Time Series Analysis
Lecture 2

Regression With Time Series, an Intro to Exploratory Time Series Data Analysis, and Time Series Smoothing

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Autoregressive Models Part 1a
Mathematical Formulation and Properties

Autoregressive Models

Autoregressive models are found on the idea that the current value of a series x_t can be explained as a function of p past values $x_{t-1}, x_{t-2}, \cdots, x_{t-p}$, where p is called the order of the autoregressive model and determines the number of steps into the past needed to *forecast* the current value.

A stationary autoregressive model of *order* p, $AR(p)$ takes the form

$$x_t = \alpha + \phi_1 x_{t-1} + \phi_2 x_{t-2} + \cdots + \phi_p x_{t-p} + \omega_t$$

where

x_t is a *stationary* series

$\phi_1, \phi_2, \cdots, \phi_p \; (\phi_p \neq 0)$ are constants (i.e. unknown parameters to be estimated)

ω_t is a *Gaussian white noise series with mean zero and variance* σ_w^2, and

$\alpha = \mu/(1 - \phi_1 - \phi_2 - \cdots - \phi_p)$

Autoregressive Models

To thoroughly study the properties of the autoregressive model, we first consider the first-order model $AR(1)$:

$$x_t = \phi x_{t-1} + \omega_t$$

$$\begin{aligned} x_t &= \phi x_{t-1} + \omega_t \\ &= \phi\left(\phi x_{t-1}\right) + \omega_t \\ &= \phi^2 x_{t-2} + \left(\omega_t + \phi\omega_{t-1}\right) \\ &\vdots \\ &= \phi^k x_{t-p} + \sum_{j=0}^{k-1} \phi^j \omega_{t-j} \end{aligned}$$

Autoregressive Models

Continue to iterate backward, the $AR(1)$ model can be written as a **linear process:**

$$x_t = \sum_{j=0}^{\infty} \phi^j w_{t-j}$$

provided that

1. $|\phi| < 1$
2. x_t is stationary

In other words, the linear process defined above exists in the mean square sense:

$$\lim_{k \Rightarrow \infty} E\left(x_t - \sum_{j=0}^{k-1} \phi^j w_{t-j}\right) = \lim_{k \Rightarrow \infty} \phi^{2k} E\left(x_{t-k}^2\right) = 0$$

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