MATHEMATICS

GRADE 7 STUDENT TEXT BOOK

South Nations Nationalities and Peoples Regional Education Bureau

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First Edition, 2014 E.C.

South Nations Nationalities and Peoples Regional Education Bureau

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Unit 1

Basic Concepts of Sets

Learning Outcomes:

At the end of this unit, learners will be able to:

- Understand the concept of set.
- Describe the relation between two sets.
- Perform two operations (intersection and union) on sets.

Introduction

The idea of a set is familiar in everyday life. In your surrounding there are different groups of objects or individuals. For example group of all grade 7 students in your school, group of all teachers in your school is a collection of individuals. In this unit you will learn about some sets and properties of sets.

1.1.Sets and Elements of Sets

In our daily life things are grouped together with a certain property in common such as family members, a collection of clothes, fingers of hands, collection of students in a class, a herd of cattle, a flock of sheep, a swarm of bees,etc.all these groups are well defined.

Definition: A Set is a well-defined collection of objects or individuals. The objects in the set are called its elements or members of the set.

By well-defined, we mean that, given a collection and an object or individual, we have to say that the object or the individual is in the collection or not without any ambiguity. Repeating elements in a set do not add new elements to the set.

For Example, {b, b, b} is the same as {b}.

Example 1.1

Identify each of the following collections are a set or not

a. The collection of whole numbers less than 6.
b. The group of short students in a certain school.
c. The collection of vowel letters in the English alphabet.
d. A group of rich people in Hawassa.

Solution:

a. The collection of whole numbers less than 5 is a well-defined collection. The whole numbers 1, 2, 3, 4, and 5 are in collection and anything other than these is not in the collection.

b. The group of short students in a certain school is not well-defined. It is because shortness is relative someone may be short for some group, but not for others.

c. The vowels in the English alphabet are a, e, i, o, u are well-defined; anything other than these five letters is not in the collection.

d. The group of rich people in Hawassa is not well-defined because richness is relative

Note:

1. Sets are denoted by capital letters like A, B, C, etc, and elements of a set are denoted by small letters like a, b, c, d, x, y, z, etc.

2. The symbol {. . . .} is used to group the members of a set called braces or curly brackets with elements separated by commas.

3. Given a set A and an object x,

i. If x is an element of A, then we denote this relation by "$x \in A$" and read as "x is an element of set A" or "x belongs to set A".

ii. If x is not an element of A, then we denote the relation by "$x \notin A$" and read as "x is not an element of set A" or "x does not belong to set A".

Example 1.2

If A is the set of natural numbers less than5, then A can written as A= {1,2,3,4,} In this case, $1\in A$,$2\in A$,$3\in A$,and $4\in A$,but$5\notin A$and also $7 \notin A$ and so on. Every object in a set is unique. The same object cannot be included in the set more than once. For example, if A= {1, 2, 3, 4, 5} then every element of A is included not more than once. We do not write A= {1, 1,2,3,4, 5} or A= {1, 2,2,3,4, 5} or
A= {1, 2,3,3,4, 5} and so on.

Example 1.3

Determine the set X as the set of all days in a week.

Solution:

There are seven days in a week: Monday, Tuesday, Wednesday, Thursday, Friday, Saturday and Sunday.

Therefore= {Monday, Tuesday, Wednesday,Thursday,Friday,Saturday, Sunday}.

Example 1.4

Let M be the set of multiples of 2 between 1 and 9; determine the set M and members of the set M.

Solution:

M = {2, 4, 6, 8,} and 2, 4, 6and 8 are the members of set M.

Exercise 1.1

1. Which of the following collections are sets? Justify your answer.

   a. The beautiful birds in Hawassa.
   b. The natural numbers less than 1000.
   c. The prime factors of 2500.
   d. The months of a year.
   e. Students in grade 6 that are 12 years of age in Ethiopia.

2. Identify each of the following statements are true or false

   a. $-1 \in \{ -2, -1, 0, 1, 2 \}$
   b. $b \in \{ a, b, c, d, e, f \}$
   c. $6 \notin \{\text{factorsof48}\}$
   d. $3 \notin \{1, 2, \{3\}, 4\}$.
   e. $\{3\} \in \{3\}$.

1.1.1. Ways to Describe Sets

i.Listing or Roster Method

Listing or roster notation also called enumeration method is a list of elements separated by commas enclosed in curly braces. The curly braces are used to indicate that the elements written between them belong to that set. The elements in such types of sets can be completely or partially listed and the elements follow certain pattern.

Example 1.5

Let P be the set of all prime numbers less than 9. Write P using listing method.

Solution: The prime numbers less than 9 are 2, 3, 5 and 7.

Therefore, P= {2, 3, 5, 7} is tabulation or complete listing method.

Example 1.6

If A is the set of natural numbers less than 10, then write B using listing or roster method.

Solution:

The natural numbers less than10 are numbers from1to9. Instead of writing all the 9 numbers, you can list the first three numbers to indicate that the numbers follow a certain pattern followed by three dots to indicate that the numbers are continuing up to the last number, which is 9.

Thus, A= {1, 2, 3… 9} is a partial listing method.

ii.Verbal Method

There are times when it is not practical to list all the elements of a set. In such cases, it is better to describe the set using verbal method.

Example 1.7

Describe each of the following sets

a. The set of letters in the English alphabet.

b. The set of Grade 6 students in Ethiopia in 2014 E.C.

Solution:

There are 26 letters in the English alphabet and instead of listing all the English alphabets, you can write the set as {the English alphabets}.

It is very difficult to list all Grade 6 students in Ethiopia in 2014 E.C. and the better way to write it is as {Grade 6 students in Ethiopia in 2014 E.C.}.

ii.Set builder method

It is called method of defining property.

It is another way to specify a set that enables us to decide whether or not any given objects belong to the set.

Example 1.8

a. F is the set of all females who are living in Hawassa can be described as F= {x/x is a female living in Hawassa} which is read as "F is the set of x such that x is a female living in Hawassa".

b. Let A={2, 3, 4, 5, 6} can be described as $A=\{x|\ x\in \mathbb{N} \text{ and } 2<x<7\}$

Exercise 1.2

1. Describe each of the following sets by the listing method.

   a. The set of prime numbers between 4 and 25.

   b. The set of capital cities of the regional states in Ethiopia.

c. The set of all subjects that a grade 7 student is learning.

d. The set of even natural numbers.

2. Describe each of the following sets by a verbal method

a. {1, 2, 3, … , 100}

b. {1, 3, 5, 7, …,}

c. {January, February, March, …, December}

d. {2, 3, 5, 7, 11, …}

1.1.2. Some special Types of Sets

Empty set

Definition: A set which contains no element is called empty set (null or void). Empty set is denoted by ∅ or {}.

Example 1.9

a. A set of whole numbers that is odd and even.

b. A group of birds which have four legs.

c. X is the set of whole numbers that are even and prime.

d. The set of cats that can fly.

Solution:

a. There is no whole number which is both even and odd. So A is empty set, that is, A={ } or ∅

b. There is no bird which has four legs.

c. The only whole number that is both even and prime is 2.

Therefore, X= {2} is not empty set.

Definition: A set G is said to be a finite set if G contains exactly n elements for some positive integer n or G = ∅. A set that is not finite is called infinite set.

Notation: If G is a finite set, then the number of elements in G is denoted by n (G).

Example 1.10

Identify each of the following sets as finite or infinite.

a. M is the set of all weeks with 9 days.
b. L= {1, 2… 200}.
c. K is the set of all natural numbers that are multiples of 3.

Solution:

a. For every week there is 7 days. Therefore, M = {} and hence M is a finite set, with n (M) = 0.
b. L contains 200 elements. Thus, L is a finite set and n (L) = 200.
c. M = {4, 8, 12 …} and M does not contain finite elements. Therefore, M is an infinite set.
d. K = {3, 6, 9 . . .} and K does not contain finite elements. Therefore, K is an infinite set.

Exercise 1.3

Identify each of the following sets as finite set, infinite set or empty set

a. D is the set of integers less than 0
b. S is the set of natural numbers less than 1.
c. Q is the set of grade 7 teachers who are 13 years old.
d. X is the set whole numbers between -2 and 10.

1.2. The Relation among sets

i.Subsets

Activity 1.1: What **relationships do you observe?**

a. M= { –2, -1, 0, 1,2 } and N ={ –2, -1,0,1,2 }

b. P={ 1, 2, 3} and Q={ 1, 2, 3,4,5 }

Definition:

Given two sets A and B if every element of A is an element of B, then we say that A is a subset of B, and we denote this relation by A⊆B. If set A is not subset of set B. we denote by A⊈ B.

Note: For any set,

a. Empty set is a subset of every set or ∅⊆A

b. Every set is the subset of itself or A⊆A

Example 1.11

a. List all subsets of the set A = {a, b, c, d}. How many are there?

b. List all subsets of the set L = {1, 2, 3}. How many are there?

Solution:

a. The subsets of A are {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c},{a, b,d}, {b, c, d}, {a, b, c}, {a, b, c, d} and ∅.

There are 16 subsets of the set A = {a, b, c, d}.

b. The subsets of L are {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, {1, 2, 3}, and Ø.

There are 8 subsets of the set N = {1, 2, 3}.

ii.Proper subset

Definition: Set A is a proper subset of set B if every element of set A is an element of the set B but there exists at least one element in B which is not an element of the set A. It is denoted by A⊂B. Which is read as 'A is a proper subset of B'. That is, A⊂B means A⊆B but B ⊈ A.

Example 1.12

Given set X = {a, b, c}, the sets ∅, {a}, {b}, {c}, {a, b}, {a, c} and {b, c} are proper subsets of set X but {a, b, c} is not proper subsets of set X. i.e. is $X \not\subset X$.

Fact: If a set A is finite with n elements, then

i. The number of subsets of A is 2^n.

ii. The number of proper subsets of A is $2^n - 1$.

i. Power set

Definition: Let A be any set, the power set of A, denoted by P (A), is the set of all subsets of A. That is, P (A) ={S/S⊆A}.

Example 1.13 Let L= {-1, 1,}, then the subsets of L are ∅, {-1}, {1}, and L. Therefore, P (A) = {∅, {-1}, {1}, L}.

Equality of Sets

Definition: Given two sets A and B, if every element of A is also an element of Band if every element of B is also an element of A, then the sets A and Bare said to be equal. We write this as A = B.

∴ A = B, if and only if A ⊆ B and B ⊆ A i.e. they have the same number of elements and their elements are the same.

If A and B are not equal, we write $A \neq B$.

Equivalence of sets

Definition: Two sets A and B are said to be equivalent, written as $A \leftrightarrow B$ or $(A \sim B)$, if there is a one to one correspondence between them.

Observe that two finite sets A and B are equivalent, if and only if $n(A) = n(B)$.

Example:

1. Let A= {1, 2, 3, 4} and B= {1, 4, 2, 3}. Since two sets contain exactly the same/identical elements, A=B.
2. Let A= {a, b, c, d} and B= {1, 2, 3, 4} since two sets contain equal number of elements, $A \leftrightarrow B$.

Exercise 1.4

1. State whether each of the following statements is true or false. If it is false justify your answer.

   a. Every equal set are equivalent sets.

   b. $\{a, b, c, d\} \subseteq \{b, a, d, c\}$

   c. $\{3\} \subseteq \{\{3\}\}$

   d. $\emptyset \subseteq \{\{3\}\}$

   e. $\{0,1\} \nsubseteq \{\{0,1\},0,1\}$.

2. Let Q={0, 1, 2} then find all

   i. Subsets of Q

   ii. Proper subsets of Q

iii. Power sets of Q

3. Let A= {0, {1, 2}} then find all (i-iii) in question 2 above.
4. Which of the following pairs represent equal sets and which of them represent equivalent sets?

   i. {∅} and {1}

   ii. K={x ∈ℕ|x is a factor of 6} and H={ x∈ℕ| 0<x<7}

   iii. A = The set of even natural numbers between 1 and 7

   B = The set of even divisors of 12 less that 12.

1.3.Operations on Sets

As there are operations on the sets of numbers there are operations on the sets. Here we consider only two operations among the operations of sets. These are the intersection of sets and the union of sets. We will see this one by one in detail as follows.

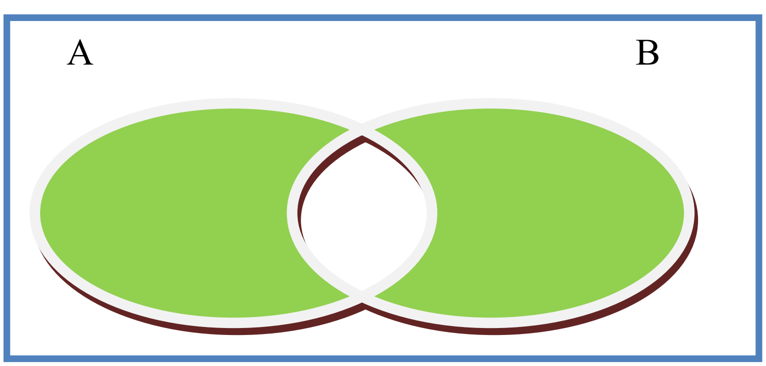
1.3.1. The intersection of sets

Definition: The intersection of two sets A and B, denoted by A∩B and read as "A *intersection* B" is the set of all elements common to both set A and set B.

That is,

$A \cap B = \{x | x \in A \text{ and } x \in B\}$

Using the Venn diagram, $A \cap B$ can be represented by the white region:



Example: Let $A = \{1,3,5,6,7,8\}$ and $B = \{2,3,6,8,9\}$. Then $A \cap B = \{3,6,8\}$.

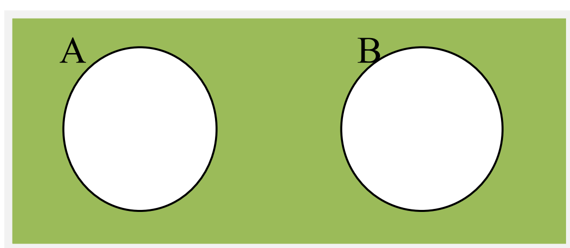
Example: Let $S = \{a, b, c, d\}$ and$T = \{f, b, d, g\}$. Then $S \cap T = \{b, d\}$

Example: Let $W = \{4,6,8\}$ and $X = \{1,3,5,7\}$. Here these two sets have no elements in common, so their intersection is empty set, that is $W \cap X = \emptyset$.

Definition: Two or more sets that have no common element are called disjoint sets.

Sets A and B are disjoint sets, if $A \cap B = \emptyset$.

In Venn diagram, the disjoint sets A and B can be represented as:



Properties of the intersection of sets

For any sets $A, B,$ and C and the universal set U

i. Commutative property: $A \cap B = B \cap A$

ii. Associative property: $(A \cap B) \cap C = A \cap (B \cap C)$

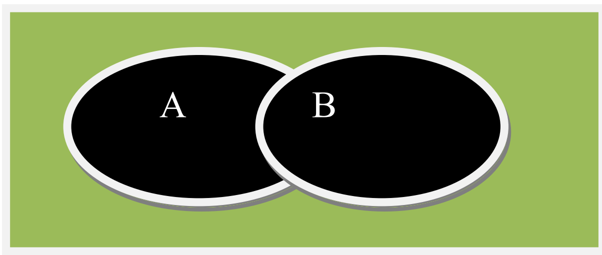
iii. Identity property: $A \cap U = A$

1.3.2. The union of sets

Definition: The union of two sets A and B, denoted by A∪B and read as

"A *union* B" is the set of all elements that are members of set A and set B or both of the sets. That is, $A \cup B = \{x | x \in A \text{ } or \text{ } x \in B\}$

We can represent $A \cup B$ using Venn diagram as follows:



Example: Let $A = \{a, b, c, d, e\}$

$B = \{c, d, e, f, g\}$, then $A \cup B = \{a, b, c, d, e, f, g\}$

Example: a. $\{a, b\} \cup \{c, d, e\} = \{a, b, c, d, e\}$

b. $\{1,2,3,4,5\} \cup \emptyset = \{1,2,3,4,5\}$

Properties of the union of sets

For any sets $A, B,$ and C and the universal set U

i. Commutative property: $A \cup B = B \cup A$

ii. Associative property: $(A \cup B) \cup C = A \cup (B \cup C)$

iii. Identity property: $A \cup \emptyset = A$

Exercise 1.5

1. Given $A = \{a, b, \{c\}\}, B = \{b, c\}$,and $C = \{\{c\}, d\}$,find:

   a. $A \cap B$ b. $A \cap C$ c. $B \cap C$ d. $A \cap (B \cap C)$

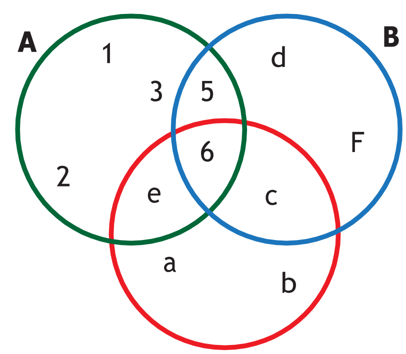
2. Given $A = \{1,2,\{3\}\}, B = \{2,3\}$,and $C = \{\{3\}, 4\}$,find:

   a. $A \cup B$ c. $A \cup C$ e. $A \cup (B \cup C)$

b. $B \cup C$ d. $(A \cup B) \cup C$

3. Determine whether each of the following statements is true or false:
   a. If $x \in A$ *and* $x \in B$,then $x \in (A \cap B)$.
   b. If $x \notin A$ *and* $x \in B$,then $x \in (A \cap B)$.
   c. If $x \notin A$ *and* $x \notin B$,then $x \notin (A \cup B)$.
   d. For any set,A, $A \cup \emptyset = A$.
   e. If $A \subseteq B$,then $A \cup B = B$
   f. If $x \in (A \cap B)$,then $x \in A$ *and* $x \in B$.
   g. If $A \subseteq B$,then $A \cap B = A$
   h. For any set,A, $A \cap A = A$.
   i. If $A \cup B = \emptyset$,then $A = \emptyset$ and $B = \emptyset$
4. Let $U = \{0,1,2,3,4,5,6,7,8,9\}$ be the universal set and let
   $A = \{0,2,3,5,7\}$, $B = \{0,2,4,6,8\}$ and
   $C = \{x | x \text{ is a factor of } 6\}$,
   Find i. $A \cap C$ ii. $B \cap C$ iii. $A \cap (B \cap C)$
5. Use the Venn diagram given below to find each of the following set relations.

a. $A \cap (B \cap C)$. c. $A \cap B$ e. $A \cup (B \cap C)$.
b. $A \cup B$ d. $A \cup C$ f. $B \cap C$



Summary on Unity 1

- A set is a well-defined collection of objects and the objects of a set are called *elements* of the set. Elements of a set are also called members of the set.
- Sets can be described by listing method or verbal method.
- A set that has no element is called an *empty set* or *vacuous set* and denoted by ∅ or {}.
- A set A is called *finite* if and only if it is the empty set or has exactly *n* elements, where *n* is a natural number. Set that is not finite is called an *infinite set.*
- Two sets A and B are said to be *equal* if every element of one is an element of the other.
- A set A is a subset of B if every element of set A is an element of set B.
- For any two sets A and B,
    a. The union of A and B, denoted by AUB, is the set of all elements that are members of either A or B.
    b. The union of A and B, denoted by AUB, is the set of all elements that are members both A and B.

Review Exercises on Unit 1

1. Determine whether each of the following is set or not.
   a. The collection of all tall students in your class.
   b. The collection of all whole numbers divisible by 2.
   c. The collection of model farmers in your locality.
   d. The collection of odd numbers.
   e. The collection of all intelligent students in south region.
2. Which of the following represent equal sets?
   a. A= $\{a, b, c, d, e\}$
   b. B= $\{a, e, i, o, u\}$
   c. F = The first five letters in the English alphabet.
   d. V = The vowel letters in the English alphabet.
3. Rewrite each of the following statements using correct notation of elements and sets.
   a. A is a set whose elements are 1,2,3,4 and 5.
   b. 6 is not an element of set A.
   c. W is the set of all natural numbers between 3 and $\sqrt{21}$.
4. Which of the following pairs of sets are equivalent?
   a. {1,2,3,4,5} and {m,n,o,p,q}
5. $\{x \in \mathbb{N} | x$ is a factor of $6\}$ and The set of natural number less than 9
6. If X=$\{a,b,c,d,e,f,g,h,i,j\}$,Y=$\{b,d,f,h\}$ and Z = $\{a,b,e,f,g,h\}$,then find each of the following sets:

   a. X∩Y   c. ZUY   e. (XUY)UZ
   b. Y∩X   d. YUZ   f. (X ∩Y)∩Z
7. Identify each of the following as finite or infinite.
   a. A=All integers less than 20.
   b. B=All students in your class who are older than 20 years of age.
   c. C=All natural numbers less than 1.

Unit 2

Integers

Learning Outcomes:

At the end of this unit, learners will be able to:

- Understand the concept of integers
- Represent integers on a number line
- Identify the commutative, associative and distributive properties of operation of integers
- Perform the operations addition and subtraction on integers.
- Apply integers in the real-life situation

Introduction

In the previous grades you had already learnt about whole numbers and natural numbers. These numbers together with negative numbers form another set of numbers called **Integers**. In this unit you will learn about integers and the four fundamental operations on integers.

2.1.Revision of whole and natural numbers

By the end of this section you should be able to:

- Describe whole and natural numbers
- Identify the relation between whole and natural numbers

Activity 2.1.1

1. List down the first three smallest natural numbers.
2. List down the first five smallest whole numbers.
3. What is the smallest and greatest natural number?
4. What is smallest and the greatest whole number?
5. Define:
   a. Natural number
   b. Whole number

Definition 2.1: Natural number is a set of numbers denoted by $\mathbb{N}$ and is described by $\mathbb{N} = \{1, 2, 3, \dots\}$

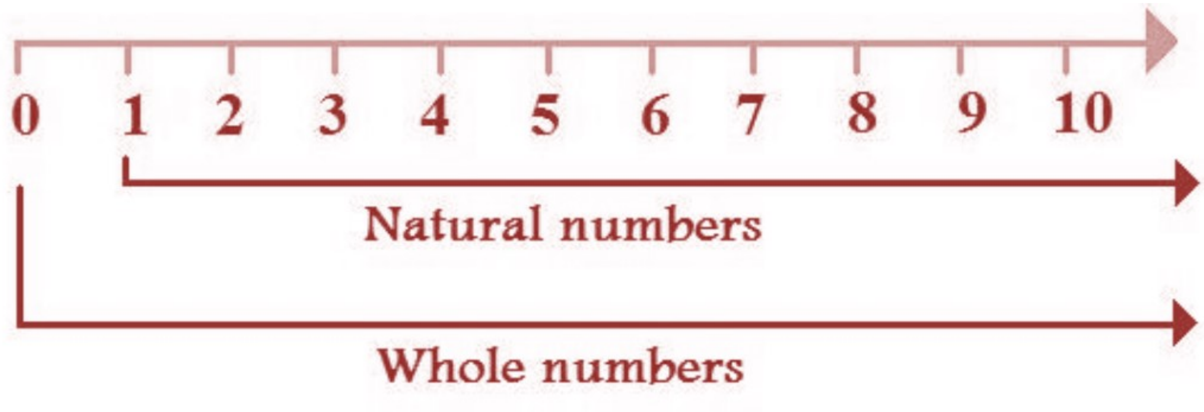
Natural numbers are also called counting numbers.

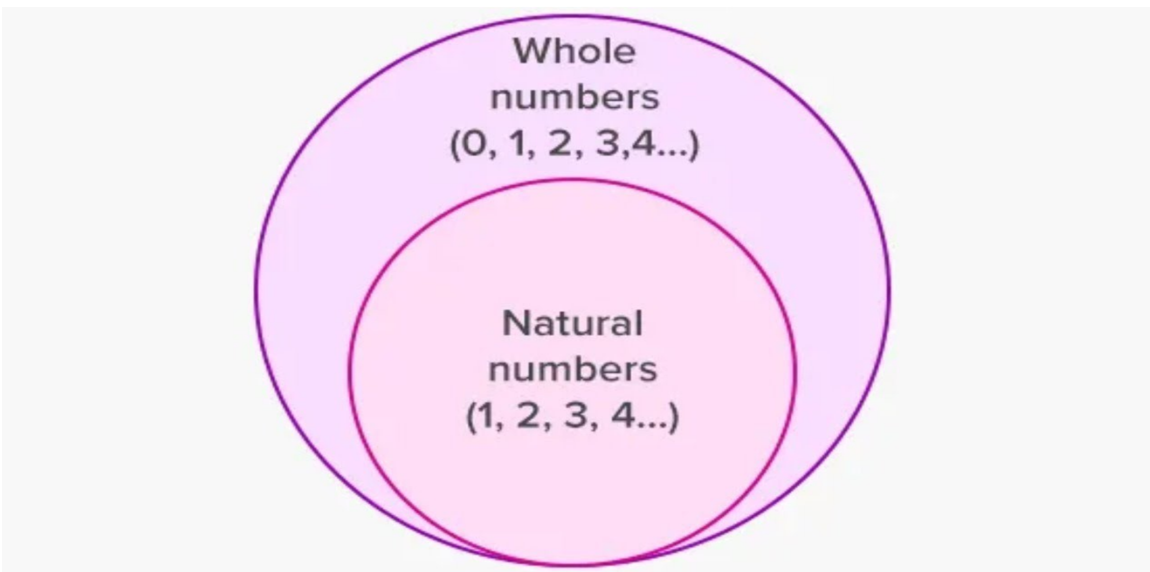
Definition 2.2: Whole numbers are numbers contains natural number and zero.

Whole number is denoted by $\mathbb{W}$ and is described by

$\mathbb{W} = \{0, 1, 2, 3, ., ., .\}$

Relationship between natural numbers and whole numbers is described below


0
1
2
3
4
5
6
7
8
9
10
Natural numbers
Whole numbers


Whole numbers (0, 1, 2, 3,4...)
Natural numbers (1, 2, 3, 4...)


0
1, 2, 3, 4, 5, ...
Whole Numbers (A)
Natural Numbers (B)

Revision of operations on natural numbers and whole numbers

Example 1. Find the following sums

$a.\ 452+323$

$b.\ 3234+598$

$c.\ 423+5672$

Solution:

a. $\begin{array}{r} 452 \\ \underline{+323} \\ \underline{\underline{=775}} \end{array}$ b. $\begin{array}{r} 3234 \\ \underline{+598} \\ \underline{\underline{=3832}} \end{array}$ c. $\begin{array}{r} 5672 \\ \underline{+432} \\ \underline{\underline{=6104}} \end{array}$

Example 2. Find the following differences

a. $453-223$
b. $4567-352$
c. $456-456$

Solution:

a. $\begin{array}{r} 453 \\ \underline{-223} \\ \underline{\underline{=230}} \end{array}$ b. $\begin{array}{r} 4567 \\ \underline{-352} \\ =\underline{\underline{4215}} \end{array}$ c. $\begin{array}{r} 456 \\ \underline{-456} \\ \underline{\underline{=0}} \end{array}$

Example 3. Find the product of the following numbers.

a. $46\times 44=2024$
b. $72\times 78=5616$
c. $123\times 342=42066$

Example 4. Find the quotient of the following

a. $455 \div 5 = 91$

b. $4824 \div 12 = 402$

Note:

- The sum of any two natural numbers is always natural number.
- The sum of any two whole numbers is always whole number.
- The product of any two natural numbers is always natural number.
- The product of any two whole numbers is always whole number

Exercise 2.1.1

Calculate the following

a. $345 + 892$

b. $342 + 435$

c. $6543 - 234$

d. $23 + (654 - 345)$

e. $5427 \div 9$

f. 342×345

2.2.Introduction to integer

By the end of this section you should be able to:

- Apply real-life applications of integers in terms of temperature, altitude and money to express positive and negative numbers
- Define the set of integers
- Indicate integers on the number line.
- Describe the relations, among natural numbers, whole numbers and integers $\mathbb{N} \subset \mathbb{W} \subset \mathbb{Z}$

Activity 2.2.1

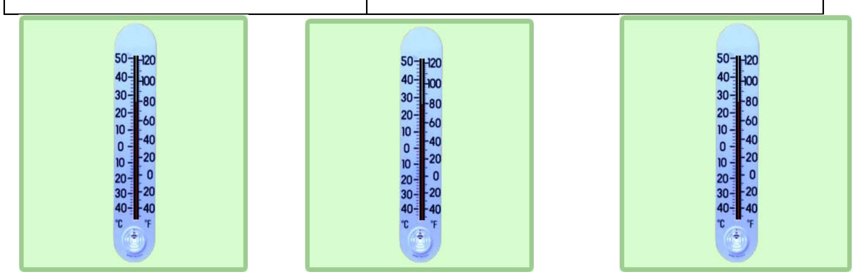
Discuss with your friends/partners

Figure 2.2.1

Anders Celsius the Swedish astronomer who lived between 1701 and 1744 A.D. He devised a way of measuring temperature which was adjusted and improved after his death.

1. The table below shows that the temperature recorded on three consecutive days for Hawassa city

Days	Temperature in degree Celsius
Monday	$16\ ^{0}$c above zero
Tuesday	6^{0}c below zero
Wednesday	1^{0}c below zero



Monday Tuesday Wednesday

Figure 2.2.2 Thermometer

a. What was the temperature on each day using the sign plus or minus?

b. How far did the temperature from zero in each day as shown in table and figures above?

c. What was the coldest temperature?

d. What was the hottest temperature?

2. What is the number that comes after zero?

3. What is the number that comes before zero?

Note: Very cold temperatures below zero can be described by number with **minus** sign which is called **negative numbers.**

The temperatures above zero can be described by a number with **plus** sign which is called **positive numbers**.

Example 1 The temperature 5^0C below zero can be written as -5^0C and

The temperature 5^0C above zero can be written as +5^0C or 5^0C

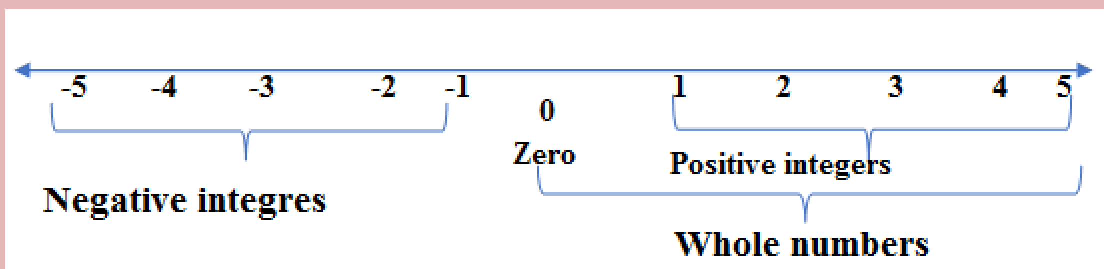
Note: positive numbers can be written without plus sign. For instance, $+2$ simply written as 2

Do you understand the difference of the concepts of the numbers below zero and above zero?

Definition 2.3

- An integer is a set of numbers consisting of whole numbers and negative numbers. The set of integers is denoted by

$$\mathbb{Z} = \{\ldots 4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$$



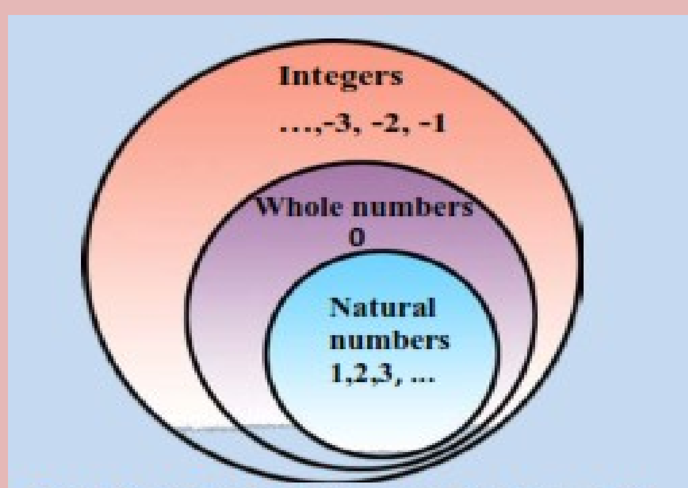
Negative integer is denoted by $\mathbb{Z}^-$ and positive integer is denoted by $\mathbb{Z}^+$

The set of positive integers is the same as the set of natural numbers. $(i.e. \mathbb{Z}^+ = \mathbb{N})$

Zero is an integer which is neither positive nor negative.

Integers, whole numbers and natural numbers are related as follows: in Venn diagram

$$\mathbb{N} \subset \mathbb{W} \subset \mathbb{Z}$$



Note that decimals and fractions are not integers.

The number -4 is read as negative four and 4 is read as positive four or four

Example 2: a. List some Positive integers

b. List five negative integers

c. Give example of integer which is neither positive nor negative

d. List four examples of numbers which are not integers

Solution a. $9, 54, 380, 6284, 89123$ are Positive integers.

b. $-9, -54, -125\ -647, -32564$, are negative integers

c. 0

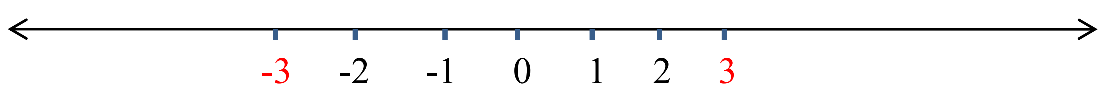
d. $2.5, \frac{3}{4}, 1.08, \frac{10}{7}$ are not integers

Example 3: Represent the following numbers on number line

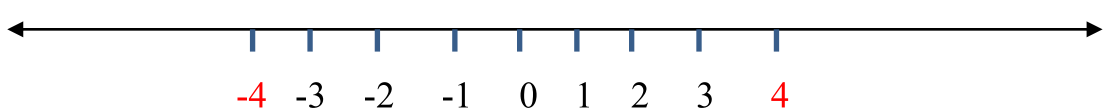
a. From -3 to 3
b. From -4 to 4
c. From -6 to 6

solution: First draw horizontal line and mark the numbers on the number line

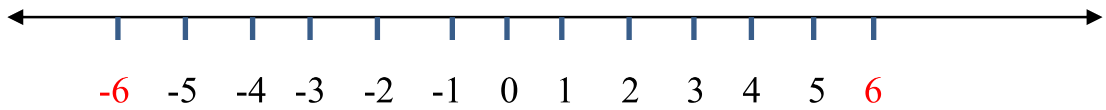
a.



b.



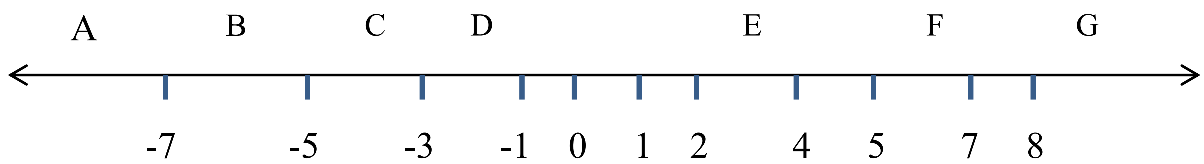
c.



Integer numbers represent on a number line

Exercise 2.2.1

1. Give the numbers represented by the letters on the number line below



2. Represent the numbers 1, -1, 5, -5, 2 and -2 on a number line
3. Write down four numbers which are not integer.
4. Write the following integers in word

   a. -3 b. 21 c. -132 d. 99

5. write the following numbers as numeral

   a. Positive four C. negative fifty-three

   b. Negative twenty d. one hundred one

6. Represent the following fact by using a numeral and $+$ and $-$ signs.

   a. A loss of Birr 100

   b. A rise of 10^0c temperature

   c. A walk of 7 km forward

   d. Five minutes late

   e. 6^0c below zero

   f. 21^0c above zero

7. What is the relationship between natural number and positive integer?
8. Explain the relationship between whole number, natural number and integer

9. From the numbers listed below select integers

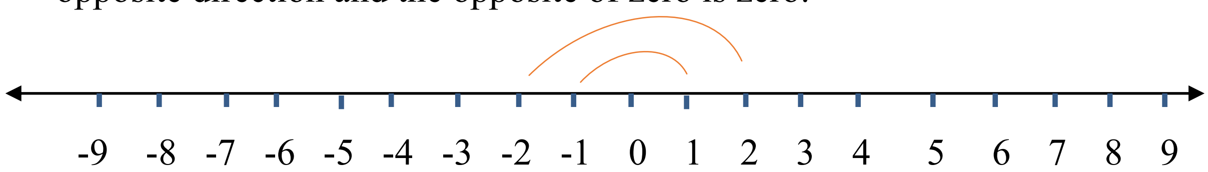
a. $2\frac{1}{3}$ d. 1.02 g. 0.25

b. 0 e. 12 h. 455

c. $\frac{2}{5}$ f. -294

Opposite numbers

Definition 2.4 **Opposite numbers** are numbers that are at the same distance from zero in opposite direction and the opposite of zero is zero.



Using the above number line, the opposite of some integers is listed in the table below

Number	-10	-9	-8	-7	-6	1	2	3	4	5
Opposite number	10	9	8	7	6	-1	-2	-3	-4	-5

Table 1

Example 4: find the opposite of the following numbers

a. 12 b. -9 c. 413 d. -564

Solution

a. The opposite of 12 is -12
b. The opposite of 0 is 0
c. The opposite of 413 is -413
d. The opposite of -564 is 564

Note:

i. The opposite of a is $-a$

ii. For integer $a, -(-a) = a$

iii. The opposite of positive number is negative number

iv. The opposite of negative number is positive number

v. The opposite of zero is zero.

Exercise 2.2.2

1. Find the opposite of each integer given below.

   a. 70   b. -23   c. -170   d. 0

2. Find the opposite of the following

   a. Gain of Birr 120

   b. 15^0c above zero

   c. A walk of 5 km backward

3. Ahmed saw on weather report that Hawassa's temperature was 9 ^{0}C above zero.

   a. write Hawassa's temperature as an integer.

   b. If the temperature in Bale is opposite of Hawassa's temperature. What is the temperature in Bale?

2.3.Comparing and Ordering Integers

By the end of this section you should be able to:

- Compare and order integers using a number line
- Determine the predecessor and successor of a given integer.

Activity 2.3.1

Discuss with your friends/partners

1. On October, the lowest temperature of three different cities was recorded as follows

City	Temperature in ^0c
Debire Birhan	4^0c below zero
Addis Ababa	4^0c above zero
Adama	10^0c above zero

   a. Which city has the lowest (minimum) temperature?
   b. Which city has the highest temperature?
   c. Indicate the above temperatures on number line and arrange the temperatures from the smallest to largest

2. Indicate each of the following pairs of numbers on number line and circle the number which is found to the right of the other.

   a. 0 and 6 b. 0 and -6 c. -6 and -4 d. 6 and 4

3. Circle the bigger number from the following pairs of numbers.

   a. 0 and 6 b. 0 and -6 c. -6 and -4 d. 6 and 4

To compare integers, draw number line and indicate the numbers on number line then,

- The number which is to the right of the other is bigger number.
- The number which is to the left of the other is smaller number.

We use the symbols " $>$ "," $<$ "," $=$ " to compare numbers.

- $a > b$ means a is greater than b
- $a < b$ means a is less than b
- $a = b$, means a is equal to b

Consider the following number line

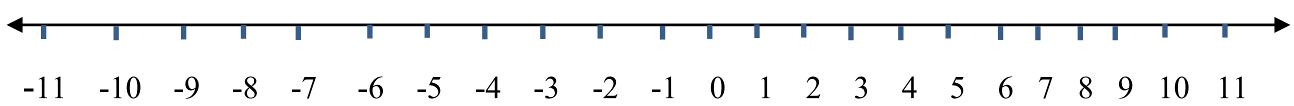
- a is to the left of b, hence a is smaller than b $(i.e. a < b)$
- b is to the right of a, hence b is bigger than a $(i.e. b > a)$

Example 1. For each pair of numbers below select the number which is to the right of the other

a. $3\ and\ 6$ c. $-3\ and\ -6$

b. $10\ and\ 5$ d. $-10\ and\ -5$

Solution: First draw a number line and mark the numbers



a. 6 is to the right 3

b. 10 is to the right of 5

c. -3 is to the right of -6

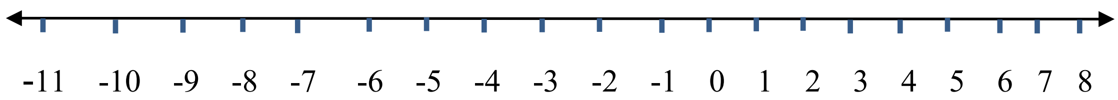
d. -5 is to the right of -10

Example 2. Insert " $>$ " or " $<$ " to express the corresponding relationship between the following pairs of integers

a. -10 ___ 0 c. 0 ___ -6

b. 0 ___ -3 d. -8 ___0

Solution: use a number line to compare numbers



a. -10 is to the left of 0, hence -10 is smaller than 0, symbolically, $-10 < 0$

b. 0 is to the right of -3, hence 0 is greater than -3, symbolically $0 > -3$

c. 0 is to the right of -6, hence 0 is greater than -6, symbolically $0 > -6$

d. -8 is to the left of 0, hence -8 is smaller than 0, symbolically $-8 < 0$

Note:

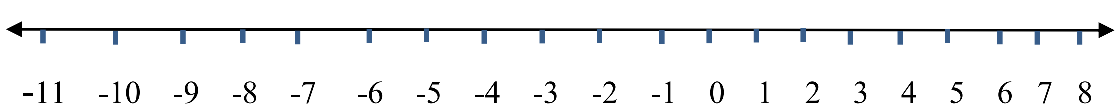
- All negative integers are to the left of 0, hence every negative integer is less than 0
- 0 is to the right of any negative integer, hence 0 is greater than any negative integer.

Example 3: Insert " $>$ " or " $<$ " to express the corresponding relationship between the following pairs of integers

a. -4 ___ 2 c. 8 ___ -6

b. 5 ___ -8 d. -10 ___ 4

Solution: use a number line to compare numbers



a. -4 is to the left of 2, hence -4 is smaller than 2 symbolically$(-4 < 2)$

b. 5 is to the right of -8, hence 5 is greater -8 , symbolically$(5 > -8)$

c. 8 is to the right of -6, hence 8 is greater -6, symbolically $(8 > -6)$

d. -10 is to the left of 4, hence -10 is smaller than 4, symbolically $(-10 < 4)$

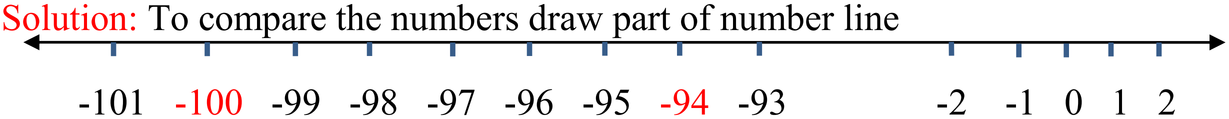
Note:

- All negative integers are to the left of positive integers; hence every negative integer is less than any positive integer.
- All positive integers are to the right of any negative integer; hence every positive integer is greater than any negative integer.

Example 4. Compare the following integers using the sign $" > "" = " \, or \, " < "$

a. -94 ___ -100

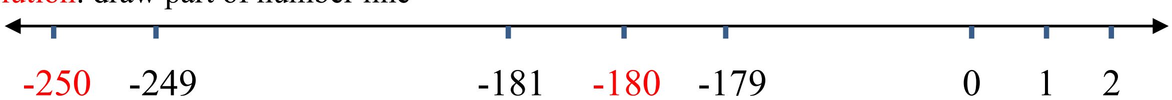
Solution: To compare the numbers draw part of number line



-94 is to the right of -100, hence -94 is bigger $(-94 > -100)$

b. -250 ___ -180

Solution: draw part of number line



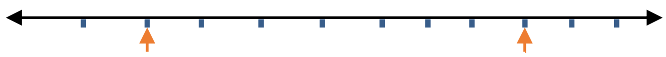
-250 Is to the left of -180, hence -250 is smaller $(-250 < -180)$

c. -40 ___ -40

This two numbers are the same numbers; hence they are equal $(-40 = -40)$

Example 5. Name all integers which lie between -5 *and* 2

Solution: Draw a number line



-6 -5 -4 -3 -2 -1 0 1 2 3 4

The integers that lie between -5 *and* 2 are $-4, -3, -2, -1, 0$ *and* 1

Example 6. List all integers which lie between -4 *and* -3

Solution: Draw a number line

-6 -5 -4 -3 -2 -1 0 1 2 3 4

There is no integer between -4 *and* -3

Exercise 2.3.1

1. Insert " > "" = " *or* " < " to express the corresponding relationship between the following pairs of integers

a. -150 ___ 0	e. 65 ___ -20	i. -120___-80
b. 0 ___ -300	f. -340 ___ -341	j. -360___185
c. -1200 ___ -74	g. -23___ -23	k. 0____13
d. -34 ___34	h. -8___2	l. 14___0

2. From each pair of numbers below circle the number which is to the right of the other.

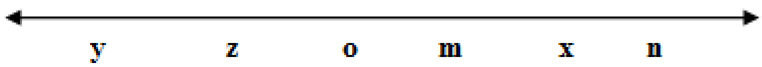
a. 3 *and* 6	e. -87 *and* -99
b. -3 *and* -6	f. -360 *and* -1
c. 10 *and* 5	g. 35 *and* -45
d. -1005 *and* -105	h. 0 *and* -12

3. List all integers which lie between the following pairs of numbers

a. -2 *and* 3	c. -98 *and* -95

b. -1 *and* 1 d. -10 *and* -9

4. The five integers $x, y, z, n,$ *and* m are represented on the number line below



Using " $<$ "*or*" $>$ " fill in the blank space

a. z _____ x c. z _____ n

b. m _____ x d. 0 _____ z

5. Answer the following questions
   a. What is the greatest negative integer?
   b. What is the smallest positive integer?
   c. What is the greatest integer?
   d. What is the smallest integer?

Successor and predecessor of integers

Successor of an integer is an integer that comes after the given integer

Example 7: Find the successor of the given integers

a. 5 b. 0 c. -4 d. -99 e. 99

Solution:

a. The successor of 5 is 6.
b. The successor of 0 is 1.
c. The successor of -4 is -3.
d. The successor of -99 is -98.
e. The successor of 99 is 100.

Predecessor of an integer is an integer that comes before the given integer.

Example 8: Find the predecessor of the given integers

a. 5 b. 0 c. -4 d. -99 e. 99

Solution:

a. The predecessor of 5 is 4.

b. The predecessor of 0 is -1.

c. The predecessor of -4 is -5.

d. The predecessor of -99 is -100.

e. The predecessor of 99 is 98.

Ascending and Descending order of integers

Ascending order means arranging the given numbers in increasing order. (i.e. arranging from the smallest to the largest number)

Example 9: Arrange the following set of numbers in ascending order

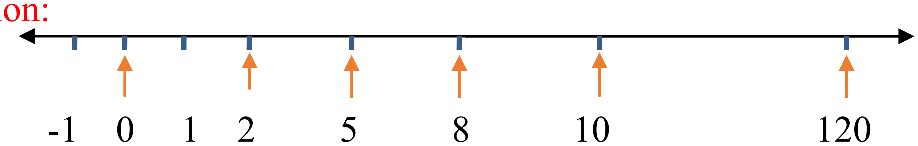
a. $10,\ 2,5,\ 0,\ 8,\ 120$

b. $-40,\ -8,0,\ -12,\ -10$

c. $-256,\ 31,\ 0,\ -25,13,\ -52,\ 103$

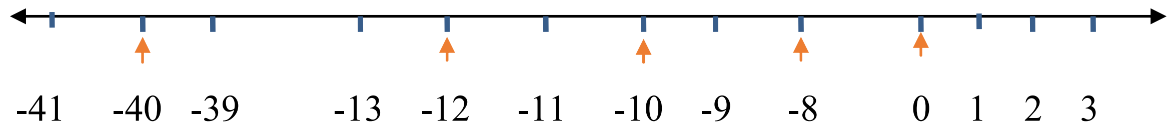
Solution:

a.



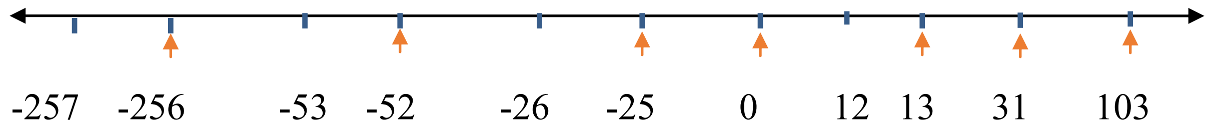
$0,\ 2,\ 5,\ 8,\ 10,120$

b.



$-40, -12, -10, -8, 0$

c.



$-256, -52, -25, 0, 13, 31, 103$

Descending order means arranging the given numbers in decreasing order. (i.e. arranging from the largest to the smallest)

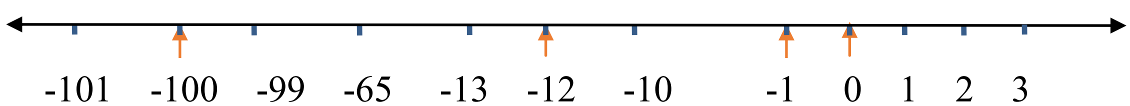
Example 10: Arrange the following set of numbers in descending order

a. $0, -65, -100, -12, -1$

b. $55, -75, 0, 15, -35$

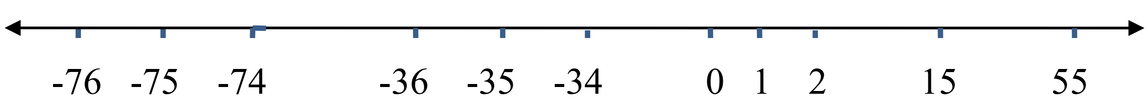
Solution:

a.



$0, -1, -12, -65, -100$

b.



$55, 15, 0, -35, -75$

Exercise 2.3.2

1. Find the successor of the following integers

   a. 20 b. -16 c. 999 d. -999

2. Find the predecessor of the following integers

   a. 1000 b. -77 c. -999 d. -1000

3. Arrange the following numbers in ascending order

   a. $0, -78, 17, -24, 71$

   b. $-230, -157, -400, -350, 50$

4. Arrange the following numbers in descending order

   a. $-9, -99, 69, 59, -89$

   b. $200, -300, -757, -445, 400$

5. Abebe, Almaz, Hailu and Derartu played basketballs. The results are shown in the table below

	First play	**Second play**	**Final result**
Abebe	Loss 5 basket balls	Won 7 basket balls	Won 2 baskets (+2)
Almaz	Won 5 basket balls	Loss 5 basket balls	
Hailu	Loss 3 basket balls	Loss 2 basket balls	
Derartu	Won 1 basket balls	Won 2 basket balls	

a. Complete the final result of the players in the above table

b. Who was the winner?

c. Who was the loser?

d. Arrange the final result of the players in descending order.

Project work:

Register the daily temperature of Addis Ababa, Adama, Hawassa, Debire Birhan, Gambela, Semera, and Bahir Dar from Ethiopian mass media in a given day and answer the following:

a. Graph the temperature of each city on number line
b. Which city was coldest?
c. Which city was warmest?

2.4. Addition and subtraction of Integers

By the end of this section you should be able to:

- Find the sum of integers.
- Find the difference between two integer

Addition of integers

Activity: 2.4.1

1. Hussen, Almaz, Hailu and Derartu are selling different articles. Their profit or loss is shown in the table below

	First sell	Second sell	Total profit/ loss
Hussen	Loss Birr 15	Profit Birr 17	
Almaz	Profit Birr 15	Loss Birr 15	
Hailu	Loss Birr 13	Loss Birr 20	
Derartu	Profit Birr 10	Profit Birr 12	

2. Find the sum of the following using number line
   a. $4 + (-8)$ c. $-3 + (-4)$
   b. $-2 + 5$ d. $-4 + 0$
3. Find the sum of the following integers
   a. $3 + 2$ c. $-3 + 2$ e. $-10 + 10$
   b. $-20 + 0$ d. $4 + (-3)$

Rules: for addition of integers on number line

1. Start the arrow from the first addend.
2. Move the arrow to the right with the same magnitude as the second addend, if the sign of the second addend is positive.
3. Move the arrow to the left with the same magnitude as the second addend, if the sign of the second addend is negative.
4. The sum of the integers is at the point where the arrow ends.

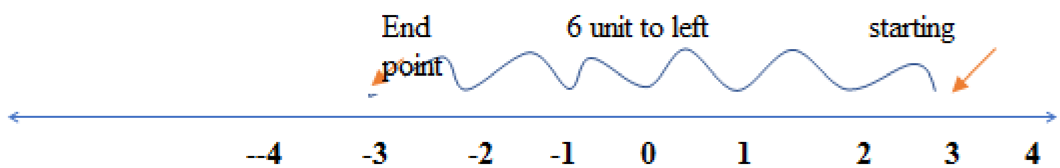
Example 1: find the sum of the following integers using number line

a. $3+(-6)$

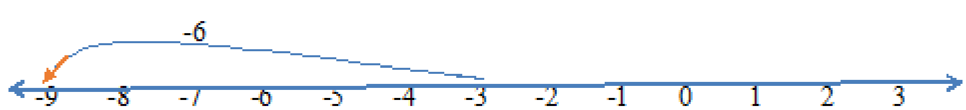
b. $-3+(-6)$

c. $-3+6$

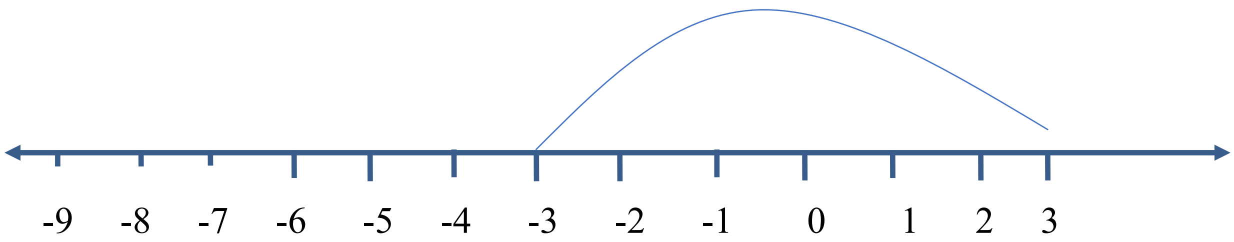
Solution



a. $3+(-6)=-3$



b. $-3+(-6)=-9$



c. $-3 + 6 = 3$

Example 2 add the following numbers

a. -4+4 b. 6+(-6) c. -1+1 d. 2+(-2)

Solution:

a. -4+4=0

b. 6+(-6)=0

c. -1+1=0

d. 2+(-2)=0

Note: the sum of any two opposite integers is zero

Addition of positive and negative integers without using number line

To find the sum of positive and negative integers, follow the following steps

Step 1: Take the sign of a number with greater magnitude.

Step 2: Find the difference of magnitude of the numbers.

Step 3: put the sign in front of the difference

Example 3: Find the sum of the following integers

a. $22 + (-43)$

Solution: $43 > 22$, the sign of 43 is $(-)$,hence the sum is $(-)$

$22 + (-43) = -$ (43-22)

$= -21$

b. $625 + (-214)$

Solution: $625 > 214$, 625 is $(+)$,hence the sum is $(+)$

$625 + (-214) = +(625 - 214)$

$= 411$

c. $-1300 + 560$

Solution: $1300 > 560$, 1300 is $(-)$,hence the sum is $(-)$

$-1300 + 560 = -(1300 - 560)$

$= -740$

Addition of two negative numbers

To find the sum of two negative integers: -

Step 1: The sign of the sum is always negative.

Step 2: Find the sum of magnitude of the numbers.

Step 3: **Put** the negative sign in front of the sum

Example 4: **Find** the sum of the following

a. $-15 + (-35) = -(15 + 35) = -50$

b. $-36 + (-40) = -(36 + 40) = -76$

c. $-120+(-76)=-(120+76)=-196$

Example 5: A person saved Birr 200 and then he spent Birr 150. How much money does he have now?

Solution:

Saved 200 can be represented by + 200

Spent 150 can be represented by -150

We get: $200\ +\ (-150) =\ +(200\ -150) =\ 50$

Therefore, he has left Birr 50.

Example 6: A submarine at 21m below sea level suddenly moves up about 6m. At what depth is the submarine located now?

Solution:

21m below sea level can be represented by - 21

Moves up about 6m can be represented by +6, then

We get -21 + +6=-(21 – 6)

= -15

The submarine is 15m below sea level.

Property of addition of Integers

I. The sum of two integers is an integer

Example:

a. $2+5=\ 7\ ,7\ is\ integer$

b. 4 + (-6) = *-2, -2 is an integer*

c. $-4+(-5)=-9\,,-9\ is\ integer$

d. $-3+5=2\ \ ,2\ is\ integer$

II. Addition has commutative property: for any two integers m and n

$m+n\ =n+m$

Example 1: $5+(-3)$ $\qquad\qquad$ $-3+5$

$=+(5-3)$ $\qquad\qquad$ $=+(5-3)$

$=2$ $\qquad\qquad$ $=2$

Therefore, $5+(-3)=\ -3+5$

III. Associative property: for any three integers x, y and z

$x+(y+z)=(x+y)+z$

Example: $3+(4+(-5))$ $\qquad\qquad$ $(3+4)+(-5)$

$=3+(-1)$ $\qquad\qquad$ $=7+(-5)$

$=2$ $\qquad\qquad$ $=2$

Therefore, $3+(4+(-5))\ =\ (3+4)+(-5)$

IV. Property of zero:

For any integer number $x, x+0=\ x=0+x$

V. Property of opposites:

for any integer x , $x+(-x)=0$

Exercise 2.4.1

1. Find the sum of the following integers?

a. $250+(-250)$ $\qquad\qquad$ c. $160+(-160)$

b. $-170+170$ d. $-15+15$

2. Find the sum of the following using number line.

a. $-3+7$ d. $-10+10$

b. $-4+(-5)$ e. $-5+(-2)$

c. $6+(-3)$ f. $-7+4$

3. Find the sum of the following integers

a. $23+(-10)$ d. 230+(-63) g. $12+(-23)$

b. $-30+40$ e. -325+234 h. 2250+(-457)

c. -560+(-50) f. -50+(-45) +62 i. -20+(-10) +(210)

4. You are Birr 5 in debt. You borrow Birr 12 more. What is the total amount of your debt?
5. An airplane takes off and then climbs 2500 meter. After 20 minutes, the airplane descends 150 meters. What is the airplane's current height?

Subtraction of Integers

Activity: 2.4.2

1. Write the following differences in the form of sum.
   a. $3-6=3+(-6)$
   b. $-3-5=$
   c. $4-2=$
   d. $-5-(-6)=$
2. Find the following differences by expressing in the form of sum
   a. $5-6=5+(-6)=-1$

b. $-2-2=$

c. $4-2=$

d. $-7-(-6)=$

Subtraction of integer

Subtracting an integer b from a is the same as adding opposite of b to a

i.e. for any integers a and b, $a-b=a+(-b)$

Example 1: Find the difference of the following integer by expressing in the form of sum.

a. $5-2$

b. $4-7$

c. $2-6$

d. $6-2$

Solution:

a. $5-2$, subtracting 2 means adding the opposite of 2

$$5-2\ =5+(-2)$$
$$=3$$

b. $4-7,$ subtracting 7 means adding the opposite of 7

$$4-7=4+(-7)$$
$$= -(7-4)$$
$$=-3$$

c. $2-6$, subtracting 6 means adding the opposite of 6

$$2-6=2+(-6)$$
$$=-(6-2)$$

$= -4$

d. $6 - 2$, subtracting 2 means adding the opposite of 2

$6 - 2 = 6 + (-2)$

$= +(6 - 2)$

$= 4$

Example 2:

Find the difference of the following integer by expressing in the form of sum.

a. $25 - (-30)$

b. $-12 - (-18)$

c. $-55 - (-20)$

d. $8 - (-9)$

Solution:

a. $25 - (-30)$, subtracting -30 means adding the opposite of -30

$25 - (-30) = 25 + 30$

$= 55$

b. $-12 - (-18)$, subtracting -18 means adding the opposite of -18

$-12 - (-18) = -12 + 18$

$= +(18 - 12)$

$= 6$

c. $-55 - (-20)$, subtracting -20 means adding the opposite of -20

$-55 - (-20) = -55 + 20$

$= -(55 - 20)$

$= -35$

d. $8 - (-9)$, subtracting -9 means adding the opposite of -9

$8-(-9)=8+9 \quad = \ 17$

Note: for any two-integer a and b

$$a-(-b)=a+b$$

Example 3:

Find the difference of the following integer by expressing in the form of sum.

a. $-21-12$

b. $-38-44$

c. $-24-35$

d. $-248-125$

Solution:

a. $-21-12$, subtracting 12 means adding the opposite of 12

$$-21-12 \quad = -21+(-12)$$
$$= -(21+12)$$
$$= -33$$

b. $-38-44$, subtracting 44 means adding the opposite of 44

$$-38-44 \quad = -38+(-44)$$
$$= -(38+44)$$
$$= -82$$

c. $-24-35$, subtracting 35 means adding the opposite of 35

$$-24-35 \quad = -24+(-35)$$
$$= -(24+35)$$
$$= -59$$

d. $-248-125$, subtracting 125 means adding the opposite of 125

$$-248-125 \quad = -248+(-125)$$

$$= -(248 + 125)$$

$$= -373$$

Note: For any two-integer a and b

$$-a - b = -(a + b)$$

Exercise2.4.2

1 express the following differences in the form of sum

a. $3 - (-10)$ c. $9 - 12$ e. $-178 - 57$

b. $-5 - (-6)$ d. $25 - 17$ f. $-365 - 1000$

2. Find the following differences.

a. $0 - 3$ c. $-75 - 15$ e. $465 - (-778)$

b. $7 - (-8)$ d. $-1789 - (-1000)$ f. $25 - 75$

3. The melting point dry ice is -109^0F .The boiling point of dry ice is 109^0F then, how many degrees is the boiling point above the melting point.

2.5.Multiplication and Division of Integer numbers

By the end of this section you should be able to:

- Solve problems on multiplication of integers.
- Identify the commutative and associative property of multiplication.
- Identify the distributive property of multiplication over addition.

- Divide integers whose quotient as expressed in decimals (2 decimal places)

Multiplication of Integer numbers

Activity 2.5.1

1. Express each of the following product as sum and find the product
   a. 5×2 c. (-5) ×4
   b. 9×3 d. (-6) × 2
2. Express each sum as product
   a. 0+0+0+0 c. (-2) + (-2) + (-2)
   b. 3+3+3+3+3 d. (-10) +(-10) +(-10) +(-10) +(-10) +(-10)

3. Multiply the following
   a. 2×4 c. 3×51 e. 4 × (-25)
   b. 0×121 d. -10 × 2

The product of two positive integers is positive integer

Example1 find the product of the following

a. 3×5 b. 11×6

Solution using similar ways as product of natural number and whole number

a. 3×5 = 5+5+5 =15
b. 11×6 =66

Example 2 Find the product of the following numbers expressing in the form of Sum

a. $4\times(-5)$ b. -13×3 c. -1×6 d. -365×2

Solution:

a. $4 \times (-5) = -5 + (-5) + (-5) + (-5)$

$= -20$

b. $-13 \times 3 = -13 + (-13) + (-13)$

$= -39$

c. $-1 \times 6 = -1 + (-1) + (-1) + (-1) + (-1) + (-1)$

$= -6$

d. $-365 \times 2 = -365 + (-365)$

$= -730$

Note: for above examples, you can see that the product of positive and negative integer is negative integer.

Example 3 Multiply the following

a. $4 \times (-3)$

b. $12 \times (-11)$

c. -10×267

Solution a. $4 \times (-3)$ → the result is negative sign (-)

$\Rightarrow 4 \times 3 = 12$

$so,\ 4 \times (-3) = -12$

b. $12 \times (-11)$ → The result is negative sign (-)

$\Rightarrow 12 \times 11 = 132$

$so, \ 12 \times (-11) = -132$

c. $-10 \times 267 \rightarrow$ The result is negative sign (-)

-(10 x267)= - 2670

Example 4: find the product of the following

a. $-2 \times 2 = -4$

b. $-2 \times 1 = -2$

c. $-2 \times 0 = 0$

d. $-2 \times (-1) = ?$

In the above example, the product pattern is $-4, -2, 0, \ldots$, hence $-2 \times (-1) = 2$

Note: The product of two negative integers is positive integer

Example 5: Multiply

a. $-3 \times (-5)$

b. $-21 \times (-1)$

c. $-6 \times (-13)$

d. $-11 \times (-8)$

Solution: - a. $-3 \times (-5) \rightarrow$ The result is positive sign $(+)$

$\Rightarrow 3 \times 5 = 15$

$so, -3 \times (-5) = 15$

b. $-21 \times (-1) \rightarrow$ The result is positive sign $(+)$

$\Rightarrow 21 \times 1 = 21$

$so, -21 \times (-1) = 21$

c. $-6 \times (-13) \rightarrow$ The result is positive sign $(+)$

$\Rightarrow 6 \times 13 = 78$

$so, -6 \times (-13) = 78$

d. $-11 \times (-8) \rightarrow$ The result is positive sign $(+)$

$\Rightarrow 11 \times 8 = 88$

$so, -11 \times (-8) = 88$

Summary of product of integers

Two factors	Sign of Product	Example
Both are positives	Positive	$3 \times 5 = 15$
Opposite sign$(+ \times -)$	Negative	$-3 \times 5 = -15$ or $5 \times (-3)$
Both are negatives	Positive	$-3 \times (-5) = 15$

Exercise 1.5.1

1. Multiply

a. 6×12 c. -7×13 e. $-17 \times (-103)$

b. 13×15 d. $16 \times (-25)$ f. $-170 \times (-345)$

Properties of multiplication of integer numbers

Activity 2.5.2

Which of the following statements are true and which are false

a. $2 \times 4 = 4 \times 2$

b. $21 \times (-3) = -3 \times 21$

c. $5 \times (3 \times 2) = (5 \times 3) \times 2$

d. $-5 \times (3+7) = (-5 \times 3) + (-5+2)$

1. Commutative properties of multiplication

For any two integers a and b,

$a \times b = b \times a$ ... Commutative property of multiplication

Example 1 $-2 \times 5 = 5 \times (-2)$

-10 $= -10$

$-12 \times (-20) = -20 \times (-12)$

240 $= 240$

2. Assciative properties of multiplication

   For any three integers numbers a, b, c, then

   $a \times (b \times c) = (a \times b) \times c$ ... Associative property

Example 2 $-2 \times (5 \times 6) = (-2 \times 5) \times 6$

$-2 \times 30 = -10 \times 6$ (First multiply the bracket)

$-60 = -60$

3. Distributive properties of multiplication over addition

Let a, b, & c be any integers, then

$a \times (b+c) = (a \times b) + (a \times c)$ ... Distributive property

Example 3 $-6 \times (5+3) = (-6 \times 5) + (-6 \times 3)$

$-6 \times (8) = (-30) + (-18)$ (Apply operations in the bracket)

$-48 = -48$

4. Properties of zero and 1 on multiplication

Note: The product of any integer and zero is zero.

Let 'a' be any integer, then $a \times 0 = 0$ or $0 \times a = 0$

Example 4 Find the product

a. 2×0 $\quad$ $c. -122 \times 0$

b. 0×49 $\quad$ $d. 0 \times (-2999)$

Solution a. $2 \times 0 = 0$

b. $0 \times 49 = 0$

c. $-122 \times 0 = 0$

d. $0 \times (-2999) = 0$

Note: The product of any integer and 1 is the number itself.

Let 'a' be any integer then $a \times 1 = a$ or $1 \times a = a$

Example 5 $a.$ $9 \times 1 = 9$

$b. -25 \times 1 = -25$

$c. 1 \times (-291 = -291$

Exercise 2.5.2

1. State the properties for the following

a. $2 \times (-3 + 54) = (2 \times (-3)) + (2 \times 54)$

b. -525 $\times 955 = 955 \times (-525)$

c. $2 \times (-13 \times 5) = (2 \times (-13)) \times 5$

d. $5 \times 1 = 5$

e. $0 \times (-8) = 0$

2. Simplify each of the following pairs and compare their result

   a. $3\times(-2\times3)$ and $(3\times(-2))\times3)$

   b. $259 \times 0 \; and \; 0 \times 259$

   c. $-6 \times (5 + 2) \; and \; (-6 \times 5) + (-6 \times 2)$

   d. $-5 \times \left(-3 + (-8)\right) \; and \; \left(-5 \times (-3)\right) + (-5 \times (-8)$

3. Copy and complete the following table below

a	B	C	$a \times b$	$b \times a$	$a \times (b \times c)$	$(a \times b) \times c$
-7	4	-3				
9	-2	5				
-6	-8	-12				

Multiplication of three or more integers

Activity 2.5.3.

1. Multiply the following numbers and notice the sign of the product.

   a. $-4 \times 5 \times 3 \times 1$

   b. $-4 \times 5 \times (-3) \times 1$

   c. $-4 \times (-5) \times (-3) \times 1$

   d. $-4 \times (-5) \times (-3) \times (-1)$

2. Multiply the following numbers

   a. $-4 \times 5 \times 0 \times 1$

   b. $-4 \times 5 \times (-3) \times 1 \times 125 \times 0$

What do you conclude from a and b?

The following properties are helpful in simplifying products with three or more factors:-

- The product of an even number of negative factors is positive.
- The product of an odd number of negative factors is negative.
- Product of integers with at least one factor 0 is zero.
- If you multiply an integer a by -1, then you get the opposite of a (i.e. -1xa=-a)
- When you multiply a number by a variable, you can omit the multiplication sign and keep the number in front of the variable (i.e. $3 \times y = 3y$)

Example 1: Determine the sign of product

a. $-1 \times 5 \times (-6)$

b. $-265 \times (-81) \times 35 \times (-68)$

c. $-5 \times (-2) \times 3 \times (-8) \times (-4) \times 7$

Solution:

a. $The\ number\ of\ negative\ factors\ is\ 2\ (even), then\ the\ product\ is\ positive$

b. $The\ number\ of\ negative\ factor\ is\ 3\ (odd), then\ the\ product\ is\ Negative$

c. $The\ number\ of\ negative\ factor\ is\ 4\ (even)$ then the product is positive

Example 2 Find the product of the following

a. $-3 \times 5 \times (-4)$

b. $-5 \times (-5) \times 3 \times (-8)$

c. $-5 \times (-2) \times 3 \times (-8) \times (-4) \times 0$

Solution:

a. $The\ number\ of\ negative\ factor\ is\ 2\ (even), then\ the\ product\ is\ positive$

$(-3 \times 5) \times (-4) = -15 \times (-4) = 60$

b. $The\ number\ of\ negative\ factor\ is\ 3\ (odd), then\ the\ product\ is\ Negative$

$\Rightarrow -5 \times (-5) \times 3 \times (-8) = (-5 \times (-5)) \times (3 \times (-8))$

$= 25 \times (-24)$

$= 600$

c. $There\ is\ one\ factor\ 0\ in\ the\ product, hence\ the\ product\ is\ 0$

$-5 \times (-2) \times 3 \times (-8) \times (-4) \times 0 = 0$

Exercise 2.5.3

1. Find the sign of the product
   a. $12 \times (-50) \times (-61)$
   b. $(-125) \times (-3) \times (-52)$
   c. $(-1) \times (-2) \times (-3) \times (-4) \times (-5)$
   d. $(3) \times (-6) \times (-9) \times (-12)$
2. Find the product
   a. $9 \times (-3) \times (-6)$
   b. $(-12) \times (-13) \times (-5)$
   c. $(-1) \times (-2) \times (-3) \times (-4) \times (-5)$

d. $(3) \times (-6) \times (-9) \times (-12)$

e. $15625 \times (-2) \times 0 \times (-15625) \times 98$

Division of integers

Activity 2.5.4

1. Check the quotient of the following using product, the first is done for you
   a. 20÷4 =5, hence $20 = 5 \times 4$
   b. -36÷9 = -4
   c. $84 \div (-7) = -12$
   d. $-8 \div (-2) = 4$
   e. $-7 \div 2$
2. Referring the above activity
   a. The quotient of negative and positive numbers is $(Positive/negative)$
   b. The quotient of positive and negative numbers is$(Positive/negative)$
   c. The quotient of negative and negative numbers is$(Positive/negative)$

Division is an inverse operation of multiplication

For example, divide 12 by 3 means find a number that gives product 12 when it is multiplied by 3. The number is 4 and $12 \div 3 = 4$, because $4 \times 3 = 12$

In the division $12 \div 3 = 4$

12 is dividend

3 is divisor and 4 is quotient

Note: For any number $a, b, and\ c\ where\ b \neq 0$, $a \div b = c,$ if and only if

$a = b \times c$

- $a \div b$ read as a divided by b
- $a \div b$ is also denoted by $\frac{a}{b}$ or a/b

Example 1: Find the quotient by converting in to products

a. $8 \div 2$ b. $12 \div (-4)$ c. $-10 \div (-5)$ d. $-8 \div 4$

Solution:

a. $8 \div 2 =$ ___, means $8 = 2 \times ?$, the number is 4, because $8 = 2 \times 4$

Hence, $8 \div 2 = 4$

b. $12 \div (-4) =$ ___, means $12 = -4 \times ?$, the number is -3, because

$12 = -4 \times -3$

Hence, $12 \div (-4) = -3$

c. $-10 \div (-5) =$ ___, means $-10 = -5 \times ?$, the number is 2, because

$-10 = -5 \times 2$,

Hence, $-10 \div (-5) = 2$

d. $-8 \div 4 =$ ___, means $-8 = 4 \times ?$, the number is -2, because

$-8 = 4 \times (-2)$

Hence, $-8 \div 4 = -2$

From the above example,

- The quotient of two positive numbers is positive
- The quotient of two negative numbers is positive
- The quotient positive and negative integers is negative

To divide integers without converting in to multiplication follow the following procedure:

1. If the sign of dividend and divisor are the same:
   - The sign of the quotient is $(+)$
   - Find the magnitude of the quotient
   - Put the sign in front of the quotient
2. If the sign of dividend and divisor is different:
   - The sign of the quotient is $(-)$
   - Find the magnitude of the quotient
   - Put the sign in front of the quotient

Example 2 calculate the quotient of the following

a. $-368 \div 8$ c. $175 \div (-7)$ e. $-1 \div 4$

b. $\frac{-264}{12}$ d. $\frac{8211}{-21}$ f. $24 \div (-5)$

Solution a. $-368 \div 8$ $\rightarrow$ The result is negative sign $(-)$

$368 \div 8 = -48$

$\Rightarrow -368 \div 8 = -46$

b. $\frac{-264}{12}$ $\rightarrow$The result is negative sign $(-)$

$264 \div 12 = 22$

$\Rightarrow \frac{-264}{12} = -22$

c. $175 \div (-7)$ $\rightarrow$The result is negative sign $(-)$

$175 \div 7 = 25$

$\Rightarrow 175 \div (-7) = -25$

d. $\frac{8211}{-21}$ →The result is negative sign $(-)$

$8211 \div 21 = 391$

$\Rightarrow \frac{8211}{-21} = -391$

Example 3 Calculate the quotient of the following

a. $-1 \div 4$

b. $-24 \div (-5)$

c. $-5 \div 4$

d. $-150 \div (-20)$

Solution a. $-1 \div 4$ →The result is negative sign $(-)$

$1 \div 4 = 0.25$

$\Rightarrow -1 \div 4 = -0.25$

b. $-24 \div (-5) \rightarrow$ The result is negative sign $(-)$

24÷5 =4.8

$-24 \div (-5) = 4.8$

c. $-5 \div 4$ →The result is negative sign $(-)$

$5 \div 4 = 1.25$

$\Rightarrow -5 \div 4 = -1.25$

d. $-150 \div (-20)$ →The result is positive sign $(+)$

$150 \div 20 = 7.5$

$\Rightarrow -150 \div (-20) = 7.5$

Example 4 Find the quotient of the following

a. $-387 \div (-9)$ b. $\frac{-1984}{-16}$

Solution a. $-387 \div (-9)$ →The result is positive sign $(+)$

$$387 \div 9 = 43$$

$\Rightarrow -387 \div (-9)$=43

b. $\frac{-1984}{-16}$ →The result is positive sign $(+)$

$1984 \div 16 = 124 \quad \Rightarrow \frac{-1984}{-16} = 124$

Division of integers is summarized in the following table

Dividend and divisor	**Sign of quotient**	**Example**
Both positive	Positive	$15 \div 3 = 5$ or $\frac{15}{3} = 3$
Opposite sign$(+\&-)$	Negative	$15 \div (-3) = -5$ or $\frac{15}{(-3)} = -5$
Both negative	Positive	$-15 \div (-3) = 5$ or $\frac{-15}{(-3)} = 5$

Properties of division of integers

Properties	Examples
1.The quotient of any two integers is not necessarily an integer	$20 \div 8 = 2.5 = \frac{5}{2}$
2. A non-zero integer divided by itself is 1	$20 \div 20 = 1$
3. An integer divided by zero is not defined. i.e Any integer cannot be divided by zero	$\frac{a}{0}$ *is not defined* (*undefined*)
4. Zero divided by any non-zero integer is zero.	$\frac{0}{20} = 0$
5. An integer divided by 1 is itself the number.	$\frac{20}{1} = 20$

Exercise2.5.4

1. Identify dividend, devisor and quotient of the following

   a. $-96 \div 12 = -8$ b. $\frac{-56420}{-124} = 455$

2. Find the quotient by converting in to products

   a. $18 \div 3$ b. $24 \div (-4)$ c. $-40 \div (-5)$ d. $-44 \div 4$

3. Divide

   a. $-130 \div 5$ e. $768 \div 0$

   b. $0 \div (-4965)$ f. $\frac{-17622}{-99}$

   c. $(-1048) \div (-1048)$ g. $0 \div 0$

   d. $\frac{-2574}{33}$

4. Fill the blank space

   a. $56 \div$ ------ $= 7$ c. -------- $\div (-18) = 46$

b. $-1092 \div$ ------- $=14$ d. -------- $\div (55) = -101$

5. Divide the following

a. $-12 \div 5$ c. $-26 \div (-8)$

b. $-37 \div (-2)$ d. $45 \div (-4)$

2.6.Even and Odd integers

By the end of this section you should be able to:

- Describe even and odd integers
- Identify even and odd integers

Activity 2.6.1

Discuss with your friends/partners

1. Group the following objects by two
   a. Picture of six balls
   b. Picture of five books
   c. Picture of 4 rulers
   d. Picture of 3 pens
2. From activity 1 a and c, explain what you observe
3. From activity 1 b and d, explain what you observe
4. What do we call numbers that can be grouped by two (divisible by two) without leaving remainder?
5. What do we call numbers that can be grouped by two leaving one remainder?
6. Identify the following numbers as even and odd
   6, 5, 4,3
   Even integers = _________
   Odd integers = _________

Even integer is an integer that can be divisible by 2 without leaving remainder.

Example 1: The following are single digit even integers

$$-8, -6, -4, -2, 0, 2, 4, 6, and\ 8$$

Odd integer is an integer that cannot be divisible by 2.

When you divide any odd integer by 2, its remainder is 1.

Example 2: The following are single digit odd integers

$$-9, -7, -5, -3, -1, 1, 3, 5, 7, and\ 9$$

Many digits even and odd integers

For many digits number:

If the unit digit is $0, 2, 4, 6, or\ 8$, then the number is even integer, otherwise it is odd.

Example 3: Identify the following numbers as even and odd integers

$$20, -16, 31, -23, 14, -15, 345, -252$$

Solution: The unit digit of even integer is $0, 2, 4, 6, or\ 8$, hence $20, -16, 14\ and$ -252 are even integers. The rest $31, -23, -15, and\ 345$ are odd integers

Example 4: list three consecutive even integers next to 31

Solution: 32, 34, and 36

Example 5: list the next three consecutive even integers greater than -6

Solution: $-4, -2, and\ 0$

Exercise 1.6.1

1. Identify the following integers as even and odd

   $-36, 55, 0, -71, 103, -180$

2. Identify the following integers as even and odd

   $-3675, 5568, -7109, 1034, -1807$

3. List the next three consecutive even integers greater than 143
4. List the next three consecutive odd integers greater than -54
5. List an even integer which is neither positive nor negative
6. What is the smallest positive even integer
7. What is the greatest odd negative integer
8. What is the greatest three-digit positive integer
9. What is the smallest two-digit negative integer

Addition of even and odd integers

Example 1: add the following integers

a. $-40 + 36$ b. $24 + (-18)$ c. $-4 + 4$ d. $246 + 344$

Solution:

a. $-40 + 36 = -(40 - 36) = -4$

b. $24 + (-18) = +(24 - 18) = 6$

c. $-4 + 4 = 0$

d. $246 + 344 = 590$

From above example, the sum of two even odd is always ______(even/odd)

Example 2: add the following integers

a. $31+25$ b. $-45+23$ c. $11+(-33)$ d. $-21+(-15)$

Solution:

a. $31+25=56$

b. $-45+23=-(45-23)=-22$

c. $11+(-33)=-(33-11)=-22$

d. $-21+(-15)=-(21+15)=-36$

From above example, the sum of two odd integers is always ______(even/odd)

Example 3: add the following integers

b. $8+19$ b. $-4+(-23)$ c. $17+(-32)$ d. $-21+30$

Solution:

a. $8+19=27$

b. $-4+(-23)=-(4+23)=-27$

c. $17+(-32)=-(32-17)=-5$

d. $-21+30=+(30-21)=9$

From above example, the sum of odd and even integer is always ____(even/odd)

Subtraction of even and odd integers

Example 4: subtract the following integers

a. $8-6$ b. $12-20$ c. $-10-10$ d. $-6-(-30)$

Solution:

a. $8-6=2$

b. $12-20=12+(-20)=-(20-12)=-8$

c. $-10-10=-10+(-10)=-(10+10)=-20$

d. $-6-(-30)=-6+30=+(30-6)=24$

From the above example, we can conclude that $even-even=$ _____

Example 5: Subtract the following integers

a. $7-5$ b. $11-23$ c. $-15-13$ d. $-9-(-11)$

Solution:

a. $7-5=2$

b. $11-23=11+(-23)=-(23-11)=-12$

c. $-15-13=-15+(-13)=-(15+13)=-28$

d. $-9-(-11)=-9+11=+(11-9)=2$

From the above example, we can conclude that $odd-odd=$ _____

Example 6: subtract the following integers

a. $12-5$ b. $31-50$ c. $-18-15$ d. $-22-(-9)$

Solution:

a. $12-5=7$

b. $31-50=31+(-50)=-(50-31)=-19$

c. $-18-15=-18+(-15)=-(18+15)=-33$

d. $-22-(-9)=-22+9=-(22-9)=-13$

From the above example, we can conclude that

$odd-even=$ _____

$even-odd=$ _____

Product of even and odd integers

Example 1: Multiply the following integers

a. 12×4 b. $0 \times (-6)$ c. -6×4 d. -2×10 e. $-8 \times (-4)$

Solution:

a. $12 \times 4 = 48$ c. $-6 \times 4 = -24$

b. $0 \times (-6) = 0$ d. $-2 \times 10 = -20$ e. $-8 \times (-4) = 32$

From the above example, we can conclude that

$$even \times even = \_\_\_\_\_$$

Example 2: Multiply the following integers

a. 2×3 b. $0 \times (-5)$ c. -7×4 d. -3×8 e. $-9 \times (-4)$

Solution:

a. $2 \times 3 = 6$ c. $-7 \times 4 = -28$

From the above example, we can conclude that

$$odd \times even = \_\_\_\_\_$$
$$even \times odd = \_\_\_\_\_$$

b. $0 \times (-5) = 0$ d. $-3 \times 8 = -24$ e. $-9 \times (-4) = 36$

Example 3: Multiply the following integers

c. 5×3 b. $1 \times (-5)$ c. -7×11 d. -13×3 e. $-9 \times (-5)$

Solution:

a. $5 \times 3 = 15$ c. $-7 \times 11 = -77$

b. $1 \times (-5) = -5$ d. $-13 \times 3 = -39$ e. $-9 \times (-5) = 45$

From the above example, we can conclude that

$$odd \times odd = \_\_\_\_$$

Exercise 2.6.2

1. Fill in the blank space with correct answer
   a. $even + even = \_\_\_\_$
   b. $even - odd = \_\_\_\_$
   c. $odd \times even = \_\_\_\_$
   d. $even + odd + even = \_\_\_\_$
   e. $odd - even - odd + odd = \_\_\_\_$
   f. $odd \times odd \times even = \_\_\_\_$
   g. $even \times odd + even = \_\_\_\_$
   h. The difference between any two consecutive even integers is equal to __.
   i. The difference between any two consecutive odd integers is equal to ___.

Summary for unit two

- Natural number is a set of numbers denoted by $\mathbb{N}$ and is described by $\mathbb{N} = \{1, 2, 3, \ldots\}$
- Whole numbers are numbers contains natural number and zero denoted by $\mathbb{W}$ and is described by $\mathbb{W} = \{0, 1, 2, 3, \ldots\}$
- An integer is a set of numbers consisting of whole numbers and negative numbers. The set of integers is denoted by

$\mathbb{Z} = \{\ldots 4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$

- Every negative integer is less than any positive integer.
- Every positive integer is greater than any negative integer.
- Successor of an integer is an integer that comes after the given integer
- Predecessor of an integer is an integer that comes before the given integer.
- The sum of any two integers is always an integer.
- The difference of any two integers is always an integer.
- The sum of two opposite integers is zero.
- The product of positive and negative integer is negative integer.
- The product of an even number of negative factors is positive.
- The product of an odd number of negative factors is negative.
- The product of any two integers is always an integer.
- Any negative integer divided by negative integer is always positive
- Even integer is an integer that can be divisible by 2 without leaving remainder.
- Odd integer is an integer that cannot be divisible by 2.

Review Exercise for unit 2

1. Represent the following integers on number line
   a. $-12, 3, 6, 8, -5, -9$
   b. $-11, 6, -4, -8$
   c. $-2, 3, 4, -3, -10$
2. Show the relation between $\mathbb{N}, \mathbb{W}, \mathbb{Z}$?
3. List down 3 example of non-integer numbers?
4. Find the opposite of the following integers?
   a. -80 d.1002 g.-1020

b.34 e. 835 h.-90

C.-34 f.543 i. -768

5. List down all integers that lie between the following pair of integers

a. -70 and -60 d.-12 and -10

b.-34 and -34 e. 0 and 7

C.-3 and -4 f.-12 and 0

6. Compare the following pair of integers using the sign $<, =,$ and $>$

a. -65_____0 d.1002_____-1002 g.-1020_____ 220

b.34 _____-678 e. -835_____-835 h.-90 _____-65

C.-34 _____-304 f. 543_____-56 i. -768_____34

7. If $-5 > x$, then list down four possible values of x, where x is an integer.

8. Arrange the following integers in descending and ascending order

$-1, -5, 1, -51, -69, 101, -101, 21, -81$

9. Find the following sum

a. 0+(-59) d-545+511 g.-1020 +20

b.26+(-52) e.-215+(-349) h.-90+(-489)+315

C.-95+124 f.121+(-290) i. -768+2001

10. Find the following difference

a. $0 - 592$ d. 513-(-109)

b. 75-105 e.-1099-(-412)

C.-89-482 f.299-1251-(-191)

11. Multiply

a. 10×(-24) d. -4 × (-25) ×14(-1)

b. -45×(-12) e. -9×(-55) ×998×0×32

C.- 9×(-24) f. -1×(-1) ×1×(-1)×(-1)×(-1)×(-1)

12. Name the property of the following for any integers m, n, &p.

a. $m \times n = n \times m$

b. $m \times (n + p) = (m \times n) + (m \times p)$

C. $m \times (n \times p) = (m \times n) \times p$

d. $0 \times m = 0$

e. $1 \times m = 1$

13. Find the quotient of the following

a. 1080÷8 d. 0÷492 g. 1÷20

b. -214÷(-214) e. - 492÷0 i. -54 ÷4

c. -19225÷-1 f. -7632÷36 h.-110÷(-20)

14. Write four consecutive negative even integers.

15. Write all odd integers between -106and -96

16. Decide whether the following statements are Even or odd

a. The sum of any two even integers

b. The difference of any two odd integers

c. The product of any two even integers

d. The product of any three odd integers

e. The sum of even and odd integers

f. The difference of odd and even integers

g. The product of odd and even integers

17. Find three integers whose sum is zero.

18. List examples of a pair of integers whose sum is zero.

19. Abel was playing a two-round game in which he could gain or loss points. During the first round he lost 30 points. During the second round gained 13 points. What was his net (total) score at the end of the game?

Unit 3

Ratio, proportion and percentage

Unit Outcomes:

At the end of this unit, students will able to:

- Understand the notions of ratio and proportions.
- Solve problems involving ratio and proportion
- Describe a percentage
- Solve problems involving percentages.
- Relate fractions, decimals and percentages to real life situations
- Apply the concept of percentage in solving real life problems

Introduction

In this unit you will learn the basic mathematical concepts that will be applied in business, like profit, loss, simple and compound interest.

The unit has three sections. The first section deals with the concept of ratio and proportion. The second section deals with percentages and the third section is about application of ratio, proportion and percentage. Here you will see how to calculate loss, profit, simple interest, compound interest, income tax, VAT, and turnover tax.

3.1.Ratio and proportion

By the end of this section you should be able to:

- Explain the notation of ratio
- Visualizes the ratio of 2 given numbers.
- Expresses ratios in their simplest forms.
- Finds the missing term in a pair of equivalent ratios.
- Solve problems on proportion

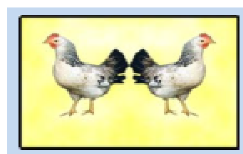
3.1.1 Ratio

Activity 3.1.1

Discuss with your friends

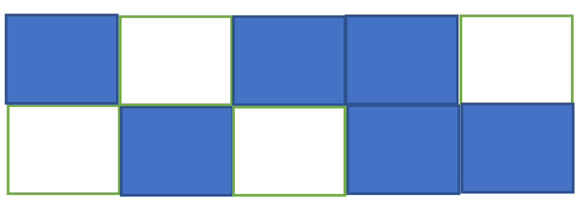
1. write a simple ratio for each of the following
   a. Cats to hens

Figure. 3.1 cats



   b. What are the ratio books to pens to pencils?

2. consider the following figure



   a. What is the ratio of shaded part to unshaded part?
   b. What is the ratio of shaded part to the whole part?
3. Can you define a ratio based on the above activities in your own words?

4. What is the ratio of girls to boys in your class?

Definition 3.1: The method of comparing two or more quantities of the same kind and in the same units by division is known as a ratio.

- The symbol to denote the ratio is:
- The ratio of one quantity a to another quantity b is usually denoted by $a:b$.Where, a is called antecedent and b is called consequent.
- The ratio of a to b can be written as any one of the following forms:

  i. $a:b \rightarrow$ read as a ratio b or a to b

  ii. $\frac{a}{b} \rightarrow$ read as a over b

  iii. $a \div b \rightarrow$, a divided by b, provided that $b \neq 0$

- In most cases the ratio $a:b$ is written in simplified form, where a and b are natural numbers.
- To write the ratio of two quantities, the quantities must have the same units of measurement.
- Quantities having different units of measurement cannot be compared in the form of ratio.

Example 1: In a class there are 18 boys and 24 girls.

a. What is the ratio of boys to girls

b. What is the ratio of girls to boys?

c. What is the ratio of girls to the total number of students in the class?

d. What is the ratio of boys to the total number of students in the class?

Solution: Total number of students= $number\ of\ boys + number\ of\ girls$

$= 18 + 24 = 42$

a. Ratio of boys to girls $= \frac{number\ of\ boys}{number\ of\ girls}$

$= \frac{18}{24} = \frac{18 \div 6}{24 \div 6} = \frac{3}{4}\ or\ 3:4$

b. Ratio of girls to boys $= \frac{number\ of\ girls}{number\ of\ boys}$

$= \frac{24}{18} = \frac{24 \div 6}{24 \div 6} = \frac{4}{3}\ or\ 4:3$

c. Ratio of girls to total number of students $= \frac{number\ of\ girls}{total\ number\ of\ students}$

$= \frac{24}{42} = \frac{24 \div 6}{42 \div 6} = \frac{4}{7}\ or\ 4:7$

d. Ratio of boys to total number of students $= \frac{number\ of\ boys}{total\ number\ of\ students}$

$= \frac{18}{42} = \frac{18 \div 6}{42 \div 6} = \frac{3}{7} or\ 3:7$

Example 2 Write down the ratio of the first number to the second 0ne, in the Simplest form

a. 4 to 8 b. 54 to 128 c. 48 to 6 d. 540 to 48

Solution:

a. Ratio of 4 to 8 $= \frac{4 \div 4}{8 \div 4} = \frac{1}{2} or\ 1:2$

b. Ratio of 54 to 128 $= \frac{54 \div 2}{128 \div 2} = \frac{27}{64} or\ 27:64$

c. Ratio of 48 to 6 $= \frac{48 \div 6}{6 \div 6} = \frac{8}{1}\ or\ 8:1$

d. Ratio of 540 to 48 $= \frac{540\div 2}{48\div 2} = \frac{270\div 2}{24\div 2} = \frac{135\div 3}{12\div 3} = \frac{45}{4}$ or $45:4$

Example 3 Find the ratio of the following and express simplest form of fraction

a. $\frac{3}{4}$ to $\frac{5}{3}$

b. $\frac{2}{13}$ to $\frac{15}{16}$

c. 3.2 to 1.2

d. $\frac{7}{9}$ to 0.7

Solution:

a. Ratio of $\frac{3}{4}$ *to* $\frac{5}{3} = \frac{\frac{3}{4}}{\frac{5}{3}} = \frac{3}{4}\times\frac{3}{5} = \frac{9}{20} = 9:20$

b. Ratio of $\frac{2}{13}$ to $\frac{15}{16} = \frac{\frac{2}{13}}{\frac{15}{16}} = \frac{2}{13}\times\frac{16}{15} = \frac{32}{195} = 32:195$

c. Ratio of 3.2 to 1.2 $= \frac{3.2}{1.2} = \frac{32}{10}\times\frac{10}{12} = \frac{32\div 4}{12\div 4} = \frac{8}{3} = 8:3$

d. Ratio of $\frac{7}{9}$ to 0.7 $= \frac{\frac{7}{9}}{0.7} = \frac{\frac{7}{9}}{\frac{7}{10}} = \frac{7}{9}\times\frac{10}{7} = \frac{10}{9} = 10:9$

Example 4. Find the ratio of the following quantities

a. 65 cents to 5 Birr
b. 12 meters to 200cm
c. 700 grams to 6kg
d. 600 seconds to 8 minutes

Solution: First change the quantities in to the same units

a. 1 Birr= $100cent\ then\ 5\ Birr\ = 500cent$

The ratio of 65 cents to Birr 5= Ratio of 65 cents to 500cent= $\frac{65\ \cancel{cents}}{500\ \cancel{cents}}$

$= \frac{65\div 5}{500\div 5} = \frac{13}{100} = 13:100$

b. $1m = 100cm\ then\ 12m = 1200cm$

The ratio of 12m to 200cm= Ratio of $1200cm$ to $200cm = \frac{12\cancel{00}\ cm}{2\cancel{00}\ cm}$

$$= \frac{12 \div 2}{2 \div 2} = \frac{6}{1} = 6:1$$

c. $1kg = 1000 grams\ then\ 6kg = 6000 grams$

The ratio of 700gram to 6kg = Ratio of 700 gram to 6000gram

$$= \frac{7\cancel{00}\ \cancel{gram}}{60\cancel{00}\ \cancel{gram}}$$

$$= 7:60$$

d. $1minute = 60seconds\ then\ 8minute = 480\ second$

The ratio of 600 second to 8minutes = Ratio of 600 second to 480 second

$$= \frac{60\cancel{0}\ \cancel{second}}{48\cancel{0}\ \cancel{second}}$$

$$= \frac{60 \div 12}{48 \div 12} = \frac{5}{4} = 5:4$$

Exercise 3.1.1

1. Write down the ratio of the first number to the second one in the simplest form

a. 48 and 80

b. 360 and 72

c. 4.8 and 9.6

d. 13.6 and 10.2

e. $\frac{2}{21}$ and $\frac{8}{21}$

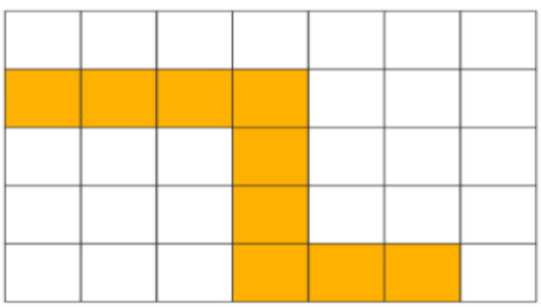
f. $2\frac{11}{12}$ and $1\frac{2}{3}$

> Note:
>
> 1 liter = 1000 milliliters
>
> 1 hectare = 10,000 square meters
>
> 1 day = 24 hours

2. Express the following ratios as in their lowest term:

   a. 4 Birr to 16 cents
   b. 2 liters to 2250 milliliters
   c. 5 days to 100 hours
   d. 3 hours and 30minutes to 180 minutes
   e. 3.5 kg to 6500 grams
   f. 3 hectares to 3000 square meters

3. Using the figure below answer the following questions



   a. What is the ratio of shaded part to unshaded part?
   b. What is the ratio of shaded part to the whole part?
   c. What is the ratio of unshaded part to the whole part?

Dividing the given quantity in the given ratio

Examples:

1. Divide Birr 300 with ratio 3: 2

Solution: let the first part is $3x$ and the second part is $2x$, then

$$3x + 2x = 300$$

$$\frac{\cancel{5}x}{\cancel{5}} = \frac{300}{5}$$

$$x = 60$$

Therefore, the first part is $3x = 3 \times 60 = 180\ Birr$ and the second part is

$2x = 2 \times 60 = 120\ Birr$

2. The ratio of two numbers is 7:3 and their sum is 50, then find the two numbers.

Solution: let the first number is $7x$ and the second number is $3x$, then

$$7x + 3x = 50$$

$$\frac{\cancel{10}x}{\cancel{10}} = \frac{50}{10}$$

$$x = 5$$

Therefore, the first number is $7x = 7 \times 5 = 35$ and the second number is

$3x = 3 \times 5 = 15$

3. Two numbers have ratio 13:10. If their difference is 84, then find the larger number.

Solution: let the larger number is $13x$ and the smaller number is $10x$, then

$$13x - 10x = 84$$

$$\frac{\cancel{3}x}{\cancel{3}} = \frac{84}{3}$$

$$x = 28$$

Therefore, the larger number is $13x = 13 \times 28 = 364$

4. A bag contains red and white balls; the ratio of red to white balls is 3:4. If the bag contains 36 white balls, then find the number of red balls.

Solution: the number of red balls is $3x$ and the number of white balls is $4x$

Number of White balls $= 4x = 36$

$$\frac{4x}{4} = \frac{36}{4}$$

$$x = 9$$

Therefore, the number of red balls is $3x = 3 \times 9 = 27$

5. The ratio of the measures of the angles of a triangle is 1:2:3. Then find the measure of each angle.

   Solution: let the angles of the triangle are $1x, 2x\ and\ 3x$

   $1x + 2x + 3x = 180^0$… Sum of interior angle of a triangle is 180^0

   $$\frac{\cancel{6}x}{\cancel{6}} = \frac{180^0}{6}$$

   $$x = 30^0$$

   Therefore: the first angle is $1x = 1 \times 30^0 = 30^0$

   Second angle is $2 \times 300 = 60^0$

   Third angle is $3x = 3 \times 30^0 = 90^0$

6. If $a, b, and\ c$ are numbers such that $a: b: c = 3: 4: 5$ and $b = 20$, then find $a + b + c$

   Solution: let $a = 3x, b = 4x\ and\ c = 5x$

   $b = 4x = 20$

   $\frac{\cancel{4}x}{\cancel{4}} = \frac{20}{4}, x = 5$

Therefore, $a = 3x = 3 \times 5 = 15, c = 5x = 5 \times 5 = 25$

And $a + b + c = 15 + 20 + 25 = 60$

Exercise 3.1.2

1. Find two numbers whose ratio is 3 to 5 and whose sum is 192.

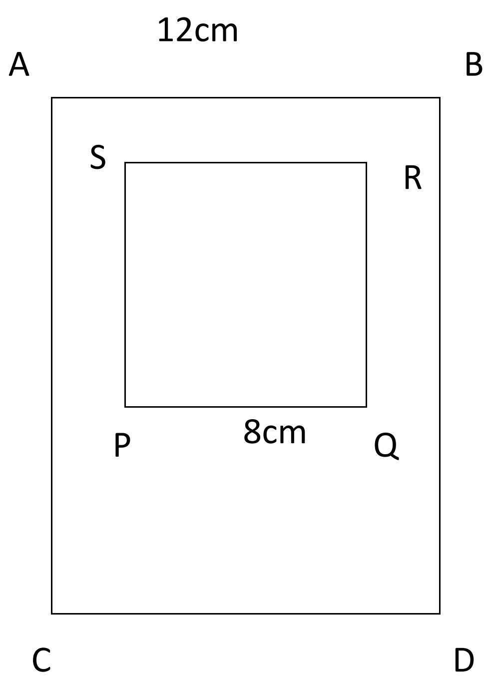
2. A wire of length 240 cm is cut into 3 pieces, in the ratio 1:2:5. Find the length of each piece.

3. Aster, Fatuma, Mohammed and Yared contribute the sum of money to Renaissance dam in the ratio 1:3:5:7. If the largest amount contributed is Birr 1050. Calculate the amount contributed by each person.

4. Two numbers have ratio 12:5. Their difference is 98. Find the larger number.

5. If $a:b = 3:2$, find the value of $\frac{a+b}{a-b}$.

6.. Find the ratio of the areas of the squares ABCD to that of PQRS where AB = 12cm and PQ = 8cm



7. $x, y\ and\ z$ are numbers such that $x:y:z = 1:2:4$ and $y = 48$. Find $x + y + z$

3.1.2 Proportion

Activity 3.1.2

Discuss with your friends

1. Compare the following ratios
   a. $4:6$ and $6:9$
   b. $\frac{3}{6}$ and $\frac{40}{80}$
2. What result did you get from above activities?
3. What do you call equality of two ratios?

Definition 3.2: proportion is equality of two ratios.

Example 1 compare the ratio $6:10$ and $48:80$, do they form proportion?

Solution: the simplest form of $6:10$ is $\frac{6}{10}\frac{\div 2}{\div 2} = \frac{3}{5} = 3:5$

The simplest form of $48:80$ is $\frac{48}{80}\frac{\div 16}{\div 16} = 3:5$

Therefore, $6:10 = 48:80$ is proportion

Example 2: compare the ratio $1:3$ and $5:10$, do they form proportion?

Solution: $1:3$ is in simplest form

the simplest form of $5:10$ is $\frac{5}{10}\frac{\div 5}{\div 5} = \frac{1}{2} = 1:2$

therefore, the two ratios are not equal $(i.e. 1:3 \neq 5:10)$

the two ratios do not form proportion.

Note:

- If the four quantities $a, b, c, and\ d$ are in proportion, then $a: b = c: d$.

- The proportion $a:b = c:d$ can be written as $\frac{a}{b} = \frac{c}{d}$, here $a\ and\ d$ are called extremes (end terms) and $b\ and\ c$ are said to be means (middle terms)
- In proportion the product of means is equal to the product of extremes.

$(i.e. if\ \frac{a}{b} = \frac{c}{d}, then\ a \times d = b \times c)$. you can easily remember this by applying, cross multiplication as follows

If $\frac{a}{b} = \frac{c}{d}$, then $a \times d = b \times c$ or

$$\text{If } \overbrace{a:\underbrace{b = c}_{b \times c}:d}^{a \times d}, \text{then } a \times d = b \times c$$

Example 3: show that $20, 45, 8\ and\ 18$ are in order of proportion

Solution: to show they are in order of proportion $(i.e. 20:45 = 8:18)$

Use cross multiplication (product of means = product of extremes)

Product of means $= 45 \times 8 = 360$

Product of extremes $= 20 \times 18 = 360$

Since, product of means is equal to product of extremes. The numbers $20, 45, 8\ and\ 18$ are in order of proportion.

Example 4: Are the numbers $3, 5, 6\ and\ 12$ in order of proportion?

Solution: check $3:5 = 6:12$?

Product of means $= 5 \times 6 = 30$

Product of extremes $= 3 \times 12 = 36$

Product of means $\neq$ product of extremes, hence the numbers $3, 5, 6$ and 12 are not in order of proportion.

Example 5: If the numbers $3, 5, x$ and 20 are in order of proportion, then find the value of x.

Solution: $3:5 = x:20$

Product of means $=$ product extremes

$$\frac{\cancel{5}x}{\cancel{5}} = \frac{3\times 20}{5}$$

$$x = 12$$

Example 6: In proportion $\frac{3}{5} = \frac{9}{y}$, find the value of y

Solution: use cross multiplication

$$\frac{3}{5} = \frac{9}{y} \quad , \frac{\cancel{3}y}{\cancel{3}} = \frac{5\times 9}{3}, y = 15$$

Example 7: The distance from Addis Ababa to Adama is $75km$. If a map scale is $1:1{,}000{,}000$, then find the distance between the two cities on a map, in centimeters.

Solution:

Map scale $=$ Map distance : ground distance

$$1:1{,}000{,}000 = x:7{,}500{,}000\ cm$$

$$\frac{\cancel{1{,}000{,}000}x}{\cancel{1{,}000{,}000}} = \frac{1\times 7{,}5\cancel{00{,}000}cm}{1{,}0\cancel{00{,}000}}$$

$$x = \frac{75\ cm}{10}$$

$$x = 7.5cm$$

Note:

Map scale=map distance: ground distance

1km = 100, 000cm

75km = 7,500,000 cm

Therefore, the distance between the two cities on a map $7.5cm$

Exercise 3.1.3

1. From each pair of ratios below circle the given pair of ratios that are in proportion.

   a. $\frac{2}{3}$ and $\frac{4}{260}$ c. $\frac{4}{5}$ and $\frac{12}{20}$

   b. $\frac{1}{3}$ and $\frac{5}{20}$ d. $\frac{8}{4}$ and $\frac{2}{1}$

2. Find the unknown terms in each of the following

   a. $\frac{3}{5} = \frac{x}{20}$ c. $\frac{8}{m} = \frac{3.2}{4}$

   b. $x:18 = 2:9$ d. $\frac{3}{5} = \frac{18}{n} = \frac{y}{50}$

3. Show that the numbers $14, 21, 2, and\ 3$ are in order of proportion.

4. Given the proportion $10:18 = 35:63$, then find

   a. The sum of means   c. the sum of extremes

   b. The product of means   d. the product of extremes

5. A map scale is $1:1{,}000{,}000$. If the distance between the two cities on a map is $3.5cm$, then find the actual distance between the cities in kilometers.

Direct and inverse proportionality

A. Direct proportionality

Definition 3.3: y is said to be directly proportional to x (written as $y \propto x$), if there is constant k such that $y = kx \text{ or } k = \frac{y}{x}$.

The number k is called constant of proportionality.

If $y \propto x$, then as x increases y also increases or as x decreases y also decreases. And the ratio $\frac{y}{x}$ is constant.

Example1: the following table shows the number of kilograms of sugar bought and the price paid in birr.

$Sugar\ in\ kg(x)$	1	2	3	4
$Price\ in\ Birr(y)$	30	60	90	120

a. Is y directly proportional to x, if so what is the constant of proportionality?

b. Write the formula relating y and x.

c. Using the formula calculate the price of $6.5\ kg$ sugar.

d. How much kg sugar will be bought with Birr 315.

Solution:

a. As x increases y also increases and the ratio $\frac{y}{x}$ is constant,

$k = \frac{30}{1} = \frac{60}{2} = \frac{90}{3} = \frac{120}{4} = 30$, hence $y \propto x$

b. $\frac{y}{x} = k$, implies $\frac{y}{x} = 30$ or $y = 30x$

c. $y =?$, when $x = 6.5kg$

$y = 30x$

$y = 30 \times 6.5$

$y = birr\ 195$

d. $x =?$, when $y = 315\ Birr$

$y = 30x$

$\frac{315}{30} = \frac{\cancel{30}x}{\cancel{30}}$

$x = 10.5kg$

Example 2: if$y \propto x$, then find the value of m.

x	12	6
y	48	m

Solution: since $y \propto x$, the ratio $\frac{y}{x}$ is constant

$$\frac{48}{12} = \frac{m}{6}$$

$$\frac{\cancel{12} \times m}{\cancel{12}} = \frac{48 \times 6}{12}$$

$$m = 24$$

Example 3: If 5kg sweet potato costs Birr 120, then calculate the cost of 8kg sweet potato.

Solution: as the amount of sweet potato increases, the cost also increases

Hence, the cost of sweet potato is directly proportional to amount of sweet potato.

$Sweet\ potato\ in\ (kg)$	5	8
$Cost\ in\ (Birr)$	120	x

$$\frac{120}{5} = \frac{x}{8}$$

$$\frac{\cancel{5} \times x}{\cancel{5}} = \frac{120 \times 8}{5}$$

$$x = 192\ Birr$$

Therefore, the cost of 8kg sweet potato is Birr 192

Exercise 3.1.4

1. If y is directly proportional to x; $y = 24$ when $x = 6$,then find
   a. the constant proportionality.
   b. the formula relating $y\ and\ x$
   c. the value y ,when $x = 3$

d. the value x ,when $y = 15$

2. Which of the following table shows direct proportional relationship between x and y?

a.

x	8	5	6
y	24	15	18

b.

x	3	7	4
y	6	14	12

3. What is the value of m from the given table below if $y \propto x$?

x	14	7
y	21	m

y is directly proportional to x, If $x = 20$ *when* $y = 160$ then what is the value of x *when* $y = 3.2$

4. If a car travels 120 km in 2 hours, then how long will it take for the car to travel 330 km (assuming the car is moving at constant speed)?

B. Inverse proportionality

Definition 3.4: y is said to be inversely proportional to x (written as $y \propto \frac{1}{x}$), if there is constant k such that $y = \frac{k}{x}$ *or* $k = y \times x$.

The number k is called constant of proportionality.

If $y \propto \frac{1}{x}$, then as x increases, y decreases or as x decreases, y increases. And the product $y \times x$ is constant.

Example1: The following table shows the time taken for a car to move a distance of 5,000m at various speed.

$time\ in\ second(x)$	1,000	500	250	200
$speed\ in\ m/s(y)$	5	10	20	25

a. Is y inversely proportional to x, if so what is the constant of proportionality?

b. Write the formula relating y and x.

c. Using the formula calculate the time taken by the car to cover the given distance moving at a speed of 8 meter/second.

Solution:

a. As x increases, y decreases and the product $y \times x$ is constant,

$k = 5 \times 1000 = 10 \times 500 = 20 \times 250 = 25 \times 200 = 5000$, hence $y \propto \frac{1}{x}$

b. $y \times x = k$, implies $y \times x = 5000$

c. $x = ?$, when $y = 8\ m/s$

$y \times x = 5000$

$\frac{\cancel{8} \times x}{\cancel{8}} = \frac{5000}{8}$

$x = 625\ seconds$

Example 2: If $y \propto \frac{1}{x}$, then find the value of m.

x	12	6
y	15	n

Solution: since $y \propto \frac{1}{x}$, the product $y \times x$ is constant

$\frac{\cancel{6} \times n}{\cancel{6}} = \frac{12 \times 15}{6}$

$n = 30$

Example 3: If 8 persons do a certain job in 9 hours, then how long will it take for 12 persons to do the same job. (Assume all persons do the job at the same rate)

Solution: As the number of person increases, the time taken to finish the job decreases.

Hence, the number of persons is inversely proportional to time taken to finish the job

Number of persons	8	12
Time taken to do the job(hr.)	9	x

$$12 \times x = 8 \times 9$$

$$\frac{\cancel{12} \times x}{\cancel{12}} = \frac{8 \times 9}{12}$$

$$x = 6\ hr.$$

Therefore, it takes $6\ hr$ for 12 persons to do the job.

Exercise 3.1.5

1. If $y \propto \frac{1}{x}$ and $y = 6$ when $x = 4$, then find the constant of proportionality.
2. y is inversely proportional to x. If $x = 25$, then $y = 8$. What is the value of y when $x = 10$?
3. Which of the following table is inversely proportional in the relationship between x and y?

a.

x	2	4	8
y	36	18	9

b.

X	5	10	15
Y	20	10	5

4. It takes 8 days for 35 laborers to harvest coffee on a plantation. How long will 20 laborers take to harvest coffee on the same plantation?

5. x is inversely proportional to y.If $x = 25$ then $y = 15$. Find y when $x = 30$

6. 100 men working in a factory produce 6000 articles in 10 days. How long it takes (assume all workers do the job at the same rate)

   a. 50 men to produce 6000 articles

   b. 150 to produce 6000 articles

7. A contractor appoints 36 workers to build a wall. They could finish the task in 12 days. How many days will 16 workers take to finish the same task?

3.2.Revision on Percentages

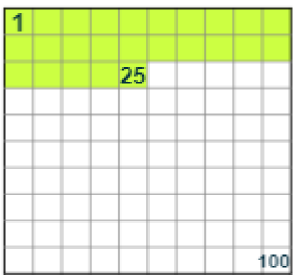
By the end of this section you should be able to:

- Visualizes percent and its relationship to fractions, ratios, and decimal numbers using bar models.
- Identifies the base, percentage, and rate in a problem.

Activity 3.2.1

Discuss with your friends

1. consider the following

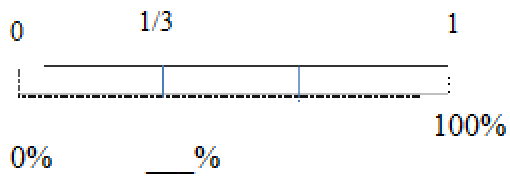


   a. count the green part and the total part
   b. What is the ratio of green part to total part?
   c. express the above ratio in percent form
   d. define percent in your own words

2. write the following numbers in percent form

a. $\frac{81}{100}$ b. $\frac{2}{100}$ c. 0.75 d. $\frac{1}{5}$

3. Use the following bar model to relate the given fraction and percent



Definition 3.5: The word percent means "for every hundred" or "per 100". We use the symbol $\%$ to denote percent.

For example,

5 Percent symbolically written as 5% and

5% Means 5 per 100 or $\frac{5}{100}$

The concept of percent is widely used in a variety of mathematical applications.

- Interest
- Sales tax
- Change of Profit and loss
- Inflation

Converting a given percent in fractional form

Example 1: change 30% in to fraction

Solution: 30% means 30 per 100

So, $30\% = \frac{30}{100} = \frac{3}{10}$

Note: To change a given percent to fraction form

- Drop the percent sign from the number

➢ Put 100 as the denominator of the number and write the fraction in the lowest term.

Example 2: Convert each of the following percent form in to fraction:

a. 1% b. 45% c. 90% d. 4.8%

Solution:

a. $1\% = \frac{1}{100}$

b. $45\% = \frac{45 \div 5}{100 \div 5} = \frac{9}{20}$

c. $90\% = \frac{90}{100} = \frac{9}{10}$

d. $4.8\% = \frac{4.8}{100} = \frac{4.8 \times 10}{100 \times 10} = \frac{48 \div 8}{1000 \div 8} = \frac{6}{125}$

Converting a given percent in decimal form

To convert a given percent in to decimal form:

First write the percent in fraction form, and then convert it in to decimal by division.

Example 3: Express each of the following percent form into decimal form.

a. 30% b. 7.38% c. 114% d. 11%

Solution:

a. $30\% = \frac{30}{100} = 0.3$

b. $7.38\% = \frac{7.38}{100} = 0.0738$

c. $114\% = \frac{114}{100} = 1.14$

d. $11\% = \frac{11}{100} = 0.11$

Exercise 3.2.1

1. Convert each of the following percent forms to fractions?

   a. 60%   c. 125%   e. $5\frac{3}{4}\%$

   b. 2.6%   d. $\frac{3}{4}\%$ f. 0.045%

2. Convert each of the following percent forms in to decimals?

   a. 80%   c. 12%   e. $5\frac{1}{4}\%$

   b. 26%   d. $\frac{2}{5}\%$ f. 0.45%

Converting a given number to percent

Example 4**:** convert $\frac{2}{5}$ to percent form

Solution: to convert it in to percent, try to write equivalent form of the number, so that its denominator is 100.

$$\frac{2}{5}=\frac{2\times 100}{5\times 100}=\frac{40}{100}=40\%$$

Note: To convert a given number to percent form, multiply the given number by $\frac{100}{100}$ *or by* 100%

Example 5: Convert each of the following fractions as percent form**:**

a. $\frac{1}{2}$ b. $\frac{4}{5}$ c. $\frac{3}{4}$ d. $\frac{3}{8}$

Solution:

a. $\frac{1}{2}=\frac{1}{2}\times 100\%=\frac{100}{2}\%=50\%,$

b. $\frac{4}{5}=\frac{4}{5}\times 100\%=\frac{400}{5}\%=80\%$

c. $\frac{3}{4} = \frac{3}{4} \times 100\% = \frac{300}{4}\% = 75\%$

d. $\frac{3}{8} = \frac{3}{8} \times 100\% = \frac{300}{8}\% = 37.5\%$

Example 6: Express each of the following decimal into percent form

a. 0.5 b. 0.8 c. 0.24 d. 2.09

Solution:

a. $0.5 = 0.5 \times 100\% = 50\%$
b. $0.24 = 0.24 \times 100\% = 24\%$
c. $1.23 = 1.23 \times 100\% = 123\%$
d. $2.09 = 2.09 \times 100\% = 209\%$

Example 7 Express each of the following ratio into percent form

a. 75 seconds to 5minutes
b. 24 hours to 8days
c. 4 km to 500 meters
d. 5litres to 4000militers

Solution:

a. 75 seconds to 5minutes= $75:300 = \frac{75}{300} = \frac{1}{4}$

$(since\ 5\ minutes = 300seconds)$

$$= \frac{1}{4} \times 100\% = \frac{100}{4} = 25\%$$

b. 24 hours to 8 days $= 24:192 = \frac{24}{192} = \frac{1}{8}$ $(since\ 8\ days\ =\ 192hours)$

$$= \frac{1}{8} \times 100\% = \frac{100}{8} = 12.5\%$$

c. 4 km to 500metres $= 4000: 500 = \frac{4000}{500} = 8 \quad (since\ 4km = 4000\ meters)$

$$= 8 \times 100\% = 800\%$$

d. 5liters to 4000 milliliters $= 5000: 4000 = \frac{5000}{4000} = \frac{5}{4}$

$(since\ 5L = \ 4000\ ml)$

$$= \frac{5}{4} \times 100\% = \frac{500}{4} = 125\%$$

Exercise 3.2.2

1. Convert each of the following fractions as percent form?

a. $\frac{3}{5}$

c. $2\frac{3}{4}$

b. $\frac{1}{6}$

d. $\frac{13}{6}$

2. Express each of the following decimals in percent form.

a. 0.12 b. 7.5 c.3.65 d.0.0012

3. Convert each of the following ratios in percent form?

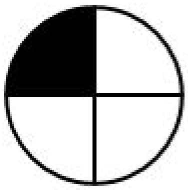
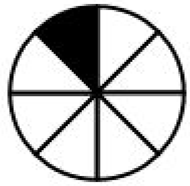
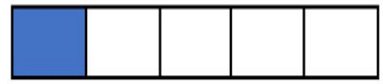
a. $2m\ to\ 5m$

c. $50mililiters\ to\ 3liters$

b. $2hr\ to\ 25minutes$

d. $5kg\ to\ 5000g$

4. Express the shaded part in percent form

a.  b.  c. 

Base, rate and percentage

Activity 3.2.2

Discuss with your friends

1. Calculate 10% of 200
2. 5% of 60 is calculated as follows

   $$5\% \; of \; 60 = \frac{5}{100} \times 60 = \frac{300}{100} = 3$$

   In this calculation

   a. 5% is called _____

   b. 60 is called ______

   c. 3 is called _______

Definition 3.6:

Base (B) is the number that represents 100% or total value of something.

Percentage (p) is the number or the amount that represents a part of the whole.

Rate (R) is the ratio of percentage to base, written as percent.

Note:

Base is usually preceded by “of” in the given statement

Rate is expressed in percent

Example: 20% of 150 is 30. In this statement determine base, rate and percentage.

Solution: The number expressed in percent is rate, hence 20% is rate

The whole part or the number preceded by of is base, hence 150 is base

The part of whole is percentage, hence 30 is percentage.

Formula relating base, rate and percentage

Example**:** calculate 5% of 120

Solution: 5% of $120 = \frac{5}{100} \times 120 = 6$

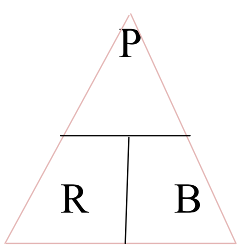
$$R \times B = P$$

From above example you can see that base, rate and percentage are related by the formula

$$\boxed{P = R \times B}$$

You can also use the following triangle to easily remember the relation between P, R and B

$$P = R \times B$$



$$B = \frac{P}{R}$$

$$R = \frac{P}{B} \times 100\%$$

Note: R is expressed in percent, so $\frac{P}{B}$ is multiplied by 100%

Example 1: calculate the following

a. 80% of $Birr\ 260$
b. 9.6% of $7.2\ liters$
c. 90% of $264\ cm$
d. 34% of $7000kg$

Solution:

a. $P = R \times B$

$P = 80\% \times 260\ Birr$

$P = \frac{80}{100} \times 260\ Birr$

$P = 8 \times 26\ Birr$

$P = Birr\ 208$

b. $P = R \times B$

$P = 9.6\% \times 7.2\ liters$

$P = \frac{9.6}{100} \times 7.2\ \ liters$

$P = 0.096 \times 7.2\ liters$

$P = 0.6912\ liters$

c. $P = R \times B$

$P = 90\% \times 264cm$

$P = \frac{90}{100} \times 264\ cm$

$P = 0.9 \times 264\ cm$

$P = 237.6\ cm$

d. $P = R \times B$

$P = 34\% \times 7000kg$

$P = \frac{34}{100} \times 7000kg$

$P = 34 \times 70kg$

$P = 2380kg$

Example 2: Eleni answered 80% of questions correctly in mathematics test. If the test has 25 questions, then how many questions she answered correctly?

Solution: $R = 80\%\ , B = 25, P = ?$

$P = R \times B$

$P = 80\% \times 25$

$P = \frac{80}{100} \times 25$

$P = 20$

Therefore, she answered 20 questions correctly

Exercise 3.2.3

1. Calculate each of the following.

a. What is 10% of 160?

b. What is 60% of Birr 300?

c. Find 4.5% of 9km.

d. Find $\frac{3}{4}\%$ of Birr 5000.

2. In a class there are 60 students. If 20% of the class are girls, then how many girls and boys are there?

3. 25% of people in Addis Ababa city watched the football final game on TV. How many people watched the football final game if the population of the city is 5000 000

Example 3: calculate the unknown variables in each of the following questions

a. $Birr\ 36\ is\ 9\%\ of\ x$

b. $15\ cents\ is\ 5\%\ of\ x$

c. $16\ minutes\ is\ 13\frac{1}{3}\%\ of\ \ T$

d. $90cm\ is\ 180\%\ of\ y$

Solution:

a. $R = 9\%\ , B = x\ and\ \ P = 36$

$$B = \frac{P}{R}$$

$$B = \frac{36}{9\%}$$

$$B = \frac{36\ birr}{9/_{100}} = 36\ birr \times \frac{100}{9} = \frac{3600\ birr}{9} = Birr\ 400$$

b. $R = 5\%\ , B = x\ and\ \ P = 15\ cents$

$$B = \frac{P}{R}$$

$$B = \frac{15}{5\%}$$

$$B = \frac{15\ cents}{5/_{100}} = 15cents \times \frac{100}{5} = \frac{1500\ cents}{5} = 300cents$$

c. $R = 13\frac{1}{3}\% = \frac{40}{3}\% = \frac{40}{300}\ , B = T\ and\ \ P = 16\ minutes$

$$B = \frac{P}{R}$$

$$B = \frac{16\ minutes}{40/_{300}} = 16\ minutes \times \frac{300}{40} = 120\ minutes$$

d. $R = 180\%, B = y\ and\ P = 90$

$$B = \frac{P}{R}$$

$$B = \frac{90cm}{180\%}$$

$$B = \frac{90\ cm}{180/_{100}} = 90cm \times \frac{100}{180} = \frac{900cm}{18} = 50cm$$

Example 4: A women saves 20% of what she earns. If she saves Birr 300 a month, how much does she earn a month?

Solution: $R = 20\%$

A part of money she earns is saved $P = Birr\ 300$

The whole amount she earns a month? $i.e. B = ?$

$$B = \frac{P}{R}$$

$$B = \frac{300\ birr}{20\%} = \frac{300\ birr}{20/_{100}} = 300\ birr \times \frac{100}{20} = Birr\ 1500$$

Therefore, the woman earns $Birr\ 1500$ a month.

Exercise 3.2.4

1. Calculate the following
   a. 6 is 24% of a number, what is the number?
   b. What is the total amount whose 35% is Birr 700.
   c. If 15% of a number is18, then what is this number?
   d. 40% of a number is 110, what is number?
2. If 30% of a man's salary is Birr 6300, what is the amount of his full salary?

3. In a class where the number of boys is 36 % of the total number of students. If there are 18 boys, then how many students are there in the class?

Example 5: calculate the following

a. What percent of 400 is 50?
b. What percent of $2kg$ is $400gm$?
c. What percent of $1hr$ is $30\ minutes$?
d. What percent of $60m$ is $72cm$?

Note:

1kg=1000gm

1hr=60 minutes

1m=100cm

Solution:

a. $B = 400, P = 50, R = ?$

$R = \frac{P}{B} \times 100\%$

$R = \frac{50}{400} \times 100\% = \frac{50}{4}\% = 12.5\%$

b. $B = 2kg = 2000gm, P = 400gm, R = ?$

$R = \frac{P}{B} \times 100\%$

$R = \frac{400\cancel{gm}}{2000\cancel{gm}} \times 100\% = \frac{400}{20}\% = 20\%$

c. $B = 1hr = 60\ miutes, P = 30\ minutes, R = ?$

$R = \frac{P}{B} \times 100\%$

$R = \frac{30\ \cancel{minutes}}{60\ \cancel{miutes}} \times 100\% = \frac{300}{6}\% = 50\%$

d. $B = 60m = 6000cm, P = 72cm, R = ?$

$R = \frac{P}{B} \times 100\%$

$R = \frac{72\cancel{cm}}{6000\cancel{cm}} \times 100\% = \frac{72}{60}\% = 1.2\%$

Example 6: In a certain village where farmers use two different types of fertilizers, fertilizer type A and fertilizer type B. 10000 farmers were asked which of the fertilizers they use. It was found that 2500 farmers use type A, 4550 farmers use type B and the remaining farmers use both types A and B.

Find the percent of the farmers that

a. Use fertilizer type A

b. Use fertilizer type B

c. use both types of fertilizers

Solution:

a. The total 10000 farmers is base, $B = 10000$

2500 is part of total farmers that use type A, hence it is percentage, $P = 2500$

$R = ?$

$$R = \frac{P}{B} \times 100\%$$

$$R = \frac{2500}{10000} \times 100\% = 25\%$$

Therefore, 25% of farmers use type A fertilizer

b. Total 10000 farmers is base, $B = 10000$

4550 is part of total farmers that use type B, hence it is percentage, $P = 4550$

$R = ?$

$$R = \frac{P}{B} \times 100\%$$

$$R = \frac{4550}{10000} \times 100\% = \frac{455}{10}\% = 45.5\%$$

Therefore, 45.5% of farmers use type B fertilizer

c. Total 10000 farmers is base, $B = 10000$ the remaining farmers that use both type of fertilizers is

$10000 - (2500 + 4550) = 2950, P = 2950$

$R = ?$

$$R = \frac{P}{B} \times 100\%$$

$$R = \frac{2950}{10000} \times 100\% = \frac{295}{10}\% = 29.5\%$$

Therefore, 29.5% of farmers use both type of fertilizer.

Exercise 3.2.5

1. Evaluate the following
   a. What percent of 60 is 30?
   b. What percent of 24 is 6?
   c. What percent of 800 is 500?
2. A Woman saves Br.300 from her monthly salary. If her monthly salary is Br.7500, then find her saving in percent?
3. The following table gives the number of students in a given class according to their age and sex.

Age	**Sex**	
	Male	**Female**
13	15	18
14	6	9
Total	21	27

   a. What percent of the class are females?
   b. What percent of the class are males?
   c. What percent of class are 13 years old?
   d. What percent of class are 14 years old female students?
4. In a class of 48 students 6 of them were absent on Monday. What percent of the class was absent and what percent of class was attended on that day?

Mixed problems containing base, rate and percentage

1. A student scored 16 out of 25 in mathematics mid exam. What is the student's score in percent?
2. There are 50 students in a class. If 8% are absent on a particular day, find the number of students present in the class.
3. In a basket of oranges, 20% of them are defective and 76 are in good condition. Find the total number of oranges in the basket.
4. Hawa was able to sell 220 kilograms of her vegetables before noon. If Hawa had 400 kg of vegetables in the morning, what percent of Kilograms of vegetables was she sold in the morning?
5. A cow gives 24 liters milk each day. If the milkman sells 75% of the milk in one day, how many liters of milk is left with him?
6. Tolosa sold 540 eggs. If these are 36% of total eggs, then how many eggs are not sold?
7. A factory has 2400 workers of which 900 are male and the rest are female. What percent of the workers are female?

3.3. Application of Ratio, Proportion and Percentage

By the end of these sections you should be able to:

- Apply the concept of percentage to solve real life problems

In our day to day life, we come across a number of situations wherein we use the concept of percent. In the following section, we discuss the application of percentage in different fields like problems in percent change, profit and loss, discount, simple interest, compound interest, Value added tax (VAT), turn of tax (TOT), and income tax.

Percent increase and decrease

Activity 3.3.1

Discuss with your friends

1. A quantity increases from 400 to 600
   a. Find the increased amount
   b. Find the ratio of the increased amount to original quantity
   c. Express the answer in b in the form of percent (find the percent increase)
   d. How do you calculate percent increase?
2. A quantity decreases from 500 to 300
   a. Find the decreased amount
   b. Find the ratio of decreased amount to original quantity
   c. Express the answer in b in the form of percent (find the percent decrease)
   d. How do you calculate percent decrease?

Note:

1. Percent increase is calculated as:

$$\text{percent increase} = \frac{\text{increased amount}}{\text{original quantity}} \times 100\%$$

2. Percent decrease is calculated as:

$$\text{percent decrease} = \frac{\text{decreased amount}}{\text{original quantity}} \times 100\%$$

Example 1: Find the percent increase from 250 to 400.

Solution:

Increased amount is $400 - 250 = 150$

Original quantity is 250

$$percent\ increase = \frac{increased\ amount}{original\ quantity} \times 100\%$$

$$percent\ increase = \frac{150}{250} \times 100\% = 60\%$$

Therefore, the percent increase from 250 to 400 is 60%

Example 2: Find the percent decrease from 400 to 250.

Solution:

decreased amount is $400 - 250 = 150$

Original quantity is 400

$$percent\ decrease = \frac{decreased\ amount}{original\ quantity} \times 100\%$$

$$percent\ decrease = \frac{150}{400} \times 100\% = 37.5\%$$

Therefore, the percent decrease from 250 to 400 is 37.5%

Example 3: The student's population of a school is 4000 in 2013 E.C. If the student's population increased to 4620 in 2014 E.C then, find the percent increase between the two years.

Solution:

Increased amount is $4620 - 4000 = 620$

Increased amount is 620

$$percent\ increase = \frac{increased\ amount}{original\ quantity} \times 100\%$$

$$percent\ increase = \frac{620}{4000} \times 100\% = 15.5\%$$

Example 4: 20% of an article is damaged and thrown away and only 20kg is left. Find its original weight.

Solution: let the original weight is x. The weight of the article decreased from x to $20kg$. Percent decrease is 20%

$$percent\ decrease = \frac{decreased\ amount}{original\ quantity} \times 100\%$$

$20\% = \frac{x-20}{x}$ → no need of multiplying by 100%, percent decrease is given.

$$\frac{20}{100} = \frac{x-20}{x},\ 20x = 100(x-20)$$

$$20x = 100x - 2000$$

$$20x - 100x = -2000$$

$$\frac{-80x}{-80} = \frac{-2000}{-80}$$

$$x = 25kg$$

Therefore, the original weight is $25kg$

Exercise 3.3.1

1. Find the percent change
   a. From 80 to 100
   b. From 800 to 500
   c. From 500 to 300
   d. From 6000 to 9000
2. While measuring a line segment 5cm, it was measured 5.2cm by mistake. Find the percent error in measuring this line segment
3. The number of student fail in mathematics test decreased from 20 to 12. What is the percent decrease?

4. Last year Samuel's salary was Birr 8000. If he gets a 10% increment this year, what is his current salary?
5. A T – shirt, discounted by 30% for a clearance sale. If the discounted price is Birr 399, then calculate
   a. The original price of T – shirt
   b. The amount of discount

3.3.1. Calculating profit and loss percentage

Activity 3.3.2

Discuss with your friends

1. A shop keeper buys a pair of shoes for Birr 500 and sells it for Birr 600. Find
   a. The cost price
   b. The selling price
   c. Profit
   d. Profit percent
2. The price at which an article is purchased is called ______.
3. The price at which an article is sold is called ______.
4. Profit is made, when selling price is ______than cost price.
5. Loss is made, when selling price is _______than cost price.

Definition:

Cost Price (C.P): The Price, at which an article is purchased, is called its cost price.

Selling Price (S.P): The Price, at which an article is sold, is called its selling price.

Profit (Gain): When $S.P > C.P$, then there is profit, and $Profit = S.P - C.P$

Loss: When $S.P < C.P$, then there is loss, and $loss = C.P - S.P$

The formula to calculate profit and loss percent

1. Profit percent:

$$Profit\ \% = \frac{Profit}{cost\ price} \times 100\% \quad or$$

$$Profit\ \% = \frac{sellig\ price - cost\ price}{cost\ price} \times 100\%$$

$$Profit\ \% = \frac{S.P - C.P}{C.P} \times 100\%$$

2. **loss percent**:

$$Loss\ \% = \frac{Loss}{cost\ price} \times 100\% \quad or$$

$$Loss\ \% = \frac{cost\ price - sellig\ price}{cost\ price} \times 100\%$$

$$Loss\ \% = \frac{C.P - S.P}{C.P} \times 100\%$$

Example 1: A shop keeper buys a jacket for Birr 500, and gives it clean, then sells it for Birr 640. Calculate his profit percent.

Solution: $C.P = 500, S.P = 640, Profit\ \% = ?$

$$Profit\ \% = \frac{S.P - C.P}{C.P} \times 100\%$$

$$Profit\ \% = \frac{640 - 500}{500} \times 100\%$$

$$= \frac{140}{500} \times 100\%$$

$$= 28\%$$

Therefore, his profit percent is 28%.

Example 2: Girma bought 200 eggs for Birr 8 each. 20 eggs are damaged and sold the remaining eggs at Birr 8.5 each; calculate his profit/loss percent.

Solution:

The cost of 1 egg is Birr 8, so the cost of 200 eggs is $200 \times 8 = Birr1600 = C.P$

20 eggs are damaged, so $200 - 20 = 180$ eggs are sold at Birr 8.5 each

$S.P = 180 \times 8.5 = Birr\ 1530$

Since, $.P < C.P$, there is loss and his loss percent is

$$Loss\ \% = \frac{C.P - S.P}{C.P} \times 100\%$$

$$Loss\ \% = \frac{1600 - 1530}{16\cancel{00}} \times 1\cancel{00}\%$$

$$= \frac{70}{16} \times 1\%$$

$$= 4.375\%$$

Therefore, his loss percent is 4.375%.

Example 3: A dealer gained 10% profit by selling an article for birr 330. What was the original price of the article?

Solution: $profit\ \% = 10\%, S.P = 330, C.P = ?$

$$profit\ \% = \frac{S.P - C.P}{C.P} \times 100\%$$

$10\% = \frac{330 - x}{x}$ $\rightarrow$ no need of multiplying by 100%, profit % is given and $C.P = x$

$\frac{10}{100} = \frac{330 - x}{x}$, $10x = 100(330 - x)$

$$10x = 33000 - 100x$$

$$10x + 100x = 33000$$

$$\frac{110x}{110} = \frac{33000}{110}$$

$$x = 300$$

Therefore, the original price of the article is $Birr\ 300$

Exercise 3.3.2

1. Find profit or Loss percent when:
   a. Cost Price = Birr 2500 and Selling Price = Birr 2000
   b. Cost Price = Birr 3500 and Selling Price = Birr 4500
2. Find cost price when:
   a. Selling Price = Birr 7950 and Gain = 6%
   b. Selling Price = Birr 980 and Loss = 12%
3. Find selling price when
   a. Cost Price = Birr 870 and Gain = 10%
   b. Cost Price = Birr 750 and Loss = 15%
4. An article was bought for Birr 2000 and sold by Birr 2200. Find the profit or loss percent.
5. A shop keeper bought a jacket for Birr 1500 and gives it clean, then sold it for Birr 1800.What is his profit percent?
6. Estifanos bought goods by Birr 5200 and sold it at a profit of 12%. Find the selling price.
7. By selling a camera for Birr 24000, a man loss 4%. Find the cost price?
8. A trader bought a TV set for Birr 2500 and sold it at a loss of 8%. what was the selling price?
9. A man bought 100 eggs for Birr 800 and sells them for Birr 10 each, then find his profit percent.

3.3.2. Simple interest

Activity 3.3.3

Discuss with your friends

1. If you deposit Birr 1000 in a bank, then
   a. After a year Birr 1000 increased by a certain amount, why?
   b. By what amount Birr 1000 increases within 1 year, if the bank pays you 10% per year for using your money?
   c. By what amount Birr 1000 increases within 2 years, if a bank pays you 10% per year on money you deposited initially?
2. Define the following by your own words
   a. Interest
   b. Simple interest

A person needs to borrow some money as a loan from credit & saving association bank, his friends, relatives etc. He promises to return it after a specified time period along with some extra money for using the money of lender.

- The amount of money borrowed or invested is called principal (p).
- The extra money paid for the use of money is called interest (I).
- The time length for which money is borrowed or lent is called Time (T).
- The total amount of money including interest at the end of time period is called Amount (A).
- The percentage charged on the total amount you borrow or save is called interest rate(R).

Example 1: If Birr 2000 is deposited in a bank at a rate of 7% per year, for 1 year then,

a. What is the principal?

b. What is the interest rate?

c. What is the time period?

d. Calculate interest paid in one year

Solution:

a. Money deposited is Birr 2000, so, principal(P) is Birr 2000

b. The interest rate(R) is 7% per year

c. The time period(T) is 1 year

d. The interest(I) in one year is 7% of Birr 2000, so $I = 7\% \times 2000$

$$I = \frac{7}{100} \times 2000 = Birr140$$

The interest paid on original principal only during the whole interest period is called **simple interest**.

Simple interest is calculated by the formula:

$$I = PRT$$

Where, $I = simple\ interest$

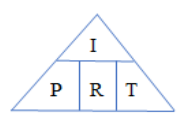
$$P = principal$$

$$T = time$$

And amount (A) is given by:

$$A = P + I$$

You can use the following triangle to easily remember the relation between $I, P, R, and\ T$



$P = \frac{I}{RT}$

$I = PRT$

$T = \frac{I}{PR}$ and $R = \frac{I}{PT} \times 100\%$

Example 2: If Birr 4000 is deposited in a bank at a rate of 7% per year, for 4 years then, calculate

a. Simple interest at the end of 4 years

b. Amount at the end of 4 years

Solution: $P = Birr\ 4000, R = 7\%\ per\ year, T = 4\ years$

a. $I = PRT$

$I = 4000Birr \times \frac{7}{100} \times 4$

$I = Birr\ 1{,}120$

b. $A = P + I$

$A = 4000 + 1120$

$A = Birr\ 5120$

Example 3: Find the simple interest on Birr 700 at 12 % per year for 3 months.

Solution: $P = Birr700\ , R = 12\%\ per\ year, T = 3\ months$, since the interest rate is per year, the time must be converted to years.

$T = 3\ months = \frac{3}{12}\ year = \frac{1}{4} year$

$I = PRT$

$I = Birr\ 700 \times \frac{12}{100} \times \frac{1}{4}$

$I = Birr\ 21$

Example 4: If Birr 1000 grows to 1200 in 4 years at simple interest rate, then calculate the simple interest rate.

Solution:

$I = PRT, so\ R = \frac{I}{PT} \times 100\%$

$R = \frac{200}{1000 \times 4} \times 100\%$ =5% per year

Exercise 3.3.3

1. Michael invests Birr 2000 in the bank that pays simple interest rate of 5% per year for 6 years. Then how much interest will he get in 6 years?

2. Birr 24000 invested at 11% simple interest per annum, then what is the amount after 8 years?
3. How long will it take for Birr 15000 to double itself, if it is invested at simple interest rate of 10% per year?
4. If Birr 20,000 grows to Birr 28000 after 20 years. Then what is the simple interest rate?
5. In how many years will Birr12, 000 yield an interest of Birr 1200 at 5% simple interest?
6. An investment earned Birr 2100 interest after 6 years. If the simple interest rate is 7% per year. What was the principal?

3.3.3 Compound interest

Activity 3.3.4

Discuss with your friends

1. If Birr 1000 is invested at a rate of 10% per year, then calculate
   a. The interest at the end of first year
   b. The interest on the second year. (take second year principal to be initial principal plus first year's interest)
   c. The interest on third year. (take third year principal to be initial principal plus first year interest plus second year interest)
   d. The interest in 3 years
2. What is compound interest?

Compound interest is the interest on a loan calculated based on the initial principal plus the accumulated interest from previous periods

Example 1: If Birr 2000 is invested at a rate of 10 % compounded annually for 3 years, then calculate

a. Compound interest for 3 years
b. Amount at the end of third year

Solution:

a. Interest for first year I_1

$P_1 = 2000, R = 10\%, T = 1\ year$ for year one only

$I_1 = P_1RT,$ so $I_1 = 2000 \times 10\% \times 1$

$$I_1 = 2000 \times \frac{10}{100} \times 1$$

$$I_1 = Birr\ 200$$

- Interest for year $2(I_2)$

Principal for second year is initial principal plus interests on previous years. $P_2 = P_1 + I_1, P_2 = 2000 + 200 = 2200, T = 1$ for year two only

$I_2 = P_2RT, I_2 = 2200 \times 10\% \times 1$

$$I_2 = 2200 \times \frac{10}{100} \times 1$$

$$I_2 = Birr\ 220$$

- Interest for year $3(I_3)$

Principal for third year is initial principal plus interests on previous years. $P_3 = P_1 + I_1 + I_2, P_3 = 2000 + 200 + 2200 = 2420,$

$T = 1$ for year three only

$I_3 = P_3RT, I_3 = 2420 \times 10\% \times 1$

$$I_3 = 2420 \times \frac{10}{100} \times 1$$

$$I_3 = Birr\ 242$$

Therefore, the interest for 3 years is $I = I_1 + I_2 + I_3$

$$I = 200 + 220 + 242 = Birr\ 662$$

b. Amount =?

$A = P + I$

$A = 2000 + 662 = Birr\ 2662$

Note:

For compound interest amount can be calculated by the formula

$$A = P(1 + R)^T$$

Where, $A = amount, R = interest\ rate, P = principal\ and$

$$T = time$$

And

$$I = A - P$$

Using the formula, the above example is done as follows

$P = 2000, R = 10\%, T = 3\ years, A = ?, I = ?$

$$A = P(1 + R)^T$$

$$A = 2000(1 + 10\%)^3$$

$$A = 2000(1 + 0.1)^3 = 2000 \times 1.1^3$$

$$A = 2000 \times 1.331$$

$$A = Birr\ 2662$$

And interest, $I = A - P = 2662 - 2000 = Birr\ 662$

Example 2: what sum of money amounts to Birr 24,200 in 2 years at 10% per annum, compounded annually?

Solution: $P = ?, A = 24{,}200, R = 10\% = 0.1, T = 2$

$$A = P(1+R)^T$$

$$24{,}200 = P(1+0.1)^2$$

$$24{,}200 = P \times 1.21, P = \frac{24200}{1.21} = Birr\ 20{,}000$$

Exercise 3.3.4

1. Find the compound interest on Birr 8000 for 2 years at 5% per annum, compounded annually.
2. Compare the simple interest and compound interest on Birr 8,000 at 10 % per annum for three years if the interest is compounded annually.
3. Find the difference between the simple and the compound interest on Birr 5000 for 2 years at 6% per annum.
4. Robel obtained a loan of Birr 250,000 from the commercial bank of Ethiopia. If the rate of interest is 8% per annum compounded annually and repaid Birr 50, 000 at the end of first year. What amount will he has to pay to the bank after 2 years to discharge his debt?
5. What sum of money will amount to Birr 21296 in 3 years at 10% per annum, compounded annually?

3.3.4 Ethiopian Income Tax, Turn over Tax, VAT

Activity 3.3.5

Discuss with your friends

1. Why do governments collect tax?
2. List out the different types of tax.

Taxes are imposed by governments on their citizens to generate income for undertaking projects to boost the economy of the country.

Tax raises the revenue of the government to cover cost of

- Administration
- Defense
- Education
- Housing
- Health etc.

We should pay tax!

To boost our country's economy

Taxes help to raise the standard of living in a country.

In this section we will try to see three different types of tax, VAT, turn over tax and income tax.

Value added tax (VAT)

Value added tax (VAT) is a tax imposed by government on sales of some goods and services.

Note:

1. In Ethiopia VAT rate is 15%
2. VAT amount $= 15\%\ of\ original\ cost$
3. Cost including VAT$=$ $original\ cost + VAT\ amount$

Example 1: the price of a machine is Birr 3000 before VAT.

a. Calculate the amount of VAT

b. Calculate the total cost of machine including VAT

Solution:

a. $VAT = 15\%\ of\ original\ cost$

$= \frac{15}{100} \times 3000$

$= Birr\ 450$

b. $Total\ cost\ of\ machine = 3000 + 450$

$= Birr\ 3450$

Example 2: The cost of TV stand including VAT is Birr 2300.

a. Calculate the cost of TV stand before VAT
b. Calculate the amount of VAT

Solution: let the cost before VAT is x

a. cost including VAT= $cost\ before\ VAT + VAT\ amount$

$2300 = x + 15\%\ of\ x$

$2300 = x + 0.15x$

$2300 = 1.15x$

$x = \frac{2300}{1.15} = Birr\ 2000$

Note:

$$cost\ before\ VAT = \frac{Cost\ \ including\ VAT}{1.15}$$

Therefore, the cost before VAT is Birr 2000

b. VAT amount $= 15\%\ of\ original\ cost$

$= \frac{15}{100} \times 2000$

$= 300\ Birr$

Turn over Tax (TOT)

Turn over tax is imposed on merchants, who are not required to register for VAT, but supply goods and services in the country.

In Ethiopia turn over tax rate is 2% on goods sold and services rendered locally.

Note:

1. A merchant his annual income below Birr 500,000 will be registered to collect turn over tax.
2. A merchant whose annual income above Birr 500,000 will be registered to collect turn over tax

Example 1: calculate turn over tax on sales of Birr 10,000

Solution: Turn over tax $= 2\%\ of\ 10{,}000$

$$= \frac{2}{100} \times 10{,}000$$

$$= Birr\ 200$$

Example 2: Elsa stationery has daily sales of Birr 205. The stationery is open 250 days per year. How much is the turn over tax payable by Elsa?

Solution: sales per year $= 205 \times 250 = Birr\ 51{,}250$

Turn over tax $= 2\%\ of\ sales$

$$= \frac{2}{100} \times 51{,}250$$

$$= Birr\ 1{,}025$$

Employment income tax

Employer deduct income tax from the employee before paying monthly salary based on the following tax rate (according to Ethiopian income tax rate)

Employment income (per month) in Birr	Employment income Tax rate
$0-600$	0%
$601-1650$	10%
$1651-3200$	15%
$3201-5250$	20%
$5251-7800$	25%
$7801-10900$	30%
$over\ 10{,}900$	35%

Note: the above table shows the tax rate for each interval of total income

Example1: the income tax for Birr 850 is calculated as follows

Solution: Birr 850 falls on 10%

- For interval $0\ to\ 600$, the tax rate is 0%, hence $Tax_{\,1} = 0$
- For interval $601\ to\ 850$, the tax rate is 10%, hence tax on this interval is

  $Tax_2 = 10\%\ of\ (850-600)$

$Tax_2 = \frac{10}{100} \times 250$

$Tax_2 = 25$

Income tax $= Tax_{\,1} + Tax_2 = 0 + 25 = Birr\ 25$

Therefore, the income tax on Birr 850 is $Birr\ 25$

Note: for interval for 601 to 850, any value above 600 up to 850 is taken, so

The difference is $850-600$

It is not $850-601$

Example 2: Alemu earns Birr 4850 per month, calculate

a. The income tax he has to pay.
b. The net income after deducting income tax

Solution:

a. Birr 4850 falls on 20%

- For interval $0\ to\ 600$, the tax rate is 0%, hence $Tax_1 = 0$
- For interval $601\ to\ 1650$, the tax rate is 10%, hence tax on this interval is

  $Tax_2 = 10\%\ of\ (1650 - 600)$

  $$Tax_2 = \frac{10}{100} \times 1050 = Birr\ 105$$
- For interval $1651\ to\ 3200$, the tax rate is 15%, hence tax on this interval is

  $Tax_3 = 15\%\ of\ (3200 - 1650)$

  $$Tax_3 = \frac{15}{100} \times 1550 = Birr\ 232.5$$
- For interval $3201\ to\ 4850$, the tax rate is 20%, hence tax on this interval is

  $Tax_4 = 20\%\ of\ (4850 - 3200)$

  $$Tax_4 = \frac{20}{100} \times 1650 = Birr\ 330$$

Income tax $= Tax_1 + Tax_2 + Tax_3 + Tax_4$

$= 0 + 105 + 232.5 + 330$

$= Birr\ 667.5$

Therefore, the income tax on Birr 4850 is $Birr\ 667.5$

b. net income is $4850 - 667.5 = Birr\ 4182.5$

Example 3: Derive a formula to calculate income tax and on Birr x, in terms of x. Where x falls on the interval $1651\ to\ 3200$

Solution: Birr x falls on 15%

- For interval $0\ to\ 600$, the tax rate is 0%, hence $Tax_1 = 0$

- For interval $601\ to\ 1650$, the tax rate is 10%, hence tax on this interval is

  $Tax_2 = 10\%\ of\ (1650 - 600)$

$$Tax_2 = \frac{10}{100} \times 1050 = Birr\ 105$$

- For interval $1651\ to\ x$, the tax rate is 15%, hence tax on this interval is

  $Tax_3 = 15\%\ of\ (x - 1650)$

$$Tax_3 = 0.15x - 247.5$$

Therefore, the income tax

$$Tax = Tax_1 + Tax_2 + Tax_3 = 0 + 105 + 0.15x - 247.5$$

$$Tax = 0.15x - 142.5$$

Exercise 3.3.5

1. Find the income tax and net income of the following employees of commercial Bank of Ethiopia
   a. Ato Ahmed with monthly salary of Birr 7500
   b. w/ro Mekdes with monthly salary Birr 11600
2. A shoe dealer purchased shoe from a shoe company that worth Birr 8000. Find the amount he should pay to the company including VAT

Summary for unit 3

- The method of comparing two or more quantities of the same kind and in the same units by division is known as a ratio.
- Proportion is equality of two ratios.
- In proportion the product of means is equal to the product of extremes.

$$(i.e.\, if\ \frac{a}{b} = \frac{c}{d}\ , then\ a \times d = b \times c)\ .$$

- y is said to be directly proportional to x (written as $y \propto x$), if there is constant k such that $y = kx\ or\ k = \frac{y}{x}$.
- y is said to be inversely proportional to x (written as $y \propto \frac{1}{x}$), if there is constant k such that $y = \frac{k}{x}\ or\ k = y \times x$.
- The word percent means "for every hundred" or "per 100". We use the symbol % to denote percent.
- Base (B) is the number that represents 100% or total value of something.
- Percentage (p) is the number or the amount that represents a part of the whole.
- Rate(R) is the ratio of percentage to base, written as percent.
- Percentage, base and rate are related by the formula

$$P = R \times B, \qquad B = \frac{P}{R} \quad \text{and} \quad R = \frac{P}{B}$$

- Percent increase is calculated as:

$$percent\ increase = \frac{increased\ amount}{original\ quantity} \times 100\%$$

- Percent decrease is calculated as:

$$percent\ decrease = \frac{decreased\ amount}{original\ quantity} \times 100\%$$

- The Price at which an article is purchased, is called its cost price.
- The Price at which an article is sold, is called its selling price.
- When $S.P > C.P$, then there is profit, and $Profit = S.P - C.P$
- When $S.P < C.P$, then there is loss, and $loss = C.P - S.P$
- Profit or loss percent is given by the formula

$$Profit\ \% = \frac{Profit}{cost\ price} \times 100\% \text{ and } Loss\ \% = \frac{Loss}{cost\ price} \times 100\%$$

- The interest paid on original principal only during the whole interest period is called simple interest.

$$Interest = principal \times Rate \times Time \quad \text{Or } I = PRT$$

- Compound interest is the interest on a loan calculated based on the initial principal plus the accumulated interest from previous periods.

$$A = P(1+R)^T \ \ and\, I = A - P$$

- **Value added tax (VAT)** is a tax imposed by government on sales of some goods and services.
- In Ethiopia VAT rate is 15%.
- Turn over tax is imposed on merchant, who are not required to register for VAT, but supply goods and services in the country.
- In Ethiopia turn over tax rate is 2% on goods sold and services rendered locally.

Review exercise for unit 3

I. Write True for the correct statements and False for the incorrect statements.

1. Ratio can be defined as “for every hundred terms”.
2. $4:3x = 32:72\ if\ x = 3$

3. The ratio of 15 meters to 2000 centimeters is 3:4
4. $Loss\ \% = \frac{loss}{cost\ price} \times 100\%$
5. If 250 divided in the ratio 3:7, the smaller part is 25.
6. Proportion is the equality of two ratios.
7. If y is inversely proportional to x, then as x increases y also increases.
8. Profit is made when the selling price is less than the cost price

II. Choose the correct answer from the given alternatives.

9. In a school, 58% of the total number of students is boys. If the Number of girls is 840, how many students are there in the class?

   a. 2000 b. 1800 c. 1500 d. 2100

10. Birr 300 is invested at 6% simple interest per annum. How long will it Take for the interest to be Birr 180?

    a. 12 years b. 10 years c. 8 years d. 24 years

11. The decimal form of $\frac{3}{4}\%$ is

    a. 0.75 b. 0.075 c. 0.0075 d. 0.057

12. The simple interest on Birr 400 invested for 4 months was Birr 12. What is the annual (yearly) rate of interest?

    a. 36 b. 4.8 c. 7 d. 9

13. If a, b and c are numbers such that $a:b:c = 1:3:4$ and $c = 28$ then, find the sum of $a + b + c$?

    a. 56 b. 60 c. 72 d. 84

14. If $A:B = 7:8$ an $B:C = 12:7$, find A: C in its simplest form.

    a. 2:3 b. 3:2 c. 19:6 d. 4:5

15. Three numbers m, n, p are in the ratio $3:6:4$. Which of the following is the value of $\frac{4m-n}{n+2p}$

a. $\frac{7}{3}$ b. $\frac{3}{7}$ c. $\frac{2}{3}$ d. $\frac{4}{3}$

16. The ratio of the number of pigs to the number of horses in a farm is 2 to 3. If there are 24 horses, what is the number of pigs?

a. 72 b. 48 c. 36 d. 16

17. $24,\ x, 18\ and\ (x-1)$ are in proportion. Find the value of x.

a. 6 b. 4 c. 12 d. 8

18. If 85% of 'a' is 'b', then which of the following is true?

a. $a = b$ b. $a < b$ c. $20a\ = 17b$ d. $a > b$

19. which one of the following is not equal to the rest?

a. 2% of 150 b. $\frac{3}{2}$% of 400 c. 5% of 60 d. 6% of 50

20. Which one of the following greater?

a. 20% of 45 b. 25% of 60 c. 2% of 800 d. 74% of 20

III. Work out problems

21. Find $\frac{7}{12}$ of 8400?

22. There are 40 questions on a final exam and Yohannes answered 75% of them correctly. How many questions correctly answered?

23. If 'b' is directly proportional to 'a'and b=30 when a=20, then

a. Find the equation relating a and b

b. Find a when b=42

24. In a given class room, the number of girls is 10 greater than the number of boys. If the ratio of the number of girls to the number of boys is 7:5 then find

 a. The number of girls

 b. The number of boys

 c. The total number of students

25. A company has 420 workers of which 189 are males and the rest are females. What percent of the workers are female?

26. A trader bought a TV set for Birr 22000 and sold it at loss of 5%. What was the selling price?

27. Mussa received a 6% pay raise. He now earns Birr 8268, what was his salary prior to the salary increase?

28. Birr 6000 is invested at a rate of 5% compound interest compounded annually, Find a. the amount at the end of 2 years b. The interest at the end of 2 years

29. A person wants to buy a car from TOYOTA Company. If the price car including VAT is Birr 5,750,000 then,

 a. What is the price of car before VAT?

 b. What is the value of VAT?

30. Calculate the income tax on salary of Birr 5200.

Derive a formula to calculate income tax and on Birr x, in terms of x. Where x falls on the interval $5251\ to\ 7800$

Unit 4

Linear equations

Unit Outcomes:

At the end of this unit, you should be able to:

- Identify variables, terms and variables in algebraic expressions
- Simplify algebraic expressions
- Develop their skills on rearranging and solving linear equations.
- Apply the rules of transformation of linear equations for solving problems
- Draw a line through the origin whose equation is given.
- Apply real-life situations in solving linear equations

Introduction

In this unit you will learn the importance of variables in mathematics, apply variables in solving linear equations, like linear equations with one variable involving brackets and linear equation involving fractions.

The unit has four sections. The first section deals with the use of variables in formula and the second section deals with solving linear equation, the third section deals with Cartesian coordinate system and the last section deals about application of linear equations. Here the student will see how to use variables in formula and how to solve linear equation with one variable involving brackets and fractions.

4.1.Algebraic terms and expressions

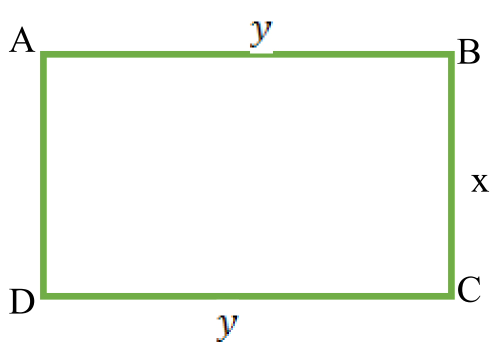
By the end of this section you should be able to:

- Describe algebraic terms and expressions
- Simplify algebraic expressions with and without brackets

4.1.1. Use of Variables in Formula

Activity 4.1.1

1. Identify the variables
   $23, x, 4, y, z, 0.5, 3x, \frac{3}{4}, a, 3b, \frac{1}{2}$
2. What is variable?
3. Express the following using variables.
   a. A number x plus 4.
   b. The difference 5 and y, where y is greater than 5.
   c. The product of 3 and x.
   d. I start with x, add 8 and then, double the result.
4. From the above figure
   a. what is the length of AB
   b. what is the length of BC
   c. Find the perimeter of the figure in terms of x and y.



Definition 4.1: a variable is any letter or a symbol that represent some unknown number or value. Such as x, y, z, n, m ...

Example1: In $x+2$, x is variable

Example2: Describe the following using variables

a. Twice a number.
b. Three times a number.
c. Two more than a number.
d. Half a number.
e. Two less than a number.
f. The product of a number and 10.
g. Product of ten and sum of two numbers.

Solution:

a. let the number is x, then twice the number $= 2x$
b. let the number is z, then three times a number $= 3z$
c. let the number y. then two more than a number $= y+2$
d. let the number is n, then half the number $= \frac{n}{2}$
e. let the number is m, then two less than a number $= m-2$
f. let the number is k, then product of a number and 10 $= 10k$
g. let the two numbers are x and y ,then product of ten and sum of two numbers $= 10(x+y)$

Example 3: If you are 13 years old now. How old will/were you

a. after x years ? b. after z years? c. y years ago?

Solution:

a. $13 + x$

b. $13 + z$

c. $13 - y$

Exercise 4.1.1

1. Identify the variables in each of the following.

   a. $2 + 4x$ c. $2y + b$

   b. $5x + x$ d. $\frac{x}{2} - p$

2. Describe the following using variables

   a. Four times a number.
   b. One third of a number.
   c. Ten more than a number.
   d. The sum of two different numbers.
   e. The product of a number and 20.
   f. Five less than a number.
   g. Ten more than twice a number.
   h. Six less than twice a number.

3. Express the following in words.

   a. $x - 1$ c. $3 + 4x$

   b. $7x$ d. $\frac{x}{3} - 1$

4. Find the value of y in the relation $y = 2x + 3$ when

   a. $x = 1$ b. $x = 5$ c. $x = -4$ d. $x = \frac{2}{3}$

4.1.2 Variables, Terms and Expressions

Activity 4.1.2

1. Which of the following are terms and which are not terms?

   a. $x + y$ b. $2x + 3$ c. $2x$ d. $-3x$ e. $\frac{3}{y}$ f. 4

2. Find the numerical coefficient of the following terms.

   a. $-3x$ b. y c. $-xy^2$ d. $\frac{xy}{4}$

3. Identify whether each pair of the following are like terms or unlike terms?

   a. $5x^2 and\ 3y^2$ b. $3x\ and\ x$ c. $2xy\ and\ yx$ d. $8x^2y\ and\ 4xy^2$

Definition 4.2: a constant (a number), a variable, product or quotient of a number and variable is called a term.

Example:

- 4 is a term
- xy is a term
- $12xy - 4$ is not a term
- $\frac{3x}{y}$ is a term
- $x - y$ is not a term

Definition 4.3: In the product of a number and variable, the numerical factor of the term is called numerical coefficient.

Example:

a. The numerical coefficient of 6ab is 6
b. The numerical coefficient of -3xy is -3
c. The numerical coefficient of x is 1.
d. The numerical coefficient of $-\frac{2ab}{3}$ is $-\frac{2}{3}$

Definition 4.4: like terms are terms whose variable and exponent of variable are exactly the same but they may differ in their numerical coefficients. Terms which are not like terms are called unlike terms.

A term which do not contain a variable is called constant term. All constant terms are like terms.

Example:

- 2 and -5 are like terms, because they are constant terms.
- $\frac{3}{5}a^2b^2$ and $-\frac{2}{3}b^2a^2$ are like terms.
- $-20abc$ and acb are like terms
- $3xyz$ and $x^2y^2z^2$ are unlike terms
- $-2a^3b^3c^3$ and $-2a^3b^2c^3$ are unlike terms
- $3xy$ and $3yx$ are like terms

Exercise 4.1.2

1. Identify whether each pair of the following are like or unlike terms.
   a. 3x and -5x
   b. 20xy and ab
   c. -80ab and $15a^2b^2$
   d. 5ab and $\frac{1}{3}$ab
   e. xy^2 and y^2x
   f. $3x^2$ and $2x^3$
   g. xyz^2 and z^2xy
   h. $\frac{3}{x}$ and $\frac{5}{x}$
2. Which of the following are terms and which are not terms?
   a. $5x - 4y$ b. $8x^2$ c. $\frac{2x-y}{5}$ d. $-9abc$ e. $\frac{8y}{x}$ f. 0
3. In each of the following expressions, determine the numerical coefficient.
   a. $2bm$
   b. $5yx^2$
   c. $-4x$
   d. $\frac{3x}{4}$
   e. $\frac{y}{2}$
   f. $-z$

Definition 4.5: Algebraic expressions are formed by using numbers, letters and the operation of addition, subtraction, multiplication and division.

Example: $12, 2x-3, a+10, 3x, \frac{3}{4}a, 3x, 3x+2a-5,$ etc. are algebraic expressions.

Note: The terms of an algebraic expression are parts of the expression that are connected by plus or minus signs.

Example: In the expression $3x+2a-5$, the terms are $3x, 2a$ and -5. and the constant term is -5.

Definition 4.6: an algebraic expression in algebra which contains one term is called a monomial.

Example: $5x, 12ab, 3xyz, 0.2ab$ ... *are monomial.*

Definition 4.7: an algebraic expression in algebra which contains two terms is called a binomial.

Example: $5x+3y, 12-ab, 3x+yz, x-b$ ... *are binomial.*

Definition 4.8: an algebraic expression in algebra which contains three terms is called a trinomial.

Example: $2x+3y-3, 12+a-b, x-y-z, ab+6+z$ *are trinomial.*

Simplifying Algebraic Expressions

To simplify any algebraic expression, follow the following basic steps

- Remove brackets using distributive property.
- Collecting like terms and
- Add or subtract like terms

Note:

1. When you add or subtract like terms, add or subtract their numerical coefficients.

   Example: $-3x + 5x = (-3 + 5)x = 2x$
2. Unlike terms cannot be added or subtracted

   Example: $5x + 2y$ cannot be added.
3. When you remove brackets using distributive property be careful of signs.

   Example:

   a. $-2(x - 4) = -2x - (-2) \times 4 = -2x + 8$

   b. $-3(-2x - 1) = -3(-2x) - (-3)(1) = 6x + 3$
4. When you multiply a number with variable, you can write the number as coefficient without multiplication sign

   Example:

   a. $4 \times y = 4y$

   b. $5 \times (3y) = 15y$

Example: Simplify the following algebraic expression

a. $3x + 2y - 4x + 5y$

b. $3(x + 4) - 5(x - 2)$

c. $20 + 4(x + 3y) - 4x - 8y$

d. $4(a + 2b) - 3(2a - b) + 6a - 7b$

e. $3(2a + 3) - 2(a + 8)$

Solution:

$a(b + c) = ab + ac$

a. $3x + 2y - 4x + 5y$

$= 3x - 4x + 2y + 5y$ …… collecting like terms

$= -x + 7y$ …… adding/subtracting coefficient of like terms

b. $3(x + 4) - 5(x - 2)$

$= 3x + 12 - 5x + 10$ ……………………… distributive

$= 3x - 5x + 12 + 10$ ………… collecting like terms

$= (3 - 5)x + 22$ ……………… combine constants

$= -2x + 22$ ……………………………simplify

c. $20 + 4(x + 3y) - 4x - 8y$

$= 20 + 4x + 12y - 4x - 8y$ ....distributive

$= 4x - 4x + 12y - 8y + 20$ …………………… collecting like terms

$= (4 - 4)x + (12 - 8)y + 20$ ………………… combine constants

$= 4y + 20$ ……………………………………… ……… simplify

d. $4(a + 2b) - 3(2a - b) + 6a - 7b$

$= 4a + 8b - 6a + 3b + 6a - 7b$ ……………… distributive

$= 4a - 6a + 6a + 8b + 3b - 7b$ ……………… collecting like terms

$= (4 - 6 + 6)a + (8 + 3 - 7)b$ ………………combine constants

$= 4a + 4b$ ……………………………………… Simplify

e. $3(2a + 3) - 2(a + 8)$

$= 6a + 9 - 2a - 16$ …………………………… distributive

$= 6a - 2a + 9 - 16$ ……………………………collecting like terms

$= (6 - 2)a + (9 - 16)$ …………………………*combine constants*

$= 4a - 7$ …………………………………… simplify

Exercise 4.1.3

1. Categorize the following expressions as monomial, binomial or trinomial and determine the constant terms in each expression.

a. 24	d. $5x - 3y - 4$	g. $z - y - 25x$
b. $5x$	e. $20w + w^2$	h. y
c. $-21t - 2x$	f. $3x - 2y + 1$	i. $2b - 4a$

2. Simplify the following expressions by collecting like terms

a. $3x + 35y - 8x$	f. $xy + ab - cd + 2xy - ab + dc$
b. $15 + 21b - 32b + 3$	g. $3x^2 + 4x + 6 - x^2 - 3x - 3$
c. $3(3x - 6y) + x + 18y - 4$	h. $5 + 2y + 3y^2 - 8y - 6 + 2y^2$
d. $3z - 12 - 11y + x + y - 48z - 3$	i. $2x^2 - 3x + 8 + x^2 + 4x + 4$
e. $-3(-a + 6) + x - (z + 4) - 2x$	j. $5(2 - x^2) - 3(x^2 - 4y)$

4.2. Solving linear equations

By the end of this section you should be able to:

- Define a linear equation
- Solve linear equations involving brackets
- Solve linear equations involving fractions

4.2.1 Linear Equations Involving Brackets

Activity 4.2.1

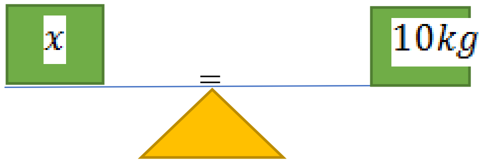
Discuss with your friends

1. Connect the following a pair of expressions by using '=' sign

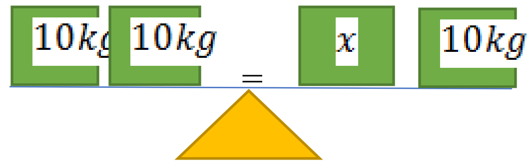
 a. $2x$ *and* 2 b. $3x - 1$ *and* 6 d. $y + 4$ *and* $5 - y$

2. Using the following balance, find the unknown variables.

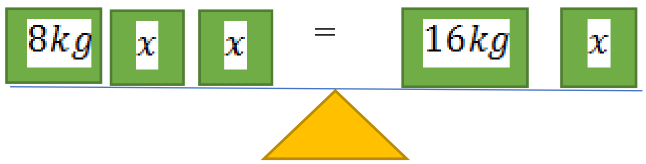
a.



c.



b.



3. What replacement of x will make the following equality is true?

   a. $x - 1 = 1$ c. $4x = 8$

   b. $x + 13 = 0$ d. $2x = 0$

4. Determine whether each the following are equations or not.

   a. $2 + x$ c. $x > 2$

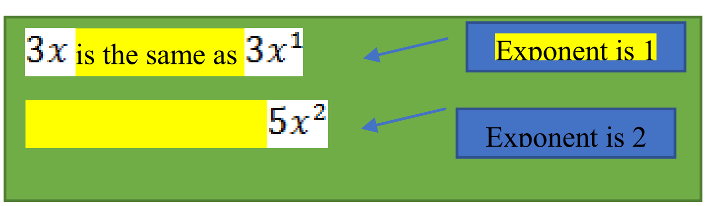
   b. $x = 2$ d. $2x - 1 = 0$

5. What is equation?

Definition 4.9: Two different algebraic expressions connected by equal $(=)$ signis called equation.

Definition 4.10: A Linear equation in one variable x is an equation which can be written in standard form$ax + b = 0$, where a and b are constant numbers with a $\neq 0$

Note: an equation of a single variable in which the highest exponent of the variable is one is a linear equation.



Example: which of the following equations are linear and which are not linear.

a. $2x = 2$ b. $x + 3 = 2x - 1$ c. $2x^2 + 3 = 2x - 1$ d. $x = 2x^3 - 6$

Solution:

a and b are linear equation, because the highest exponent of variable is one.

c and d are not linear equation, because the highest exponent of the variable is not one.

Note: All equations have two sides with respect to the equal sign. Left hand side and (L.H.S) and right-hand side (R.H.S)

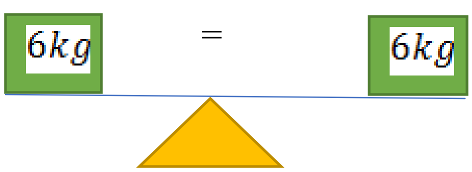
Example: In the equation $3x + 5 = x - 1$

L.H.S is $3x + 5$ and R.H.S is $x - 1$

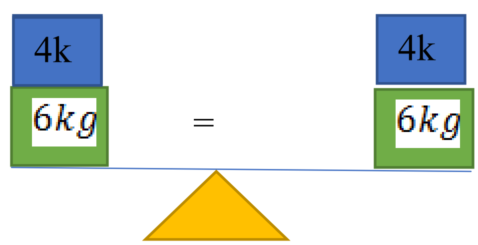
Rules of transforming equations to simpler form

Consider the following balances,

a.



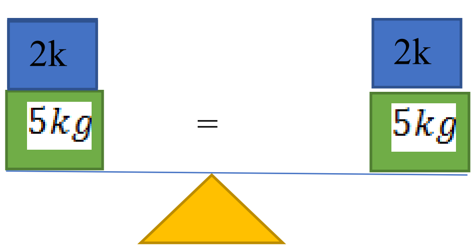
add 4 kg to both sides



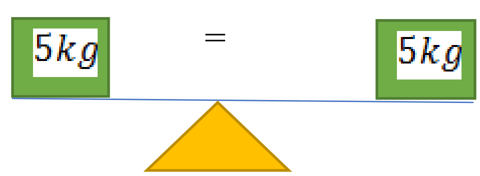
Rule 1: adding the same number to both sides of equation will not affect the equation.

i.e. $if\ a = b, then\ a + c = b + c, where\ a, b, and\ c\ are\ any\ constant\ number$

b.



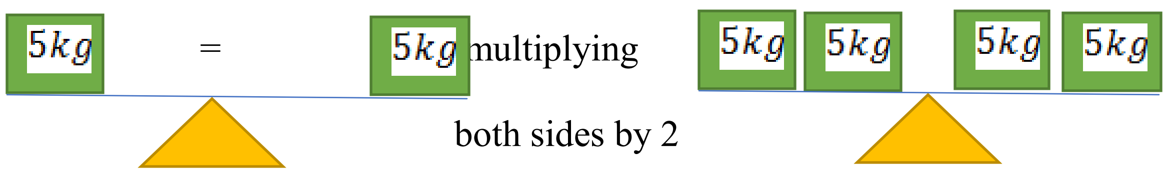
subtracting 2kg from both sides



Rule 2: subtracting the same number to both sides of equation will not affect the equation.

i.e. $if\ a = b, then\ a - c = b - c, where\ a, b, c\ any\ constant\ number$

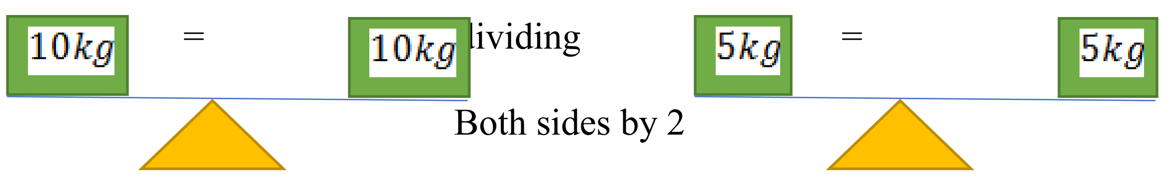
c.



Rule 3: multiplying both sides of equation with the same none zero number will not affect the equation.

$If\ a = b, then\ a \times c = b \times c, where\ a, b, c\ any\ constant\ number\ and\ c \neq 0$

d.



Rule 4: dividing both sides of equation with the same none zero number will not affect the equation.

$If\ a = b, then\ \frac{a}{c} = \frac{b}{c}, where\ a, b, c\ any\ constant\ number\ and\ c \neq 0$

Example: Find the value of x by using rules of transformation

a. $x - 10 = 6$ b. $10 - x = 6$ c. $x + 4 = -6$

Solution:

a. $x - \cancel{10} = 6$ adding 10 in both sides

$\underline{\cancel{+10} \quad +10}$

$x = 6 + 10$

$\underline{\underline{x = 16}}$

c. $x + \cancel{4} = -6$ subtracting 4

$\underline{\cancel{-4} \quad -4}$

$x = -6 - 4$

$\underline{\underline{x = -10}}$

b. $\cancel{10} - x = 6$

$\underline{\cancel{-10} \qquad -10}$ Subtracting 10

$-x = 6 - 10$

$-x = -4$

$\underline{\underline{x = 4}}$

Sum of opposite numbers is zero

$-4 + 4 = 0$

$10 + (-10) = 0$

Example: Find the value of y by using rules of transformation

a. $8y = 72$

b. $\frac{y}{5} = -2$

Solution:

a. $8y = 72$, to find the value of y divide both sides by 8

$$\frac{\cancel{8}y}{\cancel{8}} = \frac{72}{8}$$

$\underline{\underline{y = 9}}$

b. $\frac{y}{5} = -2$, to find the value of y multiply both sides by 5

$$\cancel{5} \times \frac{y}{\cancel{5}} = -2 \times 5$$

$\underline{\underline{y = -10}}$

Exercise 4.2.1

1. Decide each of the following either linear equation or not.
   a. $2y - 2$ c. $2x^2=4$ e. $3x = 0$
   b. $x - 1 = 5$ d. $x - 3 + 4x = 2x - 2$ f. $x > 0$
1. By what transformation was the second equation obtained from the first?
   a. $x + 1 = 2$ b. $-8x = 12$ c. $2x - 4 = 6$ d. $\frac{2x}{3} = 5$
   $x = 1$ $x = -1.5$ $2x = 10$ $2x = 15$
2. Find the value of unknown variable by using rules of transformation
   a. $-8x = 12$ b. $\frac{2x}{5} = 10$ c. $20 - x = 15$ d. $2x - 4 = 16$

Solving equations containing variable on both sides of the equation

Consider the equation$x - 5 = 10$, in solving this equation you add 5 on both sides and get $x = 10 + 5$, this is the same as shifting -5 to the other side of the equation and it becomes $+5$. i.e.

$x - 5 = 10 \quad +5$, so $x = 10 + 5 = 15$

Shifting -5 to the other side, becomes $+5$

Note: when you shift a variable (term) or number from one side of the equation to the other side its sign changes.

For example, in the equation$3x - 4 = 2x - 1$, shifting $2x$ to the left side you get

$-2x$

$3x - 4 = +2x - 1$ So, it becomes $3x - 2x - 4 = -1$

In solving equation containing variable on both sides, collect the terms containing variables on one side and numbers on the other side and do not forget changing the signs when you shift a term from one side to the other.

Example 3: solve the following equations and check the result

a. $2x = x + 4$
b. $2x - 8 = 4$
c. $5x - 10 = 6 - 3x$
d. $x + 5 - 4x = 3 + 2x - 6 - 4x$

Solution:

a. $2x = x + 4$

$2x - x = 4$ $(shifting\ \ x\, to\, the\, left\, it\, becomes\ -x)$

$\underline{\underline{x = 4}}$

Check: $2x = x + 4$, substitute 4 in the place of x

$2 \times 4 = 4 + 4$

$8 = 8$, (True)

Hence, $x = 4$ is the solution

b. $2x - 8 = 4$

$2x = 4 + 8$ (Shifting -8 to the right becomes$+8$)

$2x = 12$

$\frac{2x}{2} = \frac{12}{2}$ $(\,divide\ both\ sides\ by\ 2)$

$\underline{\underline{x = 6}}$

Check: $2x - 8 = 4,$ substitute 4 in place of x

$(2 \times 6) - 8 = 4$

$12 - 8 = 4$

$$4 = 4\ (True)$$

Hence, $x = 6$ is the solution

c. $5x - 10 = 6 - 3x$

$5x + 3x = 6 + 10$

$(Transform\ the\ variable\ to\ the\ left\ side\ \&\ the\ constant\ to\ the\ right\ )$

$8x = 16$

$\frac{8x}{8} = \frac{16}{8}\ (divide\ both\ sides\ by\ 8)$

$\underline{\underline{x = 2}}$

Check: $5x - 10 = 6 - 3x$, substitute 2 in place of x

$$(5 \times 2) - 10 = 6 - (3 \times 2)$$

$$0 = 0\ (True)$$

$Hence, x = 2\ is\ the\ solution$

d. $x + 5 - 4x = 3 + 2x - 6 - 4x$

$x - 4x + 5 = 3 - 6 + 2x - 4$

(Collect and simplify like terms on the same side)

$-3x + 5 = -3 - 2x$

$-3x + 2x = -3 - 5$

$-x = -8$

$\frac{-x}{-1} = \frac{-8}{-1}\ (divide\ both\ sides\ by - 1)$

$\underline{\underline{x = 8}}$

Check: $x + 5 - 4x = 3 + 2x - 6 - 4x$

$8 + 5 - (4 \times 8) = 3 + (2 \times 8) - 6 - (4 \times 8)$

$13 - 32 = 3 + 16 - 6 - 32$

$$-19 = -19(True), Hence, x = 8 \text{ is the solution}$$

Exercise 4.2.2

1. solve the following equations and check the result

a. $3x - 9 = 4x + 5$

b. $5x - 3 - 4x = 13$

c. $12x + 7 = 5 - 3x + 17$

d. $-x + 3 = x + 9$

e. $3x - 9 = -x + 19$

f. $8x + 5 = 2x + 17$

g. $10 = 3x - 5 + 2x$

h. $x - 5 = 2x + 13 + x$

Solving Linear equations involving brackets

Activity 4.2.2

Discuss with your friends

1. Simplify the following by using distributive property

a. $3(x + 2)$

b. $-(-2x + 9)$

c. $4(x - 1) + 2(1 - 2x)$

d. $4(x - 1) - 2(x - 9)$

2. Solve the following linear equations by using distributive property.

a. $3(x + 2) = 0$

$b. -(-2x + 9) = 1$

c. $4(x - 1) = 2(1 - 2x)$

d. $4(x - 1) = 2(x - 9)$

To solve linear equations involving brackets

1. Remove the brackets using distributive property
2. Collect the terms containing variables on one side and constants on the other side, do not forget to change sign while shifting the terms.

Note: For any numbers a, b and c

1. $a + (b + C) = a + b + c$
2. $a - (b + C) = a - b - c$
3. $a - (b - C) = a - b + c$
4. $a(b + c) = ab + ac$
5. $a(b - c) = ab - ac$

Example 1: Remove the bracket and simplify the following expressions

a. $3 - (2x - 5)$

b. $2y + (4 - y)$

c. $3m - 2(m - 3)$

d. $-5n + 3(4 - 2n)$

Solution:

a. $3 - (2x - 5) = 3 - 2x + 5$

$= 3 + 5 - 2x$

$= 8 - 2x$

$= -2x + 8$

Note:

1. $a - b = -b + a$
2. $8 - 2x = -2x + 8$

b. $2y + (4 - y) = 2y + 4 - y$

$= 2y - y + 4$

$= y + 4$

c. $3m - 2(m - 3) = 3m - 2m + 6$

$= m + 6$

d. $-5n + 3(4 - 2n) = -5n + 12 - 6n$

$= -5n - 6n + 12$

$= -11n + 12$

Example 2: solve each of the following equations.

a. $4(2x + 3) = 36$

c. $-10y - (-2y + 3) = 21$

b. $6(5x-7)=4(3x+7)$ d. $4(x-1)+3(x+2)=5(x-4)$

Solution:

a. $4(2x+3)=36$

$4\times 2x+4\times 3=36$ (Removing the bracket)

$8x+12=36$

$8x=36-12$ (Shifting +12 in to right side, becomes -12)

$\frac{8x}{8}=\frac{24}{8}$ (*Divide both sides by* 8)

$\underline{\underline{x=3}}$

b. $6(5x-7)=4(3x+7)$

$30x-42=12x+28$ (*Removing brackets*)

$30x-12x=28+42$ (Shifting variables to the left and constants to the right)

$\frac{18x}{18}=\frac{70}{18}$ (Divide both sides by 18)

$x=\frac{35}{9}$

c. $-10y-(-2y+3)=21$

$-10y+2y-3=21$ (Removing the brackets)

$-8y-3=21$

$-8y=21+3$

$\frac{-8y}{-8}=\frac{24}{-8}$ (Divide both sides by -8)

$\underline{\underline{y=-3}}$

d. $4(x-1)+3(x+2)=5(x-4)$

$4x-4+3x+6=5x-20$ (Removing the brackets)

$$4x + 3x - 4 + 6 = 5x - 20 \text{ (Collecting like terms in one side)}$$

$$7x + 2 = 5x - 20 \text{ (Simplify in one side)}$$

$$7x - 5x = -20 - 2$$

$$\frac{2x}{2} = \frac{-22}{2} \text{ (Divide both sides by 2)}$$

$$\underline{\underline{x = -11}}$$

Exercise 4.2.3

1. Solve each of the following equations
   a. $(x - 1) = 11$
   b. $6(8y + 3) = 4(7y + 5)$
   c. $8(2y - 6) = 5(3y - 7)$
   d. $3 - 2(2x + 1) = x - 14$
   e. $7 - (x + 1) = 9 - (2x - 1)$
   f. $5(1 - 2a) - 3(4 - 4a) = 0$
   g. $3x + 7 + 3(x - 1) = 2(2x + 6)$
   h. $5(2x + 1) + 3(3x - 4) = 4(5x - 6)$
2. Solve for x in terms of m and n for $m, n \neq 0$
   a. $m(x - 1) = 0$
   b. $m(x + n) = mn$
   c. $n(x - 2) = m + x \ , n \neq 1$
   d. $x(n - 1) = 4 - x$

4.2.1 Solving linear equation involving fractions

Activity 4.2.3

1. Find the LCM of the following numbers
   a. $2 \text{ and } 3$
   b. 4 and 6
   c. $3, 4 \text{ and } 6$
   d. $4 \text{ and } 8$
2. Multiply both side of the equation by the LCM of the denominator, simplify and Solve the equation
   a. $\frac{x}{2} = \frac{5}{2}$
   c. $\frac{x}{3} + 5 = 7$

b. $\frac{3}{4}=\frac{x}{8}$ d. $\frac{3}{4}+\frac{x}{2}=\frac{x}{3}$

Note:

To solve linear equations involving fractions, follow the following steps:

1. Find the list common multiple (LCM)of denominators
2. Multiply both sides by LCM
3. Solving linear equation by shifting the terms containing variable to one side and constants to the other side.

Example: Solve the following linear equations

a. $\frac{x+2}{4}=\frac{5}{3}$ c. $\frac{x+1}{3}+\frac{x-1}{10}=12$

b. $\frac{x}{8}+\frac{7}{4}=\frac{5}{6}$ d. $\frac{2x+3}{3}+\frac{x-1}{2}+\frac{x+1}{4}=8$

3. **Solution:** a. $\frac{x+2}{4}=\frac{5}{3}$

The LCM of 4 and 3 is 12, therefore multiply both side by 12

$12(\frac{x+2}{4})=12(\frac{5}{3})$

$3(x+2)=4(5)$ ……………..simplifying

$3x+6=20$ ………………..removing brackets

$3x=20-6$ …………………. collecting like terms

$3x=14$ ……………………..simplifying

$\frac{3x}{3}=\frac{14}{3}$…………………… dividing both side by 3

$x=\frac{14}{3}$

b. $\frac{x}{8}+\frac{7}{4}=\frac{5}{6}$

The LCM of 8, 4 and 6 of is 24

$24(\frac{x}{8}+\frac{7}{4})=24(\frac{5}{6})$………………… multiplying both sides by 24

$24\left(\frac{x}{8}\right)+24(\frac{7}{4})=24\times\frac{5}{6}$……… distributive property

$3x+42=20$ ………………simplifying

$3x=-42$ …………………collecting like term

$3x=-22$ ………………… simplifying

$\frac{3x}{3}=\frac{-22}{3}$…………………… dividing both side by 3

$x=\frac{-22}{3}$

c. $\frac{x+1}{3}+\frac{x-1}{10}=12$

The LCM of 3 and 10 is 30, therefore multiplying both sides by 30

$30(\frac{x+1}{3}+\frac{x-1}{10})=30(12)$

$30\left(\frac{x+1}{3}\right)+30(\frac{x-1}{10})=30\times 12$……… distributive property

$10(x+1)+3(x-1)=360$ ………………simplifying

$10x+10+3x-3=236$ …………………removing brackets

$10x+3x=360-10+3$ ………………… collecting like terms

$13x=353$ ……………………simplifying

$\frac{13x}{13} = \frac{353}{13}$........................ dividing both side by 13

$x = \frac{353}{13}$

Then the solution is $x = \frac{353}{13}$

d. $\frac{2x+3}{3} + \frac{x-1}{2} + \frac{x+1}{4} = 8$

The LCM of denominator is 12

$12(\frac{2x+3}{3} + \frac{x-1}{2} + \frac{x+1}{4}) = 12 \times 8$ multiplying both sides by 12

$12(\frac{2x+3}{3}) + 12(\frac{x-1}{2}) + 12(\frac{x+1}{4}) = 12 \times 8$ ....distributive property

$4(2x+3) + 6(x-1) + 3(x+1) = 96$ ....simplifying

$8x + 12 + 6x - 6 + 3x + 3 = 96$ ....removing brackets

$8x + 6x + 3x = 96 - 12 + 6 - 3$.........collecting like terms

$17x = 87$ ....simplifying

$\frac{17x}{17} = \frac{87}{17}$........................ dividing both side by 19

$x = \frac{87}{17}$

Definition 4.11: Equivalent equations are two or more equations that have the same solution.

Example 1: Show that $2x - 1 = 11$ and $x + 12 = 3x$ are equivalent equations.

Solution:

$2x - 1 = 11$ and $x + 12 = 3x$

$2x = 11 + 1$ and $x - 3x = -12$

$2x = 12$ and $-2x = -12$

$x = 6$ and $x = 6$

Hence, $2x - 1 = 11$ and $x + 12 = 3x$ are equivalent equations.

Example 2: show that $2(x-5) = x + 6\ \ and\ 2x - 4 = 10$ are not equivalent

Solution:

$2(x-5) = x + 6\ \ and\ 2x - 4 = 10$

$2x - 10 = x + 6\ \ and\ 2x - 4 = 10$

$2x - x = 6 + 10\ \ and\ 2x = 10 + 4$

$x = 16\ and\ x = 7$

Therefore, $2(x-5) = x + 6\ \ and\ 2x - 4 = 10$ do not have the same solution, so they are not equivalent.

Exercise 4.2.4

1. Solve the following
   a. $\frac{5x}{13} + \frac{5x}{26} = 1$
   b. $\frac{3x}{7} + \frac{35x}{8} = 10$
   c. $\frac{2x}{5} - \frac{2}{3} = \frac{x}{2} + 6$
   d. $\frac{-3}{5} + \frac{x}{10} = \frac{-1}{5} - \frac{x}{5}$
   e. $\frac{9w}{7} - 6 = \frac{5w}{7} + 12$
   f. $\frac{7}{10}\mathrm{x} + \frac{3}{2} = \frac{3}{5}\mathrm{x} + 2$

g. $\frac{1}{2}(4x + 6) = \frac{1}{3}(9x - 24)$

h. $\frac{n+1}{2} + \frac{n+2}{3} + \frac{n+4}{4} = 3$

i. $\frac{1}{3}(x + 6) - \frac{1}{2}(3x - 4) = 5$

2. Which of the following pairs of equations are equivalent?

a. $x = 0$ and $2x - 1 = -1$

b. $2x - 6 = 2$ and $-x = -5$

c. $5x = 10$ and $14 = 2x + 4$

d. $x = 0$ and $324x + 55 = 55$

4.3.Cartesian coordinate system

By the end of these sections you should be able to:

- Describe the Cartesian coordinate system
- Draw the four quadrants of the Cartesian plane and mark the origin, x $- axis$ and $y -$ $axis$
- Plot points on the Cartesian coordinate plane given their coordinates
- Draw graph of linear equations like $y = mx$, in a Cartesian coordinate plane using table values manually and computer applications.

4.3.1 The Four Quadrants of the Cartesian coordinate Plane

Activity 4.3.1

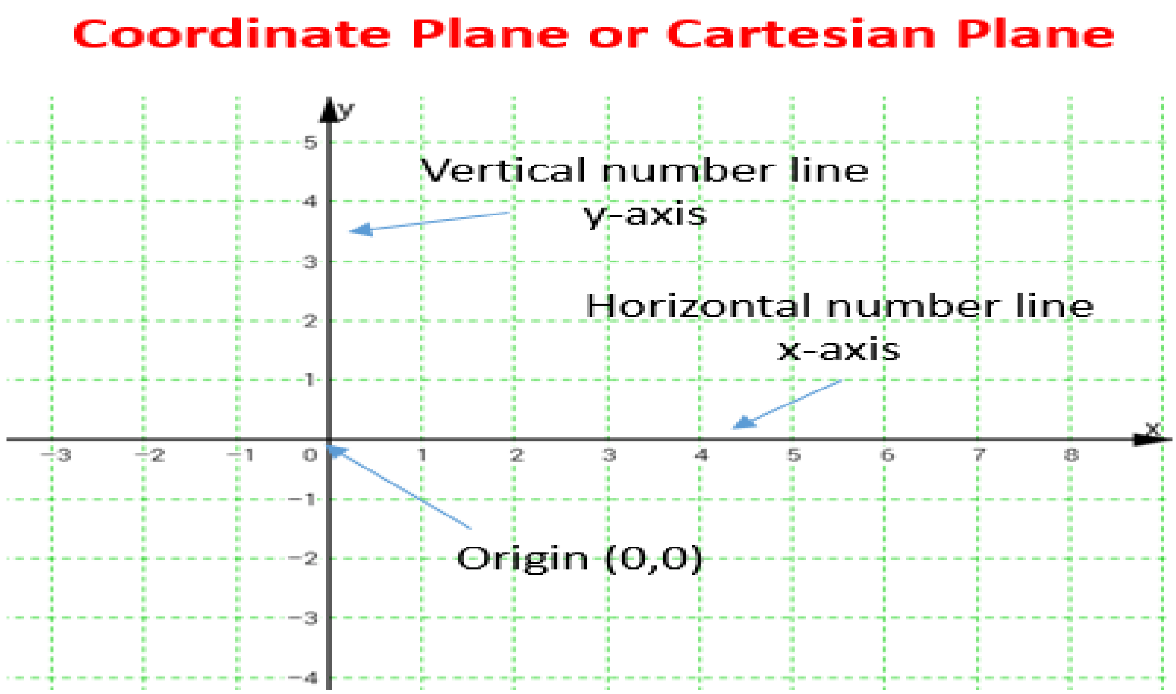
Discuss with your friends

1. Draw horizontal number line and mark numbers.
2. Draw vertical number line perpendicular to horizontal number line that pass through zero.
3. Mark the numbers above and below zero on the vertical line.

4. The horizontal number line is called ______.
5. The vertical number line is called________.
6. The two intersecting number lines (vertical and horizontal) set up a plane is called__________________.

Definition 4.12: The two perpendicularly intersecting horizontal and vertical number lines together set up a plane is called Cartesian coordinate plane.

- The horizontal number line is called x-axis.
- The vertical number line is called y-axis.
- The point where, x-axis and the y-axis intersect is called origin.



Locating points on the coordinate plane

- Points on Cartesian plane are described by two numbers (a, b) that are called **Coordinates.**
- The first number$a$, is the horizontal position of the point from the origin. It is called $x - coordinate$ (abscissa).
- The second number b is the vertical position of the point from the origin. It is called $y - coordinate$ (ordinate).

To locate a point (a, b) on a coordinate plane follow the following steps.

Step 1: Move a unit in the horizontal direction from the origin.

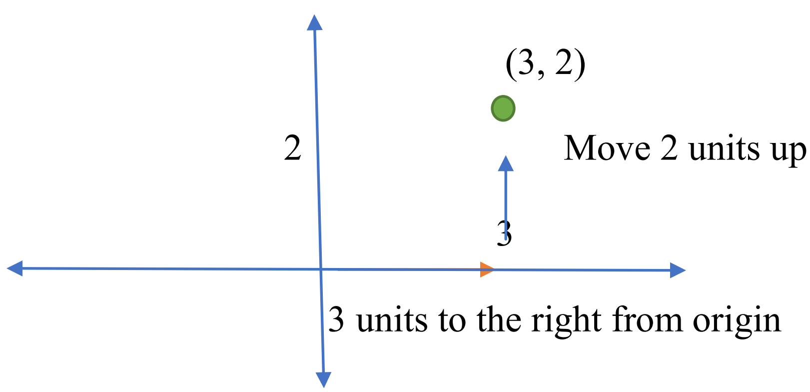
- Move a unit to the right if a is positive.
- Move a unit to the left if a is negative.

Step 2: Then, move b unit in the vertical direction

- Move b unit up if b is positive.
- Move b unit down if b is negative.

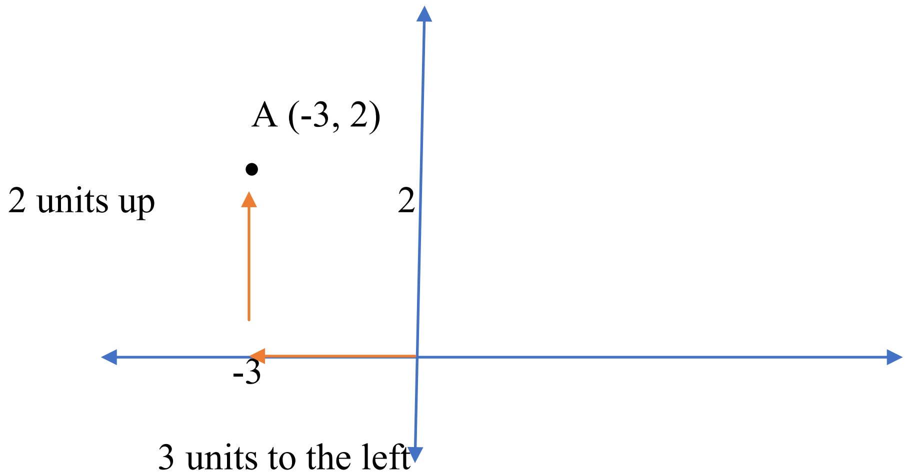
Example 1: locate the point A (3, 2) on the coordinate plane

Solution:



Example 2: locate the point A (-3, 2) on the coordinate plane

Solution: move 3 units to the left from the origin and then 2 units up.



Example 3: locate the point A (2, 0) on the coordinate plane

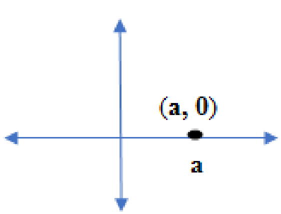
Solution: move 2 units to the right from the origin and then 0 unit in the vertical direction (no move in the vertical direction)

Example 4: locate the point A (0, -4) on the coordinate plane.

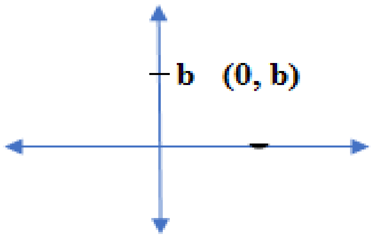
Solution: start from origin and no move on horizontal, then move 4 units down in vertical direction.

Note:

1. The coordinate of all points on the x-axis has the form $(a, 0)$



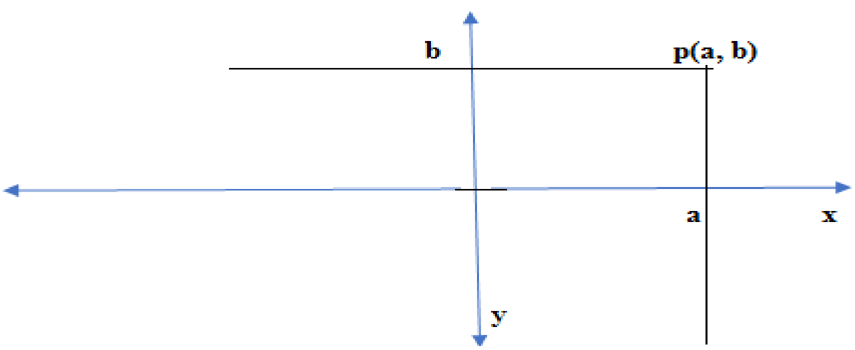
2. The coordinate of all points on the y-axis has the form $(0, b)$



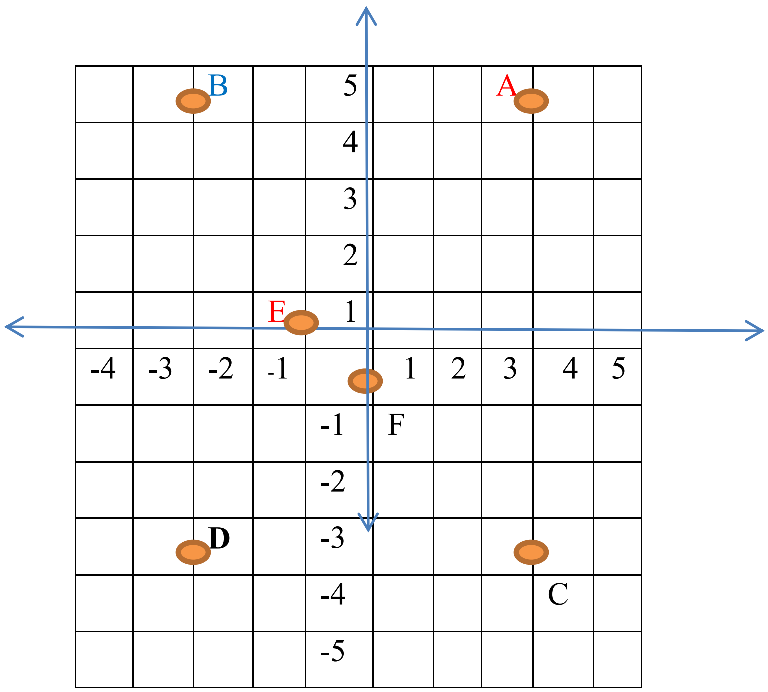
Determining the coordinate of a given point

To determine the coordinate of a point p.

- First draw vertical line passing through the point p crossing $x - axis$. Let it crosses at $'a'$,then the x coordinate of p is a
- Then, draw horizontal line passing through p crossing $y - axis$. Let it crosses at $'b'$,then the y coordinate of p is b
- So, the coordinate of p is (a, b).



Example: Determine the coordinates of the points indicated below



Solution: to determine coordinate of A, draw vertical line passing through A. it crosses x -axis at 3, hence its x coordinate is 3. Then draw horizontal line passing through A, it crosses y – axis at 4, hence its y coordinate is 4.

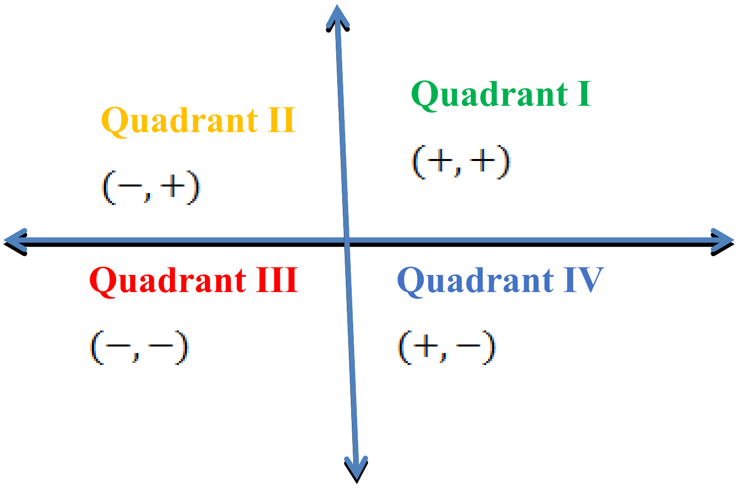
Therefore, the coordinate of A is $(3,4)$. In similar method you can determine the coordinate of the rest points.

B (-3, 4), C (3, -4), D (-3, -4), E (-1, 0), F (0, -1)

Quadrants

The *x*-axis and *y*-axis divide the Cartesian plane into 4 regions known as quadrants.

The four quadrants (1st quadrant, 2nd quadrant, 3rd quadrant, and 4th quadrant) are shown in the diagram below.



- Quadrant 1 contains positive x values and positive y values.
- Quadrant 2 contains negative x values and positive y values.
- Quadrant 3 contains negative x values and negative y values.
- Quadrant 4 contains positive x values and negative y values.

Example 2: In which quadrant or axis, the following points lie?

a. (4, 1) d. (-3, -3)
b. (-4, 1) e. (-3, 0)
c. (0, 9) f. (1. -3)

Solution: a. both x and y values are positive then (4, 1) lie 1st quadrant

b. x is negative and y is positive then (-4, 1) lie on 2nd quadrant

c. x coordinate is 0, then (0, 9) lie on y-axis

d. both x and y values are negative, then (-3, -3) lie on 3rd quadrant

e. y coordinate is 0 then (-3, 0) lie on x-axis.

f. x is positive and y is negative then (1, -3) lie on 4th quadrant

Exercise 4.3.1

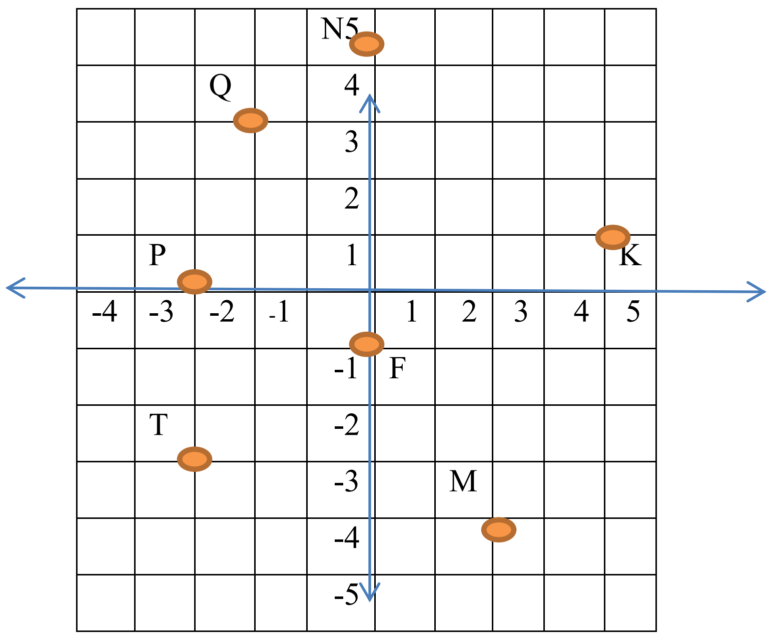
1. Locate the following points on the same Cartesian coordinate plane.

a. M (-1, 4) d. P (-3, -2) g. S (-2, 1.5)

b. N (4, 6) e. Q (-5, 0) h. T $(\frac{3}{2}, -5)$

c. O (4, -1) f. R (0. -3) i, U (0, 0)

2. Based on the coordinate plane below answer the following questions.



a. Write the coordinates of the points F, T, P, M, N, Q and k

b. Which points has the coordinate (-2, 3)

3. In which quadrant or axis lie the following coordinate points?

a. (5, 212) d. (-45, -4) g. $(\frac{5}{2}, 0)$

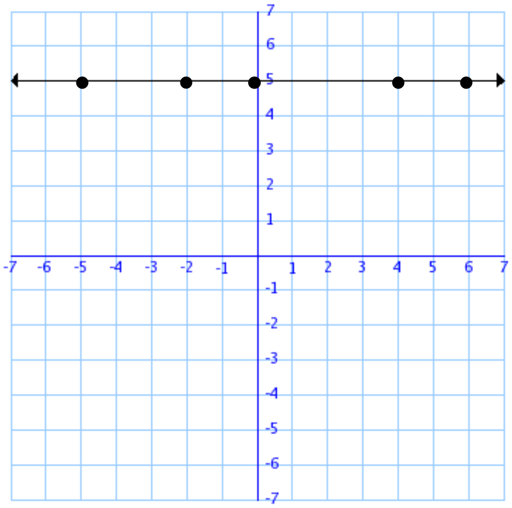
b. (-3,27) e. (10, 0) h. (35, -12.5)

c. (0, -16) f. (12. -30) i. $(-\frac{7}{4}, -\frac{13}{5})$

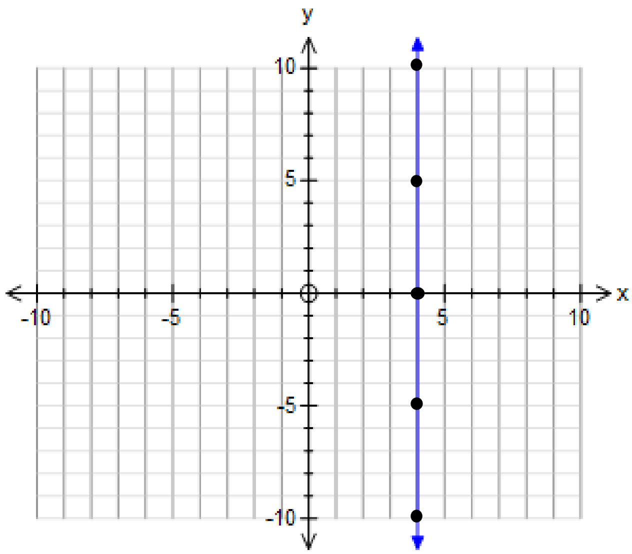
4.3.2 Coordinates and graph of linear equations

Activity 4.3.2

1. Write the coordinate of the points that lie on the lines x=4 and y=5 indicated below



$y = 5$



$x = 4$

2. Plot the following points on coordinate plane and connect them using straight line.

   $(-2,0),\ (-2,-3),\ (-2,3)$

3. Plot the following points on coordinate plane and connect them using straight line.

   $(0,-2),(-3,-2),(3,-2)$

4. What is the equation of the line in activity 2?
5. What is the equation of the line in activity 3?

Graph of an equation of the form $x = a$, where $'a'$ is constant

To draw the graph of a line $x = a$, follow the following steps:

Step 1: Prepare table of values relating x and y. In the case of $x = a$, for any different possible values of y, the value of x is constant. It is $x = a$.

Step 2: plot the coordinate of the points in step 1.

Step 3: Join all points in step 2 using straight line.

Example1: Draw the graph of the following equation of lines on the same Cartesian coordinate plane.

a. $x = -4$ b. $x - 3 = 0$ c. $x = \frac{3}{2}$

Solution:

a. $x = -4$, for any possible values of y, the value of x is -4

x	-4	-4	-4	-4	-4	-4	-4
y	-3	-2	-1	0	1	2	3
(x, y)	$(-4,-3)$	$(-4,-2)$	$(-4,-1)$	$(-4,0)$	$(-4,1)$	$(-4,2)$	$(-4,3)$

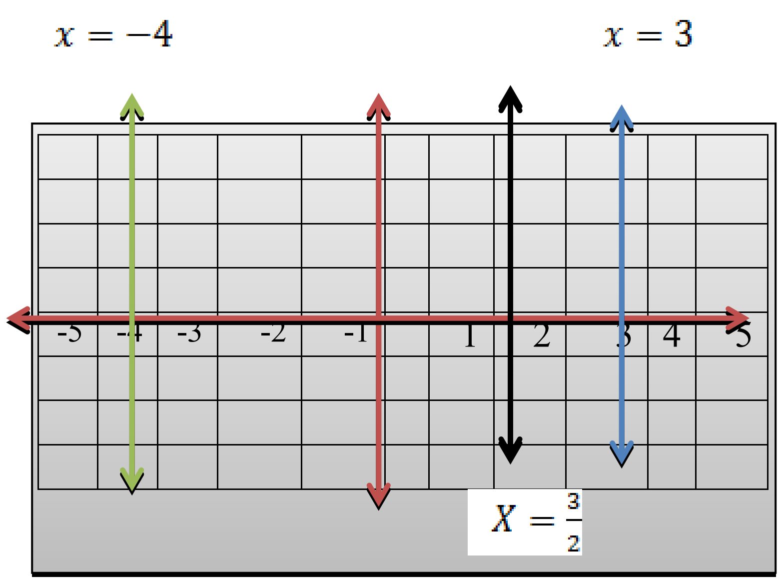
b. $x + 3 = 0$

$x = -3$, for any possible values of y, the value of x is -3

x	3	3	3	3	3	3	3
y	-3	-2	-1	0	1	2	3
(x, y)	$(3,-3)$	$(3,-2)$	$(3,-1)$	$(3,0)$	$(3,1)$	$(3,2)$	$(3,3)$

c. . $x = \frac{3}{2}$, for any possible values of y, the value of x is $\frac{3}{2}$

x	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$
y	-3	-2	-1	0	1	2	3
(x, y)	$\left(\frac{3}{2}, -3\right)$	$\left(\frac{3}{2}, -2\right)$	$\left(\frac{3}{2}, -1\right)$	$\left(\frac{3}{2}, 0\right)$	$\left(\frac{3}{2}, 1\right)$	$\left(\frac{3}{2}, 2\right)$	$\left(\frac{3}{2}, 3\right)$



Note:

- The graph of the equation of the line $x = a$, a is constant is a line parallel to y-axis at a distance of a unit from the origin.
- If a is positive, then the line lies to the right of the y-axis.
- If a is negative, then the line lies to the left of y-axis.
- If $a = 0$, then the line lies on y-axis.

Graph of an equation of the form $y = b$, where $'b'$ is constant

To draw the graph of a line $y = b$, follow the following steps:

Step 1: Prepare table of values relating x and y. In the case of $y = b$, for any different possible values of x, the value of y is constant. It is $y = b$.

Step 2: plot the coordinate of the points in step 1.

Step 3: Join all points in step 2 using straight line.

Example1: Draw the graph of the following equation of lines on the same Cartesian coordinate plane.

c. $y = -4$ b. $y = 3$ c. $y - \frac{3}{2} = 0$

Solution:

a. $y = -4$

x	-3	-2	-1	0	1	2	3
y	-4	-4	-4	-4	-4	-4	-4
(x, y)	$(-3, -4)$	$(-2, -4)$	$(-1, -4)$	$(0, -4)$	$(1, -4)$	$(2, -4)$	$(3, -4)$

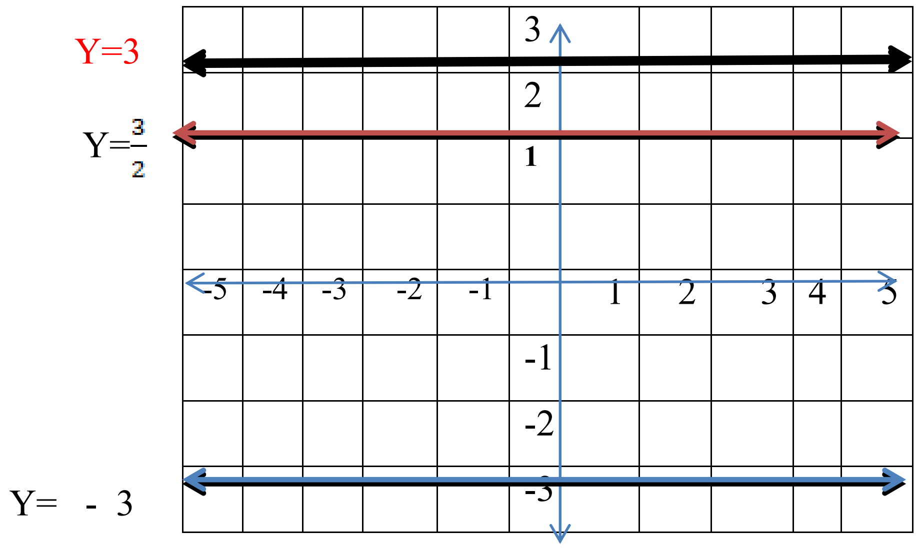
b. $y = 3$

X	-3	-2	-1	0	1	2	3
Y	3	3	3	3	3	3	3
(x, y)	$(-3,3)$	$(-2,3)$	$(-1,3)$	$(0,3)$	$(1,3)$	$(2,3)$	$(3,3)$

c. $y - \frac{3}{2} = 0$

$y = \frac{3}{2}$

x	-3	-2	-1	0	1	2	3
y	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$
(x, y)	$\left(-3, \frac{3}{2}\right)$	$\left(-2, \frac{3}{2}\right)$	$\left(-1, \frac{3}{2}\right)$	$\left(0, \frac{3}{2}\right)$	$\left(1, \frac{3}{2}\right)$	$\left(2, \frac{3}{2}\right)$	$\left(3, \frac{3}{2}\right)$



Note:

- The graph of the equation of the line y=b, b is constant is a line parallel to x-axis at a distance of b unit from the origin.
- If b is positive, then the line lies above the x-axis.
- If b is negative, then the line lies below the x-axis.
- If $b = 0$,then the line lies on x-axis.

Exercise 4.3.2

1. Draw the graph of the following equations on the same coordinate plane.

   a. $x = 7$ b. $y = 5$ c. $x = \frac{1}{2}$ d. $y = 0$

2. Which of the following points lie on a line y=1

a.(-1, 1)	d. (0, 1)	g. (2, 2)
b.(1, 1)	e. (-3. 1)	h. (-12, 1)
c.(0, 0)	f. (1, 2)	i. (1, 15)

3. A point (m, 5), (m, 6) and (m, -2) lie on the line $x = -8$, the find the value of m?

4. Draw the graph of a line that passes through the following points.
   a. (2, 3), (1, 3), (-2, 3)
   b. (-1, 0), (-1, -1), (-1, 4)
   c. (0, 0), (0, 1), (0, -2)
5. List any three points that lie on the line $x = -2$
6. List any three points that lie on the line $y = -3$

Graph of the equation of the form $y = mx$ where m is constant

Activity 4.3.3

1 If y=x, then complete the given table below.

x	-3	-2	-1	0	1	2	3
y							
(x, y)							

2. Draw the graph of the line y=x using table values above.

Steps: To draw the graph of $y = mx$

Step 1: Choose some value of x and find corresponding value of y.

Step 2: plot the coordinate of the points in step 1.

Step 3: Join all points in step 2 using straight line.

Example: Draw the graph

a. $y = 2x$
b. $y = -2x$

Solution:

a. y=2x

Choose some values of x, and determine the corresponding values of y using the equation y=2x

choose the value of $x = -3, -2, -1, 0, 1, 2, 3$

$when\ x = -3, y = 2(-3) = -6$

$when\ x = -2, y = 2(-2) = -4$

$when\ x = -1, y = 2(-1) = -2$

$when\ x = 0, y = 2(0) = 0$

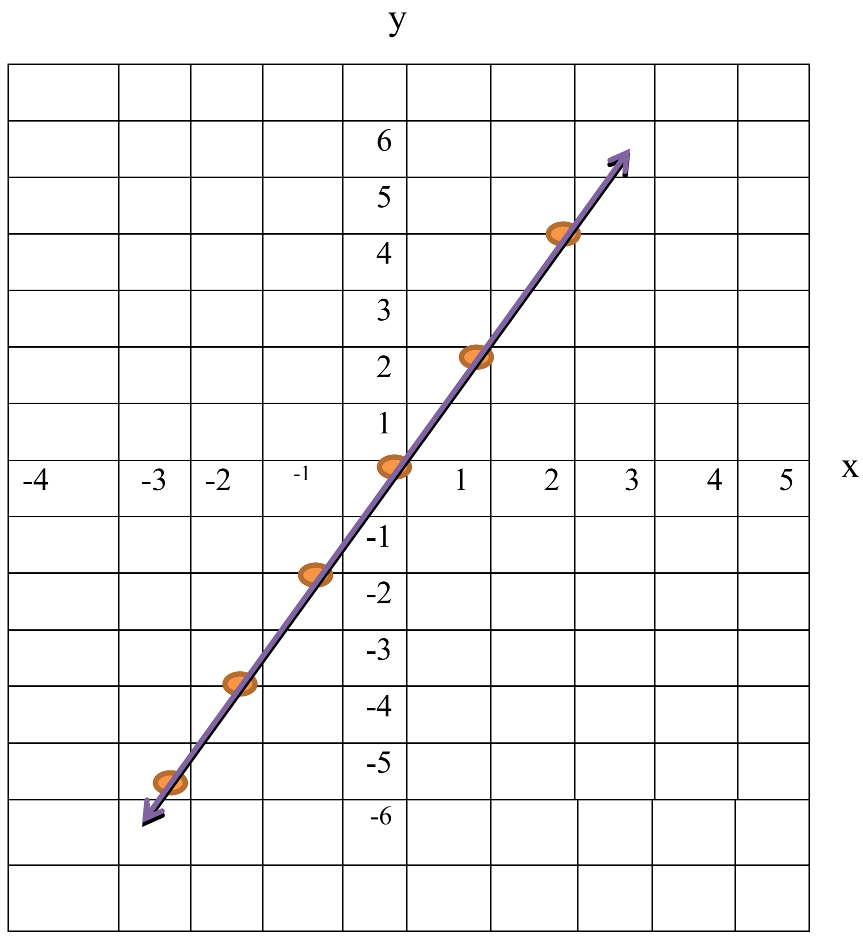
$when\ x = 1, y = 2(1) = 2$

$when\ x = 2, y = 2(2) = 4$

$when\ x = 3, y = 2(3) = 6$

x	-3	-2	-1	0	1	2	3
y	-6	-4	-2	0	2	4	6
(x, y)	$(-3, -6)$	$(-2, -4)$	$(-1, -2)$	(0,0)	(1,2)	(2,4)	(3,6)

y=2x



b. y= -2x

Choose some values of x, and determine the corresponding values of y using the equation $y = -2x$

Choose the value $x = -3, -2, -1, 0, 1, 2, 3$

$when\ x = -3, y = -2(-3) = 6$

$when\ x = -2, y = -2(-2) = 4$

$when\ x = -1, y = -2(-1) = 2$

$when\ x = 0, y = -2(0) = 0$

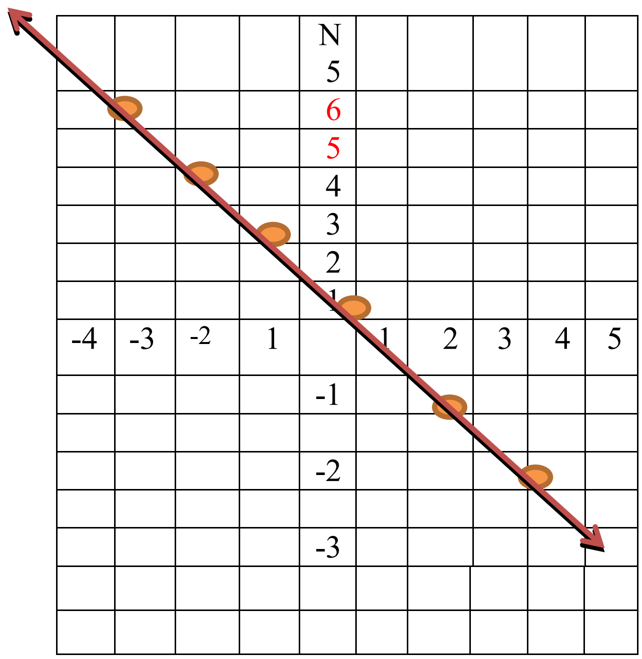
$when\ x = 1, y = -2(1) = -2$

$when\ x = 2, y = -2(2) = -4$

$when\ x = 3, y = -2(3) = -6$

x	-3	-2	-1	0	1	2	3
y	6	4	2	0	-2	-4	-6
(x, y)	$(-3,6)$	$(-2,4)$	$(-1,2)$	$(0,0)$	$(1,-2)$	$(2,-4)$	$(3,-6)$

$y = -2x$



Note:

- The graph of $y = mx$ passes through 1^{st} and 3^{rd} quadrant when $m > 0$
- The graph of $y = mx$ passes through 2^{nd} and 4^{th} quadrant when $m < 0$
- For any values of m the graph of $y = mx$ passes through the origin.
- In the equation of the line y=mx, m is called **slope** of a line.

Exercise 4.3.3

1. If $y = 4x$ then complete the table below and draw the graph of $y = 4x$

x	-3	-2	-1	0	1	2	3
Y							
(x, y)							

2. Which of the following equation of lines passes through 1^{st} and 3^{rd} quadrant

a. $y = 7x$ c. $y - 12x = 0$ e. $y + 6x = 0$

b. $y = -10x$ d. $y - x = 0$ f. $y = -\frac{3}{2}x$

3. Draw the graph of the following equations.

a. $y = 5x$ c. $y - 3x = 0$ e. $2y + 6x = 0$

b. $y = -5x$ d. $y + x = 0$ f. $y = \frac{3}{2}x$

4. If a point $(2,8)$ lie on alien $y = mx$, then find the value of m

4.4.Applications of linear Equations

Activity 4.4.1

Discuss with your friends

1. Translate each of the following phrases in to mathematical equations.
   a. A number increased by 5 is 11.
   b. A number decreased by 5 is 11.
   c. The product of a number and 5 is 30.
   d. The sum of a number and 7 is 29.
   e. Half of a number is 20.
   f. 10 less than a number is 12.
   g. 5 more than four times a number is 30.

Solving word problems

To solve word problems, follow the following important steps

1. Read the problem carefully
2. Select variables for unknown quantities
3. Write a mathematical equation

4. Solve the equation
5. Interpret the result and write the final answer in words.
6. Check the answer

Example 1: A number increased by 10 is 23. What is the number?

Solution: Let y be the unknown number

Mathematical equation $\Longrightarrow y + 10 = 23$

solve the mathematical equation $y + 10 = 23$

$y = 23 - 10$

$y = 13 \leftarrow$ solution

Hence the unknown number is 13.

Example 3: 26 less than three times a number is 46. What is the number?

Solution: Let x be the unknown number

Mathematical equation $\Longrightarrow 3x - 26 = 46$

solve the mathematical equation $3x - 26 = 46$

$3x = 46 + 26$

$3x = 72$

$x = 24 \leftarrow$ solution

Hence the unknown number is 24.

Example 3: A number decreased by 11 is 35. What is the number?

Solution: Let x be the unknown number

Mathematical equation $\Longrightarrow x - 11 = 35$

solve the mathematical equation $x - 11 = 35$

$x = 35 + 11$

$x = 46 \leftarrow$ Solution

Hence the unknown number is 46.

Example 4: The sum of 0ne seventh of a number and 15 is 19. What is the number?

Solution: Let x be the unknown number

Mathematical equation $\Longrightarrow \frac{x}{7} + 15 = 19$

Solve the mathematical equation $\frac{x}{7} + 15 = 19$

$\frac{x}{7} = 19 - 15$

$\frac{x}{7} = 4$

$x = 28 \leftarrow$ Solution

Hence the unknown number is 28.

Exercise 4.4.1

1. A number increased by 7 gives 20, what is the number?
2. A number decreased by 3 gives 13. What is a number?
3. The difference between a positive number x and 3 is 21. What is a number?
4. 15 more than twice a number is 37. What is a number?
5. A number is increased by 7, twice the result is 41. Find the number.
6. If the sum of $\frac{1}{4}$ of a number and 15 is 13. What is a number?

7. 8 less than three times a number is 55. What is a number?
8. A number is doubled and the result is increased by 8. If the final result is 36. What is a number?

Example 5: (Application involving consecutive integers)

Find three consecutive integers whose sum is 156.

Solution: Let x be the smaller integer, then

The three consecutive integers $are\ x, x+1,$ and $x+2$

Mathematical equation $\Longrightarrow x+(x+1)+(x+2)=156$

Solve the mathematical equation $x+(x+1)+(x+2)=156$

$$x+x+1+x+2=156$$

$$3x+3=156$$

$$3x=156-3$$

$$3x=153$$

$x=51 \leftarrow$ Smallest number

Hence the unknown integers are 51, 52 and 53.

Example 6: (applications involving ages)

The sum of the ages of a man and his wife is 96 years. The man is 6 years older than his wife. How old is his wife?

Solution:

- let his wife's age is y
- The man's age will be $y+6$ (he is 6 years older)
- The sum of their age is 96, so

$$y+(y+6)=96$$

$$2y+6=96$$

$$2y=96-6$$

$$2y=90$$

$y=45$ → his wife's age

Hence, the age of his wife is 45 years.

Example 7:

A farmer has sheep and hen. The sheep and hens together have 100 heads and 356 legs. How many sheep and hens does the farmer have?

Solution: Since the total heads is 100, the sheep and hen together are 100 in number.

Let the farmer has x sheep. And the number hens in this case is $100-x$

A sheep has 4 legs, so x sheep have $4x$ legs.

A hen has 2 legs, so $(100-x)$ hens have $2(100-x)$ legs

The total number of legs is 356, so

$$number\ of\ legs\ of\ sheep + number\ of\ legs\ of\ hens = 356$$

$4x + 2(100 - x) = 356$

$4x + 200 - 2x = 356$

$4x - 2x = 356 - 200$

$2x = 156$

$x = 78$

Therefore, the farmer has $x = 78\ sheep$ and $100 - x = 100 - 78 = 22\ hens$

Exercise 4.4.2

1. The sum of three consecutive integers is 345. What are the numbers?
2. The sum of two consecutive even integers is 170. What are the integers?
3. The sum of two consecutive odd integers is 144.What are the numbers?
4. The sum of the ages of a man and his wife is 83. The man is 3 years older than his wife. How old is a man and his wife?
5. In a class there are 45 students. The number of girls is 1.5 times the number of boys. How many boys and how many girls are there in the class?
6. There are 26 girls and 22 boys in class room. Every student contributes equal amount of Birr for the renaissance dam (Hidassie Gidib). If Birr 2400 is collected from all students, then what amount of Birr contributed by
   a. Each student? b. boys? c. girls?
7. A government has 87 hospitals. If the government plan to build 2 hospitals in a year. How many years will it takes so that the total number of hospitals be 109.
8. In a triangle ABC, angle A is twice as large as angle B and angle B is 20 more than angle C. What is the measure of each angle? (hint sum of interior angle of a triangle is 180^0.)

Summary for unit 4

- A variable is any letter or a symbol that represent some unknown number or value.
- a constant (a number), a variable, product or quotient of a number and variable is called a term.
- In the product of a number and variable, the numerical factor of the term is called numerical coefficient.
- like terms are terms whose variable and exponent of variable are exactly the same but they may differ in their numerical coefficients. Terms which are not like terms are called unlike terms.
- Algebraic expressions are formed by using numbers, letters and the operation of addition, subtraction, multiplication and division.
- Two different algebraic expressions connected by equal $(=)$ sign is called equation.
- A Linear equation in one variable x is an equation which can be written in standard form $ax + b = 0$, where a and b are constant numbers with a$\neq 0$.
- To transform linear equations to simpler form, you can apply the following rules.

$i.$ if $a = b$, then $a + c = b + c$, where $a, b, and\ c$ are any constant numbers.

$ii.$ if $a = b$, then $a - c = b - c$, where $a, b, and\ c$ are any constant numbers.

$iii.$If $a = b$, then $a \times c = b \times c$, where a, b, c are any constant number $and\ c \neq 0$

$iv.$ If $a = b, then\ \frac{a}{c} = \frac{b}{c}$, where a, b, c any constant number $and\ c \neq 0$

- To solve linear equations involving brackets, remember the following key properties. For any numbers a, b and c

  $a + (b + C) = a + b + c$

  $a - (b + C) = a - b - c$

  $a - (b - C) = a - b + c$

  $a(b + c) = ab + ac$

  $a(b - c) = ab - ac$

- Points on Cartesian plane are described by two numbers (a, b) that are called coordinates.
  i. The first number$a$, is the horizontal position of the point from the origin. It is called $x - coordinate$ (abscissa).
  ii. The second number b is the vertical position of the point from the origin. It is called $y - coordinate$ (ordinate).
- The graph of the equation of the line $x = a$, a is constant is a line parallel to y-axis at a distance of a unit from the origin.
- The graph of the equation of the line y=b, b is constant is a line parallel to x-axis at a distance of b unit from the origin.
- The graph of $y = mx$ passes through 1st and 3rd quadrant when $m > 0$
- The graph of $y = mx$ passes through 2nd and 4th quadrant when $m < 0$
- In the equation of the line y=mx, m is called **slope** of a line.
- For any values of m the graph of $y = mx$ passes through the origin.
- In solving word problems follow the following steps
  i. Read the problem carefully
  ii. Select variables for unknown quantities
  iii. Write a mathematical equation

iv. Solve the equation

v. Interpret the result and write the final answer in words.

vi. Check the answer

Review Exercise for unit 4

I. Write True for the correct statements and False for the incorrect statements.

1. For any integers $a, b\ and\ c, a(b+c) = ab + ac$.
2. An algebraic expression $2x^2 + 2xy$ is trinomial algebraic expression.
3. $3a^2b\ and\ 3ab^2$ are like terms.
4. If a=b, then a-b= 0
5. The equation of the line $x + y = 0$ is vertical line.
6. 4 is slope of equation of line $y - 4x = 0$.
7. The numerical coefficient of $3xy^2\ is\ 2$.
8. If $c = d$, then $a + c = d + a$.

II. Choose the correct answer from the given alternatives.

9. Which one of the following pair terms are like terms?

   a. $3x^2$ and $3x$ c. $2ab^2$ and $2a^2b$

   b. $-x^2$ and $-x$ d. $-4xy$ and $9yx$

10. Which of the following is equivalent to the equation

   $-3(2x-1) = 2x + 12$?

   a. $-6x + 3 = 2x + 12$ c. $x = -1$

   b. $-8x = 8$ d. $-6x = 2x - 4$

11. The solution of $\frac{x}{2} - x = \frac{3x}{8} - 11$ is

   a. $\frac{7}{2}$ b. $\frac{7}{11}$ c. $\frac{88}{7}$ d. $-\frac{11}{7}$

12. Which of the following value of x satisfies the equation $2x = 3x - 1$?

    a. -1 b. 1 c. 0 d. 2

13. Which of the following is the slope of the line $y - 3x = 0$?

    a. -3 b. 3 c. $\frac{1}{3}$ d. has no slope

14. The sum of the age of a girl and her brother is 34. The girl is 8 years older than her brother, how old is her brother?

    a. 21 b. 13 c. 11 d. 23

15. Which one of the following equations is equation of horizontal line?

    a. $y = 4x$ b. $y + 1 = 0$ c. $x - 2 = 0$ d. $x = 0$

III. Work out problems

16. Write the following word phrases in to algebraic expression
    a. Six less than a number
    b. Nine more than twice of a number
    c. A number increased by five
    d. 35 more than half of a number
17. Simplify the following algebraic expressions

    a. $-5x + 4 - 21x - 21$

    b. $5x - (3x - 1) - 6(7 - x)$

    c. $2(6y - x) - 10y + 2(y + x)$
18. Solve each of the following linear equations.
    a. $4x + 36 = 86 - 8x$
    b. $12x - 8 + 2x - 17 = 3x - 4 - 8x + 74$
    c. $2(6y - 18) - 102 = 78 - 18(y + 2)$
    d. $7(x + 26 + 2x) = 5(x + 7)$
    e. $\frac{2x+7}{3} - \frac{x-9}{2} = \frac{5}{2}$

f. $\frac{x+3}{6} - \frac{x-5}{4} = \frac{3}{8}x$

g. $\frac{x-2}{3} + \frac{x+3}{8} = \frac{5x}{6} - 6$

19. Draw the graph of each of the following equation of the line on the same Cartesian coordinate plane

a. $y + 4x = 0$ c. $x = 0$

b. $y + 1 = 0$ d. $y = x$

20. Find the equation of the line passes through the origin with slope 4.

21. What is the slope of a line y=$\frac{x}{2}$

22. 16 less than four times a number is 12, what is a number.

23. The sum of one fifth of a number and 12 is 30.

24. I think a number. If I subtract 8 from it and multiply the difference by 3, the result is 21. What is a number?

25. Bezawite is 8 years older than her sister. How old is Bezawit if the sum of their ages is 32.

26. In mathematics final exam Derartu scord 10 more than twice the lowest score. If Derartu scored 46 points, what was the lowest score?

Unit 5

Perimeter and Area of Plane figures

Unit Outcomes:

At the end of this unit, students will able to:

- Classifies the different kinds of triangles.
- Constructs and describe properties of four- sided figures
- Find the perimeter of triangle, parallelograms, trapezium, rhombus and composite shapes
- Derive formula for area of triangle, parallelograms, trapezium and rhombus.
- Calculate areas of triangle, parallelograms, trapezium, rhombus and composite shapes
- Apply the concept of area and perimeter of Plane figures in solving real life problems

Introduction

In this unit you will learn the different kinds of geometric figures and their properties such as triangle, rectangle, square, rhombus, and trapezium. In addition to their properties you will learn, how to calculate the area and perimeter of these geometric figures.

The unit has five sections. The first section deals with revision of triangles, second section deals with four - sided figures, the third section deals with Perimeter and Area of four-sided figures, the fourth section deals with Perimeters and Areas of triangles and the last section deals about application of the concept of area and perimeter of Plane figures in solving real life problems.

5.1.Revision of triangles

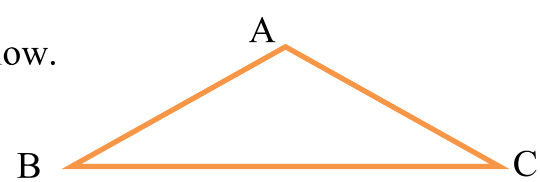
At the end of this section you should be able to:

• classify the different kinds of triangles

Activity 5.1.1

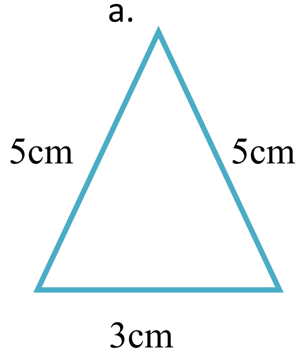
Discuss with your friends/partners

1. Define triangle
2. Name triangle below.

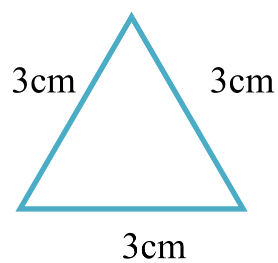


3. Classify the following triangles based on their sides.

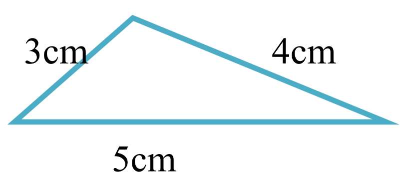
a.



b.

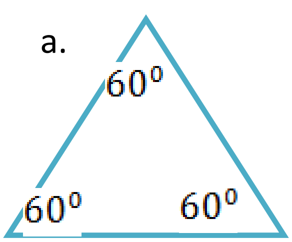


c.

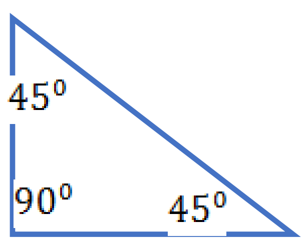


4. Classify the following triangle based on their angles

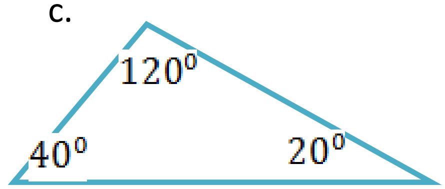
a.



b.

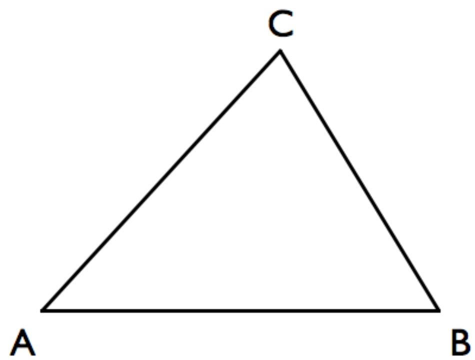


c.



5. Draw a triangle and measure the interior angle of this triangle by using protractor. Find the sum of all interior angles?

Definition 5.1: A triangle is a simple closed plane figure made of three-line segments.



- The line segments forming the triangle are sides of the triangle.

$\overline{AC}, \overline{AB}$ and $\overline{BC}$ are the sides of the triangle.

- The point where the sides of the triangle meet is called vertex (plural form vertices)

A, B and C are the vertices of the triangle.

- Angles of a triangle are formed by two sides of the triangle meeting at a point (vertex).
- The interior angles of the above triangle are:
  i. The angle at vertex A, $\angle BAC$ or $\angle A$
  ii. The angle at vertex B, $\angle ABC$ or $\angle B$
  iii. The angle at vertex C, $\angle BCA$ or $\angle C$
- The sum of interior angles of a triangle is 180^0.

$$\angle A + \angle B + \angle C = 180^0$$

- A triangle is named by using its vertices.

The above triangle is named as triangle ABC, symbolically, ΔABC. It can also be named using any other sequence of vertices like triangle BCA, symbolically, ΔBCA.

Note:

1. A triangle has three sides, three angles, and three vertices.
2. The sum of all interior angles of a triangle is always equal to 180°.

Types of triangles

Triangles can be classified in 2 different ways:

- Classification of triangle according to interior angles
- Classification of triangle based on side length

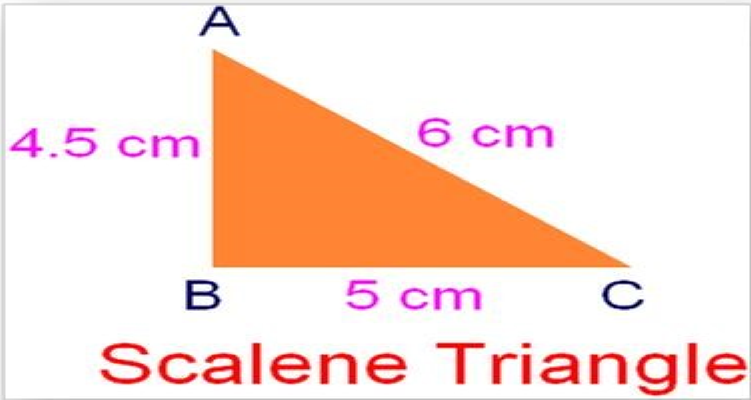
Classification of triangle based on side length

Based on side length, triangles are classified in to three:

1. Scalene Triangle
2. Isosceles Triangle
3. Equilateral Triangle

1. Scalene Triangle

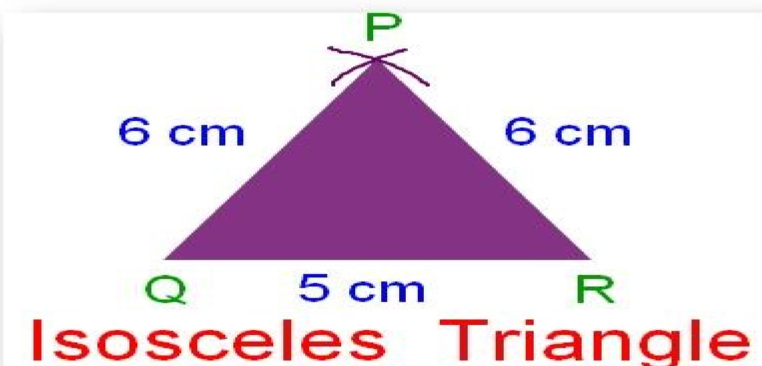
A triangle in which all the three sides are unequal in length is called a *scalene triangle.*



All sides are different in length, so ΔABC *is* scalene triangle

2. **Isosceles Triangle:**

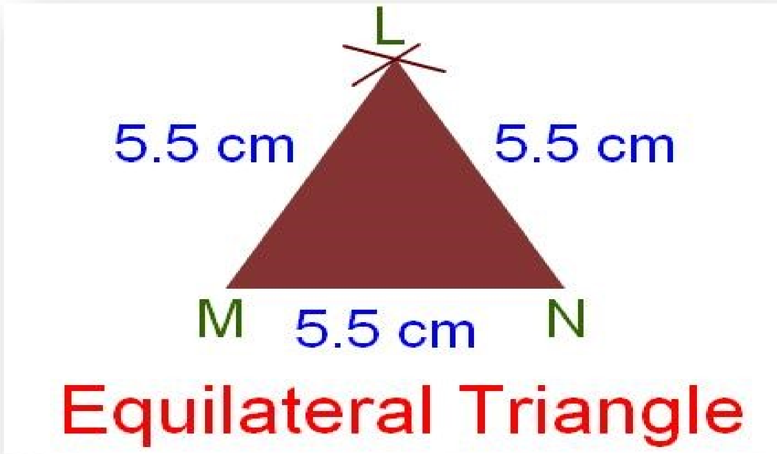
A triangle in which two of its sides are equal is called *isosceles triangle*.



In triangle ΔPQR, **PQ = PR = 6 cm, then** ΔPQR **is an isosceles**

3. **Equilateral Triangle**:

A triangle in which all its three sides are equal in length is called an *equilateral* triangle.



In triangle given above LM=MN=NL=5.5cm, then ΔLMN is equilateral triangle.

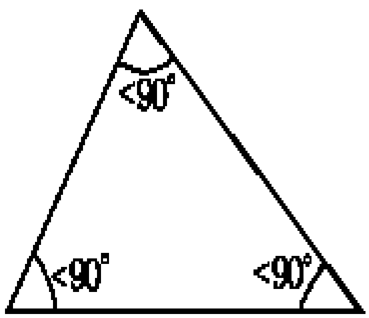
Classification of a triangle according to interior angles

Based on the measure of interior angles, triangles are classified in to three:

1. Acute Angled Triangle
2. Right-Angled Triangle
3. Obtuse Angled Triangle

1. Acute Angle Triangle

A triangle with all three angles less than 90° is an **acute angle triangle.**



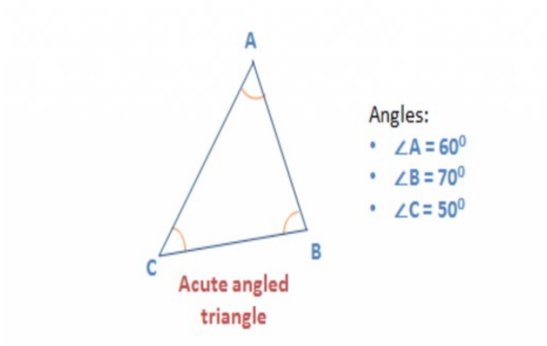


Figure 5.1.3 acute angle triangle

In the above triangle ABC, measure of all angles are less than 90^0 (acute angle), so triangle ABC is acute angle triangle

2. Right-Angle Triangle

A triangle with one angle measuring exactly 90° is a right- angle triangle.

- The other two angles of a right-angle triangle are acute angles.
- The side opposite to the right angle is the largest side of the triangle and is called the hypotenuse.

Example:

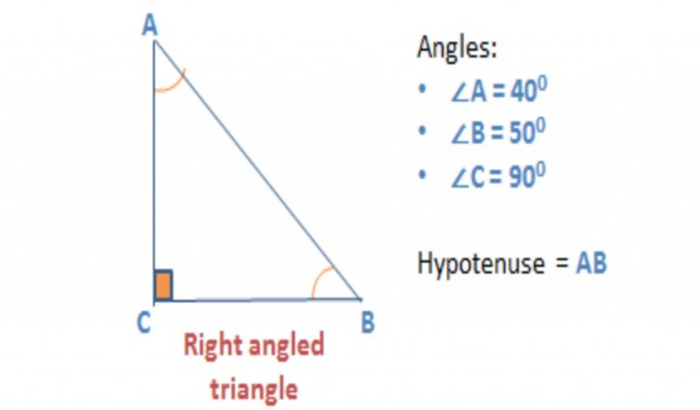


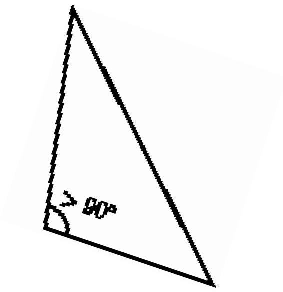
Figure 4.1.2 right angle triangle

In triangle ABC, <ACB= 90^0 is right angle, so triangle ABC is right angle triangle.

3. Obtuse angle triangle

An **obtuse angle triangle** is a triangle with one obtuse angle (greater than 90°) and two acute angles.

Example:



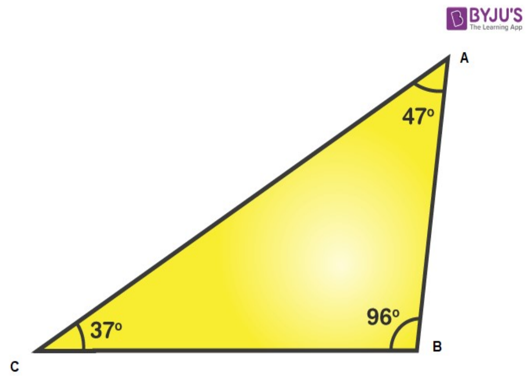


Figure 4.1.3 obtuse angle triangle

In triangle ABC, $< CBA$ $=96^0$ is obtuse angle, then Triangle ABC is obtuse angle triangle.

Exercise 5.1.1

1. Fill in the blanks:

   a. The triangle in which all its sides are equal is called_______________.

   b._____________ is a triangle in which all its sides are different length.

   c. Each angle of equilateral triangle is ____________.

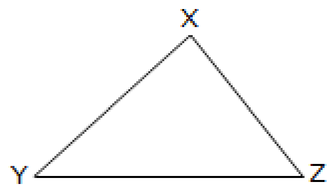
   d. _____________ is a triangle with two equal sides.

   e. In obtuse angled triangle one angle is _______than 90°.

   f. Each angle of an acute triangle is _________than 90°.

   g. The sum of interior angles of a triangle is equal to________.

2. Consider the following triangle.



a. List the sides, angles and vertices of the triangle
b. Name the triangle

3. Classify the following triangle:

a. Sides of triangle are 4 cm, 4 cm and 7 cm
b. Angles of triangle is 90°, 60° and 30°
c. Angles of triangle is 110°, 40° and 30°
d. Sides of triangle are 5 cm, 13 cm and 12 cm
e. Angles of triangle is 60°, 60° and 60°

5.2.Four - sided Figures

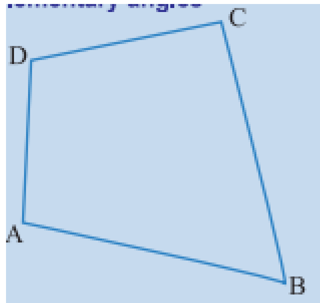
At the end of this section you should be able to:

- construct and describe properties of four-sided figures

Activity 5.2.1

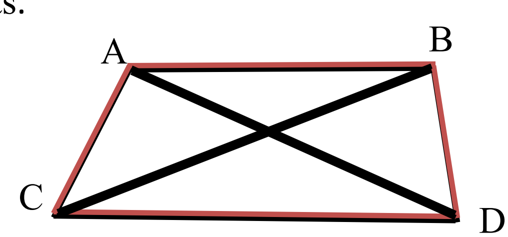
Discuss with your friends/partners:

1. considering the geometric figure below and answer the following question



a. The geometric figure has ______ sides
b. Name all its sides.
c. Name all its vertices.
d. Name all its interior angles.
e. The geometric figure is called______.
f. Name the geometric figure using the four vertices.
g. The vertex opposite to D is _____
h. Name the adjacent vertex to D.
i. Name the opposite side to $\overline{AD}$.
j. Name the adjacent side to $\overline{AD}$.

Definition: A quadrilateral is a four-sided geometric figure bounded by line segments.



- A quadrilateral is named by using its four vertices in clock wise or anti clock wise direction.
- The above quadrilateral is named as quadrilateral ACDB or BACD or CDBA and so on, but it cannot be named as ABCD or ADCB.
- The sides of the quadrilateral ABDC above are $\overline{AB}, \overline{AC}, \overline{CD}, and\ \overline{BD}$
   i. Opposite side to $\overline{AB}$ is $\overline{CD}$

ii. Opposite side to $\overline{AC}$ is $\overline{BD}$

iii. The adjacent sides to $\overline{AB}$ are $\overline{AC}$ and $\overline{BD}$

- The point where the sides of a quadrilateral meet is called Vertex (plural form vertices). The vertices of the quadrilateral ABDC above are A, B, D and C.

  i. The opposite vertex to A is D.

  ii. The vertexes adjacent to A are C and B.

- The angles formed by adjacent side of the quadrilateral are interior angles of the quadrilateral. The interior angles of quadrilateral ABDC above are $\angle A, \angle B, \angle D$ and $\angle C$

  i. The opposite angle to $\angle A$ is $\angle D$.

  ii. The angle adjacent to $\angle A$ are $\angle C$ and $\angle B$.

- A line segment that connects two opposite vertices of the quadrilateral is called diagonal. The diagonals of quadrilateral ABDC above are $\overline{AD}$ and $\overline{CB}$.

Construction and properties of trapezium

Activity 5.2.1

Discuss with your friends/partners:

Consider the quadrilateral below and answer the following questions

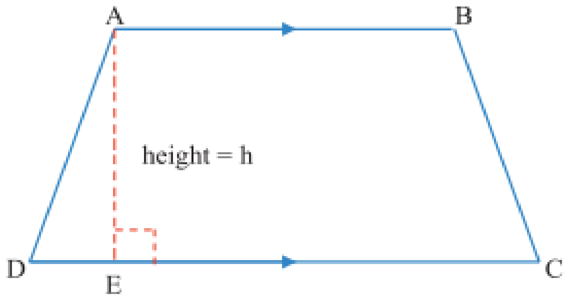
1. Which sides of this quadrilateral are parallel?
2. What do we call such type of quadrilateral?
3. Measure $\angle A, \angle B, \angle D$ and $\angle C$ and calculate

   i. $\angle A + \angle D =$ ____ iii. $\angle A + \angle B + \angle C + \angle D =$ ____

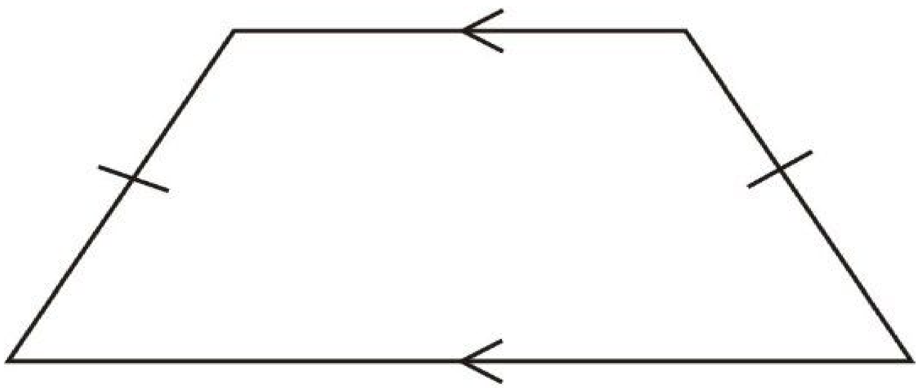
   ii. $\angle C + \angle B =$ ____

Definition:

A trapezium is a special type of quadrilateral in which exactly one pair of opposite sides are parallel.



- The parallel sides of trapezium are called the bases of trapezium. In the above trapezium, the parallel sides $\overline{AB}$ and $\overline{DC}$ are bases.
- The distance between the bases is called the height (or altitude) of the trapezium. $\overline{AE}$ is the height.
- The non-parallel sides of the trapezium are called legs of the trapezium. The non-parallel sides $\overline{AD}$ and $\overline{BC}$ are legs.
- If the legs of trapezium are congruent, then trapezium is called isosceles trapezium.



Isosceles trapezium

Construction of trapezium

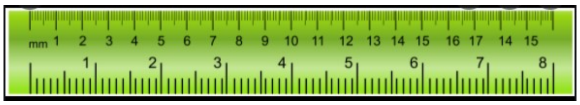
- To perform geometric construction, you need tools like:

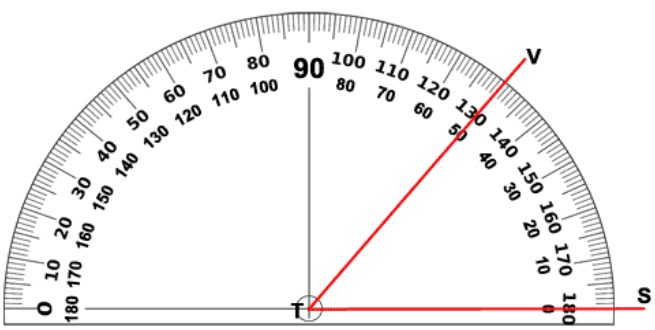
- Ruler
- Compass
- Protractor

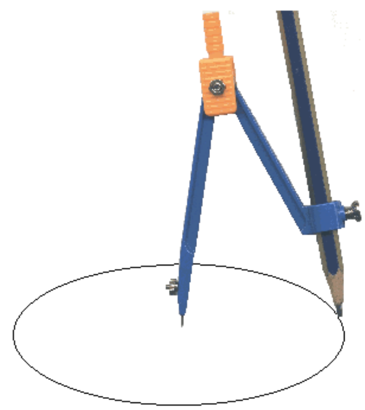
Ruler is used to construct line or line segments.

Compass is used to construct circles or arcs.

Protractor is used to measure or construct angles.







Example: construct a trapezium ABCD using the information given below.

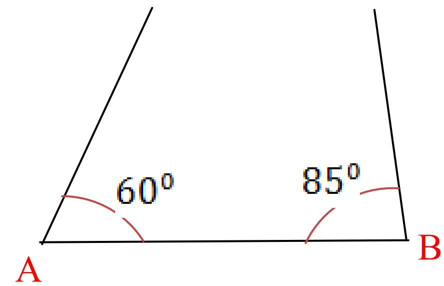
Given: $\overline{AB} \parallel \overline{CD}$, $AB = 8cm, BC = 5cm, m(\angle A) = 60^0, m(\angle B) = 85^0$

Solution:

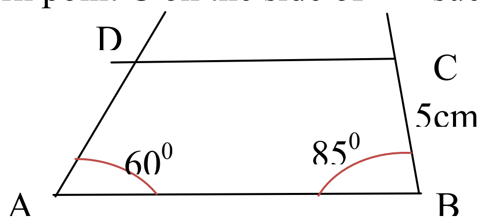
Step 1: draw a line segment $AB = 8cm$

A__________8cm__________ B

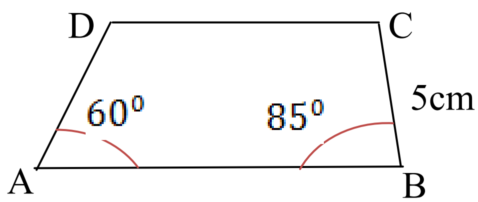
Step 2: construct $m(\angle A)$ and $m(\angle B)$ with the given measures.



Step 3: Mark point C on the side of $\angle B$ such that $BC = 5cm$



Step 4: Draw a line through C parallel to AB, intersecting the side of $\angle A$ at D

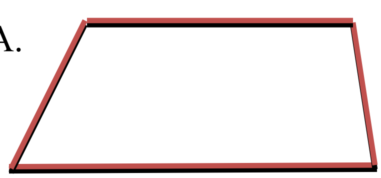


Exercise 5.2.1

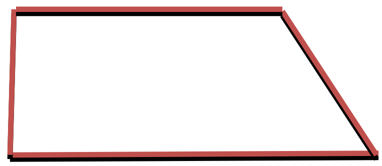
1. Fill in the blank space with the correct answer.
   a. Four-sided geometric figure is called______
   b. A line segment that joins opposite vertex of a quadrilateral is

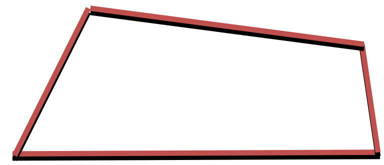
      ____
   c. The point where the sides of a quadrilateral meet is called______
   d. The tools used for geometric construction are ______, _____ and____
2. Which one of the following is **not** a trapezium?

A.

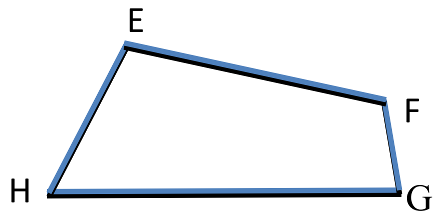


C.

B. 

D. 

3. Considering the quadrilateral below answer the following questions



a. the opposite side to FG is ______

b. the adjacent sides to HG are____, _____

c. the opposite angle to $\angle F$ is _____

d. The quadrilateral is named as __________

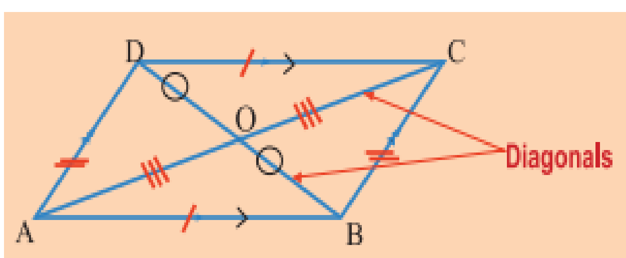
e. the diagonal of the quadrilateral are line segments joining ____ and____ or ____and ______

4. construct a trapezium EFGH using the information given below.

Given: $\overline{EF} \parallel \overline{GH}$, $EF = 10cm, FG = 6\ cm, m(\angle E) = 30^0, m(\angle F) = 80^0$

Construction and properties of parallelogram

Definition: A parallelogram is a quadrilateral in which both pair of opposite sides are parallel.



In the figure $\overline{AB} \parallel \overline{CD}\ and\ \overline{AD} \parallel \overline{BC}$, thus ABCD is a parallelogram.

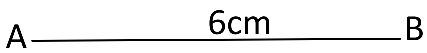
Construction of parallelogram

- Construct a parallelogram ABCD using the information given below

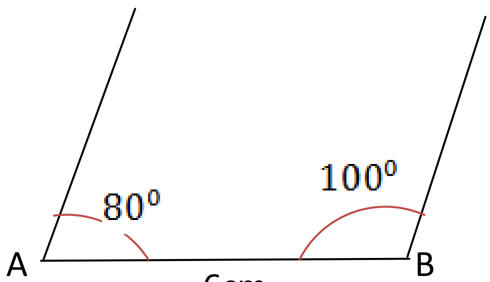
Given: $\overline{AB} \parallel \overline{CD}$ $and\ AB = 6cm, BC = 4cm, m(\angle A) = 80^0, m(\angle B) = 100^0$

Solution:

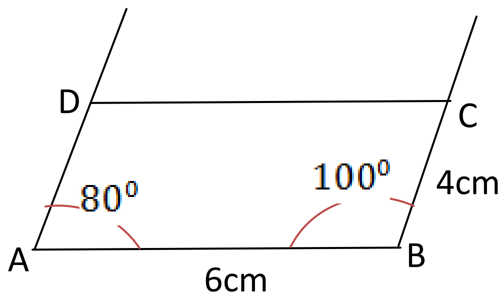
Step 1: Draw a line segment $AB = 6cm$



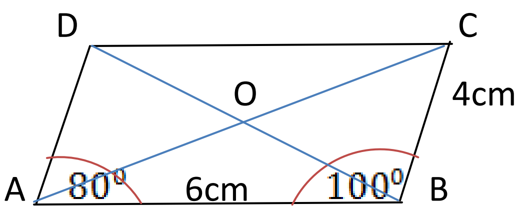
Step 2: construct $\angle A\ and\ \angle B$ so that $(\angle A) = 80^0\ and\ m(\angle B) = 100^0$



Step 3: mark point C on side of $\angle B$ such that $BC = 4cm$. And draw a line through C parallel to AB so that it meets the side of $\angle A$ at point D.



Finally, the parallelogram ABCD satisfying the given condition is



Using the constructed parallelogram above answer the following questions.

1. Measure $\angle D$ and $\angle C$. compare $\angle D$ $with$ $\angle B$, and $\angle A$ $with$ $\angle C$

From the result in 1:

- Opposite angles of a parallelogram are __________

2. Find the value of

i. $m(\angle A) + m(\angle B) =$ _____ iii. $m(\angle C) + m(\angle D) =$ _____

ii. $m(\angle B) + m(\angle C) =$ _____ iv. $m(\angle A) + m(\angle D) =$ _____

From the result in 2:

- Consecutive (or adjacent) angles of a parallelogram are __________

3. Measure side AD and DC. Compare AD with BC and AB with DC.

From the result in 3:

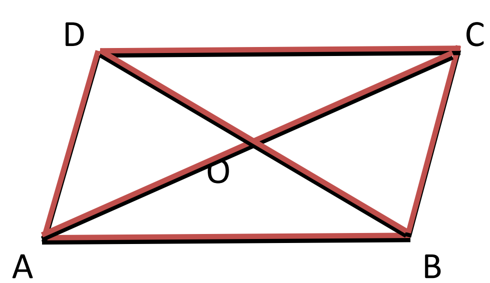
- Opposite sides of a parallelogram are __________

4. Construct diagonals AC and DB, so that they intersect at O. Compare the length of DO with OB and AO with OC.

From the result in 4:

- The diagonals of parallelogram __________ each other.

From the activity above the properties of parallelogram is summarized as follows:



1. Opposite sides of parallelogram are congruent.

   $i.e. AD = BC \; and \; AB = DC$

2. Opposite angles of a parallelogram are congruent.

$i.e. m(\angle A) = m(\angle C)$ and $m(\angle B) = m(\angle D)$

3. Consecutive (or adjacent) angles of a parallelogram are supplementary.

$i.e. m(\angle A) + m(\angle B) = 180^0, m(\angle B) + m(\angle C) = 180^0$

4. The diagonals of parallelogram bisect each other.

$i.e. AO = OC$, and $DO = OB$

Note:

Bisect means "divides exactly in to two equal parts

Supplementary angles are angles whose sum is 180^0

Congruent means equal in length.

Construction and properties of special parallelograms

Rectangle, Rhombus and square are special parallelograms. They all satisfy the properties of parallelogram.

A. Rectangle

Definition: rectangle is a parallelogram with all its angles are right angle.

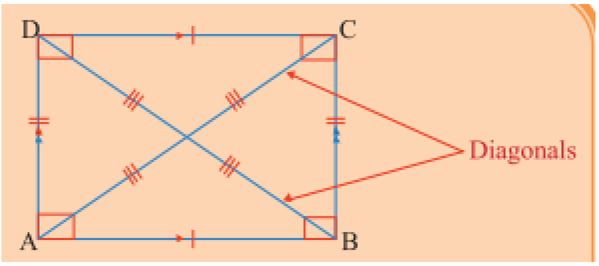


Fig. rectangle

Construction of Rectangle

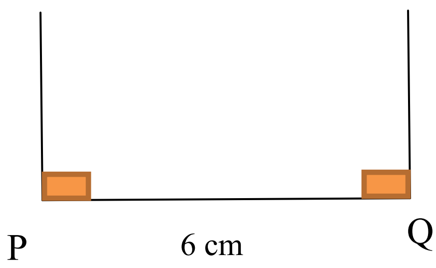
- Construct a rectangle PQRS using the information given below

Given: $\overline{PQ} \parallel \overline{RS}$, $PQ = 6cm, QR = 7cm, m(\angle P) = 90^0$

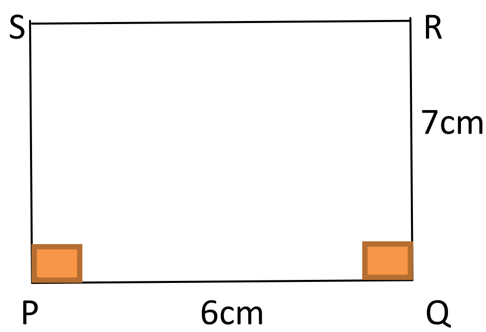
Solution:

Step 1: construct a line segment PQ with length 6 cm.

Step 2: construct $m(\angle P) = 90^0$ and $m(\angle Q) = 90^0$



Step 3: mark point R and S, such that $PS = QR = 7cm$ and connect RS parallel to PQ



Using the constructed rectangle above answer the following questions.

1. Does it satisfy all properties of parallelogram?
2. Compare the diagonals PR and SQ.

The properties of rectangle are summarized as follows:

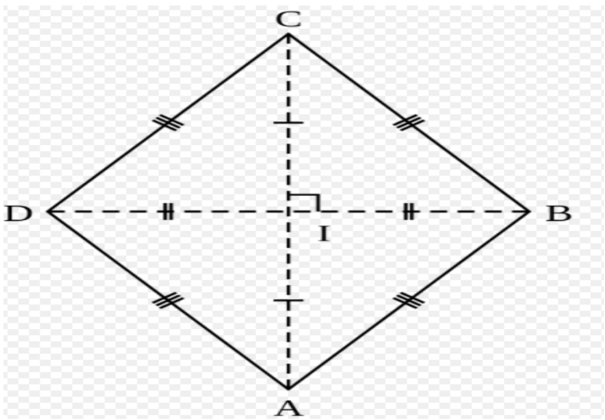
1. Rectangle satisfies all properties of parallelogram.
2. The diagonals of rectangle are equal in length and bisect each other.
3. All angles of rectangle are right angle.

Note:

1. Right angle is an angle that measures 90^0
2. All rectangles are parallelogram, but all parallelograms are not rectangles
3. A quadrilateral with congruent diagonals is not necessarily rectangle
4. A parallelogram with congruent diagonals is rectangle.

B. Rhombus

Definition: A rhombus is a parallelogram in which all its sides are congruent.



Properties of rhombus

i. All properties of parallelogram are properties of rhombus.

ii. All sides of rhombus are congruent $(i.e. AB = BC = CD = AD)$

iii. The diagonals of rhombus are perpendicular to each other (i.e. $AC \perp DB$)

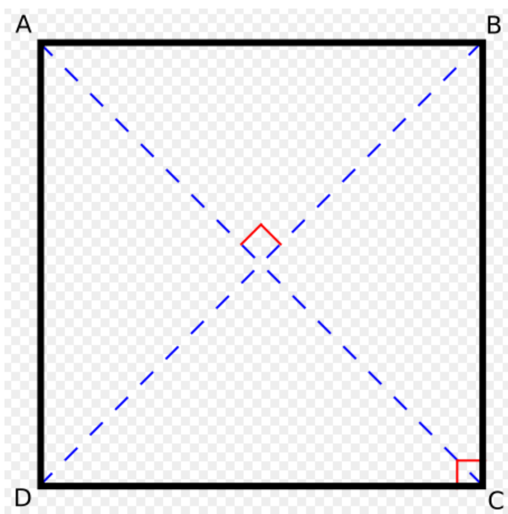
iv. The diagonal of rhombus bisects the angles at the vertices. For example, in the above rhombus $\angle CDA$ is bisected by the diagonal DB, so that $m(\angle CDB) = m(\angle ADB)$.

Note:

1. All rhombs are parallelogram, but all parallelograms are not rhombus.
2. The diagonals of rhombus are perpendicular and bisect each other.

C. Square

Definition: square is a parallelogram with four congruent sides and four right angles.

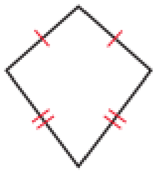


Properties of square

1. Square satisfies all properties of parallelogram, rectangle and rhombus.
2. The diagonals of square are:
   - Perpendicular to each other
   - Bisect each other and
   - Congruent
3. The diagonals bisect the angles at the vertices. Hence, the diagonal forms 45^0 with its side.

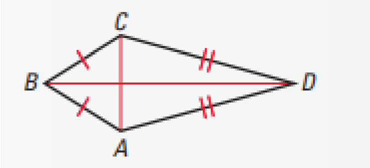
D. Kite

Definition: Kite is a quadrilateral that has two pairs of consecutive congruent sides, but opposite sides are not congruent.



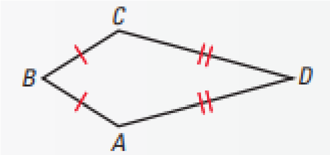
Properties of kite

1. The diagonals of kite are perpendicular to each other. But they do not bisect each other.



$\overline{AC} \perp \overline{BD}$

2. One pair of opposite angles of kite are congruent.



$\angle A \cong \angle C, \angle B \ncong \angle D$

Exercise 5.2.2

1. Write true if the statement is correct and write false if the statement is wrong.
   a. All rectangles are parallelogram.
   b. The diagonals of rhombus are congruent
   c. The opposite sides of kite are congruent
   d. All squares are rhombus
   e. A quadrilateral with congruent diagonals is necessarily rectangle

f. All trapeziums are quadrilateral

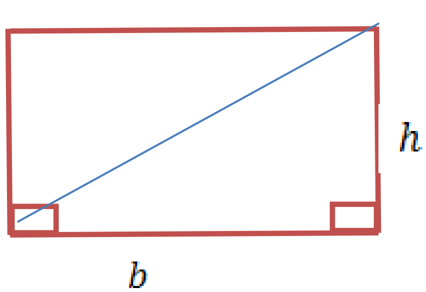
2. Fill in the blank space with the correct answer
   a. A quadrilateral with both pair of opposite sides are parallel is_______
   b. A quadrilateral with adjacent sides congruent, but opposite sides are not congruent is______
   c. A rhombus with congruent diagonals is ________
   d. A quadrilateral with exactly one pair of opposite sides parallel is_______
   e. A parallelogram with all four angles are right angle, but the diagonals are not perpendicular is _________
   f. A parallelogram in which its diagonals are congruent, perpendicular and bisect each other is_________
3. Answer the following questions
   a. Describe similarity and difference of rhombus and kite.
   b. Describe similarity and difference of square and rectangle.

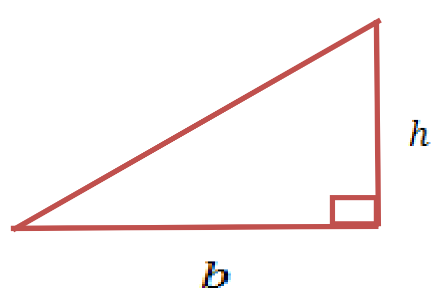
5.3.Area and perimeter of triangle

- solve routine and non-routine problems invoving Perimetre and area of triangle.

a. Area of right angled triangle

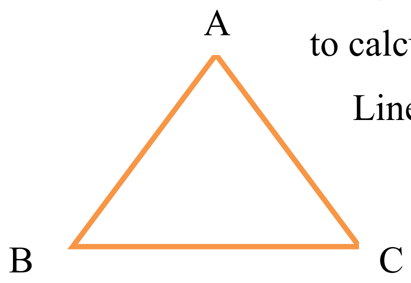
he formula of area of right angled triangle is derived from area of rectangle. The diagonal of a rectangle divides the rectangle in to two equal right-angled triangles.





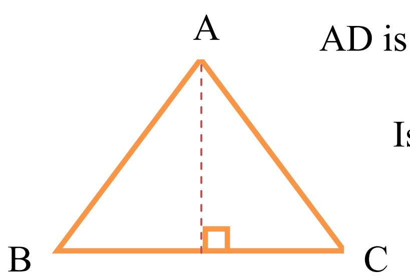
$A = bh$ the area of triangle is half of the area of Rectangle, so the area of triangle is: $A = \frac{1}{2}bh$

b. Area of acute angled triangle



to calculate area of such triangle, draw perpendicular Line from one of the vertices to the opposite side.

The perpendicular line segment from the vertex to the opposite side is called altitude or height(h) and the opposite side is the base(b).



AD is height(h) and BC is base(b). The area of ΔABC Is the sum of area of $\Delta ABD \text{ and } \Delta ADC$

Area of $\Delta ABC = area\ of\ \Delta ABD + area\ of \Delta ADC$

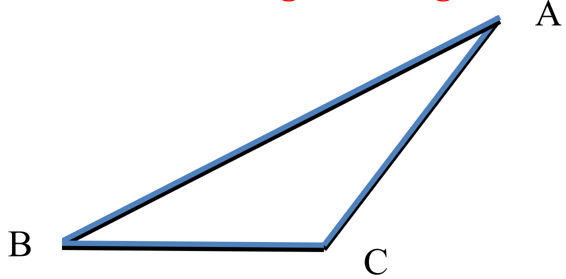
$$A = \frac{1}{2}BD \times AD + \frac{1}{2}AD \times DC$$

$$A = \frac{1}{2}AD \times (BD + DC)$$

$$A = \frac{1}{2}AD \times BC \ldots\ldots\ldots\ldots\ldots\ldots(BD + DC = BC)$$

$$A = \frac{1}{2}bh \ldots\ldots\ldots where\ AD\ is\ height(h)\ and\ BC\ is\ base(b)$$

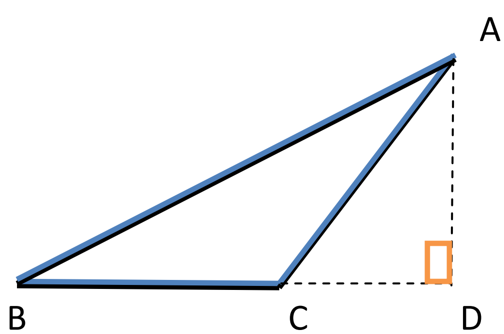
c. Area of obtuse angled triangle



to calculate area of obtuse angled

Draw height from vertex A to side

Extension of base BC



Area of ΔABC is equal to area of

ΔABD minus area of ΔADC

Area of $\Delta ABC = area\ of\ \Delta ABD - area\ of \Delta ADC$

$$A = \frac{1}{2}BD \times AD - \frac{1}{2}AD \times DC$$

$$A = \frac{1}{2}AD \times (BD - DC)$$

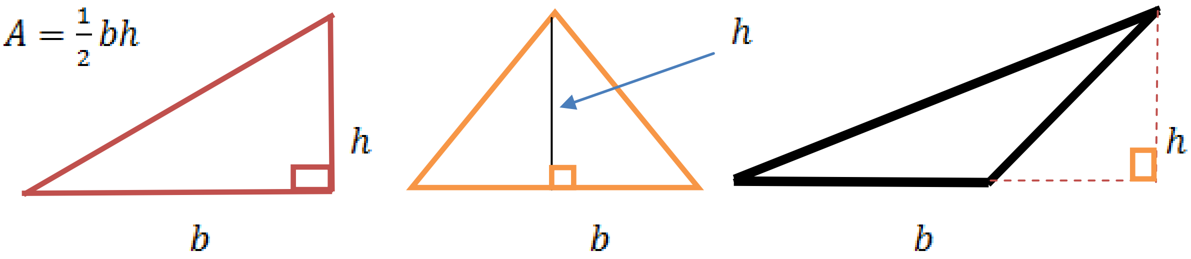
$$A = \frac{1}{2}AD \times BC \ldots\ldots\ldots\ldots\ldots\ldots (BD - DC = BC)$$

$$A = \frac{1}{2}bh \ldots\ldots\ldots where\ AD\ is\ height(h)\ and\ BC\ is\ base(b)$$

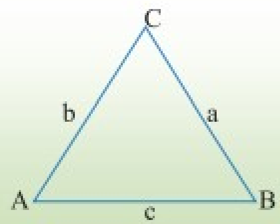
Note:

The area of triangle whose base b and altitude(height) to this base h is given by

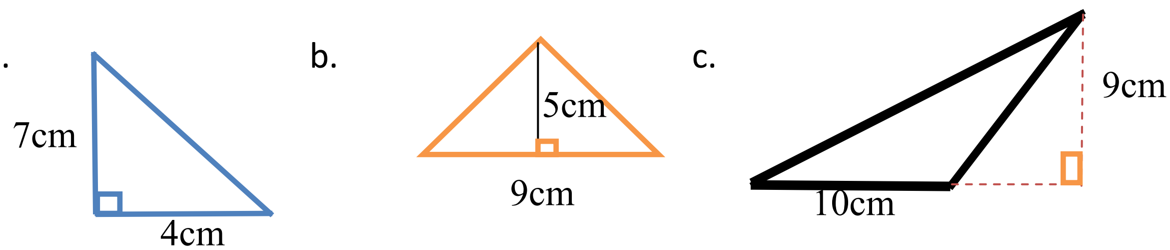
$A = \frac{1}{2}bh$



Note: If the lengths of the sides of a triangle are a, b and c, then the perimeter p of the triangle is p = a + b + c.



Example 1: Calculate the area of the following triangles.



Solution:

a. $A = \frac{1}{2}bh$

$A = \frac{1}{2} \times 7cm \times 4cm$

$A = \frac{28\ cm^2}{2}$

$A = 14cm^2$

b. $A = \frac{1}{2}bh$

$A = \frac{1}{2} \times 9cm \times 5cm$

$A = \frac{45\ cm^2}{2}$

$A = 22.5cm^2$

c. $A = \frac{1}{2}bh$

$A = \frac{1}{2} \times 10cm \times 9cm$

$A = \frac{90\ cm^2}{2}$

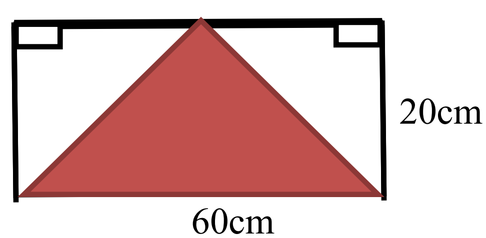
$A = 45cm^2$

Example 2: Using the figure below, calculate

a. the area of rectangle

b. the area of shaded region

c. the area of unshaded region



Solution:

a. Area of rectangle $= bh$

$= 60cm \times 20cm$

$\underline{\underline{= 1200cm^2}}$

b. Area of shaded part $= \frac{1}{2}bh$

$= \frac{1}{2} \times 60cm \times 20cm$

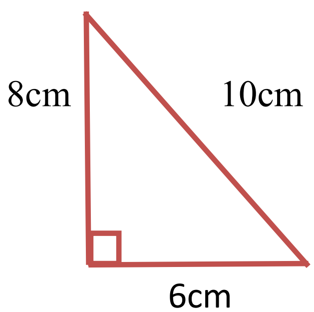
$\underline{\underline{= 600cm^2}}$

c. Area of unshaded part= area of rectangle minus area of shaded part

$= 1200\ cm^2 - 600cm^2$

$\underline{\underline{= 600cm^2}}$

Example 3: what is the area and perimeter of the triangle below



Solution: $A = \frac{1}{2}bh$

$A = \frac{1}{2} \times 6cm \times 8cm$

$A = \frac{48\ cm^2}{2} = 24cm^2$

Perimeter of the triangle is sum of each side:

$P = 8cm + 10cm + 6cm = 24cm$

Example 4: the area of a triangle is 64 cm^2. If the base is 16 cm long, then calculate the height of the triangle.

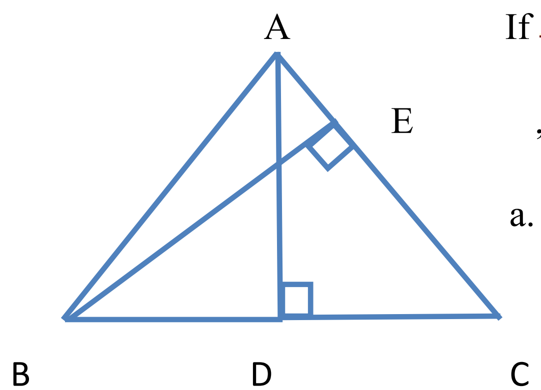
Solution: $A = 64cm^2, b = 16cm, h = ?$

$A = \frac{1}{2}bh$

$$64cm^2 = \frac{1}{2} \times 16cm \times h$$

$$\frac{64cm^2}{8cm} = \frac{8cm \times h}{8cm}, h = 8cm$$

Example 5: based on the triangle below answer the following questions.



If $AD = 12cm, BC = 8cm$ and $AC = 6cm$

, then calculate

a. Area of ΔABC

b. the length BE

Solution:

a. $A(\Delta ABC) = \frac{1}{2}bh$

$= \frac{1}{2} \times BC \times AD$ ..... Using base BC and height AD

$= \frac{1}{2} \times 8cm \times 12cm$

$= 48cm^2$

b. Using the other base and height, area of ΔABC is

$A(\Delta ABC) = \frac{1}{2}bh$

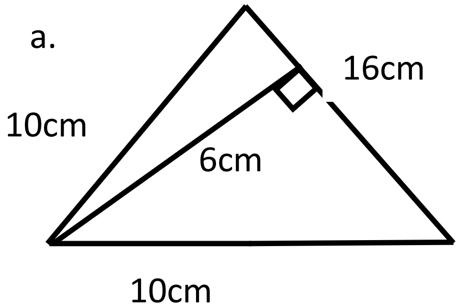
$A(\Delta ABC) = \frac{1}{2} \times AC \times BE$ .... Using base AC and height BE

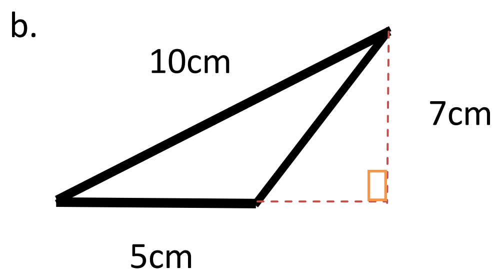
$48cm^2 = \frac{1}{2} \times 6cm \times BE$

$\frac{48cm^2}{3cm} = \frac{3cm \times BE}{3cm}$, $BE = 16cm$

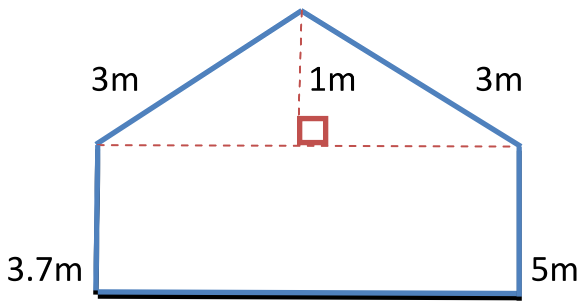
Exercise 5.3.1:

1. Calculate the area and perimeter of the following triangles.

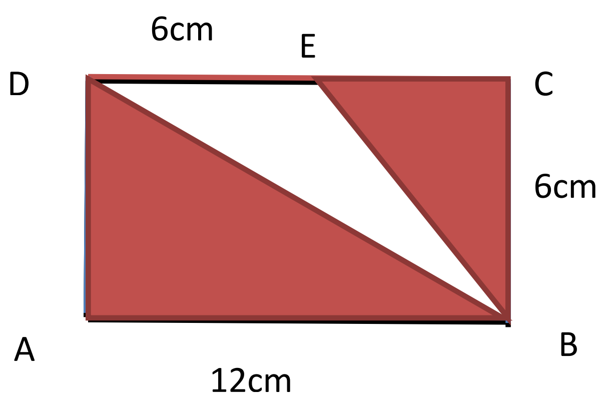




2. The area of a triangle is 63 cm^2. If its height is 9cm long, then calculate the length of its base.
3. The figure below represents a wall of a certain building. Find the area and perimeter of the wall



4. Based on the figure below, answer the following questions



a. Calculate the area of shaded region

b. Calculate the area of unshaded part

5.4.Perimeters and Areas of four sided figures

At the end of this section you should be able to:

- solve routine and non-routine problems invoving Perimetre and area of four sided figures.
- Solves routine and non-routine problems involving area of composite figures formed by any two or more figures such as triangle, square, and rectangles.

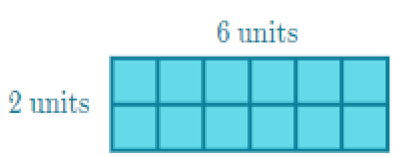
Introduction

In grade 6 you have learnt about perimeter and area of different closed figures. In this section you will also learn how to calculate, the area and perimeter of rectangle,square,parallelogram, trapezium, rhombus and kite. In ordre to have an obiviuos understang about the topics you need to follow the fowing activities.

Activity 5.4.1

Discuss with your friends/partners

1. Take a metre scale and measure the length of all corners of your classroom. Tell your classmates the value you get.
2. Multiply only the value of length and width. What do you think about the value you get?
3. Considering the following rectangle answer the following questions?

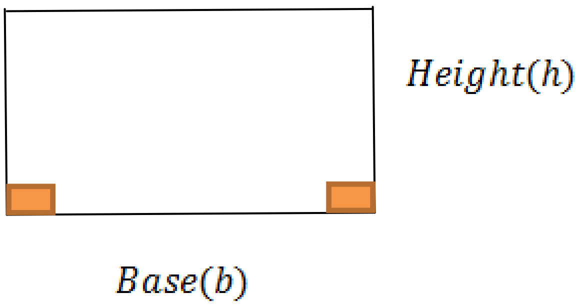


a. What is the area of the rectangle above in square units?

b. What is the perimeter of the rectangle?

4. What is area of a closed region?
5. What is perimeter of a closed region?
6. How do you calculate area and perimeter of a rectangle?

1. Area and perimeter of rectangle



- Area of rectangle is the product of its base and height.

$$Area\ of\ rectangle = base \times height$$

$$A = b \times h$$

- The perimeter of rectangle is calculated as:

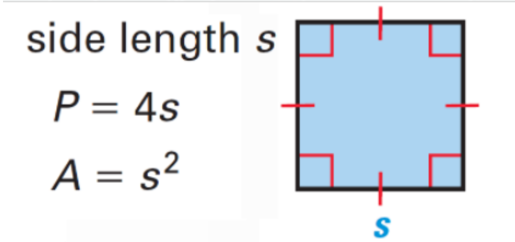
$$perimeter = b + h + b + h$$

$$P = 2b + 2h$$

$$P = 2(b + h)$$

2. Area and perimeter of square

- Square is a rectangle whose base and height are equal.
- The area of square whose side length $'S'$ is $A = S^2$ and its perimeter is $P = 4S$



Example: calculate the area and perimeter of the rectangle given below

$8cm$

$6cm$

$A = b \times h$

$A = 6cm \times 8cm = 48cm^2$

$P = 2(b + h)$

$P = 2(6 + 8) = 2 \times 14 = 28cm$

Example: calculate the area and perimeter of square whose side length is $10m$

Solution: $S = 10m, A = ?, P = ?$

$A = S^2$

$A = (10m)^2 = 100m^2$

$P = 4S$

$P = 4 \times 10m = 40m$

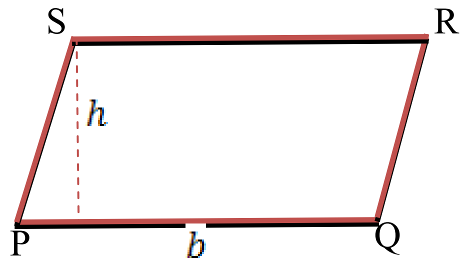
3. Perimeter and area of parallelogram

Activity 5.4.2

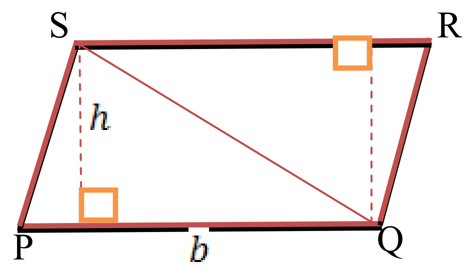
Discuss with your friends/partners

1. What is parallelogram?
2. list its properties of parallelogram
3. Consider the parallelogram below, where PQ is base(b) and the altitude(h) from S to PQ

a.



b.



i. Construct altitude from Q to SR, and diagonal SQ as in b above

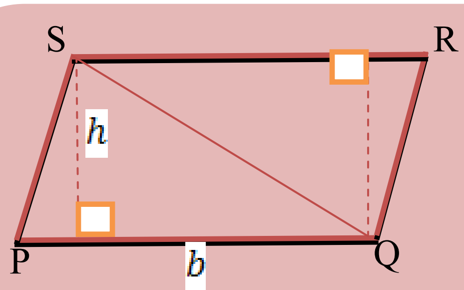
ii. What is the area of ΔPQS in terms of b and h

iii. What is the area of ΔQRS in terms of b and h

iv. How do you find the area parallelogram using areas of ΔPQS and ΔQRS

v. From a, b, c, the formula to calculate area of parallelogram is_____.

As you have done in the above activity the formula to calculate area of parallelogram is derived as follows:



$$A(PQRS) = A(\Delta PQS) + A(\Delta QRS)$$

$$A(PQRS) = \frac{1}{2} \times PQ \times h + \frac{1}{2} \times RS \times h$$

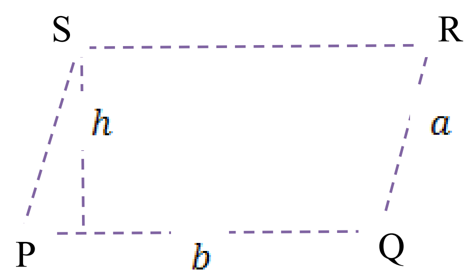
$$A(PQRS) = \frac{1}{2}bh + \frac{1}{2}bh$$

$$A(PQRS) = bh$$

Note:

1. The area of parallelogram with base b and altitude h is given by the formula:

$A = bh$



2. The perimeter of parallelogram is calculated by adding all its sides

$P = PQ + QR + RS + PS$

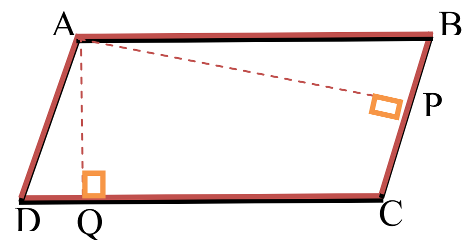
$$P = b + a + b + a$$

$$P = 2a + 2b$$

$$P = 2(a + b)$$

Example 1: In the figure given below,AP, AQ are altitude of the paralellogram ABCD.If AQ = 4cm CD = 5cm and AP = 8cm,then calculate

a. Area of the parallelogram
b. Length of BC
c. Perimetre of the paralelogram



Solution:

a. $A = bh$

$A = DC \times AQ$…. using base DC
And height AQ

$A = 5cm \times 4cm$

$A = 20cm^2$

b. $A = bh$

$A = BC \times AP$…. using base BC

And height AP

$$\frac{20cm^2}{8cm} = \frac{BC \times 8cm}{8cm}$$

$$BC = 2.5cm$$

c. perimeter of the parallelogram is:

$P = AB + BC + CD + AD$

$P = 5cm + 2.5cm + 5cm + 2.5$, $AB = DC = 2.5cm, AD = BC = 2.5cm$

$P = 15cm$

Example 2: The area of parallelogram is $42cm^2$.If the base of the trapezium is $6cm$, then find the corresponding altitude.

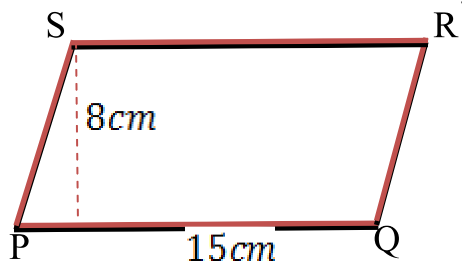
Solution: $A = 42cm^2, b = 6cm, h = ?$

$A = bh$

$\frac{42cm^2}{6cm} = \frac{6cm \times h}{6cm}$, $h = 7cm$

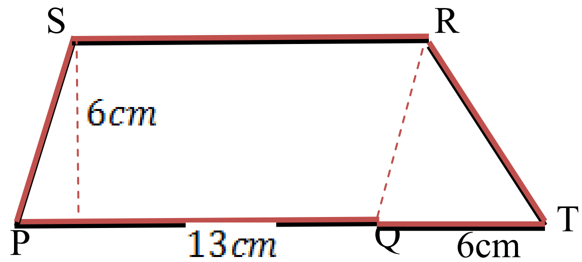
Exercise 5.4.1

1. Calculate the area of the following parallelogram



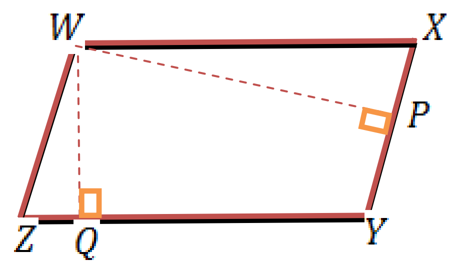
2. ABCD is a parallelogram of area $18cm^2$. Find the length of the corresponding altitudes, if $AB = 5cm$

3. Calculate the area of the region $PTRS$, if PQRS is parallelogram



Example 1: In the figure given below,WQ, WP are altitude of the paralellogram WXYZ.If WP = 6cm, XY = 5cm and WQ = 8cm,then calculate

a. Area of the parallelogram
b. Length of ZY
c. Perimetre of the paralelogram



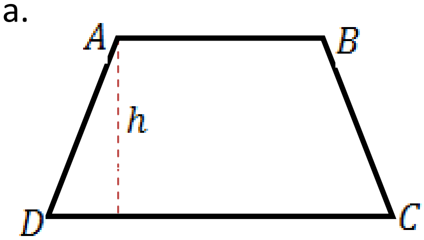
4. Perimeter and area of trapezium

Activity 5.4.3

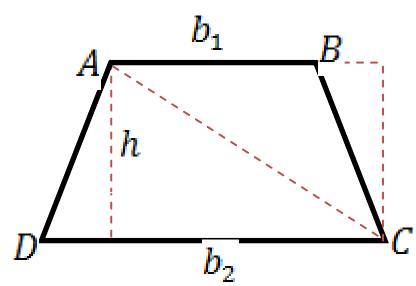
Discuss with your friends/partners

1. What is trapezium?
2. Consider the trapezium below

a.



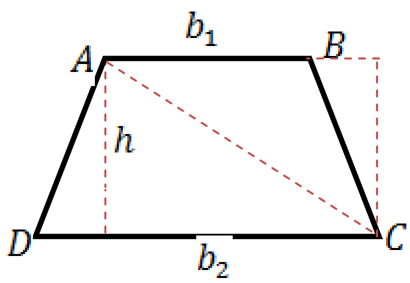
b.



i. Construct diagonal AC and altitude from c to base AB (its extension) as in b above.
ii. What is the area of ΔADC in terms of b_2 and h
iii. What is the area of ΔABC in terms of b_1 and h
iv. The area of trapezium ABCD is the sum of the area of ΔADC and ΔABC

v. From a, b, c the formula to calculate area of trapezium is _____.

As you have done in the above activity the formula to calculate area of trapezium is derived as follows:



- The area of trapezium ABCD is the sum of the area of ΔADC and ΔABC

$$A(ABCD) = A(\Delta ADC) + A(\Delta ABC)$$

$$= \frac{1}{2} \times b_2 \times h + \frac{1}{2} \times b_1 \times h$$

$$= \frac{1}{2} h(b_1 + b_2)$$

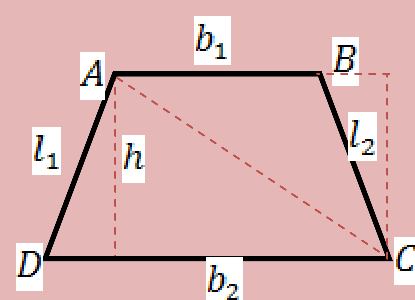
$$A = \frac{h}{2}(b_1 + b_2)$$

Note:

1.Area of trapezium with base b_1 and b_2 and altitude h is given by the formula:

$$A = \frac{h}{2}(b_1 + b_2)$$

2.



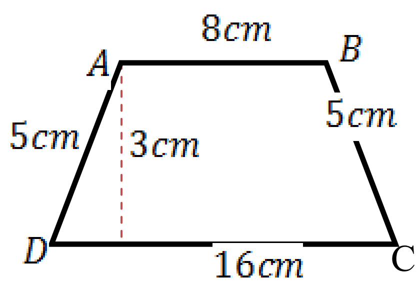
The perimeter of trapezium is the sum of the two base and its legs:

$perimeter = AB + BC + DC + AD$

$P = b_1 + b_2 + l_1 + l_2$

Example 1: calculate the area and perimeter of the trapezium given below

Solution:



$A = \frac{h}{2}(b_1 + b_2)$

$A = \frac{3}{2}(8 + 16)$

$A = \frac{3}{2} \times 24$

$A = 36cm^2$

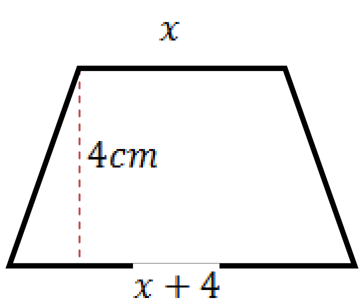
$P = b_1 + b_2 + l_1 + l_2$

$P = 8 + 16 + 5 + 5$

$P = 34cm$

Example 2: The area of trapezium is $48cm^2$ and its height is $4cm$. If one of the base is $4cm$ more than the other base, then calculate each base of the trapezium.

Solution: let the smaller base is x, and the larger base is $x + 4$



$A = 48cm^2$

$A = \frac{h}{2}(b_1 + b_2)$

$48 = \frac{4}{2}(x + x + 4)$

$48 = 2(2x + 4)$

$4x + 8 = 48$

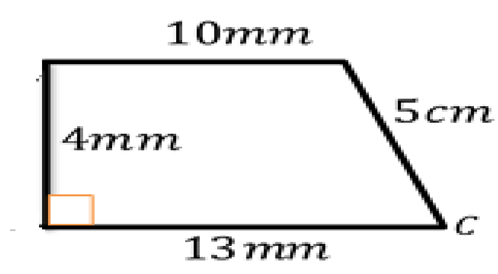
$4x = 40$,

$x = 10cm$

Therefore, the bases of trapezium are Smaller base $x = 10cm$ and larger base is $x + 4 = 10 + 4 = 14cm$.

Exercise 5.4.2

1. Calculate the area and perimeter of the following trapezium



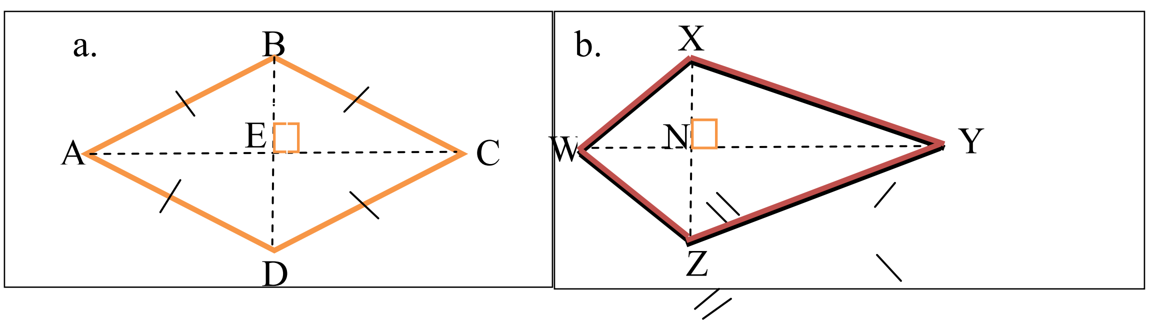
2. The area of trapezium is $170cm^2$. If its height and one of the bases are 17cm and 12 cm respectively, then calculate the other base of the trapezium.
3. One of the bases of trapezium exceeds the other by $2cm$. If the altitude and area of trapezium are $6cm$ and $42\ cm^2$ respectively, then calculate the larger base of the trapezium.

5. Perimeter and area of rhombus and kite

Activity 5.4.4

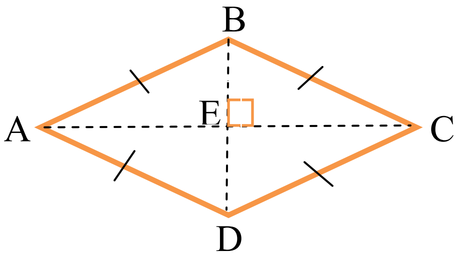
Discuss with your friends/partners

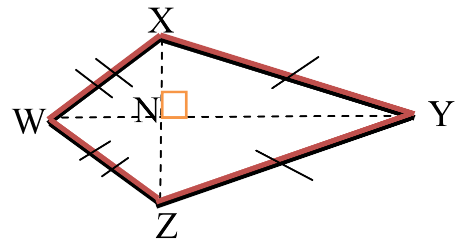
1. Define rhombus and list its properties.
2. Define kite and list its properties
3. Consider the following rhombus and kite



i. Name the diagonals of the rhombus
ii. Name the diagonals of kite
iii. Calculate the area of rhombus by adding area of $\Delta ABC\ and\ \Delta ADC$
iv. Calculate the area of kite by adding area of $\Delta WXY\ and\ \Delta WZY$
v. Using the above result generalize the formula to calculate area of rhombus and kite.

As you have done in the above activity the formula to calculate area of trapezium is derived as follows



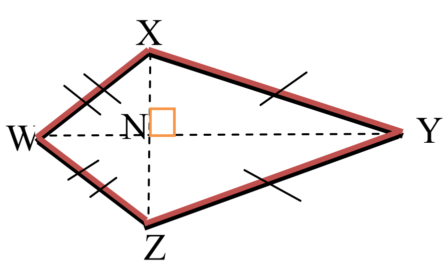


Area of rhombus is the sum of area ΔABC and ΔADC	Area of kite is the sum of area ΔWXY and ΔWZY
$A = A(\Delta ABC) + A(\Delta ADC)$ $A = \frac{1}{2}AC \times BE + \frac{1}{2}AC \times ED$ $A = \frac{1}{2}AC(BE + ED)$ $A = \frac{1}{2}AC \times BD$ … since $BE + ED = BD$ $A = \frac{1}{2}d_1 d_2$, where AC is diagonal 1(d_1) and BD is diagonal 2(d_2)	$A = \Delta WXY + \Delta WZY$ $A = \frac{1}{2}WY \times NX + \frac{1}{2}WY \times NZ$ $A = \frac{1}{2}WY(NX + NZ)$ $A = \frac{1}{2}WY \times XZ$, since $NX + NZ = XZ$ $A = \frac{1}{2}d_1 d_2$, where WY is diagonal1(d_1) and XZ is diagonal 2(d_2)

Note: The area of rhombus and kite is given by the formula $A = \frac{1}{2}d_1 d_2$, where d_1 an d_2 are diagonals. The perimeter of rhombus and kite is calculated by adding the length of all the four sides.

Example 1: Calculate the perimeter and area of the kite given below,

if $WX = 6cm, XY = 10cm, XZ = 8cm \text{ and } WY = 16cm$



Solution:

$A = \frac{1}{2} d_1 d_2$

$A = \frac{1}{2} XZ \times WY$

$A = \frac{1}{2} \times 8 \times 16$

$A = 64\ cm^2$

$P = WX + WZ + XY + YZ$

$P = 6 + 6 + 8 + 8$, since $WX = WZ = 6cm$

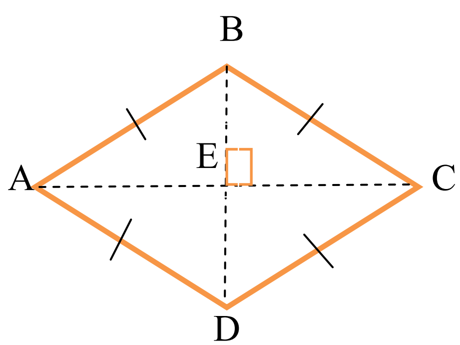
And $XY = ZY = 8cm$

$P = 18cm$

Exercise 5.4.3

1. Calculate the area and perimeter of the following rhombus if

$AB = 5cm, AC = 8cm\ and\ BD = 6cm.$



2. Calculate the area of kite, whose diagonals are 12cm and 16cm.
3. Calculate the perimeter of kite, whose adjacent sides are 18mm and 10mm.
4. The area of rhombus is $144cm^2$. If one of its diagonals is 18cm long, then calculate the length of the other diagonal.

5.5.Circumference and Area of a crcle.

At the end of this section you should be able to:

- Define circumference and area of a circle

Calculate the circumference and area of a circle.

Activity 5.5.1

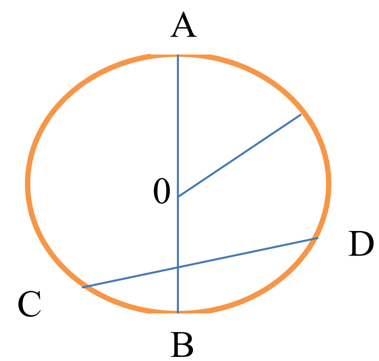
1. Have you every observed circular object in your locality? If yes, tell your classmates loudly.
2. Can you tell your classmates how to draw a circle?
3. What are the materials you use to draw a circle?
4. Draw a circle and indicate different parts of a circle.

5.5.1. Revision on the parts of a circle

In grade 6 you had learnt about parts of circles. It is necessary to note the following parts of a circle to clearly remember what you learnt in your previous class.

1. A circle is the set of all points on a plane that are at equal distance from the center of a given point called $center\ of\ a\ circle$.
2. A chord of a circle is any line segment F that joins two points on the circle.$\overline{AB}\ and\ \overline{CD}$
3. A diameter (d) is a line segment that passing through the center and joining any two points on the circle.
4. A radius (r) of a circle is the distance from the center to any point on the circle.

   OA, OB and OF are radii of circle

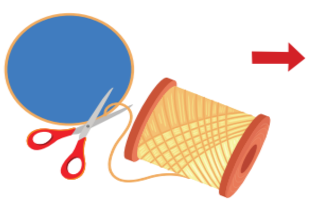


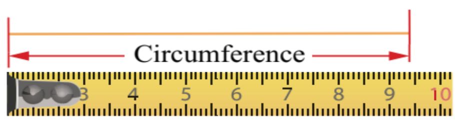
5.5.2. Circumference of circle

As you have seen before, the distance all-round the plane figure is perimeter. The analogy of perimeter for a circle is named as circumference of a circle.

Activity 5.5.2

1. Materials needed: ruler, string, scissor and circular objects of various sizes.
   a. Bring a circular object.
   b. Use a ruler to measure the diameter of your circular object. Record your finding.
   c. Wrap a string around the circular object once. Mark the string where it meets itself.
   d. Lay the string out straight, measure and then record the length of the string with your ruler. Can you call the circumference of the circle?
   e. Divide the measure of the circumference obtained in (d) by the measure of the diameter obtained in (b). Record your answer.
   f. Repeat this activity with circular objects of various sizes.
2. Compare the results you obtained in (e) and (f). What do you observe or find?
3. How is the circumference related to the diameter?
4. What is your conclusion?





Circumference of a Circle

For any circle with diameter d, or radius r, its circumference is given by:

$\boldsymbol{C = \pi d \text{ or } C = 2\pi r}$

Example: Find the circumference of a circle with diameter 12m.

Solution:

$C = \pi d$ and $d = 12m$

Then, $C = \pi d = \pi \times 12m = 3.14 \times 12m \approx 53.38m,$

Therefore, the circumference of the circle is $12\pi m$ or approximately 53.38m.

Example: The circumference of the circular basement of the building is 220m. Find the radius of the basement. (Use $\pi = \frac{22}{7}$)

Solution:

Given: $C = 220m$

$C = 2\pi r$

\ $220m = \left(2 \times \frac{22}{7} \times r\right) = \frac{44}{7}r$

$220m \times \frac{7}{44} = \frac{44}{7}r \times \frac{7}{44}$

$35m = r$

Therefore, the radius of the building is $35m$.

Example: The figure below is formed a semicircle and a straight line. What is its perimeter? (Use $\pi = \frac{22}{7}$)

Solution: Diameter of circle, $d = 6m$

Perimeter of a semicircle = Halve of the circumference of a circle

$= \frac{1}{2} \times \pi \times d$

$= \frac{1}{2} \times \frac{22}{7} \times 6m = 11m$

Perimeter of a semicircle = $\frac{1}{2}$ of the circumference of a circle + length diameter

$= 11m + 6m = 18m$

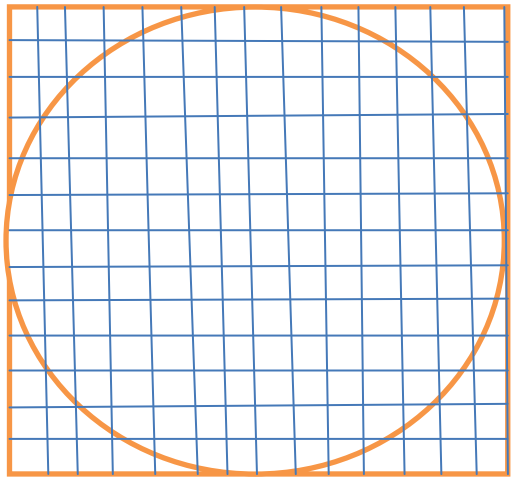
5.5.3. Area of a Circle

From your previous grade lesson you could remember what to mean area of a plane figure. It is the number of square units that cover the plane figure. We can approximate the area of a circle by using square grid. That is, dividing a circle in to small pieces of unit squares (a square of area$1cm^2$).

Number of squares covered by the quadrant is 39.

Area of a quadrant =39 $\times$ $1cm^2$=39cm^2

Area of a circle =39cm^2 $\times$ 4 = 156cm^2

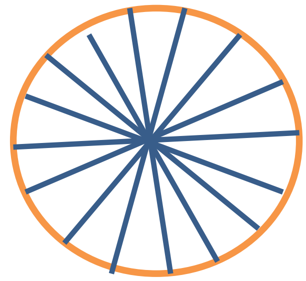


The following activity will help you find the way of calculating the accurate area of a circle.

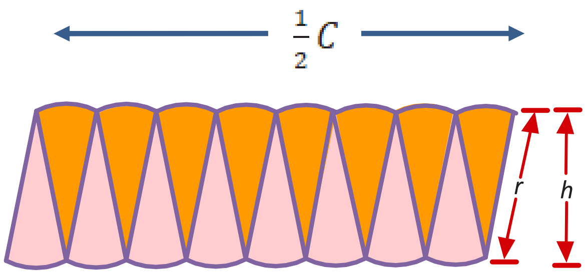
Activity 5.5.3

Materialsneeded:paper,compass,rulerorstraightedge,scissors,pencil.

1. Draw a circle and several radii that separate the circle into**equal-sized** sections. (Let r units represent the length of the radius and C units represent the circumference of the circle).



2. Cut out each section of the circle.
3. Reassemble the sections in the form of a parallelogram.



4. Whatis the base length of this "parallelogram"?Howabout length of its height?
5. Find the area of the parallelogram. Remember area of a parallelogram is:A= bh.
6. How could you use this formula to find the area of acircle?
   - The base of the parallelogram shown on the activity above is half the circumference of the circle

$$\frac{1}{2}C = \frac{1}{2} \times 2\pi r$$

   - The height of the parallelogramis the length of the radius.Substitute this information into the formula for the area of a parallelogram.

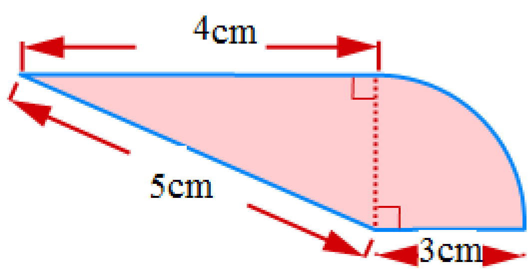
Area of acircle = area of the parallelogram $= \frac{1}{2}C \times r = \pi r \times r = \pi r^2$

Exercise 5.5.1

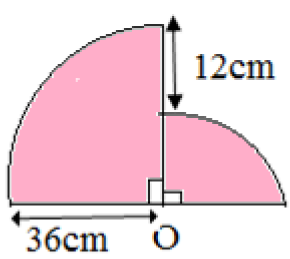
1. Complete the following table(Use $\pi = \frac{22}{7}$)

Radius	Diameter	Circumference	Area
12cm			
		16cm	
			$\frac{22}{7}\text{cm}^2$
	24cm		

2. If the diameter of a circle is three times as long as the diameter of another circle, explain how the circumference of two circles relates to each other.
3. Find the perimeter and area of the following figure. ($\pi = \frac{22}{7}$)



4. What happens to the area of a circle if the radius of the circle is doubled?
5. Calculate the perimeter and area of the figure given below.



5.6.Applications of Perimeter and Area of Plane Figures.

At the end of this section you should be able to:

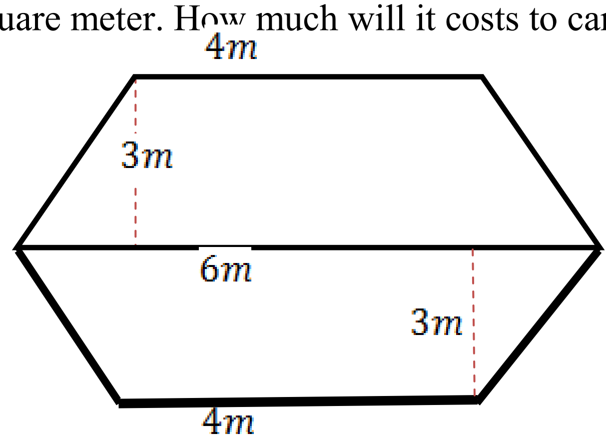
- Apply the concept of perimeter and area of plane figures in real life problems.

In this section you will learn some of the application of area and perimeter in our real world.

The concept of area and perimeter is applied:

- In decorating the wall of a house, so that to estimate the cost of painting
- To estimate the fencing material required in fencing a garden.
- To estimate the area of roofing material.
- Making various furniture.
- To estimate carpet size for floor.

Example 1: the diagram below shows the floor plan of a hotel. Carpet costs Birr 100 per square meter. How much will it costs to carpet the hotel.



Solution: First calculate the area of the floor

Area of the floor is the sum of the areas of the two trapeziums.

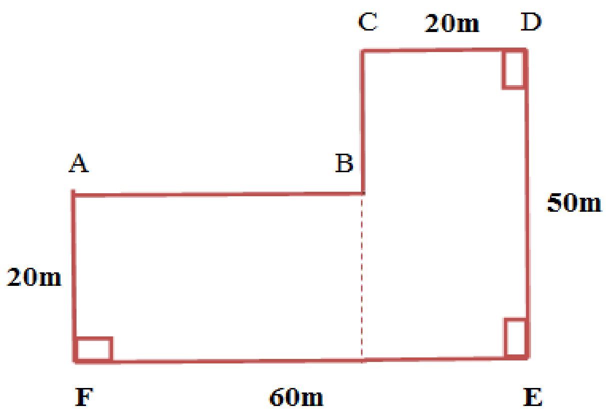
$$A = \frac{h}{2}\,(b_1 + b_2) + \frac{h}{2}\,(b_1 + b_2)$$

$A = \frac{3}{2}(4 + 6) + \frac{3}{2}(4 + 6)$ $\quad A = 15 + 15 = 30m^2$

The cost of $1m^2$ carpet is Birr 100, so the cost of $30m^2$ is $Birr\ 30 \times 100 = 3000$

Hence, it costs Birr 3000 to carpet the floor.

Example2: A farmer wants today fence a plot of land shown below



a. Calculate the length of fencing material required
b. If the cost of 1m fencing material is Birr 250, then calculate the cost to fence the plot of land

Solution:

a. $P = AB + BC + CD + DE + EF + FA$

$P = 40 + 30 + 20 + 50 + 60 + 20$

$P = 220m$

$BC = DE - AF = 50 - 20 = 30$

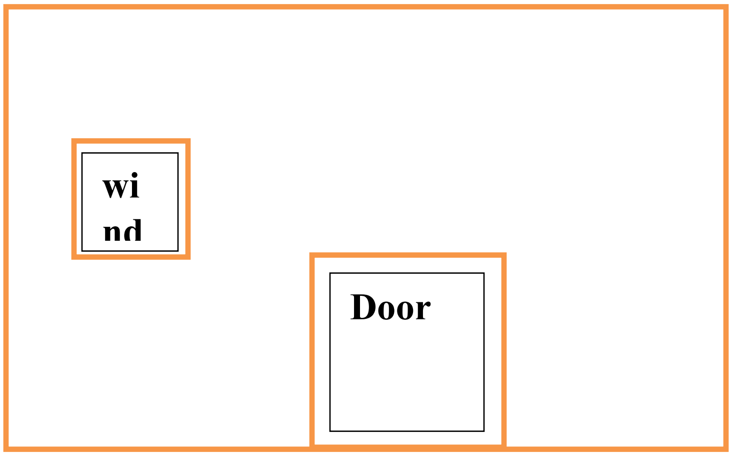
And

$CD = EF - AB = 60 - 40 = 20$

Therefore, 220m of fencing material is required.

b. The cost of 1m fencing material is Birr 250, so the cost of 220m fencing material is $Birr 250 \times 220 = Birr\ 55{,}000$

Example 3: The following diagram shows the front part of a building



If you paint the wall of the building, then

a. calculate the area of the building painted.
b. If it costs Birr200 to paint $1\ m^2$ wall, then how much it costs to paint the wall?

Solution:

a. Note that the area of window and door will not be painted, so the area of wall painted is equal to the are of bigger rectangle minus the area of door and window

$$painted\ area = 8 \times 3 - 1 \times 1.5 - 2 \times 1.7$$
$$= 24 - 1.5 - 3.4$$
$$= 19.1m^2$$

b. it costs Birr200 to paint $1\ m^2$. So, to paint $19.1m^2$, it costs

$$Birr\ 200 \times\ 19.1 = Birr\ 3820$$

Example 4: A class room has length of 9m and width of 6m. The flooring is to be replaced by terazo tiles of size 30cm by 30cm. how many terazo tiles are needed to cover the class room.

Solution: Area of class room, $A = 9m \times 6m = 54m^2$

$$= 540{,}000cm^2 \quad ,1\ m^2 = 10{,}000\ cm^2$$

Area of terazo tile, $A = 30cm \times 30cm = 900cm^2$

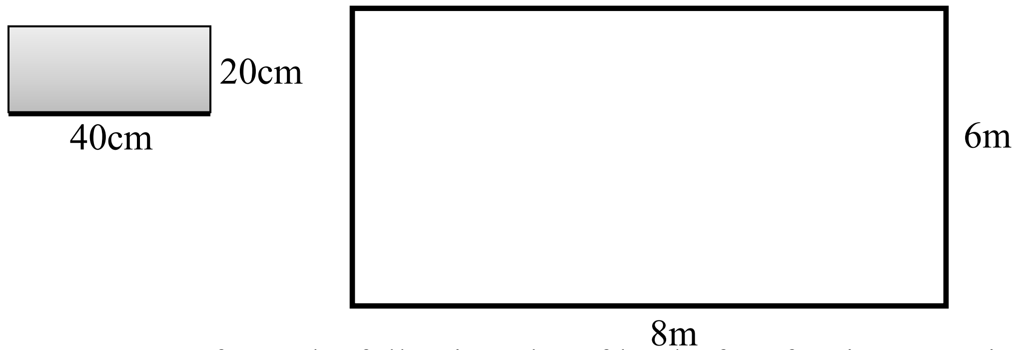
Number of terazo tiles= $\frac{\text{Area of class room}}{\text{Area of terazo tile}} = \frac{540{,}000cm^2}{900cm^2} = 600$

Therefore, 600 terazo tiles are required for flooring

Exercise 5.6.1

1. The length and width of rectangular football field are 100m and 80m respectively. $1\ m^2$ artificial grass costs Birr 500, then how much it costs to cover the field by artificial grass.
2. The following diagram shows one side of a building. If this part will be covered by HCB (hollow concrete block), then how many HCB is required?

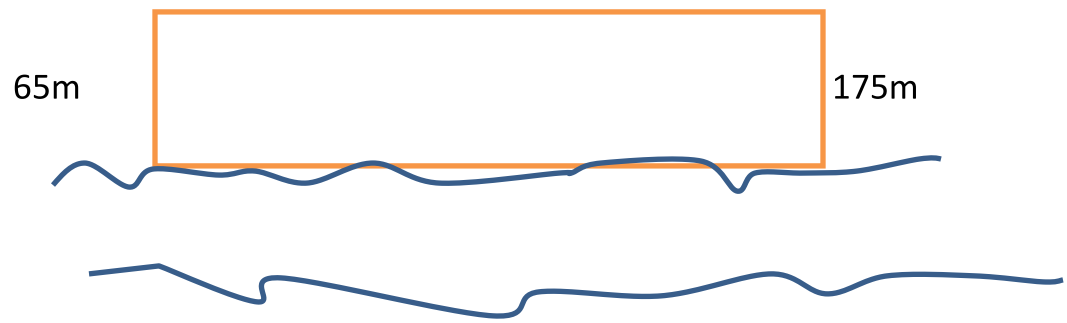
(the size of HCB is 20cm by 40cm, and $1\ m^2 = 10{,}000\ cm^2$)



A farmer wants to fence the following plot of land. If no fencing material is not required along river side, then calculate

a. The length of fencing material required
b. If the cost of $1\ m$ fencing material is Birr 150, then calculate the cost to fence the land.

3. A rectangular flower bed measures 10m by 6m. It has a path 2m wide around it. Find the area of the path.



Summary for unit 5

- Triangles based on their sides and angles classified as follows

Based on sides	Based on angles
1. Scalen triangle	1. Acute angled triangle
2. Isosceles triangle	2. Right angled triangle
3. Equilateral triangle	3. Obtuse angled triangle

- A quadrilateral is a four-sided geometric figure bounded by line segments.
- A trapezium is a special type of quadrilateral in which exactly one pair of opposite sides are parallel.
- A parallelogram is a quadrilateral in which both pair of opposite sides are parallel.
- rectangle is a parallelogram with all its angles are right angle.
- square is a parallelogram with four congruent sides and four right angles.
- A rhombus is a parallelogram in which all its sides are congruent.
- Kite is a quadrilateral that has two pairs of consecutive congruent sides, but opposite sides are not congruent.
- Formula to calculate area and perimeter of different geometric figures

1. **Triangle**	4. **Rhombus**
$A = \frac{1}{2}bh \; and \; P = a + b + c$	$A = \frac{1}{2}d_1d_2 \;\; and \; P = 4S$
2. Rectangle	5. **Kite**
$A = bh \;\; and \; P = 2(b + h)$	$A = \frac{1}{2}d_1d_2 \;\; and \;\; P = 2(a + b)$
3. **Square**	6. **Trapezium**
$A = \; S^2 \; and \; P = 4S$	$A = \frac{h}{2}(d_1 + d_2\,) \;\; and \;\; P = a + b + b_1 + b_2$

Review exercise for unit 5

I. Write True for the correct statements and False for the incorrect statements.

1. If the adjacent sides of a parallelogram are congruent, then the parallelogram is rhombus
2. If the adjacent sides of a parallelogram are congruent, then the parallelogram is square.
3. The sides of rectangle make 45^0 with its sides.
4. The diagonals of kite bisect each other.
5. If the diagonals of a parallelogram are perpendicular bisector of each other, then the parallelogram is necessarily rhombus.
6. All squares are rhombus.
7. All rectangles are squares.

II. Fill in the blank space with the correct answer

1. A triangle with all its sides are equal is called_________
2. A triangle with one right angle is called_______
3. A quadrilateral with adjacent sides congruent, but opposite sides are not congruent is called______
4. A parallelogram with all four sides congruent is necessarily ______
5. A rhombus with four right angles is _________

III. Choose the correct answer

1. The area of triangle with base 4cm and height 3cm is _____

   A. $12cm^2$ B. $6cm^2$ C. $7cm^2$ D. $14cm^2$

2. The area of rhombus whose diagonals are 12cm and 18cm is____

   A. $108cm^2$ B. $216cm^2$ C. $80cm^2$ D. $160cm^2$

3. A toilet room has length of 2m and width of 2m. The flooring is to be tiled by ceramic tiles of size 20cm by 20cm. how many ceramic tiles are needed to cover the class room?

   A. 300 B. 150 C. 200 D. 100

4. The area of rectangular field is $480m^2$. If the length of base of the field is 12m, then the perimeter of the field is ______

   A. $40m$ B. $52m$ C. $104m$ D. $80m$

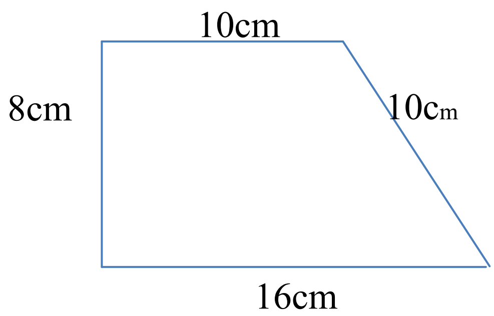
5. The perimeter of kite with its opposite sides are 5cm and 12cm is _____

   A. $60cm$ B. $34cm$ C. $17m$ D. $30cm$

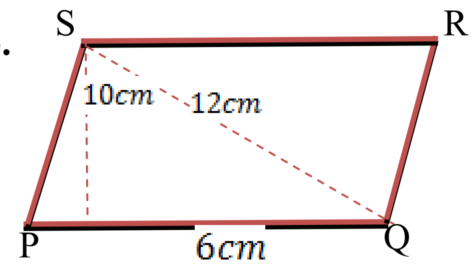
IV. Work out

1. Calculate the area and perimeter of the following figures.

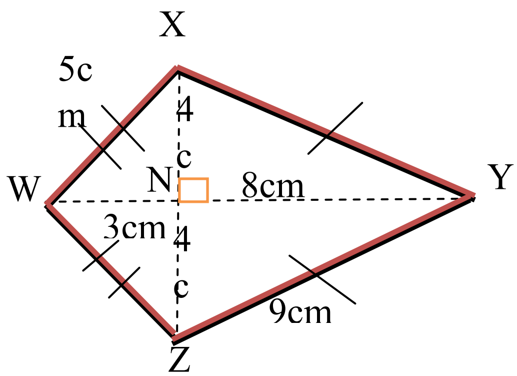
a.



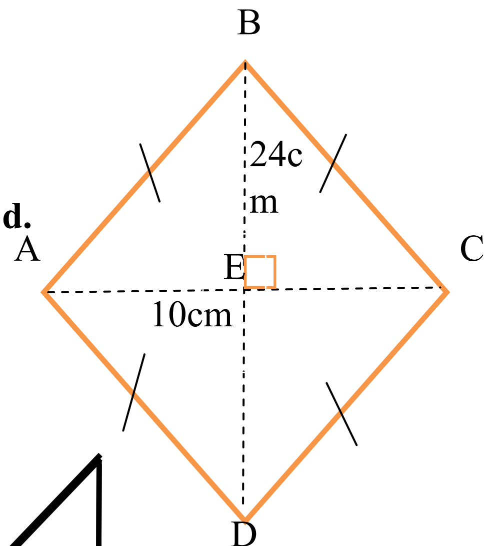
b.



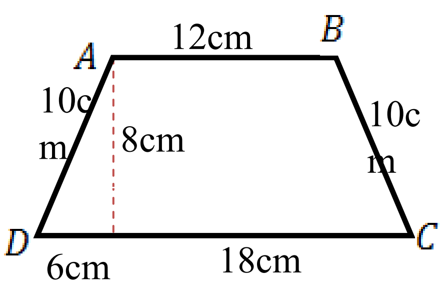
c.



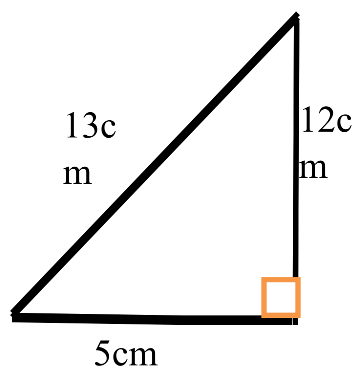
d.



e.



f.



Unit 6

Congruency of plane figures

Unit Outcomes:

At the end of this unit, you will able to:

- Identify congruent triangles by using the tests for congruency (SSS, SAS, ASA).
- Apply real-life situations in solving geometric problems

Introduction

In this unit you will learn about congruence of geometric figures. The unit has two sections. The first section deals with the congruence of geometric shapes. The main focus of this section is to enable you identifying a given triangle is congruent to the other using congruence test for triangles. In the second section you learn the application of congruence of figures in your daily life.

6.1.Congruent of Plane Figures

At the end of this section you should be able to:

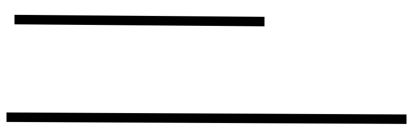
- Explain the concept of congruency of triangles
- Check the congruence of given triangles by tracing, cutting and overlapping.
- Apply real-life applications of congruency of triangles

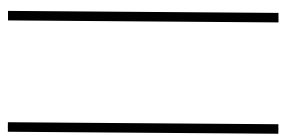
6.1.1 Definition and Illustration of Congruent Figures

Activity 6.1.1

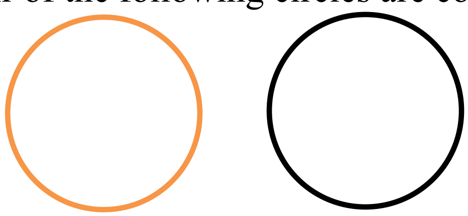
Discuss with your friends/partners

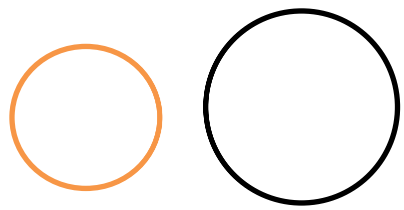
1. Which pair of the following line segments are congruent?

a. 

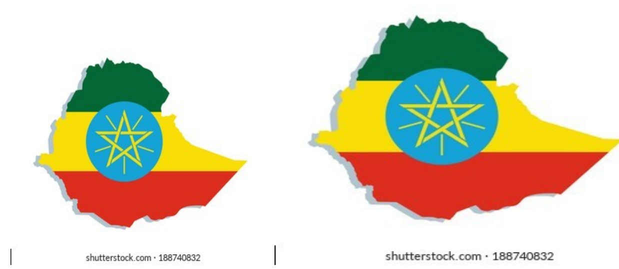
b. 

2. Which pair of the following circles are congruent?

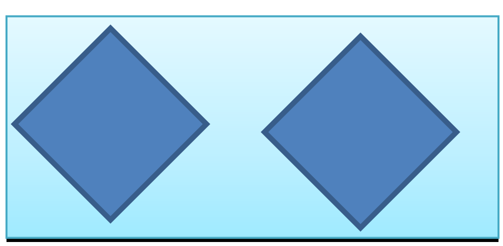
a. 

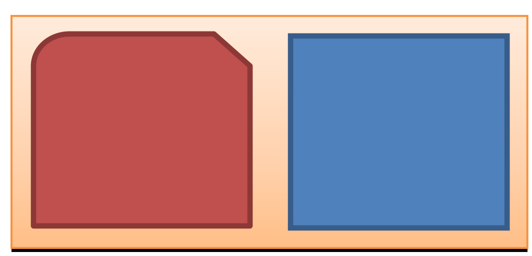
b. 

3. Are the two maps below congruent?



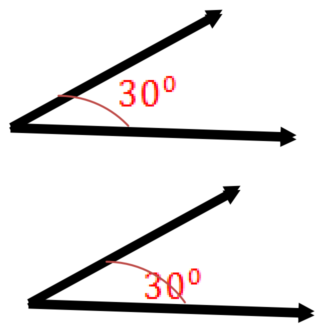
4. Which pair of the following figures are congruent?

a 

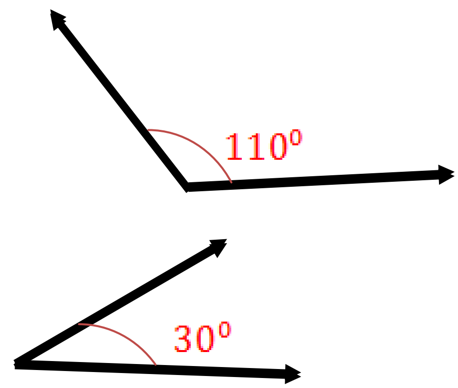
b 

5. Which pair of angles are congruent?

a.



b.



6. when do we say that two figures are congruent?
7. When do we say that two-line segments are congruent?
8. When do we say that two circles are congruent?
9. When do we say that two angles are congruent?

Definition 6.1: Congruent figures are figures that have the same size and shape. Congruent figures are exact copies of one another.

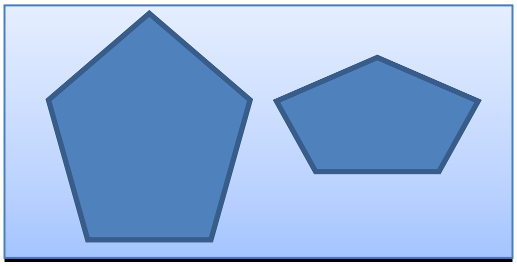
Note:

1. Two figures are congruent, if one figure cover the other completely and exactly.
2. Two-line segments are congruent if they have the same length.
3. Two circles are congruent if they have the same radius.
4. Two angles are congruent if they have the same measure.

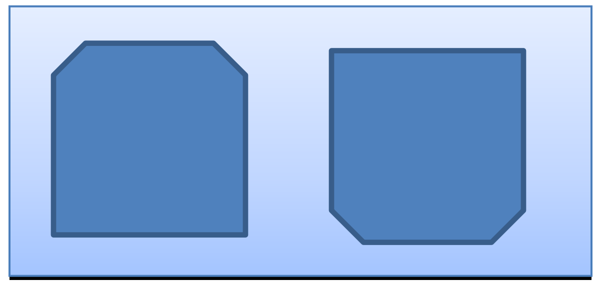
Exercise 6.1.1

1. Which pair of the following figures are congruent?

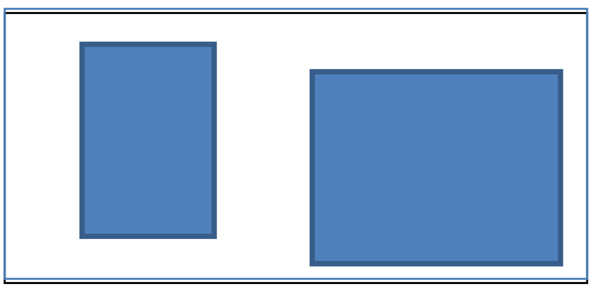
a.



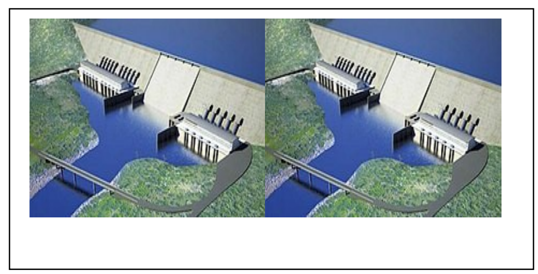
c.



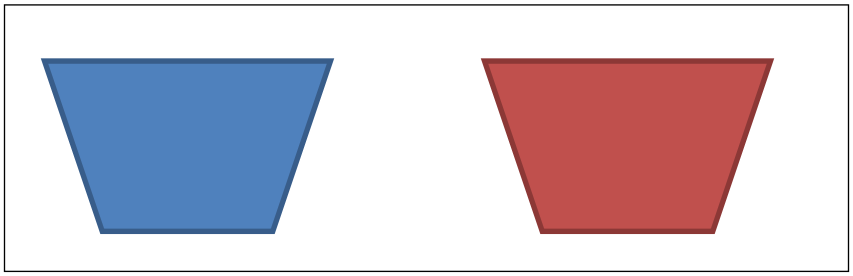
c.



d.



2. Write true if the statement is correct and write false if the statement is wrong.
   a. All line segments are congruent
   b. All circles are congruent
   c. All rectangles are congruent
   d. Two circles having the same diameter are congruent
   e. Two-line segments having the same length are congruent
   f. Two angles that have the same measure are congruent
   g. If two figures have the same shape, then they are congruent.
   h. If two triangles are congruent, then they have the same area.
3. Answer the following questions using the following figures



a. If you put one of the figures above over the other, does it cover completely and exactly?

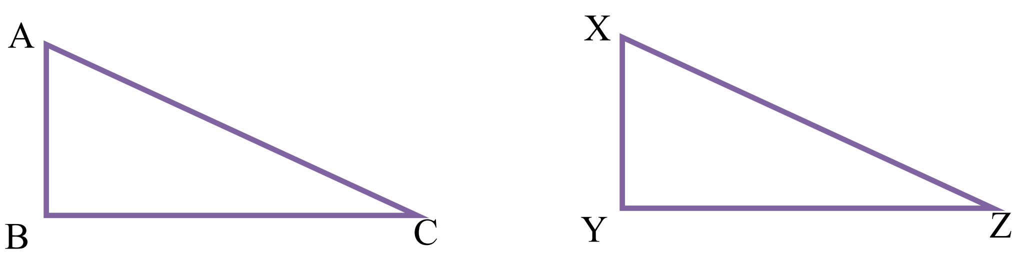
b. What can you say about the figures?

6.1.2 Congruency of Triangles

Activity 6.1.2

Discuss with your friends/partners

1. Consider the following two triangles and measure each side and each angle of the two triangles and fill the following blank space
   a. Length of AB congruent to ________
   b. Length of BC congruent to ________
   c. Length of AC congruent to ________
   d. $\angle A$ is congruent to ________



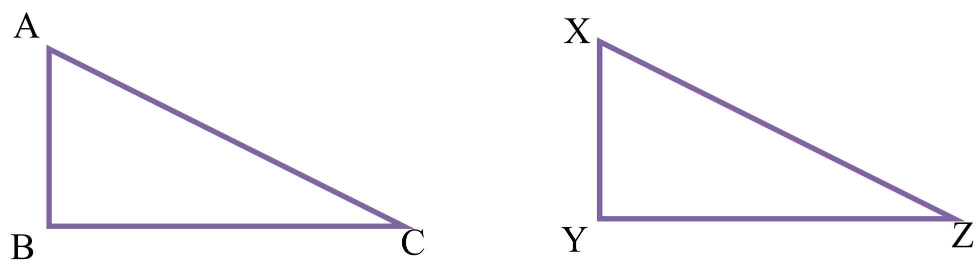
   e. $\angle B$ is congruent to ________
   f. $\angle C$ is congruent to ________

2. If you put ΔABC on ΔXYZ, in such a way that
    a. AB on XY, BC on YZ and AC on XZ, then does ΔABC covers completelyΔXYZ?
    b. What can you say about the two triangles?
3. If ΔABC is congruent to ΔXYZ, then
    a. The corresponding side to AB (the side that will fit or cover side AB completely) is ______
    b. The corresponding side to AC (the side that will fit or cover side AC completely) is ______
    c. The corresponding side to BC (the side that will fit or cover side BC completely) is ______
    d. The corresponding angle to $\angle A$ (the angle that fit $\angle A$) is ____
    e. The corresponding angle to $\angle B$ (the angle that fit $\angle B$) is ____
    f. The corresponding angle to $\angle C$ (the angle that fit $\angle C$) is ____
4. When do we say that two triangles are congruent?

Definition 6 .2: Two triangles are congruent, if their corresponding parts (angles and sides) that match one another are equal.

Note:

1. Two triangles are congruent if they are copies of each other and when you place one triangle on another, they cover each other completely.
2. If ΔABC is congruent to ΔXYZ, then symbolically written as $\Delta ABC \cong \Delta XYZ$
3. If $\Delta ABC \cong \Delta XYZ$, then when you place ΔXYZ on ΔABC, it should satisfy the following six conditions.



i. X falls on A, and $\angle X \cong \angle A$, (i.e. $\angle X$ is congruent to $\angle A$)

ii. Y falls on B, and $\angle Y \cong \angle B$, (i.e. $\angle Y$ is congruent to $\angle B$)

iii. Z falls on C, and $\angle Z \cong \angle C$, (i.e. $\angle Z$ is congruent to $\angle C$)

iv. $\overline{XY}$ falls along $\overline{AB}$, and $\overline{XY} \cong \overline{AB}$ (i.e. $\overline{XY}$ is congruent to $\overline{AB}$)

v. $\overline{YZ}$ falls along $\overline{BC}$, and $\overline{YZ} \cong \overline{BC}$ (i.e. $\overline{YZ}$ is congruent to $\overline{BC}$)

vi. $\overline{XZ}$ falls along $\overline{AC}$, and $\overline{XZ} \cong \overline{AC}$ (i.e. $\overline{XZ}$ is congruent to $\overline{AC}$)

Example 1: If $\Delta ABC \cong \Delta FED$, then find the six congruent corresponding parts of the triangles.

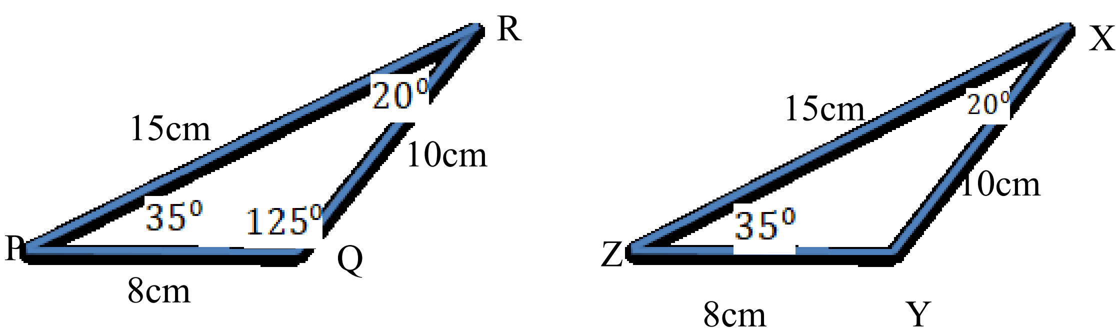
Solution:

i. $\angle A \cong \angle F$ iv. $\overline{AB} \cong \overline{FE}$

ii. $\angle B \cong \angle E$ v. $\overline{BC} \cong \overline{ED}$

iii. $\angle C \cong \angle D$ vi. $\overline{AC} \cong \overline{FD}$

Example 2: consider the following triangles, congruent angles are marked as indicated, based on the figure answer the following questions



a. $\Delta PQR \cong$ ______ b. $\Delta QRP \cong$ _____

Solution:

a. The corresponding part of $\angle P$ is $\angle Z$, The corresponding part of $\angle Q$ is $\angle Y$

The corresponding part of $\angle R$ is $\angle X$,

Therefore, $\Delta PQR \cong \Delta ZYX$

b. $\Delta QRP \cong \Delta YXZ$

Example 3: let $\Delta ABC \cong \Delta DEF$, and $m(\angle A) = 70^0, DE = 6cm$, then find $m(\angle D)$ and the length of AB

Solution:Since the triangles are congruent, their corresponding part are congruent.

So, $m(\angle A) = m(\angle D) = 70^0$ and , $DE =$ AB $= 6cm$

Exercise 5.1.2

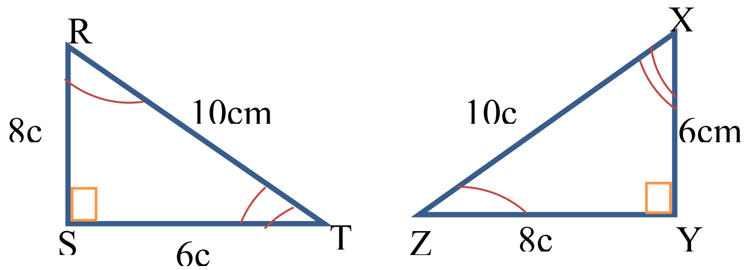
1. Write true if the statement is correct and write false if the statement is wrong
   a. If $\Delta ABC \cong \Delta EDF$, then $m(\angle A) = m(\angle D)$
   b. All equilateral triangles are congruent
   c. If $\Delta ABC \cong \Delta EDF$, then $AB = ED$
   d. If $\Delta ABC \cong \Delta EDF$, then $\angle B \cong \angle D$
   e. If $\Delta ABC \cong \Delta EDF$, then $\overline{AC} \cong \overline{DF}$
   f. Any triangle is congruent to itself.

Note:

1. AB means measure of length of $\overline{AB}$
2. $m(\angle D)$ means measure of $\angle D$

2. If $\Delta ZXY \cong \Delta EDF$, then
   a. $\Delta XYZ \cong$ ______
   b. $m(\angle D) =$ _____
   c. $\overline{FD} \cong$ _____
   d. $FD =$ ____
   e. $\angle D \cong$ ____
   f. $\Delta YXZ \cong$ _____

3. Considering the following triangles and answer the following questions, congruent angles are indicated with the same mark



a. $\Delta XYZ \cong$ _____

b. $\Delta SRT \cong$ _____

c. $\Delta ZYX \cong$ _____

6.1.3 Tests for congruency of triangles (ASA, SAS, SSS)

At the end of this section you should be able to:

- Describe each of the tests for congruence SAS, SSS and ASA
- Identify the congruence of two given triangles by using the tests for congruence SAS, SSS and ASA

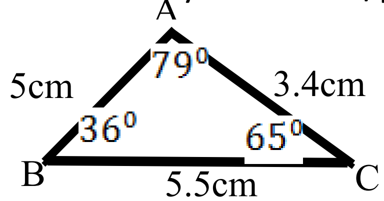
In previous discussion you have seen that, in order to say two triangles are congruent, their six corresponding part must be congruent. But, in this section, you will learn that you do not need all six pieces of information to show the triangles are congruent. By using only, the three parts of a triangle and applying the three tests for congruence (SSS, SAS, ASA), you can show whether two triangles are congruent or not.

A. side – side – side (SSS)congruence test

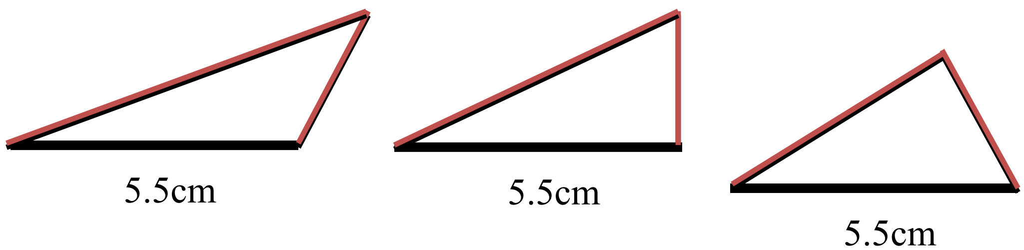
Activity 6.1.3

Read the following and discuss with your friends/partners

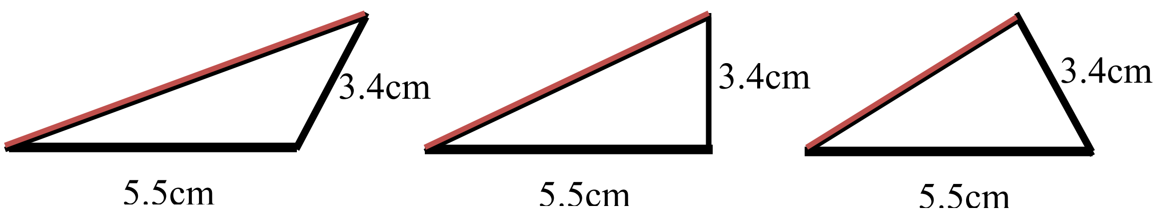
Given ΔABC,



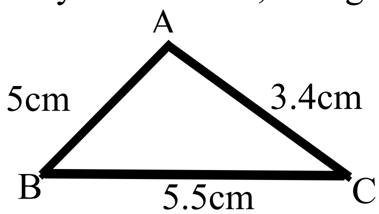
1. If only one side of ΔABC is given say 5.5cm, can you draw the exact copy of it?
    - The answer is **NO**, because you can draw a number of triangles using two another arbitrary side as follows



2. If only two sides of ΔABC is given (say 5.5cm and 3.4cm) can you draw the exact copy of it?
    - The answer is **NO**, because you can draw a number of triangles using the two sides and another arbitrary side as follows



3. If all the three sides of ΔABC is given (say BC= 5.5cm, AC= 3.4cm and AB=5cm) can you draw the exact copy of it?
    - In this case you can draw, a single triangle which the exact copy of ΔABC

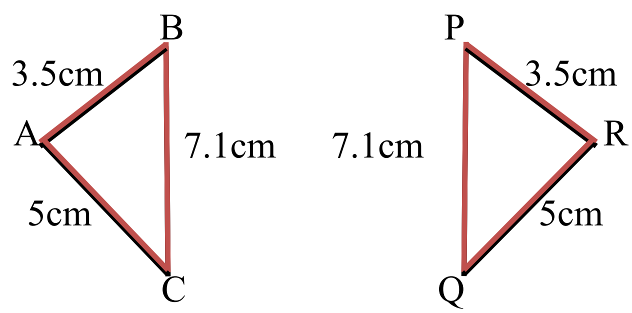


So, to draw an exact copy of ΔABC, we need the length of three sides of the triangle. We call this side – side – side congruence criterion.

Side – Side – Side (SSS) congruence test:

If the three sides of one triangle is congruent to the three corresponding sides of another triangle, then the triangles are congruent

Example 1: examine whether the two triangles are congruent or not. If yes write the congruence relation in symbolic form

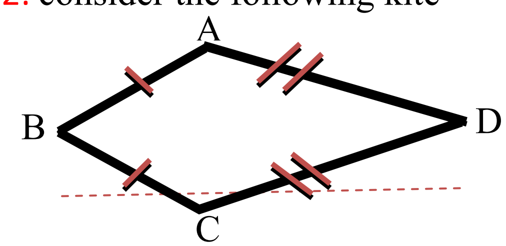


Solution:

$AB = PR = 3.5cm \;\; , AC = RQ = 5cm \; and \; BC = PQ = 7.1cm$

- The three sides of one triangle are equal to the three sides of other triangle, so by SSS congruence test the two triangles are congruent.
- You can easily see that A corresponds to R, B corresponds to P and C corresponds to Q,
- Hence, $\Delta ABC \cong \Delta RPQ$ by SSS congruence test

Example 2: consider the following kite



a. State the three pairs of equal parts in ΔABD and ΔCBD
b. Is $\Delta ABD \cong \Delta CBD$? Why or why not?
c. Does $\overline{BD}$ bisect $\angle ABC$? Give reasons

Solution:

a. three pairs of equal parts in ΔABD and ΔCBD are

$AB = CB$...... (given)

$AD = CD$ ... ...(given)

$BD = BD$ ... ...(common side for both)

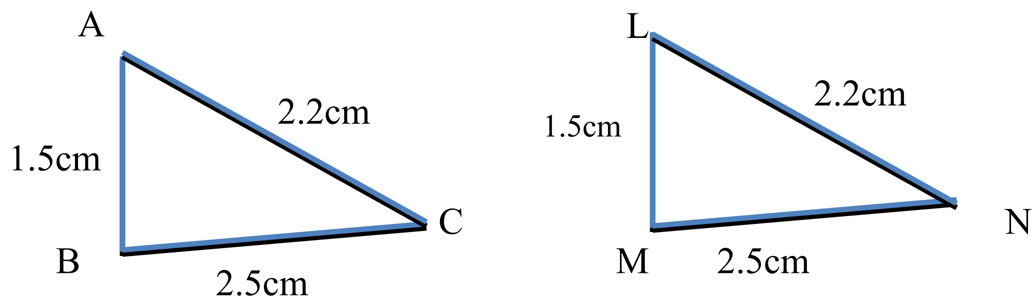
b. From a above $\Delta ABD \cong \Delta CBD$ by SSS

c. From b above $\angle ABD \cong \angle CBD$ …. Corresponding parts of congruent triangles

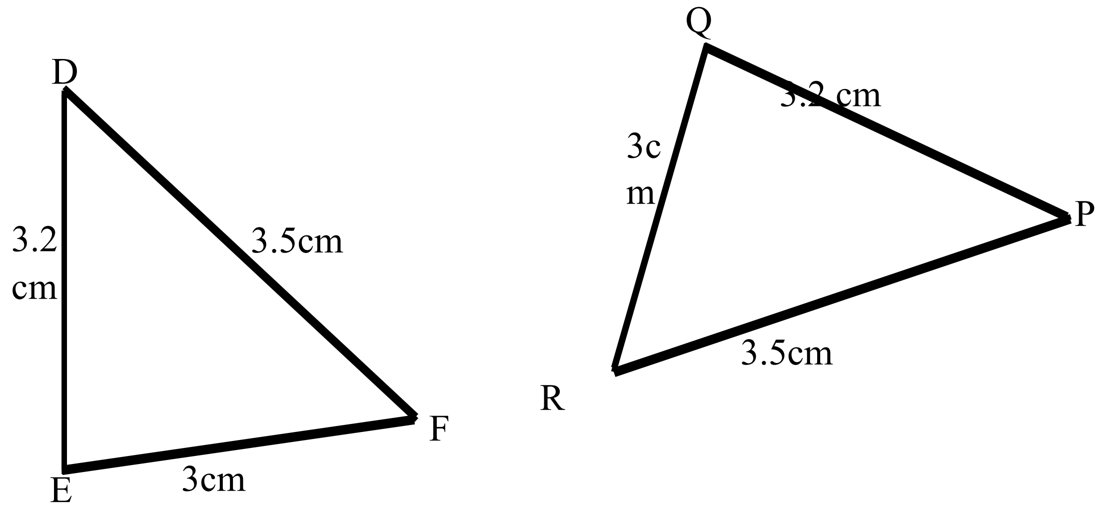
So, $\angle ABC$ is bisected by $\overline{BD}$

Exercise 6.1.3

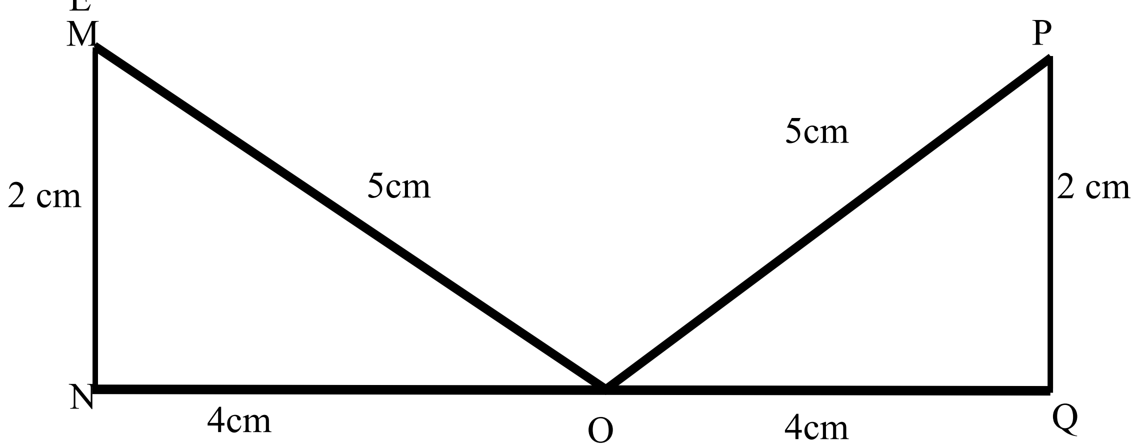
1. In the given figure below, lengths of the sides of the triangles are indicated. By applying the SSS congruence rule, state which pairs of triangles are congruent, in case of congruent triangles, write the result in symbolic form



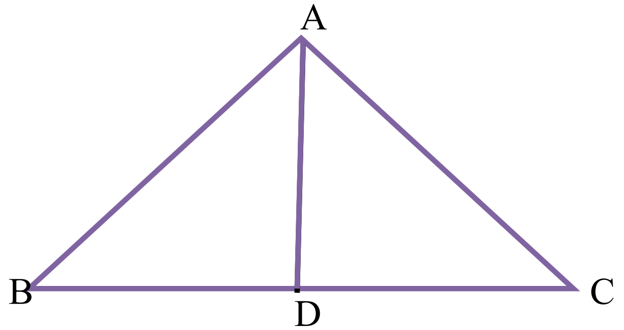
a.



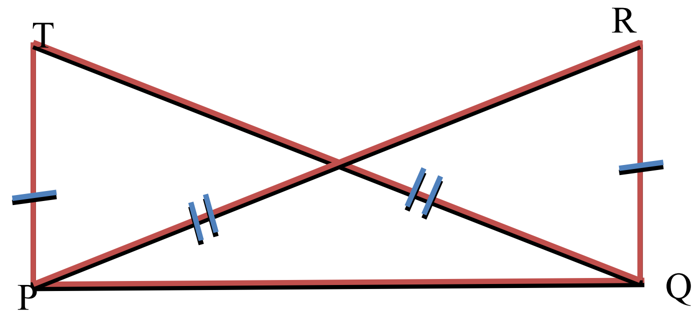
b.



2. In the figure below $AB = AC$ and D is the midpoint of BC
   a. State the three pairs equal parts in ΔADB and ΔADC
   b. Is $\Delta ADB \cong \Delta ADC$? Give your reasons
   c. Is $< B =< C$? Why?



3. In the figure, PR=QT and PT=QR. Which of the following statements is correct?
   a. $\Delta PQR \cong PQT$
   b. $\Delta PQR \cong \Delta QPT$
   c. $\Delta TPQ \cong \Delta RPQ$
   d. $\Delta PRQ \cong \Delta QTP$

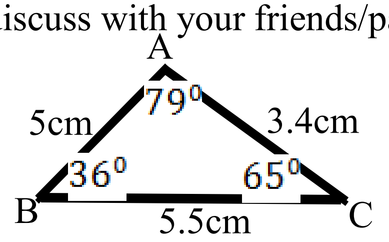


B. side – Angle – side (SSS)congruence test

Activity 6.1.4

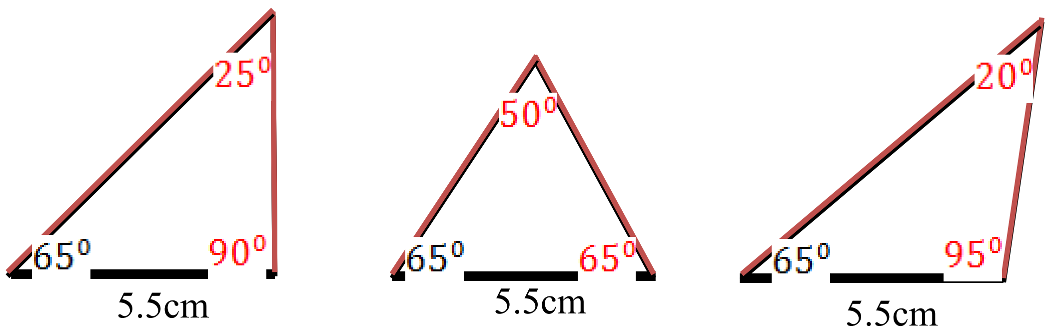
Read the following and discuss with your friends/partners

Given ΔABC,

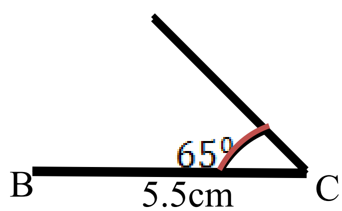


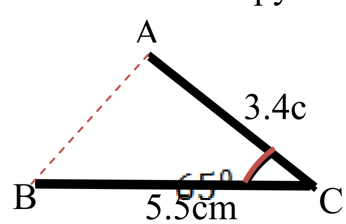
1. If only one side of ΔABC is given say 5.5cm, you can't draw the exact copy of it as you have seen in activity 5.3. can you a draw exact copy of it, if one side and one angle is given (say one side is 5.5cm and one angle is 65^0)?

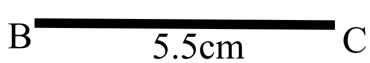
   The answer is **NO**, because you can draw a number of triangles as follows



2. If two sides and the angle between these two sides is given (say BC= 5.5cm, AC= 3.4cm and the angle between BC and AC is 65^0) can you draw the exact copy of it?
   - In this case you can draw, a single triangle which the exact copy of ΔABC as follows





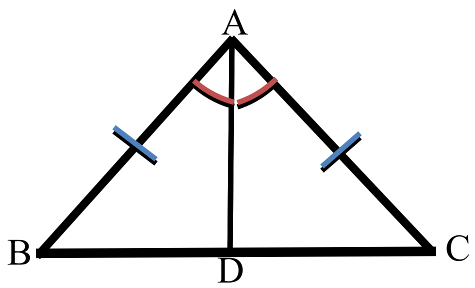


So, to draw an exact copy of ΔABC, we need the length of two sides of the triangle and the angle between the two sides. We call this side – Angle – side congruence criterion.

Side – Angle – Side (SAS) congruence test:

If the two sides and the angle included between them of a triangle are equal to two corresponding sides and the angle included between them of another triangle, then the triangles are congruent.

Example 1:



In the figure, $AB = AC$, and $\overline{AD}$ is the bisector of $\angle BAC$

i. State the three pairs of equal parts in ΔADB and ΔADC

ii. Is $\Delta ADB \cong \Delta ADC$? Give reason

iii. is $\angle B \cong \angle C$? Give reason

iv. is $BD = DC$? Give reason

Solution:

i. The three pairs of equal parts are

$AB = AC$……. (Given)

$AD = AD$……. (common sides for both)

$m(\angle BAD) = m(\angle CAD)$…. ($\overline{AD}$ bisects $\angle BAC$)

ii. yes, $\Delta ADB \cong \Delta ADC$ (by SAS)

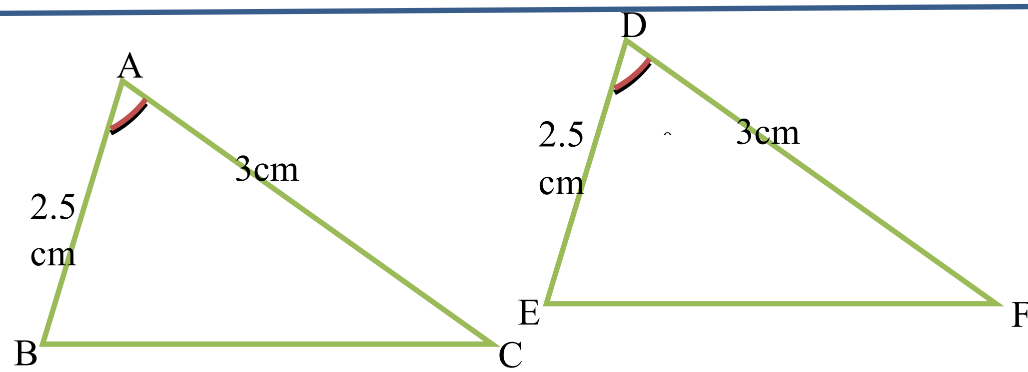
iii. $\angle B \cong \angle C$, because they are corresponding parts of congruent triangles.

iv. $BD = DC$, because they are corresponding parts of congruent triangles.

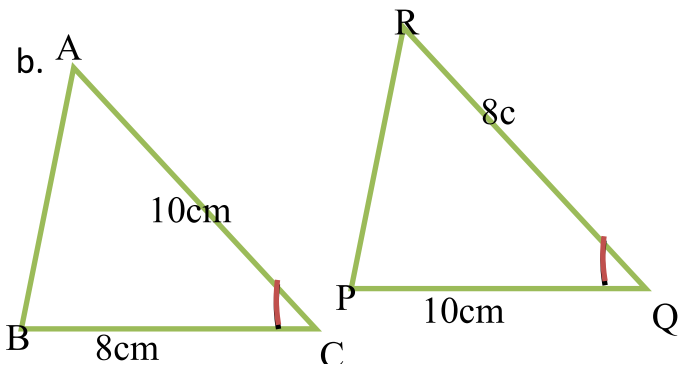
Exercise 6.1.4

1. Which angle included between side $\overline{DE}$ and $\overline{EF}$ in ΔDEF?
2. By applying SAS congruence rule, you want to establish that ΔPQR ≅ ΔFED. It is given that PQ = FE and RP = DF. What additional information is needed to be ΔPQR ≅ ΔFED?
3. In the given figure below, by applying SAS congruence rule, state the pairs of congruent triangles. In case of congruent triangles, write them in symbolic form.

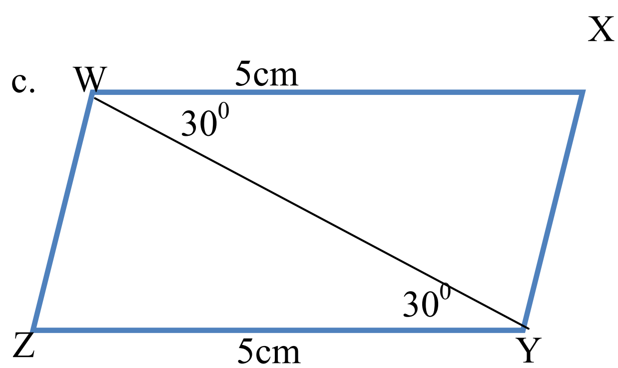
a.



b.

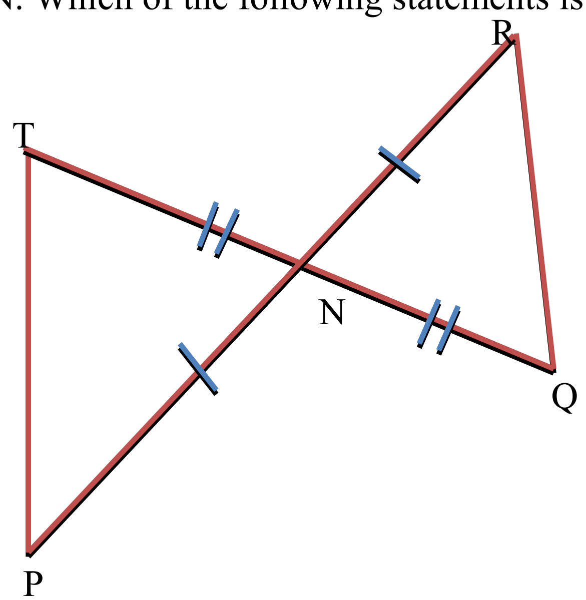


c.



4. In the given figure, PN=RN and TN=QN. Which of the following statements is correct?

   a. $\Delta PNT \cong RNQ$
   b. $\Delta PNT \cong \Delta QNR$
   c. $\Delta TPN \cong \Delta RQN$
   d. $\Delta NTP \cong \Delta NQR$



C. Angle – Side – Angle (ASA)congruence test

Activity 5.1.5

Discuss with your friends/partners

1. Can you draw the exact copy of a given triangle, if you know
   a. Only one of its angles?
   b. Only two of its angles?
   c. Two angles and any one side?
   d. Two angles and the side included between them?

Angle – Side – Angle (ASA) congruence test:

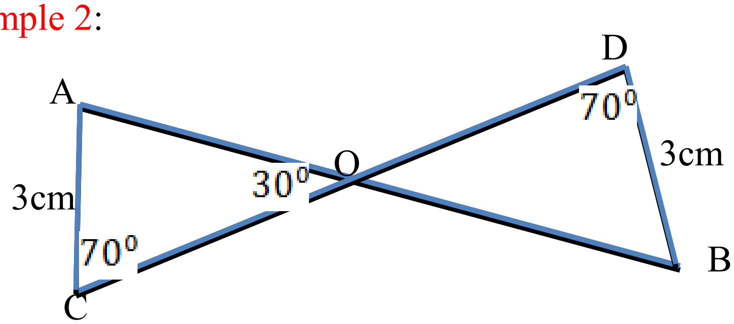
If two angles and the included side of a triangle are equal to two corresponding angles and the included side of another triangle, then the triangles are congruent.

Example 1: $\Delta ABC \cong \Delta QRP$ by ASA, and $BC = RP$ is given. what additional information is needed, to establish ASA congruence test?

Solution: For ASA congruence test, we need the two angles between which the two sides BC and RP are included. So, the additional information is

$\angle B \cong \angle R$ and $\angle C \cong \angle P$

Example 2:



Using the figure show that $\Delta AOC \cong \Delta BOD$

Solution: in $\Delta AOC, m(\angle A) + m(\angle C) + m(\angle O) = 180^0$

$$m(\angle A) + 70^0 + 30^0 = 180^0$$

$m(\angle A) = 80^0$

$m(\angle AOC) = m(\angle DOC) = 30^0$...... vertically opposite angles

in $\Delta DOB, m(\angle D) + m(\angle O) + m(\angle B) = 180^0$

$70^0 + 30^0 + m(\angle B) = 180^0$

$m(\angle B) = 80^0$

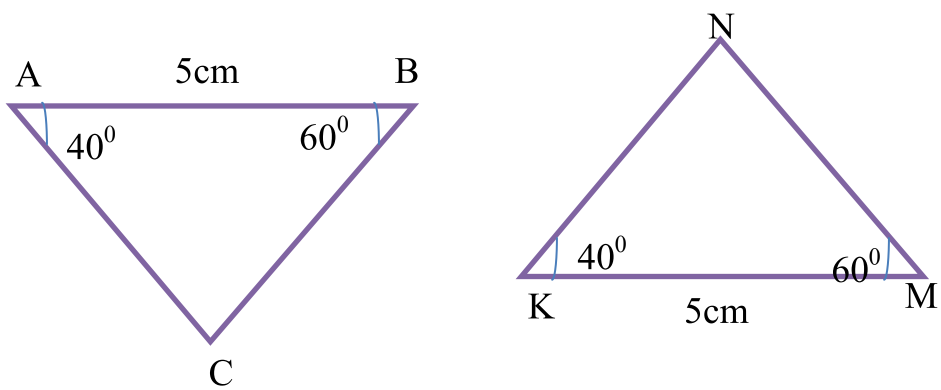
Now, $AC = DB$ and the angles that includes the sides AC and DB are also congruent (i.e. $\angle A \cong \angle B, and\ \angle C \cong \angle D$)

So, by ASA $\Delta AOC \cong \Delta BOD$.

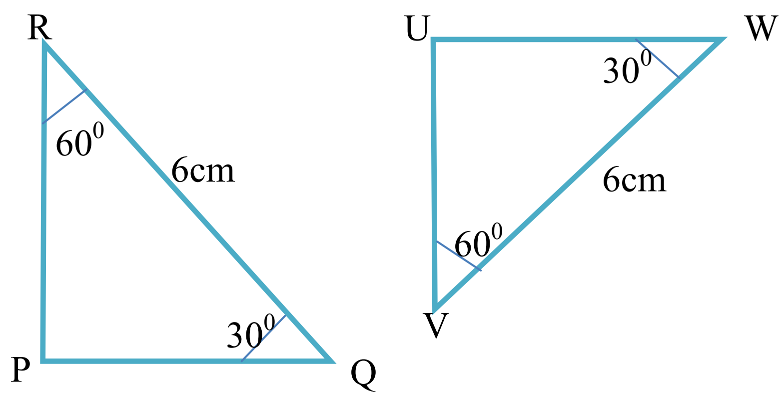
Exercise 5.1.5

1. What is the side between the angle M and N of ΔMNP.
2. You want to establish ΔDEF ≅ ΔMNP, using the ASA congruence rule. You are given that ∠D = ∠M and ∠F = ∠P. What additional information is needed to establish the congruence ΔDEF ≅ ΔMNP?
3. 3. In figure below, by applying ASA congruence rule, state which pairs of triangles are congruent and write the result in symbolic form.

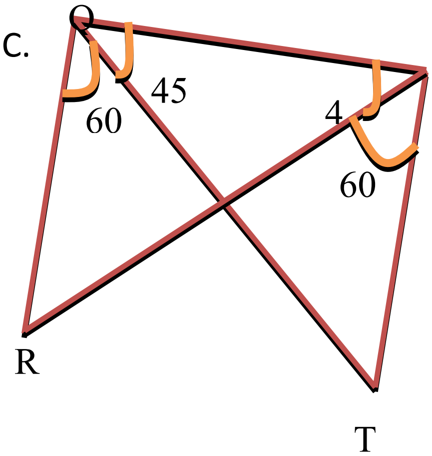
a.



b.



c.



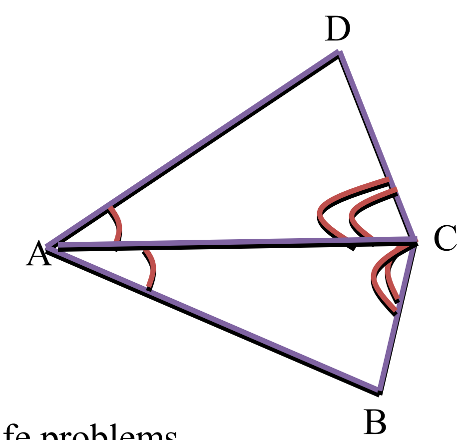
4. Given below are measurements of some parts of ΔDEF and ΔPQR. Check whether the two triangles are congruent or not, by ASA congruence rule and write it in symbolic form.

ΔDEF	ΔPQR
a. $\angle D = 70^0$, $\angle F = 85^0$, DF = 7 cm	$\angle Q = 70^0$, $\angle R = 85^0$, QR = 7 cm
b. $\angle D = 60^0$, $\angle F = 80^0$, DF = 6 cm	$\angle P = 60^0$, $\angle R = 80^0$, PR = 6cm
c. $\angle E = 80^0$, $\angle F = 30^0$, EF = 5 cm	$\angle P = 80^0$, PQ = 5 cm, $\angle R = 30^0$
d. $\angle D = 80^0$, $\angle F = 30^0$, DF = 5 cm	$\angle P = 80^0$, PR = 5 cm, $\angle R = 30^0$

5. In the figure, diagonal AC bisects $\angle DAB$ and $\angle DCB$.

   State the three pairs of equal parts in triangles BAC and DAC.

   a. Is $\Delta BAC \cong \Delta DAC$? Give reasons.
   b. AB = AD? Justify your answer.
   c. DC= CB? Give reasons



6.2.Applications

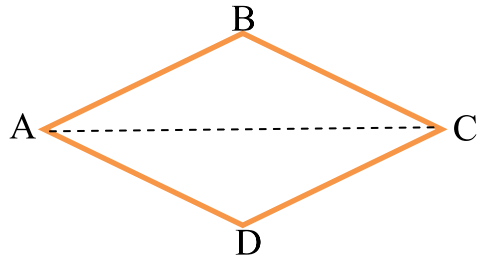
At the end of this section you should be able to:

- Apply congruency of plane figures to real life problems

In section you learn some the applications of congruence of triangles

Example 1: show that the diagonal of rhombus bisects the angles at the vertex

Solution: let ABCD is rhombus, we need to show $\overline{AC}$ bisects $\angle BAD$ and $\angle BCD$



Considering ΔABC and ΔADC

$AB \cong AD\ldots$ All sides of rhombus are congruent

$BC \cong DC\ldots$ All sides of rhombus are congruent

$AC \cong AC\ldots$ common side for both

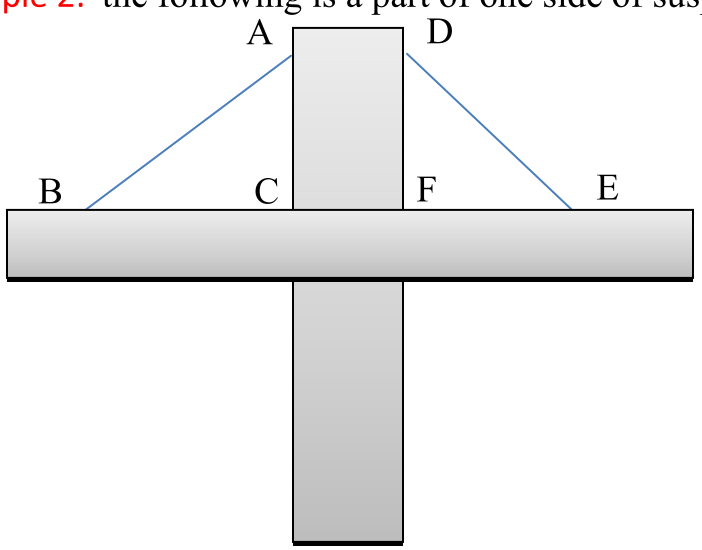
Therefore, $\Delta ABC \cong \Delta ADC$ by SSS congruence test

$\angle BAC \cong \angle DAC\ldots$ corresponding parts of congruent triangles, hence $\angle BAD$ is bisected by $\overline{AC}$.

$\angle BCA \cong \angle DCA\ldots$ corresponding parts of congruent triangles, hence $\angle BCD$ is bisected by $\overline{AC}$

In similar way you can show that $\overline{BD}$ also bisects $\angle B$ and $\angle D$

Example 2: the following is a part of one side of suspension bridge on Abay river



The length of cable AB= 5m, BC=4m, EF = 4m, AC= 3m, what should be the length of cable DE? (assume $AC \perp BC$ and $DF \perp FE$)

Solution: consider $\Delta ABC\ and\ \Delta DEF$

$BC = EF = 4m$ …. Given

$AC = DF = 3m$….. common side

$\angle C \cong \angle F$ …. Since $AC \perp BC$ and $DF \perp FE$)

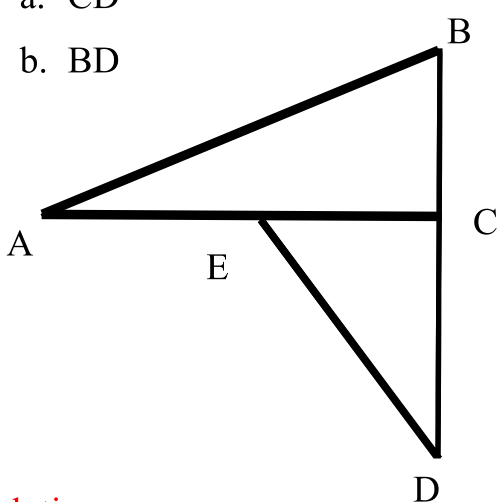
Therefore, by SAS test $\Delta ABC\ and\ \Delta DEF$ and

$DE = AB = 5m$…. Corresponding part of congruent triangles

Example 3: consider the following geometric figure that shows a plot of land to plant flowers.

If $= CE = 6m$, $\angle ABC \cong \angle CED$, $AE = 4m$ and $AC \perp BD$, then calculate the length of

a. CD
b. BD



Solution:

a. consider ΔABC and ΔCDE

$BC = CE$ …. Given

$\angle ABC \cong \angle CED$…. Given

$\angle ACB \cong \angle ECD$ ….. since $AC \perp BD$

The corresponding two angles and included side of the two triangles are congruent.

So, $\Delta ABC \cong \angle DEC$ by ASA.

$CD = AC$ … corresponding side of congruent triangles

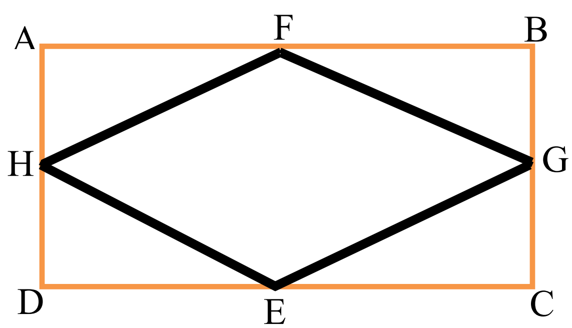
Therefore, $CD = AC = AE + EC = 4m + 6m = 10m$

b. $BD = BC + CD$

$= 6m + 10m$

$= 16m$

Example 4: in the figure ABCD is rectangle and F, G, E, and H are the mid points on side AB, BC, CD, and AD respectively.



Show that

a. $\Delta FAH \cong \Delta FBG$
b. $\Delta ECG \cong \Delta EDH$
c. $\Delta FAH \cong \Delta EDH$
d. $HF = FG = EG = EH$

Solution:

a. consider ΔFAH and ΔFBG

$FA = FB$….. since F is midpoint of AB

$AH = HD$…. Since AD = BC and, H is midpoint of AD, G is midpoint of BC

$m(\angle A) = m(\angle B) = 90^0$…. Angles of rectangle

Therefore, $\Delta FAH \cong \Delta FBG$ by SAS

b. consider ΔECG and ΔEDH

$EC = ED$….. since E is midpoint of CD

$CG = HD$…. Since AD = BC and, H is midpoint of AD, G is midpoint of BC

$m(\angle C) = m(\angle D) = 90^0$…. Angles of rectangle

Therefore, $\Delta ECG \cong \Delta EDH$ by SAS

c. consider ΔFAH and ΔEDH

$FA = ED$….. since AB =DC, and F is midpoint of AB, E is midpoint DC

$AH = HD$…. Since H is midpoint of AD

$m(\angle A) = m(\angle D) = 90^0$…. Angles of rectangle

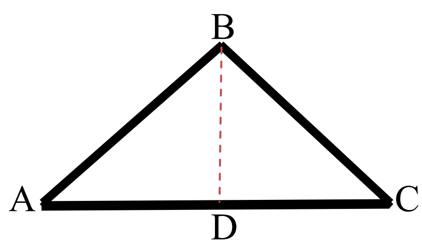
Therefore, $\Delta FAH \cong \Delta EDH$ by SAS

d. From a, b, and c you can see that $\Delta FAH \cong \Delta FBG \cong \Delta ECG \cong \Delta EDH$

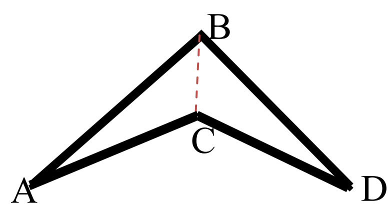
Hence, $HF = GF = GE = HE$ since the corresponding sides of congruent triangles

Exercise 6.2.1

1. Show that the diagonal of rectangle divides the rectangle in to two congruent triangles
2. In the figure below ΔABC is isosceles triangle with $AB = BC$ and BD bisects $\angle ABC$, then show that D is the mid-point AC



3. A portion of roof truss has the following shape.
   If $AB = BD$, and $\angle ABC \cong \angle DBC$, then show that $\Delta ABC \cong \Delta DBC$.



Summary for unit 6

- Congruent figures are figures that have the same size and shape. Congruent figures are exact copies of one another.
- Two triangles are congruent, if their corresponding parts (angles and sides) that match one another are equal.
- Two triangles are congruent if they are copies of each other and when you place one triangle on another, they cover each other completely.
- If ΔABC is congruent to ΔXYZ, then symbolically written as $\Delta ABC \cong \Delta XYZ$
- If $\Delta ABC \cong \Delta XYZ$, i. $\angle X \cong \angle A$, $\angle Y \cong \angle B$ and $\angle Z \cong \angle C$

  i. $\overline{XY} \cong \overline{AB}$, $\overline{YZ} \cong \overline{BC}$ and $\overline{XZ} \cong \overline{AC}$
- If the three sides of one triangle is congruent to the three corresponding sides of another triangle, then the triangles are congruent by Side – Side – Side (SSS) congruence rule.
- If the two sides and the angle included between them of a triangle are equal to two corresponding sides and the angle included between them of another triangle, then the triangles are congruent by Side – Angle – Side (SAS) congruence rule.
- If two angles and the included side of a triangle are equal to two corresponding angles and the included side of another triangle, then the triangles are congruent by Angle – Side – Angle (ASA) congruence rule.

Review exercise for unit 6

I. Write True if the statement is correct and False if it is incorrect

1. Congruent objects are exact copies of one another.
2. Two-line segments are congruent if they have equal lengths.
3. Two angles are congruent if their measures are equal.

4. Two triangles are congruent if the three sides of the one triangle are equal to the three corresponding sides of the other.
5. Two triangles are congruent if the three angles of the one triangle are equal to the three corresponding angles of the other.
6. If two triangles are congruent, then they have the same perimeter.
7. If two squares have equal length of side then they are congruent.
8. If $\Delta DEF \cong RTS$ then $< E =< R$.
9. Every triangle is congruent to itself.
10. $< Y$ is between the side YZ and XZ of ΔXYZ.

II. Choose the correct answer from the given alternatives

11. Which congruence rule do you use in the following?

    AC = DF, AB = DE, BC = EF, So, ΔABC ≅ ΔDEF

    a. SSS b. ASA c. SAS d. AAS
12. Which congruence rule do you use in the following?

    ZX = RP, RQ = ZY, ∠PRQ = ∠XZY, so, ΔPQR ≅ ΔXYZ

    a. SSS b. ASA c. SAS d. AAS
13. Which congruence rule do you use in the following?

    ∠MLN = ∠FGH, ∠NML = ∠GFH, ML = FG, So, ΔLMN ≅ ΔGFH

    a. SSS b. ASA c. SAS d. AAS
14. If$\Delta ABC \cong \Delta PQR$, then which of the following is not true?

    a. BC=QR b. <C=<Q c. <ABC=<PQR d. BC=PQ
15. Diagonal rectangle intersects in to congruent triangles, which congruent criteria show these congruency

    a. SSS b. ASA c. SAS d. AAS
16. Which one of the following is not true about congruent figures?

A. Any two rhombuses are congruent.

B. Any two congruent figures have the same area.

C. Any two congruent figures have the same size and shape.

D. Any two congruent circles have the same length of radius.

III. Work out

17. Show and explain using a different congruence rule for triangle why a diagonal of a square divides the square into two congruent triangles.

18. Recall that a kite is a quadrilateral with two pairs of adjacent and congruent sides. Will one of the diagonals of a kite divide the kite into two congruent triangles? Show and explain your answer using congruence rule.

19. Two triangles ABC and PQR are such that; AB = 3.5 cm, BC = 7.1 cm, AC = 5 cm, PQ = 7.1 cm, QR = 5 cm and PR = 3.5 cm. Check whether the two triangles are congruent or not and explain your answer using congruence rule.

20. Given that ΔABC$\cong$ ΔPQR such that $<$B= (2x + 30)°, ∠Q = 55° and find the value of x.

21. Describe the rule of congruence in two triangles given by;
In Δ ABC, AB = 7 cm, BC = 5 cm, ∠B = 50° and In Δ DEF, DE = 5 cm, EF = 7 cm, ∠E = 50°

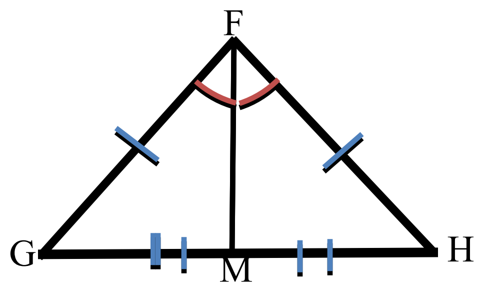
22. ΔFGH is an isosceles triangle with FG = FH and FM is one of its height of triangle

a. State the three pairs of equal parts in ΔFGM andΔFHM.

b. is ΔFGM $\cong$ FHM? Why or why not?

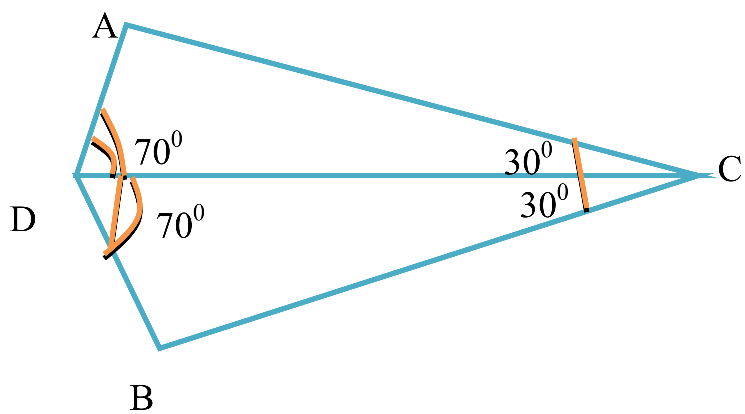
c. is $< G =< H$? Why or why not?

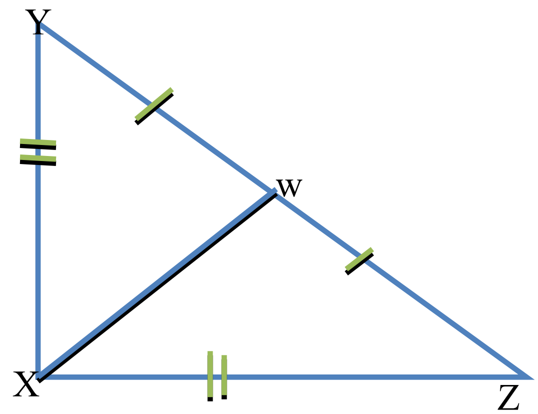
d. is $< GFM = \angle FHM$? Why or why not?



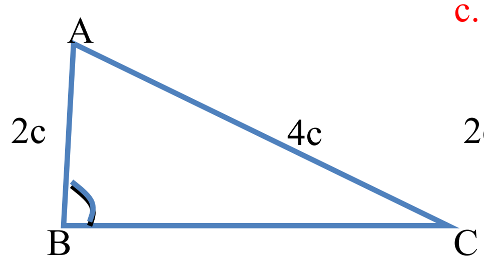
23. In the figure below, by applying congruence rule, state which pairs of triangles are congruent. In case of congruent triangles, write the result in symbolic form.

a.

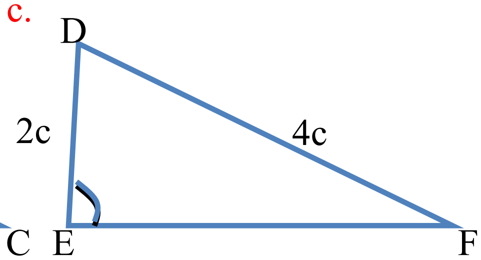




b.



c.



Unit 7

Data Handling

Unit Outcomes: At the end of this unit, you will able to:

- Organize data using frequency tables for a given data
- Construct and Interpret data from pie charts
- Calculate Mean, Mode, Median and range of a given data
- Apply the concept of data handling to organize and interpret real life problems

Introduction

Collection of data from a group of things helps us to understand more about these things in the group. To do this the collected data should be presented systematically pictorially so as to analysis them. From this unit you will learn how to collect simple data and present them pictorially and do some calculation on them to study their nature or property.

7.1.Organization of data using frequency table

At the end of this topic, you will able to:

- Collect simple data from their environment using tally mark.
- Organizes data in a frequency distribution table.

Activity 7 .1.1

1. Define the word 'Data'.
2. Where do you get data?

3. The following data shows the age of students in a certain class.

15	13	13	13	15	16	14	13	14	15	12
17	15	13	13	14	15	13	16	14	13	14
15	13	16	15	16	13	12	13	15	14	14

Copy and Complete the table given below using the above information

Age of students	Tally	Number of students (frequency)
12	||	2
13		
14		
15		
16		
17		
Total number of students		

Definition: Data is a collection of facts, such as numbers, words, measurements, observations or even just descriptions of things.

You can collect data:

- ✓ by using a questionnaire.
- ✓ by making observations and recording the results.
- ✓ by carrying out an experiment.
- ✓ from records or data base
- ✓ from the internet

One ways of presenting the data is tally chart or frequency table.

Note: A tally chart is a simple way of recording and counting frequencies

- A tally chart or frequency table is a quick and easy way of recording data.
- Frequency is the number of times a data value occurs.

Example 1: Draw tally chart or frequency table using the following data.

In a school, 40 students were asked what size of shoe does they dressed on Monday. Here are the results:

33	32	3 4	37	37	3 5	3 8	36	37	36
36	38	35	33	36	37	38	38	31	37
36	35	37	36	39	37	36	35	38	33
34	33	37	37	38	35	34	37	39	36

Solution:

Number of friends	Tally	Frequency(Number of students)
31	|	1
32	|	1
33	||||	4
34	|||	3
35	𝍸	5
36	𝍸 |||	8
37	𝍸 𝍸	10
38	𝍸 |	6
39	||	2

Note: The above tally chart or frequency table in example 1, shows the frequencies of the different size of shoe (how often each size occurred).

Tally marks are grouped in five to make them easier to count: 𝍸 𝍸 is easier to count than ||||||||||.

- ✓ | represents 1 member of the group (sample).
- ✓ 𝍸 Represents 5 members of the group (sample).

Example 2: Consider the following data collected from the scores of 45 students in a mathematics final exam. Show this information more clearly by drawing a tally chart

36	38	32	37	34	25	28	30	33
28	32	38	32	36	34	34	38	36
32	25	37	35	29	27	32	35	37
36	31	35	36	33	24	31	34	22
27	31	29	32	37	26	34	35	32

Solution:

Score	Tally	Frequency (Number of students)
22	|	1
24	|	1
25	||	2
26	|	1
27	||	2
28	||	2
29	||	2
30	|	1
31	|||	3
32	~~||||~~ ||	7
33	||	2
34	~~||||~~	5
35	||||	4
36	~~||||~~	5
37	||||	4
38	|||	3
Total number of students		45

Exercise 7.1.1

1. Show the following information more clearly by drawing a tally chart or frequency table.

The following are weights in kg of 40 students in a class

40	45	45	46	44	43	45	47	46	49	
44	51	47	45	44	46	46	43	44	50	
48	43	45	46	44	44	47	43	44	45	
45		43	45	46	44	47	45	46	44	47

2. The table below shows the favorite color of grade 7th students.

White	Red	White	Yellow	Green	black	Green	Blue
Green	White	Black	Red	Yellow	Blue	Blue	Red
Yellow	Blue	White	Blue	Green	White	White	White
Yellow	Blue	Green	Green	White	Blue	Black	Red
Red	Blue	Yellow	Red	Green	White	White	Green

Show the above information (table) by tally chart.

3. For each of the following recorded data shows monthly average temperature in degree Celsius for Addis Ababa city, then display the information in a tally chart or frequency table.

21	19	19	20	20	21	22	22	23	22
22	21	20	19	19	20	21	22	22	21
19	21	21	22	20	23	22	23	24	23

7.2. Construction and Interpretation of line graphs and pie charts

At the end of this topic, you will able to:

- Construct line graphs and pie chart to represent organized data by using the given data or by collecting data from their environment
- Interpret simple pie charts

7.2.1 Line graphs

Activity 7.2.1

Discuss with your friends

1. a. Copy and complete the following table.

X	0	1	2	3	4	5
Y	1	3	5	7	9	11
(x ,y)						

   b. Plot the above points on a Cartesian coordinate plane and connect by straight line.

2. The following table represents score of students. Draw the graph of this information.

score	4	5	6	7	8	9	10
Number of students	2	6	8	12	7	5	5

3. Define line graph by your own words.

Definition: Line graph is a graph that uses lines to connect individual data points on a Cartesian coordinate plane.

Note: The line graph is most commonly used to represent two related facts. It use data point "markers," which are connected by straight lines or smooth curves.

The following points are important to making a line graph

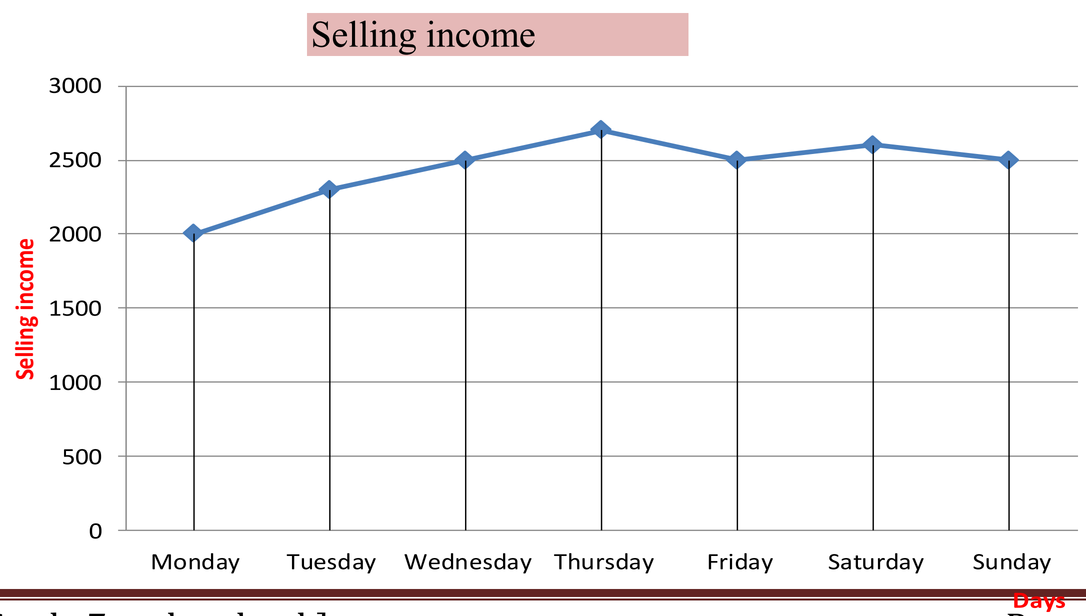
1. Construct a Cartesian coordinate plane and label an appropriate scale.
2. Make a table of data arranged in order pairs and mark the points on a Cartesian coordinate plane
3. Connect the points by a straight line or smooth curve.

Example 1: The following table shows selling income of a shopkeeper in a week. Draw a line graph of the information given below.

Days	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Selling income in Birr	2000	2300	2500	2700	2500	2600	2500

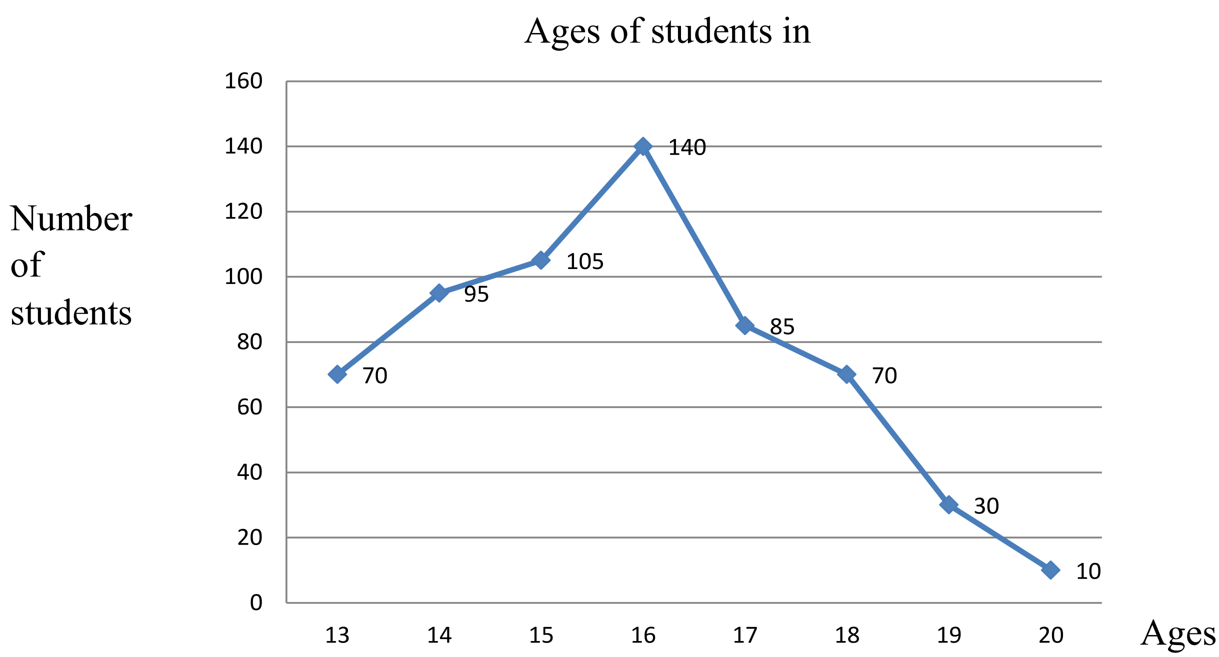
Solution: To draw the line graph of the above information

1. Draw a Cartesian coordinate plane and label the days horizontally and the selling income vertically in appropriate scale.
2. Mark the points (Monday, 2000), (Tuesday, 2300), (Wednesday, 2500), (Thursday, 2700), (Thursday, 2500), (Friday), (Saturday, 2600) and (Sunday, 2500)
3. Connect all points by smooth curve



Example 2: The graph shows the number of student's age from 13 to 20 in a

School



a. What is the title of the graph?
b. At what age do the number of students highest?
c. At what age do the number of students lowest?
d. Prepare a table that contains ages and the number of students.

Solution: a. Ages of students in school

b. 16

c. 20

d.

Ages	13	14	15	16	17	18	19	20
Number of students	70	95	105	140	85	70	30	10

Exercise 7.2.1

1. Draw a line graphs to represent each of the following data.

   a. The number of letters delivered to an office in one week

Days	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Number of Letters	10	0	5	8	12	15	10

   b. The temperature in Addis Ababa at midday during the last week in April

Days	Mon	Tue	Wed	Th	Friday	Sat	Sun
Temperature in ^{0}C	20	22	21	20	23	24	25

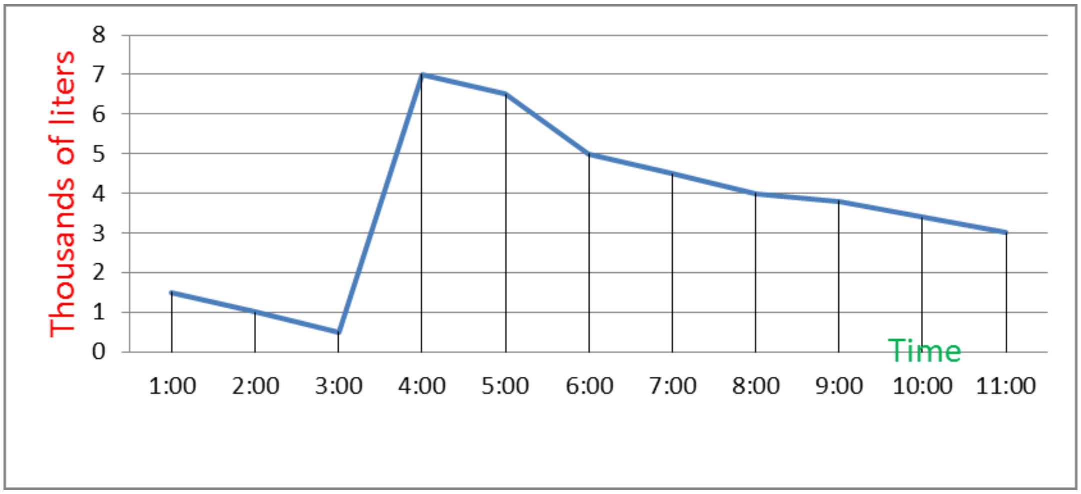
2. a. Copy and complete the table below gives some values between inches and centimeters. (Hint 1 inch = 2.54 centimeters).

Inches	1	2	6	10	20	30	40
Centimeters	2.54	5.08					

   b. Draw the graph from of the relation between inches to centimeters.

3. The amount of petrol (in liters) in the storage tank at a garage was measured every hour between morning 1:00 and afternoon 11:30 in one day.

   This is the shape of the line graph showing the results:



a. When was the amount of petrol in the tank at its lowest?

b. What happened to the amount of petrol between 3:00 and 4:00

c. What can you say about the sales like between 7:00 and 11:00?

d. Give a reason for your answer to (question c).

7.2.2 Pie charts

Activity 7.2.2

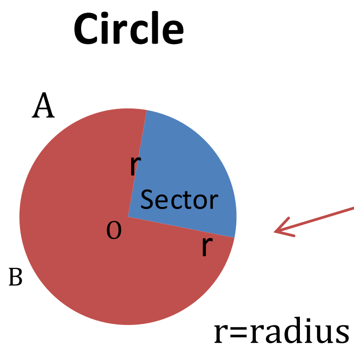
Discuss with your friends

1. Draw a circle and divide the circle with angles at the center are 90^0. 60^0, 30^0 , 180^0,
2. Look at the table given below

Age of students	Below 13	13-14	15-16	17-18	Above 18
Number of students covered degree of sector	20^0	244^0	54^0	32^0	10^0

Draw a pie chart to display the above data.

Note: The portion of a circular region enclosed between two radii and part of the circumference is called a sector of the circle.



- Radii means plural form of radius
- O is center of circle
- $\widehat{AB}$ is arc of the sector

- The size of the sector is determined by the size of the angle formed by the two radii.

Note: Mesasure angle of circle is 360^0 .

360^0 is covered 100% of circle

i.e $100\% = 360^0$ $\quad 1\% = 3.6^0$ and $1^0 = \frac{10}{36}\%$

$$Percentage = \frac{measure\ of\ angle \times total\ value}{360^0}$$

$$Measure\ of\ angle = \frac{percentage \times 360^0}{total\ value}$$

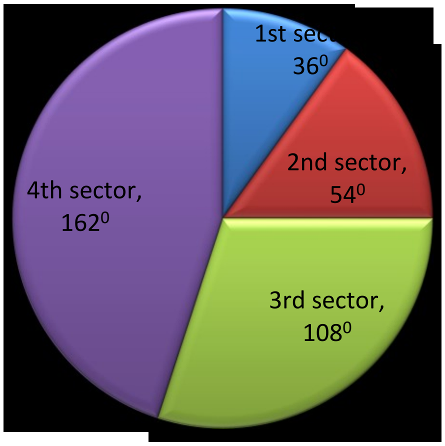
Example 1: Divide the circle in to 4 sectors with $36^0, 54^0, 108^0, 162^0$

The first sector measures $36^0 = 36^0 \times \frac{10\%}{36^0}$ =10%of a circle.

The 2^{nd} sector measures $54^0 = 54^0 \times \frac{10\%}{36^0} = 15$ %of a circle.

The 3rd sector measure $108^0 = 108^0 \times \frac{10\%}{36^0} = 30\%$ of a circle.

The 4th sector measure $162^0 = 162^0 \times \frac{10\%}{36^0} = 45\%$ of a circle.



Definition: Pie chart is a type of graph that represents the data in the circular graph.

It is a very common and accurate way of representing data especially useful for showing the relations of one item with another and one item with the whole items.

Example 2: Show the following data by using pie chart.

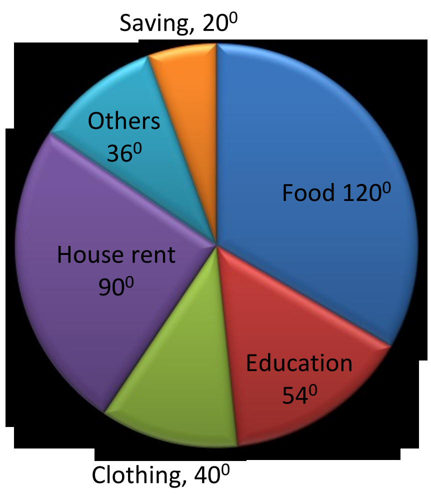
The expenditure on different budget title of a family in a month is given below in table

Budget in Birr	Food	Education	Clothing	House rent	others	Saving	Total Birr
Expenditure in Birr	2400	1080	800	1800	720	400	7200

Solution:

Measure of angle $= \frac{Expenditure\ on\ the\ given\ budget\ in\ BIrr \times 360^0}{Total\ expenditure}$

Budget	Expenditure in Birr	Measure of angle of sectors
Food	2400	$\frac{2400 \times 360^0}{7200} = 120^0$
Education	1080	$\frac{1080 \times 360^0}{7200} = 54^0$
Clothing	800	$\frac{800 \times 360^0}{7200} = 40^0$
House rent	1800	$\frac{1800 \times 360^0}{7200} = 90^0$
Others	720	$\frac{720 \times 360^0}{7200} = 36^0$
Saving	400	$\frac{400 \times 360^0}{7200} = 20^0$
Total	7200	360^0



Example 2: Show the following data by using pie chart

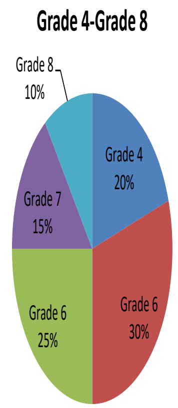
There are 1200 students in a certain school from grade 4 up to grade 8 and the percent of students are given below

Grade	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8
Number of students in %	20%	25%	15%	15%	10%

Solution:

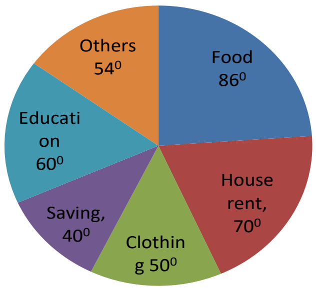
Measure of angle of sector = $\frac{\text{percentage of students} \times 360^0}{100}$

Grade	Number of students in %	Measure of angle of sectors
Grade 4	20%	$\frac{20 \times 360^0}{100} = 72^0$
Grade 5	30%	$\frac{30 \times 360^0}{100} = 108^0$
Grade 6	25%	$\frac{25 \times 360^0}{100} = 90^0$
Grade 7	15%	$\frac{15 \times 360^0}{100} = 54^0$
Grade 8	10%	$\frac{10 \times 360^0}{100} = 36^0$
Total	100%	360^0



Example 3: The pie chart given below shows W/ro Tseday's expenses and saving for last month. If the monthly income was Birr 10800, then find

a. Her food expenses
b. Her house rent
c. Her clothing expenses
d. Her saving
e. Her education expenses
f. For other expenses



Solution:

Measure of angle of sector$= \frac{each\ Expenses}{total\ amount\ of\ budget} \times 360^0$

$$each\ Expenses = \frac{measure\ of\ angle\ of\ sector \times Total\ amount\ of\ budget}{360^0}$$

a. Food expense $= \frac{measure\ of\ angle\ of\ sector \times Total\ amount\ of\ budget}{360^0}$

$$= \frac{86^0 \times 10800}{360^0} = Birr\ 2580$$

b. House rent $= \frac{measure\ of\ angle\ of\ sector \times Total\ amount\ of\ budget}{360^0}$

$$= \frac{70^0 \times 10800}{360^0} = Birr\ 2100$$

c. Clothing expense $= \frac{measure\ of\ angle\ of\ sector \times Total\ amount\ of\ budget}{360^0}$

$$= \frac{50^0 \times 10800}{360^0} = Birr\ 1500$$

d. Saving $= \frac{measure\ of\ angle\ of\ sector \times Total\ amount\ of\ budget}{360^0}$

$$= \frac{40 \times 10800}{360^0} = Birr\ 1200$$

e. Education$=\frac{measure\ of\ angle\ of\ sector \times Total\ amount\ of\ budget}{360^0}$

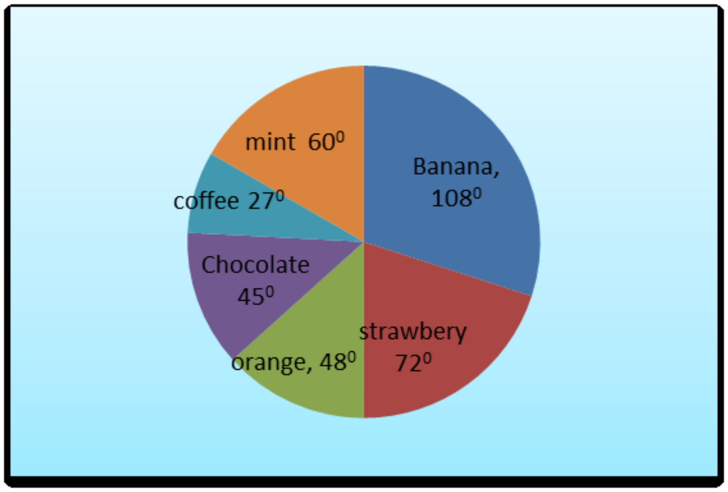
$$=\frac{60\times 10800}{360^0}=Birr\ 1800$$

f. For other$=\frac{measure\ of\ angle\ of\ sector \times Total\ amount\ of\ budget}{360^0}$

$$=\frac{54\times 10800}{360^0}=Birr\ 1620$$

Exercise 6.2.2

1. 60 students were asked to name their favorite chewing gum. The results are shown in the given figure below



a. What does the whole Circle represent?

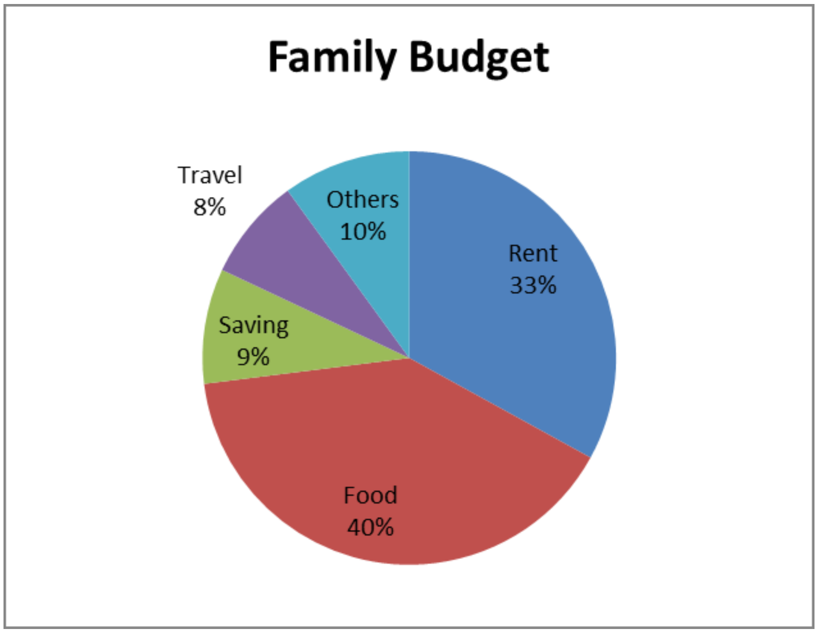
b. Which chewing gum does the largest sector represent?

c. What does the smallest sector represent?

d. Use the given angles of sectors to calculate the number of students who liked

i. Strawberry chewing gum ii. Chocolate ii. Orange

2. The following pie chart shows a family budget based on a net income of Birr 8400 per month.



a. Determine the amount spent on rent.

b. Determine the amount spent on food.

c. Determine the amount saving money.

d. How much more money is spent?

3. The budget for social development programme of wereda district is given as follows in table.

Item	Amount in Birr
Education	1,800,000
Public health	1,200,000
Community development	200,000
life skill Training for children	400,000
Total budget	3,600,000

Draw a pie chart to show the above information using percent and degree of sector.

7.3.The Mean, Mode, Median and Range of Data

By the end of these sections you should be able to:

• Calculate the mean, mode, median and range of the data.

Mean

Activity 7.3.1

1. Find the sum of the following numbers

13, 18, 13, 14, 12

2. Divide the sum of the above numbers by five
3. What do you say about your result

Definition: The mean of a given data is the sum of all values divided by the number of values: mean= $\frac{sum\ of\ all\ values}{number\ of\ values}$

That mean of for the given data $x_1, x_2, x_3, \ldots . x_n$ is

$mean(\bar{x}) = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n}$, where $\bar{x}$ mean of data

Example1: Find the mean of the following data

a. 22, 20, 14, 12, 27
b. 100,200,120,320,150,160
c. 4500,2560,5000

Solution:

a. $mean(\bar{x}) = \frac{x_1 + x_2 + x_3 + x_4 + x_5}{5}$

$= \frac{22 + 20 + 14 + 12 + 27}{5}$

$$= \frac{95}{5}$$

$$= 19$$

b. $mean(\bar{x}) = \frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6}{6}$

$$= \frac{100 + 200 + 120 + 320 + 150 + 160}{6}$$

$$= \frac{1050}{6}$$

$$= 175$$

c. $mean(\bar{x}) = \frac{x_1 + x_2 + x_3}{3}$

$$= \frac{4500 + 2560 + 5000}{3}$$

$$= \frac{12060}{3}$$

$$= 4020$$

2. The mean of five numbers is 30. Four of the numbers are 32, 28, 40 and 27, then find the value of the other numbers.

Solution:

Let x be the missing number

mean $(\bar{x}) = \frac{sum\ of\ all\ values}{number\ of\ values}$

$30 = \frac{32+28+40+27+x}{5}$

$x + 127 = 150$

$x = 150 - 127$

$x = 23$

3. The mean of 4 numbers is 15 and the mean of 6 other numbers is 20, then find the mean of all 10 numbers.

Solution:

$$mean = \frac{The\ sum\ of\ values}{Number\ of\ values}$$

$$The\ sum\ of\ 4\ numbers = mean \times number\ of\ value$$

$$= 15 \times 4 = 60$$

$$The\ sum\ of\ 6\ numbers = mean \times number\ of\ value$$

$$= 6 \times 20 = 120$$

Thus, the total sum =60+120=180

Total number of data = 4+6=10

Then the mean of all ten numbers = $\frac{total\ sum}{total\ number\ of\ data}$

$$= \frac{180}{10} = 18$$

Therefore, the mean of all ten numbers is 18

Exercise 7.3.1

1. Find the mean of the following given data
   a. 10 14 16 19 25 12
   b. 28 35 70 140 160
   c. 58 40 56 55 78 80 70 45
   d. 600 750 950 800 2000
2. If the age of 8 students are 11, 12, 14, 17, 15, 13, 14 and 16, then find the mean of age of students.

3. The mean of four numbers is 70. Three of the numbers are 38, 85 and 60, then find the value of the other numbers
4. The mean of three numbers is 18 and the mean of five other numbers is 24, then find the mean of all 8 numbers.
5. Find the value of x, so that the mean of the given data 16, 2x, 6, 10 and 4 is

Mode

Activity 7.3.2

1. Which number occurs most frequently?

a.	6	12	10	6	23	12	6	25		
b.	23	22	21	24	23	0	23	24	25	24
c.	100	500	600	700	800	900				

2. Define Mode of data by your own words.

Definition: The mode of list of data is the value which occurs most frequently.

Example 1: find the mode of given data below

a.	20	30	40	20	20	30	50		
b.	12	14	12	14	13	14	12		
c.	25	28	50	60	70	50	60	70	90
d.	21	78	90	40	20	10	15	25	35

Solution:

a. 20 occurs more frequently than any other values of data, then mode=20
b. 12 and 14 occurs three times, hence three are two modes 12 and14.
c. 50, 60 and 70 occurs two times equally, then there are two modes 50, 60 and 70.
d. Each value occurs only once, so there is no mode for the given data.

Note:

- A data that has one mode is called unimodal.
- A data that has two mode is called bimodal
- A data that has three mode is called trimodal
- If each value occurs only once, so there is no mode.

Exercise 7.3.2

1. Find the mode of the following given data below and identify it is unimodal, bimodal, trimodal and no mode.

a. 8 11 9 14 9 15 18 6 9 10

b. 24 15 18 20 18 22 24 26 18 26 24

c. 35 43 39 46 51 47 38 51 43 38 40 45

d. 123 121 119 116 130 121 131

e. 117 119 135 121 129 139 134

f. 7.2 6.2 7.7 8.1 6.4 7.2 5.4

Median

Activity 7.3.3

1. Arrange the following data in increasing order

a. 14 10 12 18 17

b. 12 18 6 8 2 4 14 4 18

2. Circle the middle value of the above arrangement of numbers?

Definition: The median is the middle value when data is arranged in order of size**.**
To find the median of list of data

I. Arrange data in increasing or decreasing order
II. Median= the middle value of arranged data
III. If the middle value is two data, then median is the mean of two middle data.

Example 1: Find the median of the following data

a. 4 5 6 10 14
b. 12 18 14 28 26 14 22
c. 8 12 18 13 16 15
d. 20 30 65 70 15 60 90 45

Solution:

a. The increasing order of given data is

$\boxed{4 \quad 5}$ 6 $\boxed{10 \quad 14}$

Then the middle value is 6

Therefore, the median = 6

b. The increasing order of given data is

$\boxed{12 \quad 14 \quad 14}$ 18 $\boxed{22 \quad 26 \quad 28}$

Then the middle value is 18

Therefore, the median =18

c. The increasing order of given data is

$\boxed{8 \quad 12}$ 13 15 $\boxed{16 \quad 18}$

Then the middle values are 13 and 15

The mean of 13 and 15 is $\frac{13+15}{2} = \frac{28}{2} = 14$

Therefore, the median =14

d. The increasing order of given data is 15 20 30 60 65 70 90

middle value is 45 and 60

The mean of 45 and 60 is $\frac{45+60}{2} = \frac{105}{2} = 52.5$

Therefore, the median $= 52.5$

Exercise 7.3.3

1. Find the median of the following list of data

   a. 14 12 24 36 23
   b. 104 112 100 150 102 160
   c. 250 300 100 150 220 230 90
   d. 450 400 670 260 450 300 420 350 150 430

2. The salaries of 8 employees in Birr who work for a small company are listed below. What is the median salary?

8000, 9000, 5500, 10,000, 14,000, 30,000, 20,000

Range

Activity 7.3.4

1. identify the lowest and the highest value from the given data below

   a. 26 29 60 70 80 100 20 15

   b. 108 150 155 183 213

2. Find the difference between the highest and the lowest of above data?
3. Define range by your own words?

Definition: The range of the listed data is the difference between the highest value and the lowest value

$Range = highest\ value - lowest\ value$

Example 1: find the range of the following data

a. 70 55 74 63 80 40

b. -800 -200 -600 0 -300

c. 12 43 -55 70 10 -200 68 90

Solution:

a. the lowest value is 40 and the highest value is 80

$$Range = highest\ value - lowest\ value$$

$$Range = 80 - 40 = 40$$

b. the lowest value is -800 and the highest value is 0

$$Range = highest\ value - lowest\ value$$

$$Range = 0 - (-800) = 800$$

c. the lowest value is -200 and the highest value is 90

$$Range = highest\ value - lowest\ value$$

$$Range = 90 - (-200) = 290$$

Exercise 7.3.4

1. Find the range of the following given data

   a. 61 23 13 90 72 30 50

   b. 46 70 85 84 83 92 94 58

   c. -900 -300 -600 -460 0 -500 -250

   d. 8.5 7.2 9.7 8.3 7.4 7.8 8.4

2. In a class of 20 students the highest score in mathematics exam was 94 and the lowest was 41. What was the range?

7.4.Application of Data Handling

By the end of these sections you should be able to:

- Apply the concept of data handling to organize and interpret real life problems

Example 1: The exam scores of 9 grade seventh students are listed below. Find the mean, mode, median and range of given data.

82, 92, 78.5, 91, 92, 89, 95, 100, 86

Solution:

I. $mean(\bar{x}) = \frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9}{9}$

$= \frac{82 + 92 + 78.5 + 91 + 92 + 89 + 95 + 100 + 86}{9}$

$= \frac{805.5}{9}$

$= 89.5$

II. 92 occurs more frequently than any other values of data, then mode=92

III. The increasing order of data is

78.5 82 86 89 91 92 92 95 100

The middle value is 91, therefore the medina is 91

IV. The lowest value is 78.5 and the highest value is 100

Range= highest –lowest= 100-78.5

= 21.5

Example 2: The ages of 10 college students are listed below. Find the mean, mode, median and range for age of students.

18, 24, 20, 35, 19, 23, 26, 23, 19, 20

Solution:

I. $mean(\bar{x}) = \frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}}{10}$

$= \frac{18 + 24 + 20 + 35 + 19 + 23 + 26 + 23 + 19 + 20}{10}$

$= \frac{227}{10}$

$= 22.7$

II. 19, 20, and 23 occurs twice than any other values of data, then modes are 19, 20 and 23

III. The increasing order of data is

18 19 19 20	20 23	23 24 26 35

The middle values are 20 and 23

The mean of 20 and 23 is $\frac{20+23}{2} = \frac{43}{2} = 21.5$

Therefore, the median $= 21.5$

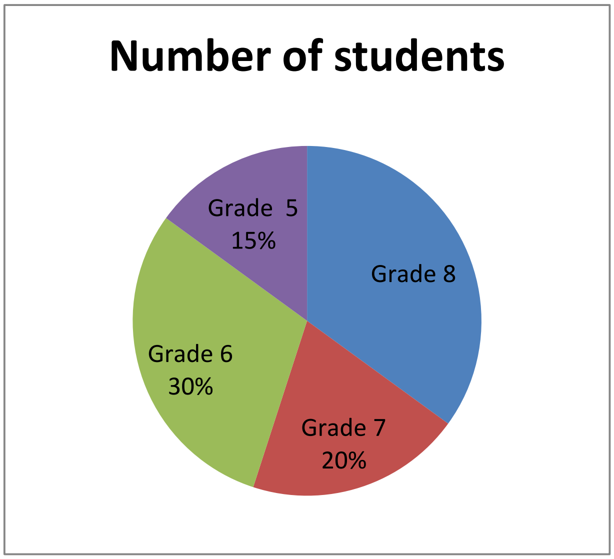
IV. The lowest value is 18 and the highest value is 35

Range= highest –lowest= 35-18

= 17

Example 3: The pie chart shown below is the number of students in a certain school.

There are 1200 students in the school, then what is the number of students in grade 6, 7 and 8?



Solution:

Grade 6= $1200 \times \frac{30}{100} = 360$

Grade 7= $1200 \times \frac{20}{100} = 240$

Grade 8= 100%-30%-15%-20%=35%

$$= 1200 \times \frac{35}{100} = 420$$

Example 4: The table shows the daily sales in Birr grocery of different categories of items for five days.

Day	Mon	Tues	Wed	Thurs	Fri
Drinks	3000	4500	1500	4000	6500
Food	4000	5000	3500	3000	5000

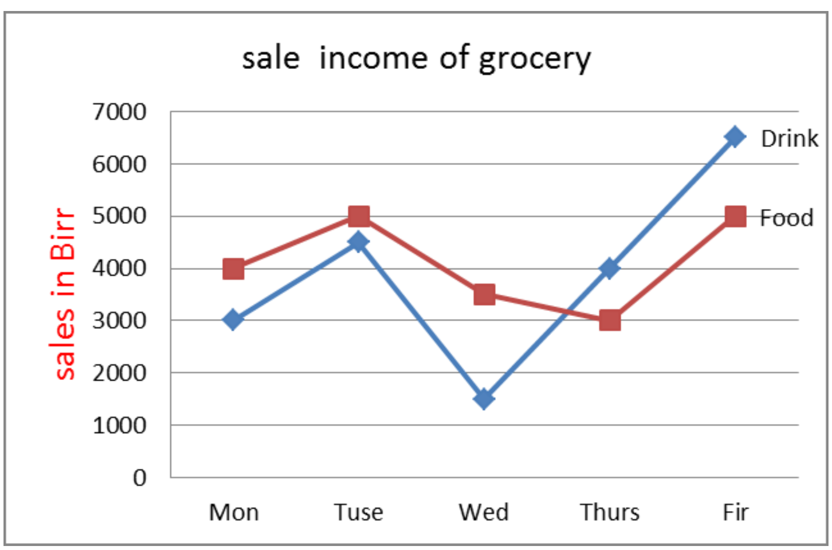
a .Construct a line graph for the frequency table.

b. On what days were the sales for drinks better than the sales for food?

c. What is the total earnings for food and drinks on Wednesday?

Solution:

a.



b. Thursday and Friday

c. Sale in Wednesday =Birr 1500+Birr 3500= Birr5000

Exercise 7.4.1

1. The height (in centimeters) of the members of a school football team has been listed below. 142,140,130,150,160,135,158,132. Find the mean mode, median and range of the above given data.
2. The table shows the daily earnings of a store for five days in Birr.

Day	Mon	Tues	Wed	Thurs	Fri
Earnings	300	450	200	400	650

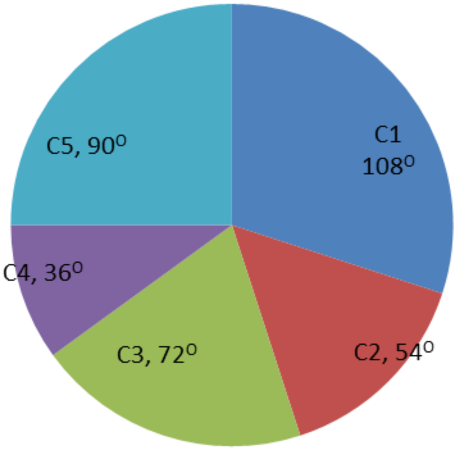
a. Construct a line graph for the frequency table.

b. On which days were the earnings above Birr 400

3. The number of points scored in Ethiopian premier league of football games is listed below. Find the mean, mode, median and range of scores?

   13 18 23 9 3 18

4. 3000 students appeared for an examination from five different centers C1, C2, C3, C4 and C5 of a city. From the given pie chart, find the number of students appearing for the examination from each center



Summary for unit 7

- A tally chart or frequency table is a quick and easy way of recording data.
- Line graph is a graph that uses lines to connect individual data points on a Cartesian coordinate plane
- The line graph is most commonly used to represent two related facts. It use data point "markers," which are connected by straight lines or smooth curves
- Pie chart is a type of graph that represents the data in the circular Graph.
- The portion of a circular region enclosed between two radii and part of the circumference (an arc) is called a sector of the circle
- The mean of given data is the sum of all values divided by the number of values: mean= $\frac{sum\ of\ all\ values}{number\ of\ values}$
- The mode of list of data is the value which occurs most
- A data that has one mode is called unimodal.
- A data that has two mode is called bimodal
- A data that has three mode is called trimodal
- If each value occurs only once, so there is no mode
- The median is the middle value when data is arranged in order of size.
- That is $range = highest\ value - lowest\ value$

Review exercise for unit 7

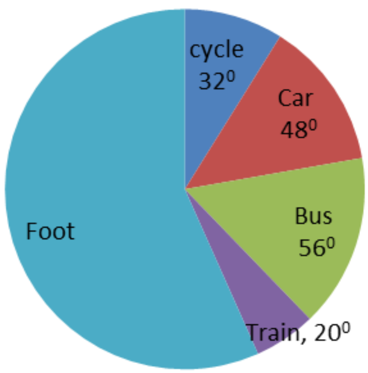
I. Write True if the statement is correct and False if the statement is incorrect

1. Mean is the difference between maximum and minimum value.
2. The mean of 1,3,5 and 7 is 5.

3. A data which has two modes is called bimodal
4. Pie chart is called circular graph.
5. 360^0 is 100% of a circle.

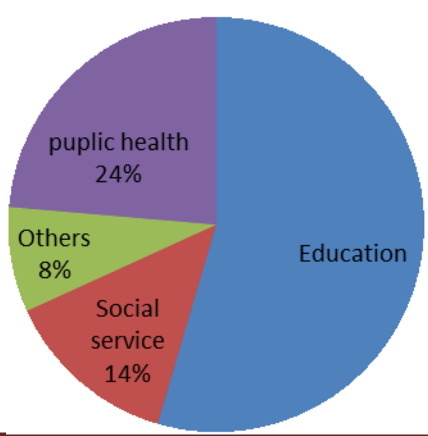
II. Work out

6. 1440 students were asked how they travelled to school. The pie chart shows the results of this survey; Find
   a. How many of the students travelled to school by bus.
   b. How many students travelled on foot?



7. The total expenditure of a sub city is Birr 720,000,000. The pie chart as shown below then how much money was spent on
8.

a. Education
b. Public health
c. Social service
d. Others



9. If the mean of 7 numbers is 20 and the mean of 5 other numbers is 44, then find the mean of all 12 numbers?
10. Find the mean median, mode and range of each set of numbers below.
    a. 3, 4, 7, 3, 5, 2, 6, 10
    b. 8, 10, 12, 14, 7, 16, 5, 7, 9, 11
    c. 17, 18, 16, 17, 17, 14, 22, 15, 16, 17, 14, 12

11. The weights, in kilograms, of the 8 sport club members at a school are:
    45, 53, 47, 46, 44, 46, 47, 48
    a. What is the modal weight?
    b. Calculate the mean and range of weight.
12. Students are arranged in eleven classes. The class sizes are
    23, 24, 24, 26, 27, 28, 30, 24, 29, 24, 27
    a. What is the modal class size?
    b. Calculate the mean class size.
13. Find mean , mode, median of the following number
    a. 49 70 24 76 97 32 32 31
    b. 73 93 72 44 16 43 22 29
14. If the mean of x, x+3, x-5,2x and 3x is 8, then find the value x?
15. 1200 students were interviewed to find out which of five clubs they Preferred. The results as follows

Club	Girls club	HIV club	Maths club	Sport club	Mini media club
Number of students	180	150	240	270	360

Draw a pie chart to represent the above information.

16. Use the information given to find the value of n in each of the following of numbers.

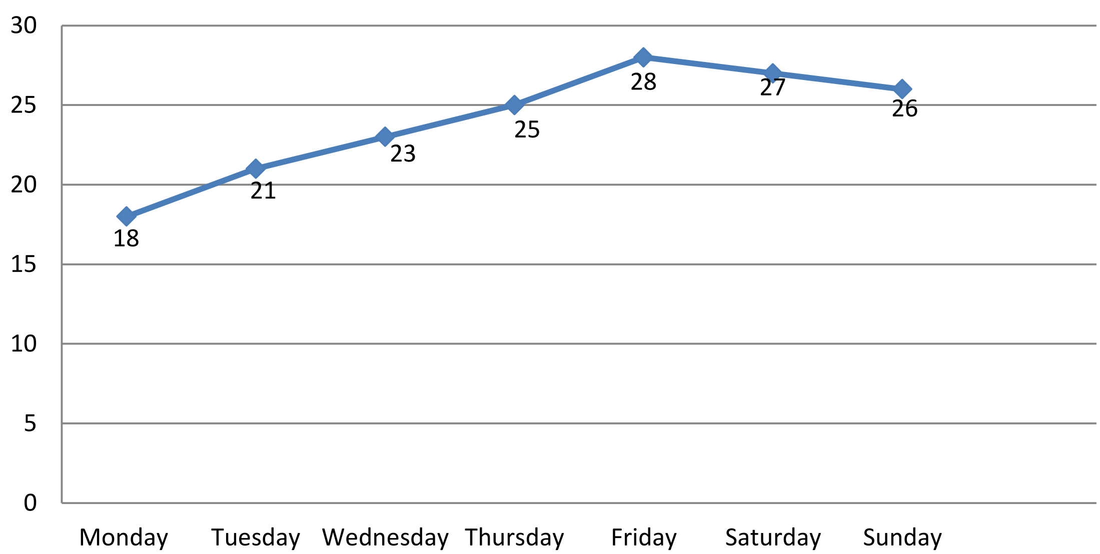
a. 5, 7, 4, 1, n, 5: the mean is 6

b. 3, 1, 4, 5, 4, n : the mode is 4

c. 1, 7, 2, 1, n, 4, 3: the modes are 1 and 2

d. 2.6, 3.5, n, 6.2: the mean is 4

17. The line graph given below shows weekly recorded Addis Ababa's temperature in degree Celsius



a. What is the range of temperature?

b. What is mean of temperature?

c. When was the temperature highest?