Time Series Analysis

Lecture 3

Autoregressive Models and Moving Average Models

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Autoregressive Models Part 3

Expression in Lag Operators

Backshift (or Lag) Operators: An Introduction

Backshift Operator:
A very useful concept is the backshift operator because it and its associated characteristic polynomials can be used to study the properties of $AP(p)$ models (and the $ARIMA(q)$ models in general)

$$\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p$$

Using the backshift operator, the autoregressive model can be re-written as:

$$\phi(B)x_t = \omega_t$$

The equation $\boxed{\phi(B) = 0}$ is called a *characteristic equation*, and it provides a powerful tool to check if a process is stationary. In particular, if the roots of the characteristic equation *all* exceed unity in absolute value, then the process x_t is *stationary*.

Examples of Using the Backshift Operators

Examples:

1. The random walk model $x_t = x_{t-1} + \omega_t$ has $\phi = 1$ and $\theta = 1 - \mathbf{B}$ with root $\mathbf{B} = 1$. Thus, this is a non-stationary process.

2. The AR(1) model $x_t = \frac{1}{2}x_{t-1} + \omega_t$ has the characteristic equation $\boxed{1 - \frac{1}{2}B = 0}$ and the root of $B = 2 > 1$. Thus, this AR(1) model is stationary.

3. Consider the AR(2) model $x_t = x_{t-1} - \frac{1}{4}x_{t-2} + \omega_t$. Expressed using the backshift operator, $\frac{1}{4}\left(B^2 - 4B + 4\right)x_t = \omega_t$, or $\frac{1}{4}\left(B - 2\right)^2 x_t = \omega_t$. The corresponding characteristic equation is $\phi(B) = \frac{1}{4}\left(B - 2\right)^2 = 0$, so the root is $B = 2 > 1$. Thus, the AR(2) model is stationary.

4. Consider another AR(2) model $x_t = \frac{1}{2}x_{t-1} + \frac{1}{2}x_{t-2} + \omega_t$, which can be expressed as $-\frac{1}{2}\left(B^2 + B - 2\right)x_t = \omega_t$. The corresponding polynomial $\phi(B) = -\frac{1}{2}(B-1)(B+2)$ has roots $B = 1, -2$. With the unit root $B = 1$, this AR(2) model is *non-stationary*.

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