

# LOGIC AND ARITHMETIC

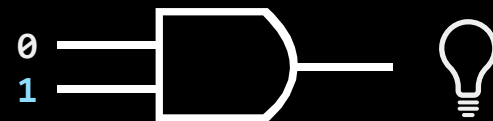
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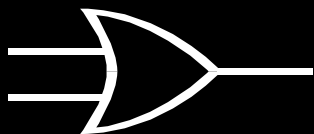
logic gates are the parts a computer is made of.

combined in the right way, they enable the basic  
arithmetic operations a computer can do:  
add, subtract, divide, and multiply.

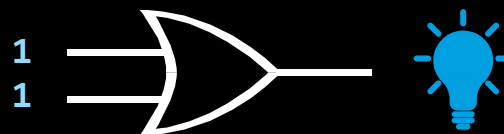
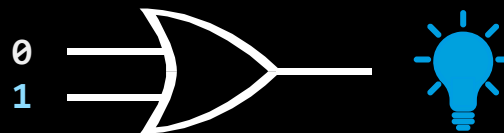
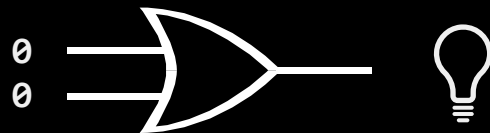


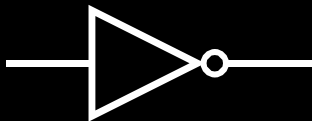
| AND | 0 | 1 |
|-----|---|---|
| 0   | 0 | 0 |
| 1   | 0 | 1 |



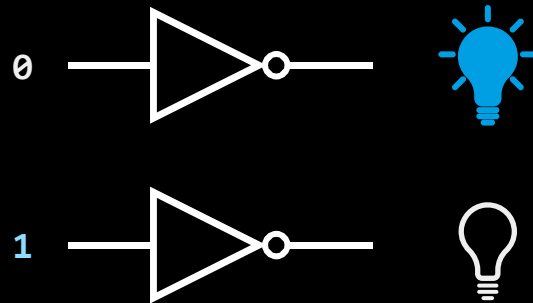


| OR | 0 | 1 |
|----|---|---|
| 0  | 0 | 1 |
| 1  | 1 | 1 |



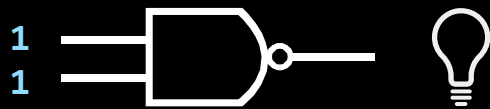
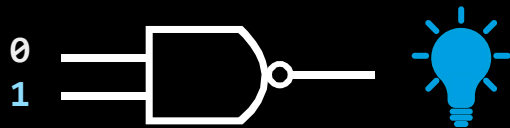
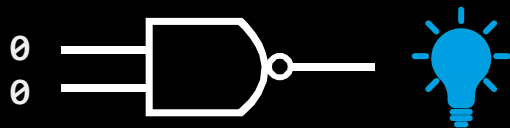


| NOT | 0 | 1 |
|-----|---|---|
|     | 1 | 0 |



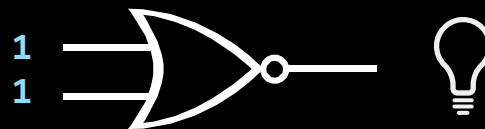
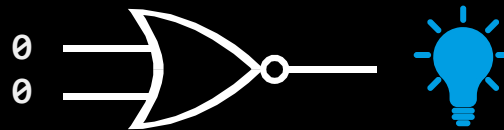


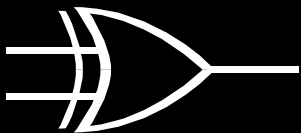
| NAND | 0 | 1 |
|------|---|---|
| 0    | 1 | 1 |
| 1    | 1 | 0 |



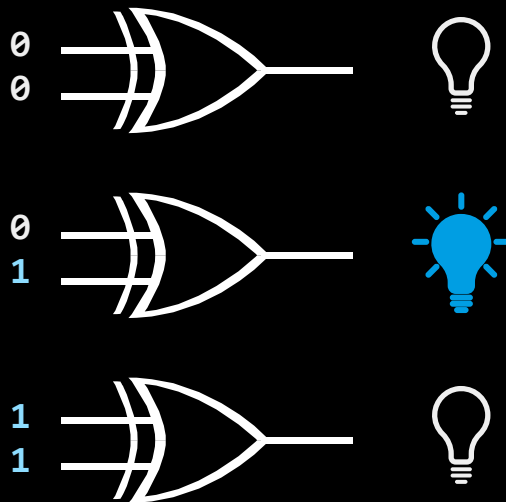


| NOR | 0 | 1 |
|-----|---|---|
| 0   | 1 | 0 |
| 1   | 0 | 0 |





| XOR | 0 | 1 |
|-----|---|---|
| 0   | 0 | 1 |
| 1   | 1 | 0 |





building a logic gate

both inputs off



both inputs off



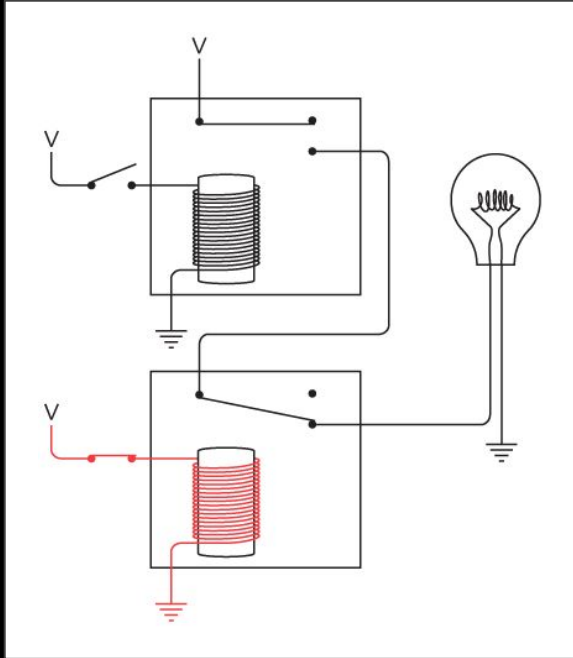
one input on



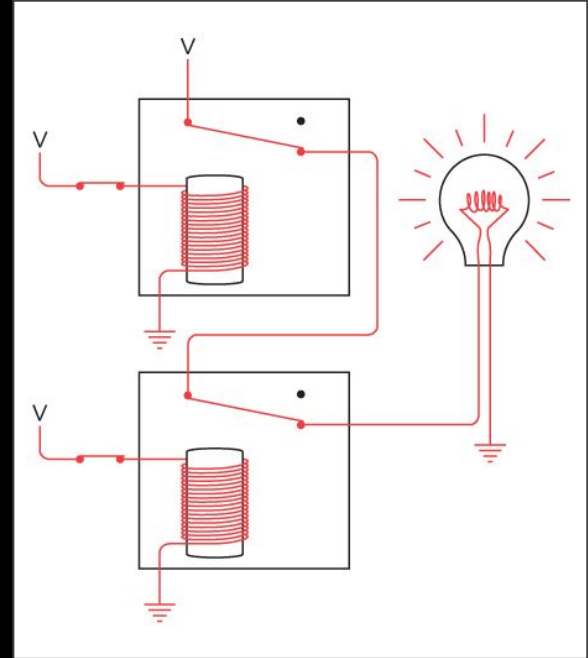
the other input on



the other input on



both inputs on



binary addition

when you come right down to it, **addition** is just about the only thing that computers do.

if we can build something that adds, we're well on our way to building something that uses addition to also subtract, multiply, divide, calculate mortgage payments, guide rockets to Mars, play chess, and foul up our phone bills.

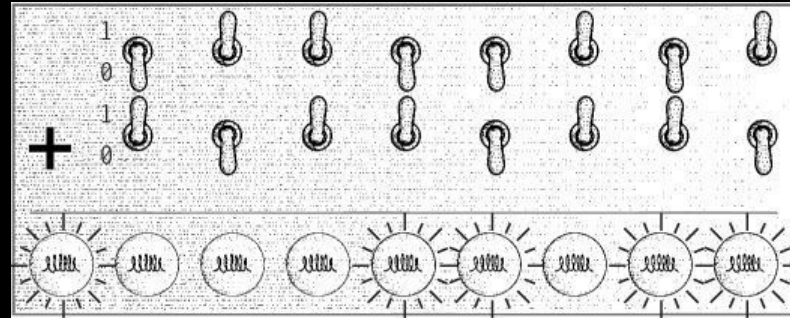
(Charles Petzold)

an 8-bit binary adding machine





an 8-bit binary adding machine



maximum result:

$$255 + 255 = 510$$

**OR**

1 1111 1110

How can we add two binary numbers?

$$\begin{array}{r} 1\ 1\ 1\ 0 \\ +\ 1\ 0\ 1\ 0 \\ \hline \end{array}$$

How can we add two binary numbers?

$$\begin{array}{r} 1\ 1\ 1\ 0 \\ +\ 1\ 0\ 1\ 0 \\ \hline 0 \end{array}$$

How can we add two binary numbers?

$$\begin{array}{rcccc} & 1 & 1 & 1 & 0 \\ + & 1 & 0 & 1 & 0 \\ \hline & & & 0 & 0 \end{array}$$

How can we add two binary numbers?

$$\begin{array}{rcccc} & 1 & 1 & 1 & 0 \\ + & 1 & 0 & 1 & 0 \\ \hline & & & & 0 & 0 \end{array}$$

How can we add two binary numbers?

$$\begin{array}{r} 1\ 1\ 1\ 0 \\ +\ 1\ 0\ 1\ 0 \\ \hline 0\ 0\ 0 \end{array}$$

How can we add two binary numbers?

$$\begin{array}{r} 1\ 1\ 1\ 0 \\ +\ 1\ 0\ 1\ 0 \\ \hline 0\ 0\ 0 \end{array}$$

How can we add two binary numbers?

$$\begin{array}{r} 1\ 1\ 1\ 0 \\ +\ 1\ 0\ 1\ 0 \\ \hline 1\ 0\ 0\ 0 \end{array}$$



How can we add two binary numbers?

$$\begin{array}{r} 1110 \\ + 1010 \\ \hline 11000 \end{array}$$

adding two bits

| + | 0 | 1 |
|---|---|---|
| 0 |   |   |
| 1 |   |   |

| + | 0 | 1 |
|---|---|---|
| 0 | 0 |   |
| 1 |   |   |

| + | 0 | 1 |
|---|---|---|
| 0 | 0 | 1 |
| 1 |   |   |

| + | 0 | 1 |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 1 |   |

| + | 0 | 1  |
|---|---|----|
| 0 | 0 | 1  |
| 1 | 1 | 10 |

| + | 0 | 1  |
|---|---|----|
| 0 | 0 | 1  |
| 1 | 1 | 10 |

OR

| + | 0  | 1  |
|---|----|----|
| 0 | 00 | 01 |
| 1 | 01 | 10 |



| + | 0 | 1  |
|---|---|----|
| 0 | 0 | 1  |
| 1 | 1 | 10 |

OR

| + | 0  | 1  |
|---|----|----|
| 0 | 00 | 01 |
| 1 | 01 | 10 |

the digit on the right, we call the **sum bit**  
the left digit, we call ...?

| + | 0 | 1  |
|---|---|----|
| 0 | 0 | 1  |
| 1 | 1 | 10 |

OR

| + | 0  | 1  |
|---|----|----|
| 0 | 00 | 01 |
| 1 | 01 | 10 |

the digit on the right, we call the **sum bit**  
the left digit, we call the **carry bit**

| + | 0   | 1   |
|---|-----|-----|
| 0 | 0 0 | 0 1 |
| 1 | 0 1 | 1 0 |

| + | 0   | 1   |
|---|-----|-----|
| 0 | 0 0 | 0 1 |
| 1 | 0 1 | 1 0 |



sum bit

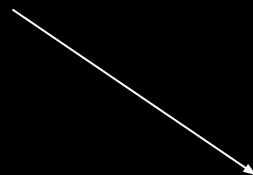
| + | 0 | 1 |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 1 | 0 |

| + | 0   | 1   |
|---|-----|-----|
| 0 | 0 0 | 0 1 |
| 1 | 0 1 | 1 0 |



sum bit

| + | 0 | 1 |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 1 | 0 |



carry bit

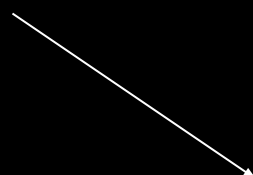
| + | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |

| + | 0   | 1   |
|---|-----|-----|
| 0 | 0 0 | 0 1 |
| 1 | 0 1 | 1 0 |



sum bit

| + | 0 | 1 |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 1 | 0 |



carry bit

| + | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |

doesn't this look familiar?

| + | 0   | 1   |
|---|-----|-----|
| 0 | 0 0 | 0 1 |
| 1 | 0 1 | 1 0 |



sum bit

| + | 0 | 1 |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 1 | 0 |



carry bit

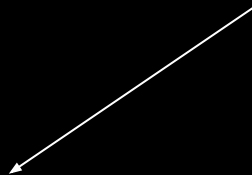
| + | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |



| AND | 0 | 1 |
|-----|---|---|
| 0   | 0 | 0 |
| 1   | 0 | 1 |

doesn't this look familiar?

| + | 0   | 1   |
|---|-----|-----|
| 0 | 0 0 | 0 1 |
| 1 | 0 1 | 1 0 |

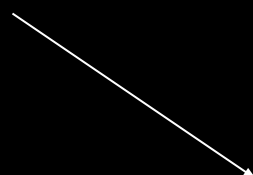


sum bit

| + | 0 | 1 |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 1 | 0 |



| XOR | 0 | 1 |
|-----|---|---|
| 0   | 0 | 1 |
| 1   | 1 | 0 |



carry bit

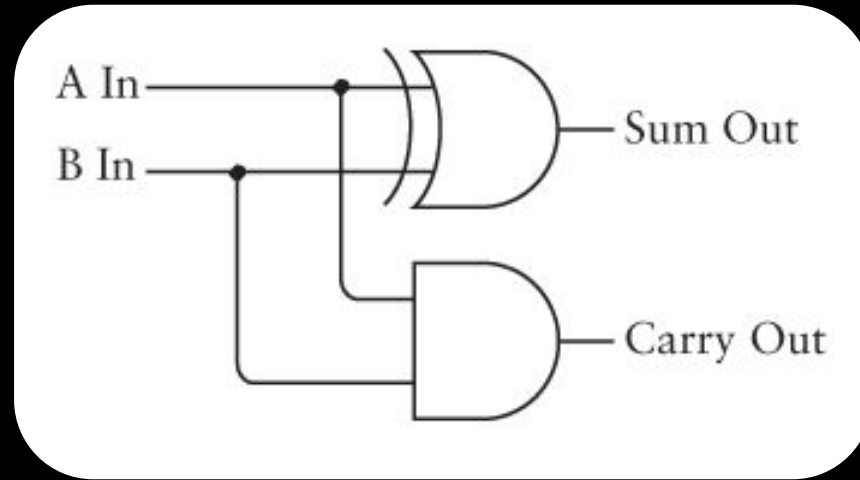
| + | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |



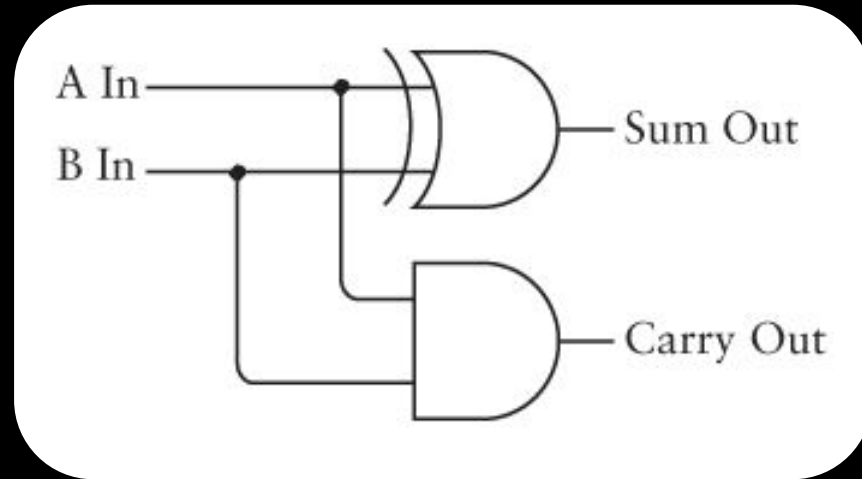
| AND | 0 | 1 |
|-----|---|---|
| 0   | 0 | 0 |
| 1   | 0 | 1 |

doesn't this look familiar?

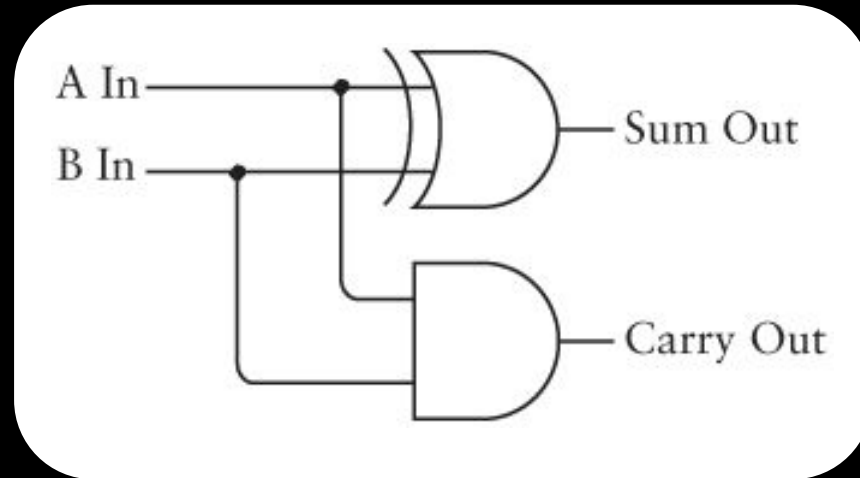




bits A and B  
are inputs

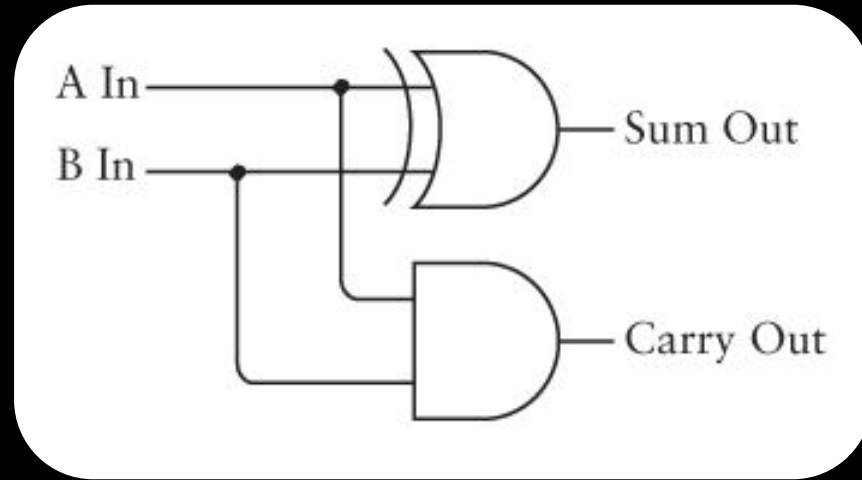


bits A and B  
are inputs



XOR gate  
computes sum bit

bits A and B  
are inputs

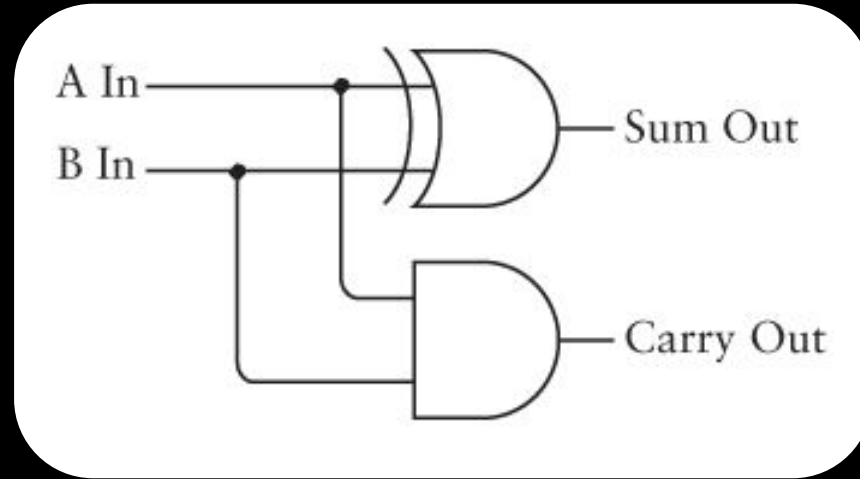


XOR gate  
computes sum bit

AND gate  
computes carry bit

## half adder

bits A and B  
are inputs



XOR gate  
computes sum bit

AND gate  
computes carry bit

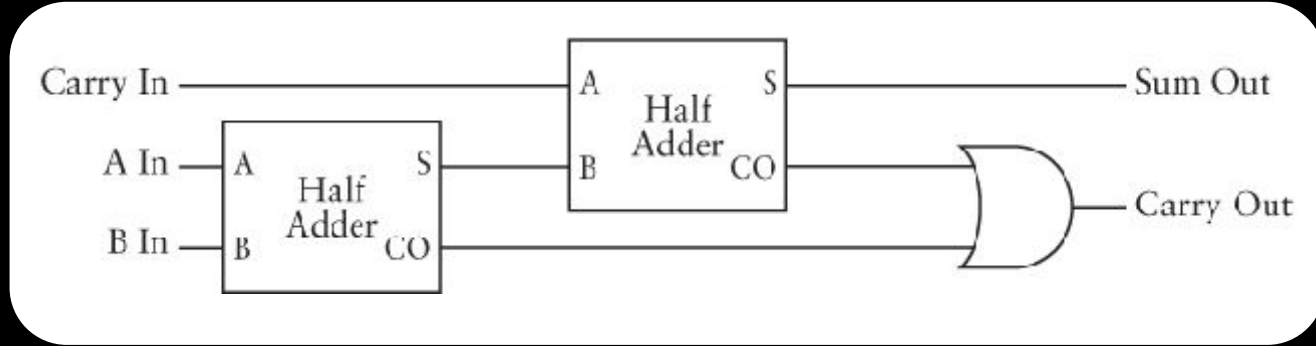


## half adder



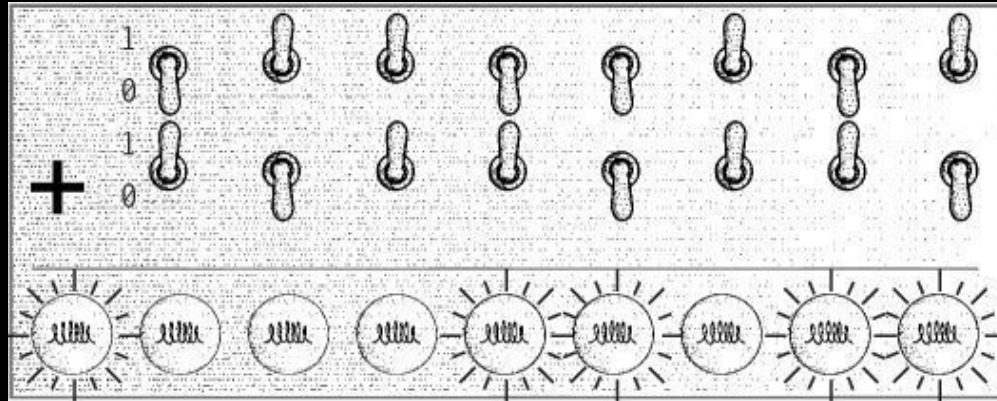
the half adder has no input for a carry bit.

two half adder wired together make a full adder.





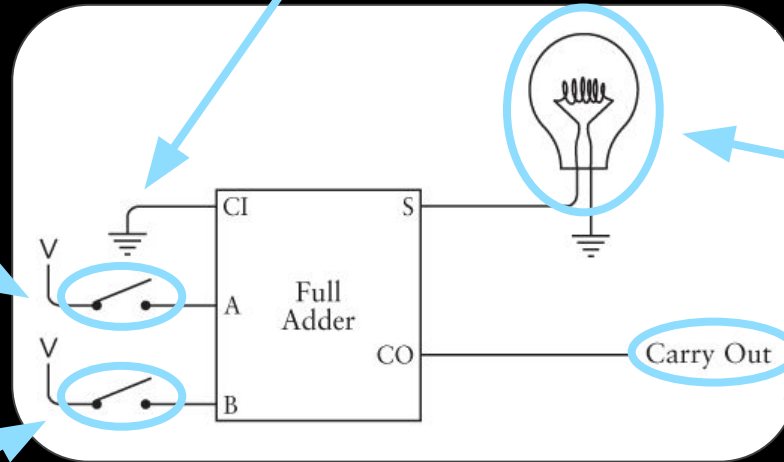
so how to build this?





no carry bit for 1st digits

switch 1st byte,  
1st digit (1)

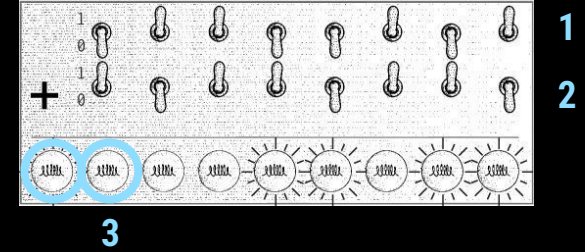


result for rightmost digit (3)

switch 2nd byte, 1st digit (2)

this is carried to the next full adder

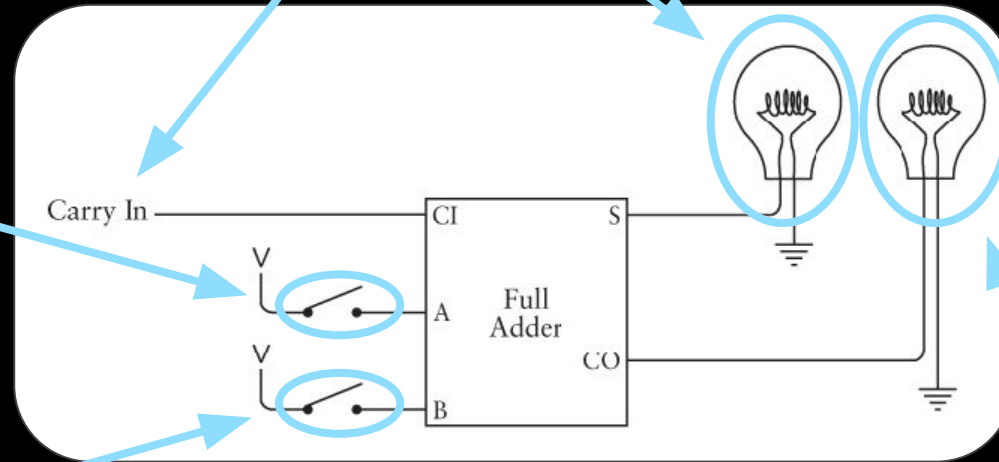




result for 8th digits (3)

carry bit from 7th digits

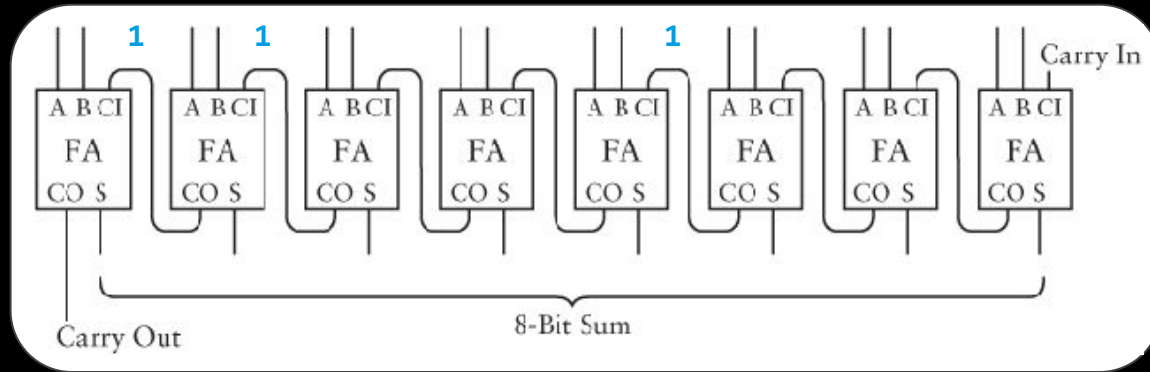
switch 1st byte, 8th digit (1)



switch 2nd byte, 8th digit (2)

result for the 9th digit (4)

|   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 |



|   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 |
|---|---|---|---|---|---|---|---|---|