Time Series Analysis Lecture 4

Mixed Autoregressive Moving Average (ARMA) Models Autoregressive Integrated Moving Average (ARIMA) Models Seasonal ARIMA (SARIMA) Models

datascience@berkeley

Seasonal ARIMA Introduction and Mathematical Formulation

Seasonal ARIMA: Introduction

- So far we have been side-stepping the issue of seasonal effects.
- The dependence on the past often tends to occur most strongly at multiples of some underlying seasonal lag *s*.
- ARIMA models can be extended to include seasonal effects.
- A seasonal ARIMA (SARIMA) model uses differencing at a lag equal to the number of season(s) to remove additive seasonal effects.
- A seasonal ARIMA model is formed by including <u>additional</u> seasonal terms in the ARIMA models, as written below.

where m is the number of periods per season.

Seasonal ARIMA: Mathematical Formulation

- Uppercase notation is for the <u>seasonal parts</u> of the model.
- Lowercase notation for the <u>nonseasonal parts</u> of the model.
- The seasonal part of the model consists of terms that are very similar to the nonseasonal components of the model, but they involve backshifts of the seasonal period.
- The general-form seasonal ARIMA model can be written as below.

The seasonal ARIMA $(p, d, q)(P, D, Q)_s$ model can be most succinctly expressed using the backward shift operator

$$\Theta_P(\mathbf{B}^s)\theta_p(\mathbf{B})(1-\mathbf{B}^s)^D(1-\mathbf{B})^dx_t = \Phi_Q(\mathbf{B}^s)\phi_q(\mathbf{B})w_t$$
 where Θ_P , θ_p , Φ_Q , and ϕ_q are polynomials of orders P , p , Q , and q

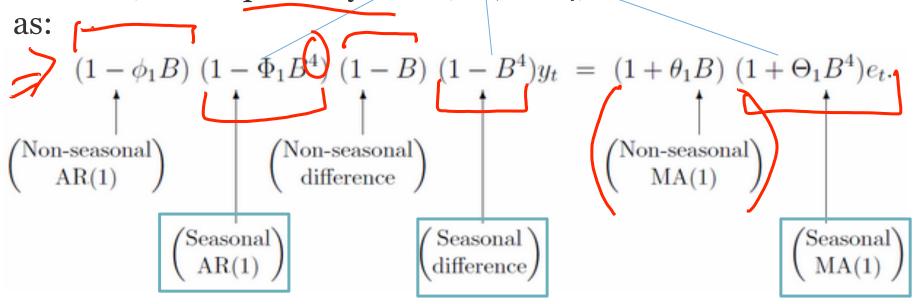
Seasonal ARIMA: Explanation of the Formula

The seasonal ARIMA $(p, d, q)(P, D, Q)_s$ model can be most succinctly expressed using the backward shift operator

$$\Theta_P(\mathbf{B}^s)\theta_p(\mathbf{B})(1-\mathbf{B}^s)^D(1-\mathbf{B})^dx_t = \Phi_Q(\mathbf{B}^s)\phi_q(\mathbf{B})w_t$$

where Θ_P , θ_p , Φ_Q , and ϕ_q are polynomials of orders P, p, Q, and q

For example, an ARIMA(1,1,1)(1,1,1)₄ model (without a constant) is for quarterly data (i.e., m=4) and can be written



Seasonal ARIMA: Examples

- (a) A simple AR model with a seasonal period of 12 units, denoted as ARIMA(0, 0, 0)(1, 0, 0)₁₂, is $x_t = \alpha x_{t-12} + w_t$. Such a model would be appropriate for monthly data when only the value in the month of the previous year influences the current monthly value. The model is stationary when $|\alpha^{-1/12}| > 1$.
- (b) It is common to find series with stochastic trends that nevertheless have seasonal influences. The model in (a) above could be extended to $x_t = x_{t-1} + \alpha x_{t-12} \alpha x_{t-13} + w_t$. Rearranging and factorising gives $(1 \alpha B^{12})(1 B)x_t = w_t$ or $\Theta_1(B^{12})(1 B)x_t = w_t$, which, on comparing with Equation (7.3), is ARIMA(0, 1, 0)(1, 0, 0)₁₂. Note that this model could also be written $\nabla x_t = \alpha \nabla x_{t-12} + w_t$, which emphasises that the change at time t depends on the change at the same time (i.e., month) of the previous year. The model is non-stationary since the polynomial on the left-hand side contains the term (1 B), which implies that there exists a unit root B = 1.

Seasonal ARIMA: Examples (2)

(c) A simple quarterly seasonal moving average model is x_t = (1 - βB⁴)w_t = w_t - βw_{t-4}. This is stationary and only suitable for data without a trend. If the data also contain a stochastic trend, the model could be extended to include first-order differences, x_t = x_{t-1} + w_t - βw_{t-4}, which is an ARIMA(0, 1, 0)(0, 0, 1)₄ process. Alternatively, if the seasonal terms contain a stochastic trend, differencing can be applied at the seasonal period to give x_t = x_{t-4} + w_t - βw_{t-4}, which is ARIMA(0, 0, 0)(0, 1, 1)₄.

Berkeley school of information