Time Series Analysis

Lecture 3

Autoregressive Models and Moving Average Models

datascience@berkeley

Moving Average Models

Mathematical Formulation and Derivation of Properties

Moving Average Models: An Introduction

A moving average model of order q is a linear combination of the current and past q white noises:

$$x_t = \omega_t + \theta_1 \omega_1 + \cdots + \theta_{t-q} \omega_{t-q} \tag{3.0.2}$$

t-1

where $\{\omega_t\}$ is a white noise sequence, each of which has zero mean and variance σ^2_ω. Also, consider x_t as a demeaned series. That is, $x_t - \mu$.

Expressed in backshift operators,

$$\tilde{x}_t = \left(1 + \theta_1 B + \theta_2 B^2 + \cdots + \theta_q B^q\right) = \boxed{\theta_q\left(B\right)\omega_t} \tag{3.0.3}$$

$\theta_q()$ is a polynomial of order q.

- Since the MA(q) process is a finite linear combination of white noises, the moving average processes are **stationary with constant mean, variance, and autocovariance**.

- In other words, the stationarity condition for a moving average process is met regardless of the values of its parameters.

Moving Average Models: An Introduction (2)

- A moving average process is a function of both current and past shocks. These shocks are theoretical, and more importantly, they are unobservable!

- So, we really cannot use moving average models for forecasting, as forecasting generally requires an established statistical relationship between current and past values.

- The question becomes, **"Can we express a moving average model as a function of current and past observable values?"**

- This leads to the concept of **invertibility**.

The Invertibility Condition

- In particular, if a moving average process is invertible, then it can be "inverted" and expressed as a function of current shock and lagged values of the series. We call this form of expression of MA models as an **autoregressive representation**.

In general, an MA(q) process is *invertible* when the roots of $\theta_q(B)$ all exceed 1 in absolute value.

- Invertibility condition is needed because of its practical importance.

- If a MA model is invertible, it can be expressed as autoregressive representation.

The Invertibility Condition (2)

- With the autoregressive representation, one can use it for real-world forecasting because forecasting requires the linkage between the present observations to the past observations.

- With this linkage (or a model), we can extrapolate to form a forecast based on present and past observations.

- For this reason, we will restrict our focus on invertible processes.

- Let’s look at the mathematical formulation and the key properties of MA models.

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