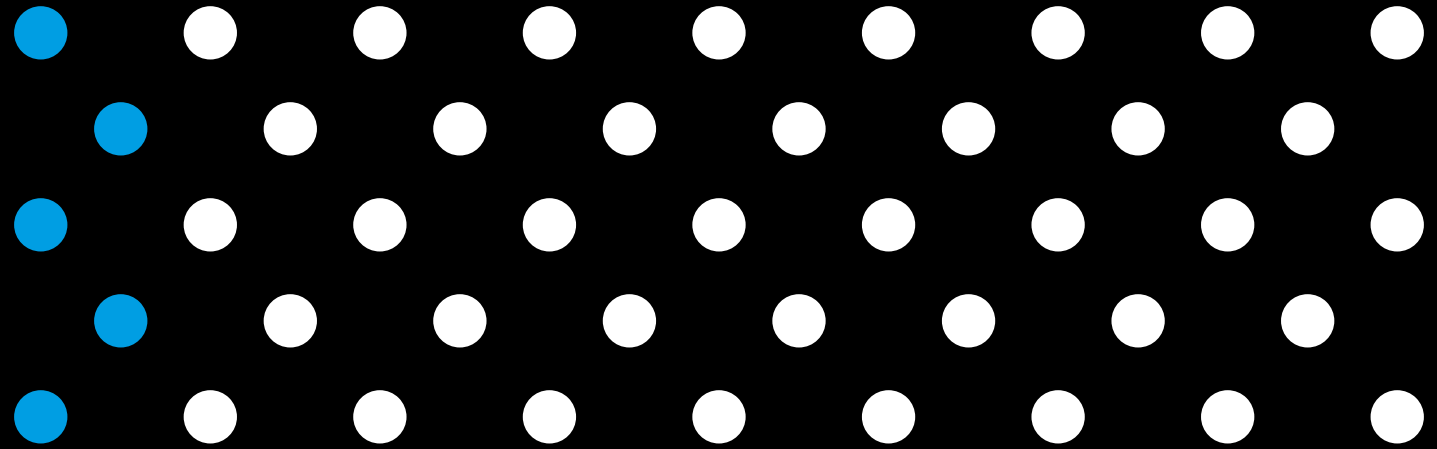
0. MOTIVATION
1. PROBLEM SOLVING
2. INFORMATION
3. COUNTING
4. BITS
5. CODES
6. ALGORITHMS
7. COMPUTERS
8. ARITHMETIC
9. MEMORY
10. ANALOG VS. DIGITAL

MOTIVATION

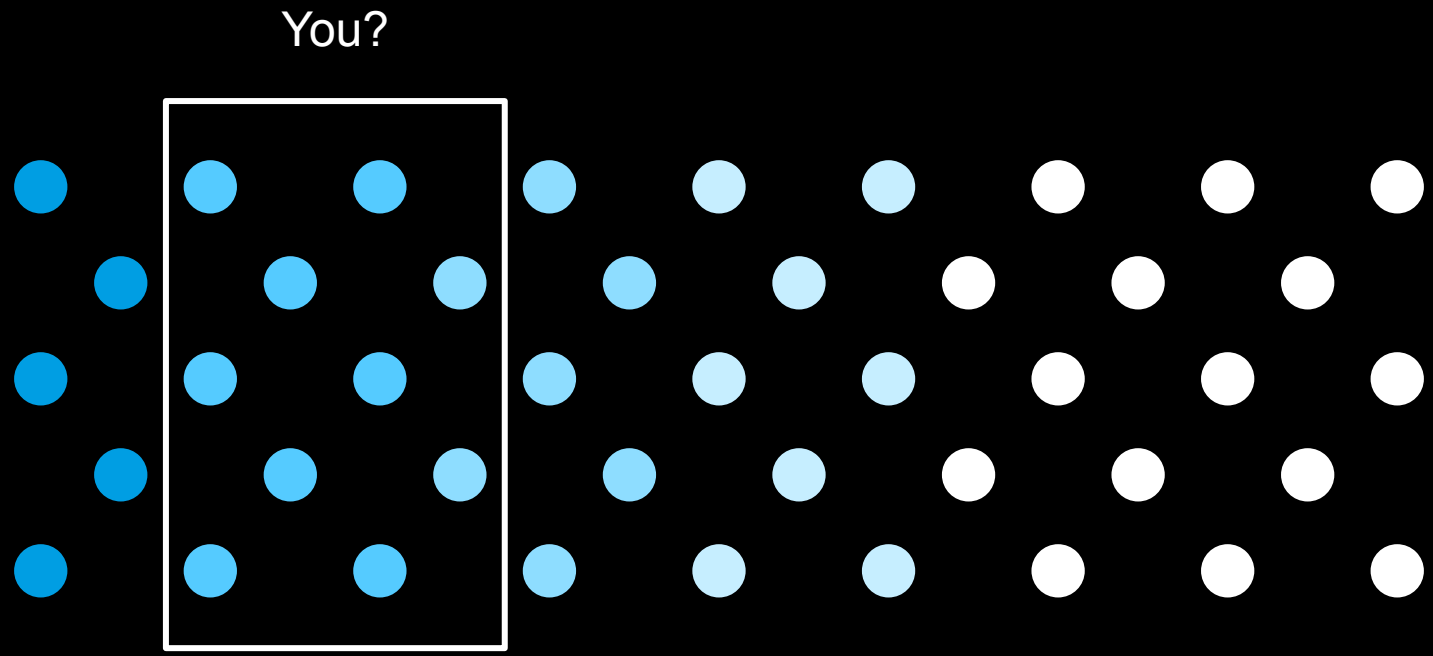
a few experts

digitally uneducated society



digitally illiterate society with a few experts

collective understanding



society with a distributed and high degree of digital education

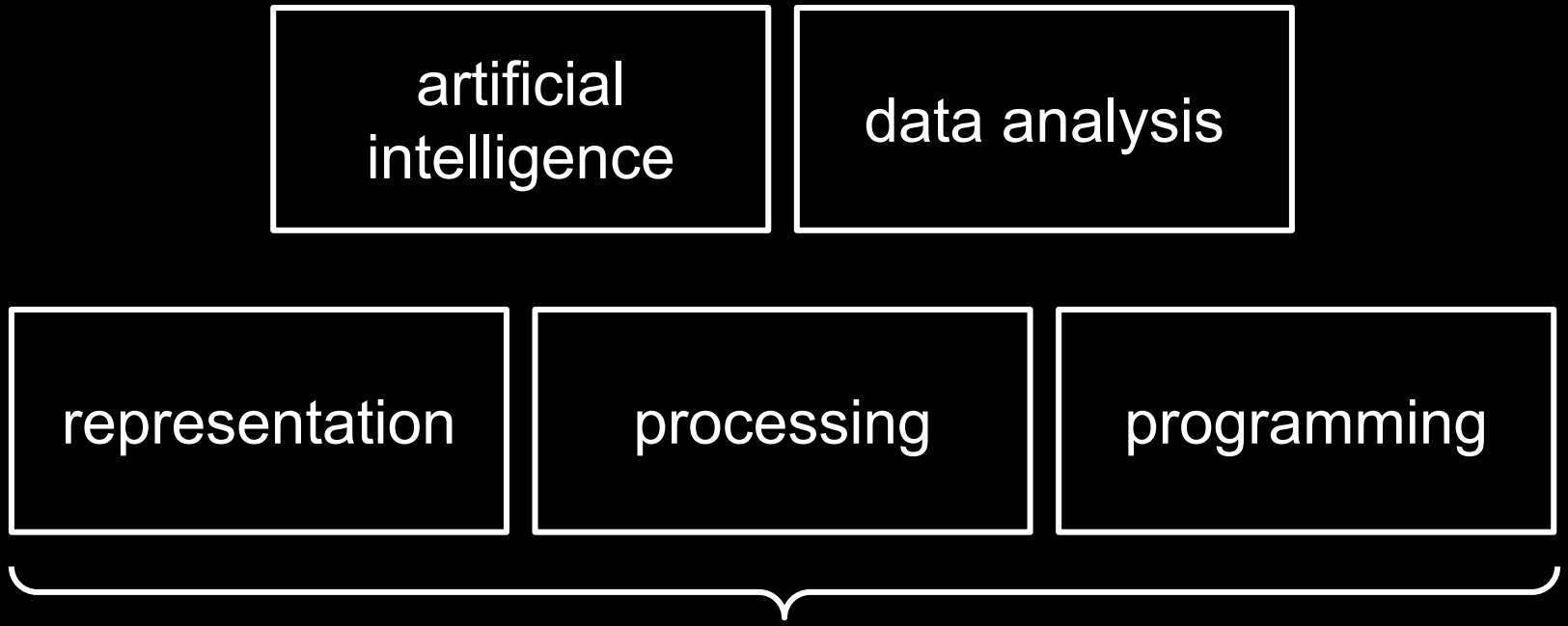
artificial Intelligence

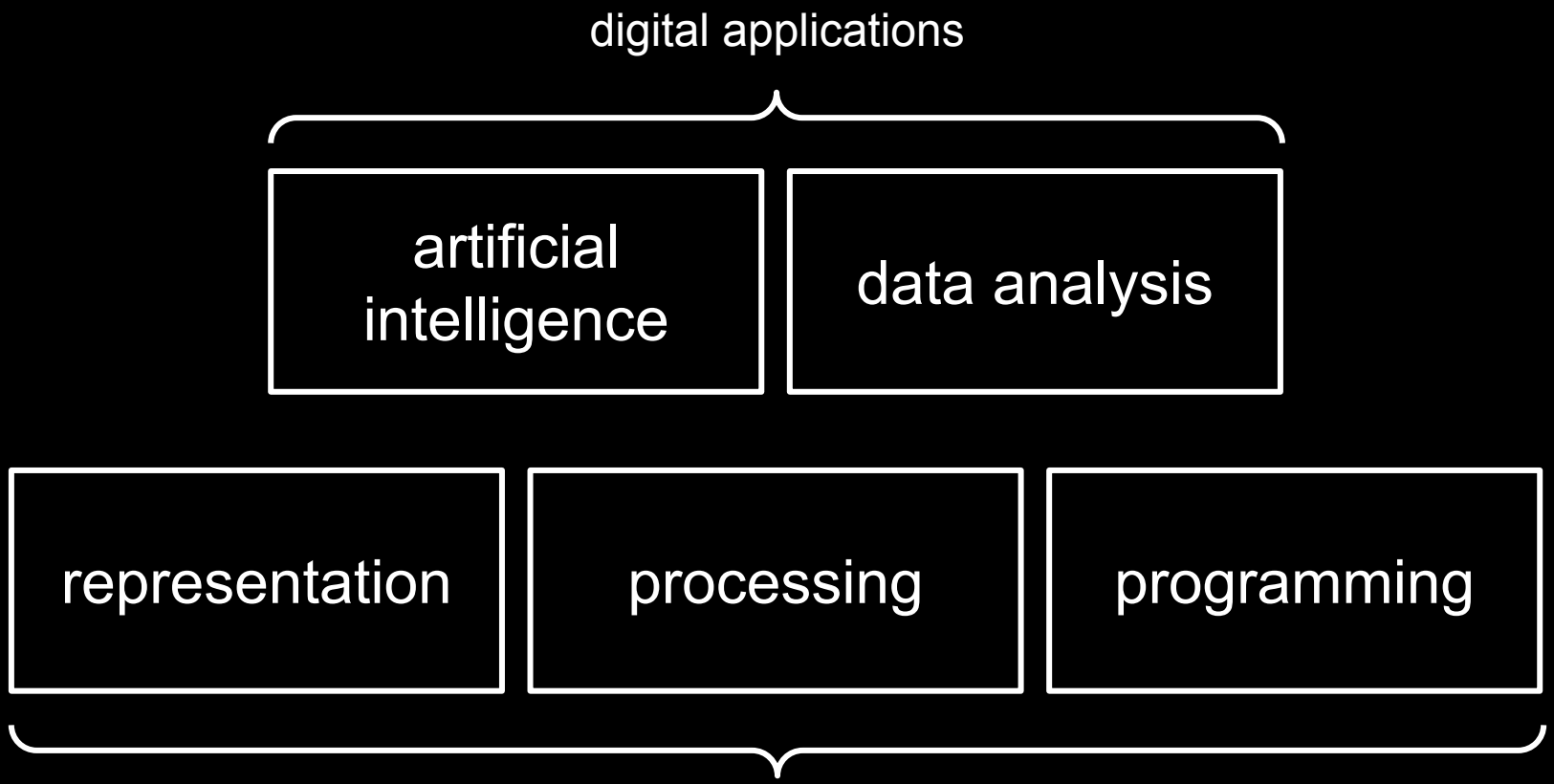
data analysis

representation

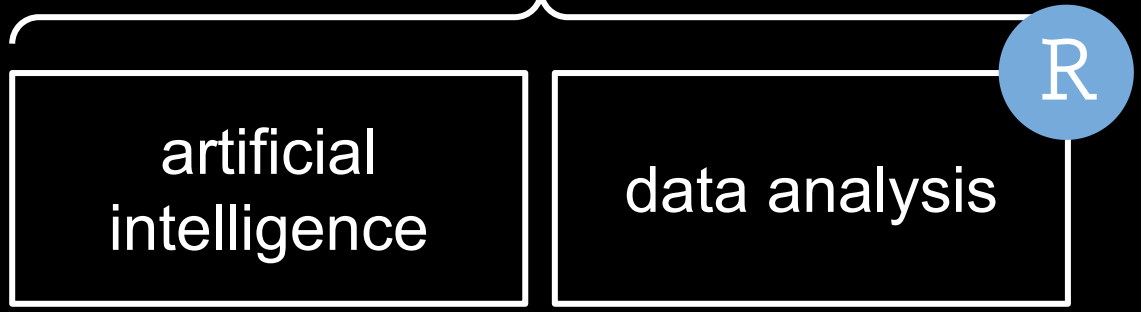
processing

programming


artificial intelligence
data analysis
representation
processing
programming
digital fundamentals


digital applications
artificial intelligence
data analysis
representation
processing
programming
digital fundamentals

digital applications

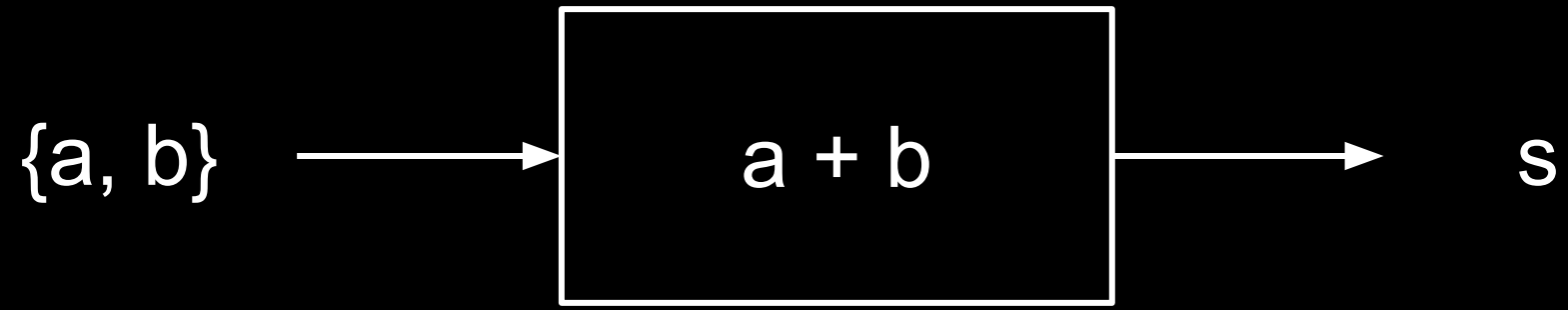


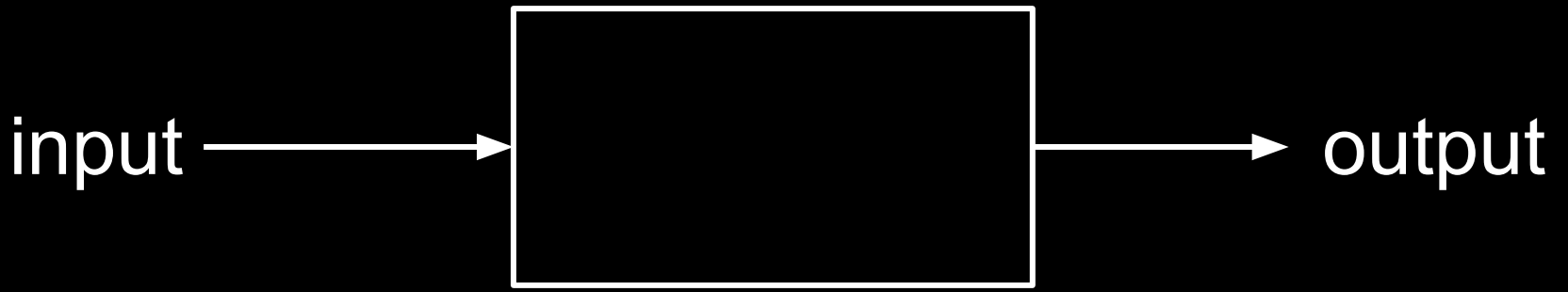
digital fundamentals

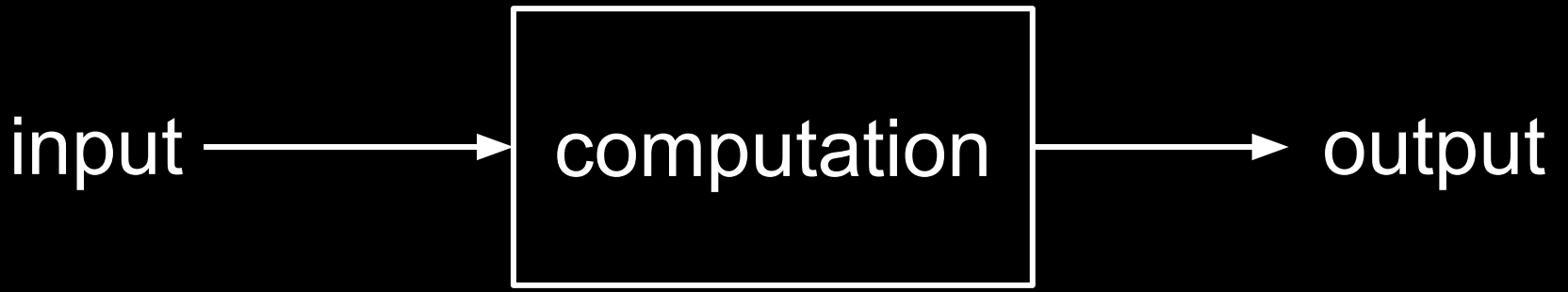
PROBLEM SOLVING

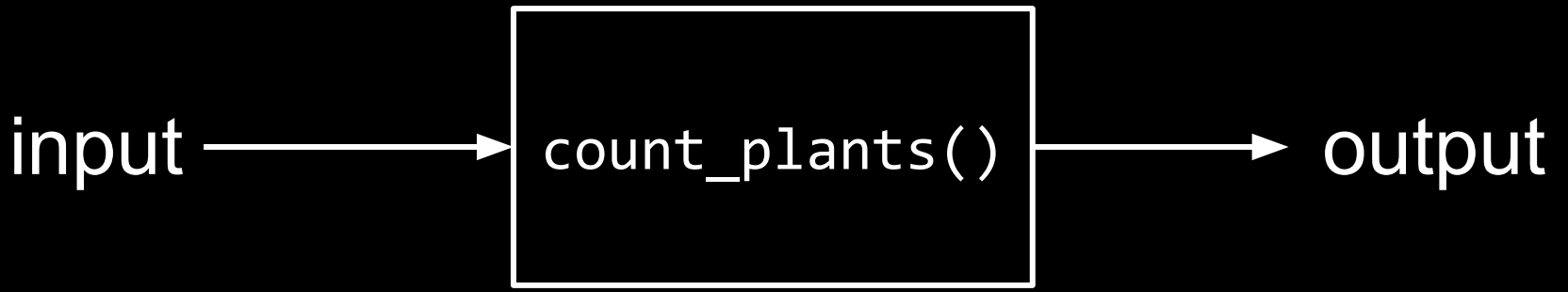
a model for solving problems

a model for solving problems




input
output


input
computation
output


input
count_plants()
output

```
count_plants()
```

42

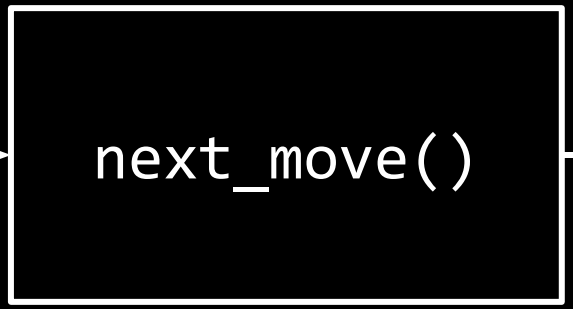
processing of information

```
count_plants()
```

42

representation of information

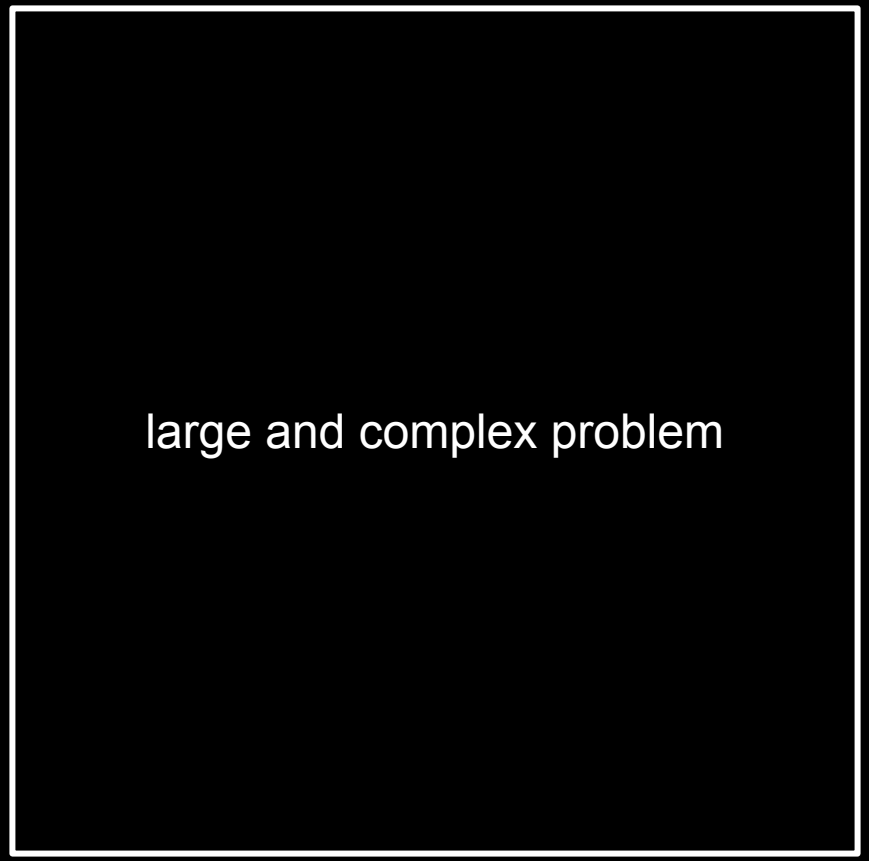
input
ChatGPT
output

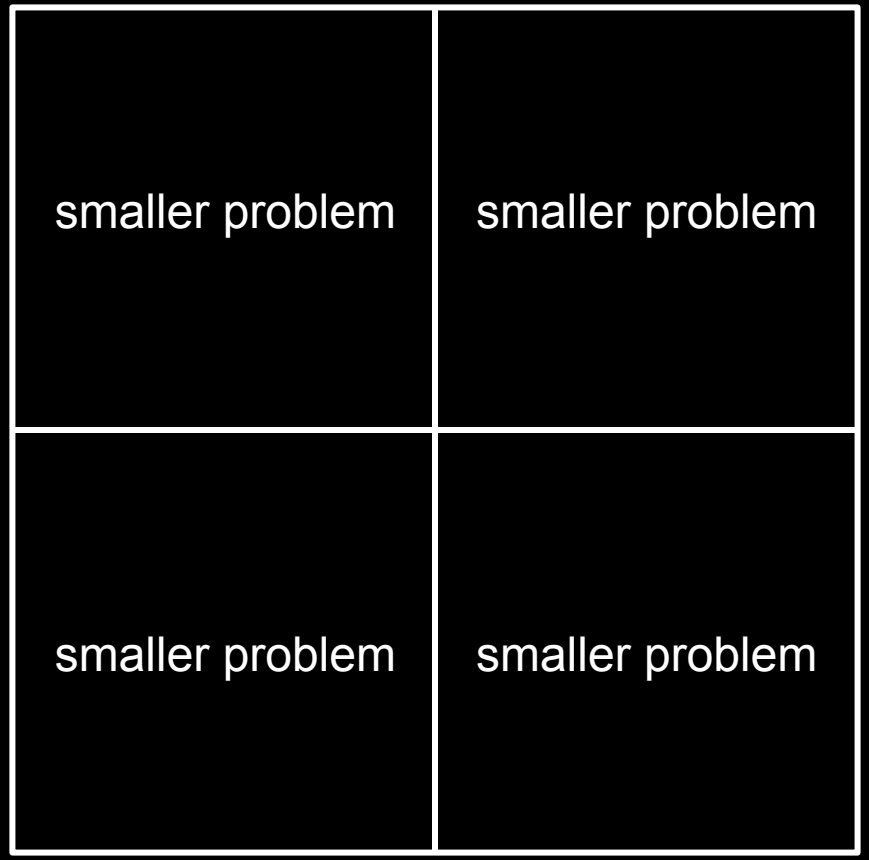


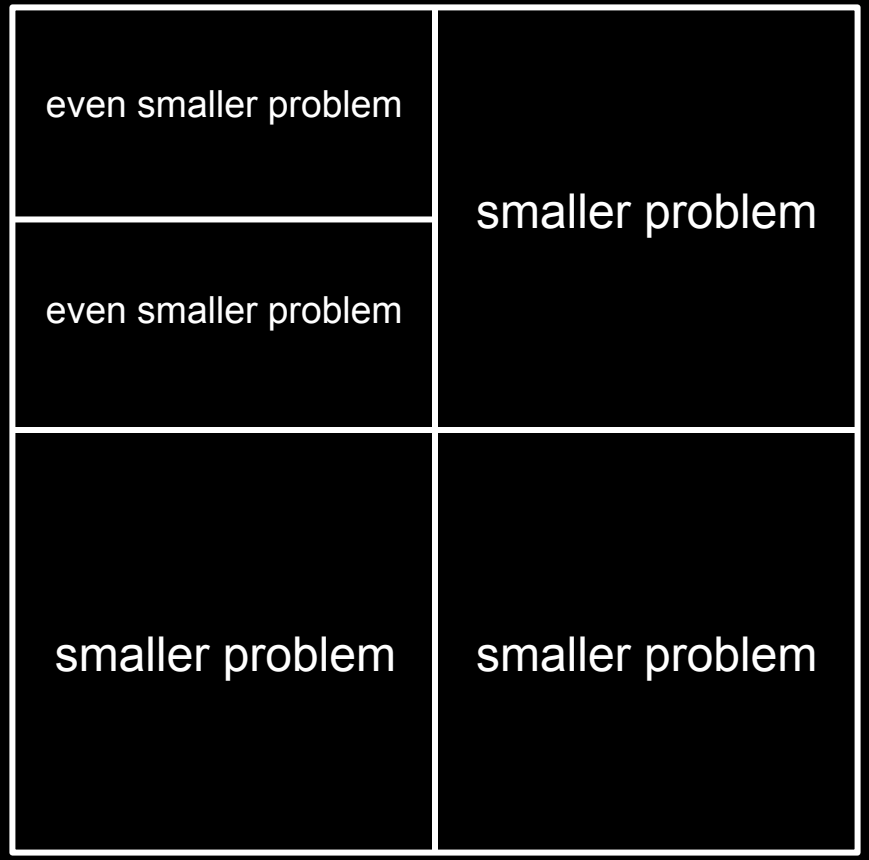
E2 → E4

problem solving strategies

divide and conquer


large and complex problem


smaller problem
smaller problem
smaller problem
smaller problem


even smaller problem
even smaller problem
smaller problem
smaller problem
smaller problem

sorted list +
element
search()
yes / no

is 67 a prime number?

```
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97
```

linear search

```
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97
```

linear search

~~2,~~ 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

linear search

~~2,~~ ~~3,~~ 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

linear search

~~2~~, ~~3~~, ~~5~~, ~~7~~, ~~11~~, ~~13~~, ~~17~~, ~~19~~, ~~23~~, ~~29~~, ~~31~~, ~~37~~, ~~41~~,
~~43~~, ~~47~~, ~~53~~, ~~59~~, ~~61~~, 67, 71, 73, 79, 83, 89, 97

19 steps… can’t we do better?

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

binary search

67 != 41

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, ~~41~~,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

binary search

67 > 41

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, ~~41~~,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

binary search

67 > 41

~~2~~, ~~3~~, ~~5~~, ~~7~~, ~~11~~, ~~13~~, ~~17~~, ~~19~~, ~~23~~, ~~29~~, ~~31~~, ~~37~~, ~~41~~,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

binary search

~~2~~, ~~3~~, ~~5~~, ~~7~~, ~~11~~, ~~13~~, ~~17~~, ~~19~~, ~~23~~, ~~29~~, ~~31~~, ~~37~~, ~~41~~,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

67 != 71

binary search

~~2~~, ~~3~~, ~~5~~, ~~7~~, ~~11~~, ~~13~~, ~~17~~, ~~19~~, ~~23~~, ~~29~~, ~~31~~, ~~37~~, ~~41~~,
43, 47, 53, 59, 61, 67, ~~71~~, 73, 79, 83, 89, 97

67 != 71

binary search

~~2~~, ~~3~~, ~~5~~, ~~7~~, ~~11~~, ~~13~~, ~~17~~, ~~19~~, ~~23~~, ~~29~~, ~~31~~, ~~37~~, ~~41~~,
43, 47, 53, 59, 61, 67, ~~71~~, ~~73~~, ~~79~~, ~~83~~, ~~89~~, ~~97~~

67 < 71

binary search

~~2~~, ~~3~~, ~~5~~, ~~7~~, ~~11~~, ~~13~~, ~~17~~, ~~19~~, ~~23~~, ~~29~~, ~~31~~, ~~37~~, ~~41~~,
43, 47, 53, ~~59~~, 61, 67, ~~71~~, ~~73~~, ~~79~~, ~~83~~, ~~89~~, ~~97~~

67 != 59

binary search

~~2~~, ~~3~~, ~~5~~, ~~7~~, ~~11~~, ~~13~~, ~~17~~, ~~19~~, ~~23~~, ~~29~~, ~~31~~, ~~37~~, ~~41~~,
~~43~~, ~~47~~, ~~53~~, ~~59~~, 61, 67, ~~71~~, ~~73~~, ~~79~~, ~~83~~, ~~89~~, ~~97~~

67 > 59

binary search

~~2~~, ~~3~~, ~~5~~, ~~7~~, ~~11~~, ~~13~~, ~~17~~, ~~19~~, ~~23~~, ~~29~~, ~~31~~, ~~37~~, ~~41~~,
~~43~~, ~~47~~, ~~53~~, ~~59~~, 61, 67, ~~71~~, ~~73~~, ~~79~~, ~~83~~, ~~89~~, ~~97~~

67 = 67

binary search

~~2~~, ~~3~~, ~~5~~, ~~7~~, ~~11~~, ~~13~~, ~~17~~, ~~19~~, ~~23~~, ~~29~~, ~~31~~, ~~37~~, ~~41~~,
~~43~~, ~~47~~, ~~53~~, ~~59~~, ~~61~~, 67, ~~71~~, ~~73~~, ~~79~~, ~~83~~, ~~89~~, ~~97~~

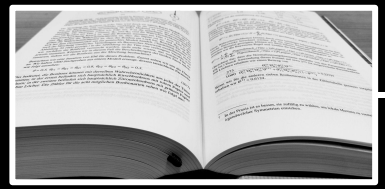
67 = 67

3 splits → much better

~~2~~, ~~3~~, ~~5~~, ~~7~~, ~~11~~, ~~13~~, ~~17~~, ~~19~~, ~~23~~, ~~29~~, ~~31~~, ~~37~~, ~~41~~,
~~43~~, ~~47~~, ~~53~~, ~~59~~, ~~61~~, 67, ~~71~~, ~~73~~, ~~79~~, ~~83~~, ~~89~~, ~~97~~

67 = 67

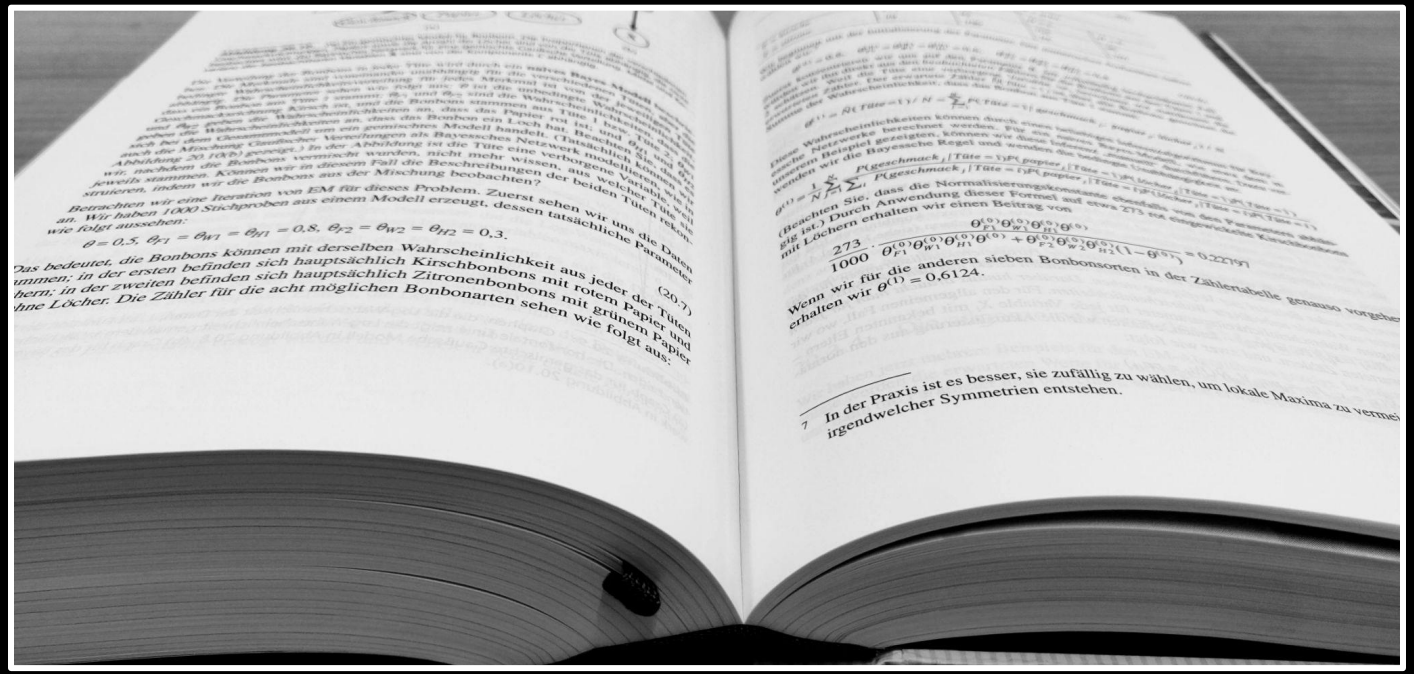
how efficient are linear and binary search in general?



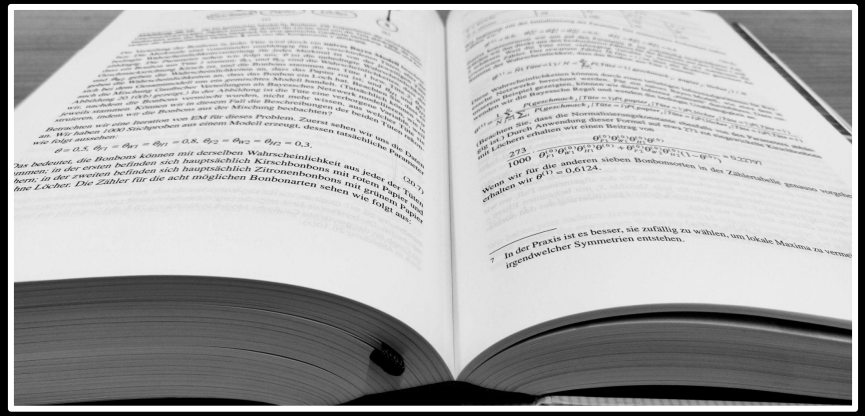
`count_words()`

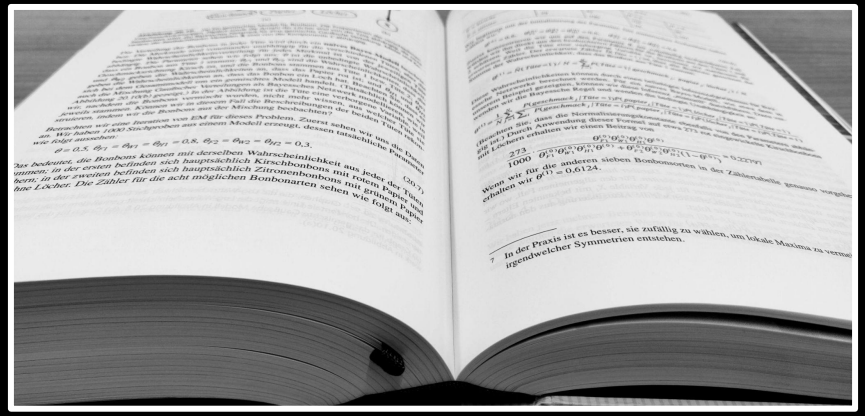
word count

how many words are in the book?

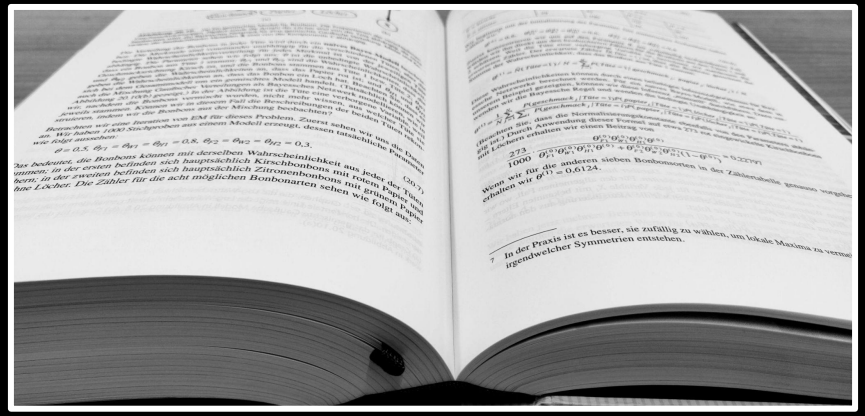


strategies, anyone?



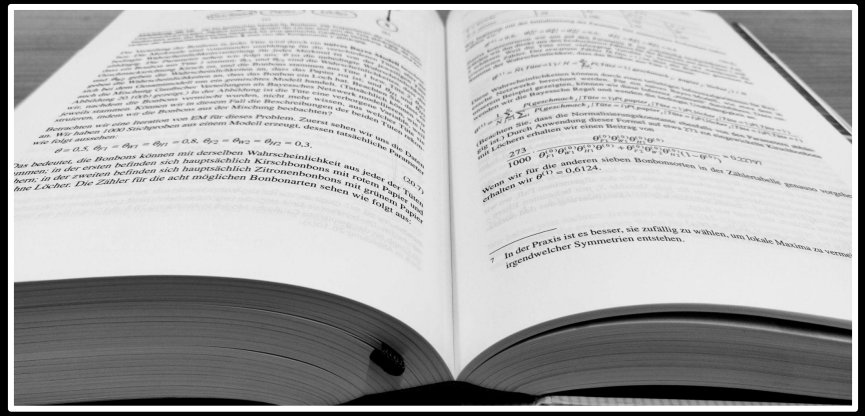


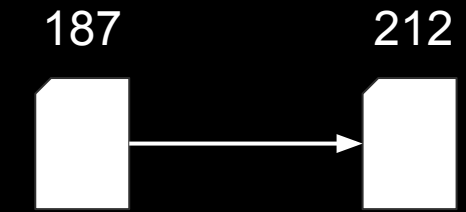
page 1

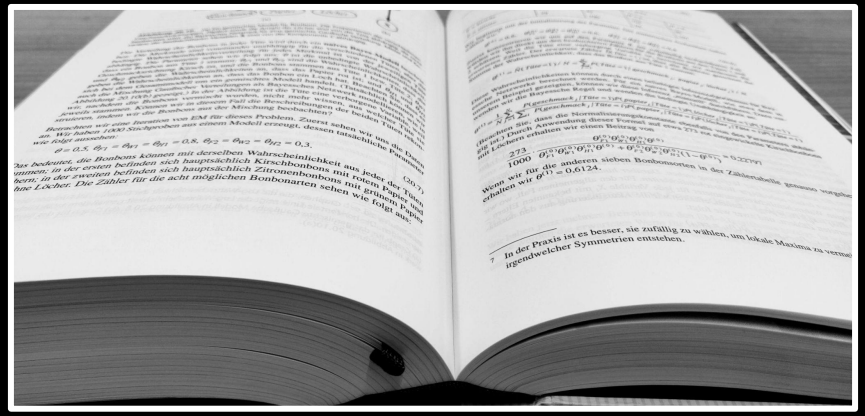


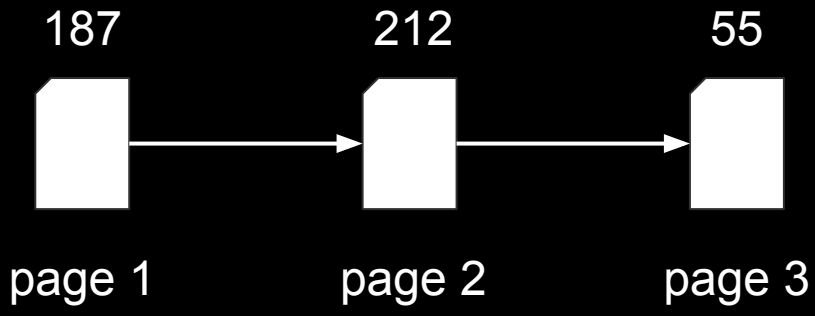
187

page 1




187
212
page 1
page 2

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212
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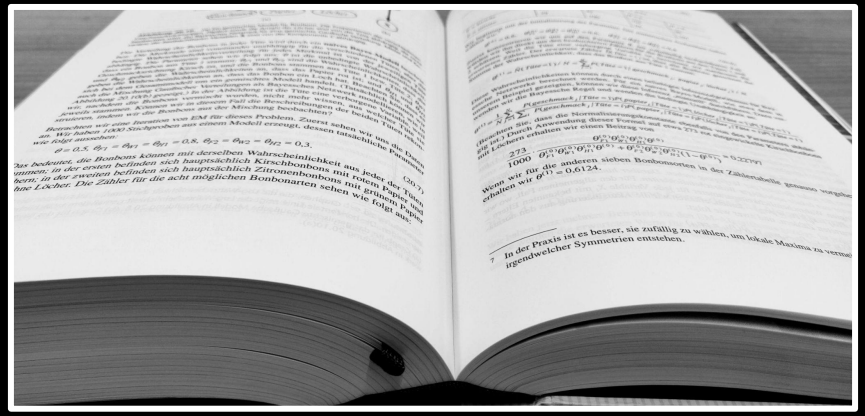
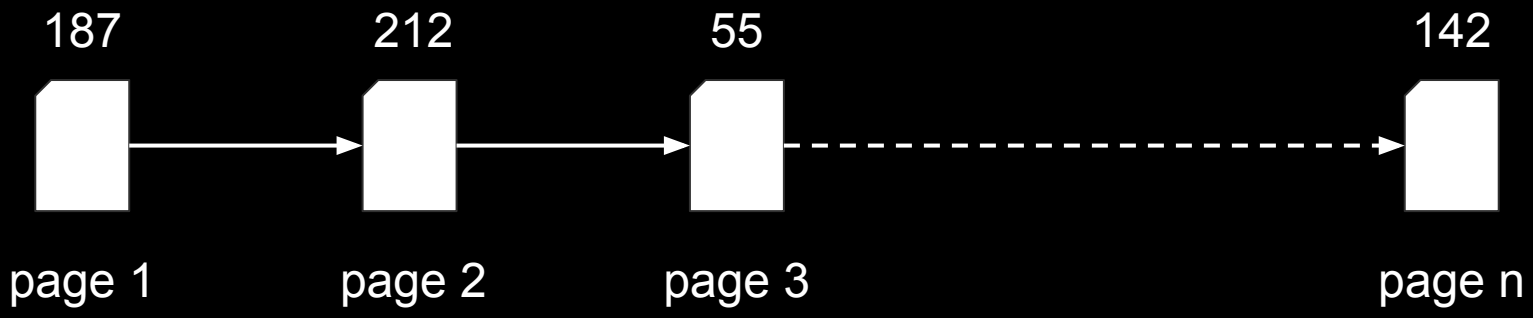
page 2
page 3

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212
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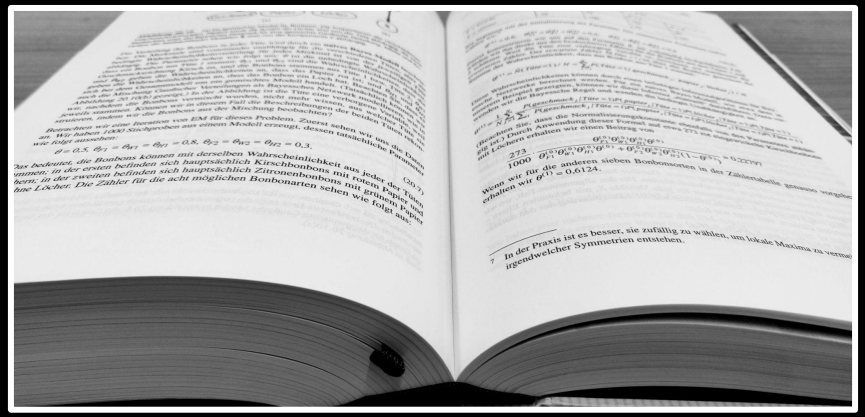
142
page 1
page 2
page 3
page n



n = 1327 pages

Ø 2:23 minutes per page

~ 52.34 hours

divide and conquer

+

?

divide and conquer

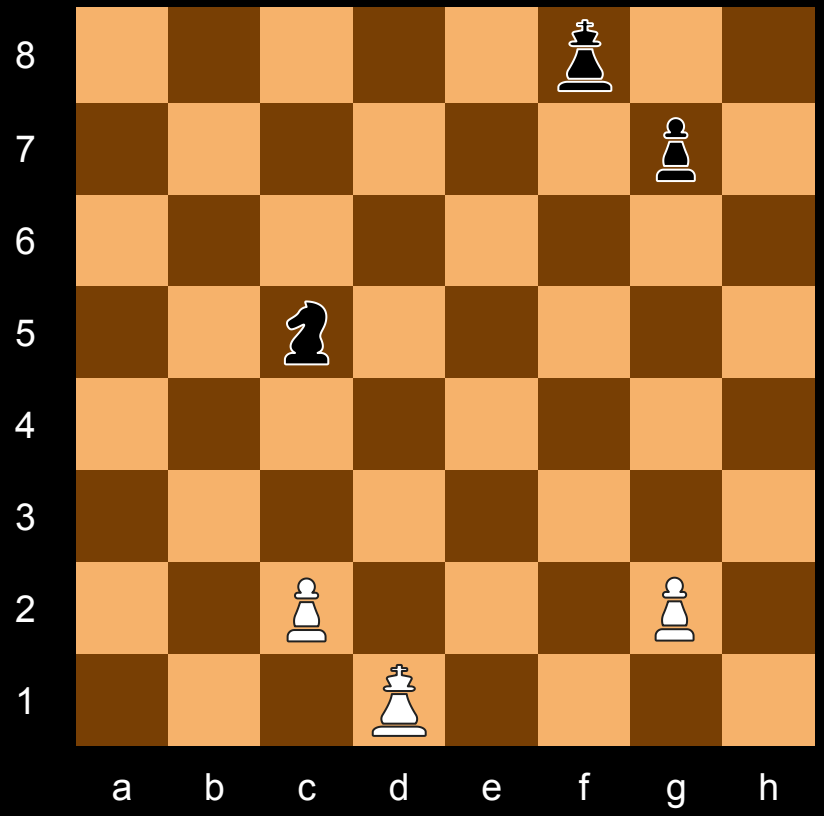
+

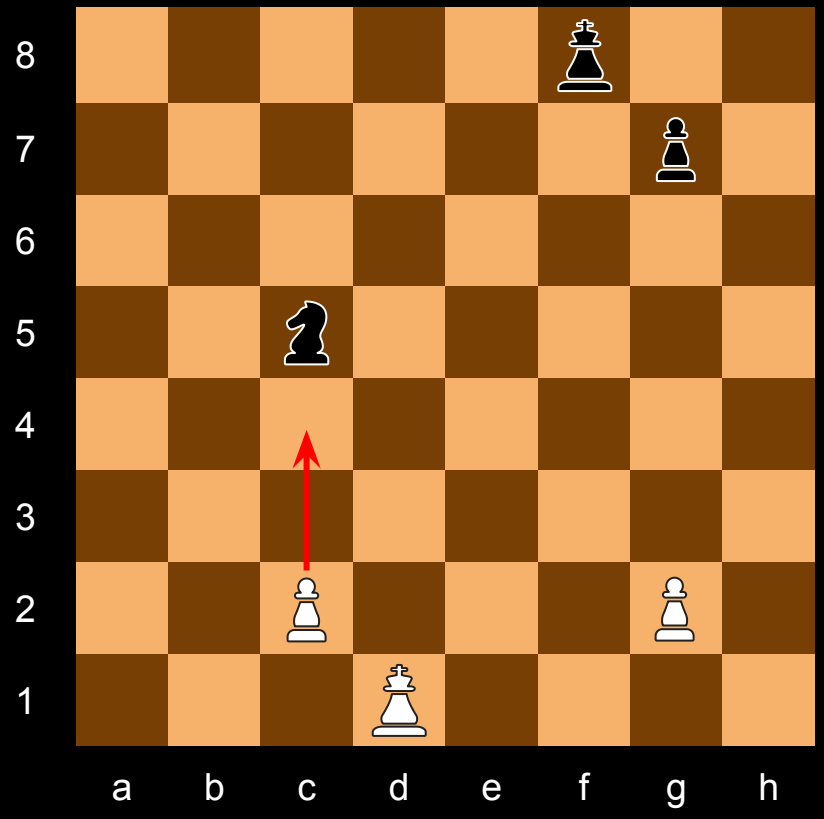
distribution and parallelization

recursion

INFORMATION

Image created with DALLE-3


8
7
6
5
4
3
2
1
a b c d e f g h


8
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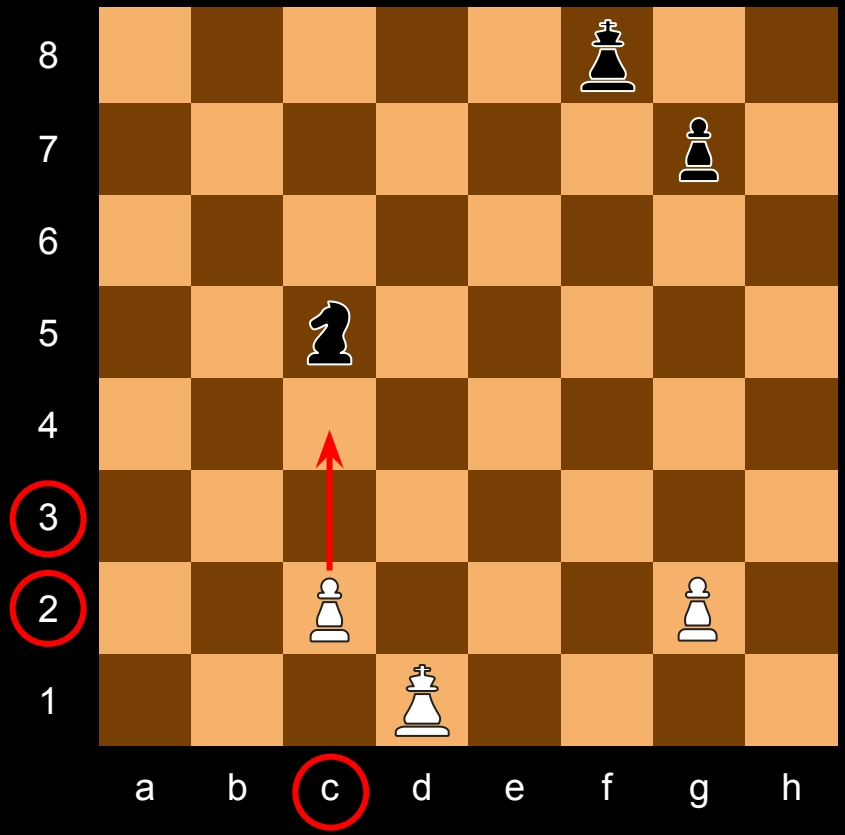
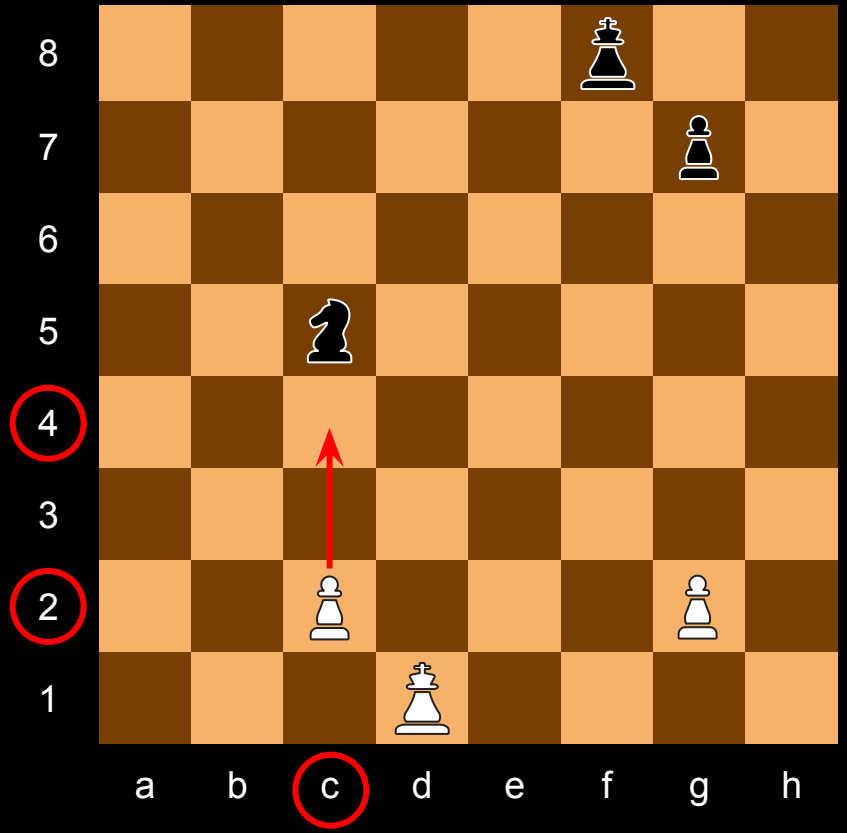
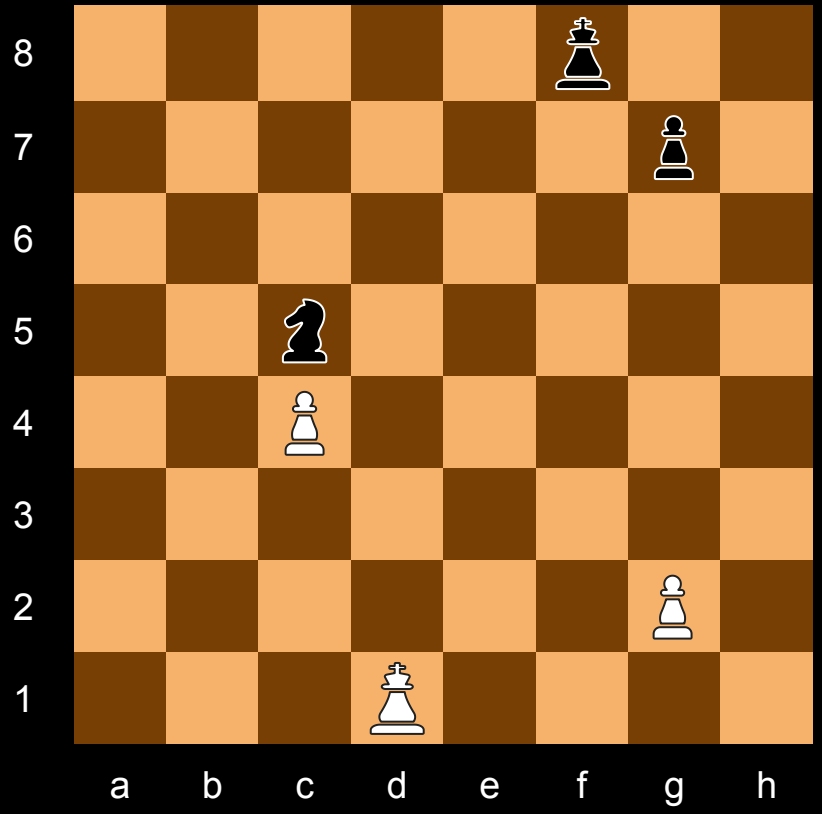
a b c d e f g h


8
7
6
5
4
3
2
1
a b c d e f g h

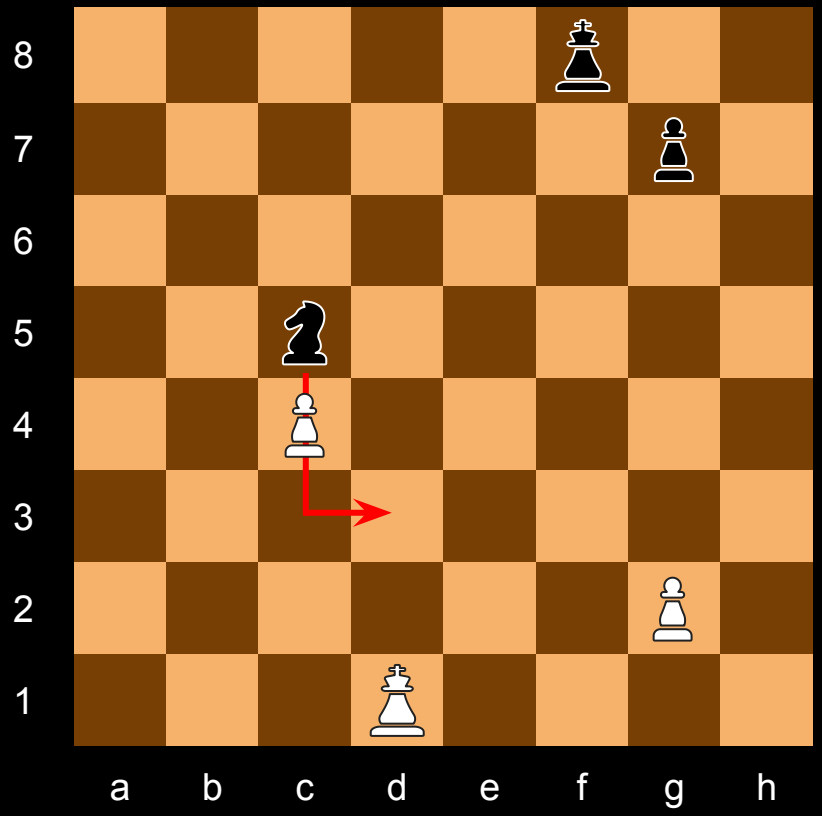
c2 → c4



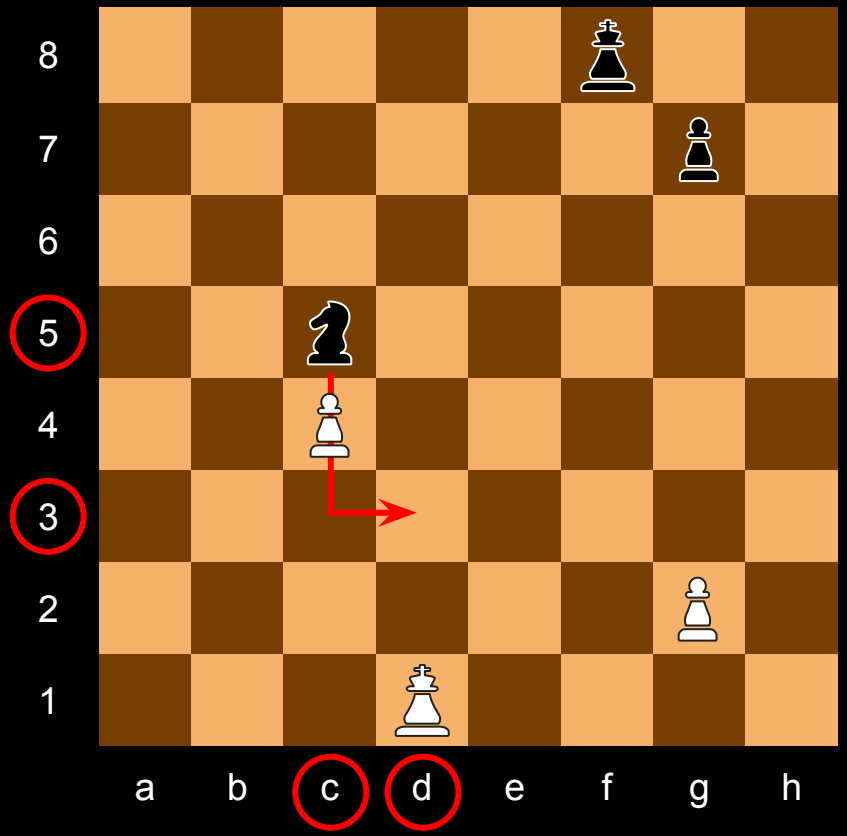
c2 → c4



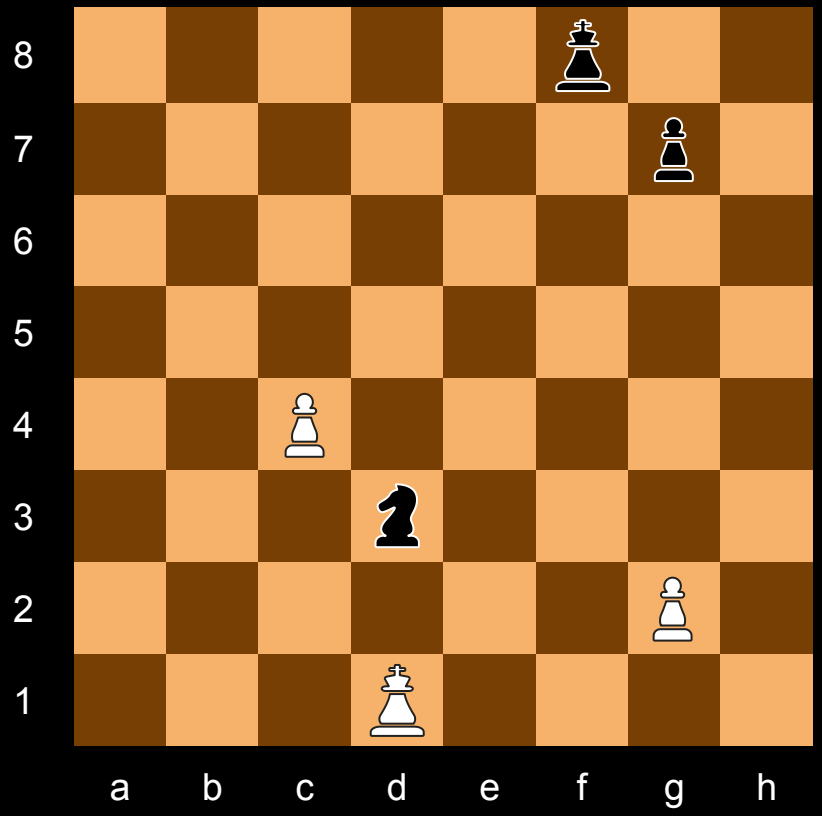
c2 → c4

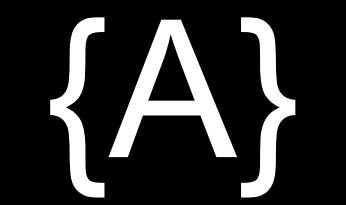


c2 → c4
c5 → d3



c2 → c4
c5 → d3
…




{A}

{A}

A A

{A, B}

_ _

{A, B}

A A

{A, B}

A B

{A, B}

B A

{A, B}

B B

{A, B}

AA, AB, BA, BB

{A, B, C}

_ _

{A, B, C}

AA, AB, BA, BB,
AC, BC, CA, CB, CC

{A, B, C, D}

_ _

{A, B, C, D}

AA, AB, BA, BB, AC, BC, CA, CB,
CC, AD, DA, BD, DB, CD, DC, DD

{A, B, C, D, E}

_ _

{A, B, C, D, E}

AA, AB, BA, BB, AC, BC, CA, CB, CC,
AD, DA, BD, DB, CD, DC, DD, AE, EA,
BE, EB, CE, EC, DE, ED, EE

with length n = 2

# symbols	# messages
1	1
2	4
3	9
4	16
5	25

with length n = 2

# symbols		# messages
1		1
2	f(x)	4
3	→	9
4		16
5		25

{A, B}

_ _ _

COUNTING

1 2 3

1	2	3
10^2	10^1	10^0

1	2	3
10^2	10^1	10^0

$= 1 \times 10^2 + 2 \times 10^1 + 3 \times 10^0$

$= 1 \times 100 + 2 \times 10 + 3 \times 1$

$= 123$

4	1	2	3
?	10^2	10^1	10^0

4	1	2	3
?	10^2	10^1	10^0

$$= 4 \times 10^3 + 1 \times 10^2 + 2 \times 10^1 + 3 \times 10^0$$

4	1	2	3
?	10^2	10^1	10^0

$= 4 \times 10^3 + 1 \times 10^2 + 2 \times 10^1 + 3 \times 10^0$

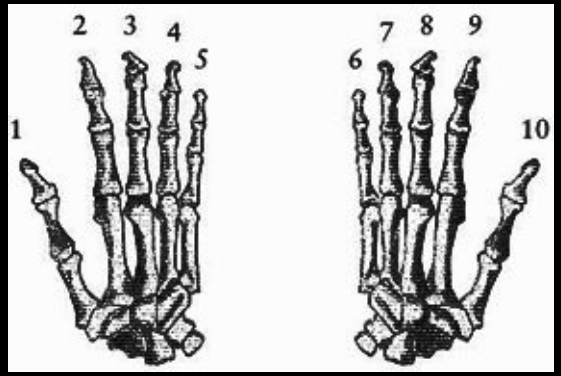
$= 4 \times 1000 + 1 \times 100 + 2 \times 10 + 3 \times 1$

4	1	2	3
?	10^2	10^1	10^0

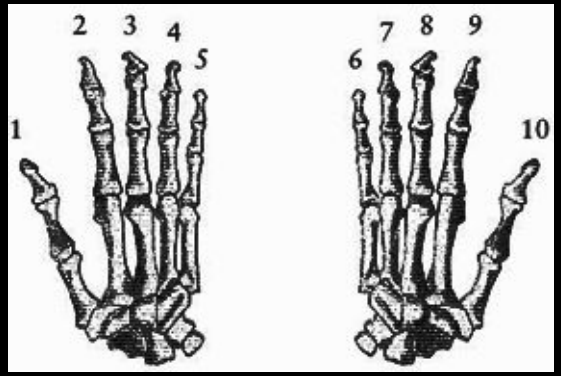
$= 4 \times 10^3 + 1 \times 10^2 + 2 \times 10^1 + 3 \times 10^0$

$= 4 \times 1000 + 1 \times 100 + 2 \times 10 + 3 \times 1$

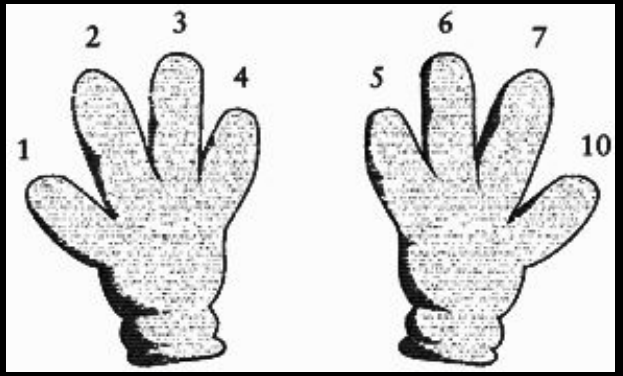
$= 4123$



Human Hand



Human Hand



Cartoon Character’s Hand

1 2 3 (octal)

1	2	3	(octal)
8^2	8^1	8^0	

1	2	3
8^2	8^1	8^0

(octal)

$$= 1 \times 8^2 + 2 \times 8^1 + 3 \times 8^0$$

1	2	3
8^2	8^1	8^0

(octal)

$$= 1 \times 8^2 + 2 \times 8^1 + 3 \times 8^0$$

$$= 1 \times 64 + 2 \times 8 + 3 \times 1$$

1	2	3
8^2	8^1	8^0

(octal)

$= 1 \times 8^2 + 2 \times 8^1 + 3 \times 8^0$

$= 1 \times 64 + 2 \times 8 + 3 \times 1$

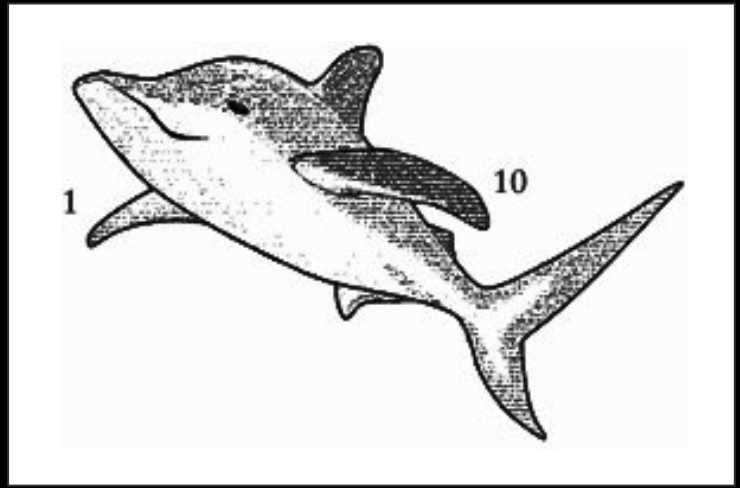
$= 83$ (decimal)

decimal
octal
8
?

decimal
octal
?
7

decimal
octal
16
?

decimal
octal
?
100



What now?

0, 1, …

0, 1, 10, …

0, 1, 10, 11, …

0, 1, 10, 11, 100, …

0, 1, 10, 11, 100, 101, …

0, 1, 10, 11, 100, 101, 110

1 1 0 (binary)

1	1	0	(binary)
2^2	2^1	2^0	

1 1 0 (binary)

2^2 2^1 2^0

$= 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$

1 1 0 (binary)

2^2 2^1 2^0

$= 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$

$= 1 \times 4 + 1 \times 2 + 0 \times 1$

1	1	0	(binary)
2^2	2^1	2^0	

$= 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$

$= 1 \times 4 + 1 \times 2 + 0 \times 1$

$= 6$ (decimal)

2 3 4 5 6

0, 1, 10, 11, 100, 101, 110

Place Value Systems

$$N = d_n * R^{n-1} + \ldots + d_1 * R^1 + d_0 * R^0$$

$d \in \{ 0, 1, \ldots R-1 \}$

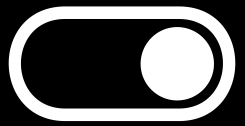
n = Number of digits

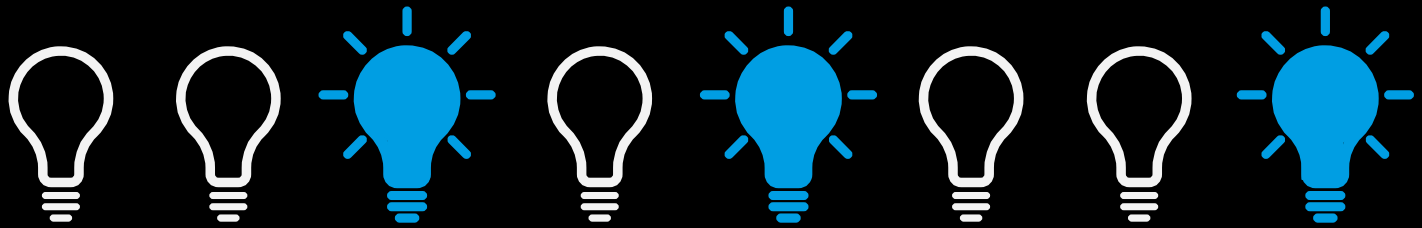
Place Value Systems

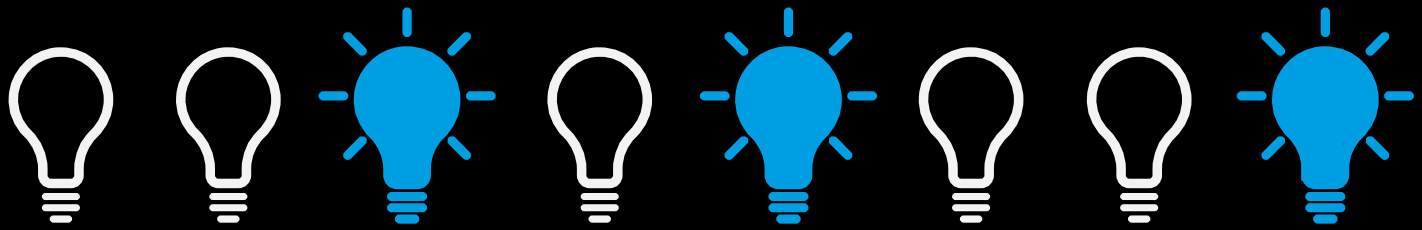
$$R \geq 2$$

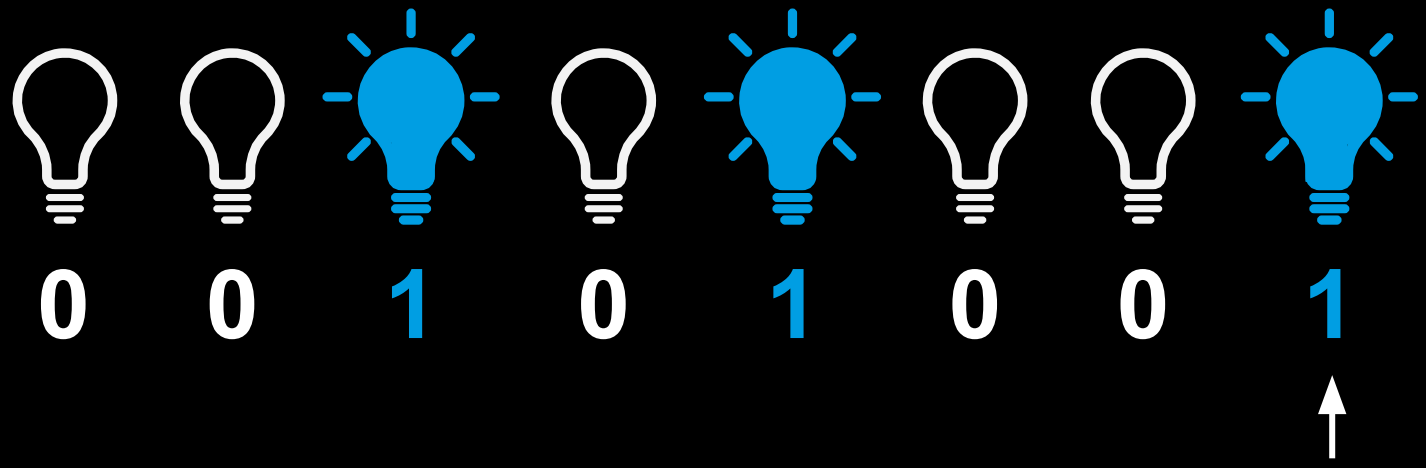
BITS

Why do computers think binary?

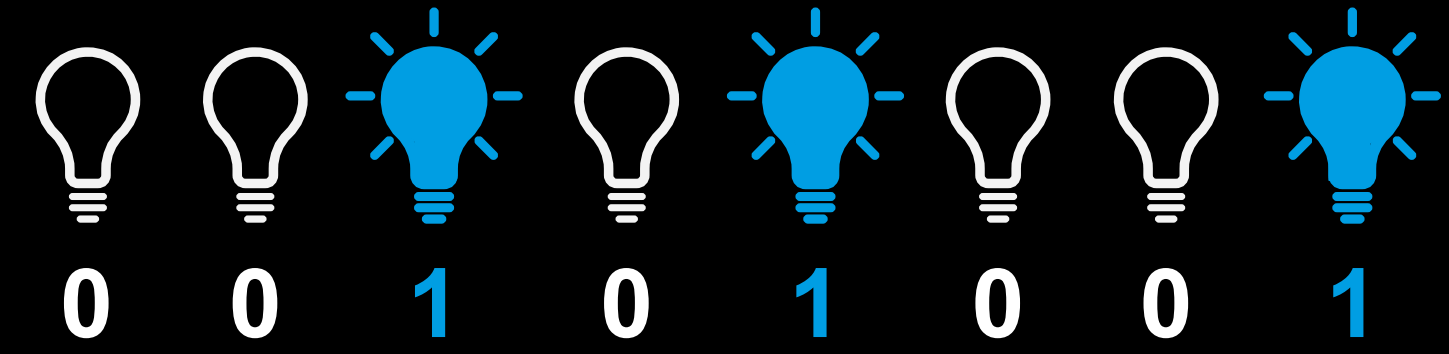





0 0 1 0 1 0 0 1

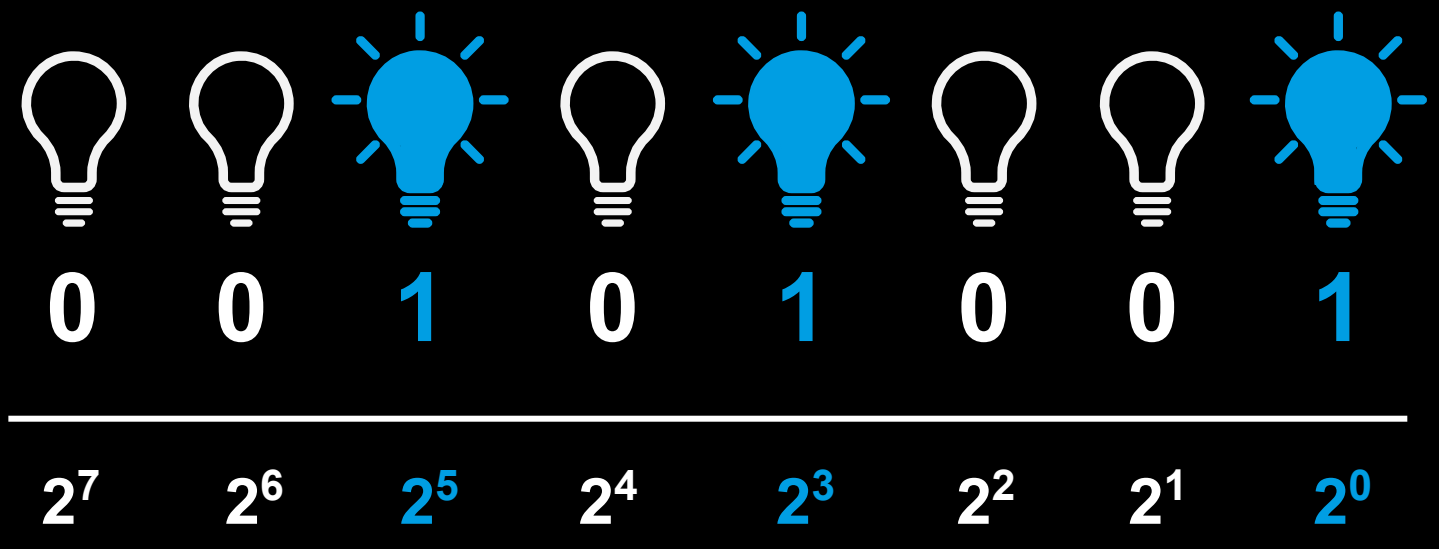


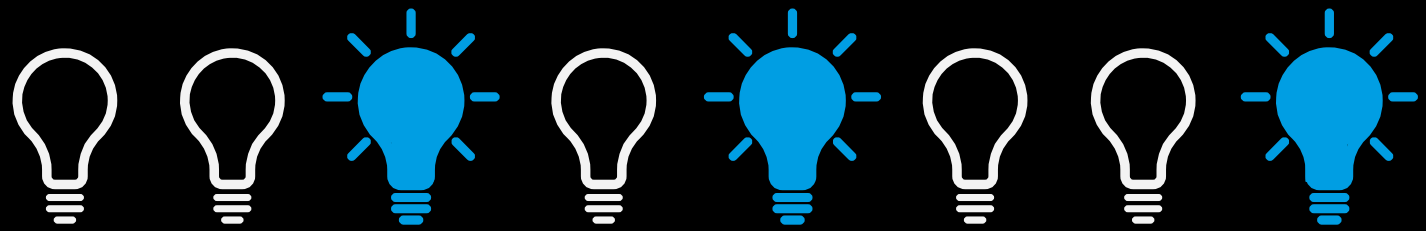
A **Bit** (binary digit)



A **Bit** (binary digit)

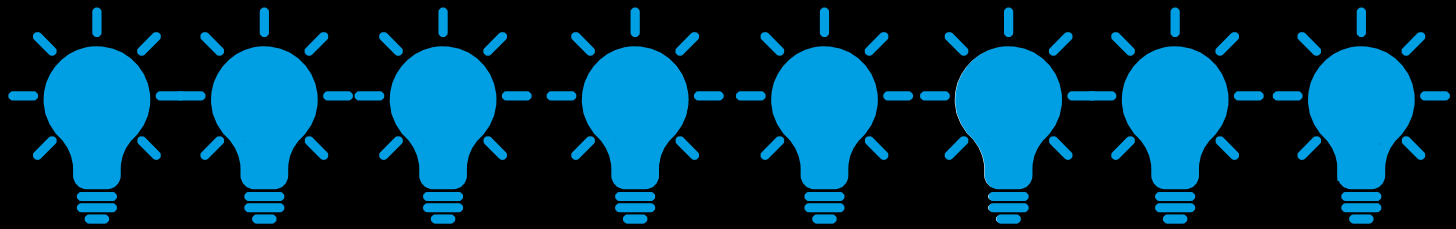
A **byte** (8 bits)


0 0 1 0 1 0 0 1
2^7 2^6 2^5 2^4 2^3 2^2 2^1 2^0



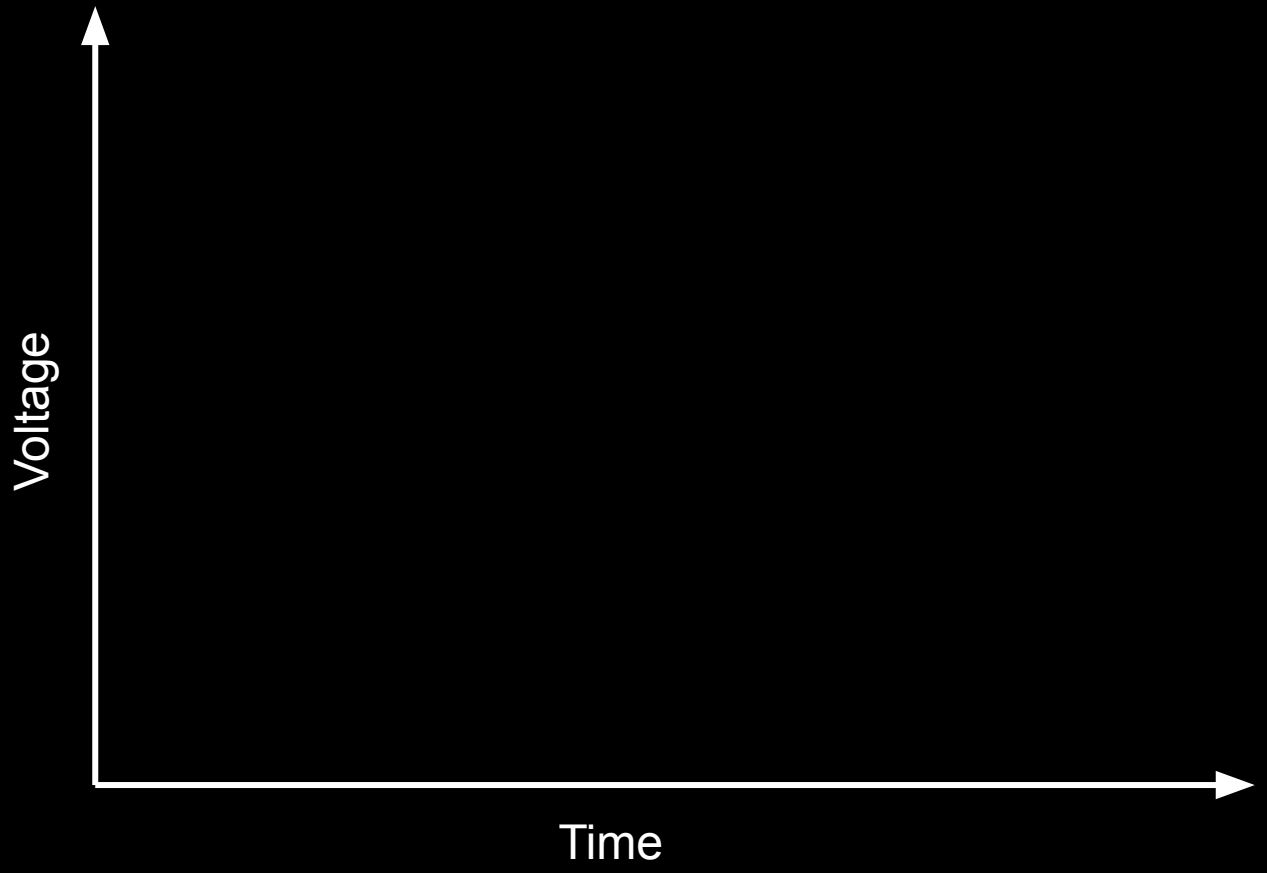
0	0	1	0	1	0	0	1

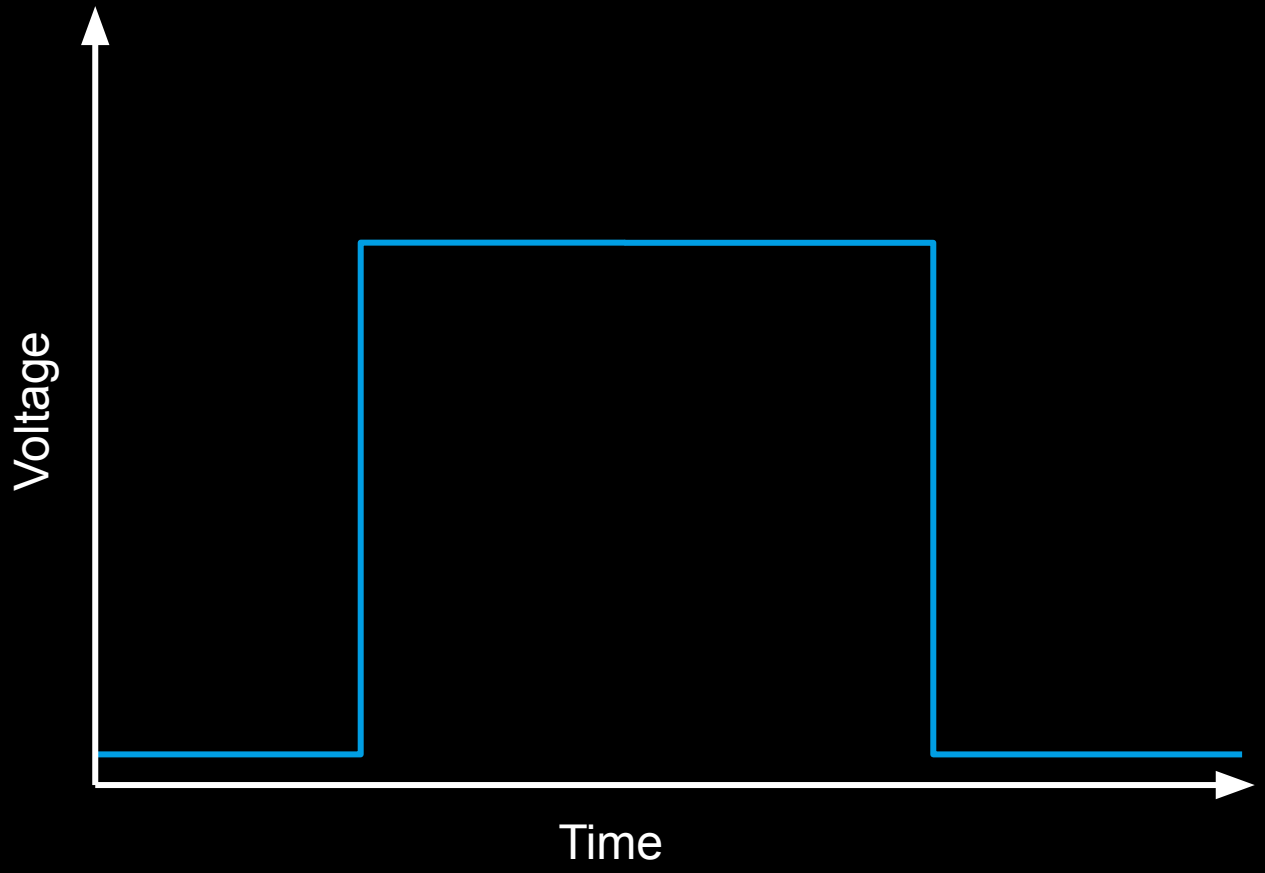
2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
128	64	32	16	8	4	2	1

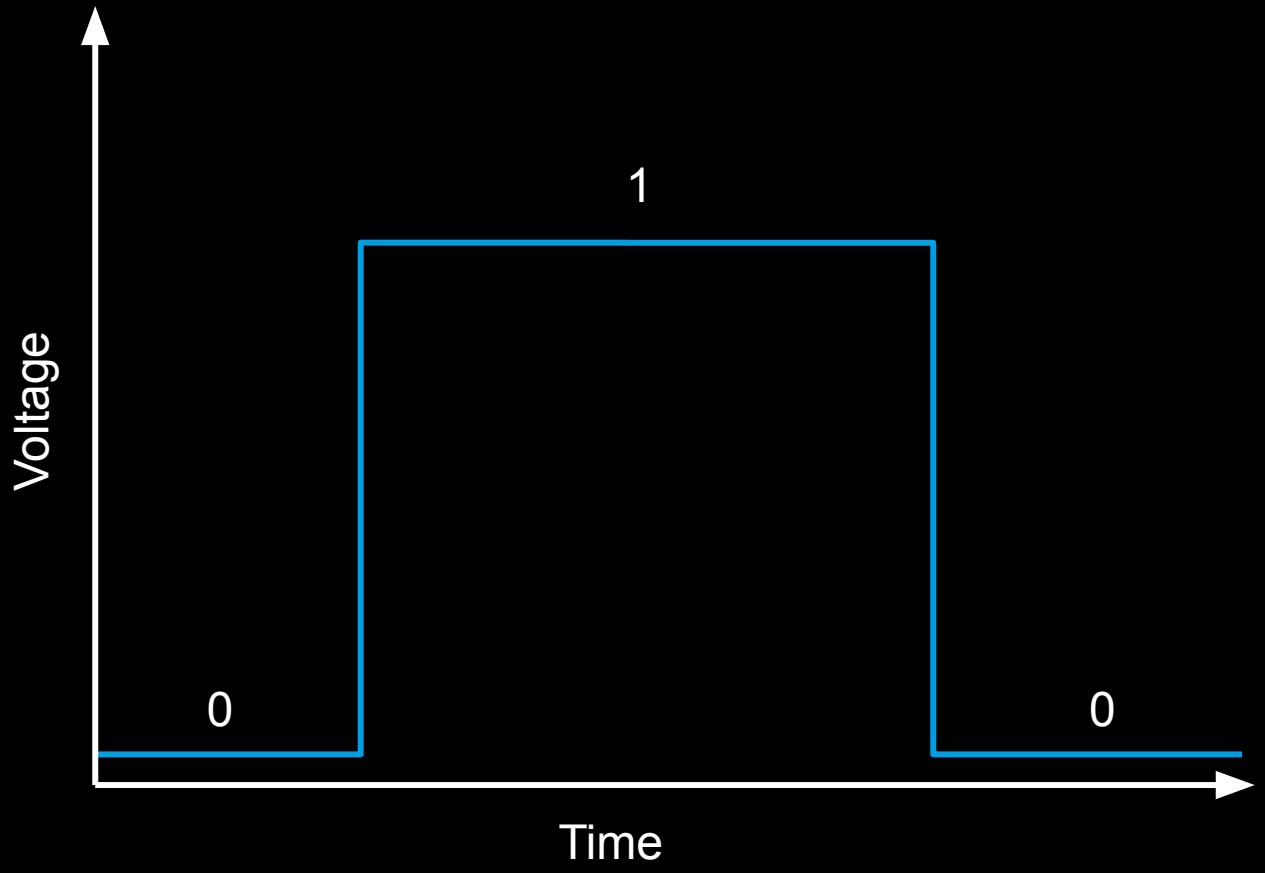


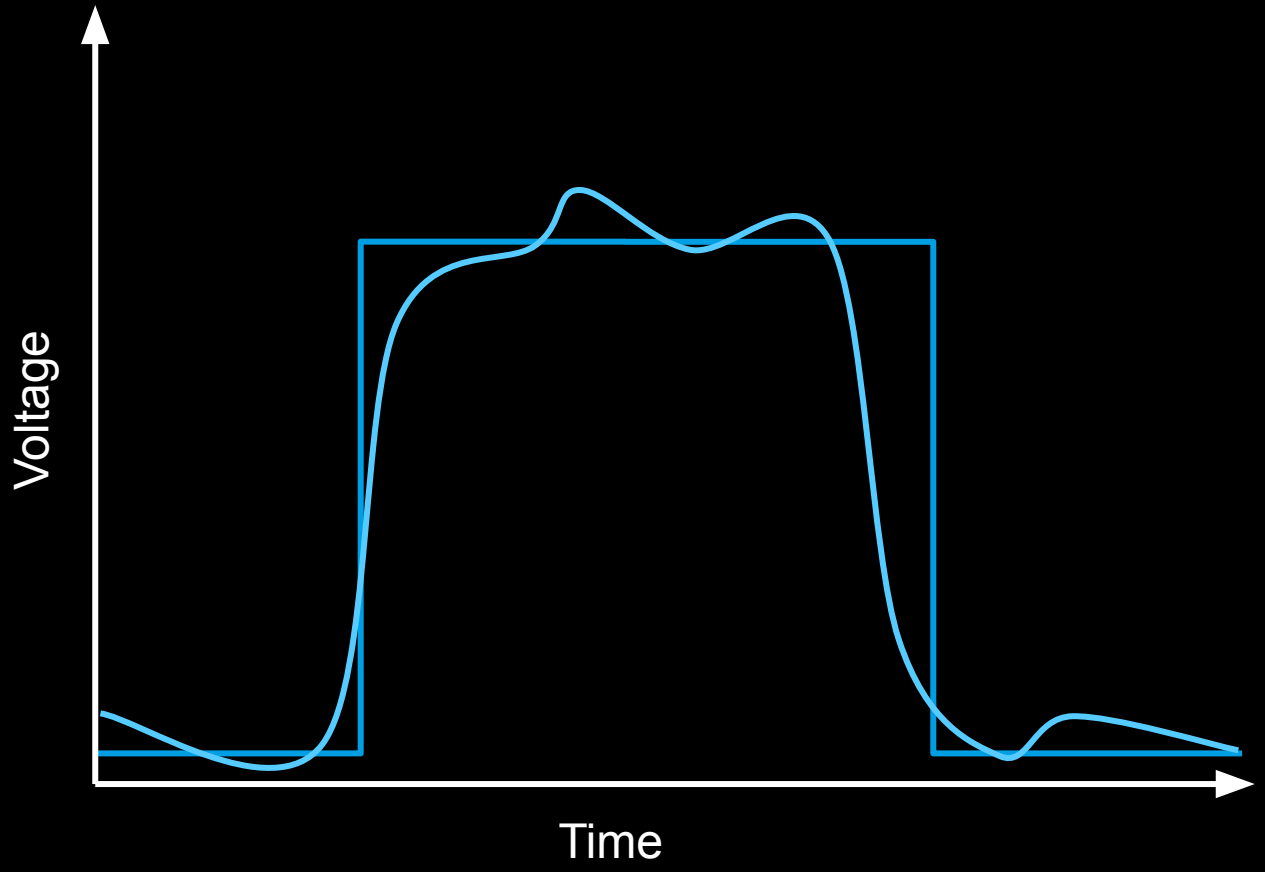
What can we store in one byte?

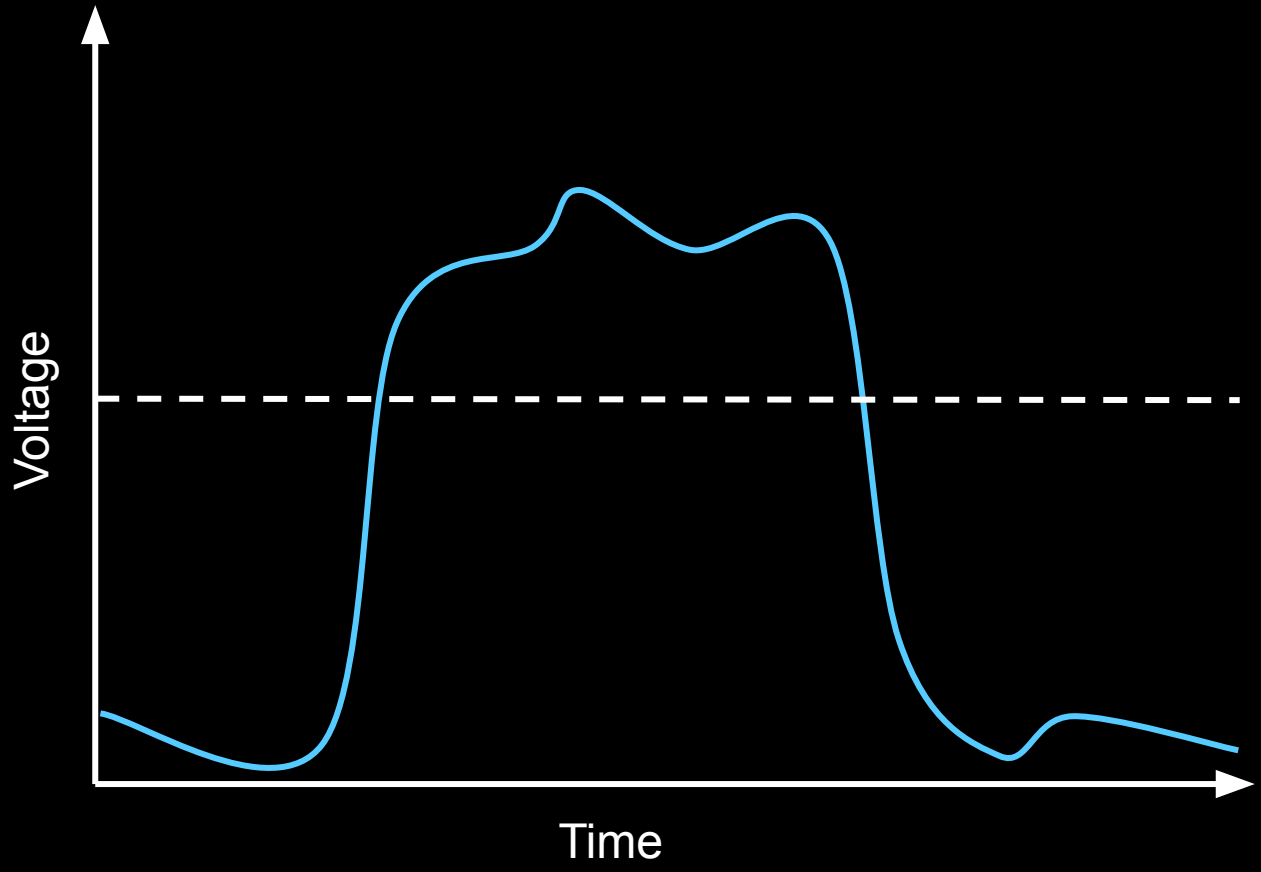
Are we stuck with binary?

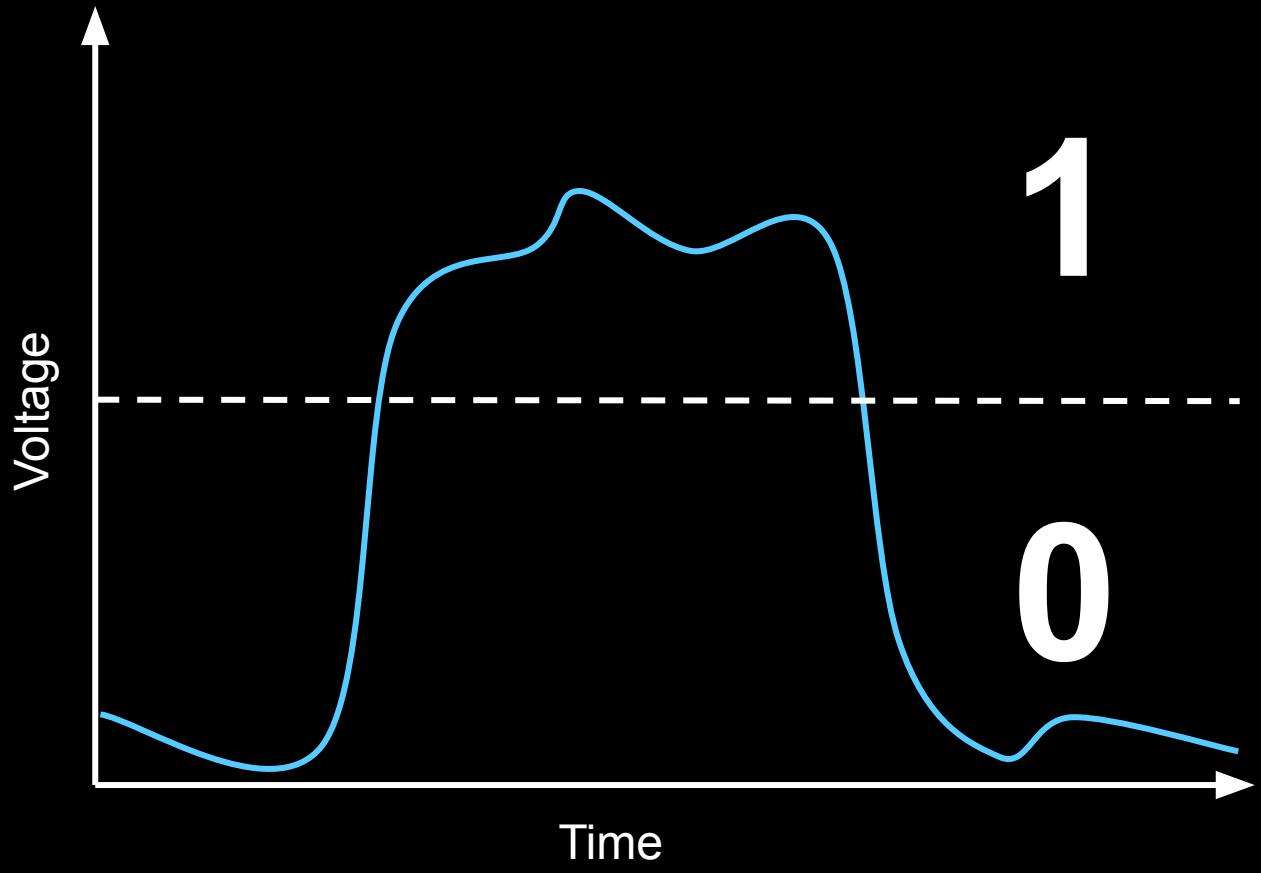

Voltage
Time

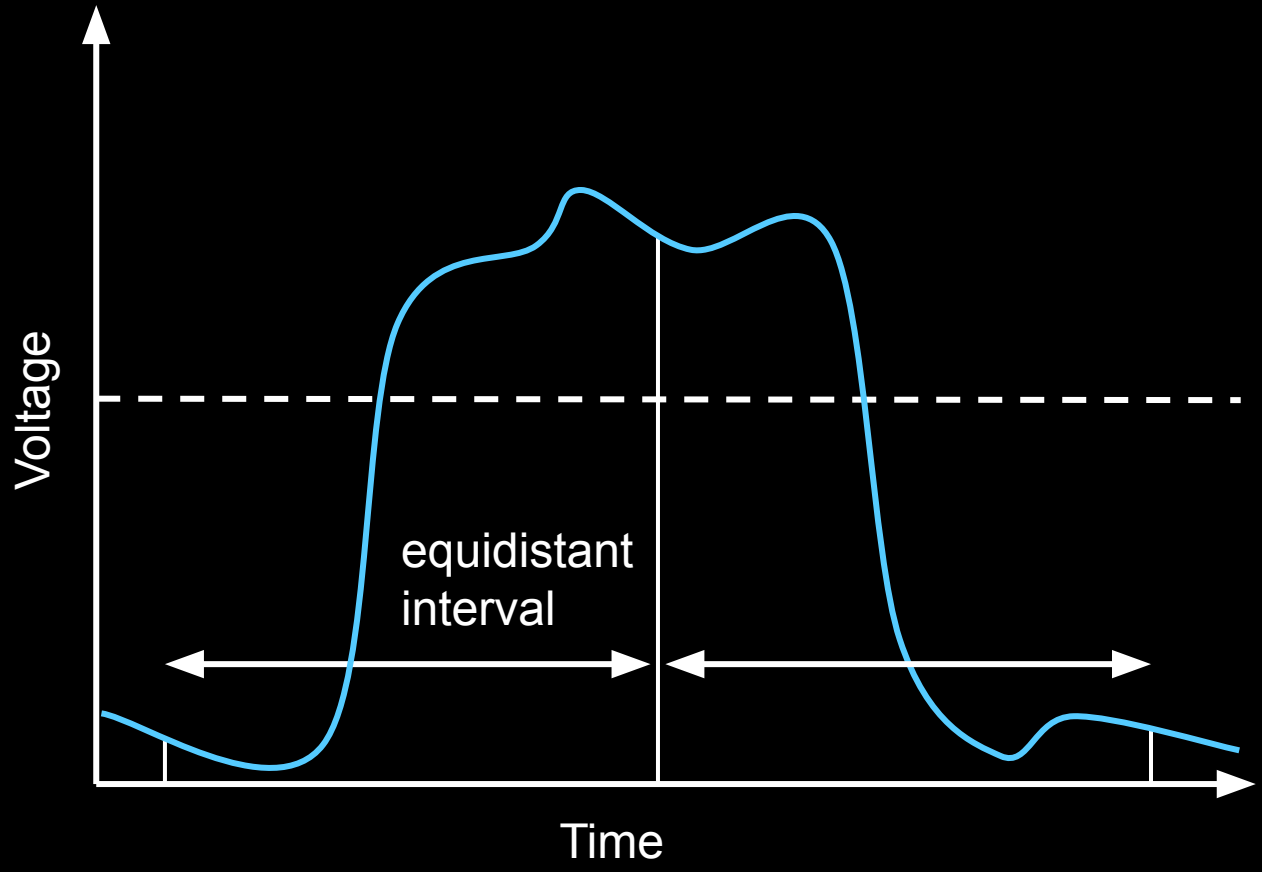

Voltage
Time

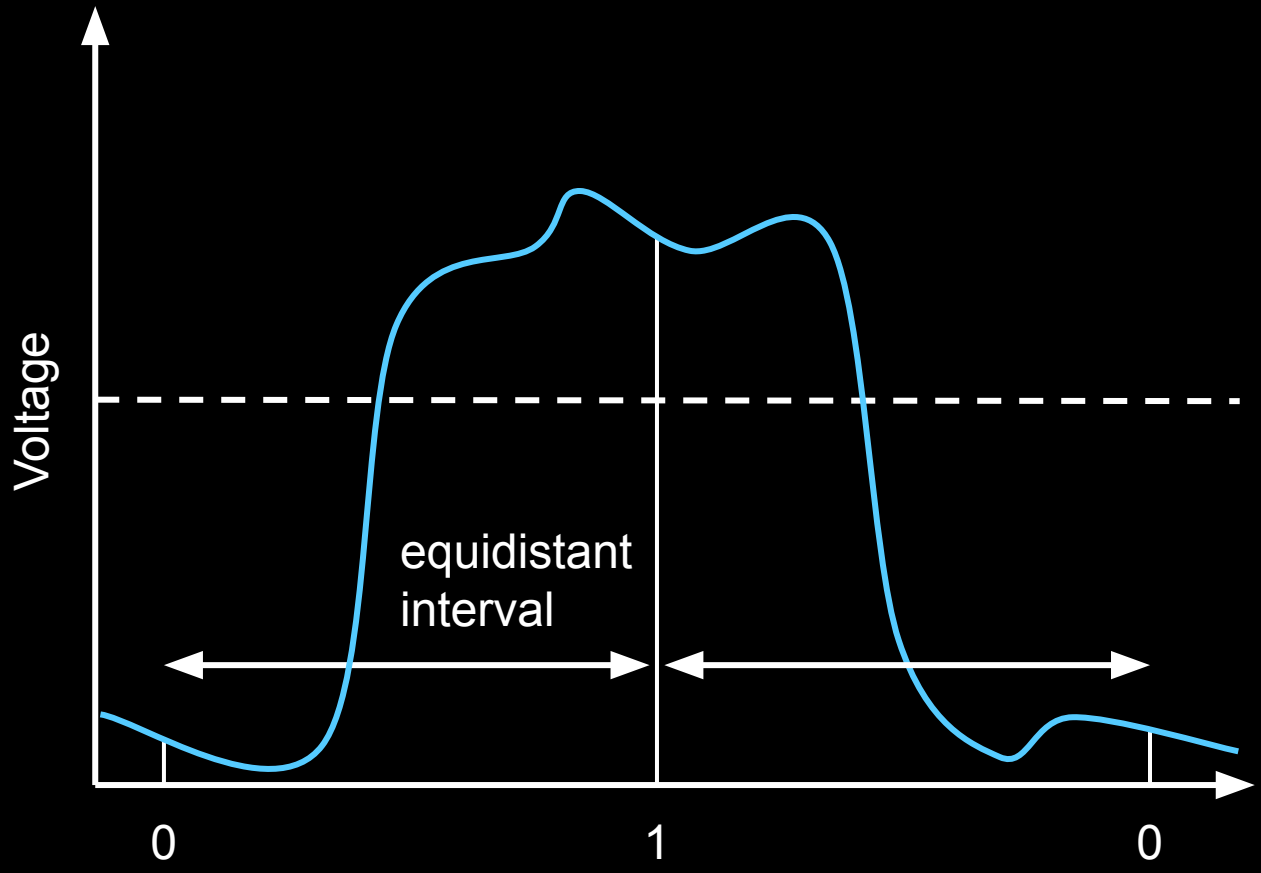

1
Voltage
0
0
Time

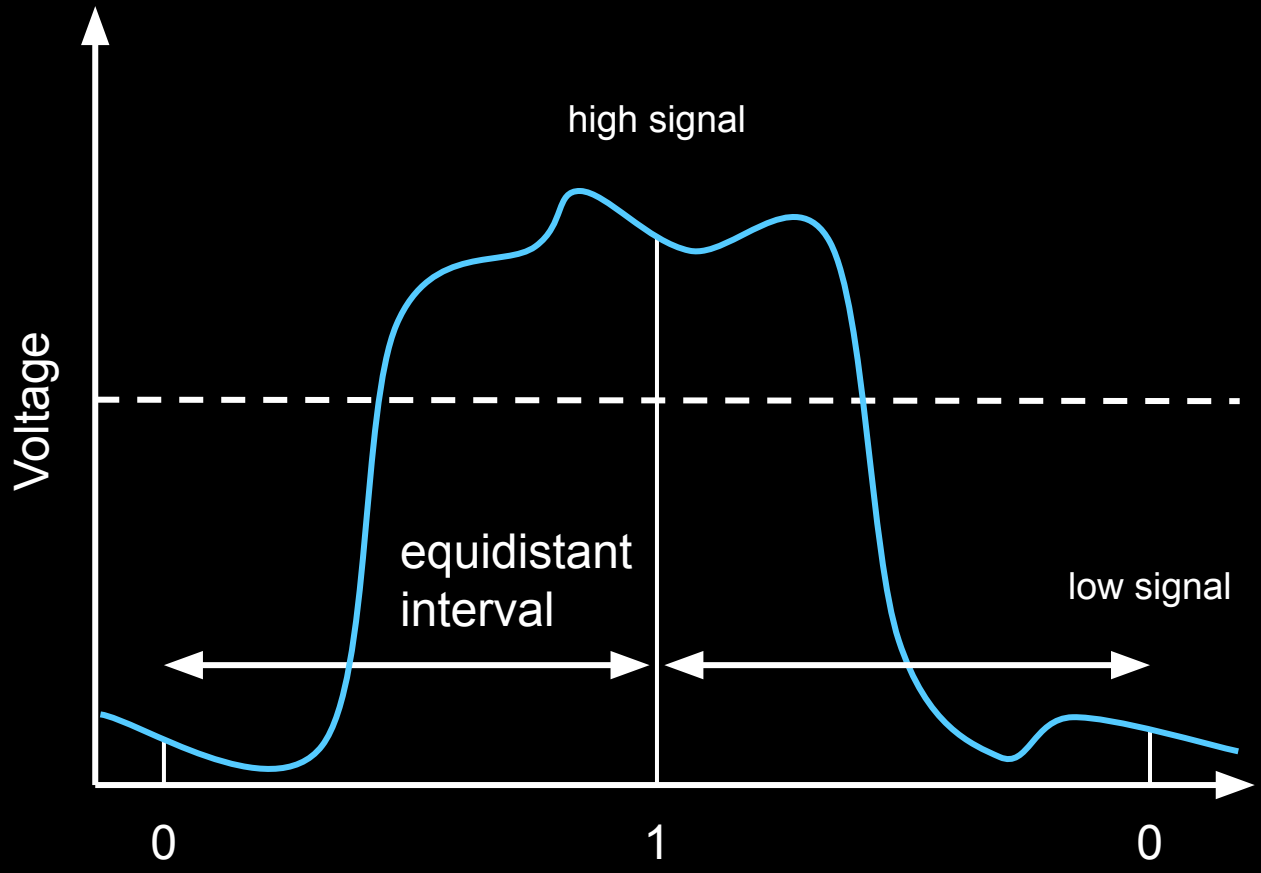

Voltage
Time


Voltage
Time


Voltage
Time
1
0

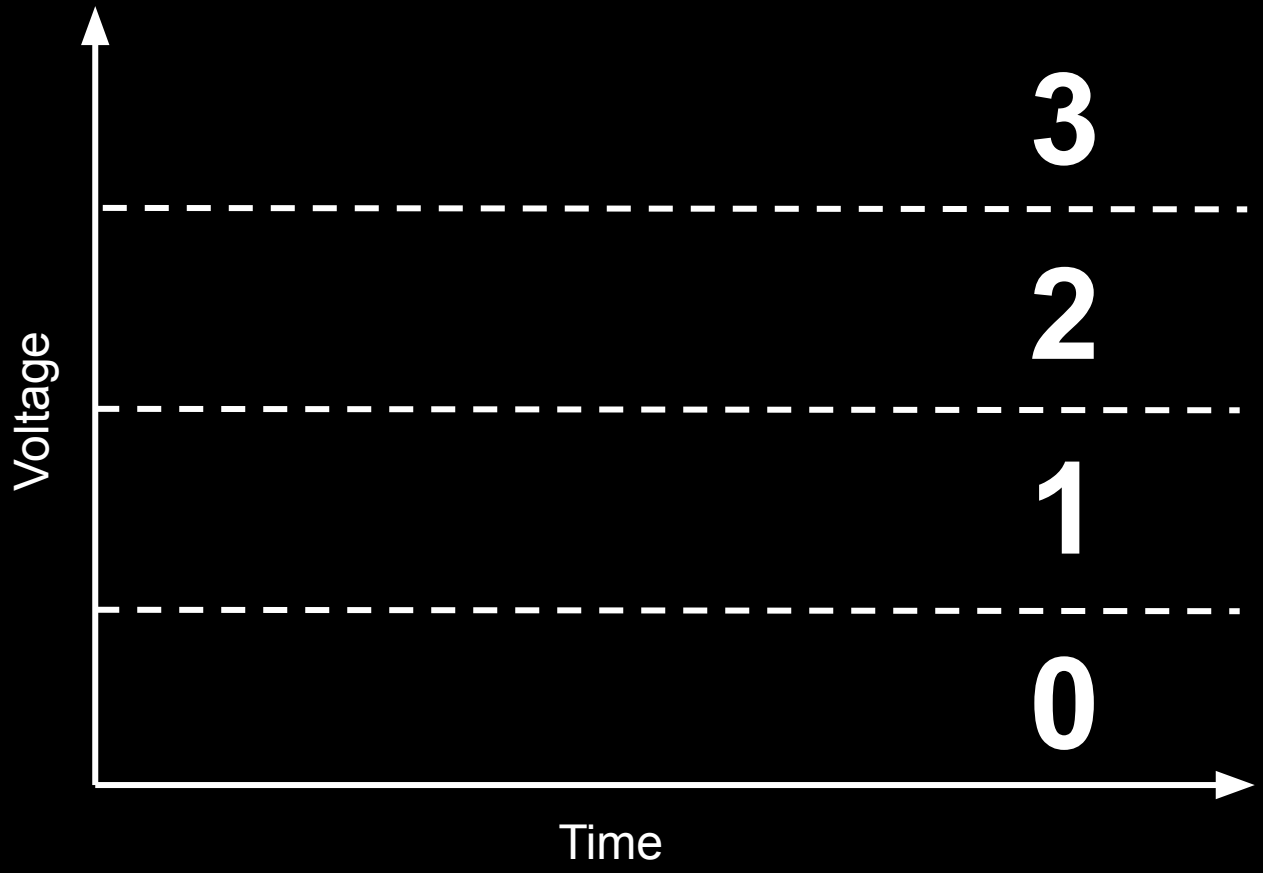

Voltage
equidistant
interval
Time

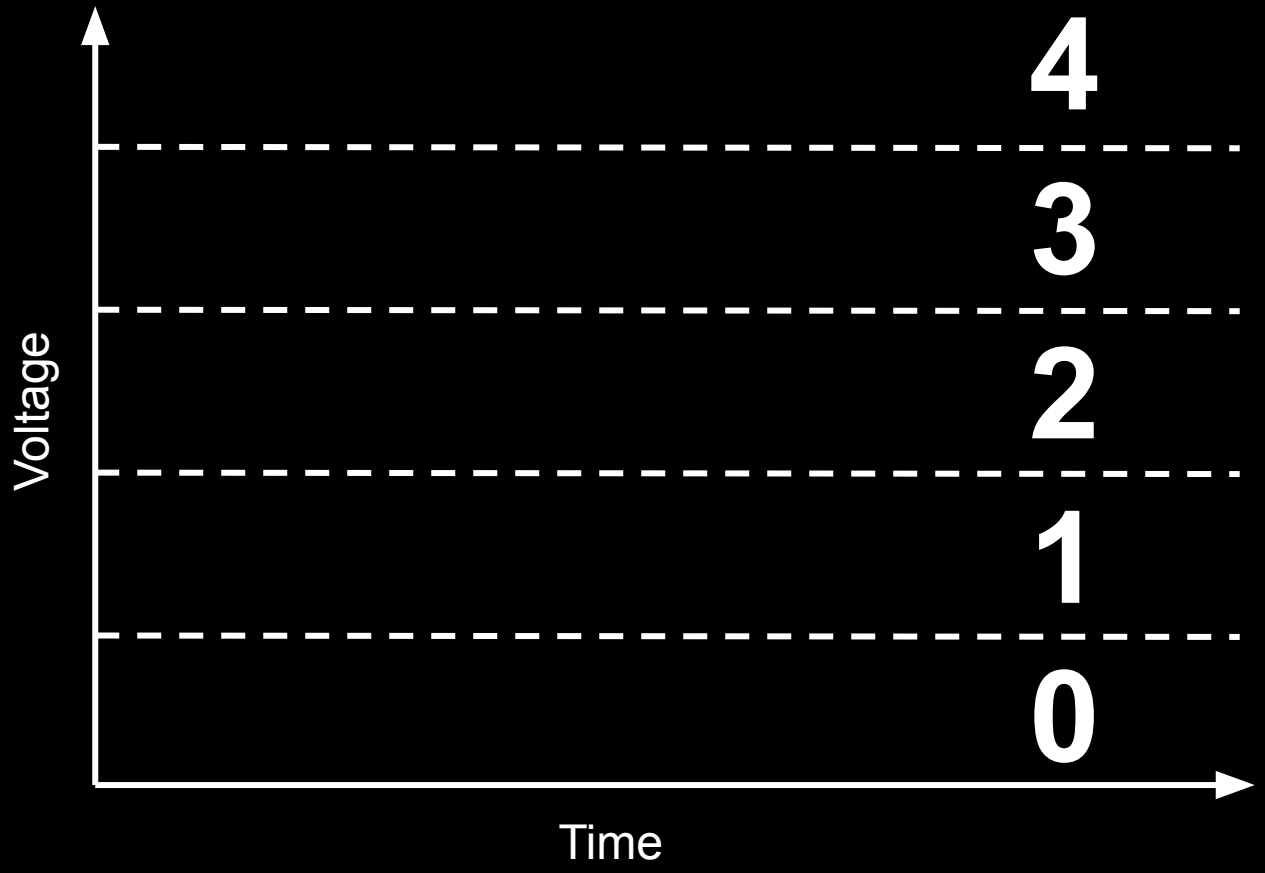

Voltage
equidistant
interval
0
1
0

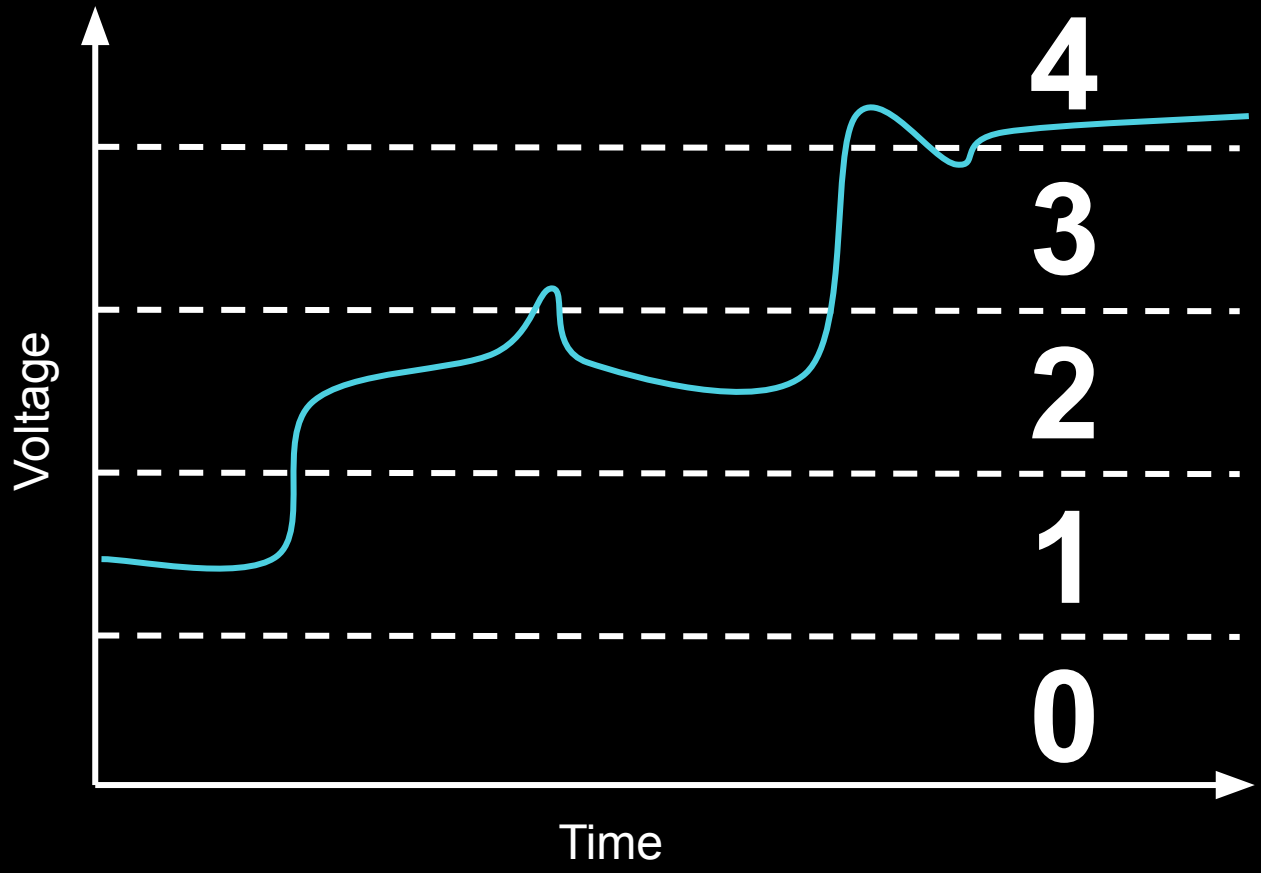

high signal
Voltage
equidistant
interval
low signal
0
1
0

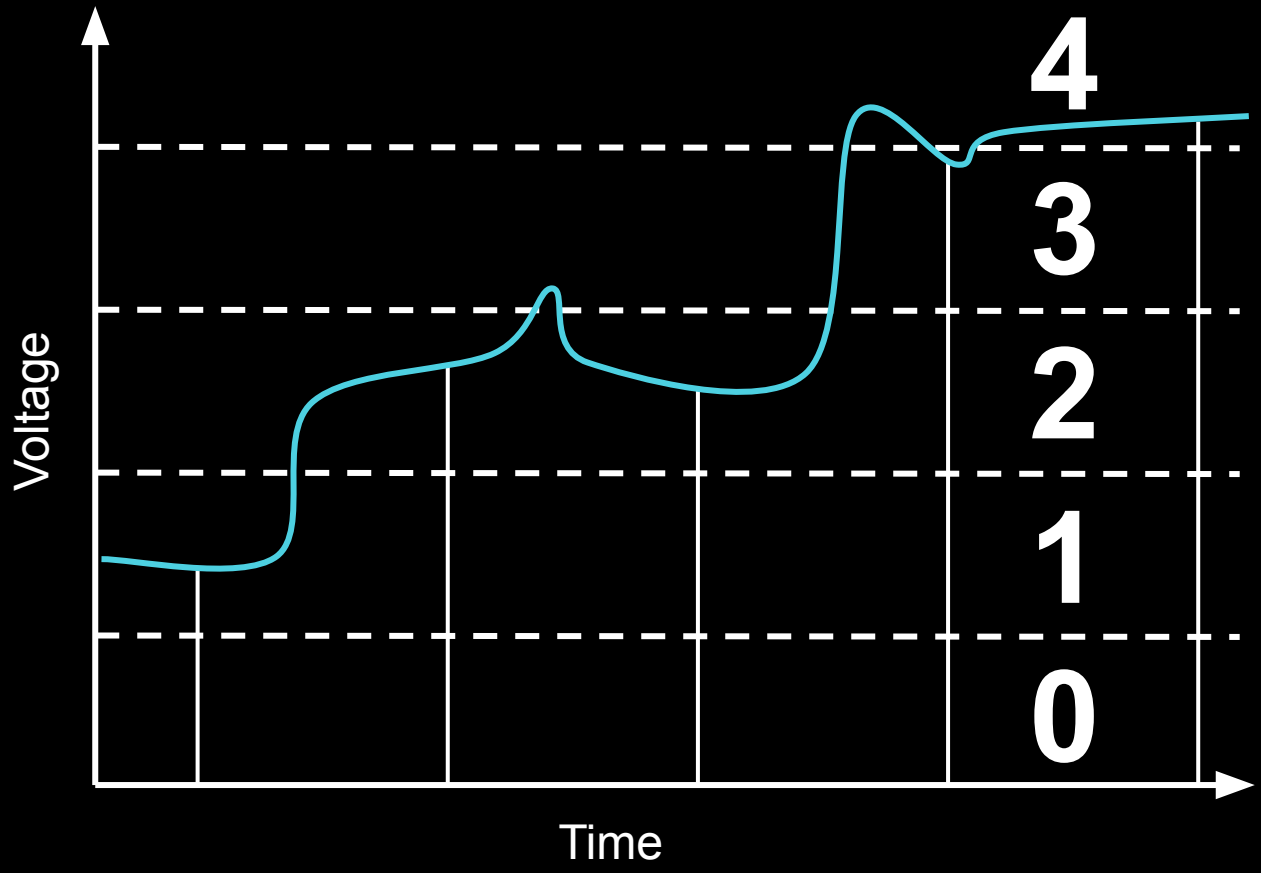
What about ternary?

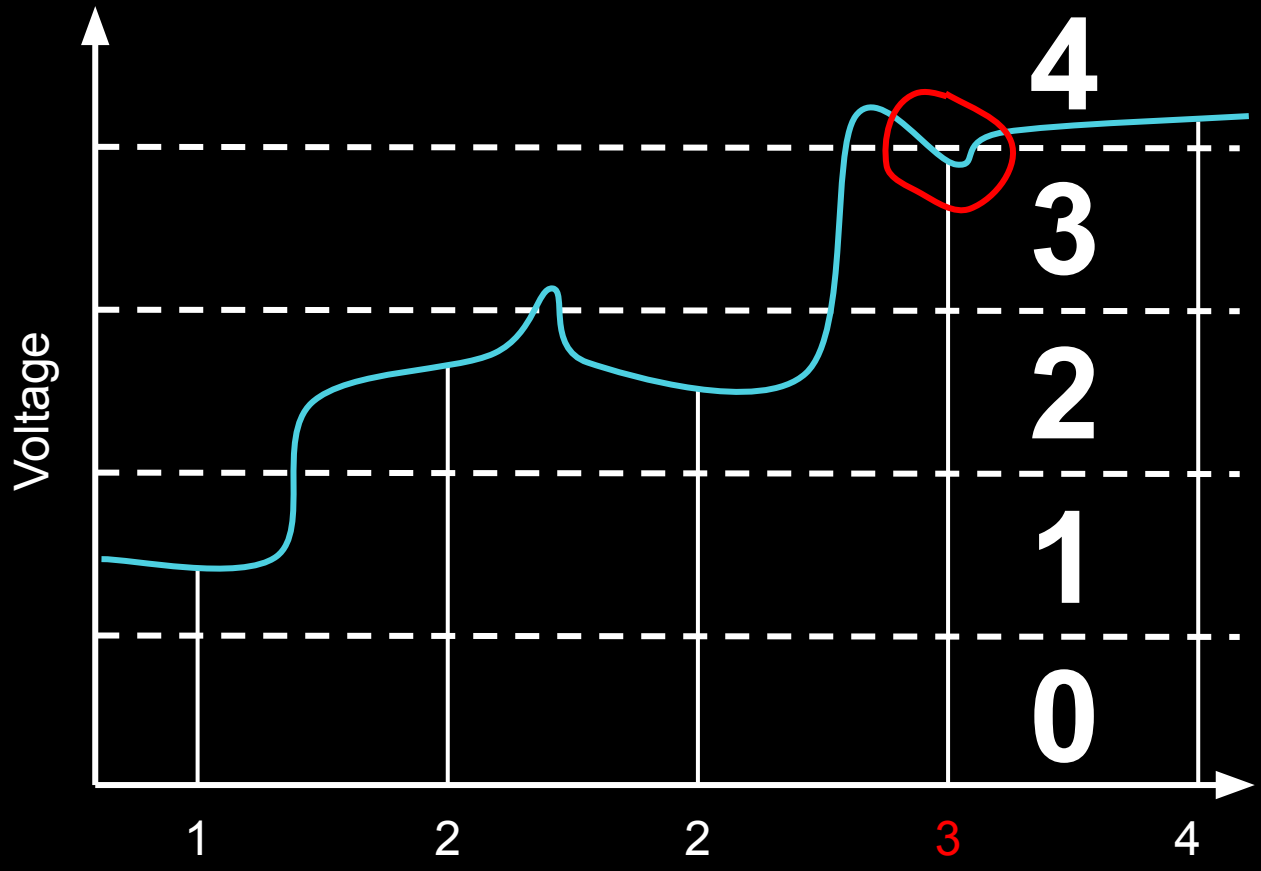
2
1
0
Voltage
Time


3
2
1
0
Voltage
Time

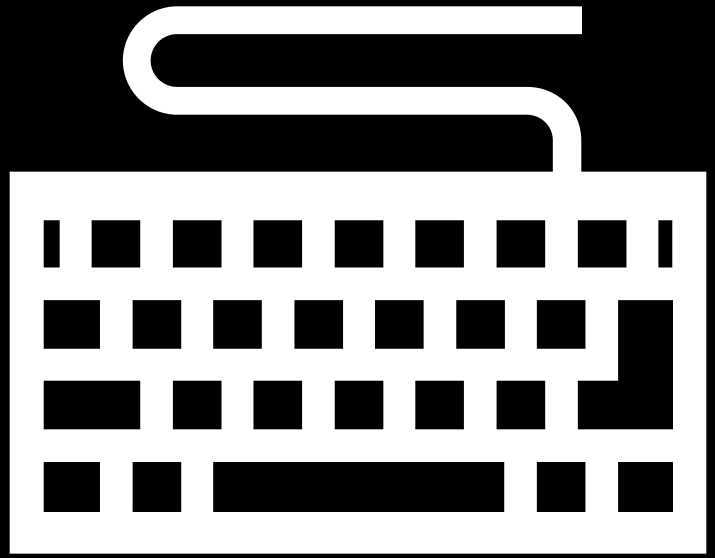

4
3
2
1
0
Voltage
Time


4
3
2
1
0
Voltage
Time


4
3
2
1
0
Voltage
Time


Voltage
4
3
2
1
0
1
2
2
3
4

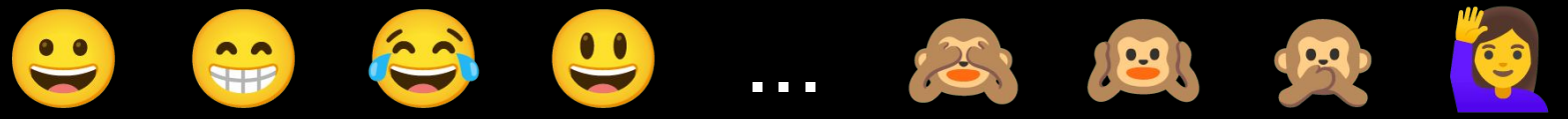
CODES



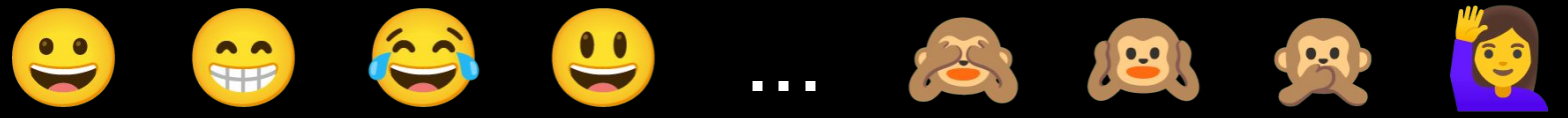
A	B	C	D	...	a	b	c	d
65	66	67	68		97	98	99	100

ASCII Code

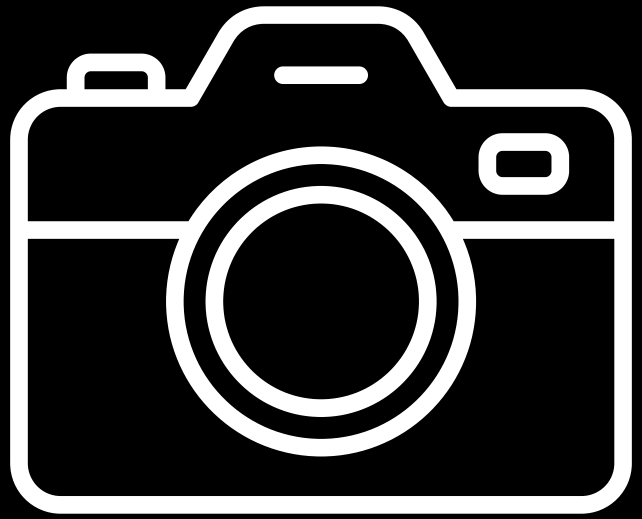
A	B	C	D	...	a	b	c	d
65	66	67	68		97	98	99	100

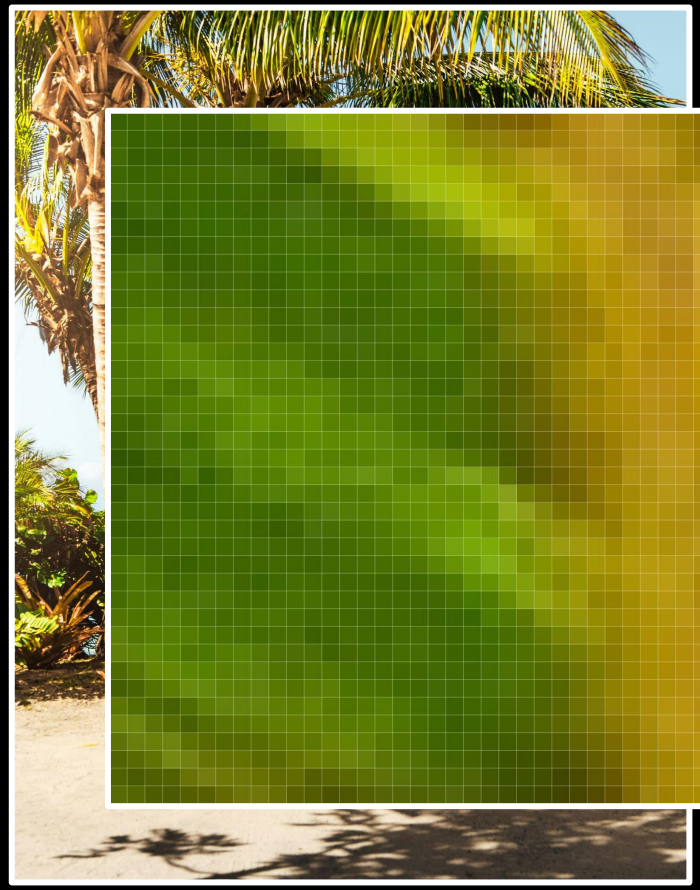

1F600
1F601
1F602
1F603
...
1F648
1F649
1F64A
1F64B

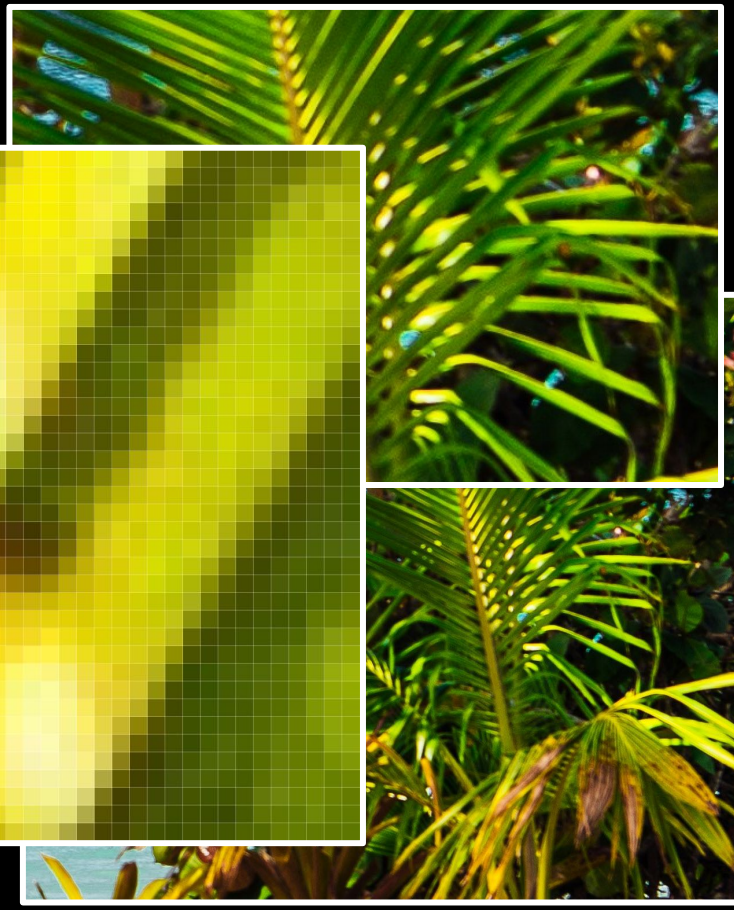
Unicode

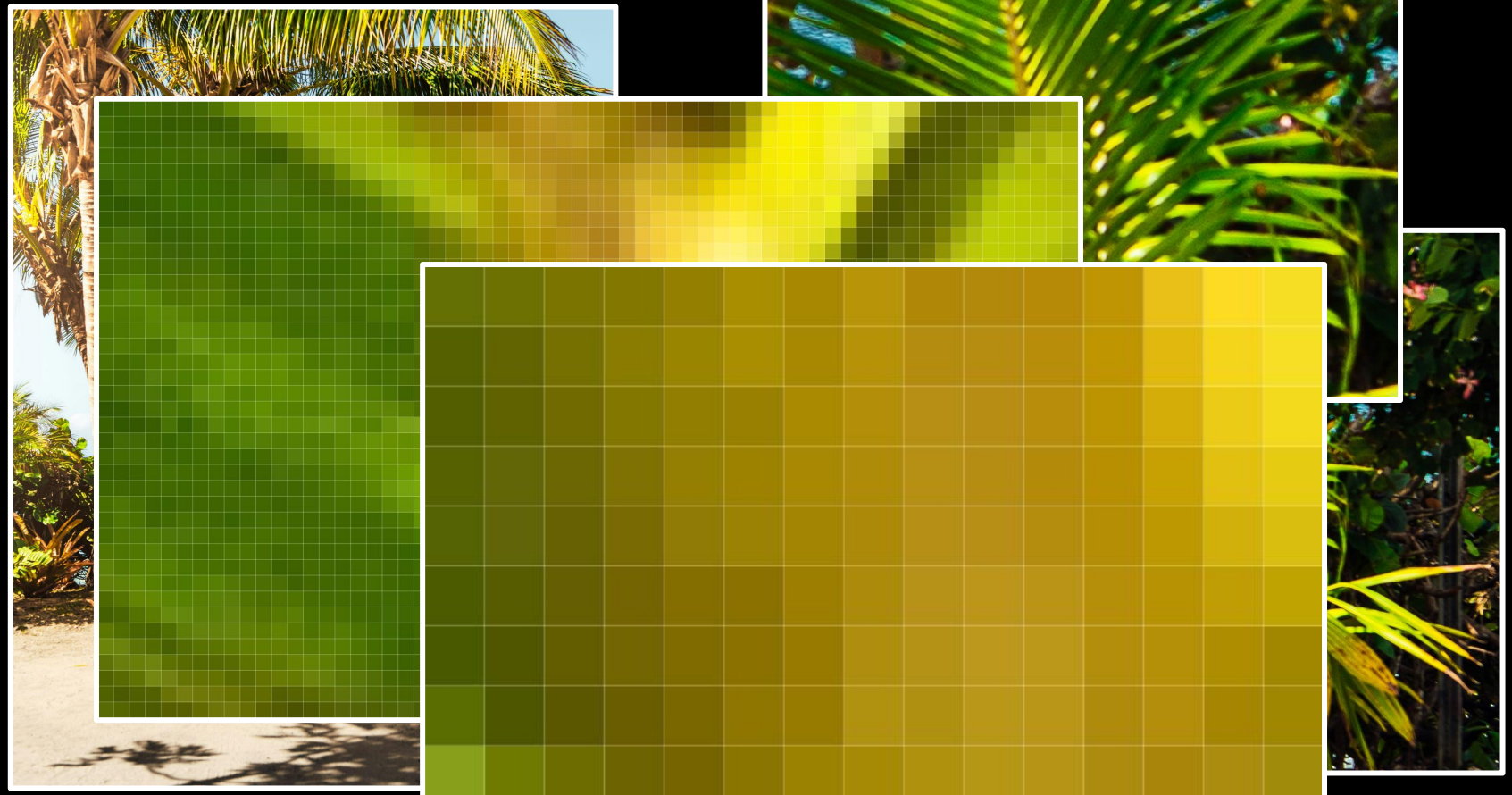


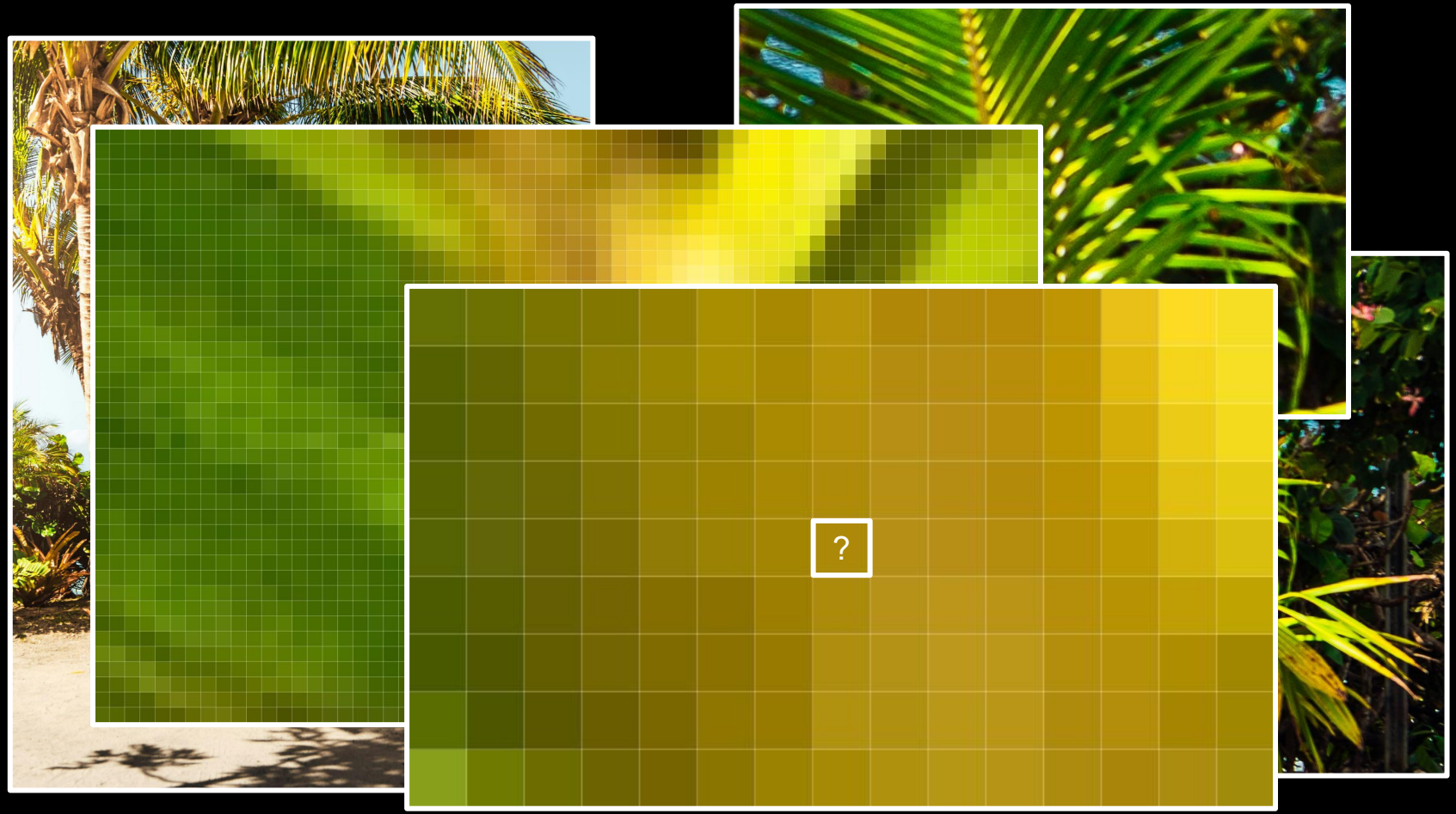
1F600 1F601 1F602 1F603 ... 1F648 1F649 1F64A 1F64B

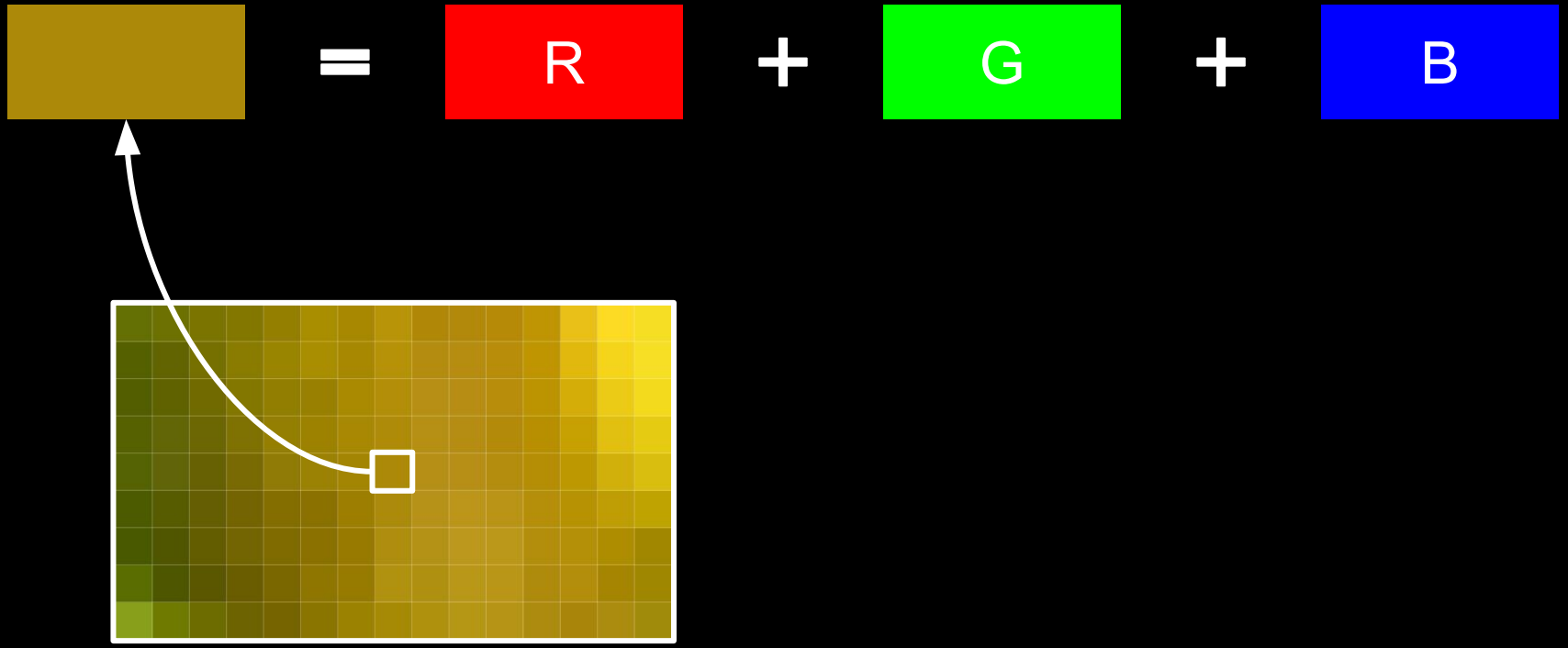


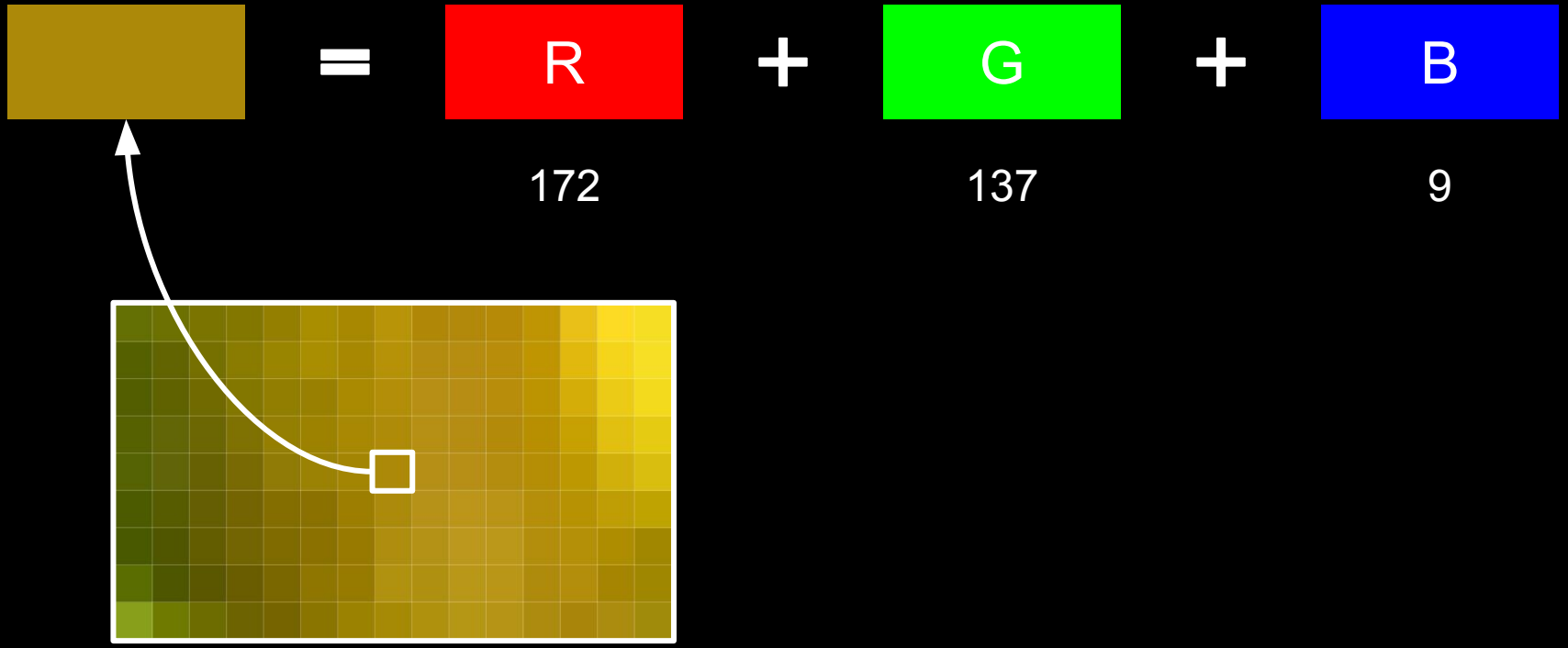


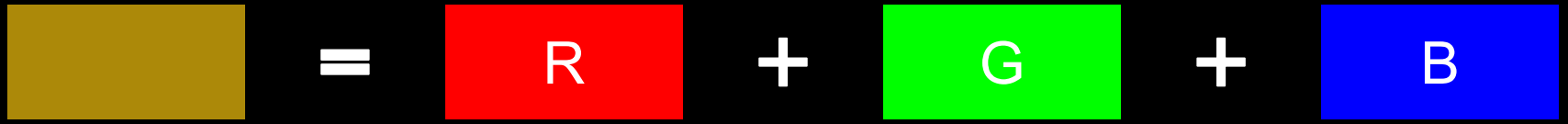




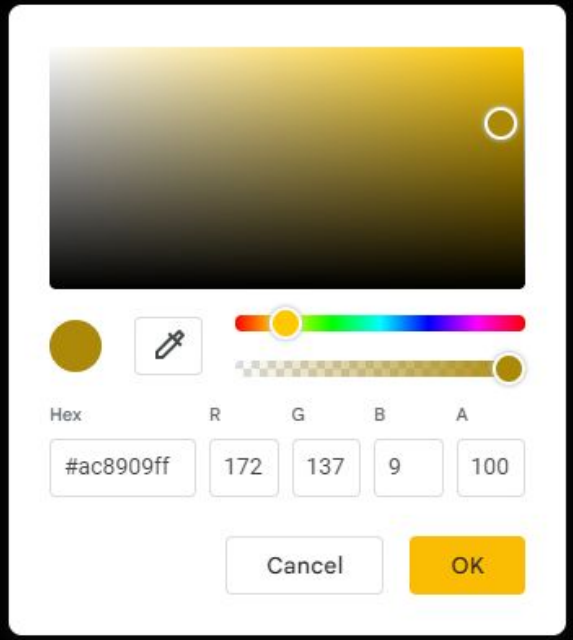



=
R
+
G
+
B


=
R
+
G
+
B
172
137
9



#AC8909


Hex
R
G
B
A
#ac8909ff
172
137
9
100
Cancel
OK

= R + G + B

	R	G	B
	172	137	9
#AC8909	AC	89	09

=

R

+

G

+

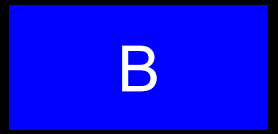
B

	172	137	9
#AC8909	AC	89	09
	10101100	01011001	00001001

possible colors?

R

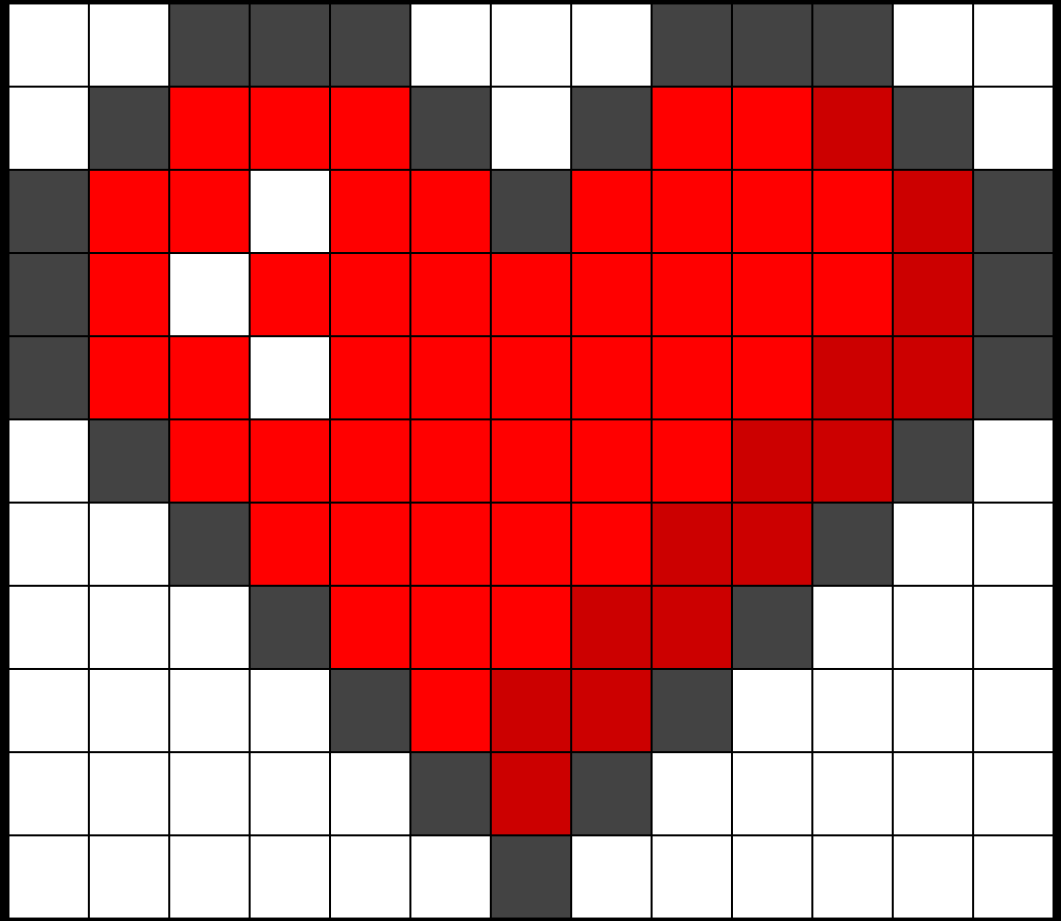
2^7 2^6 2^5 2^4 2^3 2^2 2^1 2^0



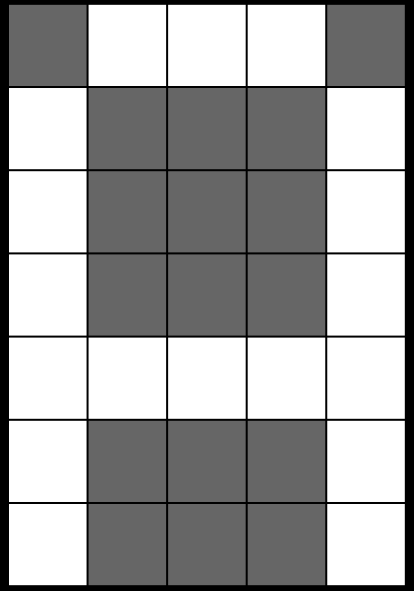
2^{23} 2^{22} 2^{21} 2^{20} 2^{19} 2^{18} 2^{17} 2^{16} 2^{15} 2^{14} 2^{13} 2^{12} 2^{11} 2^{10} 2^{9} 2^{8} 2^{7} 2^{6} 2^{5} 2^{4} 2^{3} 2^{2} 2^{1} 2^{0}

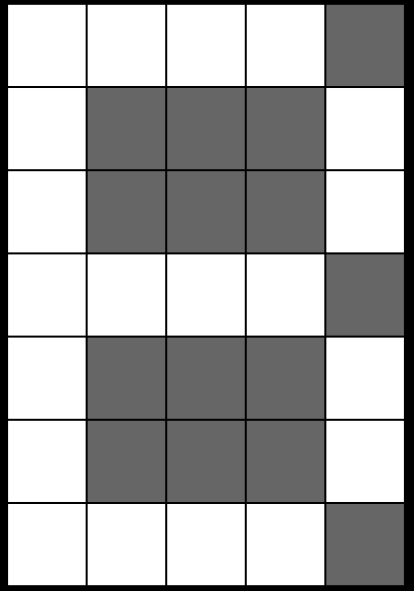
R
G
B
2^{23} 2^{22} 2^{21} 2^{20} 2^{19} 2^{18} 2^{17} 2^{16} 2^{15} 2^{14} 2^{13} 2^{12} 2^{11} 2^{10} 2^{9} 2^{8} 2^{7} 2^{6} 2^{5} 2^{4} 2^{3} 2^{2} 2^{1} 2^{0}
8.388.608
+
8.388.607
=
16.777.215

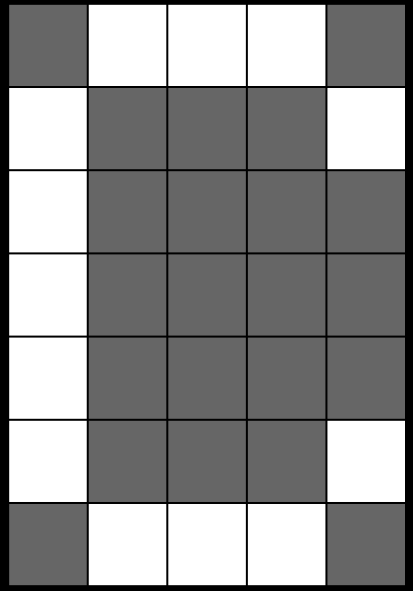
R
G
B
2^{23} 2^{22} 2^{21} 2^{20} 2^{19} 2^{18} 2^{17} 2^{16}
2^{15} 2^{14} 2^{13} 2^{12} 2^{11} 2^{10} 2^{9} 2^{8}
2^{7} 2^{6} 2^{5} 2^{4} 2^{3} 2^{2} 2^{1} 2^{0}
256
×
256
×
256

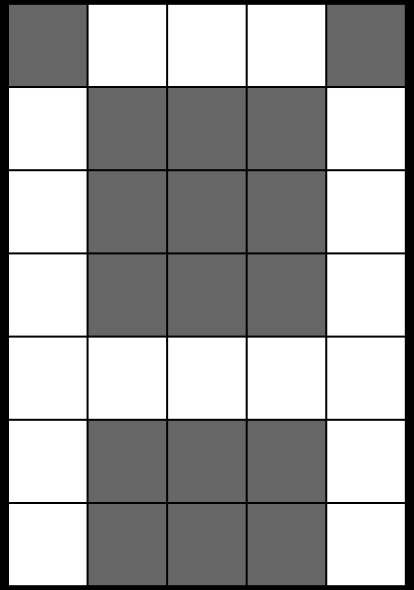


compression?

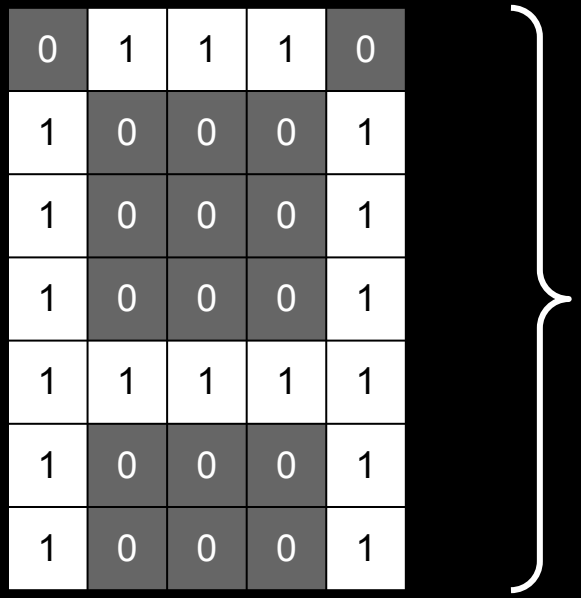








0	1	1	1	0
1	0	0	0	1
1	0	0	0	1
1	0	0	0	1
1	1	1	1	1
1	0	0	0	1
1	0	0	0	1



0 1 1 1 0 1 0 0 0 1 1 0 0 0 1 1 0 0
0 1 1 1 1 1 1 1 0 0 0 1 1 0 0 0 1

5

7

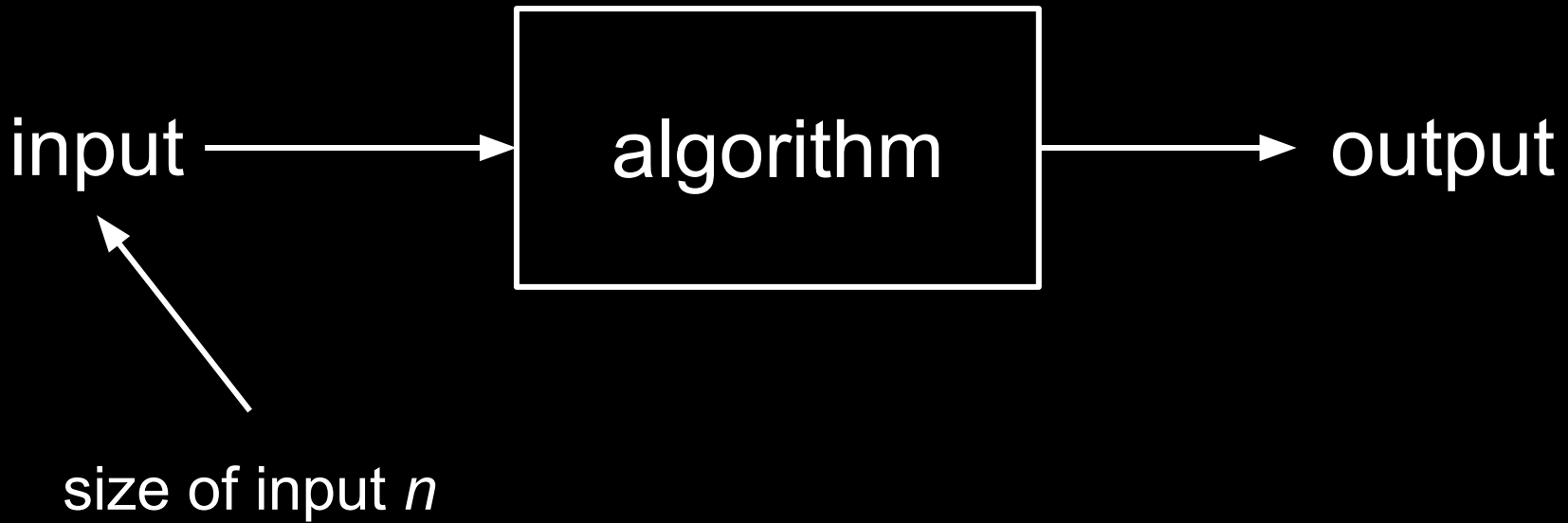
0	1	1	1	0
1	0	0	0	1
1	0	0	0	1
1	0	0	0	1
1	1	1	1	1
1	0	0	0	1
1	0	0	0	1

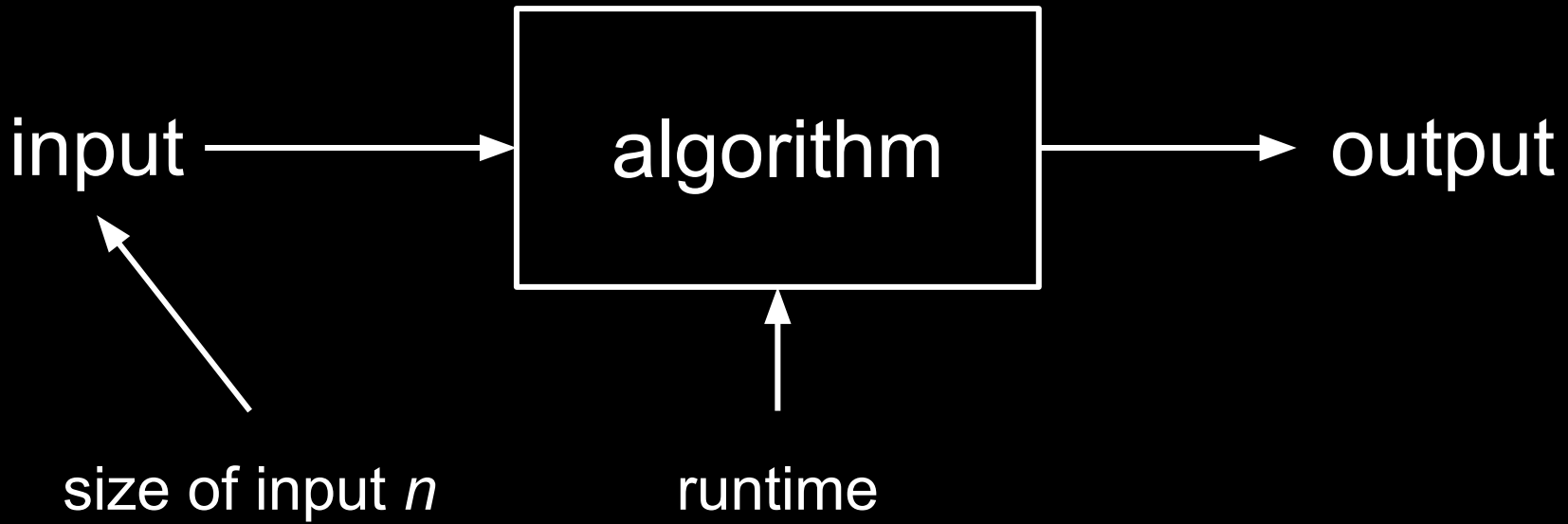
0 1 1 1 0 1 0 0 0 1 1 0 0 0 1 1 0 0
0 1 1 1 1 1 1 1 0 0 0 1 1 0 0 0 1

ALGORITHMS

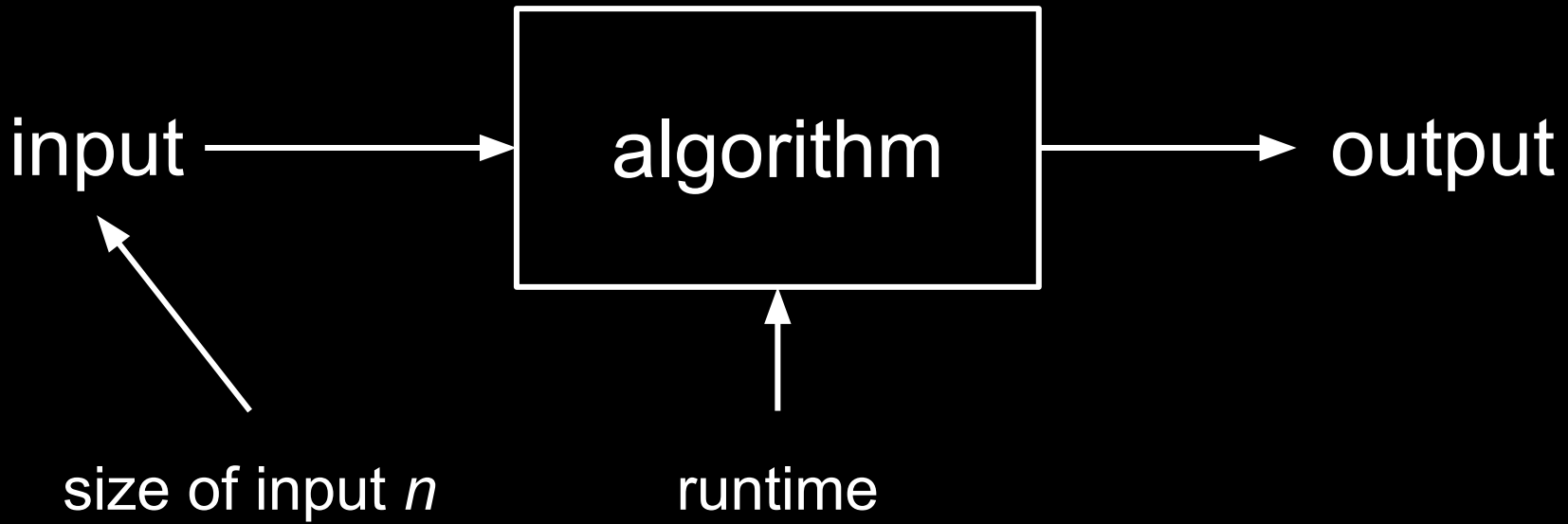
complexity

input
algorithm
output


input
algorithm
output
size of input *n*


input
algorithm
output
size of input *n*
runtime

O(*n*)



%%algorithms_size_runtime%%

COMPUTERS

ARITHMETIC

MEMORY

ANALOG VS. DIGITAL