

Time Series Analysis

Lecture 4

Mixed Autoregressive Moving Average (ARMA) Models

Autoregressive Integrated Moving Average (ARIMA) Models

Seasonal ARIMA (SARIMA) Models

ARIMA Model: Simulation

ARIMA: Algebraic Manipulation Before Simulation

- Following the approach in the last few lectures, we simulate an ARIMA model and examine its patterns exhibited in a time series plot, ACF, and PACF.
- The simulated model acts as a “true” model, so we can estimate the model and examine how close the parameter estimates are to the “true” model parameters.
- Consider a model taken from an example in (CM2009) section 7.2.4 on page 140:

$$\begin{aligned}
 y_t &= 0.5y_{t-1} + y_{t-1} - 0.5y_{t-2} + \omega_t + 0.3\omega_{t-1} \\
 (y_t - y_{t-1}) &= 0.5(y_{t-1} - y_{t-2}) + \omega_t + 0.3\omega_{t-1} \\
 (y_t - y_{t-1}) - 0.5(y_{t-1} - y_{t-2}) &= \omega_t + 0.3\omega_{t-1} \\
 \nabla y_t - 0.5 \nabla y_{t-1} &= \omega_t + 0.3\omega_{t-1}
 \end{aligned}$$

ARIMA: Algebraic Manipulation Before Simulation

- The equation can be rearranged and factored as an ARIMA(1,1,1) model:


$$(1 - 0.5B) \nabla y_t = (1 + 0.3B)\omega_t$$

$$\left(\nabla y_t = 0.5 \nabla y_{t-1} + \omega_t + 0.3\omega_{t-1} \right)$$

- Note that after the first difference, the model becomes a stationary ARMA(1,1) model.

Simulated Data


```
set.seed(898)
x1 <- w <- rnorm(100)
for (i in 3:100) x1[i] <- 0.5*x1[i-1] + x1[i-1] - 0.5*x1[i-2] + w[i] + 0.3*w[i-1]
```



Red bracket annotations are present under the for loop equation, highlighting the terms: $0.5 \times x1[i-1]$, $x1[i-1]$, $-0.5 \times x1[i-2]$, $w[i]$, and $0.3 \times w[i-1]$.

Basic structure and descriptive statistics of the data

```
> str(x1)
num [1:100] -0.579 -0.0823 0.0254 -0.1728 0.254 ...
> summary(x1)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-0.9      5.0     12.5     11.5    15.9     24.3
```



A red bracket annotation is present under the summary output, highlighting the values: -0.9, 5.0, 12.5, 11.5, 15.9, and 24.3.

Data Visualization

Fig 1: Simulated Series

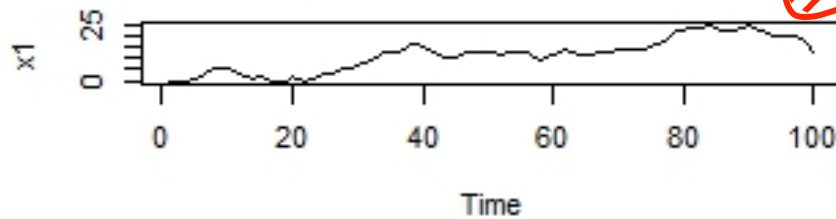


Fig 2: First Difference of the Simulated Series

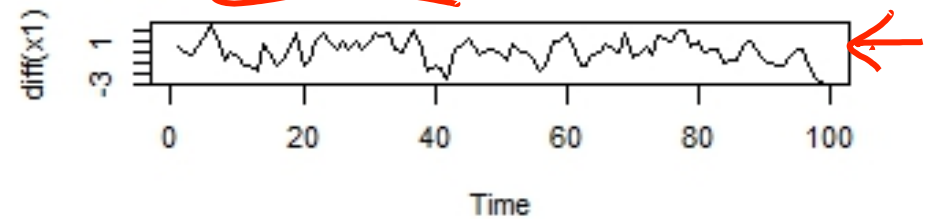


Fig 3: ACF of the Simulated Series

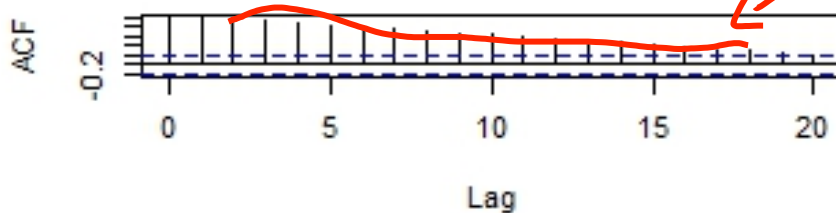


Fig 4: ACF of the Differenced Simulated Series

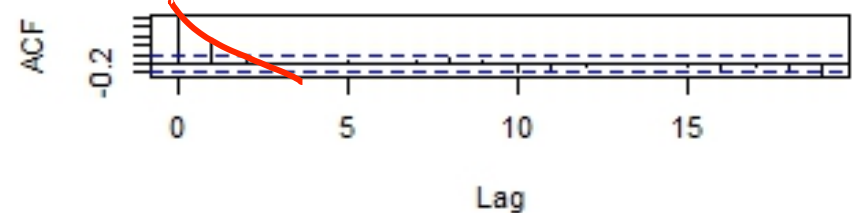


Fig 5: PACF of the Simulated Series

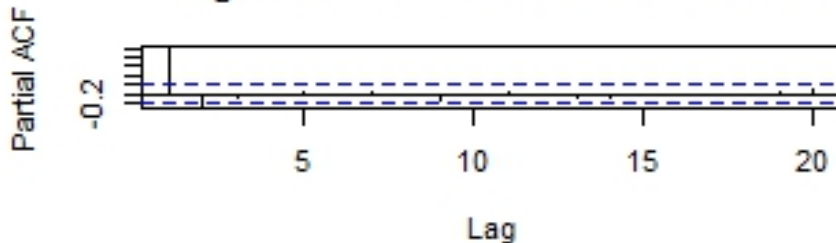
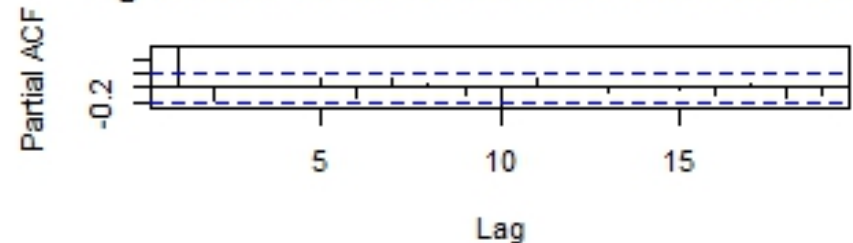


Fig 6: PACF of the Differenced Simulated Series



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