AE4320 Assignment:

Aerodynamic Model Identification using the Two-Step Approach

Assignment description:
This project concerns the aircraft aerodynamic model identification using flight test data with the two-step approach learnt in the lecture. Accurate estimation of aircraft position, airspeed body components and attitude with a GPS/IMU/Airdata integrated system for a short range flying region with wind is performed in the first step of the two-step approach applying the Extended or Iterated Extended Kalman Filters EKF/IEKF). The sensor integration structure is defined by using tightly coupled IMU and loosely coupled GPS/Airdata. The IMU will measure the specific force and rotational rate vectors (raw data) of the aircraft, the GPS receiver will provide aircraft position, ground speed earth components and attitude (processed data), and the airdata sensors will offer processed true airspeed, angle of attack, and side slip angle of the aircraft. After the aircraft flight trajectory has been estimated, the aerodynamic model identification will be performed in the second step.

The entire assignment consists of 4 parts. Additional information necessary to complete the assignment is provided in **Appendix A**.

Part 1: Report introduction (10 points)

You are asked to write a brief introduction (max 1 page) for your report which should contain the following items:

1. A brief discussion on the basic principles as well as the relevance of System Identification to aircraft control & simulation. (**2 points**)
2. A brief explanation of the working principles of state estimation techniques and parameter estimation methods as discussed in this course. Discuss these methods from the perspective of linearity of the system, imperfect measurements, model structure, and type of optimization problem. (**4 points**)
3. A brief explanation of the working principles of the two advanced system identification methods discussed in this course, i.e. neural networks and multivariate splines. Mention the role of basis functions, cost functions, and optimization methods. Name an advantage and disadvantage of both methods. (**4 points**)

Part 2: Data pre-processing (17 points)

You are asked to pre-process the given data into a form that is usable in the state estimation part:

1. Generate aircraft position using the given simulated airspeed **components in the navigation frame** and assumed wind velocity. Clearly (graphically) present the results. (**5 points**)
2. Generate GPS, IMU and airdata measurements using Eqs. (5) and (6) using the flight data provided. (**5 points**)
3. Design an integrated GPS/IMU/Airdata navigation system using the accelerometers, rate gyros, GPS receiver and airdata sensors by constructing a navigation model with 18 states (position,

velocity, attitude and IMU biases and wind components), 6 inputs and 12 measurements. (**7 points**)

Part 3: Kalman Filter (35 points)

You are asked to design and use an extended or iterated extended Kalman filter on the pre-processed data. Use the information provided in Appendix A in this part.

1. Create an Extended or Iterated Extended Kalman filters (EKF/IEKF) for this aircraft flight path reconstruction problem. Clearly present in the report the different steps in your EKF/IEKF. (**5 points**)
2. Clearly show that the Kalman filter estimates for the accelerometer and gyro biases are correct or provide a sound scientific argumentation when they are not. (**10 points**)
3. Use your EKF/IEKF to estimate all aircraft states. Clearly show the difference between the 'raw' sensor measurements and the filtered measurements. (**5 points**)
4. Clearly demonstrate how the estimates of the aircrafts states and IMU sensor biases change if standard deviations for the angle of attack and side slip angle measurement noises are increased to 2.0 degrees and 3.5 degrees, respectively. (**5 points**)
5. Show (graphically) that your Kalman filter functions properly by discussing convergence and filter innovation. (**10 points**)

Part 4: Aerodynamic Model Identification (38 points)

You are asked to identify and validate a simple aerodynamic model using the estimated states and estimated forces and moments from the state estimation part. Use the information provided in Appendix A in this part.

1. Compute the aerodynamic forces and moments from the trajectory and sensor parameters found using the EKF/IEKF. Clearly indicate how you performed the numerical differentiation to calculate the angular acceleration components required for the moment calculations. (**3 points**)
2. Formulate an OLS parameter estimator for the parameters of the aerodynamic model given in the assignment. Clearly indicate the structure of your linear regression model and estimator. (**5 points**)
3. Estimate the parameters of your aerodynamic model using the OLS estimator. Clearly indicate how (and why) you used the different data files in this process. (**5 points**)
4. Formulate an alternative estimator (WLS, RLS, MLE) for the parameters of the aerodynamic model given in the assignment, and show how the resulting parameter estimates differ (or not) from those estimated using OLS (**5 points**).
5. Validate your aerodynamic model by using the estimated model output and measured data. Provide a model residual analysis. Additionally, prove the statistical quality of your parameter estimates. Which parameter(s) are you most/less sure about? Clearly indicate why! (**10 points**)
6. Provide an indication which model terms are most dominant, and which have the least influence. Introduce an alternative model structure and compare its validation with the validation given in **4.3**. (**10 points**)

Appendix A: Background Information

The model to be used for the aircraft trajectory estimation consists of five parts:

1. Kinematic equations of an aircraft (flat and non-rotating earth);
2. IMU measurement model (raw measurements);
3. GPS observation model (processed measurements);
4. Airdata observation model (processed measurements)
5. Error models (covariance matrices of process and measurements noises).

Kinematic equations:

$$\dot{\underline{x}}(t) = \underline{f}[\underline{x}(t),\underline{u}(t),t], \quad \underline{x}(t_0) = \underline{x}_0 \tag{1}$$

The kinematic equations for aircraft trajectory estimation in a short range can be simplified by using assumption of a flat and non-rotating earth. These differential equations can be found in the lecture notes: aircraft equations of motion with aircraft position coordinates, air speed components and attitude angles:

$$\begin{aligned}
\dot{x} &= \left[u\cos\theta + (v\sin\phi + w\cos\phi)\sin\theta\right]\cos\psi - (v\cos\phi - w\sin\phi)\sin\psi + W_{x_E} \\
\dot{y} &= \left[u\cos\theta + (v\sin\phi + w\cos\phi)\sin\theta\right]\sin\psi + (v\cos\phi - w\sin\phi)\cos\psi + W_{y_E} \\
\dot{z} &= -u\sin\theta + (v\sin\phi + w\cos\phi)\cos\theta + W_{z_E} \\
\dot{u} &= A_x - g\sin\theta + rv - qw \\
\dot{v} &= A_y + g\cos\theta\sin\phi + pw - ru \\
\dot{w} &= A_z + g\cos\theta\cos\phi + qu - pv \\
\dot{\varphi} &= p + q\sin\phi\tan\theta + r\cos\phi\tan\theta \\
\dot{\theta} &= q\cos\phi - r\sin\phi \\
\dot{\psi} &= q\frac{\sin\phi}{\cos\theta} + r\frac{\cos\phi}{\cos\theta}
\end{aligned} \tag{2}$$

with $\underline{x} = col(x, y, z, u, v, w, \phi, \theta, \psi, W_{x_E}, W_{y_E}, W_{z_E})$ and $\underline{u} = col(A_x, A_y, A_z, p, q, r)$.

***Observation model*:**

$$\begin{aligned}
&\underline{z}(t) = \underline{h}[\underline{x}(t), \underline{u}(t), t]; \\
&\underline{z}_m(t) = \underline{z}(t) + \underline{v}(t), \quad t = t_i, \qquad i = 1, 2, \ldots
\end{aligned} \tag{3}$$

where the observed variables contain GPS position, velocity, attitude, and true airspeed, angle of attack and side slip angle from airdata sensors.

$\underline{z} = col(x, y, z, u, v, w, \phi, \theta, \psi, V, \alpha, \beta)$,

$\underline{z}_m = (x_{GPS}, y_{GPS}, z_{GPS}, u_{GPS}, v_{GPS}, w_{GPS}, \phi_{GPS}, \theta_{GPS}, \psi_{GPS}, V_m, \alpha_m, \beta_m)$

$\underline{v} = col(v_x, v_y, v_z, v_u, v_v, v_w, v_\varphi, v_\theta, v_\psi, v_V, v_\alpha, v_\beta)$.

IMU input model:

In this integrated sensor system (tightly/loosely coupled integration) Inertial Measurement Unit (IMU) is served as the system input with simplified inertial sensor models in a discrete form:

$$
\begin{aligned}
A_{x_m} &= A_x + \lambda_x + w_x \\
A_{y_m} &= A_y + \lambda_y + w_y \\
A_{z_m} &= A_z + \lambda_z + w_z \\
p_m &= p + \lambda_p + w_p \\
q_m &= q + \lambda_q + w_q \\
r_m &= r + \lambda_r + w_r
\end{aligned}
\tag{4}
$$

Note that λ_z will be biased if an incorrect value of the gravitational constant is used!

A GPS receiver with multiple antennas offers 9 observables. They are the position coordinates, ground speed components, and attitude angles:

$$
\begin{aligned}
x_{GPS} &= x \\
x_{GPSm}(t_i) &= x(t_i) + v_x(t_i) \\
y_{GPS} &= y \\
y_{GPSm}(t_i) &= y(t_i) + v_y(t_i) \\
z_{GPS} &= z \\
z_{GPSm}(t_i) &= z(t_i) + v_z(t_i) \\
u_{GPS} &= [u\cos\theta + (v\sin\phi + w\cos\phi)\sin\theta]\cos\psi - (v\cos\phi - w\sin\phi)\sin\psi + W_{x_E} \\
u_{GPSm}(t_i) &= u_{GPS}(t_i) + v_u(t_i) \\
v_{GPS} &= [u\cos\theta + (v\sin\phi + w\cos\phi)\sin\theta]\sin\psi + (v\cos\phi - w\sin\phi)\cos\psi + W_{y_E} \\
v_{GPSm}(t_i) &= v_{GPS}(t_i) + v_v(t_i) \\
w_{GPS} &= -u\sin\theta + (v\sin\phi + w\cos\phi)\cos\theta + W_{z_E} \\
w_{GPSm}(t_i) &= w_{GPS}(t_i) + v_w(t_i) \\
\phi_{GPS} &= \phi \\
\phi_{GPSm}(t_i) &= \phi_{GPS}(t_i) + v_\phi(t_i) \\
\theta_{GPS} &= \theta \\
\theta_{GPSm}(t_i) &= \theta_{GPS}(t_i) + v_\theta(t_i) \\
\psi_{GPS} &= \psi \\
\psi_{GPSm}(t_i) &= \psi_{GPS}(t_i) + v_\psi(t_i)
\end{aligned}
\tag{5}
$$

Airdata sensors give three observables. They are the true airspeed, angle of attack and side slip angle.

$$
\begin{aligned}
&V=\sqrt{u^2+v^2+w^2}\\
&V_m(t_i)=V(t_i)+v_V(t_i)\\
&\alpha=\tan^{-1}\frac{w}{u}\\
&\alpha_m(t_i)=\alpha(t_i)+v_\alpha(t_i)\\
&\beta_m=\tan^{-1}\frac{v}{\sqrt{u^2+w^2}}\\
&\beta_m(t_i)=\beta(t_i)+v_\beta(t_i)
\end{aligned}
\tag{6}
$$

Error models for the input noise vector $\underline{w}$ and output noise vector $\underline{v}$ are give as:

$$
\begin{aligned}
&E\{\underline{w}(t_i)\}=0\\
&E\{\underline{w}(t_i)\underline{w}^T(t_i)\}=Q\delta_{ij}; \qquad Q=diag(\sigma^2_{w_x},\sigma^2_{w_y},\sigma^2_{w_z},\sigma^2_{w_p},\sigma^2_{w_q},\sigma^2_{w_r})
\end{aligned}
\tag{7}
$$

$$
\begin{aligned}
&E\{\underline{v}(t_i)\}=0\\
&E\{\underline{v}(t_i)\underline{v}^T(t_j)\}=R\delta_{ij}; \quad R=diag\left(\sigma^2_{v_x},\sigma^2_{v_y},\sigma^2_{v_z},\sigma^2_{v_u},\sigma^2_{v_v},\sigma^2_{v_w},\sigma^2_{v_\varphi},\sigma^2_{v_\theta},\sigma^2_{v_\psi},\sigma^2_{v_V},\sigma^2_{v_\alpha},\sigma^2_{v_\beta}\right)
\end{aligned}
\tag{8}
$$

The input and output noises are assumed to be uncorrelated:

$$
E\{\underline{w}(t_i)\underline{v}^T(t_j)\}=0, \tag{9}
$$

The standard deviations of the input and output noises are assumed to be constant and known.

Complete model

The complete model can next be written in the form:

$$
\underline{\dot{x}}(t)=\underline{f}[\underline{x}(t),\underline{u}_m(t),\underline{\theta},t]+G[\underline{x}(t)]\underline{w}(t), \qquad \underline{x}(t_0)=\underline{x}_0 \tag{10}
$$

$$
\begin{aligned}
&\underline{z}(t)=\underline{h}[\underline{x}(t),\underline{u}_m(t),\underline{\theta},t]\\
&\underline{z}_m(t_i)=\underline{z}(t_i)+\underline{v}(t_i), \qquad\qquad t=t_i, \quad i=1,2,\ldots
\end{aligned}
\tag{11}
$$

The flight test data of $\underline{u}=col(a_x,a_y,a_z,p,q,r)$ is given with the assignment.

The real biases of accelerometers and rate gyros are assumed as:

$$
\begin{aligned}
&\lambda_{x_r}=0.02\ \ m/s^2\\
&\lambda_{y_r}=0.02\ \ m/s^2\\
&\lambda_{z_r}=0.02\ \ m/s^2\\
&\lambda_{p_r}=0.003\ \ \deg/s\\
&\lambda_{q_r}=0.003\ \ \deg/s\\
&\lambda_{r_r}=0.003\ \ \deg/s
\end{aligned}
\tag{12}
$$

These biases need to be estimated in the first step of the two step identification approach.

The noise standard deviations for the accelerometers and rate gyros are assumed to be the same level as stated in (12).

The wind components in the earth reference frame are assumed as constants. They take the amplitudes of 2, -8, and 1 m/s in order to generate the actual flight test data. However, these wind components are also unknowns and have to be estimated in the first step of the two step approach.

The GPS position measurement noise standard deviation is 2.5 meters. The GPS velocity measurement noise standard deviation is 0.02 m/s. The GPS attitude measurement noise standard deviation is 0.05 degrees.

The standard deviations for the true airspeed, angle of attack and side slip angle measurement noises are 0.1 m/s, 0.1 degrees and 0.1 degrees, respectively.

Other data of the aircraft is listed as follows

$$I_{xx} = 11{,}187.8\ kg\,m^2, \quad I_{yy} = 22{,}854.8\ kg\,m^2, \quad I_{zz} = 31{,}974.8\ kg\,m^2, \quad I_{xz} = 1{,}930.1\ kg\,m^2$$

$$\text{mass}: 4{,}500\,kg, \quad \text{wing span: } b = 13.3250\ m, \quad \text{wing area}: S = 24.9900\,m^2, \quad \text{chord}: \ \bar{c} = 1.9910$$

The dimensionless aerodynamic force body components can be calculated according to

$$\begin{aligned} C_X &= \frac{X}{\frac{1}{2}\rho V^2 S} = \frac{mA_x}{\frac{1}{2}\rho V^2 S} \\ C_Y &= \frac{Y}{\frac{1}{2}\rho V^2 S} = \frac{mA_y}{\frac{1}{2}\rho V^2 S} \\ C_Z &= \frac{Z}{\frac{1}{2}\rho V^2 S} = \frac{mA_z}{\frac{1}{2}\rho V^2 S} \end{aligned} \tag{13}$$

The dimensionless aerodynamic moment components can be calculated with

$$\begin{aligned} C_l &= \frac{L}{\frac{1}{2}\rho V^2 Sb} = \frac{\dot{p}I_{xx} + qr(I_{zz} - I_{yy}) - (pq + \dot{r})I_{xz}}{\frac{1}{2}\rho V^2 Sb} \\ C_m &= \frac{M}{\frac{1}{2}\rho V^2 S\bar{c}} = \frac{\dot{q}I_{yy} + rp(I_{xx} - I_{zz}) + (p^2 - r^2)I_{xz}}{\frac{1}{2}\rho V^2 S\bar{c}} \\ C_n &= \frac{N}{\frac{1}{2}\rho V^2 Sb} = \frac{\dot{r}I_{zz} + pq(I_{yy} - I_{xx}) + (qr - \dot{p})I_{xz}}{\frac{1}{2}\rho V^2 Sb} \end{aligned} \tag{14}$$

The aerodynamic force can be modeled as:

$$
\begin{aligned}
C_X &= C_{X_0} + C_{X_\alpha}\alpha + C_{X_{\alpha^2}}\alpha^2 + C_{X_q}\frac{q\bar{c}}{V} + C_{X_{\delta_e}}\delta_e + C_{X_{T_c}}T_c \\
C_Z &= C_{Z_0} + C_{Z_\alpha}\alpha + C_{Z_q}\frac{q\bar{c}}{V} + C_{Z_{\delta_e}}\delta_e + C_{Z_{T_c}}T_c \\
C_m &= C_{m_0} + C_{m_\alpha}\alpha + C_{m_q}\frac{q\bar{c}}{V} + C_{m_{\delta_e}}\delta_e + C_{m_{T_c}}T_c
\end{aligned}
\tag{15}
$$

The aerodynamic moments can be modeled as:

$$
\begin{aligned}
C_Y &= C_{Y_0} + C_{Y_\beta}\beta + C_{Y_p}\frac{pb}{2V} + C_{Y_r}\frac{rb}{2V} + C_{Y_{\delta_a}}\delta_a + C_{Y_{\delta_r}}\delta_r \\
C_l &= C_{l_0} + C_{l_\beta}\beta + C_{l_p}\frac{pb}{2V} + C_{l_r}\frac{rb}{2V} + C_{l_{\delta_a}}\delta_a + C_{l_{\delta_r}}\delta_r \\
C_n &= C_{n_0} + C_{n_\beta}\beta + C_{n_p}\frac{pb}{2V} + C_{n_r}\frac{rb}{2V} + C_{n_{\delta_a}}\delta_a + C_{n_{\delta_r}}\delta_r
\end{aligned}
\tag{16}
$$