Time Series Analysis

Lecture 3

Autoregressive Models and Moving Average Models

datascience@berkeley

Moving Average Models

Key Properties Recap

The "Memory" of Moving Average Process

Let's recap the functional form of the variance and autocorrelation function:

The Autocovariance function of an MA(q) process $(k = 0, 1, \dots, q)$ is

$$\gamma_k = E\left[(\omega_t + \theta_1\omega_{t-1} + \cdots + \theta_q\omega_{t-q})(\omega_{t-k} + \theta_1\omega_{t-k-1} + \cdots + \theta_q\omega_{t-k-q})\right]$$
$$= \theta_k E[\omega_{t-k}^2] + \theta_1\theta_{k+1}E[\omega_{t-k-1}^2] + \cdots + \theta_{q-k}\theta_q E[\omega_{t-q}^2]$$

since the ω_t are uncorrelated, and $\gamma_l = 0$ for $k > q$.

Therefore, the variance of the process is

$$\gamma_0 = \left(1 + \theta_1^2 + \theta_2^2 + \cdots + \theta_q^2\right)\sigma_\omega^2$$

The autocovariance of the process is

$$\gamma_k = \begin{cases} (\theta_k + \theta_1\theta_{k+1} + \theta_2\theta_{k+2} + \cdots + \theta_{q-k}\theta_q)\,\sigma_\omega^2 & k = 1, 2, \dots, q \\ 0 & k > q \end{cases}$$

The autocorrelation function is

$$\rho_k = \begin{cases} \frac{\theta_k + \theta_1\theta_{k+1} + \theta_2\theta_{k+2} + \cdots + \theta_{q-k}\theta_q}{1 + \theta_1^2 + \theta_2^2 + \cdots + \theta_q^2} & k = 1, 2, \dots, q \\ 0 & k > q \end{cases}$$

The "Memory" of Moving Average Process

- The autocorrelation function shows that a MA(q) has a "memory" of only **q** periods. If q=1, then the model has memory of only one period.

- It means that the current value is not affected by the values older than q periods.

- This has an important implications on model identification (if the underlying data generating process is indeed a MA(q) process):

> The ACF of a MA(q) model drops off completely after **q** periods.

- This means that the ACF provides a considerable amount of information about the order of dependence if the process is a moving average process.

Berkeley SCHOOL OF INFORMATION