

# PROBLEM SOLVING

[BACK](#)

# Pólya's approach to problem-solving



image source: <http://doi.org/10.3932/ethz-a-000099441>

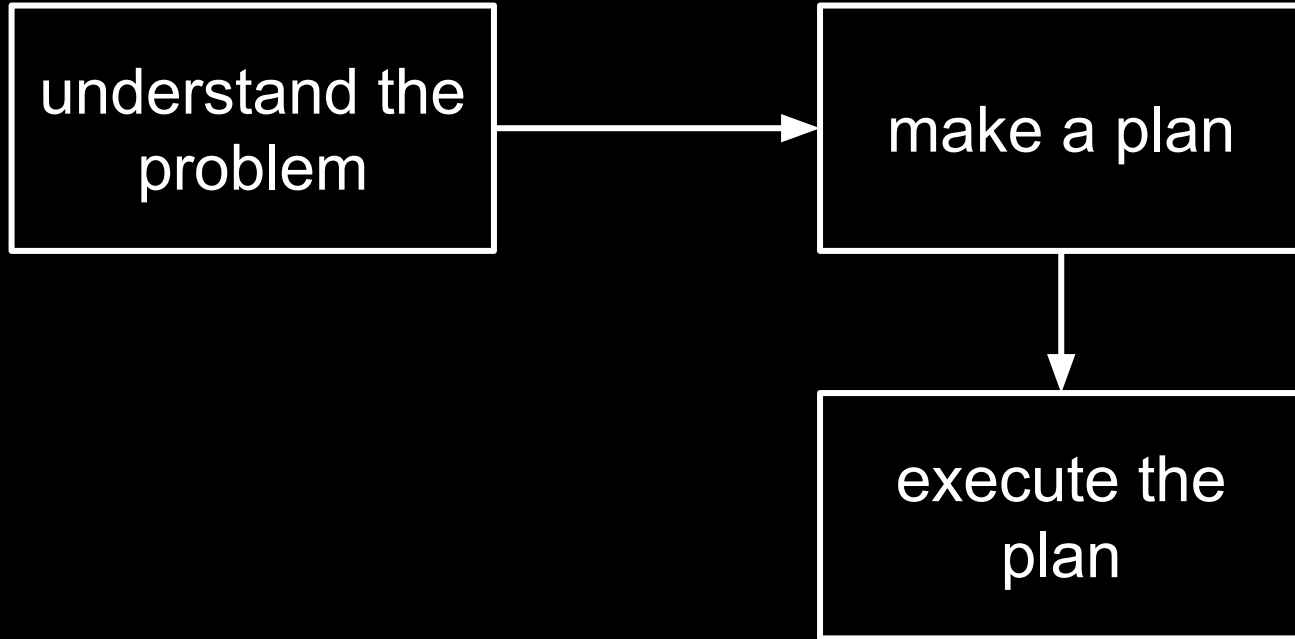
# Pólya's approach to problem-solving

understand the  
problem

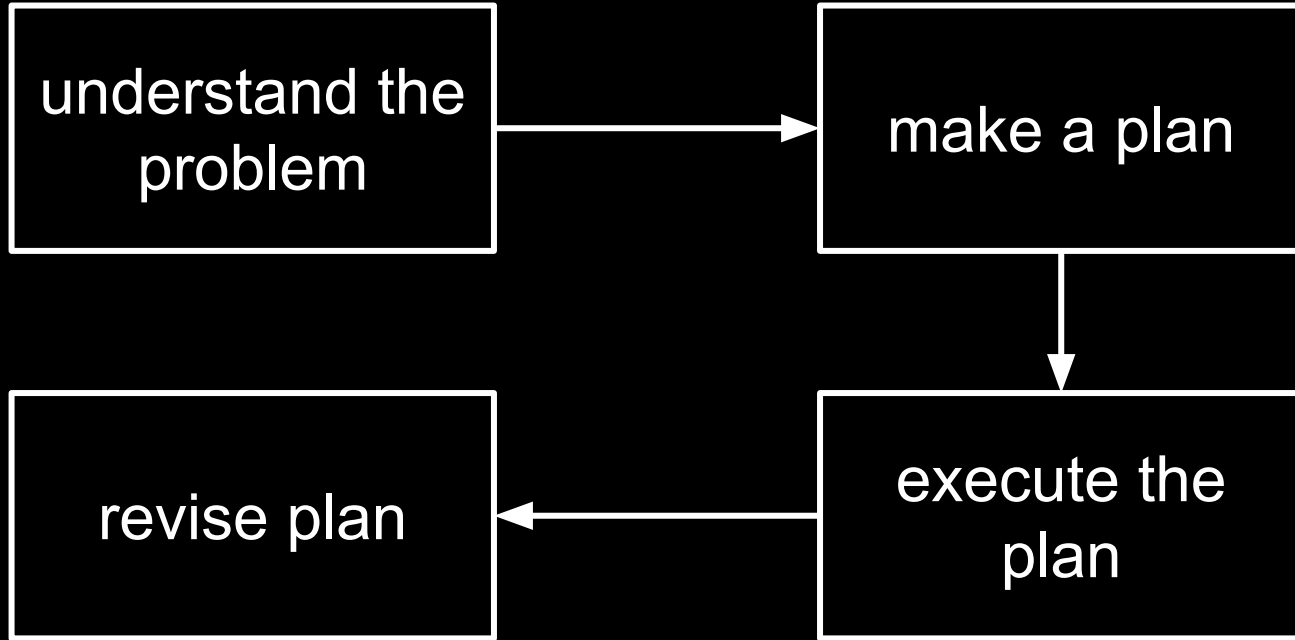
# Pólya's approach to problem-solving



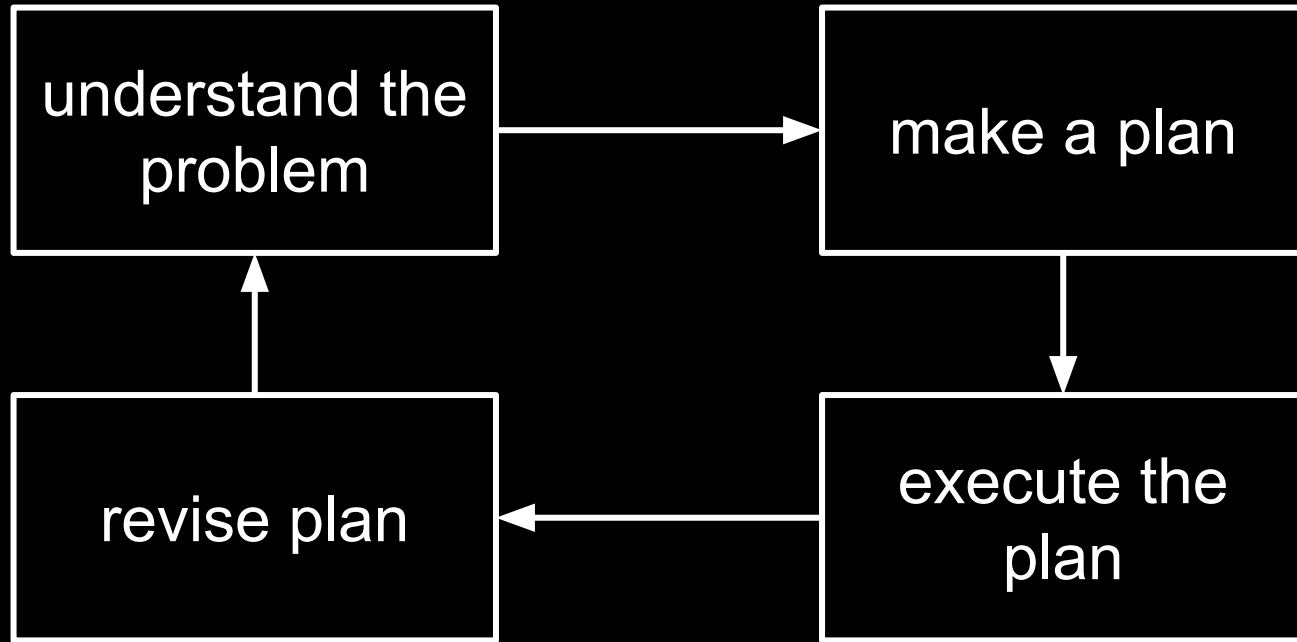
# Pólya's approach to problem-solving



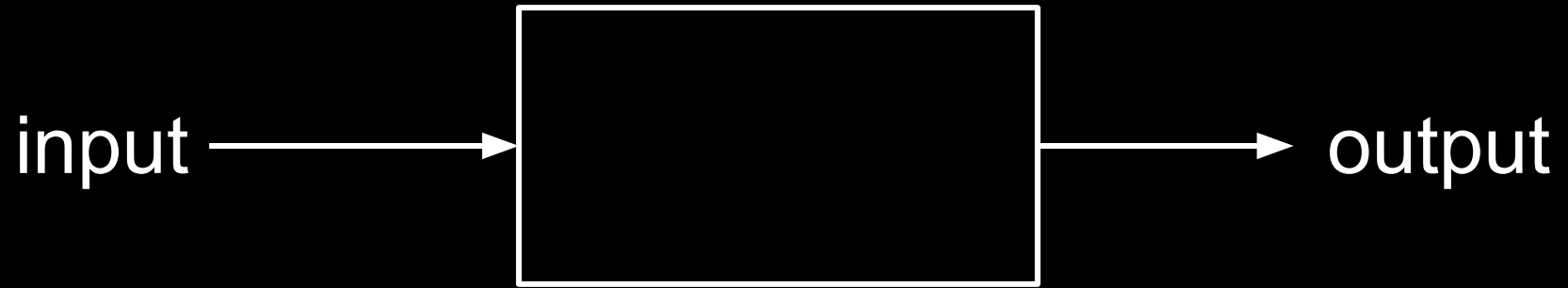
# Pólya's approach to problem-solving



# Pólya's approach to problem-solving



a model to represent problems

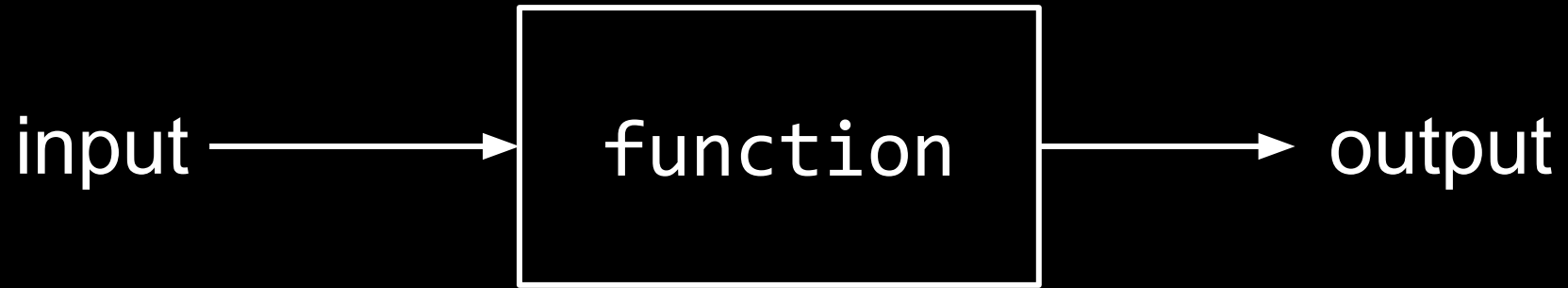




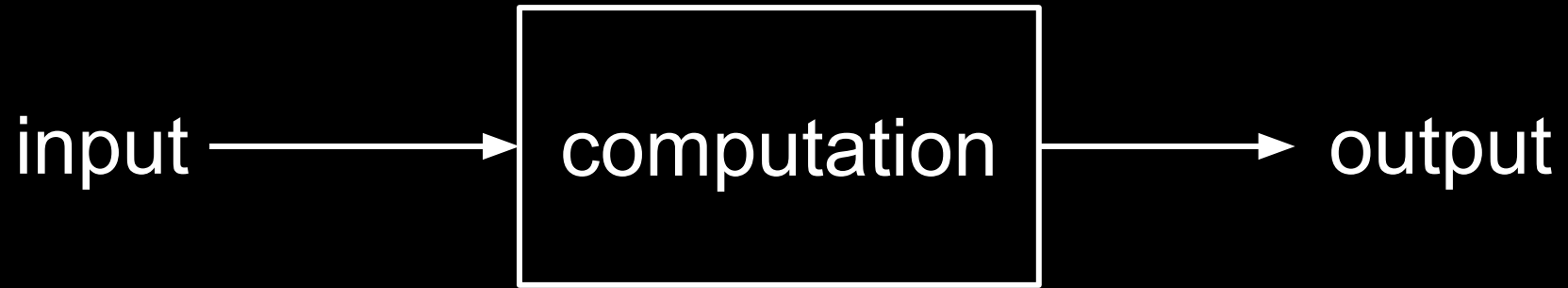
a model to represent problems



a model to represent problems

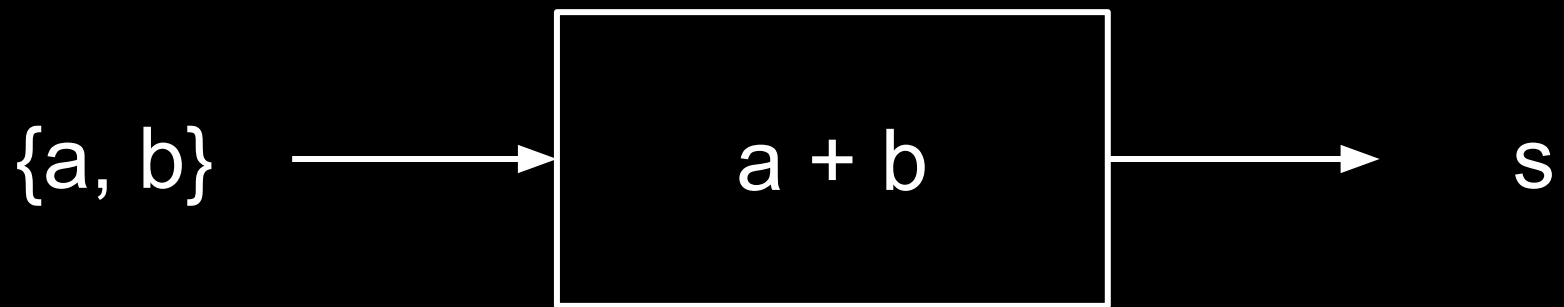


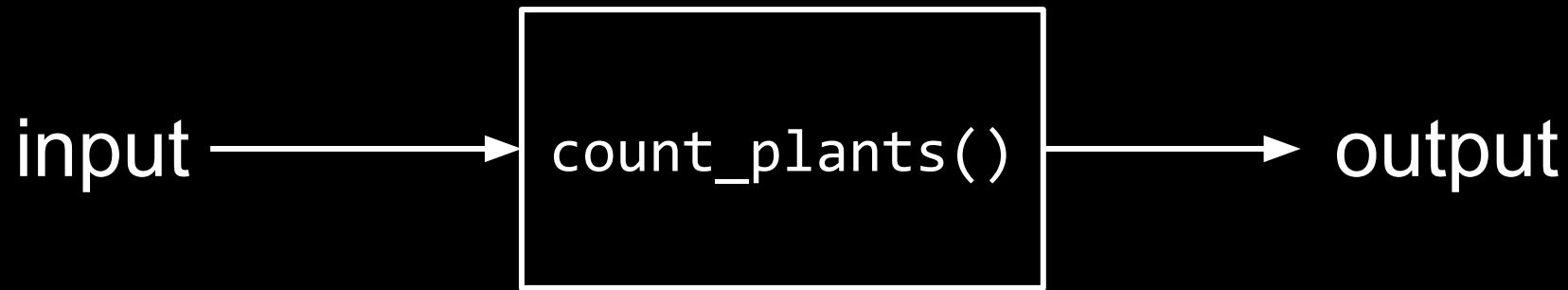
a model to represent problems



the rainbow experiment as an  
input - processing - output - problem









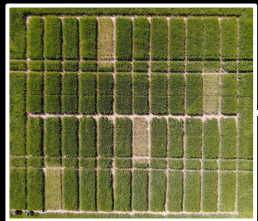
output



42



processing of  
information



`count_plants()`

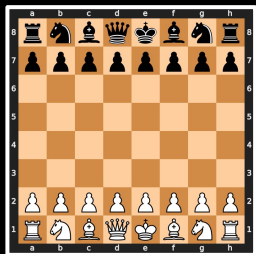
42

representation of  
information



next\_move()

E2 → E4



1: 0R	9: 0P	57: 1R
2: 0N	10: 0P	58: 1N
3: 0B	11: 0P	59: 1B
4: 0K	12: 0P	60: 1K
5: 0Q	13: 0P	61: 1Q
6: 0B	14: 0P	62: 1B
7: 0N	15: 0P	63: 1N
8: 0R	16: 0P	64: 1R

...

representation of information



problem solving strategies

problem decomposition

large and complex problem

less complex  
subproblem

less complex  
subproblem

less complex subproblem



less complex  
subproblem

less complex  
subproblem

less complex subproblem

less complex subproblem

less complex subproblem

divide and conquer

large and complex problem of type A

smaller problem  
of type A

smaller problem  
of type A

smaller problem  
of type A

smaller problem  
of type A

even smaller problem of type A	even smaller problem of type A
even smaller problem of type A	even smaller problem of type A
even smaller problem of type A	even smaller problem of type A
even smaller problem of type A	even smaller problem of type A

sorted list +  
element



search()



yes / no

is 67 a prime number?

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,  
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

## linear search



2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,  
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97



## linear search



2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,  
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

linear search



2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,  
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

linear search



2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,  
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

## linear search

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,  
~~43~~, ~~47~~, ~~53~~, ~~59~~, 61, 67, 71, 73, 79, 83, 89, 97  
↑

19 steps... can't we do better?

2, 3, 5, 7, 11, ~~13~~, 17, ~~19~~, ~~23~~, ~~29~~, ~~31~~, ~~37~~, ~~41~~,  
~~43~~, ~~47~~, ~~53~~, ~~59~~, ~~61~~, 67, 71, 73, 79, 83, 89, 97

↑

large and complex  
problem

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,  
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

large and complex  
problem

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,  
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

smaller  
problem

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41

smaller  
problem

43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

## binary search

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,  
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97



binary search

67 != 41



2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, ~~41~~,  
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

binary search

67 > 41



2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,  
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

binary search

67 > 41



2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,  
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

## binary search

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,  
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97



67 != 71

## binary search

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,  
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97



67 != 71

## binary search

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,  
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97



$67 < 71$

## binary search

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,  
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97



67 != 59

## binary search

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,  
~~43~~, 47, ~~53~~, 59, 61, 67, ~~71~~, ~~73~~, 79, ~~83~~, ~~89~~, 97



67 > 59



## binary search

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,  
~~43~~, ~~47~~, ~~53~~, ~~59~~, 61, 67, ~~71~~, ~~73~~, ~~79~~, ~~83~~, ~~89~~, 97



67 = 67

## binary search

2, 3, 5, 7, 11, ~~13~~, 17, 19, ~~23~~, ~~29~~, 31, 37, 41,  
~~43~~, 47, ~~53~~, 59, ~~61~~, 67, ~~71~~, ~~73~~, 79, ~~83~~, ~~89~~, 97



67 = 67

3 splits → much better

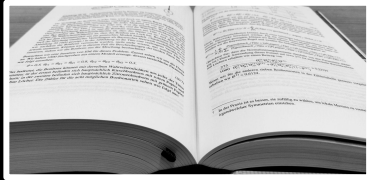
2, 3, 5, 7, 11, ~~13~~, 17, 19, ~~23~~, 29, 31, 37, 41,  
~~43~~, 47, ~~53~~, 59, ~~61~~, 67, ~~71~~, ~~73~~, 79, ~~83~~, ~~89~~, 97



67 = 67



how efficient are linear and  
binary search in general?



`count_words()`

word count

$\theta = 0.5$ ,  $\theta_{T1} = \theta_{W1} = \theta_{H1} = 0.8$ ,  $\theta_{T2} = \theta_{W2} = \theta_{H2} = 0.3$ .

Zus bedeutet, die Bonbons können mit derselben Wahrscheinlichkeit aus jeder der Tüten ummen; in der ersten befinden sich hauptsächlich Kirschbonbons mit rotem Papier und Zitronebonbons mit grünem Papier, in der zweiten befinden sich hauptsächlich Zitronenbonbons mit grünem Papier ohne Löcher. Die Zähler für die acht möglichen Bonbonarten sehen wie folgt aus:

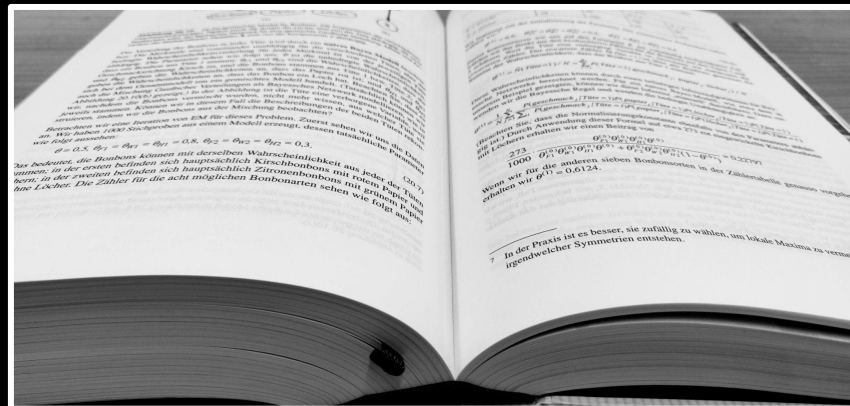
(20.7)

Wenn wir für die anderen sieben Bonbonsorten in der Zählertabelle genauso vorgehen erhalten wir  $\theta(1) = 0.6124$ .

In der Praxis ist es besser, sie zufällig zu wählen, um lokale Maxima zu vermeiden.

7 In der Praxis ist es besser, sie zufällig zu wählen, um lokale Maxima zu vermeiden, wenn Symmetrien entstehen.

strategies, anyone?



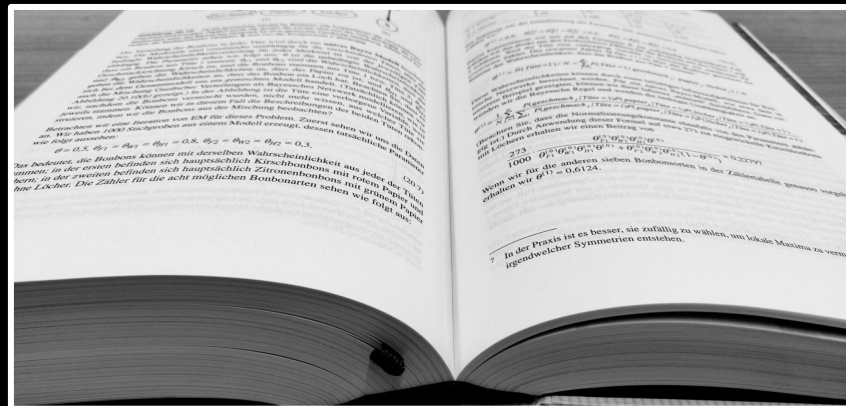
Das bedeutet, die Bonbons können mit derselben Wahrscheinlichkeit aus jeder der Urnen  
kommen, in der ersten befinden sich hauptsächlich Zitronenbonbons mit einem Loch, in der  
zweiten hauptsächlich Zitronenbonbons mit einem Loch, in der dritten hauptsächlich Zitronenbonbons  
ohne Loch. Die Zähler für die acht möglichen Bonbonurten sehen wie folgt aus:

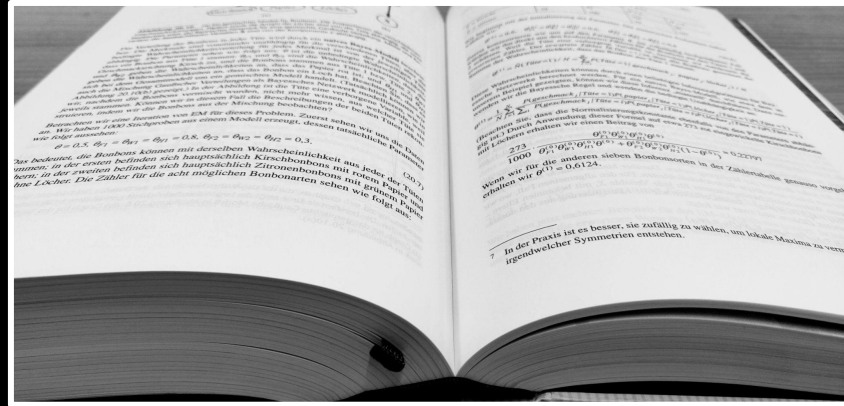
$$\theta_1 = 0.5, \theta_2 = 0.5, \theta_3 = 0.5, \theta_4 = 0.5, \theta_5 = 0.5, \theta_6 = 0.5, \theta_7 = 0.5, \theta_8 = 0.5.$$

Wenn wir für die anderen sieben Bonbonurten in der Zählertabelle gemessen werden  
erhalten wir  $\theta^{(1)} = 0.6124$ .

In der Praxis ist es besser, sie zufällig zu wählen, um lokale Maxima zu vermeiden  
irgendwelcher Symmetrien entstehen.



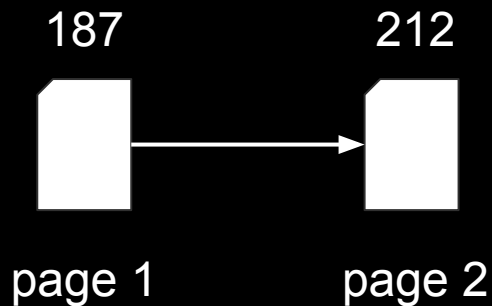
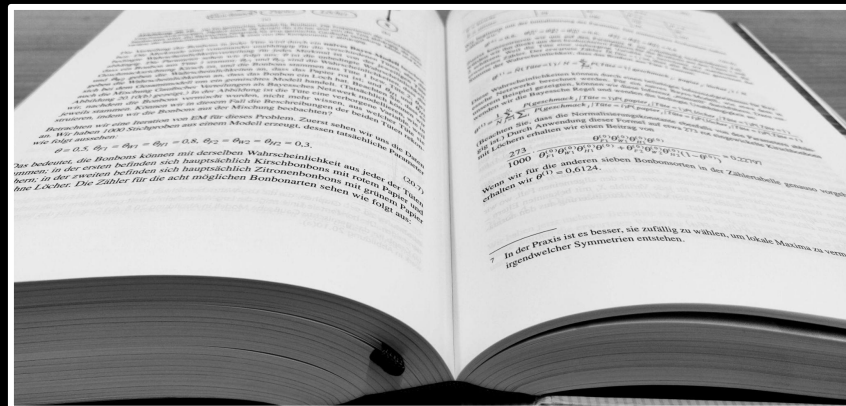


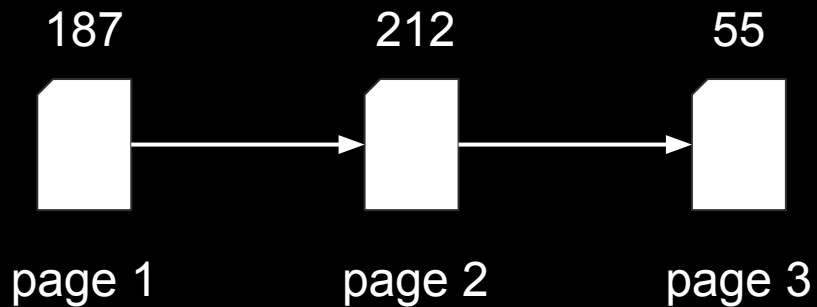
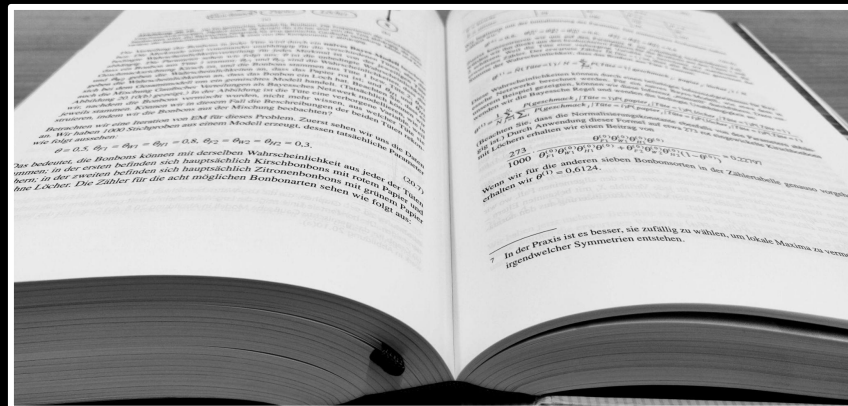


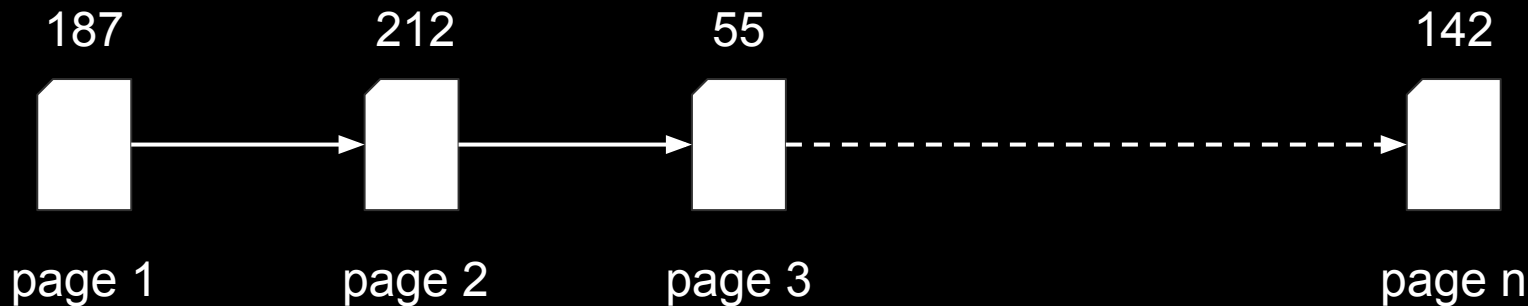
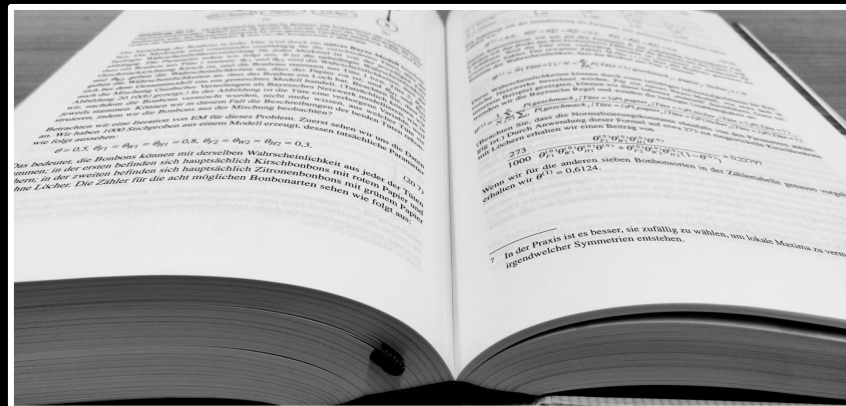
187



page 1









n = 1327 pages

Ø 2:23 minutes per page

~ 52.34 hours

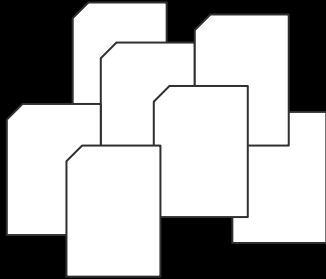


divide and conquer

+

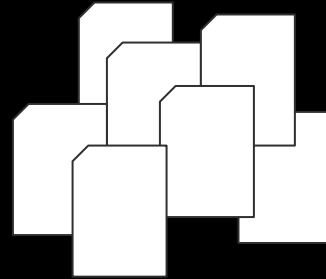
?

pages 1 - 700



student 1

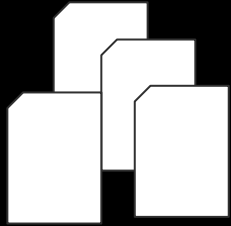
pages 701 - 1327



student 2

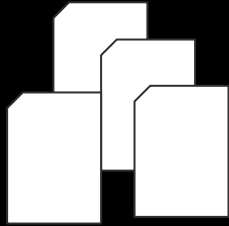


pages 1 - 350



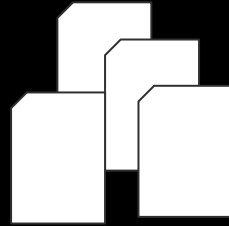
student 1

pages 351 - 700



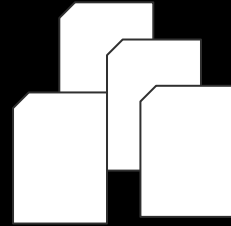
student 2

pages 701 - 1050



student 3

pages 1051- 1327



student 4

divide and conquer

+

distribution and parallelization

