Time Series Analysis
Lecture 4

Mixed Autoregressive Moving Average (ARMA) Models

Autoregressive Integrated Moving Average (ARIMA) Models

Seasonal ARIMA (SARIMA) Models

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Nonstationary Models: An Introduction

Nonstationary Models

- In the previous lectures, we focused on stationary time series models and studied primarily series that are covariance stationary. In this lecture, we shift the focus to **nonstationary time series models**, as many time series encountered in practice are nonstationary due to the existence of trends or seasonal effects.

- Fortunately, many of the nonstationary series in the real world can be converted to a covariance stationary series using simple differencing, especially first-order differencing. This modeling strategy leads to the celebrated **Box-Jenkins Approach** to derive **A**uto**R**egressive **I**ntegrated **M**oving **A**verage, or ARIMA, models.

Nonstationary Model

- The term "integrated" comes from the fact that the differenced series need to be aggregated to recover the original series, the underlying process is called an "integrated" series.

- The ARIMA process can accommodate seasonal terms, giving rise to seasonal ARIMA (SARIMA) model, which will also be discussed in this lecture.

- Not all nonstationarity can be dealt with using differencing. For instance, volatility clustering, which occurs in many financial and macroeconomic time series, leading to **conditionally heteroskedaticity** is more appropriately modeled using an Autoregression Conditional Heteroskedastic (ARCH) or a Generalize ARCH (GARCH) model.

Nonstationary Model

- Without seasonal effects, <u>first differencing</u> can remove both (stochastic and deterministic) trends. An example of stochastic trends includes random walks, and an example of deterministic trend includes a linear trend:

Recall that a Random Walk takes the following form:

$$y_t = y_{t-1} + \omega_t$$

Taking a first difference transforms the model to the following form:

$$\nabla y_t = \underbrace{y_t - y_{t-1}}_{} \cancel{= \omega_t} = \omega_t$$

which is a mean-zero, stationary white-noise series.

Nonstationary Model

On the other hand, a linear trend with white noise errors

$$y_t = a + bt + w_t$$

can be transformed into a stationary moving average (MA(1)) process with first differencing:

$$\nabla y_t = y_t - y_{t-1} = b + (w_t + w_{t-1})$$

Another transformation we can take is to subtract the trend from the series, which gives a white noise process, which may be a more sensible approach:

$$y_t - (a + bt) = w_t$$

In practice, do not just blindly take first differencing or first differencing in log or even higher-level differencing. Always first investigate the series using various graphical techniques and statistical tests before estimating a model on some transformation of the original series.

Also, when making transformations, always ask what the meaning/ interpretation is of the transformed series.

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