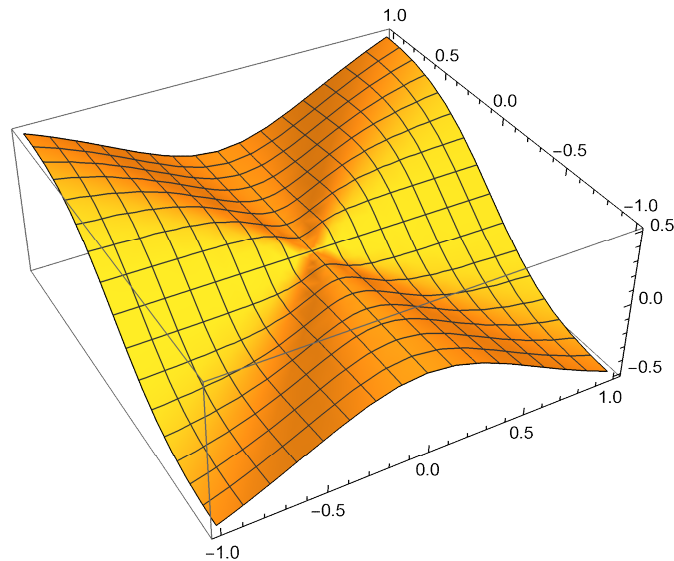In[2]:= `f := (y * x^2) / (x^2 + y^2)`

In[•]:= `Plot3D[f, {x, -1, 1}, {y, -1, 1}]`

Out[•]=

Compute the partial derivative with respect to x and evaluate it along a line through the origin at an angle θ.

In[24]:= `Simplify[D[f, x] /. {x → u * Cos[θ], y → u * Sin[θ]}]`

Out[24]= $2\,\text{Cos}[\theta]\,\text{Sin}[\theta]^3$

We see that the partial derivative depends on direction but not on how far we are from the origin. Clearly the partial derivative varies with the direction so it is not continuous in a neighborhood of the origin.

Can do the same with the derivative with respect to y.

In[26]:= `Simplify[D[f, y] /. {x → u * Cos[θ], y → u * Sin[θ]}]`

Out[26]= $\text{Cos}[\theta]^2\,\text{Cos}[2\,\theta]$