Time Series Analysis

Lecture 4

Mixed Autoregressive Moving Average (ARMA) Models

Autoregressive Integrated Moving Average (ARIMA) Models

Seasonal ARIMA (SARIMA) Models

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Random Walk, Integrated Process, an Introduction to ARIMA Process

The Variance of Random Walk Process

- Assuming the process started at some time 0 with value y0, we can write the random walk process as

$$y_t = y_0 + \sum_{i=1}^{t} \epsilon_i$$
$$E(y_t) = y_0$$
$$var(y_t) = t\sigma^2$$

- Note that the variance grows without bounds over time.
- Likewise, for random walk with drift, we can express the process as

$$y_t = t\delta + y_0 + \sum_{i=1}^{t} \epsilon_i$$
$$E(y_t) = y_0 + t\delta$$
$$var(y_t) = t\sigma^2$$

- The mean grows by the speed of drift term, and the variance grows without bounds over time.

Integrated Process: An Introduction

- However, a first differencing can transform the nonstationary random walk process to a stationary white noise process.

- White noise is the simplest I(0) process, and the random walk is the simplest I(1) process, where I(1) means the process is a differenced one. This is called integrated process of order 1.

- In practice, I(0) and I(1) cases find themselves having the most applications. One reason is that the results are hard to explain once a series is differenced too many times.

Random Walk Process

A time series y_t follows an ARIMA(p, d, q) process if the d^{th} differences of the y_t series is an ARMA(p, q) process. Mathematically, using lag operator, it can expressed as

$$\phi_p(B)(1 - B)^d y_t = \theta_q(B)\omega_t$$

where ϕ_p and θ_q are polynomials of orders p and q discussed in the previous lectures.

Writing an **ARIMA(p,d,q)** may seem too abstract, and whenever a model is presented this way, you may get a feel of the model by making simple cases, such as a low-order **ARIMA(p,d,q)** model.

Next, two such examples are shown, but you should create more examples of your own. Once an example is created, use R (or Python) to simulate some realizations of the model.

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