From Thermodynamics to Complex Systems*

Yaneer Bar-Yam

New England Complex Systems Institute, Cambridge, MA 02139

(Dated: February 26, 2019)

Our objective in this book is to consider the dynamics of complex systems. While, as discussed in the previous section, we will use the principles of thermodynamics to help us in this analysis, another important reason to review thermodynamics is to recognize what complex systems are not. Thermodynamics describes macroscopic systems without structure or dynamics. The task of thermodynamics is to relate the very few macroscopic parameters to each other. It suggests that these are the only relevant parameters in the description of these systems. Materials and complex systems are both formed out of many interacting parts. The ideal gas example described a material where the interaction between the particles was weak. However, thermodynamics also describes solids, where the interaction is strong. Having decided that complex systems are not described fully by thermodynamics, we must ask, Where do the assumptions of thermodynamics break down? There are several ways the assumptions may break down, and each one is significant and plays a role in our investigation of complex systems. Since we have not yet examined particular examples of complex systems, this discussion must be quite abstract. However, it will be useful as we study complex systems to refer back to this discussion. The abstract statements will have concrete realizations when we construct models of complex systems.

The assumptions of thermodynamics separate into space-related and time-related assumptions. The first we discuss is the divisibility of a macroscopic material. Fig. 1 illustrates the property of divisibility. In this process, a small part of a system is separated from a large part of the system without affecting the *local* properties of the material. This is inherent in the use of extensive and intensive quantities. Such divisibility is not true of systems typically considered to be complex systems. Consider, for example, a person as a complex system that cannot be separated and continue to have the same properties. In words, we would say that complex systems are formed out of not only interacting, but also interdependent parts. Since both thermodynamic and complex systems are formed out of interacting parts, it is the concept of interdependency that must distinguish them. We will dedicate a few paragraphs to defining a sense in which "interdependent" can have a more precise meaning.

We must first address a simple way in which a system may have a nonextensive energy and still not be a complex system. If we look closely at the properties of a material, say a piece of metal or a cup of water, we discover that its surface is different from the bulk. By separating the material into pieces, the surface area of the material is changed. For macroscopic materials, this generally does not affect the bulk properties of the material. A characteristic way to identify surface properties, such as the surface energy, is through their dependence on particle number. The surface energy scales as $N^{2/3}$, in contrast to the extensive bulk energy that is linear in N. This kind of correction can be incorporated directly in a slightly more detailed treatment of thermodynamics, where every macroscopic parameter has a surface term. The presence of such surface terms is not sufficient to identify a ma-
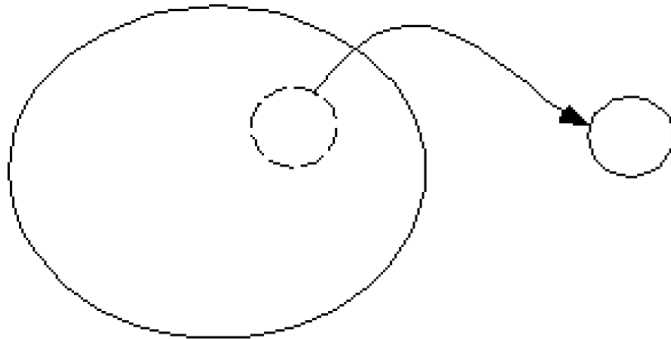

FIG. 1: The assumption that the local properties of a system are unaffected by subdivision applies also to the case where a small part of a much larger system is removed. The local properties, both of the small system and of the large system are assumed to remain unchanged. Even though the small system is much smaller than the original system, the small system is understood to be a macroscopic piece of material. Thus it retains the same local properties it had as part of the larger system.

terial as a complex system. For this reason, we are careful to identify complex systems by requiring that the scenario of Fig. 1 is violated by changes in the local (i.e., everywhere including the bulk) properties of the system, rather than just the surface.

It may be asked whether the notion of "local properties" is sufficiently well defined as we are using it. In principle, it is not. For now, we adopt this notion from thermodynamics. When only a few properties, like the energy and entropy, are relevant, "affect locally" is a precise concept. Later we would like to replace the use of local thermodynamic properties with a more general concept—the behavior of the system.

How is the scenario of Fig. 1 violated for a complex system? We can find that the local properties of the small part are affected without affecting the local properties of the large part. Or we can find that the local properties of the large part are affected as well. The distinction between these two ways of affecting the system is important, because it can enable us to distinguish between different kinds of complex systems. It will be helpful to name them for later reference. We call the first category of systems complex materials, the second category we call complex organisms.

Why don't we also include the possibility that the large part is affected but not the small part? At this point it makes sense to consider generic subdivision rather than special subdivision. By generic subdivision, we mean the ensemble of possible subdivisions rather than a particular one. Once we are considering complex systems, the effect of removal of part of a system may depend on which part is removed. However, when we are trying to understand whether or not we have a complex system, we can limit ourselves to considering the generic effects of removing a part of the system. For this reason we do not consider the possibility that subdivision affects

the large system and not the small. This might be possible for the removal of a particular small part, but it would be surprising to discover a system where this is generically true.

Two examples may help to illustrate the different classes of complex systems. At least superficially, plants are complex materials, while animals are complex organisms. The reason that plants are complex materials is that the cutting of parts of a plant, such as leaves, a branch, or a root, typically does not affect the local properties of the rest of the plant, but does affect the excised part. For animals this is not generically the case. However, it would be better to argue that plants are in an intermediate category, where some divisions, such as cutting out a lateral section of a tree trunk, affect both small and large parts, while others affect only the smaller part. For animals, essentially all divisions affect both small and large parts. We believe that complex organisms play a special role in the study of complex system behavior. The essential quality of a complex organism is that its properties are tied to the existence of all of its parts.

How large is the small part we are talking about? Loss of a few cells from the skin of an animal will not generally affect it. As the size of the removed portion is decreased, it may be expected that the influence on the local properties of the larger system will be reduced. This leads to the concept of a robust complex system. Qualitatively, the larger the part that can be removed from a complex system without affecting its local properties, the more robust the system is. We see that a complex material is the limiting case of a highly robust complex system.

The flip side of subdivision of a system is aggregation. For thermodynamic systems, subdivision and aggregation are the same, but for complex systems they are quite different. One of the questions that will concern us is what happens when we place a few or many complex systems together. Generally we expect that the individual complex systems will interact with each other. However, one of the points we can make at this time is that just placing together many complex systems, trees or people, does not make a larger complex system by the criteria of subdivision. Thus, a collection of complex systems may result in a system that behaves as a thermodynamic system under subdivision—separating it into parts does not affect the behavior of the parts.

The topic of bringing together many pieces or subdividing into many parts is also quite distinct from the topic of subdivision by removal of a single part. This brings us to a second assumption we will discuss. Thermodynamic systems are assumed to be composed of a very large number of particles. What about complex systems? We know that the number of molecules in a cup of water is not greater than the number of molecules in a human being. And yet, we understand that this is not quite the right point. We should not be counting the number of water molecules in the person, instead we might count the number of cells, which is much smaller. Thus appears the problem of counting the number of components of a system. In the context of correlations in materials, this was briefly discussed at the end of the last section. Let us assume for the moment that we know how to count the number of components. It seems clear that systems with only a few components should not be treated by thermodynamics. One of the interesting questions we will discuss is whether in the limit of a very large number of components we will always have a thermodynamic system. Stated in a simpler way from the point of view of the study of complex systems, the question becomes how large is too large or how many is too many. From the thermodynamic perspective the question is, Under what circumstances do we end up with the thermodynamic limit?

We now switch to a discussion of time-related assumptions. One of the basic assumptions of thermodynamics is the ergodic theorem that enables the description of a single system using an ensemble. When the ergodic theorem breaks down, as discussed in the previous section, additional fixed or quenched variables become important. This is the same as saying that there are significant differences between different examples of the macroscopic system we are interested in. This is a necessary condition for the existence of a complex system. The alternative would be that all realizations of the system would be the same, which does not coincide with intuitive notions of complexity. We will discuss several examples of the breaking of the ergodic theorem later. The simplest example is a magnet. The orientation of the magnet is an additional parameter that must be specified, and therefore the ergodic theorem is violated for this system. Any system that breaks symmetry violates the ergodic theorem. However, we do not accept a magnet as a complex system. Therefore we can assume that the breaking of ergodicity is a necessary but not sufficient condition for complexity. All of the systems we will discuss break ergodicity, and therefore it is always necessary to specify which coordinates of the complex system are fixed and which are to be assumed to be so rapidly varying that they can be assigned equilibrium Boltzmann probabilities.

A special case of the breaking of the ergodic theorem, but one that strikes even more deeply at the assumptions of thermodynamics, is a violation of the separation of time scales. If there are dynamical processes that occur on every time scale, then it becomes impossible to treat the system using the conventional separation of scales into fast, slow and dynamic processes. As we will discuss in Section 1.10, the techniques of renormalization that are used in phase transitions to deal with the existence of many spatial scales may also be used to describe systems changing on many time scales.

Finally, inherent in thermodynamics, the concept of equilibrium and the ergodic theorem is the assumption that the initial condition of the system does not matter. For a complex system, the initial condition of the system does matter over the time scales relevant to our observation. This brings us back to the concept of correlation time. The correlation time describes the length of time over which the initial conditions are relevant to the dynamics. This means that our observation of a complex system must be shorter than a correlation time. The spatial analog, the correlation length, describes the effects of surfaces on the system. The discussion of the effects of subdivision also implies that the system must be smaller than a correlation length. This means that complex systems change their internal structure—adapt—to conditions at their boundaries. Thus, a suggestive though incomplete summary of our discussion of complexity in the context of thermodynamics is that a complex system is contained within a single correlation distance and correlation time.

* Adapted from Dynamics of Complex Systems Section 1.3.6.