Time Series Analysis

Lecture 4

Mixed Autoregressive Moving Average (ARMA) Models

Autoregressive Integrated Moving Average (ARIMA) Models

Seasonal ARIMA (SARIMA) Models

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Mathematical Formulation and Properties of ARMA Models

Mathematical Formulation of ARMA(p,q) Models

A time series $x_t : \cdots -2, -1, 0, 1, 2, \ldots$ is called a mixed autoregressive moving average process of order (p,q), **ARMA(p,q)**, if it is stationary and takes the following functional form

$$x_t = \phi_1 x_{t-1} + \cdots + \phi_p x_{t-p} + w_t - \theta_1 w_{t-1} - \cdots - \theta_q w_{t-q} \quad (4.1.1)$$

where $\phi_p \neq 0, \theta_q \neq 0$, and $\sigma_w^2 > 0$. Also, we implicitly assume that the series x_t is demeaned: $x_t - \mu$. To simplify notations, we do not use $\tilde{x}$ where $\tilde{x} = x_t - \mu$

The parameters p and q are called autoregressive and the moving average orders.

To incorporate a non-zero mean, μ into the model, we set $\alpha = \mu(1 - \phi_1 - \cdots - \phi_p)$ and re-write the model as

$$x_t = \alpha + \phi_1 x_{t-1} + \cdots + \phi_p x_{t-p} + w_t + \theta_1 w_{t-1} + \cdots + \theta_q w_{t-q} \quad (4.1.2)$$

where w_t is assumed to be a Gaussian white noise series with mean zero and variance σ_w^2.

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