Sample Case: Olympic Badminton

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Using Statistics To Conclude Which Team Is Better

At which point does winning a sport match becomes more about luck than skill? Making 1-2 mistakes too many at a critical point of a game can make a difference between Olympic gold and silver medals. Thus, watching Tokyo Olympic Games made me curious, **are gold medalists really better than silver or bronze medalists?** If so, how can we prove/test that?

The Plan

The rough idea is to view each point in the match as a Bernoulli trial with the probability of team A winning the point as p while that of team B is $(1-p)$.

The distribution of the total score of each team in a match can then be modeled using binomial distribution. Having observed scores as a distribution allows for further hypothesis testing.

Hypothesis Testing

The next step will then be performing a binomial test on the total score of each team in the game. In this case, the H_0 is $p = 0.5$ or claiming that both teams are equally good. The H_1 is thus $p \neq 0.5$ or claiming that one team being better than the other.

The binomial test is equivalent to checking the probability of having the match score value be equal or more extreme than the value observed given that $p = 0.5$ or

$$P(observation|p = 0.5)$$

And if this probability is less than the threshold, we cannot reject the hypothesis that $p = 0.5$ As for the threshold, some reasonable number such as 20% would suffice. This would be equivalent to stating that we can afford to falsely crown one team as the winner 20% of the time despite both teams being equally good.

Assumptions

1. Both teams play their best for evey single point, regardless of the point difference or the set they play in.
2. The probability of winning a point is constant throughout the game even as both teams get more exhausted.

A Really Close Game

I will first pick one match to work on before trying to extend the idea to other matches.

The sample match used here is: https://olympics.com/tokyo-2020/olympic-games/en/results/badminton/results-mixed-doubles-fnl-000100-.htm

This is a badminton Mixed Doubles Gold Medal Match where the scores and competitors are:

```
data.frame(
  "team" = c("A", "B"),
  "origin" = c("CHN", "CHN"),
  "player_name" = c("ZHENG Si Wei, HUANG Ya Qiong", "WANG Yi Lyu, HUANG Dong Ping"),
  "match" = c(1,2),
  "game_1" = c(17,21),
  "game_2" = c(21,17),
  "game_3" = c(19,21)
)
```

```
##   team origin                  player_name match game_1 game_2 game_3
## 1    A    CHN ZHENG Si Wei, HUANG Ya Qiong     1     17     21     19
## 2    B    CHN WANG Yi Lyu, HUANG Dong Ping     2     21     17     21
```

Team A's total score is $17 + 21 + 19 = 57$ Team B's total score is $21 + 17 + 21 = 59$ A total of 116 points were played.

```
a_score <- 57
b_score <- 59
total_score <- 116
a_p <- 57/116
a_b <- 59/116
```

Binomial Distribution

Binomial distribution models the number of observed positive outcome given the probability and number of trials. Simulating 10 samples of 116 trials using `0.5` as the probability. These are the score we expect to see if the two teams which are equally good played 10 matches against each other.

```
rbinom(10,116,0.5)
```

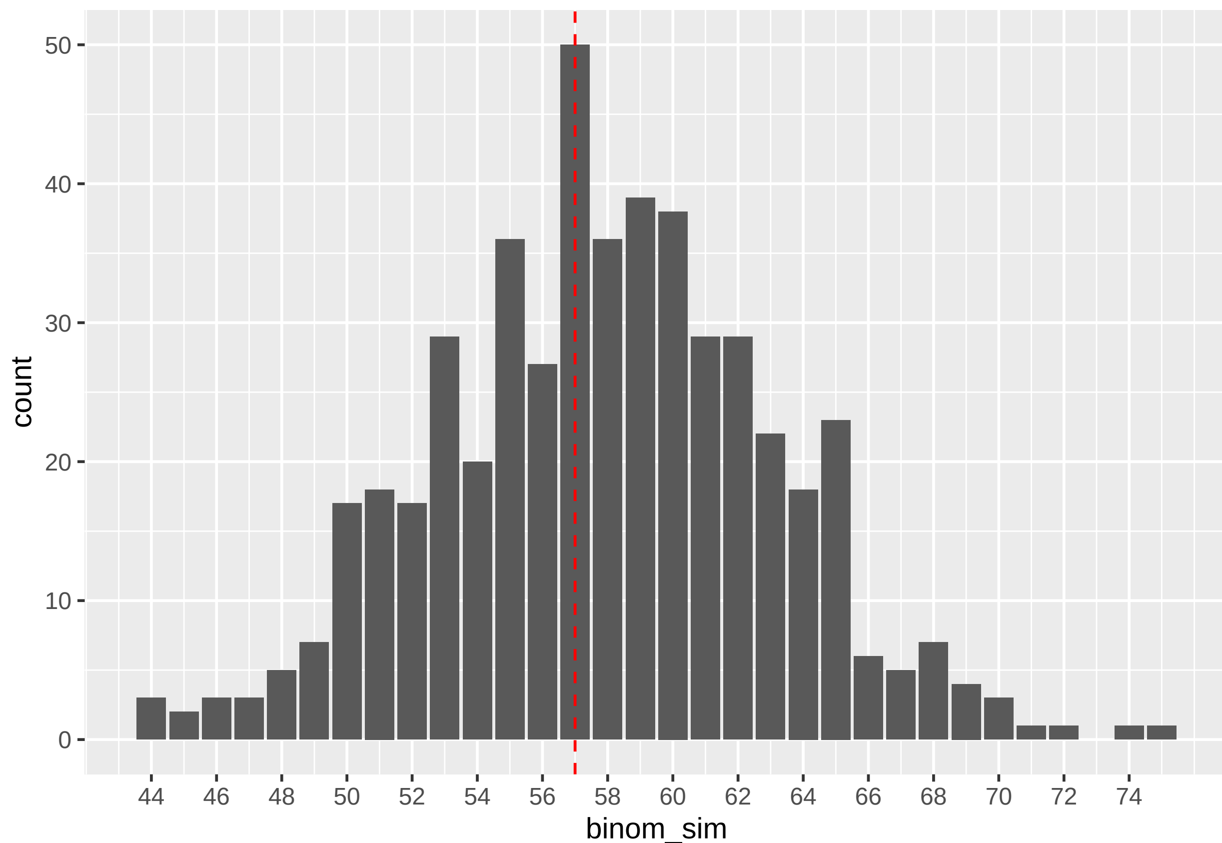
```
##  [1] 57 62 57 58 59 62 54 49 50 64
```

Simulation

Simulating 500 matches. The red dotted line denotes the frequency that we observe team A scoring 57 points. We can see that the observed 57 points is quite close to the center of the distribution.

```
binom_sim <- rbinom(500,116,0.5)
max_binom_sim <- max(binom_sim)
min_binom_sim <- min(binom_sim)
plot <- ggplot() +
  aes(binom_sim) +
```

```
  geom_bar() +
  scale_x_continuous(breaks = seq(min_binom_sim,max_binom_sim,2)) +
  geom_vline(xintercept = 57, linetype="dashed", color = "red")
plot
```



CDF

Next, we calculate the probability of observing team A getting 57 points or less.

$$P(X \leq 57)$$

which is

$$\sum_{i=0}^{57} \binom{116}{i} p^i (1-p)^{116-i})$$

which is

```
pbinom(57, 116, 0.5)
```

```
## [1] 0.4630389
```

This means that we have ~0.46 probability of observing such a score (57 to 59 over 3 sets) if the two teams are equally good.

Visualizing The Results

```r
binom.test(57, 116, 0.5, alternative = "less")
```

Verifying with R's built in binomial test

```
##
##  Exact binomial test
##
## data:  57 and 116
## number of successes = 57, number of trials = 116, p-value = 0.463
## alternative hypothesis: true probability of success is less than 0.5
## 95 percent confidence interval:
##  0.0000000 0.5715209
## sample estimates:
## probability of success
##              0.4913793
```

Another Game

The sample match used here is: https://olympics.com/tokyo-2020/olympic-games/en/results/badminton/results-men-s-doubles-qfnl-000100-.htm

This is a badminton Men's Doubles Quarterfinal Match where the scores and competitors are:

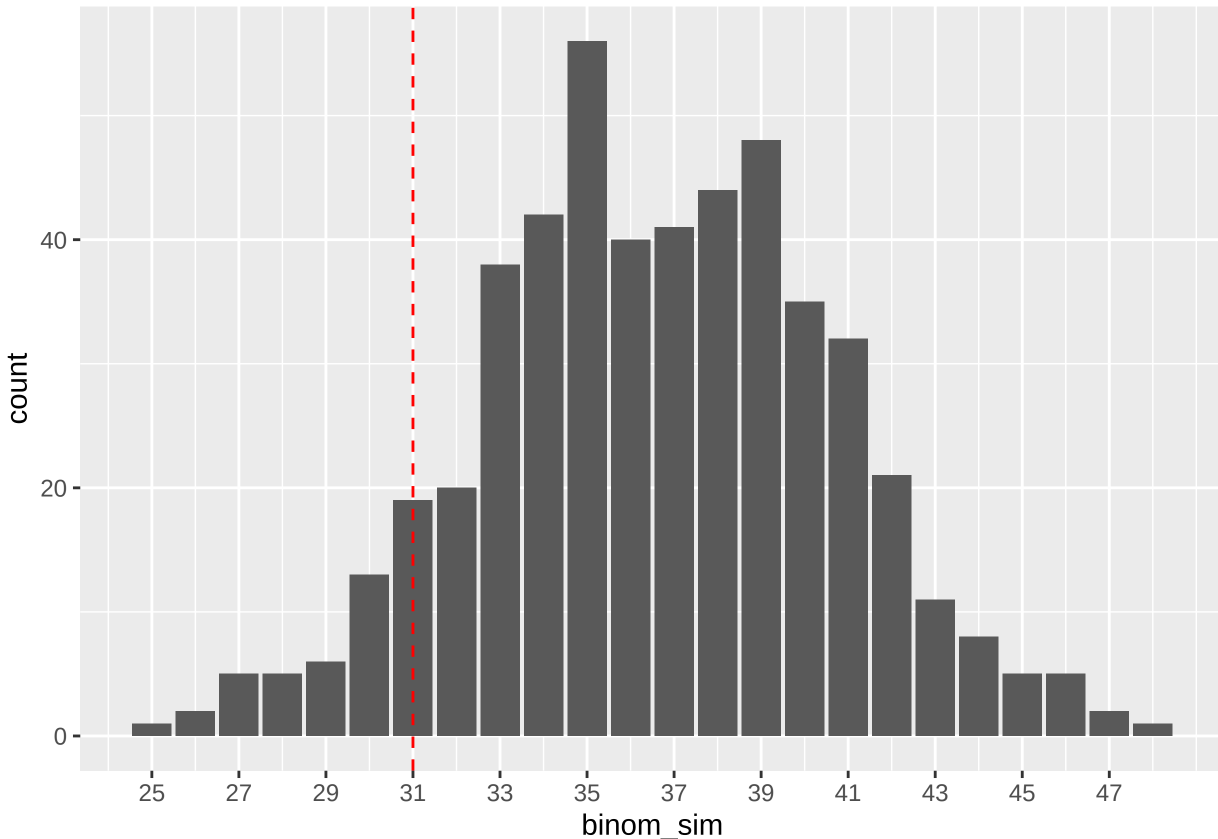
```r
data.frame(
  "team" = c("A", "B"),
  "origin" = c("INA", "MAS"),
  "player_name" = c("GIDEON Marcus Fernaldi, SUKAMULJO Kevin Sanjaya", "CHIA Aaron, SOH Wooi Yik"),
  "match" = c(0,2),
  "game_1" = c(14,21),
  "game_2" = c(17,21)
)
```

```
##   team origin                                     player_name match game_1
## 1    A    INA GIDEON Marcus Fernaldi, SUKAMULJO Kevin Sanjaya     0     14
## 2    B    MAS                        CHIA Aaron, SOH Wooi Yik     2     21
##   game_2
## 1     17
## 2     21
```

Simulation

Simulating 500 matches. The red dotted line denotes the frequency that we observe team A scoring 14 + 17 = 31 points out of 73 points played. We can see that the observed 31 points is further to the left than the previous example.

```r
binom_sim <- rbinom(500,73,0.5)
max_binom_sim <- max(binom_sim)
min_binom_sim <- min(binom_sim)
plot <- ggplot() +
  aes(binom_sim) +
  geom_bar() +
  scale_x_continuous(breaks = seq(min_binom_sim,max_binom_sim,2)) +
  geom_vline(xintercept = 31, linetype="dashed", color = "red")
plot
```



CDF

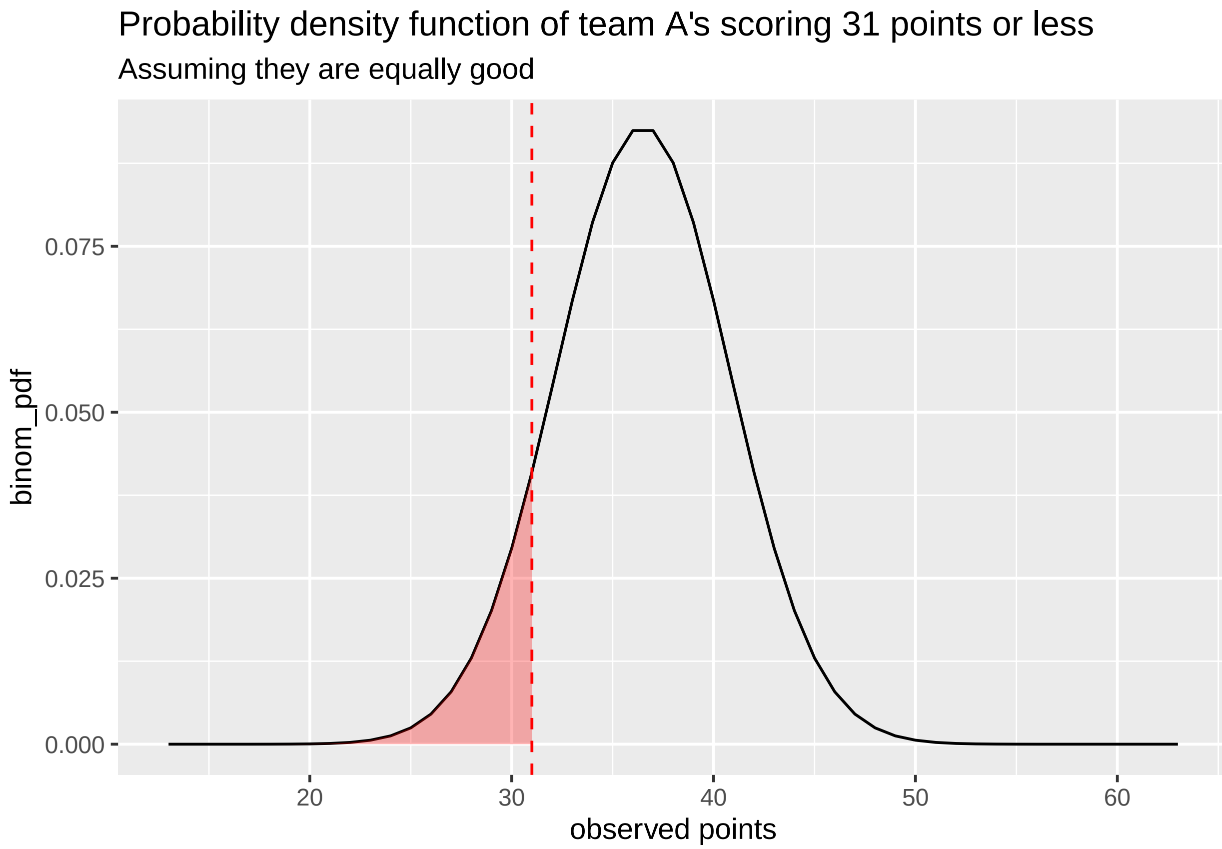
Now we look at the probability of observing team A getting 31 points or less if they played 73 points and the two teams are equally good.

```r
pbinom(31, 73, 0.5)
```

```
## [1] 0.1208224
```

This is less than our threshold of 20% and we can confidently say that team B is better than team A in this match.

Plotting the PDF



How Many Points Should They Play?

In this section we discuss a follow up question of coming up with the appropriate amount of points to be played within a game to be effective at discerning which team is better.