

PROBLEM SOLVING

[BACK](#)

Polya's approach to problem-solving



image source: <http://doi.org/10.3932/ethz-a-000099441>

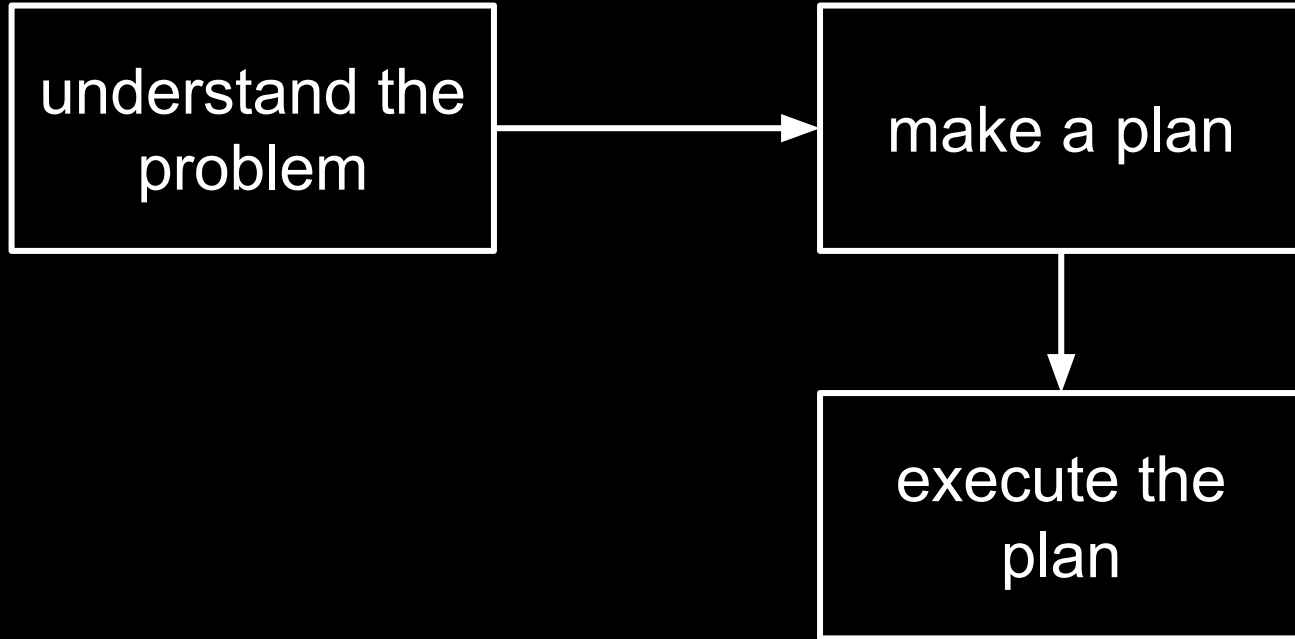
Polya's approach to problem-solving

understand the
problem

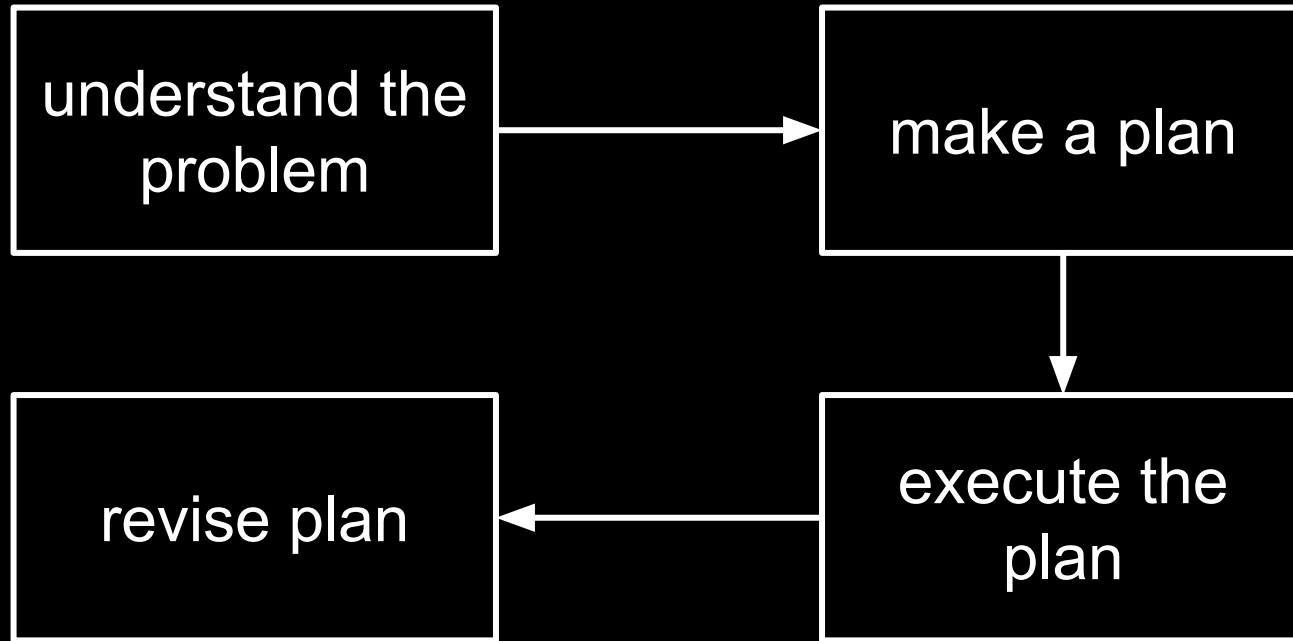
Polya's approach to problem-solving



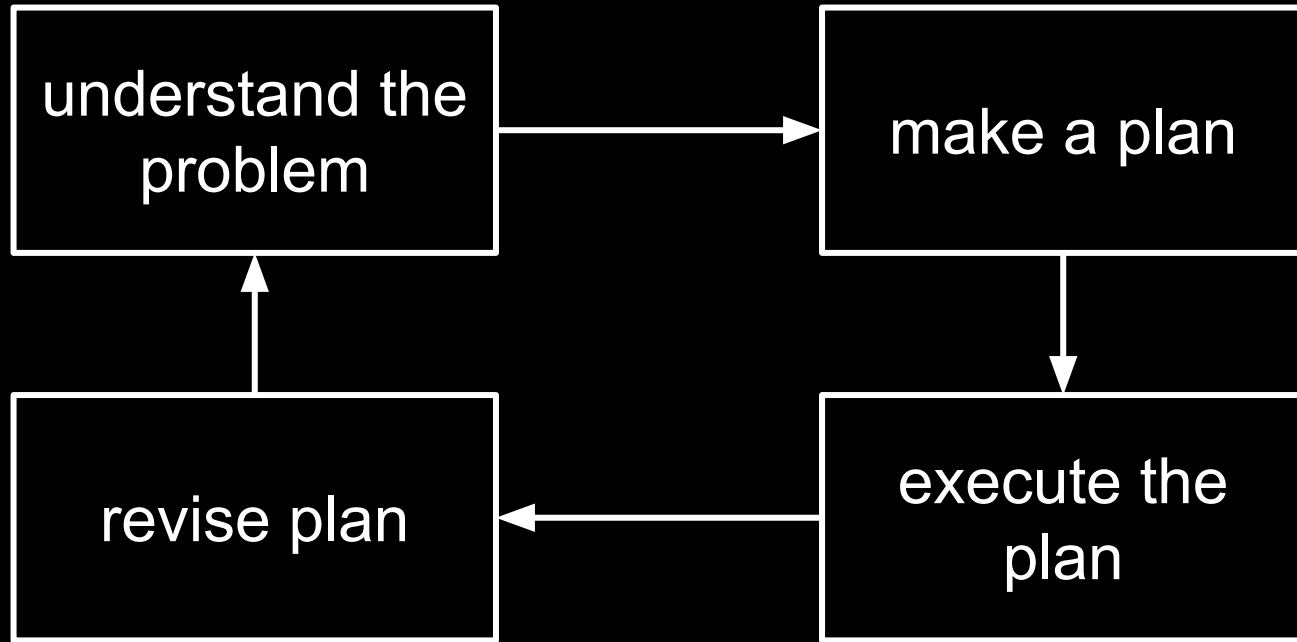
Polya's approach to problem-solving



Polya's approach to problem-solving



Polya's approach to problem-solving



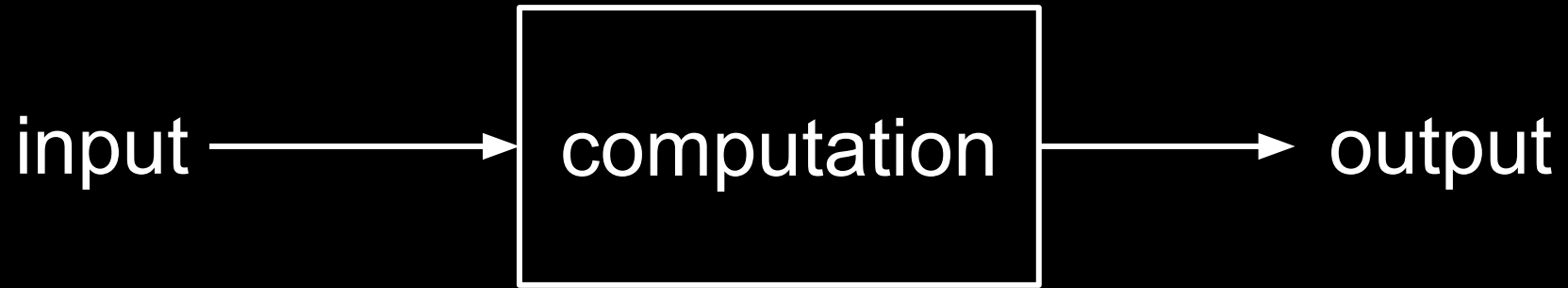
a model to represent problems



a model to represent problems



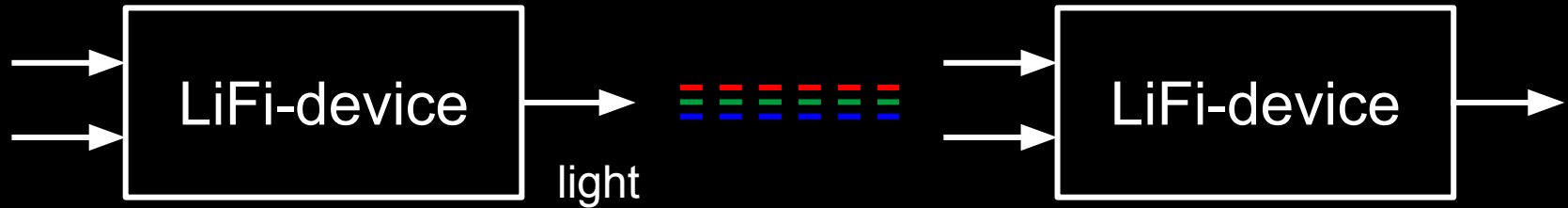
a model to represent problems

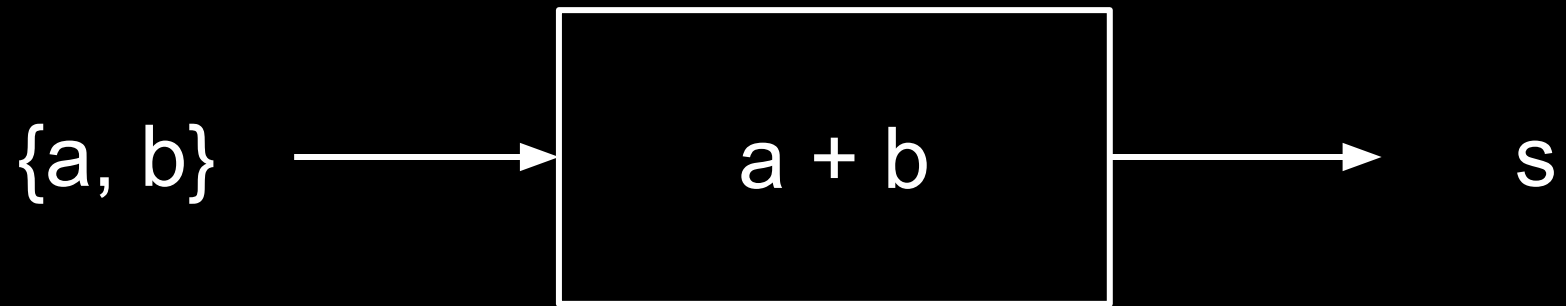


the LiFi-project as an
input - processing - output - problem



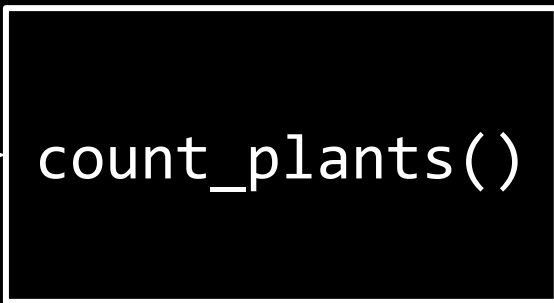
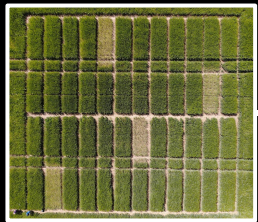
the LiFi-project as a communication problem







%%problem_solving_example_count_plants_1%%

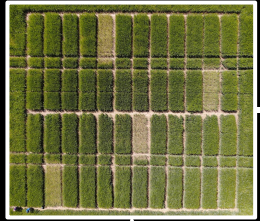


output



42

processing of
information



count_plants()

42

representation of
information



next_move()

E2 → E4



1: 0R	9: 0P	57: 1R
2: 0N	10: 0P	58: 1N
3: 0B	11: 0P	59: 1B
4: 0K	12: 0P	60: 1K
5: 0Q	13: 0P	61: 1Q
6: 0B	14: 0P	62: 1B
7: 0N	15: 0P	63: 1N
8: 0R	16: 0P	64: 1R

...

representation of information

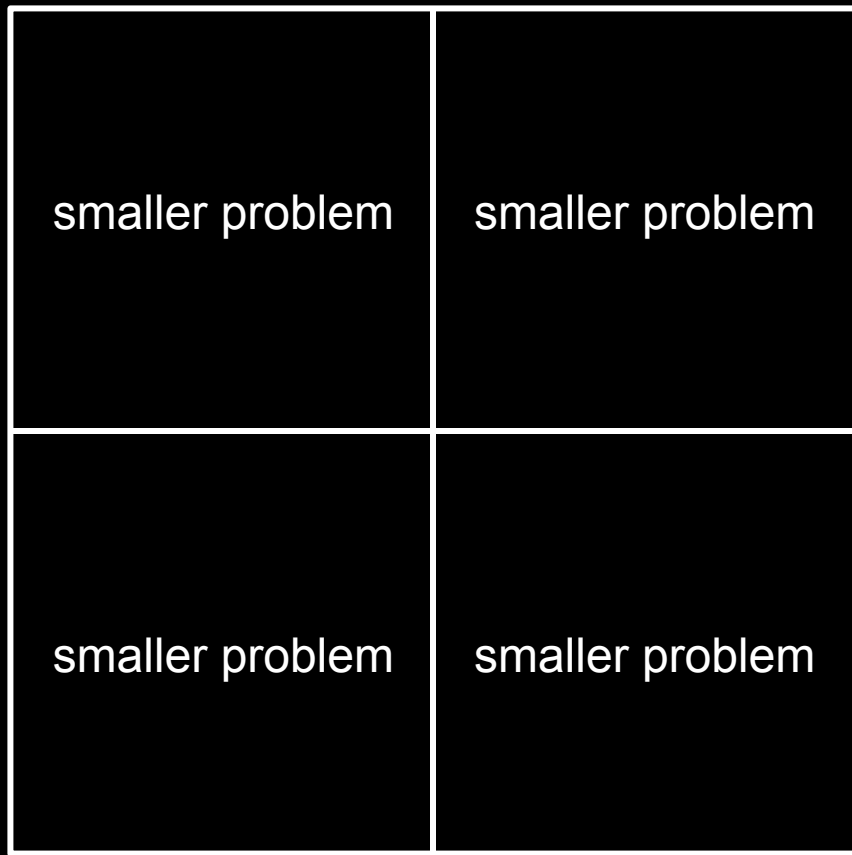


%%problem_solving_example_chatgpt%%

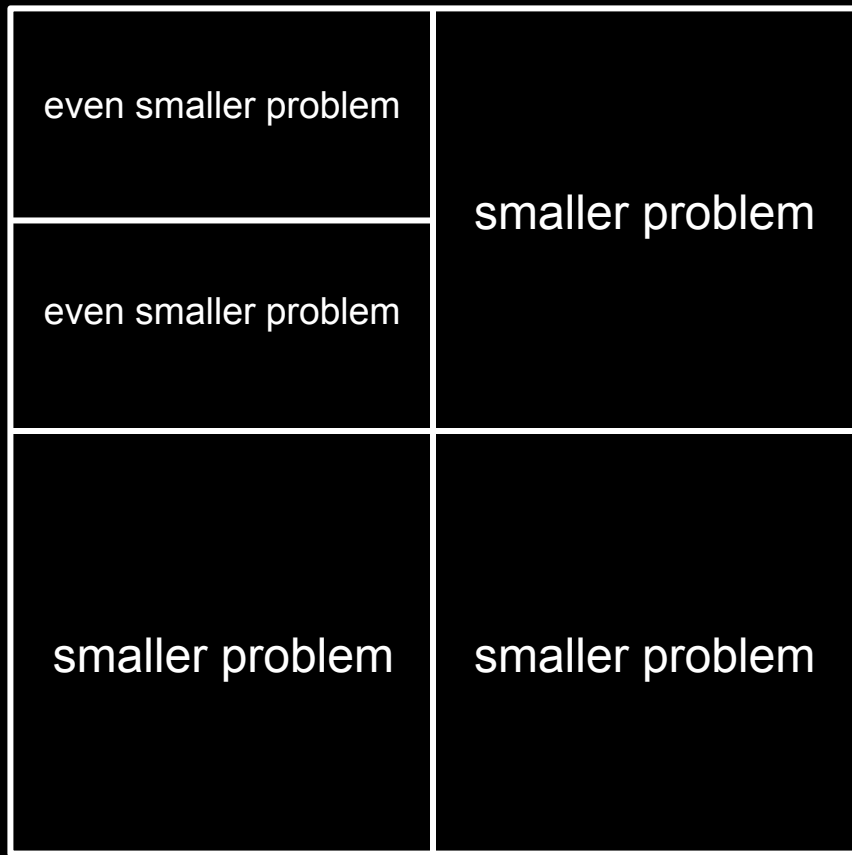
problem solving strategies

divide and conquer

large and complex problem



%%problem_solving_smaller_problems%%



%%problem_solving_even_smaller_problems%%

sorted list +
element



search()



yes / no

is 67 a prime number?

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

linear search



2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

linear search



2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

linear search



2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

linear search



2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

linear search

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

↑

19 steps... can't we do better?

2, 3, 5, 7, 11, ~~13~~, 17, ~~19~~, ~~23~~, ~~29~~, ~~31~~, ~~37~~, ~~41~~,
~~43~~, ~~47~~, ~~53~~, ~~59~~, ~~61~~, 67, 71, 73, 79, 83, 89, 97

↑

large and complex
problem

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

large and complex
problem

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

smaller
problem

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41

smaller
problem

43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

binary search

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

binary search

67 != 41



2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, ~~41~~,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

binary search

67 > 41



2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, ~~41~~,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

binary search

67 > 41



~~2~~, ~~3~~, ~~5~~, ~~7~~, ~~11~~, ~~13~~, ~~17~~, ~~19~~, ~~23~~, ~~29~~, ~~31~~, ~~37~~, ~~41~~,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

binary search

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97



67 != 71

binary search

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97



67 != 71

binary search

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97



$67 < 71$

binary search

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97



67 != 59

binary search

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97



67 > 59

binary search

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
~~43~~, ~~47~~, ~~53~~, ~~59~~, 61, 67, ~~71~~, ~~73~~, ~~79~~, ~~83~~, ~~89~~, 97



67 = 67

binary search

2, 3, 5, 7, 11, ~~13~~, 17, 19, ~~23~~, ~~29~~, 31, 37, 41,
~~43~~, 47, ~~53~~, 59, ~~61~~, 67, ~~71~~, ~~73~~, 79, ~~83~~, ~~89~~, 97



67 = 67

3 splits → much better

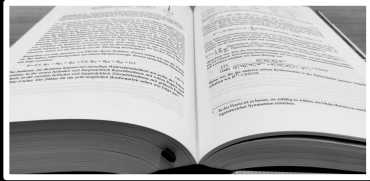
2, 3, 5, 7, 11, ~~13~~, 17, 19, ~~23~~, 29, 31, 37, 41,
~~43~~, 47, ~~53~~, 59, ~~61~~, 67, ~~71~~, ~~73~~, 79, ~~83~~, ~~89~~, 97



$$67 = 67$$



how efficient are linear and
binary search in general?



`count_words()`

word count

$\theta = 0.5$, $\theta_{T1} = \theta_{W1} = \theta_{H1} = 0.8$, $\theta_{T2} = \theta_{W2} = \theta_{H2} = 0.3$.

Zus bedeutet, die Bonbons können mit derselben Wahrscheinlichkeit aus jeder der Tüten ummen; in der ersten befinden sich hauptsächlich Kirschbonbons mit rotem Papier und Zitronebonbons mit grünem Papier, in der zweiten befinden sich hauptsächlich Zitronenbonbons mit grünem Papier ohne Löcher. Die Zähler für die acht möglichen Bonbonarten sehen wie folgt aus:

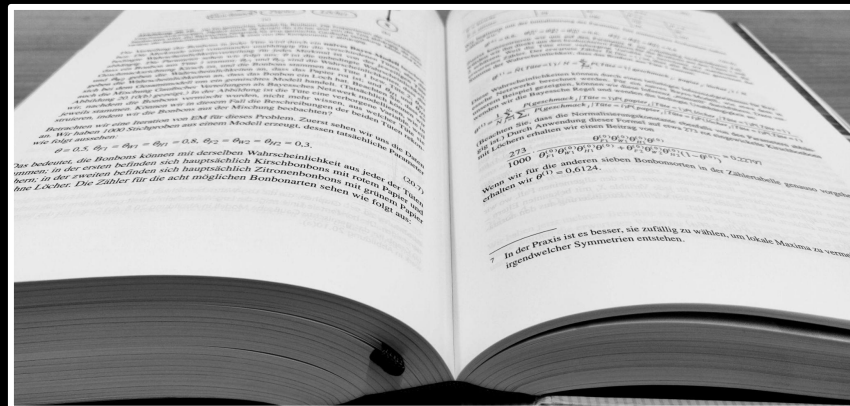
(20.7)

Wenn wir für die anderen sieben Bonbonsorten in der Zählertabelle genauso vorgehen erhalten wir $\theta(1) = 0.6124$.

In der Praxis ist es besser, sie zufällig zu wählen, um lokale Maxima zu vermeiden.

7 In der Praxis ist es besser, sie zufällig zu wählen, um lokale Maxima zu vermeiden, wenn Symmetrien entstehen.

strategies, anyone?

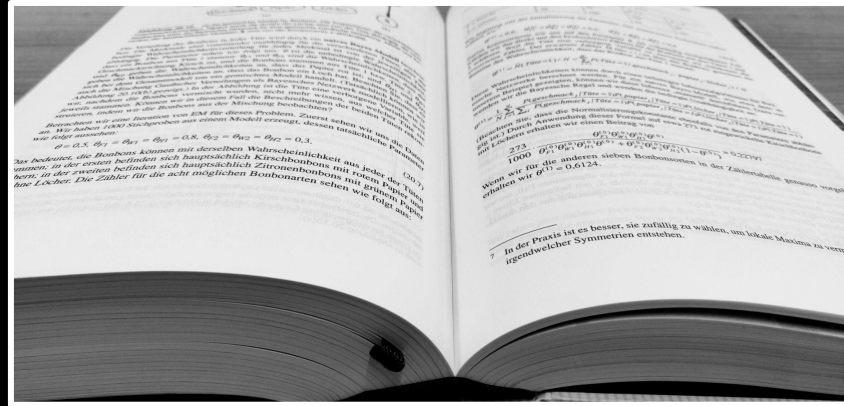


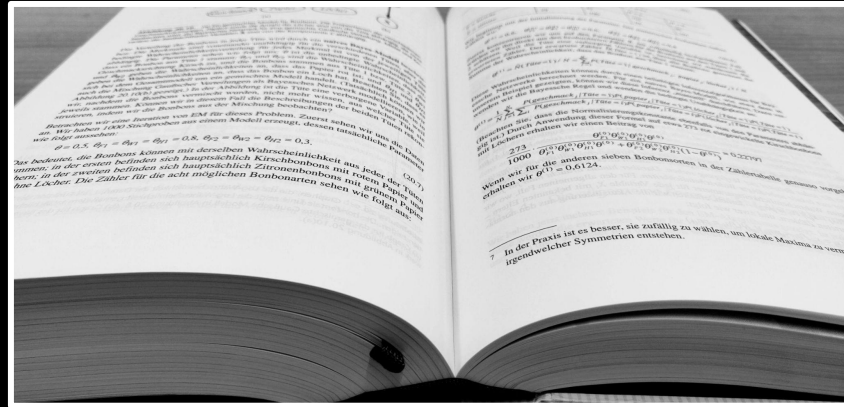
... das bedeutet, die Bonbons können mit derselben Wahrscheinlichkeit aus jeder der Urnen
kommen, in der ersten befinden sich hauptsächlich Zitronenbonbons mit einem geringen
Anteil an Limettenbonbons, in der zweiten befinden sich hauptsächlich Limettenbonbons mit
einem geringen Anteil an Zitronenbonbons. Die Zahlen für die acht möglichen Bonbonarten sehen wie folgt aus:

$$\theta_{11} = 0.8, \theta_{12} = 0.2, \theta_{21} = 0.2, \theta_{22} = 0.8$$

Wenn wir für die anderen sieben Bonbonarten in der Zählentabelle gemessen werden
erhalten wir $\theta^{(1)} = 0.6124$.

In der Praxis ist es besser, sie zufällig zu wählen, um lokale Maxima zu vermeiden
irgendwelcher Symmetrien entstehen.

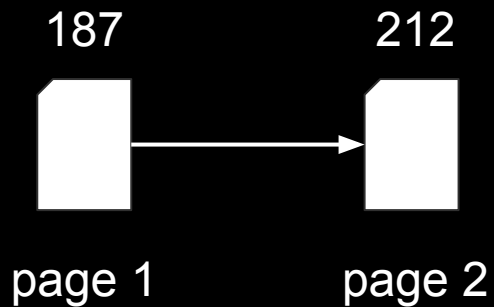
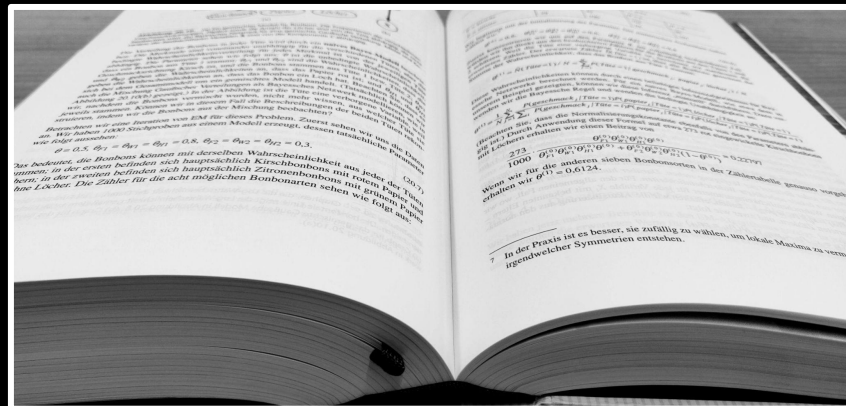


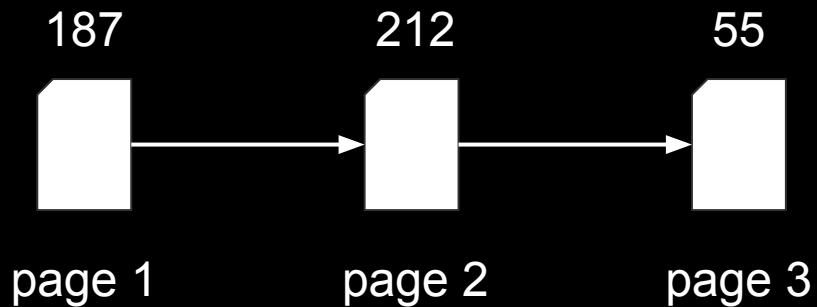
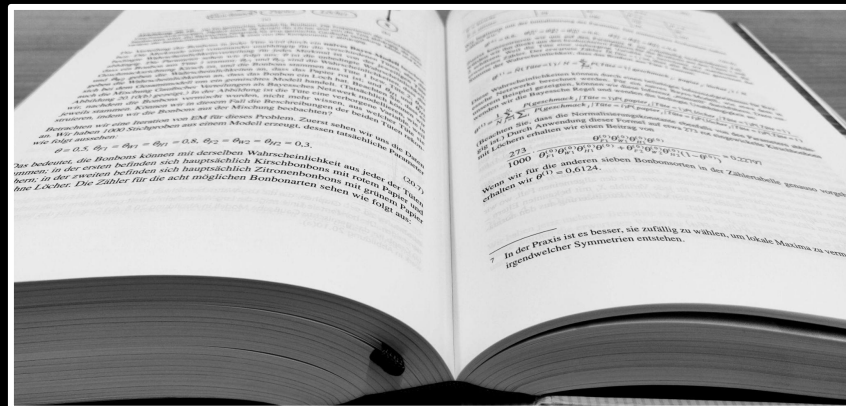


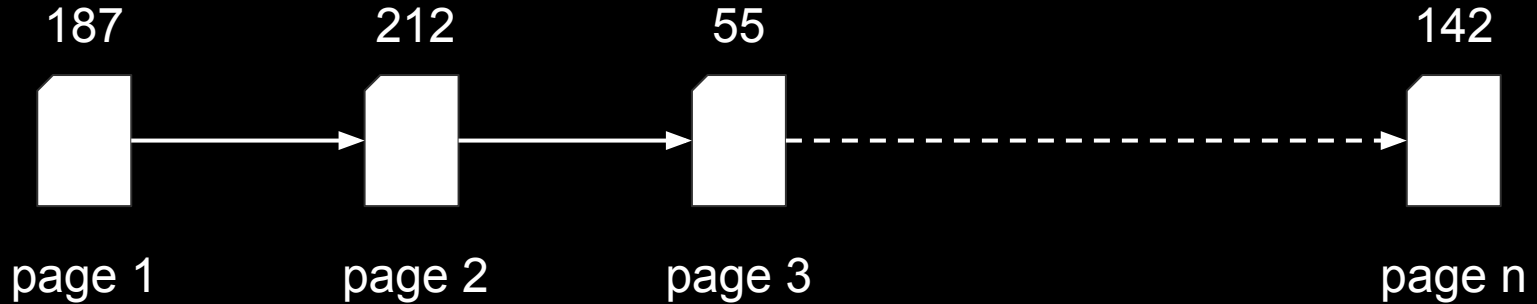
187



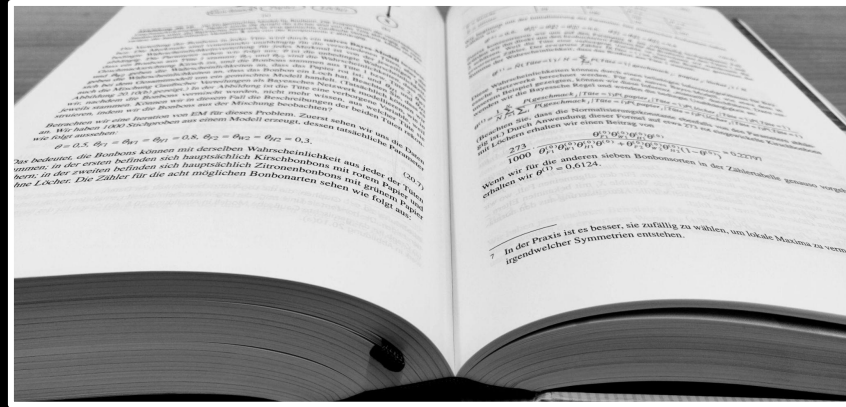
page 1







%%problem_solving_example_word_count_sequentially%%



n = 1327 pages

Ø 2:23 minutes per page

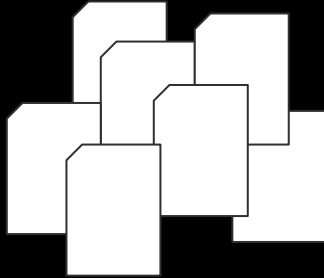
~ 52.34 hours

divide and conquer

+

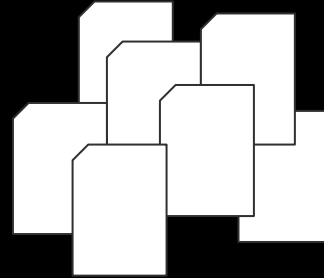
?

pages 1 - 700



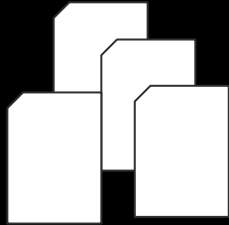
student 1

pages 701 - 1327



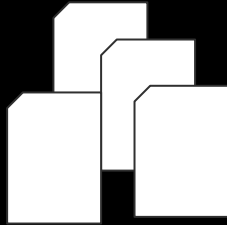
student 2

pages 1 - 350



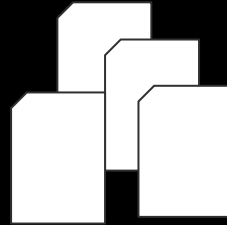
student 1

pages 351 - 700



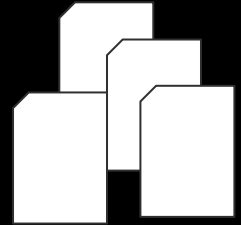
student 2

pages 701 - 1050



student 3

pages 1051- 1327

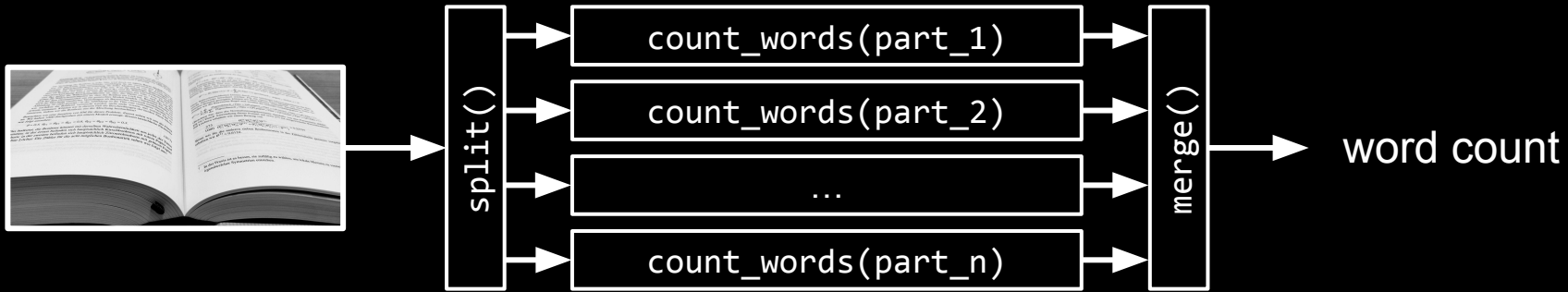


student 4

divide and conquer

+

distribution and parallelization



%%problem_solving_example_word_count_distribute_parallelize%%