Time Series Analysis
Lecture 4

Mixed Autoregressive Moving Average (ARMA) Models

Autoregressive Integrated Moving Average (ARIMA) Models

Seasonal ARIMA (SARIMA) Models

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Comparing ARMA Models and AR Models Using Simulated Series, Part 1

ACF of ARMA and AR Models Recap

- Recall that the ACF of ARMA models and AR models are hard to distinguish. Their functional forms are:

ACF of ARMA(1,1) Model

$$\rho(h) = \underbrace{\frac{(1+\theta\phi)(\phi+\theta)}{1-\phi^2}}\phi^{h-1} \qquad h \geq 1$$

ACF of AR (1) Model

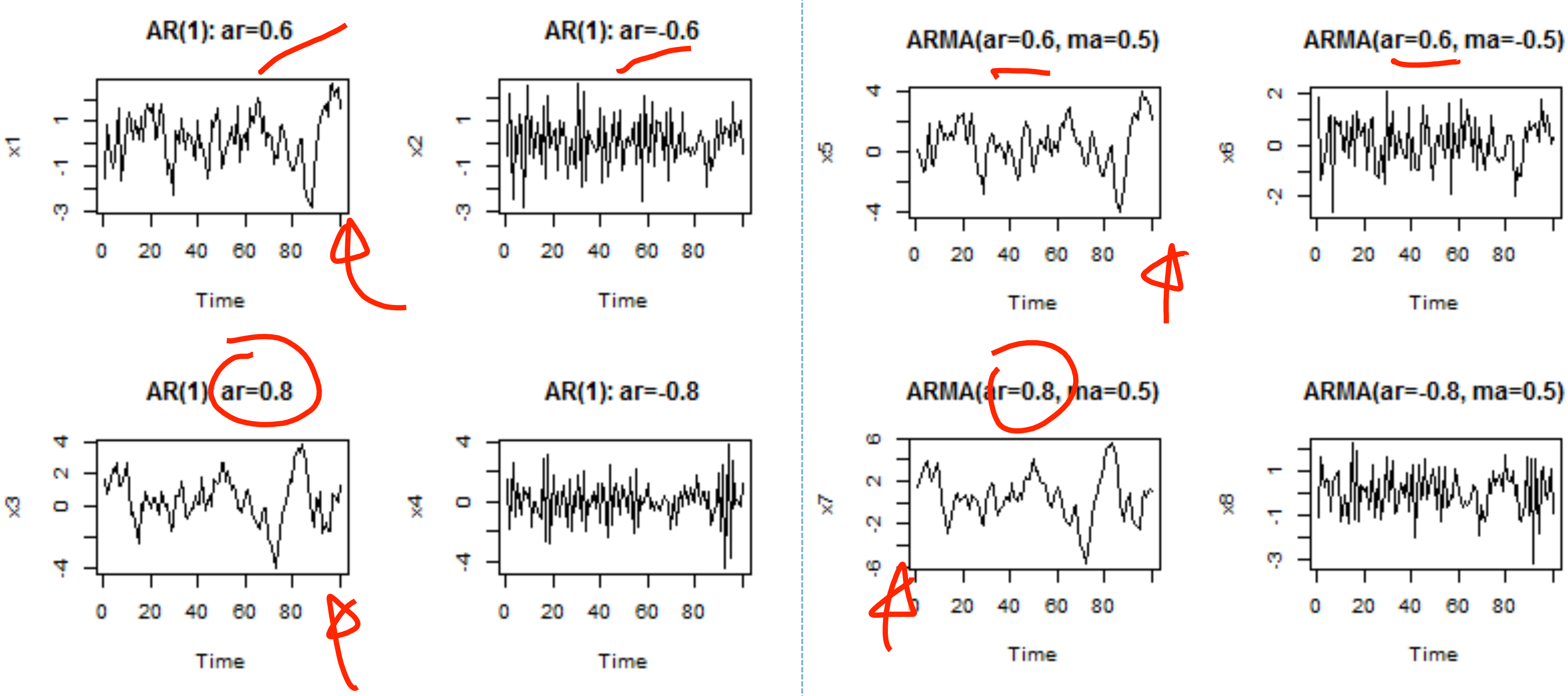
$$\rho(h) = \frac{\gamma(h)}{\gamma(0)} = \phi^h, \qquad h \geq 0$$

- Their functional forms differ only by a constant multiple.

- Therefore, using ACF alone is not sufficient for model identification.

Time-Series Plots of AR(1) and ARMA(1,1) Models

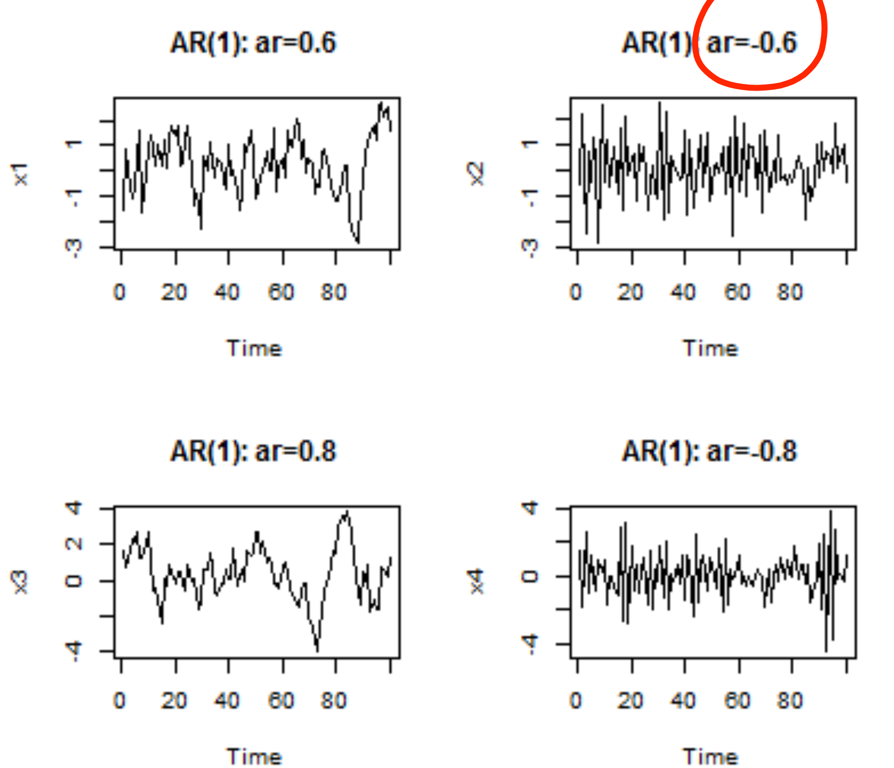
- All the simulations used in this module have 100 simulated points.
- If the AR parts of the AR(1) and ARMA(1,1) are identical, it is very difficult to distinguish between the two based only on the t-plots.

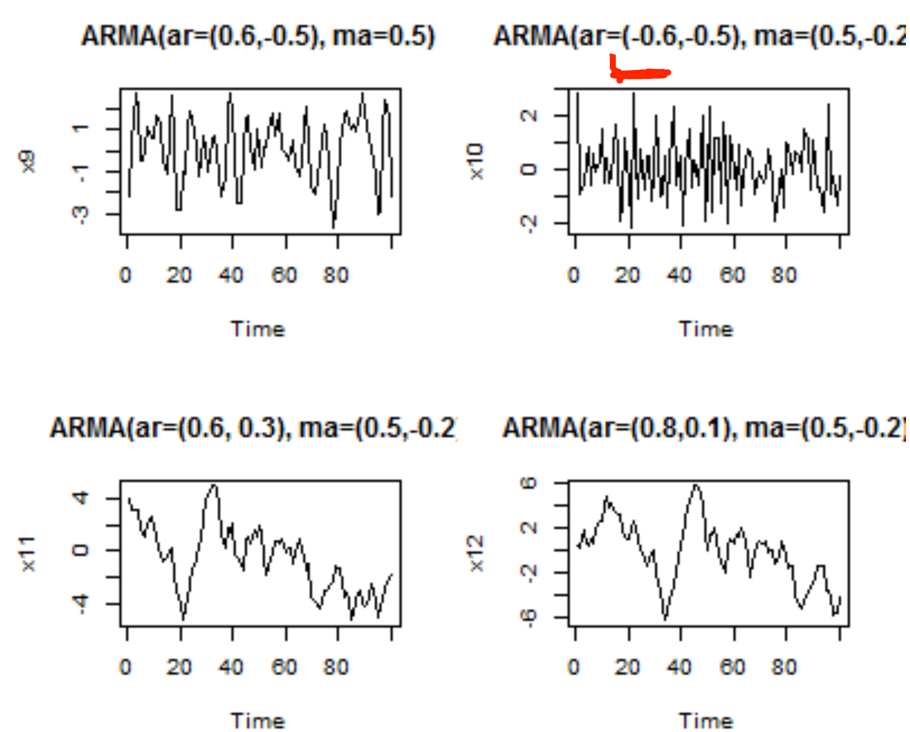


Time-Series Plots of AR(1) and ARMA(1,1) Models

- Adding an AR and/or MA term does not help to distinguish the two types of models.

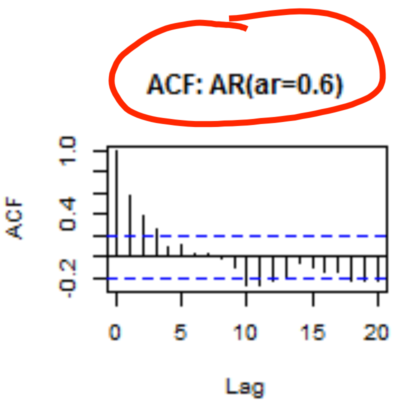
- Adding another AR term with negative coefficient makes the distinction even harder.

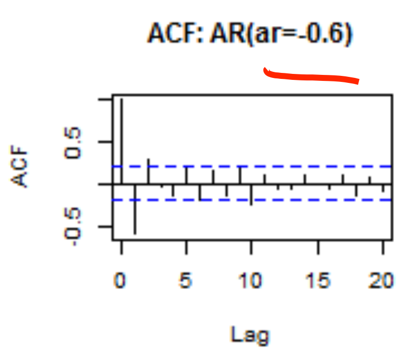


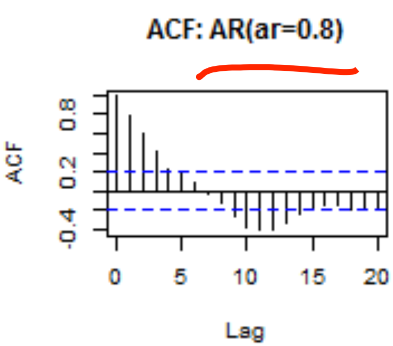


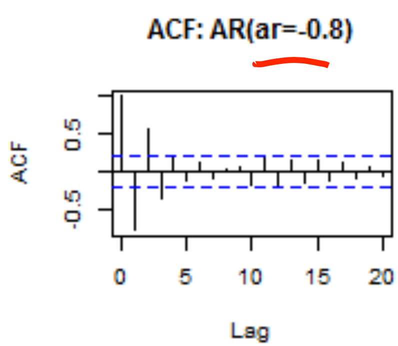
ACF of AR(1) and ARMA(1,1) Models

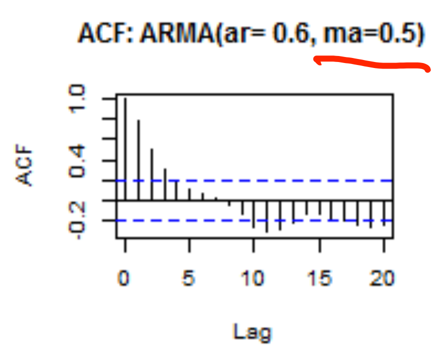
- As seen from the theoretical ACF, using a correlogram alone cannot distinguish AR and ARMA models.
- This can be seen from the empirical ACFs of the simulated AR and ARMA models.

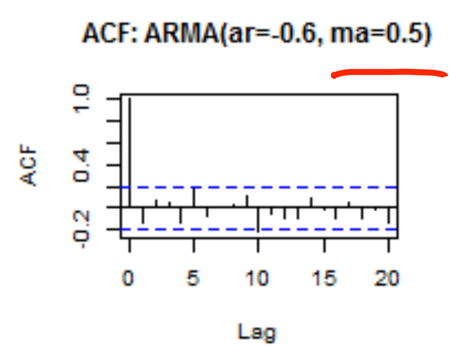


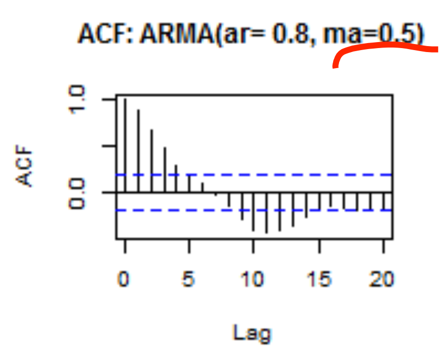


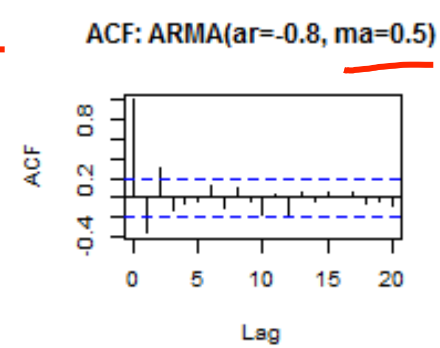






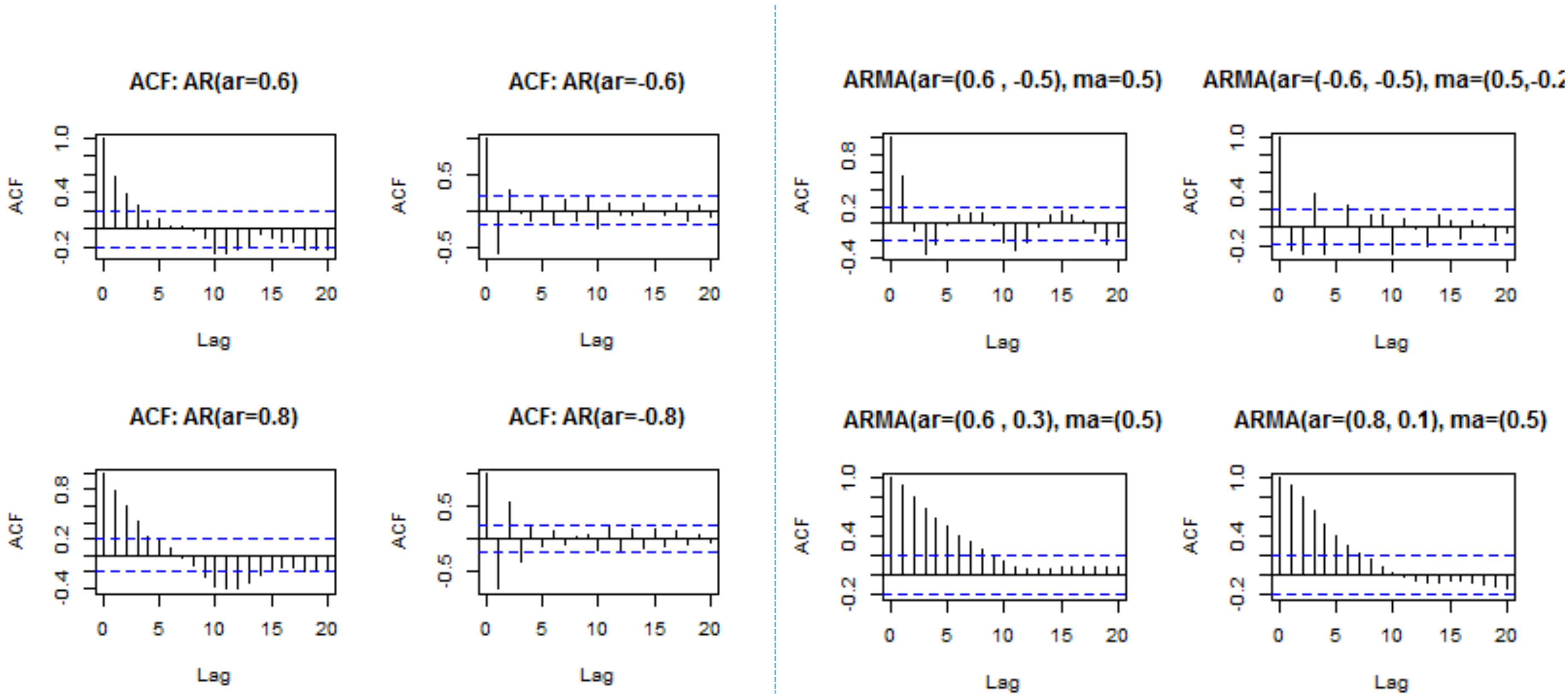






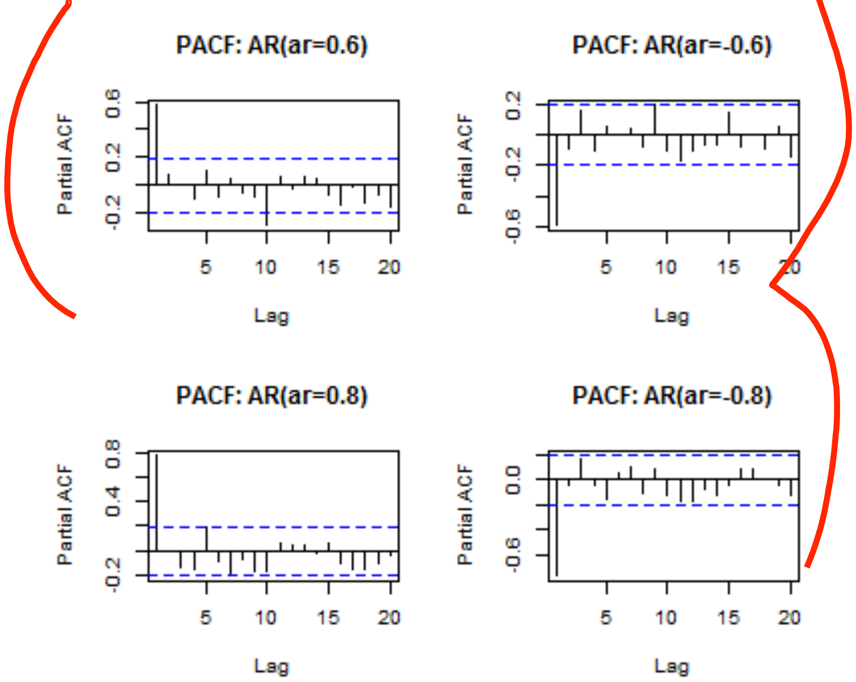
ACF of AR(1) and ARMA(1,1) Models

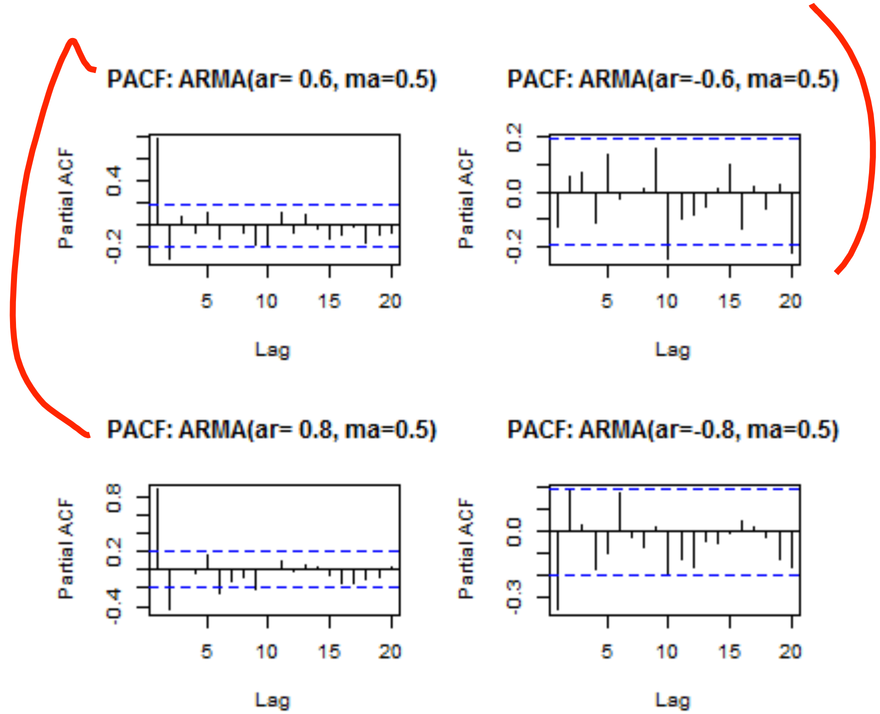
- The general patterns still look similar…
- The conclusion is that if we are analyzing the series and plot and the ACF and the ACF look like one of those below, it is not possible to tell which AR or ARMA (or even MA) models to use.



PACF of AR(1) and ARMA(1,1) Models

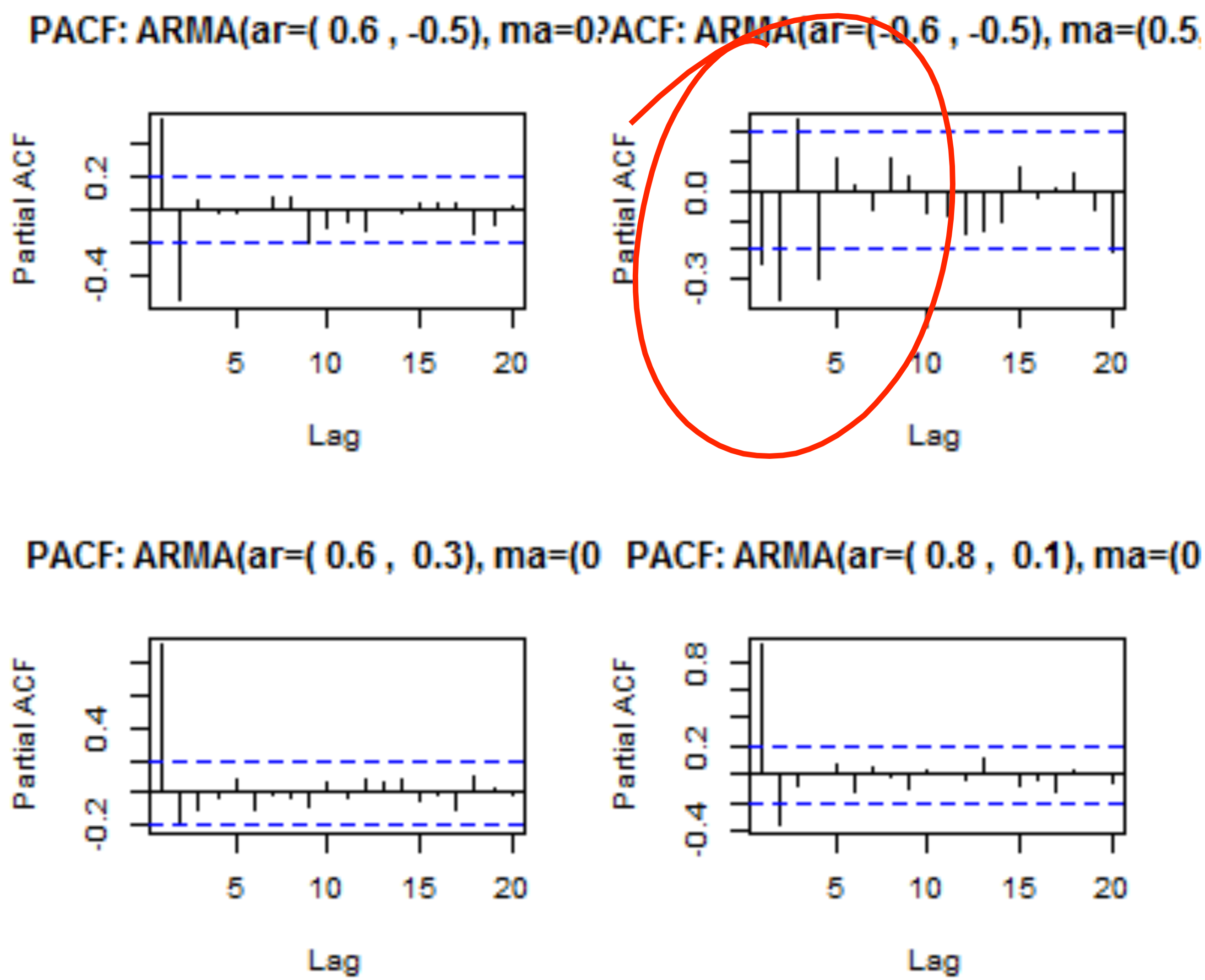
- The PACF of AR(p) models sharply drop off after p lags while the PACF of ARMA(p,q) models generally gradually decline to zero.
- Based on the graphs below, this feature is not apparent.
- Yet, I still want to show them to you because these graphs come from my simulation, and I don't want to cherry-pick graphs just to show you this feature.





PACF of AR(1) and ARMA(1,1) Models

- Similar conclusion, although the PACF do linger a little longer...



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