Lambda Expressions (Haskell)

Evaluate each of the following lambda expressions. Firstly, do it yourself on paper and then enter it into GHCi to be evaluated by Haskell.

The first two have been done for you as examples.

1. $(\lambda x \to 3 * x * x)\ 4$

   Answer

   $(\lambda x \to 3 * x * x)\ 4 = 3 * 4 * 4 = 48$

   In Haskell,

```
> (\x -> 3*x*x) 4
48
```

2. $\big(\lambda x \to (\lambda y \to (x\ (x\ y)))\big)\ (\lambda x \to 2 * x)\ 7$

   Answer

$$\big(\lambda x \to (\lambda y \to (x\ (x\ y)))\big)\ (\lambda x \to 2 * x)\ 7$$

$$= (\lambda y \to ((\lambda x \to 2 * x)\ ((\lambda x \to 2 * x)\ y))\ 7$$: Substitute outer x with $(\lambda x \to 2 * x)$

$$= ((\lambda x \to 2 * x)\ ((\lambda x \to 2 * x)\ 7)$$: Substitute y with 7

$$= ((\lambda x \to 2 * x)\ (2 * 7)$$: Substitute inner x with 7

$$= 2 * (2 * 7)$$: Substitute x with $2 * 7$

$$= 28$$

   In Haskell,

```
> (\x -> (\y -> x (x y))) (\x -> 2*x) 7
28
```

3. $(\lambda x \to 2 * x + 1)\ 5$

4. $(\lambda y \to 3)\ 4$

5. $(\lambda x \to x)\ 9$

6. $\big(\lambda x \to (\lambda y \to x * y + x)\big)\ 5\ 2$

7. $\big(\lambda x \to (\lambda y \to x\ y)\big)\ (\lambda z \to z + z)\ 6$

8. $\big(\lambda y \to (\lambda x \to 2 * (x\ 2) + (x\ y) + y - 3)\big)\ 5\ (\lambda x \to 3 * x)$

9. $(\lambda x \to x)\ (\lambda x \to x)\ 8$

10. $(\lambda x \to x\ x)\ (\lambda x \to x\ x)\ 8$