Time Series Analysis

Lecture 3

Autoregressive Models and Moving Average Models

Autoregressive Models, Part 4

Intuition of the Properties of the General AR(p) Models

Autoregressive Models in Lag Operators

For a (weakly) stationary $AR(p)$ model, its lag operator presentation is

$$\Phi(L)x_t = \alpha + \epsilon_t$$

$x_t = \alpha x_{t-1} + \varepsilon_t$

$(1-\alpha L)x_t = \varepsilon_t$

$\underbrace{1-\alpha L}$

$1-\alpha L = 0$

$\Rightarrow \alpha = \left|\frac{1}{L}\right| > 1$

where

$$\boxed{\Phi(L) = 1 - \phi_1 L - \cdots - \phi_p L^p}$$

Recall that the stationarity condition for the $AR(p)$ model is that the roots of the characteristic polynomial $\Phi(z)$ all lie outside of the unit circle (or equivalently, the roots of $z^p - \phi_1 z^{p-1} - \cdots - \phi_p$ all lie inside the unit circle.

Inverse of the

$\frac{1}{\alpha} = L < 1$

The mean of this model is

$$E(x_t) = \frac{\alpha}{1 - \phi_1 - \cdots - \phi_p}$$

The autocovariance function of the general $AR(p)$ model can be derived using the *Yule-Walker* equations. ~~We wi~~

Key Properties of the General AR(p) Model

The general properties of an AR(1) model carries through to an AR(p) model.

1. **Stationarity condition:** An AR(p) process is covariance stationary if and only if the inverse of all roots of the autoregressive lag operator polynomial $\Phi(B)$ are inside the unit circle.

2. **ACF:** The autocorrelation function for the AR(p) process decays gradually with displacement.

3. **PACF:** The partial autocorrelation function has a sharp cut-off at **displacement p**.

Key Properties of the General AR(p) Model (2)

However, there are difference between the general AR(p) models and AR(1) models.

1. Models with higher autoregressive order allows for a richer dynamics, and the autocorrelation function displays a wider variety of patterns.
   - For example, it can display damped, monotonic decays, as in the AR(1) case with a positive coefficient, but it can also have damped oscillation that AR(1) can't have unless its coefficient is negative.

2. The richer patterns of the ACF from the higher-order autoregressive models can mimic a wider range of cyclical patterns.

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