

# Time Series Analysis

## Lecture 4

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Mixed Autoregressive Moving Average (ARMA) Models

Autoregressive Integrated Moving Average (ARIMA) Models

Seasonal ARIMA (SARIMA) Models

# Seasonal ARIMA

An Example: European Quarterly  
Retail Trade

## European Retail Trade: 1996–2011

- This is an example from the book *Forecasting: Principles and Practice* by Rob J. Hyndman and George Athanasopoulos.
- Rob Hyndman also wrote the R package *forecast*, which we use extensively in this course.
- The series is clearly nonstationary and appears to have some seasonality.
- First, take seasonal difference.

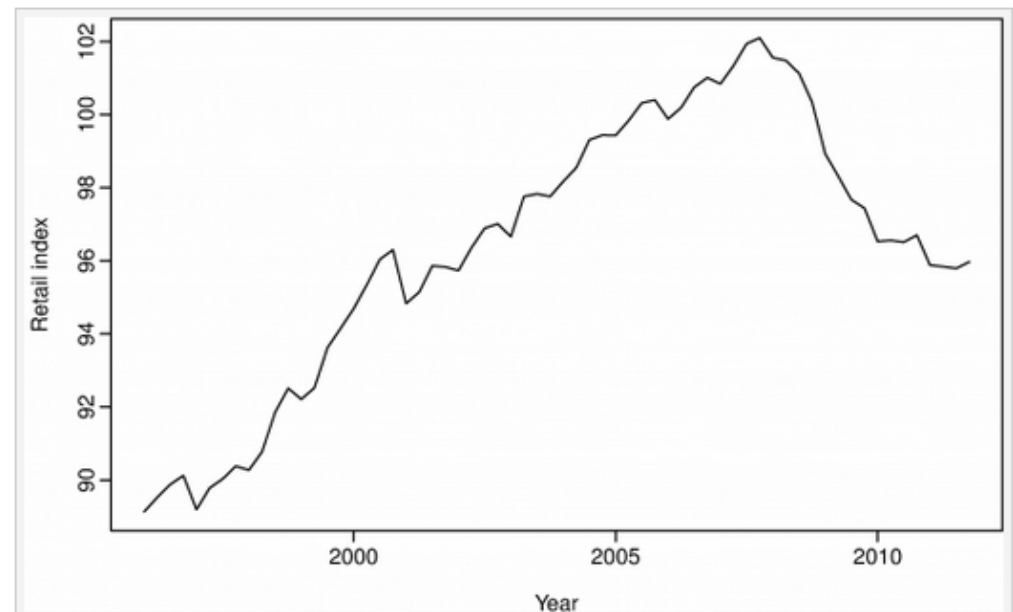
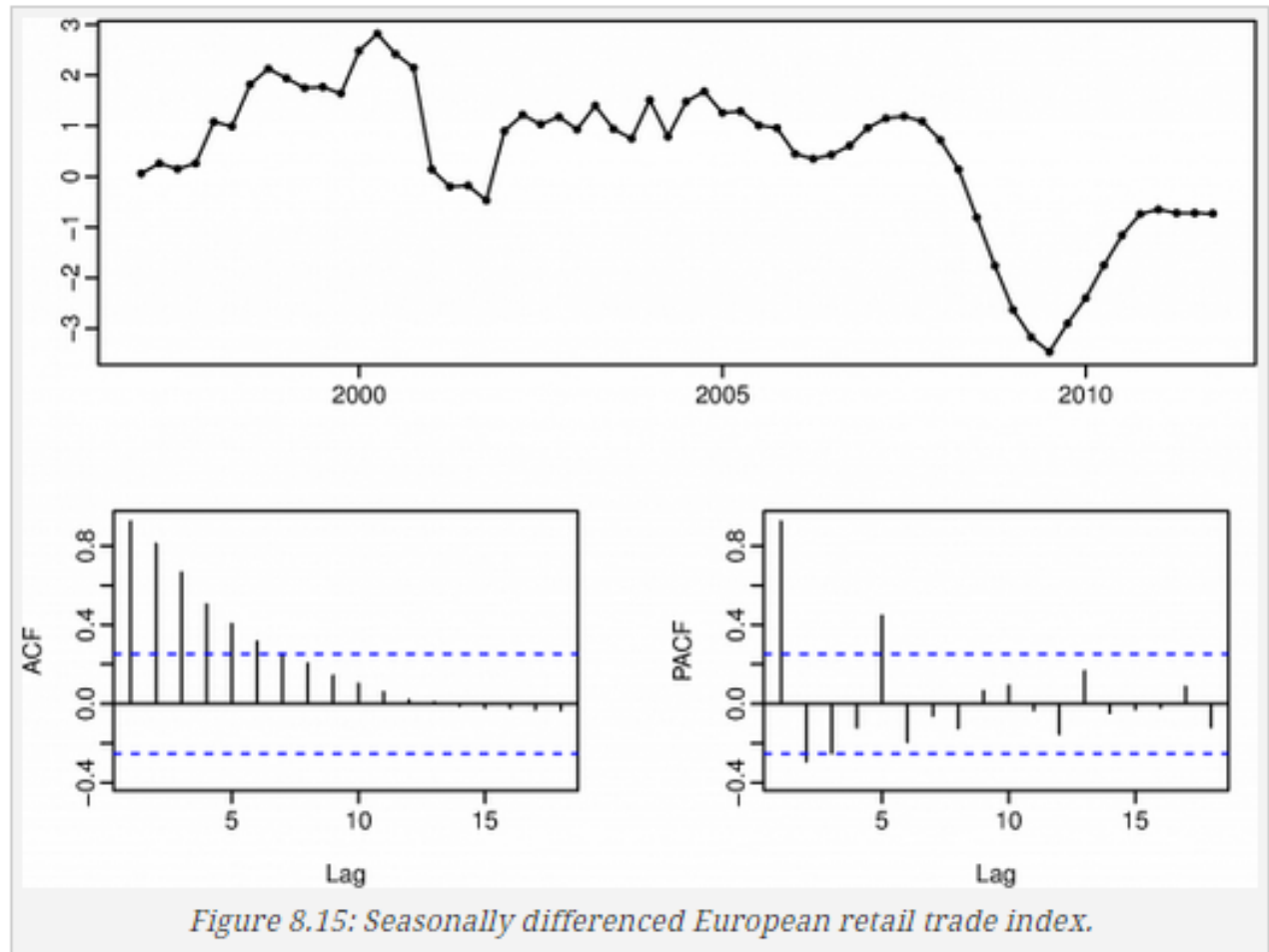


Figure 8.14: Quarterly retail trade index in the Euro area (17 countries), 1996–2011, covering wholesale and retail trade, and repair of motor vehicles and motorcycles. (Index: 2005 = 100).

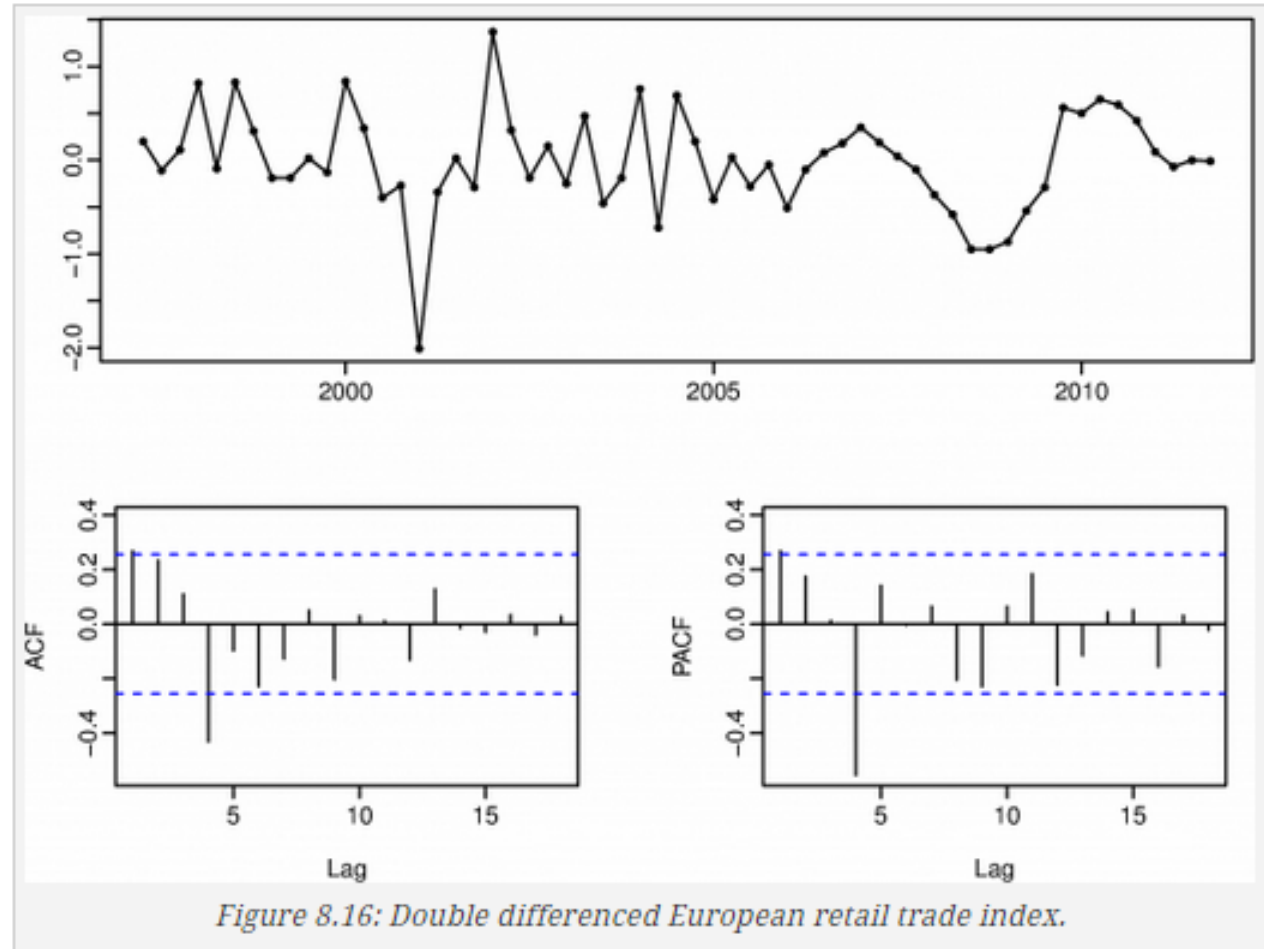
## European Retail Trade: 1996–2011

- The seasonal differenced series, along with its ACF and PACF, is shown below.
- The series still appears to be nonstationary.
- Take an additional first difference to convert it into a stationary series.



# European Retail Trade: 1996–2011

- The seasonal differenced and first differenced series, along with its ACF and PACF, is shown below.
- After the differenced transformation, model the transformed series using ARMA model based on the ACF and PACF of the transformed series.
- The spike at lag 1 in the ACF suggests a nonseasonal MA(1) component.
- The spike at lag 4 in the ACF suggests a seasonal MA(1) component.



# European Retail Trade: 1996–2011

- We will start with a model of the form  $\text{ARIMA}(0,1,1)(0,1,1)_4$ .
- The residuals of the estimated model is shown below.

A first and seasonal difference

Nonseasonal and seasonal MA(1) components

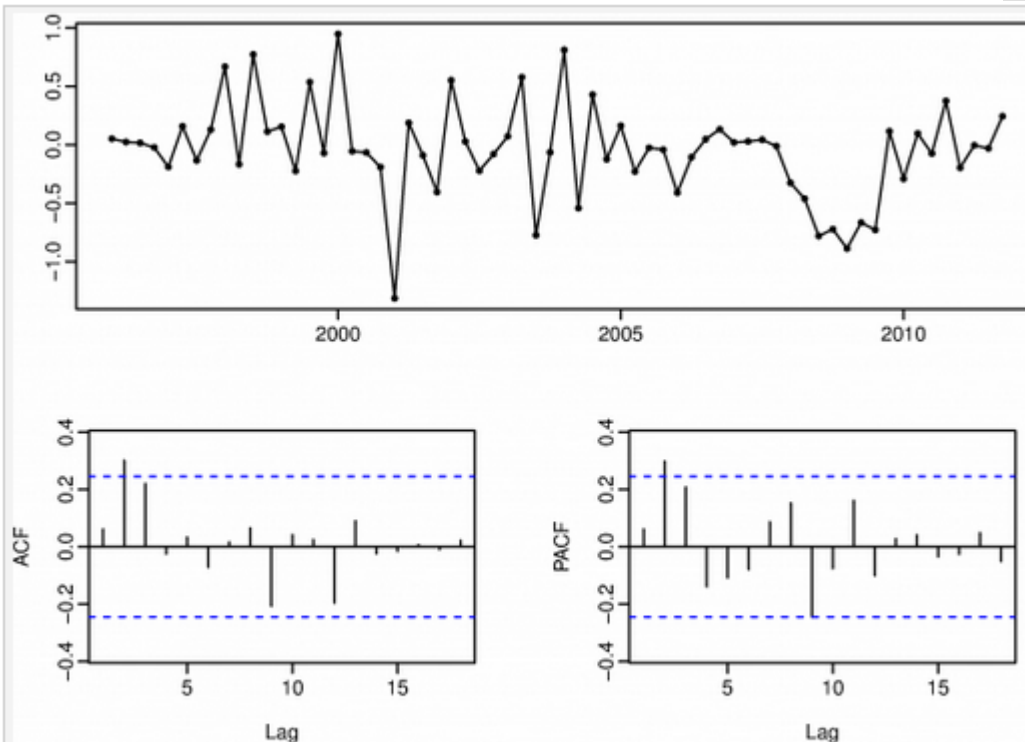
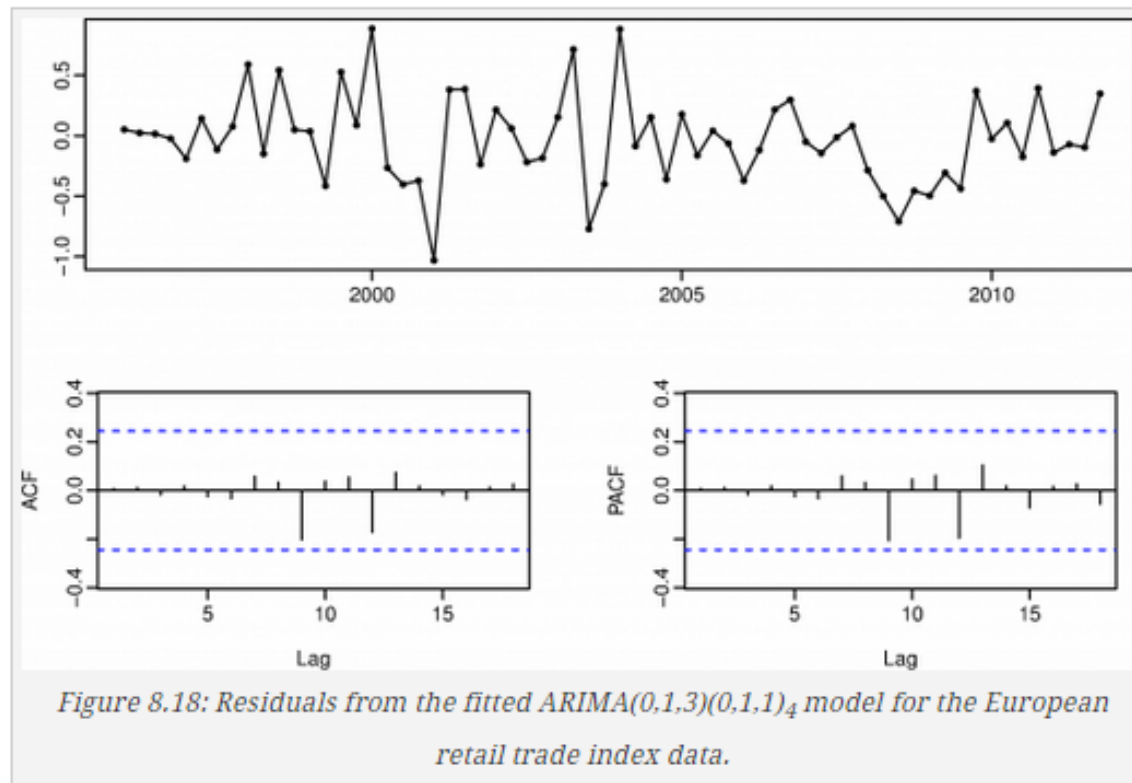


Figure 8.17: Residuals from the fitted  $\text{ARIMA}(0,1,1)(0,1,1)_4$  model for the European retail trade index data.

Both the ACFs and PACFs show significant spikes at lag 2, suggesting the need to include additional nonseasonal terms.

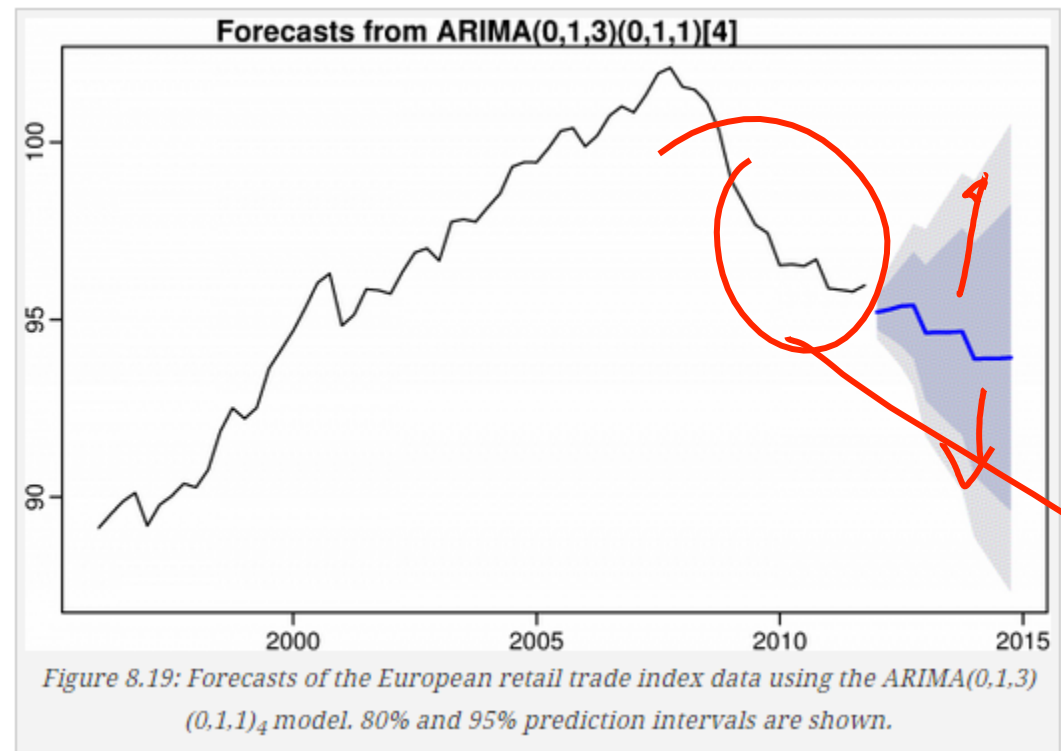
# European Retail Trade: 1996–2011

- Different models are estimated, including  $\text{ARIMA}(0,1,2)(0,1,1)_4$ ,  $\text{ARIMA}(0,1,3)(0,1,1)_4$ , and other models with AR terms, but  $\text{ARIMA}(0,1,3)(0,1,1)_4$  gave the smallest AIC value.
- Remember that AIC is not the only model fit measure. Also, we may want to use a recursive method and use out-of-sample fit measure to select a model.



# European Retail Trade (1996–2011): Forecasting

- With a candidate model that appears to satisfy the statistical assumptions of the model, let's proceed to forecasting.
- A 12-step- (i.e., 12 quarters or three years) ahead forecast is shown below.
- Note 1: The forecast follows the “most recent trend” (due to the double differencing transformation applied to the series we model).
- Note 2: The large and increasing prediction interval allows for an increase retail trade index; in fact, both increasing and decreasing trends cannot be ruled out while the mean forecast points to a decreasing trend.





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