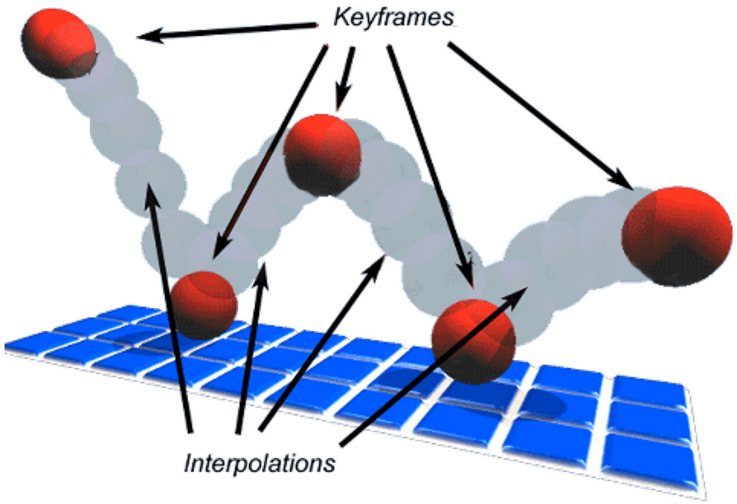
CS30800
Introduction to Computer Graphics
Lab 8 – Catmull-Rom spline interpolation

2025. 05.20 / 2025. 05. 22

Recap: animation

- Previously…

Key Frame in Animation

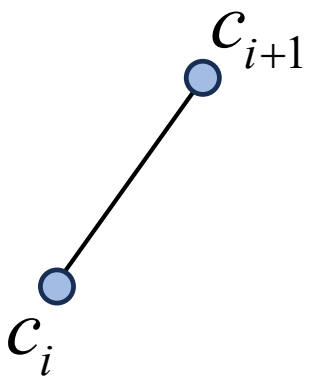


Tasks

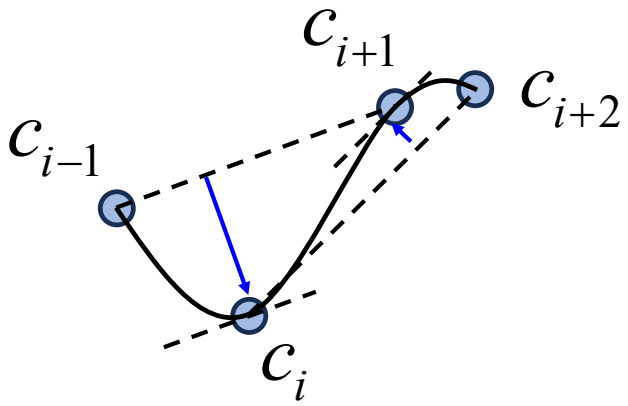
- In homework 5, you made an animation based on linear interpolation
- In homework 7, you will implement Catmull-Rom spline on this task

Catmull-Rom Spline

- We need extra points to construct Catmull_Rom Spline



Linear interpolation



Catmull-Rom Spline

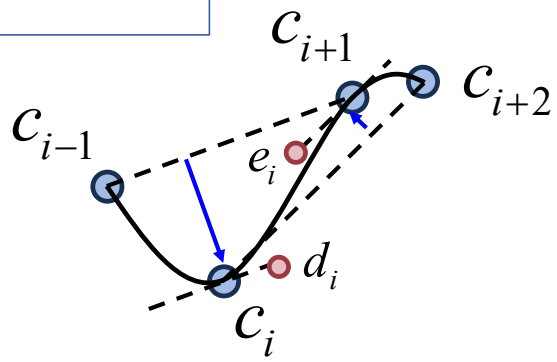
CRS Construction

- d_i, e_i can be calculated from $c_{i-1}, c_i, c_{i+1}, c_{i+2}$
- Based on Catmull-Rom spline constraint and property of cubic Bezier curve,

$$c_i' = \frac{1}{2}(c_{i+1} - c_{i-1})$$
$$c_{i+1}' = \frac{1}{2}(c_{i+2} - c_i)$$

$$c_i' = 3(d_i - c_i)$$
$$c_{i+1}' = 3(c_{i+1} - e_i)$$

$$d_i = \frac{1}{6}(c_{i+1} - c_{i-1}) + c_i$$
$$e_i = -\frac{1}{6}(c_{i+2} - c_i) + c_{i+1}$$



Catmull-Rom Spline

CRS Construction

- Final cubic Bezier curve

$$d_i = \frac{1}{6}\left(c_{i+1} - c_{i-1}\right) + c_i \qquad e_i = -\frac{1}{6}\left(c_{i+2} - c_i\right) + c_{i+1}$$

$$c(t) = c_i\left(1-t\right)^3 + 3d_i t\left(1-t\right)^2 + 3e_i t^2\left(1-t\right) + c_{i+1}t^3$$

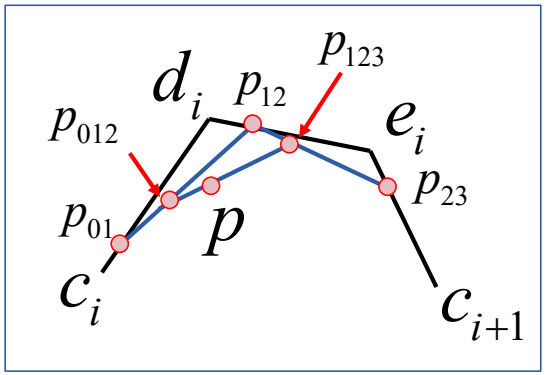
Cubic Bezier curve

Quaternion splining

- Scalar addition → quaternion multiplication
- Scalar negation → quaternion inversion
- Scalar multiplication → quaternion power

$$d_i = \frac{1}{6}(c_{i+1} - c_{i-1}) + c_i \quad \rightarrow \quad d_i = \left(\left(c_{i+1} c_{i-i}^{-1} \right)^{1/6} \right) c_i$$

$$e_i = -\frac{1}{6}(c_{i+2} - c_i) + c_{i+1} \quad \rightarrow \quad e_i = \left(\left(c_{i+2} c_i^{-1} \right)^{-1/6} \right) c_{i+1}$$



$$p_{01} = slerp(c_i, d_i, t) \qquad p_{012} = slerp(p_{01}, p_{12}, t)$$

$$p_{12} = slerp(d_i, e_i, t) \qquad p_{123} = slerp(p_{12}, p_{23}, t)$$

$$p_{23} = slerp(e_i, c_{i+1}, t) \qquad p = slerp(p_{012}, p_{123}, t)$$

Question?