

CS1510

web.mst.edu/~ricardom/

section A

CS340 - office

T/Th 3:30 - 5:00 in CS212 (computer lab)

50% tests 3 and 1 final

50% homework (lowest dropped)

graded on correctness, not length

no-compile = 10%
crashing = 25%

not -io
score = 10 (1/100)

LEAD
CS212

Tu/Th 3:30 - 5:30

max 5 ~~misses~~ absences

price@mst.edu

advisors are informed of bad academic performance

Feb 17 — Test #1

March 24 — Test #2

April 28 — Test #3

Final (^{date} published somewhere) (comprehensive)

Lecture Notes/Sample Code

++03

~~C++11~~
~~C++14~~
~~C++17~~

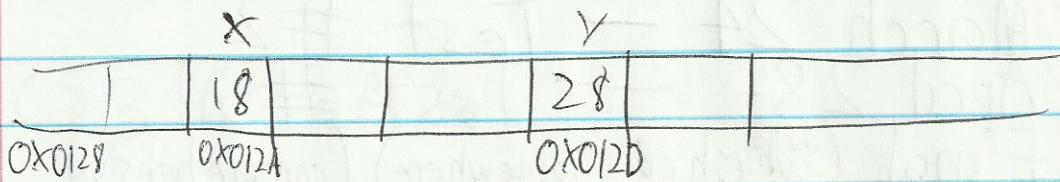
do not use jpic!

V20/2017

Tre is in Google

Minerama 4-6 today Havener

C++ Pointers



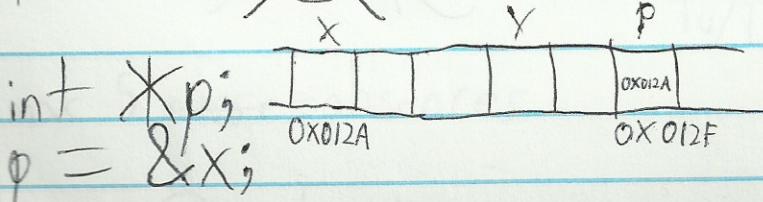
& - address-of operator

cout << &x; 0X012A

real addresses are many more digits

0x is a prefix indicating a hexadecimal number

int p;
p = &x; ~~X~~ fails



$\text{int } *p = \text{NULL};$ $\boxed{\text{p}} \boxed{0x0000}$
 ↓
 a constant

pointers carry type

```

int x = 74;
char c = 'r';
float z = 3.14;
  
```

$\text{int } *p = \text{NULL};$

$p = &x;$

$p = &c; \times$

$p = &z; \times$

& - address of

$\text{char } *q = \text{NULL};$

$q = &c;$

$\text{float } *r = \text{NULL};$

$r = &z;$

$\text{int } *s = p;$

$\text{char } *t; \quad \boxed{\text{junk}}$

$t = &t; \quad \times$
 $\text{char } * \quad \text{char } **$

with operator $=$, both sides must be the same type

```

int a[5];
int *v = &a[3];
  
```

$\text{char } *t = \text{NULL};$
 $\text{char } **z = &t;$

$\boxed{t} \boxed{z}$
 $\boxed{0x0000} \boxed{0x0128}$
 $\boxed{0x0128}$

dereference, *

cout << *q;

cout << *p;

r

74

*q = 'm';

*p = 3 * *p;

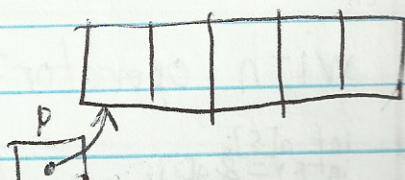
~~new operator~~ — new
— delete

new <type> — creates anonymous variable,
p = new int; returns pointer
q = new float; (dynamic memory)

p = new int; // old *p lost :(

delete p; — deletes what p points to
p = NULL; — good practice

p = new int[5];

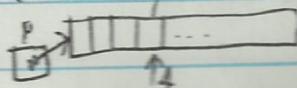


p = new int[X]; // dynamic array variable size! 0 < X < 999999
X *= 2; // *p size not changed

p[3] = 4; // brackets de reference pointer

delete [] p; // C++ remembers the size of the p array.
You can't access it.

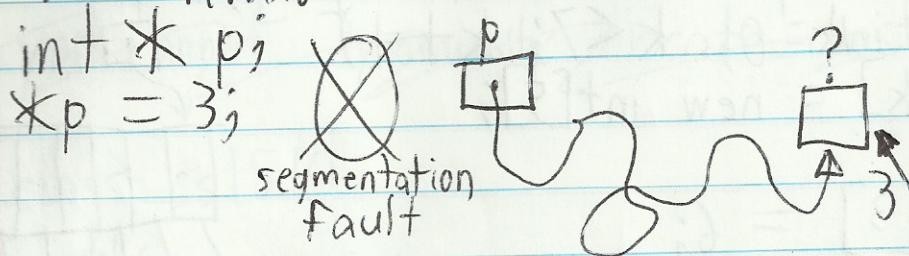
p = NULL; // good practice



Errors

- Dangling pointer
- Memory leak

Dangling pointer — when a pointer with an invalid address is used



p = new int;

delete p;

*p = 3; // doesn't crash
very difficult
to trace

Memory leak — when a dynamic variable becomes unreachable

q = new float;

q = &y; // doesn't crash
if repeated, slows computer

programs that
slow to a crawl
after running for
long periods
have memory leaks

1/25/2016

c++ pointers

• new only supports one dimension
fails $\Rightarrow \text{int} * p = \text{new int}[7][5];$

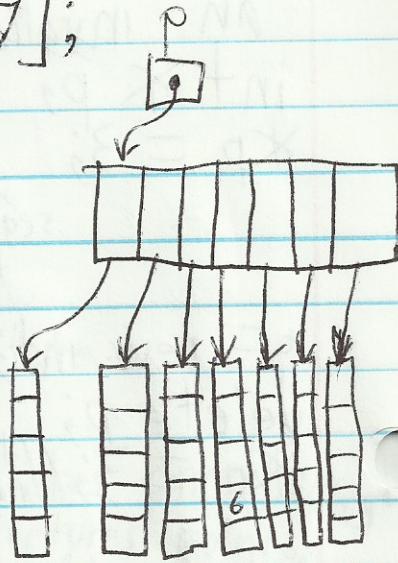
ACM

- SIG-COMP 6-7^{we} cs204

- SIG-GAME 3:30-4:30th cs207

$\text{int} ** p = \text{new int}*[7];$
 $\text{for}(\text{int } k=0; k<7; k++)$
 $p[k] = \text{new int}[5];$

$p[3][3] = 6;$



$\text{delete } [] p;$ // WRONG

• delete only supports one dimension

$\text{for}(\text{int } k=0; k<7; k++)$

$\text{delete } [] p[k];$

$\text{delete } [] p;$

$p = \text{NULL};$

- const

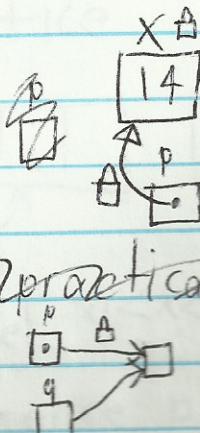
$\text{const int } x = 14;$

$\text{int} * p = \&x;$

$\text{const int } * p = \&x;$

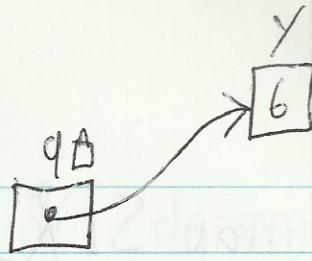
$p = \text{new int};$ // runs, not practical

$\text{int } * q = p;$



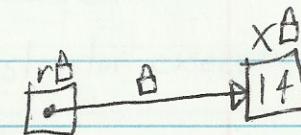
```
const int x = 14;
```

```
int y = 6;
```



```
int * const q = &y;  
*q = 7;
```

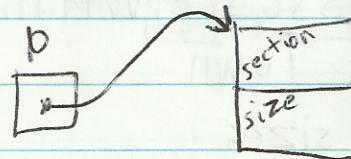
```
const int * const r = &x;
```



~~You cannot dynamically allocate constants~~

```
class cs1510  
{ public:  
    char section;  
    int size;  
};
```

```
cs1510 * p;  
p = new cs1510;
```



~~*p.section = 'a';~~
~~*p.size = 1;~~

$(*p).section = 'a';$
 $p \rightarrow section = 'a';$

Map SDIV on personal computer
— see cs1001 presentation 1

emacs save Cx CS
 exit Cx CC

- Homework #1 Posted
- LEAD Tu/Th 3:30 - 5:00 CS213

Classes with Pointers

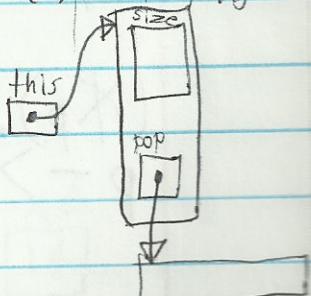
class Town

```
{ int size;
  Person* pop;
  void setSize(int n) hidden { Town* const this, int n)
    { this->size = n; // works
    }
  Town(){} // hidden
  ~Town(){} // hidden
  Town(const Town& rhs){...} // hidden
  const Town& operator=(const Town& rhs) // hidden
  {...}
```

override
with pointer
members

class Person

```
Springfield.setSize(5); Town Springfield
```



destructor is executed when a class goes out of scope

{
 Town Rolla;

} // destructor executes

copy constructor

- declaration with initialization

- passing objects by value

{
 foo(Town x)
}
}

{
 Town Rolla;
 Town Cuba = Rolla; // copy constructor executes
 Town Sedalia(Rolla); // original, this is more proper
 foo(Rolla); // called again

operator=

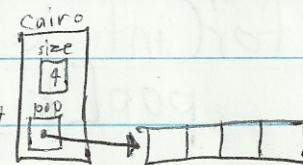
- whenever explicitly called

{
 Town Salem;
 Salem = Rolla; // operator=

```
class Town
{
    int size;
    Person* pop;
    Town()
    {
        size = 4;
        pop = new People[size];
    }
}
```

{
 Town Cairo;

} // memory leak, *pop is lost

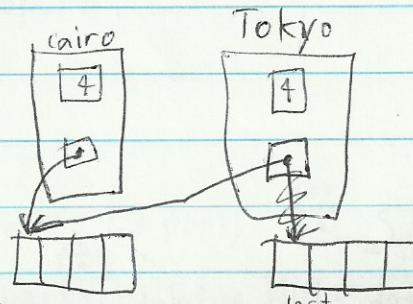


solution: destructor

```
~Town()
{
    delete [] pop;
}
```

```
{  
    Town Tokyo;  
    Town Cairo;  
    Tokyo = Cairo; // Tokyo.pop is lost
```

shallow copy



} // deletes [] * Tokyo.pop twice

"double free" followed by pages of binary numbers

Default operator=

const Town& operator=(const Town& rhs)

```
{  
    if (this != &rhs) // alias test  
    {  
        size = rhs.size;  
        pop = rhs.pop; // change this line  
    }  
    return *this;  
}
```

to this

```
delete [] pop;  
pop = new Person[size][size];  
for (int k = 0; k < size; k++)  
    pop[k] = rhs.pop[k];
```

1/30/2017

Hw1 due Friday

./a.out < in.txt

Classes with Pointers

Big-3 (new C++ has 5 things you must override)

- Destructor
- Copy Constructor
- Operator=

```
class Town
```

```
{ int size;  
  People* pop;
```

→ const Town& operator=(const Town& rhs)

if (this != rhs) // alias test

{ this->size = rhs.size;

delete [] pop;

pop = new People[size];

for (int k=0; k<size; k++)
 pop[k] = rhs.pop[k];

}

return *this;

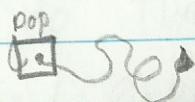
}

```
Town(const Town& rhs)
```

*this = rhs; ~~(delete [] pop;)~~

this->pop = NULL;

*this = rhs; // works, deleting NULL does nothing



Abstract Data Types & Data Structures

Planning Organizing Data

ADT — a mathematical object together with operations

Data Structures — a particular implementation of an ADT

ADT	DS
integer	int
real number	float
set	
string	string
function	
graph	
list	
stack	
queue	
tree	
map	

what is the sum of the digits
of $100!$

Building Blocks

classes

arrays

pointers

a List of size n is a sequence of
elements (usually of the same type)
 $\langle a_0, a_1, a_2, \dots, a_{n-1} \rangle$
if $n=0$, empty list $\langle \rangle$

the elements are linearly ordered

- every element has a position
- every element has a predecessor (except a_0)
- every element has a successor (except a_{n-1})

operations $l_1 = \langle c, e, r, a, b \rangle$

- $\text{size}(l_1) = 5$
- $\text{front}(l_1) = c$
- $\text{back}(l_1) = b$
- $\text{at}(l_1, 3) = a$
- $\text{find}(l_1, a) = \text{true}$

How many sig figures do float and double have

- $\text{insert}(l_1, 2, 'k') = \langle c, e, k, r, a, b \rangle$
- $\text{remove}(l_1, 3) = \langle c, e, r, b \rangle$
- $\text{insert_back}(l_1, p) = \langle c, e, r, a, b, p \rangle$

an array is a Data Structure

a list is a mathematical object

ArrayList



You cannot dynamically
resize an array

```
template<typename T>
class ArrayList
{
    T *m_data;
    int m_size;
    int m_max;

    ArrayList()
    {
        m_data = new T[4];
        m_size = 0;
        m_max = 4;
    }

    void insert_back( const T& x )
    {
        if( m_size != m_max )
        {
            m_data[m_size] = x;
            m_size++;
        }
        return;
    }

    int size() const
    {
        return m_size;
    }

    const T& at( const int pos ) const
    {
        if( 0 <= pos && pos < m_size )
            return m_data[pos];
    }

    const T& front() const
    {
        if( m_size > 0 )
            return m_data[0];
    }
}
```

2/3/2017

```
ArrayList<char> l;  
l.insert_back('g');  
l.insert_back('r');  
cout << l.front();
```

max = 4
size = 0
data = new char[4]
size++

g		
---	--	--

homework #2 posted

in class ArrayList

```
void grow()  
{  
    T* tmp = new T[m-max * 2];  
    for (int k=0; k<m-size; k++)  
        tmp[k] = m-data[k];  
    m-max *= 2;  
    delete [] m-data;  
    m-data = tmp;  
    delete tmp; // WRONG  
    return;  
}
```

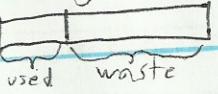
```
void insert_back( const T& x)  
{  
    if (m-size == m-max)  
        grow();  
    m-data[m-size] = x;  
    m-size++;  
    return;  
}
```

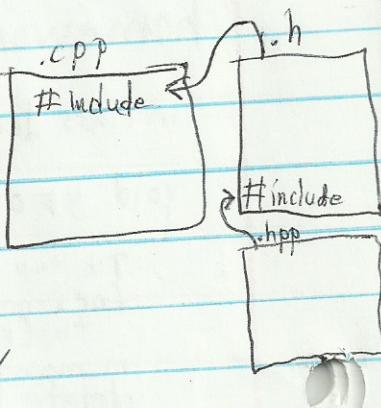
```
void insert( const T& x, const int pos)  
{  
    if (m-size == m-max)  
        grow();  
    if (pos <= m-size && 0 <= pos)  
    {  
        for (int k=m-size-1; k >= pos; k--)  
            m-data[k+1] = m-data[k];  
        m-data[pos] = x;  
        m-size++;  
    }  
    return;  
}
```

`if (m_size < m_max / 4)`
`shrink(); // halves size, leaving extra space`
 "separation of concerns"

2/6/2017

ArrayList drawbacks
 ↵ inserting and deleting
 is inefficient

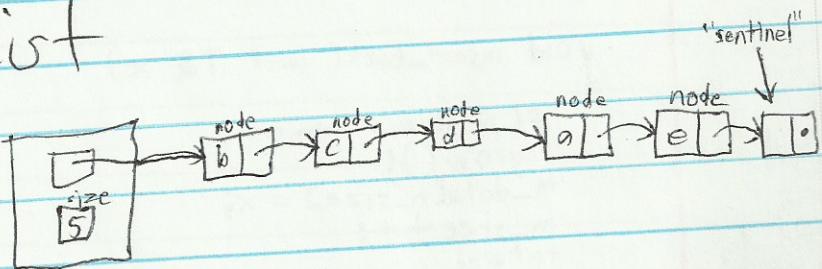
- having to get a new array
-  a lot of wasted memory



D.S. LinkedList

```

class Node
public:
    T m_data;
    Node* m_next;
}
  
```



```

class LinkedList
{
    Node* m_head;
    int size;
    LinkedList()
    {
        size = 0;
        m_head = new Node();
        m_head->m_next = NULL;
    }
    void insert_front( const T& x )
    {
        Node* tmp = new Node();
        tmp->m_next = m_head;
        tmp->m_data = x;
        m_head = tmp;
        tmp = NULL;
        m_size++;
    }
}
  
```

2/10/2017

valgrind - check for memory leaks
\$ valgrind [program to execute]

$$\int \arcsin x \, dx = x \arcsin x + \sqrt{1-x^2} + C$$
$$\int \arccos x \, dx = x \arccos x - \sqrt{1-x^2} + C$$
$$\int \arctan x \, dx = x \arctan x - \frac{1}{2} \ln(1+x^2) + C$$

Next Friday Test #1

D.S. LinkedList

```
const T& at( int pos )
{
    if( pos < m_size )
        for( Node* p = m_head;
            int k=0; k<pos; k++ )
            p = p->m_next;
        return p->m_data;
}
```

```
Node* at_ptr( int pos )
{
    if( pos < m_size )
        Node* p = m_head;
        for( int k=0; k<m_size; k++ )
            p = p->m_next;
        return p;
}
```

```
void insert(Node* p, const T& x)
```

```
{
```

```
    Node* tmp = new Node(*p); // or, with no constructor, Node* tmp = new Node();
    p->m_data = x;
    p->m_next = tmp;
    m_size++;
}
```

- 1) create new Node
- 2) copy Node pointed by p to new node
- 3) insert x and chain new node
- 4) increment size

```
bool find(const T& x)
```

```
{
```

```
    Node* p = m_head;
    while(p->m_next != NULL)
        if(p->m_data == x)
            return true;
        else
            p = p->m_next;
    }
    return false;
}
```

What if it's the last element (before the sentinel)?

```
Node // here's an option
```

```
{  
    T m_data;  
    Node* m_next;  
    Node* m_prev;
```

D.S. DoublyLinkedList (4 pointers to take care of with insert)

```
void insert(Node* p, const T& x)
```

```
{
```

```
    Node* tmp = new Node;
```

~~if(p == NULL)~~~~for~~~~p->m_prev = tmp;~~~~tmp->m_data = x;~~~~tmp->m_next = p;~~~~tmp->m_prev = p->m_prev;~~~~for~~~~((tmp->m_prev)).m_next = tmp;~~~~tmp->m_prev->m_next = tmp;~~~~p->m_prev = tmp;~~

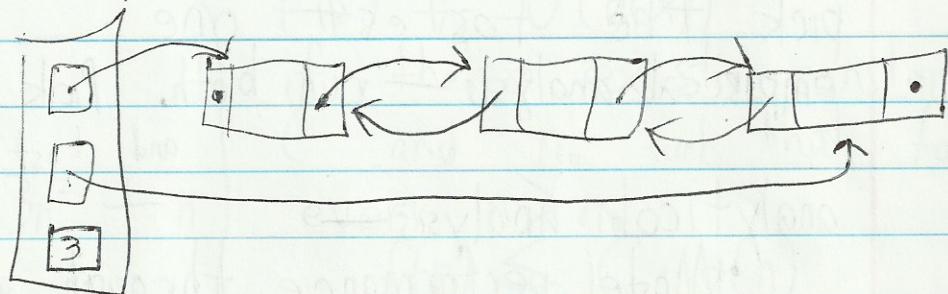
2/10/2017

Test #1 — 1 week from today

50 min

- class notes from website
- know about homework

D.S. Doubly Linked List

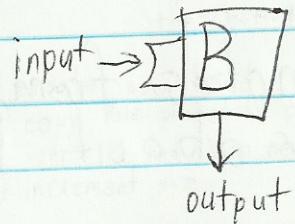
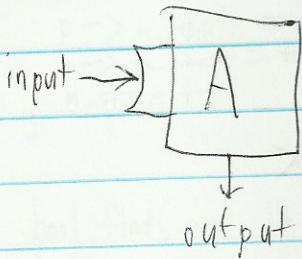


```
void insert_back( const T& x)
```

```
{ Node* tmp = new Node;  
m_back->m_next = tmp;  
m_back = tmp;  
m_back->m_next = NULL;  
m_back->m_data = x;  
m_size++; }
```

// fails for empty list

Algorithm Complexity (an intro)



pick the fastest one
empirical analysis — run both. Pick "bench marks"
and test both programs

analytical analysis —

• Model performance through a mathematical object

runtime function — returns time

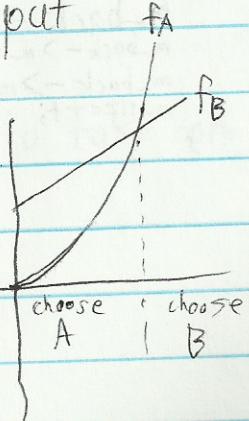
$$f(\underset{\text{input}}{\text{size of}}) \rightarrow \text{time}$$

— assume worst quality input

$$f_A(n) = \frac{1}{2}n^2 + 3$$

$$f_B(n) = 20n + 123$$

But, in general, choose B.



we're not interested in values, but the R.O.G.

Test #1 this Friday

runtime function

- rate of growth: Big-O

Big-O — given functions $f(x)$ and $g(x)$ we say that $f(x)$ is $O(g(x))$ if there exists constants C and n_0 such that for every $n > n_0$ $f(n) \leq C \cdot g(n)$

$$\left. \begin{array}{l} f_1(n) = n^2 \\ f_2(n) = 3n^2 + n \end{array} \right\} f_1 < f_2$$

$$\left. \begin{array}{l} f_1 \text{ is } O(f_2) \\ f_2 \text{ is } O(f_1) \end{array} \right.$$

ignoring constant factors,
the R.O.G. of $g(x)$
is greater than or equal
to the R.O.G. of $f(x)$

Big-O

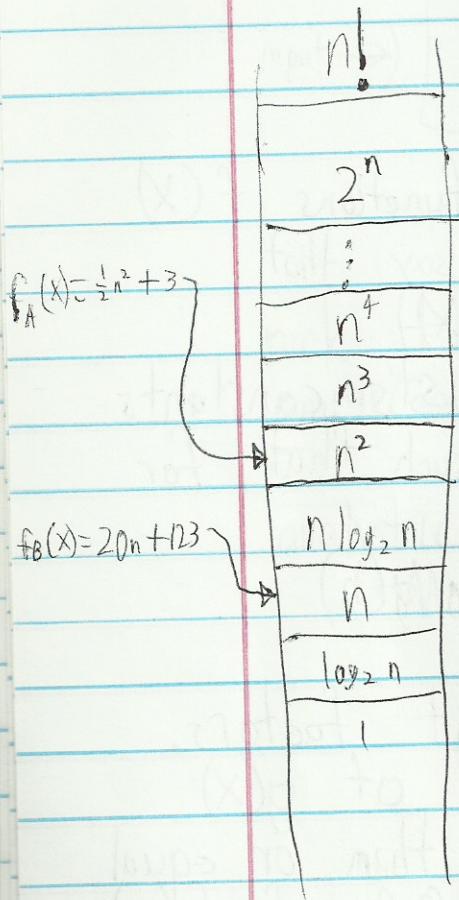
$f(x)$ is $O(g(x))$ if

$f(x)$ is $O(g(x))$ and

$g(x)$ is $O(f(x))$

R.O.G.

Ladder hierarchy



Rule 1

if $T_1(x)$ is $O(f(x))$
and $T_2(x)$ is $O(g(x))$
then $T_1(x) + T_2(x)$
is $O(f(x) + g(x))$

Rule 2

if $T_1(x)$ is $O(f(x))$
and $T_2(x)$ is $O(g(x))$
then $T_1(x) \cdot T_2(x)$
is $O(f(x) \cdot g(x))$

Rule 3

if $T_1(x)$ is $O(f(x))$
and $T_2(x)$ is $O(g(x))$
then $T_1(x) + T_2(x)$
is $O(\max(f(x), g(x)))$

Rule 4

A polynomial of degree
 k is $O(n^k)$

Getting runtime functions

— count operations

0) The cost of a statement is the # of operations in the statement

```
void Arraylist::swap(int i, int j)
```

```
{  
    T tmp;  
    tmp = data[i];  
    data[i] = data[j];  
    data[j] = tmp;  
}
```

```
return;           cost: 7 = O(1)
```

```
foo(int n, int k)
```

```
int x;  
if (n == 0)  
    x = 0;  
else {  
    x = k * k;  
    x = x / n;
```

assume worst quality

input

cost = 5 = O(1)

```
}
```

```
return x;
```

sum(int a[], int n)

```
int s=0;  
for(int k=0; k<n; k++) {  
    s = s + a[k];
```

}

return s;

cost of for loop

}

n-1

$$\text{init} + \sum_{k=0}^{n-1} (\text{check} + \text{update} + \text{body})$$

| | 3

$$\text{for loop cost} + \sum_{k=0}^{n-1} (5) = 1 + 5n$$

while loops

while (test)
body

1) assume it terminates

2) estimate # of iterations

```
linkedList * LinkedList::find( T x )  
{  
    LinkedList * p = this;  
    while( p->next != NULL ) {  
        if( p->data == x ) {  
            return p;  
        }  
        p = p->next;  
    }  
    return NULL;  
}
```

still reachable — memory leaks

iostream makes memory leaks

"no leaks are possible" — no memory leaks

concentrate on "definitely lost"

n : size of LinkedList

in worst case, find repeats n times

```

while (p->next != NULL) {
    if (p->data == x)
        return p;
}
p = p->m_next;
return NULL;

```

for(int $\overset{0}{k}=0$; $k < \overset{0}{n}$; $k \overset{0}{++}$) — $\overset{\text{2x k}}{6n^2 + 3n + 1}$
 {
 for(int $\overset{0}{j}=0$; $j < \overset{0}{n}$; $j \overset{0}{++}$)
 {
 $s = \overset{0}{s} + \overset{0}{a[k][j]}$; $i - 4 \overset{0}{=}$ $6n + 1$
 }
 }
 $O(n^2)$

Test #1 review

- C++ pointers & * →
 new delete

- destructor
- copy constructor
- operator=
- memory leaks
- dangling pointers

- Abstract Data Types
& Data Structures

- ADT List

- D.S. ArrayList

- LinkedList (D.S.)
 - DoublyLinkedList
 - (remove fn)

- Algorithm Complexity
 - Definition of Big-O

if $f(x)$ is $O(g(x))$ and $g(x)$ is $O(f(x))$
then $f(x)$ is $\Theta(g(x))$

- Complexity Hierarchy

- Place functions in Hierarchy

- Extract a runtime fn from code

Study guides in website

Algorithm Complexity

Complexity of a Problem

"the problem has a complexity of $f(n)$ "

— "the best program we know of to solve the problem has a complexity of $f(n)$ "

Moore's law

$n!$
 2^n
⋮
 n^4
 n^3
 n^2
 $n \log_2 n$
 n
 $\log_2 n$
1

Wirth's law — software

gets slower faster
than hardware gets
faster

	$1000 \frac{\text{op}}{\text{min}}$	$10,000 \frac{\text{op}}{\text{min}}$	$\times 10$
$33n$	30	300	10X
$6n^2$	12	40	4X
$2n^3$	7	17	2X
$n!$	6	7	1.15X

m-back — last element,
NOT sentinel

2/22/2017

Re-download ~~Linf~~ linkedlist.h

const Node<T>* getFirstPtr() const;

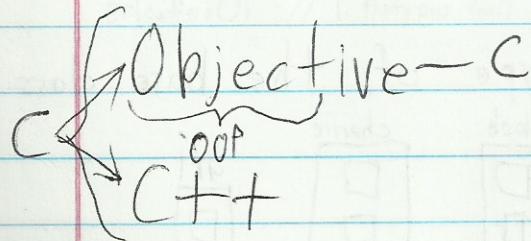
Inheritance in Object Oriented Programming

C was for Unix

Simula

Alan Kay: Smalltalk

- encapsulation public: private:
- inheritance
- polymorphism



Inheritance — the ability to declare a class as an extension of another class

class A

```
{  
    int x;  
    public:  
        int y;  
        void foo();  
};
```

"Base class" 2 variables

class B : public A

```
{  
    public:  
        int z;  
        void bar();  
        void foo();  
};  
  
void B::bar()  
{  
    x=0; x is private  
}
```

"derived class"

3 variables

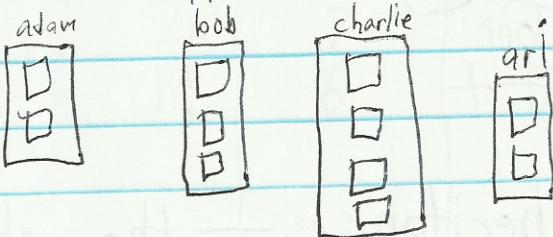
class C : public B

```
{  
    public:  
        int w;  
};
```

4 variables

- A derived class has the type of the base class

```
A adam;  
B bob;  
C charlie;  
A ari;
```



```
ari = adam;  
ari = bob;  
bob = ari; Bob  
zap(A & a);  
}
```

```
zap(charlie);  
zap(B & B);  
}
```

```
zap(charlie); // calls zap-B
```

A * p;
p = &bob;
g = &charlie;
p -> r = 6;
p -> z = 7; ~~⊗~~

bob has 2 foo() fns

bob.foo(); // foo-B

p -> foo(); // foo-A
bob.A::foo(); // foo-A

bob.bar(); ~~⊗~~

solution
class A

{ protected:
int x; // accessible in B and C, not accessible elsewhere

class Platypus : public Duck, public Otter, public Spy

{
};

Platypus Eric; // searches in this order
Eric.foo(); // 1. Platypus foo() 2. Duck foo() 3. Otter foo() 4. Spy foo()

hw3

throw an error

if you are
given a swap

with the same
IDs

C++ Inheritance

- Constructors and
Destructor are

NOT inherited

class A

{ protected:

int x;

int y;

A()

x=0;

y=0;

}

A(int i, int j)

{

x=i;

y=j;

}

class B : public A

{ int z; // calls A() before execution

z=0;

same

B(): A(0, 0)

z=0;

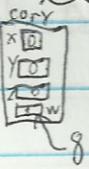
class C : public B

{ int w; // calls A(0, 0), B(), etc. before execution

w=8;

B bob(3, 7); ~~copy~~

C copy;



- the constructor of a derived class calls automatically a constructor of a base class

- the Destructor of a derived class automatically calls the destructor of a base class

class A

{ protected:

int * x;
~A(){
delete [] x;
}

class B: public A

int * z;

{ ~B() // calls ~A(), probably after execution
} delete [] z;

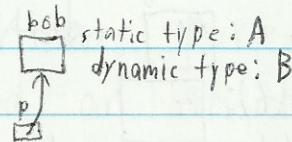
class C: public B

{ int a;

} // when a C is destructed,
// ~B() and ~A() are called

Poly morphism

B bob;
A * p = & bob;
p->foo(); // calls A::foo()
not B::foo()



static vs. dynamic type — only appears with inheritance and pointers

poly morphism — use the dynamic type of objects rather than their static type

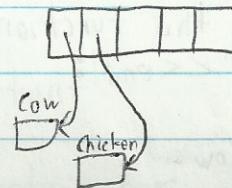
class Animal

class Chicken: public Animal

class Cow: public Animal

Animal Farm[5]; // DOES NOT ALLOW CHICKENS AND COWS
Animal * Ranch[5]; // ALLOWS CHICKENS AND COWS

Ranch[0] = new Cow;
Ranch[1] = new Chicken



for(int k=0; k<2; k++)

Ranch[k]-> speak();  Animal does not have a speak function

hw3

alias test
operator=

if we give Animal a speak function

```
class Animal
{
    void speak()
    { cout << "..." << endl;
    }
```

Ranch[0] → speak();



virtual

```
class Animal
{
    virtual void speak()
    { cout << "..." << endl;
    }
```

// you cannot have virtual variables

Ranch[0] → speak(); Moo static type: Animal

Ranch[1] → Animal::speak(); ...

static — explicitly named

dynamic — anonymous

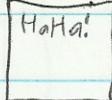
in Java, all functions are virtual

"virtual" is inherited

```
class Cow:public Animal
{
    void speak() // this function is virtual
    { cout << "Moo" << endl;
    }
```

```
class madCow:public Cow
{
    void speak()
    { cout << "Hahaha!" << endl; }
```

Ranch[0] →

Ranch[2] = new madCow;
Ranch[2] → speak(); 

class Animal { // "Abstract class"
virtual void speak() = 0; // "pure virtual"

Animal Bob; 

class Fox: public Animal

{
} // no one knows what the Fox says

Fox
Ranch[4] = new Fox; 

if a class only has "pure virtual" functions,
it is called an interface

class Cow : public Animal
{
 virtual void speak()
}{
 cout << "Moo" << endl;
}
good practice

Make Destructors Virtual

class Animal
{
 virtual ~Animal()
}

very important

you cannot have a virtual constructor

3/1/2017

A.D.T. Stack

a stack is a sequence

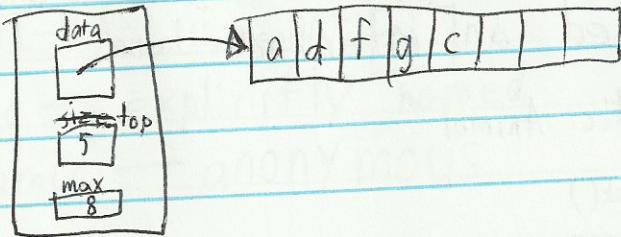
one end is the "top" $s_1 = \langle a, d, f, g, c \rangle$
operations

- $\text{top}(s)$
- $\text{push}(s, x)$
- $\text{pop}(s)$

these are the only operations

$$\begin{aligned}\text{top}(s_1) &= 'c' \\ \text{push}(s_1, 'e') &= \langle a, d, f, g, e \rangle \\ \text{pop}(s_1) &= \langle a, d, f, g \rangle\end{aligned}$$

D.S. ArrayStack



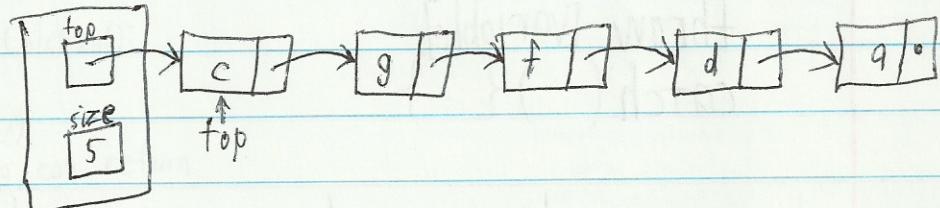
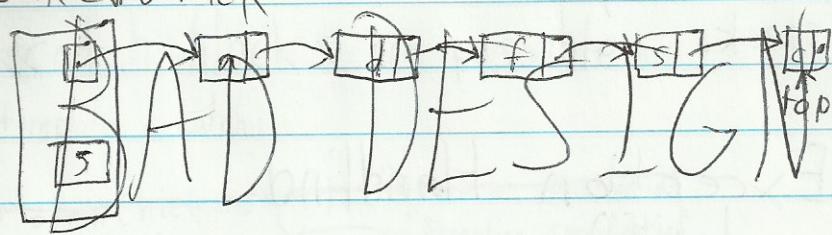
$T \& \text{top}()$

```
{  
    if(m_top != 0)  
        return m_data[m_top];  
    else  
        error();  
}
```

$\{\text{void push(const } T& x)\}$

```
if(m_max == m_top)  
    grow();  
m_data[m_top] = x;  
m_top++;  
}
```

D.S. LinkedStack



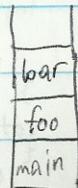
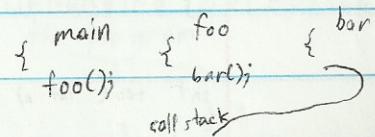
```
void top()
{
    if( m_head != NULL )
        return m_head->m_data;
    else
        error();
}
```

```
void push( const T & x ) // LinkedList::insert_front
{
    Node<T> * tmp = new Node<T>( x, m_head );
    m_head = tmp;
    m_size++;
}
```

```
void pop() // LinkedList::remove_front
{
    if( m_head != NULL )
    {
        Node<T> * tmp = m_head;
        m_head = tmp->m_next;
        delete tmp;
        m_size--;
    }
    else
        error();
}
```

balanced brackets (application of stack)

(([]))



function call stack (application of stack)

hw4 posted

stacks

Exception Handling

try { }

throw [variable]

catch() { }

to separate regular execution from
exceptional execution

int foo(int n)

{

if((n-6) != 0)

y = a / (n-6);

else

cout << "Error!"; } exceptional code

}

}

int foo(int n)

{

try {

if(n-6==0) throw n;

y = a / (n-6)

}

catch(int x)

cout << "Error!";

return 0;

}

multiple catch blocks are possible

you cannot template a catch

```
// throws std::string"Exception specification"
foo() throw Chicken"Exception specification"
{
    throw Chicken();
}
catch(int x)
{
    // catches can return
}
// no chicken catcher, throw goes to caller
//
```

bar()

{

foo(); // throws Chicken, if foo() throws, it does not return

}

catch(Chicken c) // catch(Animal a) would also catch Chickens

}

// if ~~no~~ a throw is not caught, an unhandled exception
crashes the program

```
int zap() throw string, int, grenadeException specification  
ignored by compiler
```

```
catch(...) // syntax "master catch" "default catch"
```

exception handling makes execution jump around, so
only use it for error handling

#include <stdexcept>

```
logic_error
└ invalid_argument
  └ domain_error
    └ length_error
      └ out_of_range_error
runtime_error
└ range_error
  └ overflow_error
    └ underflow_error
```

A.D.T. Queue

A queue is a sequence

$\langle a_0, a_1, a_2, \dots, a_n \rangle$

Designate one end the "front" and the other
end the "back"

- front()

end

- enqueue()

- dequeue()

$q_2 = \langle \overset{\text{front}}{c}, b, f, e, \overset{\text{back}}{d} \rangle$

$\text{enqueue}(q_2) = \cancel{\langle c, b, f, e, d \rangle}$

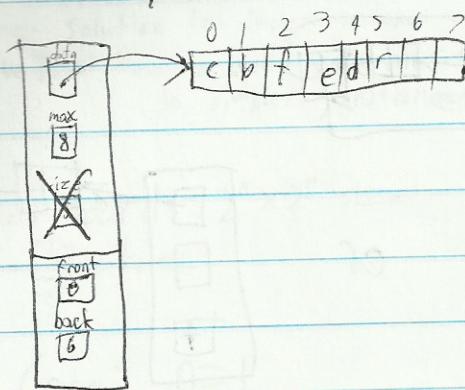
$\text{enqueue}(q_2, 'r') = \langle c, b, f, e, d, r \rangle$ (insert back)

$\text{dequeue}(q_2) = \langle b, f, e, d \rangle$ (remove front)

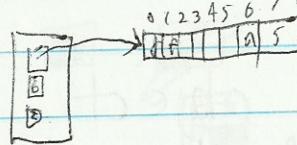
$\text{front}(q_2) = c$

"FIFO"

D.5. Array Queue



circular array



enqueue

$$m_back = (m_back + 1) \% m_max;$$

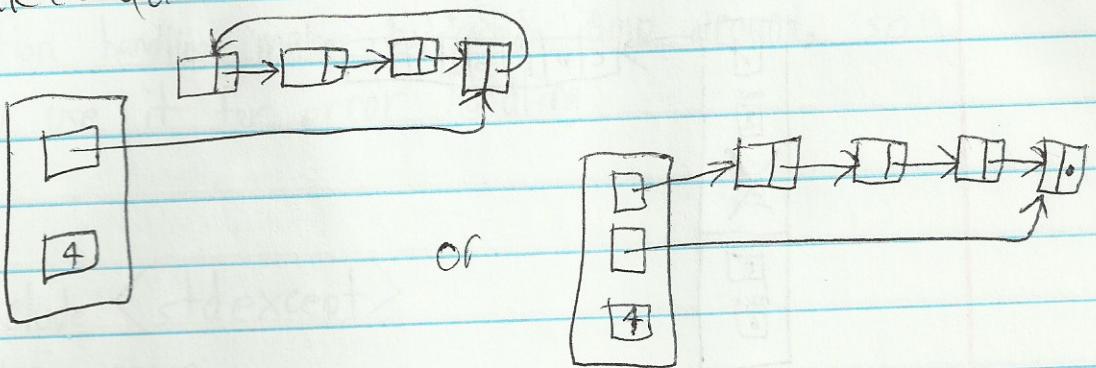
enqueue

$$\{ m_back = (m_back + 1) \% m_max; \\ \text{if } (m_back == m_front) \\ \quad \text{know();} \}$$

`++` is no longer faster than `= +1`
(compilers can detect the situation)

```
void dequeue()
{ if (m_front != m_back)
    m_front = (m_front + 1) \% m_max;
```

linked Queue



3/8/2017 Recursion

recursive object — an object which
consists or is defined in terms
of itself

- with Mathematical Objects
- Base Case (simpler or smallest object)
- Recursive Rule — How to obtain complex objects from simpler ones

$$P = \{2, 4, 8, 16, 32, \dots\}$$

Base Case: 2 is in P

Recursive Rule: $n \in P \Rightarrow 2n \in P$

Solutions to Problems (Algorithms)

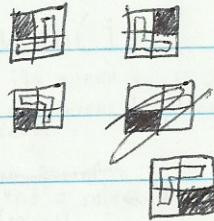
-Base Case: Solution to simplest instance of problem

-Recursive Rule: How to build solutions to complex instances from solutions to simpler instances

cover chess board $2^n \times 2^n$ size
by $\begin{smallmatrix} 2 \\ 2 \end{smallmatrix}$ tiler

Base Case: $n=1$

2×2



Recursive Rule:

Suppose a $2^n \times 2^n$ board with one hole can be solved

consider a $2^{n+1} \times 2^{n+1}$ board

the hole is in one of its four quadrants
that quadrant is $2^n \times 2^n$, so it can be
covered by tiles

place a tile in the center of the
 $2^{n+1} \times 2^{n+1}$ board across the three
quadrants without holes

now, those quadrants can be solved by the
hypothesis

recursive function - a function that calls itself

```
void foo()
{
    int x; // local only to this call. Each call gets its own x.
    foo();
}
```

Fibonacci

$$\begin{aligned}f(0) &= 1 \\f(1) &= 1 \\f(n) &= f(n-1) + f(n-2)\end{aligned}$$

```
int fibo( int n )
{
    if( n==0 || n==1 )
        return 1;
    else
        return fibo(n-1) + fibo(n-2);
}
```

any recursive function can also be written with iteration

```
int exp( b, p )
{
    int x = 1;
    for( int k=0; k<p; k++ )
        x *= b;
    return x;
}
```

$O(p)$

~~```
pow(int b, int p)
{
 if(p==1)
 return b;
 else
 if(p is even)
 return 2 * pow(b, p/2)
```~~

$O(\log_2 p)$

# Recursive Backtracking

- Some problems cannot be solved with a fixed rule computation

## n-queens problem

in a chess board of  $n \times n$

place  $n$  queens so that they do not attack each other

strategy: decompose into sequence of trial & error tasks

solve\_queen( $i$ )

```
repeat
 place i th queen in i th row (in valid spot)
 if no more queens to place
 success!
 else
 solve_queen()
 solved = solve_queen($i+1$)
 if (solved)
 success!
 else
 while places available for i queen
 fail
```

| 6x6 |   |   |   |   |   |
|-----|---|---|---|---|---|
| 0   | 1 | 2 | 3 | 4 | 5 |
| 0   | Q |   |   |   |   |
| 1   |   |   | Q |   |   |
| 2   |   |   |   |   | Q |
| 3   | Q |   |   |   |   |
| 4   |   |   | Q |   |   |
| 5   |   |   |   | Q |   |

| Board      | D <sub>1</sub> | S <sub>1</sub> | D <sub>2</sub> | S <sub>2</sub> | D <sub>3</sub> | S <sub>3</sub> | D <sub>4</sub> | S <sub>4</sub> | D <sub>5</sub> | C array of size $n$ |
|------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|---------------------|
| row<br>col | 1              | 3              | 5              | 0              | 2              | 4              |                |                |                |                     |

a place is invalid  
if it is the same  
col as a previous  
queen or  
 $b[i] - i = b[j] - j$   
or  
 $b[i] + i = b[j] + j$

bool solve\_queen( int row, Board b, int n )

```
{ for (int col = 0; col < n; col++)
 {
 if (valid(row, col, b))
 record(row, col, b)
 if (row == n)
 return true;
 else
 solved = solve_queen(row + 1, b, n)
 if (solved)
 return true;
 else
 undo(row, col, b)
 }
}
return false;
```

## ~~HW~~ Recursive Backtracking

solve  
  initialize choices

do

  select choice  
  if choice is valid  
    record choice  
    if solution incomplete  
      if solution incomplete  
       r=solve next step  
       if not r  
          undo recording

  else

    else success  
    else success

  while choices available  
  fail

used to be called "AI" — not anymore

practice — code n-queens problem before exam

## A.D.T. Tree

- A tree is a collection of elements with a hierarchical relationship between the elements

- a single element ("node") is a tree

- if  $n$  is an element ("node") and  $T_1, T_2, T_3, \dots, T_k$  are trees  
then  $n$  related to  $T_1, T_2, T_3, \dots, T_k$  is a tree  
(reculsive)

$n$  is called the root and  $T_1, \dots, T_k$  are called subtrees of  $n$

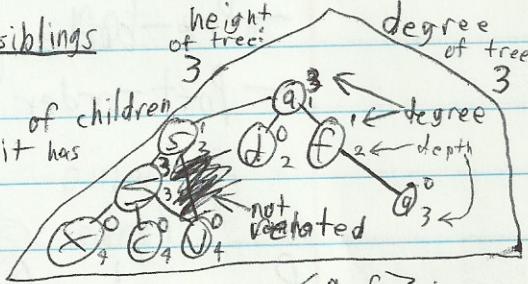
every tree has ~~a root~~ exactly one root

the root of each subtree of  $n$  is called a child of  $n$  and  
 $n$  is called the parent

nodes that have the same parent are called siblings  
a leaf is a node with no children

the degree of a node is the number of children

the degree of a tree is the highest degree of a node



tree has degree 5 — a node has at most 5 children

$\langle n, f \rangle$  is a path

a path is a sequence of nodes:  $\langle n_0, n_1, n_2, \dots, n_k \rangle$

where  $n_{i+1}$  is the parent of  $n_i$ ;  $0 \leq i \leq k$

(so we start from the bottom)

Note: there is only one path from any node to the root

the depth of a node  $n$  is the number of nodes in the path between  $n$  and the root

the height of a tree is the length of the longest path in the tree

the root of a tree has no parent

3/15/2017

Homework #4 due today

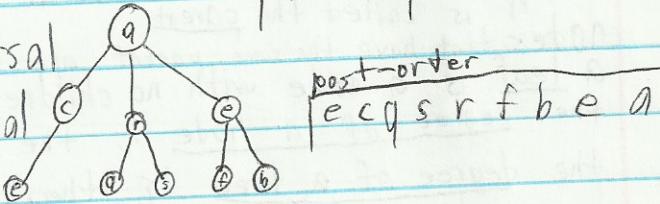
Test #2 Next Friday

A.D.T. Tree

Tree traversal

- Pre-order traversal

- Post-order traversal



pre-order output  
1acerqsefb

post-order  
fecqsrfb ea

Pre-order (tree t)

process(root of t)  
for every subtree  $t'$  of t  
pre-order( $t'$ )

Post-order (tree t)

for every subtree  $t'$  of t  
post-order( $t'$ )

~~post-order( $t'$ )~~

~~process( $t'$ )~~

process(root of t)

~~step~~

iter-pre-order(tree t)

stack s

push root of t

while s is not empty

x = top of s

process x

pop s

for every child y of x

push y into stack

what would happen if  
queue is used rather than  
stack

(if it's not post-order)

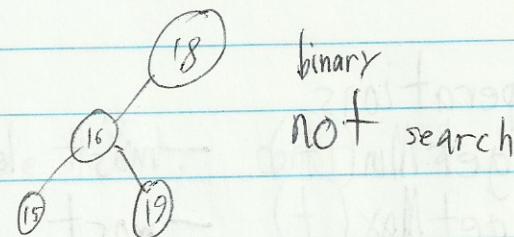
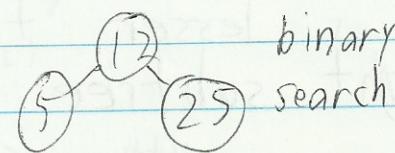
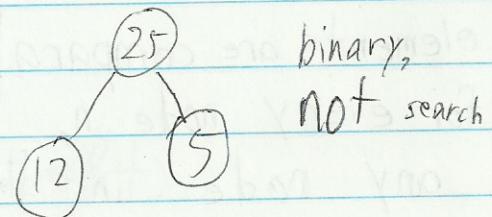
("breadth first")

# ADT BinarySearchTree

degree 2 - "binary"

search - elements are comparable

- for every node in BST,  $n$  is greater than any node in its left subtree and  $n$  is lesser than any node in its right subtree



---

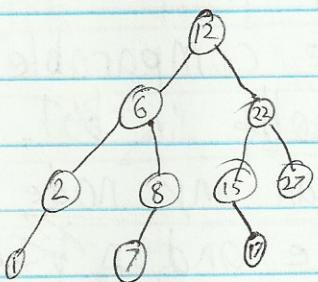
3/20/2017 Monday

Test #2 Friday

Tomorrow 9:45 AM CS203 - new cs chair candidate  
(refreshments)

# Binary Search Tree

BST



not BST



## search

- elements are comparable
- for every node  $n$ ,  $n$  is greater than any node in its left subtree and  $n$  is lesser than any node in its right subtree

## operations

$\text{getMin}(t)$  — most left element in BST

$\text{getMax}(t)$  — most right node in BST

$\text{insert}(t, x)$  — there is always only one place

$\text{remove}(t, x)$

$\text{find}(x, t)$  — similar to insert

to insert  
a node of  
a certain  
comparable  
value

## remove( $t, x$ )

case 1,  $x$  is a leaf

delete  $x$  from the tree

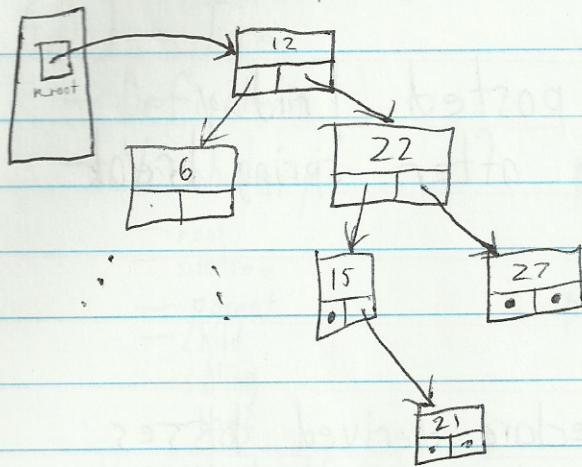
case 2,  $x$  has 1 child and it's a parent

graft  $x$ 's child to its parent

case 3,  $x$  has 2 subtrees  
take the max of the left subtree and  
the min of the right subtree

and pick one, remove that one  
and insert it in place of  $x$

# D.S. BinarySearchTree



```
class TreeNode
{
 T m_data;
 TreeNode* m_left;
 TreeNode* m_right;
};

class BSTree
{
 TreeNode* m_root;
 int m_size;
};
```

`TreeNode* t`

`getMax(TreeNode*& t)`

```
{ if(t->m_right == NULL)
 return t->m_data;
else
 return getMax(t->m_right); }
```

`insert(TreeNode*& t, const T& x)`

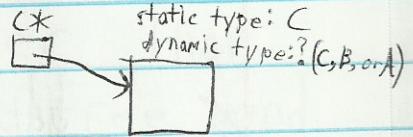
```
{ m_size++;
if(t == NULL)
 t = new TreeNode;
t->m_data = x;
t->m_right = t->m_left = NULL;
}
else if(x < t->m_data)
 insert(t->m_left, x);
else
 insert(t->m_right, x);
else if(x == t->m_data)
 m_size--;
}
```

## Test #2 Friday

HW 5 will be posted Friday  
due Friday after spring break

### C++

- inheritance - declare derived classes
  - protected:
- polymorphism
  - static vs dynamic
  - virtual
  - pure virtual
  - abstract class
- Exception handling
  - try, throw, catch



### Recursion

- write recursive functions
- Algorithm vs Code

### A.D.T. Stack

D.S. ArrayStack

D.S. LinkedStack

write operation

### A.D.T. Queue

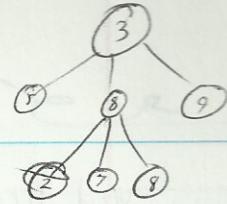
D.S. ArrayQueue

D.S. LinkedQueue

## A.D.T. Tree

- Definition
- Vocabulary

- root
- subtree
- parent
- child
- sibling
- leaf
- degree of node
- degree of tree
- path — goes up
- depth of a node
- height of a tree



traversals      pre 3582789  
post 5278893

pre-order(tree t)

process root of t  
for each subtree t' of t  
pre-order(t')

post-order

No BSTrees! (in test)

class A

{  
  ~A() // either written or generated  
}  
};

class B : public A

{  
  ~B() // either written or generated  
}  
} // calls A::~A()

```
void ArrayStack<T>::pop()
{
 if(m_top)
 m_top--;
 return;
}
```

7/3/2017

-Homework 5

don't copy/paste samples from website  
download

-Google Code Jam (same day as MegaMiner)

-MegaMiner

# DS. BinarySearchTree

class Node

```
{ m-data;
Node * m-right;
Node * m-left;
```

}

getMax()  
getMin()

find()

insert()

remove()

- if no children, simply delete and set pointer to NULL
- if one child, point pointer to child and delete
- if two children, replace with min of right subtree or max of left subtree, removing that and deleting this

```
void remove(const T& x, Node*& t)
```

```
{ if(t == NULL)
```

```
 return;
```

```
if(x < t->m_data)
```

```
 remove(x, t->m_left);
```

```
else if(x > t->m_data)
```

```
 remove(x, t->m_right);
```

```
else // x == t->m_data
```

```
{
```

```
 if(t->m_left == NULL
```

```
 && t->m_right == NULL)
```

```
 delete t; t = NULL; }
```

```
else if(t->m_left == NULL
```

```
 || t->m_right == NULL)
```

```
{
```

```
 Node* child = t->m_left;
```

```
 if(child == NULL)
```

```
 child = t->m_right;
```

```
 delete t;
```

```
 t = child;
```

```
}
```

```
else // two children
```

```
{
```

```
 t->m_data = getMax(t->m_left);
```

```
 remove(t->m_data, t->m_left);
```

```
}
```

4/5/2017

Homework #5 due Friday

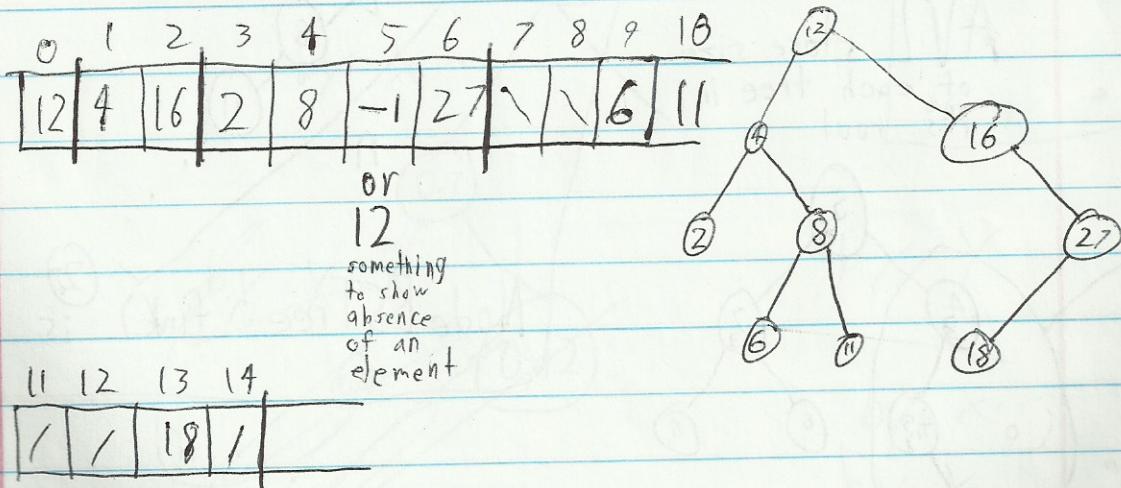
MetaMinerAE

siggame.io

ADT BSTree

DS Node

DS Array BSTree



$$\text{left}(i) = 2i + 1$$

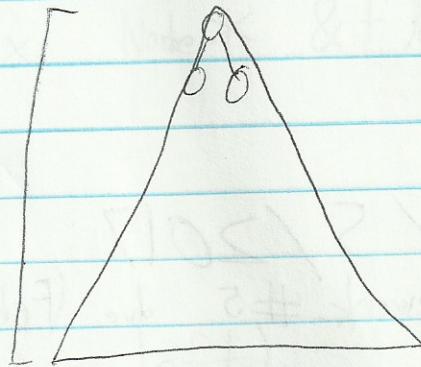
$$\text{right}(i) = 2i + 2$$

$$\text{parent}(i) = \lfloor (i - 1) / 2 \rfloor$$

$n$  elements in full tree

insert()  
remove()  
find()

$\log_2(n)$

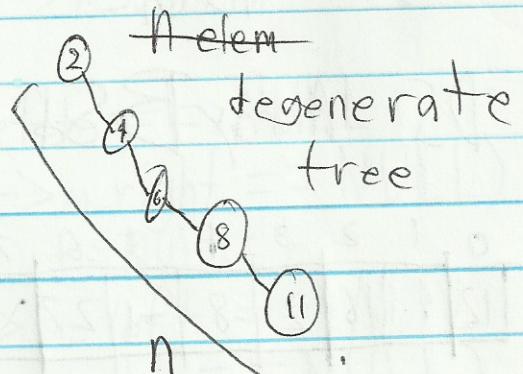


## Advanced Binary Search Trees

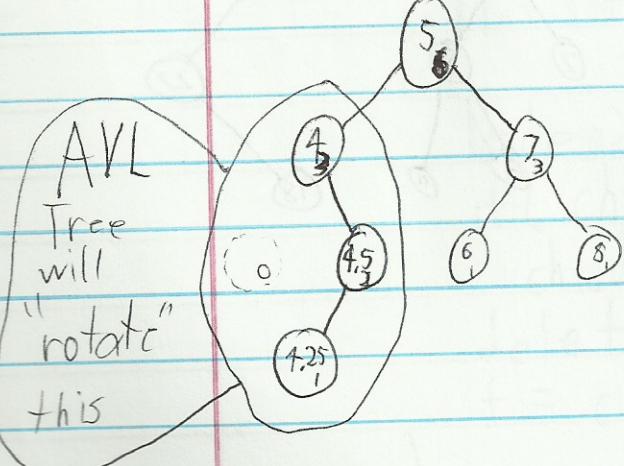
- AVL Tree
- Red-Black Tree
- Splay Tree

AVL store size  
of each tree in  
its root

NodeBSTree::find()  
is  ~~$\log_2(n)$~~



NodeBSTree::find() is  $n$



Splay Tree

rearranges a tree when find()

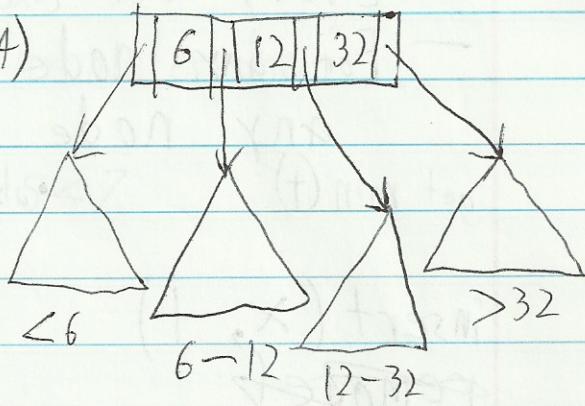
is called (if degenerate)

Red-Black "colors" each node and uses the color to prevent degeneracy

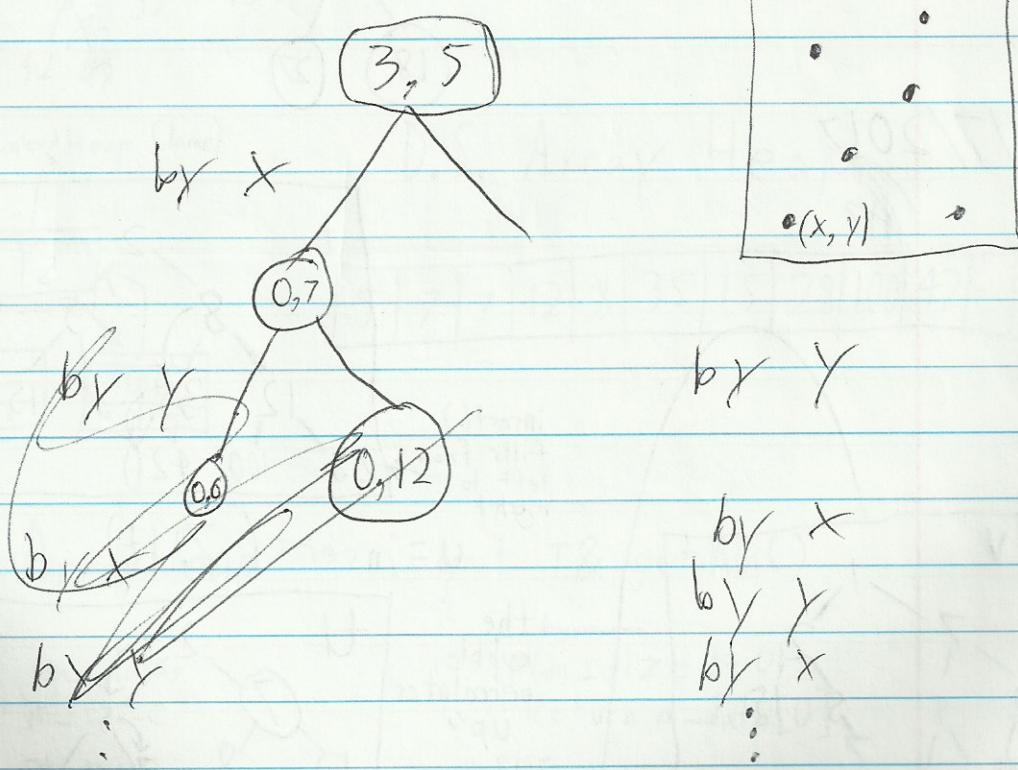
other trees • B-tree

~~AVL Tree~~

(degree 4)



• KD Tree



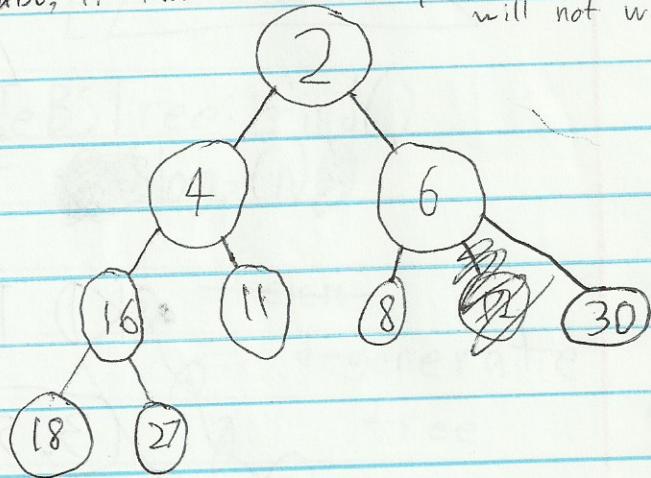
## ADT Heap

- Binary Tree (not search)
- Every level except bottom is full
- For any node  $x$ ,  $x$  is lesser than any node in its subtrees
  - get  $\min(t)$
  - also, if two elements are equal, the operations will not work

$\text{insert}(x, +)$

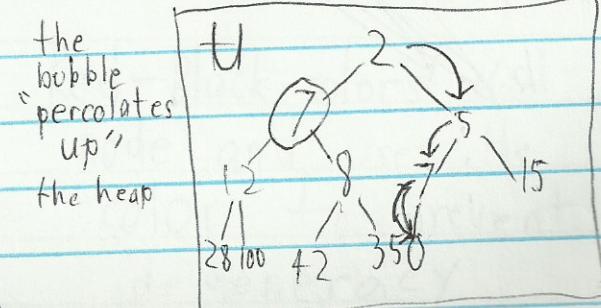
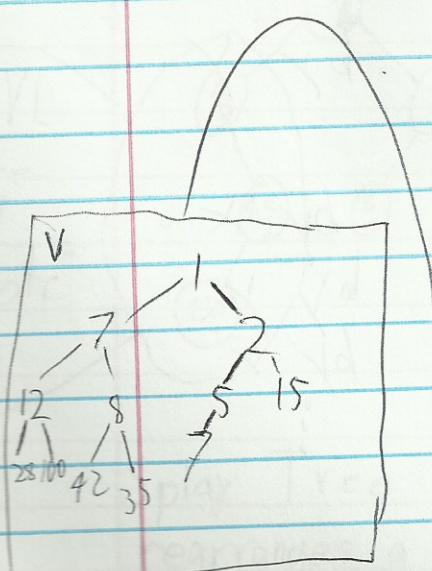
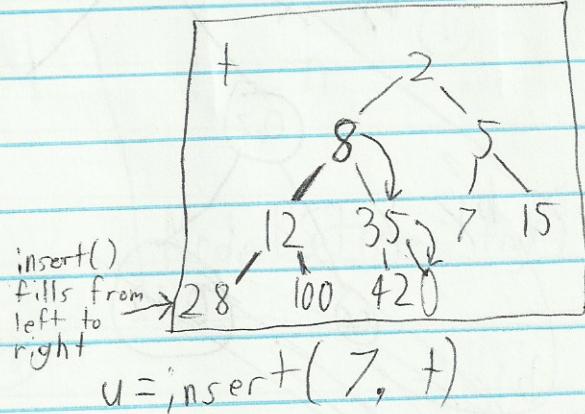
~~remove<sub>t</sub>~~

$\text{remove\_min}(+)$



4/7/2017

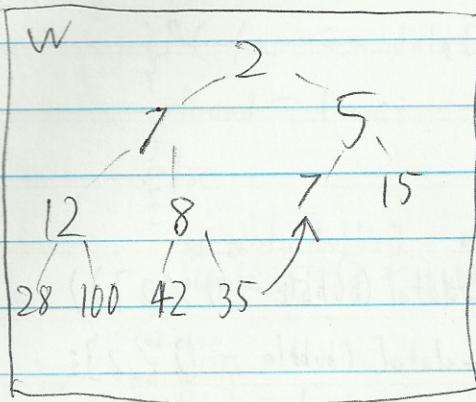
B



the  
bubble  
"percolates  
up"  
the heap

$v = \text{insert}(1, u)$

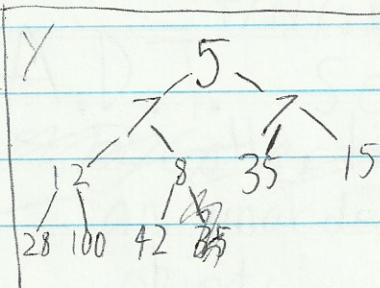
w = remove\_min(x)



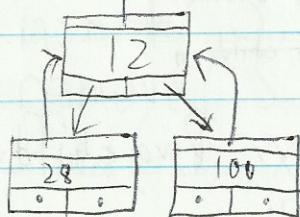
the bubble  
"percolates down"

5 < 7

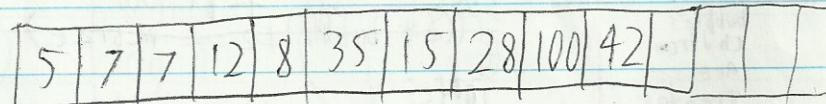
y = remove\_min(w)



Linked Heap (lame)



D.S. Array Heap



class ArrayHeap

```
 T* m_data;
 int m_size;
 int m_max;
```

};

const T& getMin()

```
 return
 if(m_size != 0)
 return m_data[0];
 else
 throw error;
```

# Array Heap

```
void insert(const T& x)
{
 if(m_size == m_max)
 grow();
 int bubble = m_size;
 while((bubble > 1) && (x < m_data[(bubble - 1) / 2]))
 {
 m_data[bubble - 1] = m_data[(bubble - 1) / 2];
 bubble = (bubble - 1) / 2;
 }
 m_data[bubble - 1] = x;
 m_size++;
}
```

```
void remove_min()
{
 bool found = false;
 int bubble = 0;
 T& tmp = m_data[m_size - 1];
 m_size--;
 while((bubble * 2) < m_size && !found)
 {
 fci;
 if((2 * bubble) + 1 < m_size) // two children
 {
 if((2 * bubble) + 2 < m_size) // two children
 {
 fci = m_data[(2 * bubble) + 1];
 if(m_data[(2 * bubble) + 2] < fci)
 fci = m_data[(2 * bubble) + 2];
 }
 }
 }
}
```

```

int fav;
fci = (bubble * 2) + 1;
if(m_data[(bubble * 2) + 2] < m_data[fci])
 fci++;
if(m_data[fci] > tmp)
 found = true;
else
{
 m_data[bubble] = m_data[fci];
 bubble = fci;
}
else
```

while conditions

- 1. bubble reaches bottom
- or 2. both of bubble's children are greater than ~~tmp~~ equal to tmp

```

 {
 if (tmp < m_data[(2 * bubble) + 1])
 found = true;
 else
 m_data[bubble] = m_data[(2 * bubble) + 1];
 bubble = (2 * bubble) + 1;
 }
} // end else
} // end while
m_data[bubble] = tmp;

```

4/12/2017

## A.D.T. Set

~~Collection~~

— an unordered collection of non-repeating objects

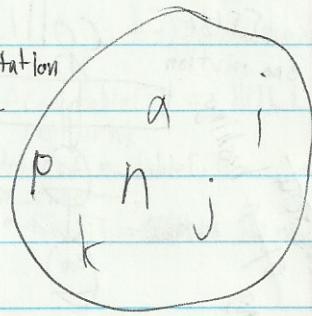
- insert( $S, x$ )
- remove( $S, x$ )
- find( $S, x$ )

tree implementation complexity

$\log n$

$\log n$

$\log n$



## A.D.T. Map

— a set of pairs  $\langle k, v \rangle$

$k$  = key

$v$  = value

— all keys must be unique

— values may appear multiple times

- insert( $M, k, v$ )
- remove( $M, k$ )
- findValue( $M, k$ )

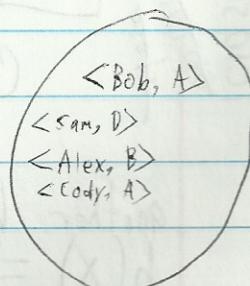
tree implementation complexity

$\log(n)$

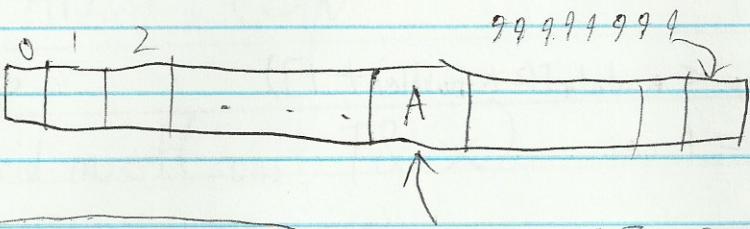
$\log(n)$

$\log(n)$

array implementation complexity  
(impractical due to memory requirements)

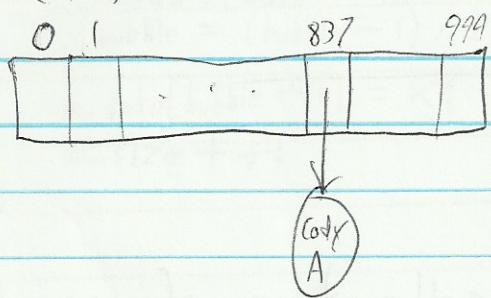


## ID Number array



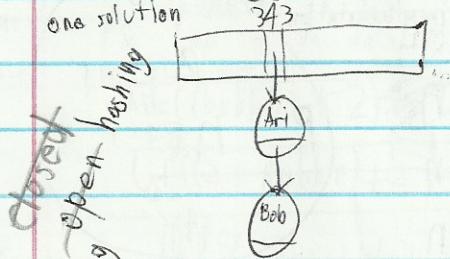
better ArrayMap  
hash function

$$h(id) = id \% 1000$$



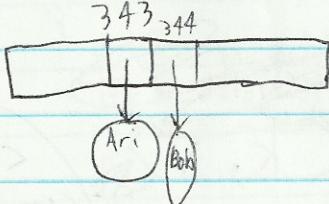
issue: collision

one solution



closed  
closed

closed  
closed  
close hashing open hashing



another hash function

$$h(x) = \text{middle 3 digits of } x^2$$

```
class student
```

```
{
 int m_id;
 string m_name;
 char m_grade;
};
```

```
class studentHashTable
```

```
{
 student * m_data;
 int m_size;
 int m_max;
 studentHashTable()
{
```

```
 m_data = new student*[100];
 for (long k = 0; k < 100; k++)
 m_data[k] = new student NULL;
 m_size = 0;
}
```

4/14/2017  
D.S. HashTable

```
class student ABT Map
{
 int m_id; ← key
 string m_name; ← value
 char m_grade;
```

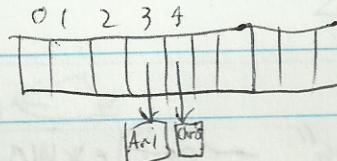
```
class studentTable
```

```
{
 student **m_table;
 int m_max;
 int m_size;
 student m_dead;
 void insert(student &x);
```

```
int hash(int id)
{
 return id % m_max;
```

linear probing

walk right until  
either  
• target is found  
or  
• NULL is found



this has problems

4/17/2017

Homework #7 posted  
due next Wednesday

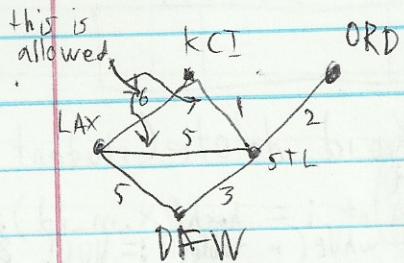
Test #3 next week

Miner LAN - ACM LAN party

Open hashing - tombstones  
close hashing - Linked List

### A.D.T. Graph

- a collection of objects together with a relationship among the objects

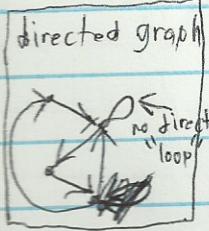


- a pair  $G = \langle V, E \rangle$   
 $V = \text{set of objects (vertices)}$   
 $E = \text{set of pairs of objects}$   
from  $\langle v, w \rangle$   
where  $v \in V$   
and  $w \in V$

$$V = \{LAX, KCI, STL, ORD, DFW\}$$

$$E = \{\langle LAX, STL \rangle, \langle STL, LAX \rangle,$$
  
$$\langle STL, ORD \rangle, \langle ORD, STL \rangle,$$
  
$$\langle KCI, STL \rangle, \langle STL, KCI \rangle,$$

⋮  
⋮  
⋮



if  $E$  is symmetric, the graph is called  
"undirected"  
otherwise the graph is called  
"directed"

$\langle w, w \rangle$  "loop" - reflexive relation

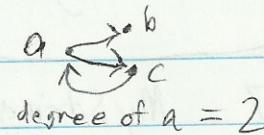
if a graph does not contain a loop, it is

called "simple"

w is "adjacent" to v if  $\langle v, w \rangle \in E$

w is a "neighbor" of v if  $\langle v, w \rangle \in E$   
note ↑  
order

the "degree" of a node is the number of nodes adjacent to the node



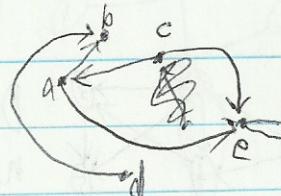
a "path" is a sequence of nodes

$\langle n_0, n_1, n_2, n_3, \dots, n_k \rangle$

such that  $n_{i+1}$  is adjacent to  $n_i$ ,  $0 \leq i < k$

A path that does not

"contains" a loop is  
called "simple"



example path:  $\langle c, a, e, e, e \rangle$

not simple not cycle

a cycle is a path where  $n_0 = n_k$

DAG - Directed Acyclic Graph

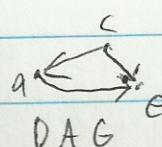
- a directed graph with

no cycles

$\langle c, a, e \rangle$   
simple not cycle

$\langle e, e \rangle$  cycle

$\langle STL, KCI, STD \rangle$  cycle



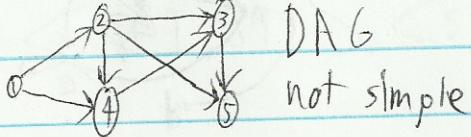
if  $G = \langle V, E \rangle$  + weigh function  
 "weighted graph"

operations

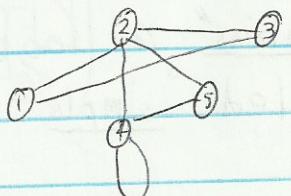
- addVertex( $G, v$ )
- addEdge( $G, v, w$ )
- neighbors( $G, v$ ) → list of neighbors of  $v$

4/19/2017 My String Map 256 ~~chars~~  
 char possibilities

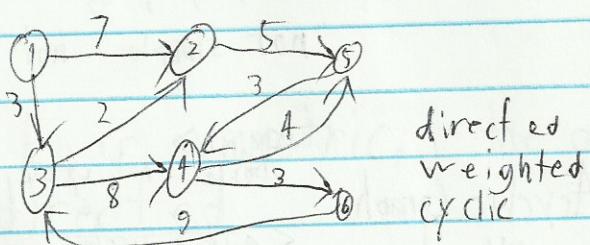
MAX CHAR VALUE  
 Use sum of chars % max for hash



DAG  
not simple



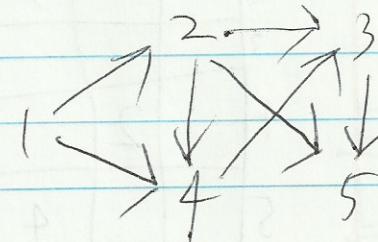
cyclic  
not simple  
not directed



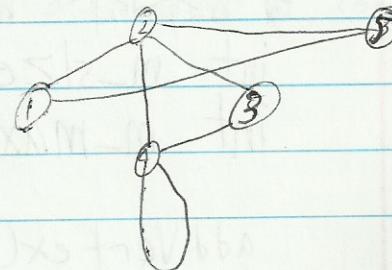
directed  
weighted  
cyclic

## P.S. Adjacency Matrix

| from | 1 | 2 | 3 | 4 | 5 |
|------|---|---|---|---|---|
| 1    |   | T | T |   |   |
| 2    |   |   | T | T | T |
| 3    |   |   |   | T |   |
| 4    |   | T |   |   |   |
| 5    |   |   |   |   |   |

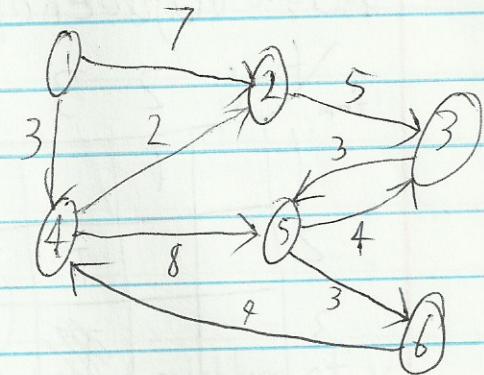


| from | 1 | 2 | 3 | 4 | 5 |
|------|---|---|---|---|---|
| 1    |   | T |   |   | T |
| 2    | T |   | T | T | T |
| 3    |   | T |   | T |   |
| 4    |   | T | T |   | T |
| 5    | T | T |   |   |   |



"symmetric matrix"

| from | to | 1 | 2 | 3 | 4 | 5 | 6 |
|------|----|---|---|---|---|---|---|
| 1    |    | 7 |   | 3 |   |   |   |
| 2    |    |   | 5 |   |   |   |   |
| 3    |    |   |   | 3 |   |   |   |
| 4    |    | 2 |   |   | 8 |   |   |
| 5    |    |   | 4 |   |   | 3 |   |
| 6    |    |   |   | 9 |   |   |   |



```
class AdjacencyMatrix
{
```

```
 T ** m_matrix;
```

```
 int m_size;
```

```
 int m_max;
```

```
 addVertex()
```

```
{
```

```
 m_size++;
```

```
 if(m_size == m_max)
```

```
 grow();
```

$n^2$

```
} addEdge(int v, int w)
```

```
{ // suppose the edge does not already exist
```

```
m_matrix[v][w] = true;
```

```
}
```

```
neighbors()
```

$n$

$$\int \frac{\ln(3x-5)}{\sin(-x+2) \sec(x+3)} dx$$

$$-\int \frac{\ln(3x-5)}{\sin(x-2) \sec(x+3)} dx$$

$$\cosh^2(x) - \sinh^2(x) = 1$$

$$\frac{d}{dx} (\operatorname{sech} x) = -\operatorname{sech} x \tanh x$$

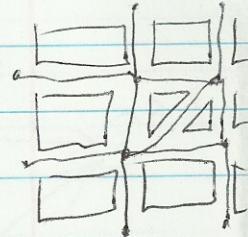
$$\frac{d}{dx} \left( \frac{2}{e^x + e^{-x}} \right) = -\frac{2}{(e^x + e^{-x})^2}$$

## Adj Matrix

~~Pro~~

Cons:

Memory use  
addVertex() inefficient  
neighbors() very inefficient  
(consider street map)

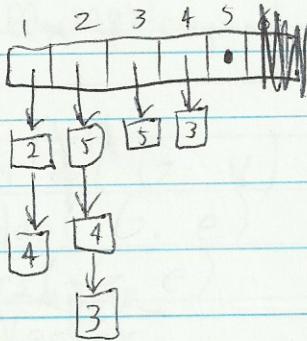


# nodes: 1000  
degree: ≤ 8

"sparse graphs" — many nodes, low node degree

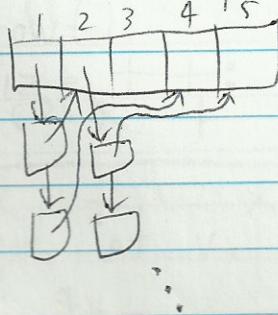
## D.S. Adjacency List

- Array of nodes, each with a LinkedList of connections

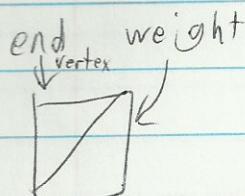
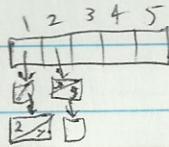


or

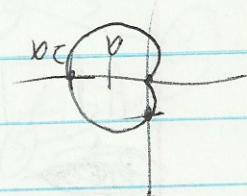
- Array of pointers to pointers to nodes



for weights, use structs



$$r = \alpha + \alpha \sin \theta$$



$$\int \sec x \, dx = \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} \, dx$$

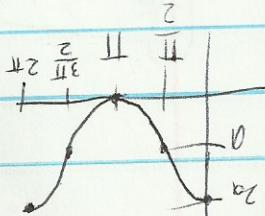
$$\text{let } u = \sec x + \tan x$$

$$du = (\sec x \tan x + \sec^2 x) dx$$

$$dx = \frac{du}{\sec x \tan x + \sec^2 x}$$

$$\int \frac{1}{u} \, du$$

$$= a + a \cos \theta \quad (\text{arclength})$$



$$\ln|u| + C$$

$$\ln|\sec x + \tan x| + C$$

$$+ \int x \cdot \frac{du}{\sec x \tan x + \sec^2 x} - \ln|u| -$$

$$np \frac{n}{1} \int -$$

~~cancel~~

$$\frac{x_2 \sec x + \csc x}{du} - = xp$$

$$xp(-\csc x \cot x - \csc^2 x) = np -$$

$$x + \csc x + \cot x = n + 2$$

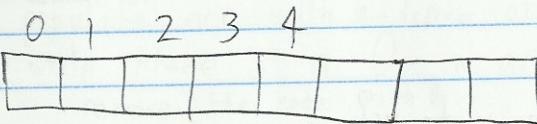
$$xp \frac{x_1 + x_2}{x + \csc x \cot x} \int$$

$$xp x_2 \int$$

Homework #7 due Wednesday  
Test #3 next Friday

## A.D.T. Graph

### D.S. AdjacencyList



class Edge  
{  
int m\_to;

class AdjacencyList  
{

LinkedList<Edge> \*m\_nodes;  
int m\_size;  
int m\_max;

• AddVertex(G, v)      complexity: n  
• AddEdge(G, e)      1  
• Neighbors(G, e)      1

AddVertex  
{

~~m\_size++;~~

if(m\_size == m\_max)

grow();

m\_size++;

}

• areNeighbors(G, v, w)  
true if (v, w) is  
in G, false otherwise

AdjList      complexity: n  
AdjMatrix      complexity: 1

addEdge( int v, int w)

{  
m\_nodes[v].insert\_back(w);

neighbors (int v)

```
{ return m_Nodes[V]; // that's a LinkedList
}
```

Graph problems.

- Has cycle?
- Is there a path from v to w?
- What is the shortest path from v to w?

Is there a cycle?

Topological sort — sort nodes of graph

$\langle n_0, n_1, n_2, n_3, \dots, n_k \rangle$

such that

if there is a path from

$n_i$  to  $n_j$

then  $j > i$

if you fail, there is a cycle

in-degree of a node  $\nabla$

the number number of edges  $\langle x, w \rangle$

(directed graphs only)

A node with in-degree 0 cannot be part of a cycle

to search for cycles, elimi-

~~root cycle~~ remove nodes with in-degree 0

algorithm:

sort = <>

Look at graph for a node with in-degree == 0

add such node to sort

remove node from graph

repeat until graph is empty or no such node exists

no cycles

cycle

version 2

allocate sort

populate arr with in-degree of nodes

while there is a 0 in arr

remove the node with 0

append the removed node to sort

if arr empty, no cycle

else, cycle

|   |   |   |   |   |   |
|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 |
| 0 | 1 | 2 | 3 | 1 |   |

Euler circuit — a <sup>circuit</sup> ~~path~~ using each edge exactly once ~~and returning to start~~

4/24/2017

Is there a path from v to w?

// Pre: the graph is a DAG

path(node v, node w)

if(v == w)

return true

for each neighbor u of v

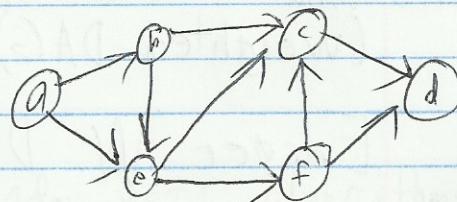
success = path(u, w)

if(success)

return true

return false

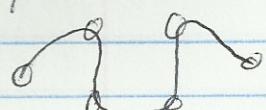
// This is called "depth first search"



DAG

to get the path, create a predecessor table

|   |   |   |   |   |   |
|---|---|---|---|---|---|
| a | b | c | d | e | f |
| a | a | b | c | b | e |



to find the path from pred,  
start at end and construct the path backward

"Breadth first search"

```

path(node v, node w)
queue Q
push enqueue v into Q
while Q is not empty
 x = front of Q
 dequeue Q
 if(v == w)
 return true
 for every neighbor y of x
 pred[y] = x
 push y into S
 return false
 }
```

path(node v, node w)

stack S

push v into S

while S is not empty

$x = \text{top of } S$

pop S

if( $x == w$ )

return true

for every neighbor y of x

$\text{pred}[y] = x$

push y into S

return false

4/26/2017

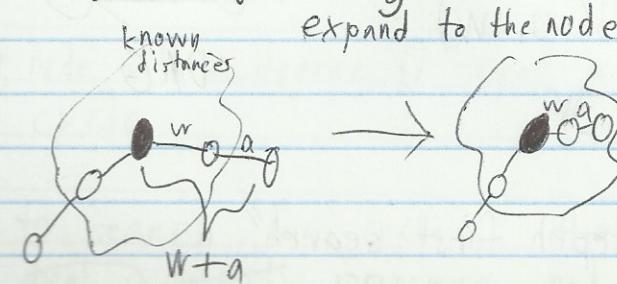
breadth first

depth first

which neighbor to explore next

problem 3 What is the shortest path between u and w?  
(weighted DAG)

Edsger W. Dijkstra



shortest-path(node v, node w)  
 heap H // the heap contains "shortest distance candidates"  
 insert  $\langle v, 0 \rangle$  into H // v-start node 0-distance  
 $\text{pred}[v] = v$   
 $\text{dist}[v] = 0$   
 while H is not empty  
 $\langle u, du \rangle = \text{getMin}(H)$  remove\_min(H)  
 if ( $du < \text{dist}[u]$ )  
 $\text{dist}[u] = du$ ;  $\text{pred}[u]$   
 for every neighbor y of u  
 insert  $\langle y, du + \cancel{+} \text{weight}(u, y) \rangle$   
 $\text{pred}[y] = u$   
 return fail

this can be applied to AI graphs

Test #3

~~D.S.~~  
 ADT Binary Search Tree  
 DS TreeNode  
 DS ArrayTree

and w?  
mallest  
stance)

ADT Heap  
 DS Array Heap - no code, yet; algorithms

ADT Map & Set  
 DS HashTable

ADT Graph  
 Terminology Traversals  
 DS Adj Matrix  
 DS Adj List

Final Exam

either

Final Exam (comprehensive)  
or  
Final Project (not a homework,  
cannot be dropped)

how to choose

diverse test scores — choose exam

consistent test scores — choose project

STL

Standard Template Library

— classes and functions for D.S.

#include <vector> (ArrayList)

```
vector<int> v1;
v1.size();
v1.push_back();
v1.push_front(); //expensive
v1.clear();
v1.front();
vector<int> v2
v1 = v2
vector<int> v3(v1);
v1 = v2
v1[k] //seg fault
```

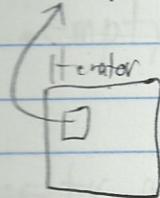
#include <list> (DoublyLinkedList)

```
list<int> l1;
l1.size();
l1.push_front(); //cheap
l1.push_back();
```

1. clear() ll, front()

ll[] ~~( )~~ - no bracket operator for list

DEQUE



++

whether  
or not 2  
iterators point  
to the same element

- -

! =

\*

1. begin() → the first element in an iterator

2. end() → the sentinel iterator (NOT last element)

```
list<int>::iterator p;
p = ll.begin();
while(p != ll.end())
{
 cout << *p << endl;
 p++;
}
```

vectors also have iterators (which is dumb)

```
for(vector<int>::iterator p = vl.begin(); p != vl.end(); p++)
 cout << *p << endl;
```

5/3/2017

put name in f project

## STL

nesting

list<vector<int>> l;  
vector<vector<int>> m;  
m[a][b]

space

  space

#include <stack>  
<queue>

stack<int> s; queue<int> q;  
(b01) s.size()  
      s.empty()  
      s.push(3)  
      s.pop()  
      s.top()  
                 q.size()  
          empty()  
         push(3)  
         pop()  
         front()  
         back()

stacks and queues have iterators

#include <set>

set<int> s; type must have operator<  
implemented as BinarySearchTree  
(red and black tree)

(b01) s.size()  
      s.empty()  
          .insert(E)  
          .find(E)  
          .erase(E)

~~later classes, STL is allowed~~

=include <queue>

priority-queue<int> q; // Max heap

- push(3)
- pop()
- top()
- size()
- empty()

← required

priority-queue<int,

class widget

{ int x

float y

class widget Comparator

operator()

compareWidget(widget a, widget b)

==  
=

↓ min\_heap

like greater<int>  
↓ or (default) less<int>  
↑ max\_heap

priority-queue<widget, vector<widget>, widgetComparator>

for final, Dr. Morales has  
a super tester

XOR → final project due Thursday @ Midnight

final exam → Wednesday 3:00 - 5:00  
email Morales  
for this option

G-3 Schrank

#include <map>

map<string, char> grades;

bool grades.size()  
grades.empty() const

access: [ ]

grades["Alice"] = 'A';  
grades["Bob"] = 'B';

cout << grades["Bob"] << endl;

grades.erase("Bob");  
grades.find("Zebra");

in later classes, STL is allowed  
in later classes, Python is allowed, other languages are allowed

=include <algorithm> // very useful, worth a read  
`vector<string> v;`

`find(v.begin(), v.end(), "Bob");` ⇒ iterator  
if not found, v.end() is returned

`count(v.begin(), v.end(), "(at");`

`equal(v1.begin(), v1.end(), v2.begin(), v2.end())`

`reverse(v.begin(), v.end())` // not compatible with list

`random.shuffle(v.begin(), v.end())`

`sort(v.begin(), v.end())`

`make_heap(v.begin(), v.end(), cmp)`

`push_heap(v, /, /, /)`

`pop_heap(/, /, /)`

STL is not in final exam

recruiters like:

• DDM "ninjas" — Price

• AI — Tavritz

• E/C "Evolutionary Computing" —? (probably Tavritz)

next semester, Morales will teach "Deep learning"