Time Series Analysis

Lecture 3

Autoregressive Models and Moving Average Models

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Autoregressive Models

Simulation of AR(2) Models

Use an AR(2) Model to Illustrate the Properties

Consider an AR(2) model of the specification:

$$y_t = 1.5y_{t-1} - 0.9y_{t-2} + \epsilon_t$$

The corresponding lag operator polynomial is

$$\left(1 - 1.5B + 0.9B^2\right)$$

The roots of this polynomial can be easily found in **R** using the **polyroot** function: `polyroot(c(1, -1.5, 0.9))`.

The result is two complex conjugate roots **0.83 +/- 0.65i** .

```
> polyroot(c(1, -1.5,0.9))
[1] 0.8333333+0.6454972i 0.8333333-0.6454972i
```

abs(polyroot(c(,)))

1.054

Use an AR(2) Model to Illustrate the Properties

The inverse roots are `0.75+/-0.58i`:

```
> 1/polyroot(c(1, -1.5,0.9))
[1] 0.75-0.5809475i 0.75+0.5809475i
```

0.948

Both of these roots are close to 1 but nevertheless inside the unit circle.

Therefore, the process is **covariance stationary**.

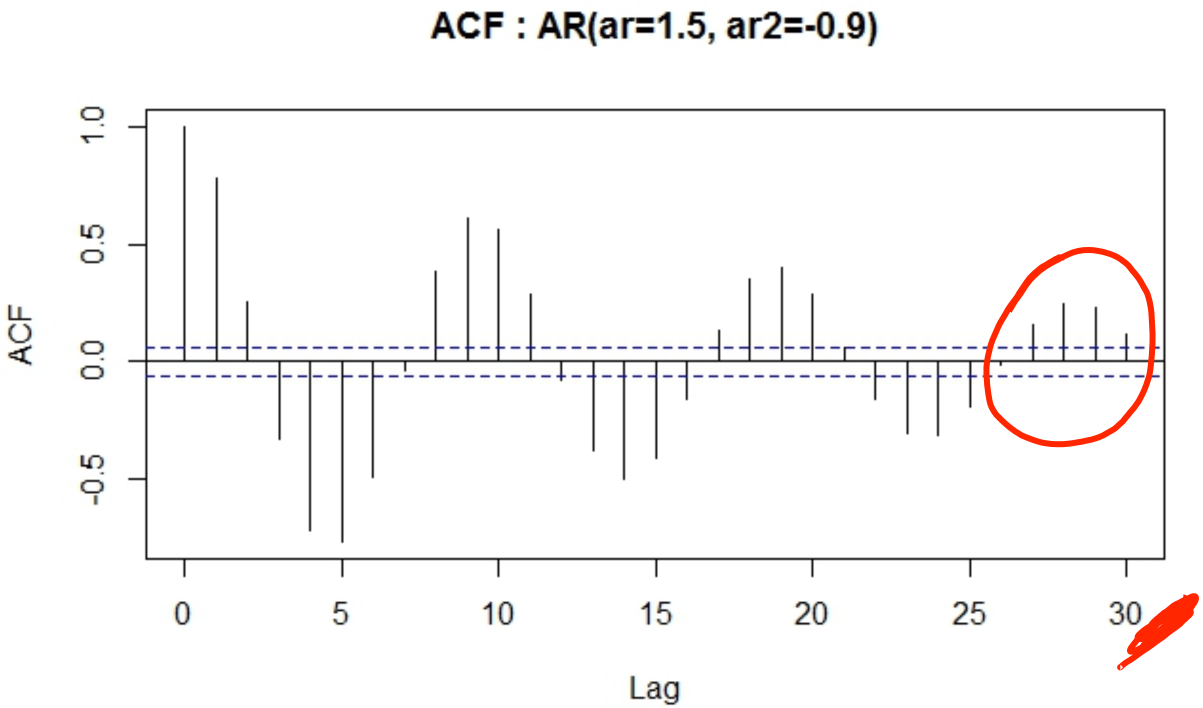
The autocorrelation function of an AR(2) process is

$$\boxed{\begin{aligned} &\rho(0) = 1 \\ &\rho(1) = \frac{\phi_1}{1-\phi_2} \\ &\rho(2) = \phi_1\rho(\tau-1) + \phi_2\rho(\tau-2), \quad \tau = 2, 3, \ldots \end{aligned}}$$

Use an AR(2) Model to Illustrate the Properties

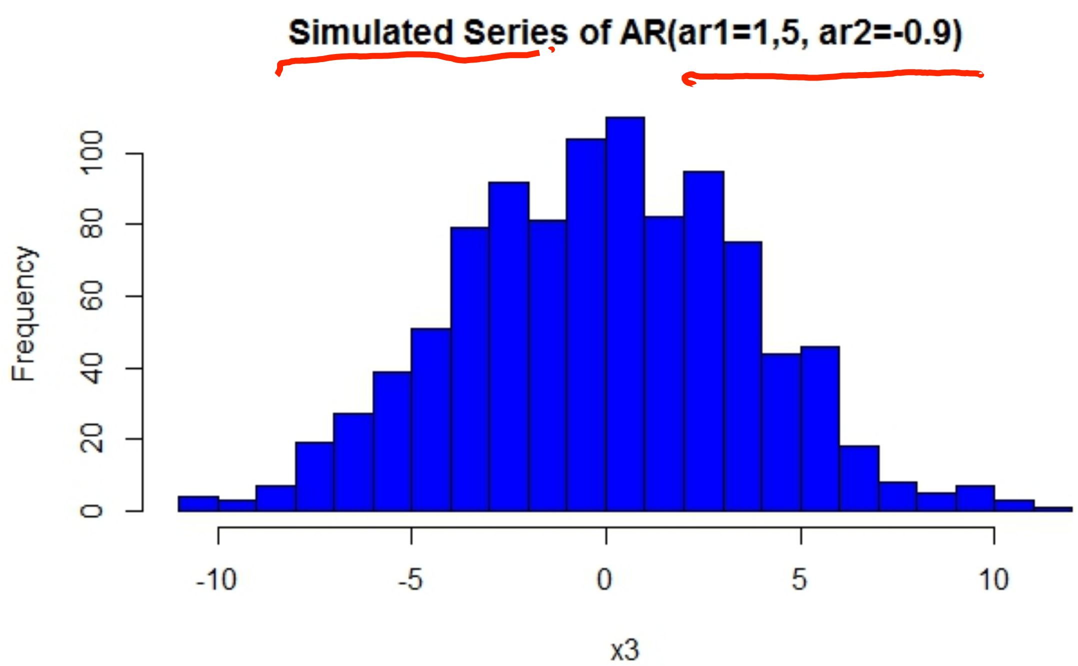
- Because the roots are complex, the autocorrelation function oscillates.

- Because the roots are close to one, the autocorrelation function oscillates slowly.



Use an AR(2) Model to Illustrate the Properties

- The histogram looks fairly symmetric.
- The summary statistics are displayed below the histogram.

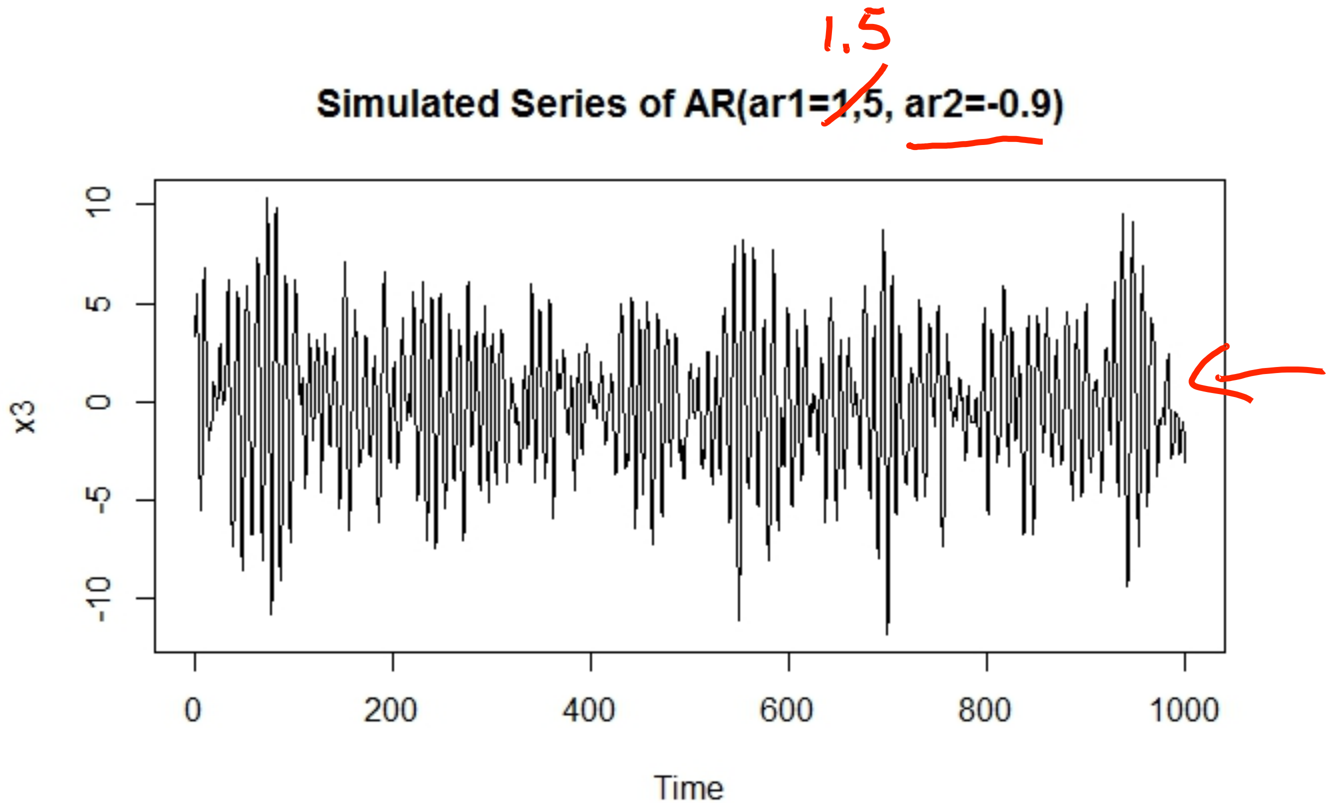



```
> str(x3)
 Time-Series [1:1000] from 1 to 1000: 3.34 5.44 5.03 2 -2.86 ...
> summary(x3)
    Min.   1st Qu.    Median      Mean   3rd Qu.      Max. 
-11.7900   -2.5550   -0.1784   -0.1619    2.3410   10.3400 
```

Use an AR(2) Model to Illustrate the Properties

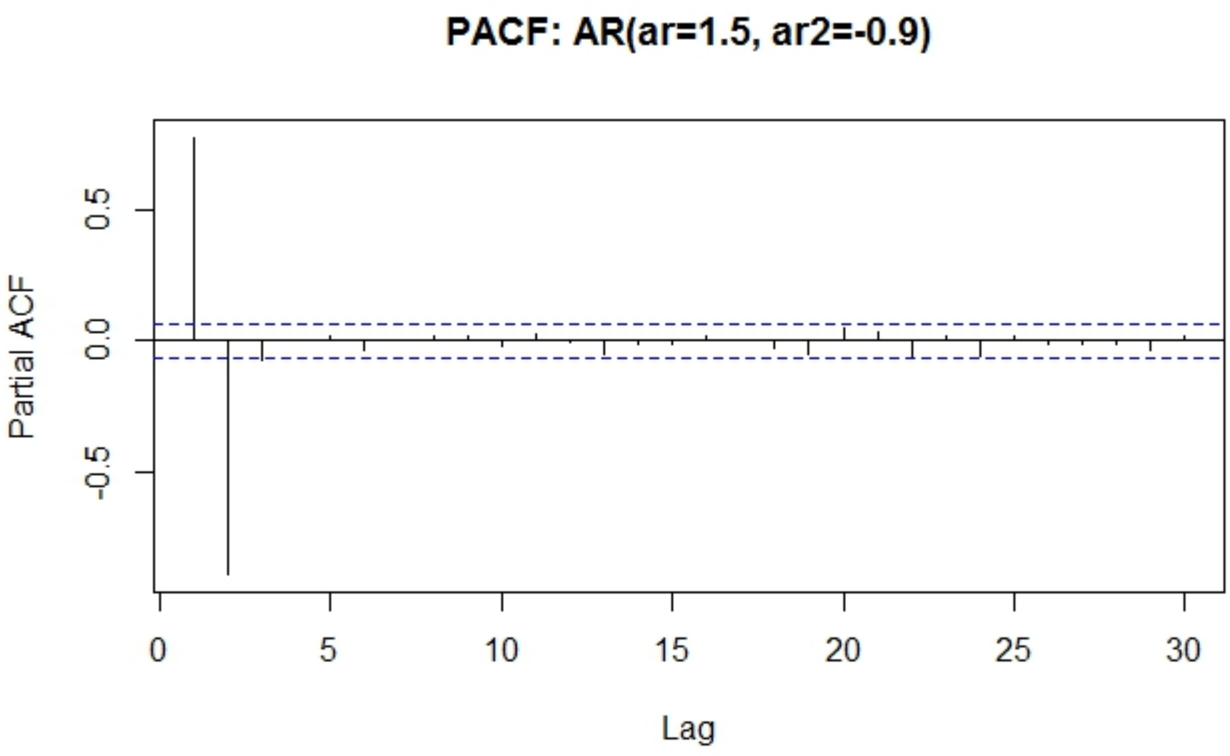
- The time series plot shows that the series display strong fluctuations

- The magnitude of these fluctuations change over time



Use an AR(2) Model to Illustrate the Properties

- As in the AR(1) model in which the PACF has a sharp cut-off at **displacement 1**, the PACF of the simulated AR(2) process has a sharp cut-off at **displacement 2**.



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