BME 355: Hill-Type Muscle Models

Millard, Uchida, Seth & Delp (2013) Flexing Computational Muscle: Modeling and Simulation of Musculotendon Dynamics, *Journal of Biomechanical Engineering 135*

Agenda

- Hill muscle model and extensions needed for musculoskeletal simulations
- A recent example of a Hill-type muscle model
- The model within simple systems
  - Isometric contraction
  - Suspended mass

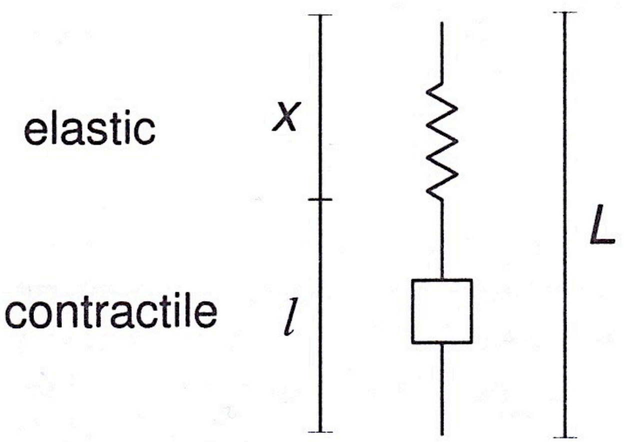
- Next class we will integrate the Hill-type model with a link-segment model

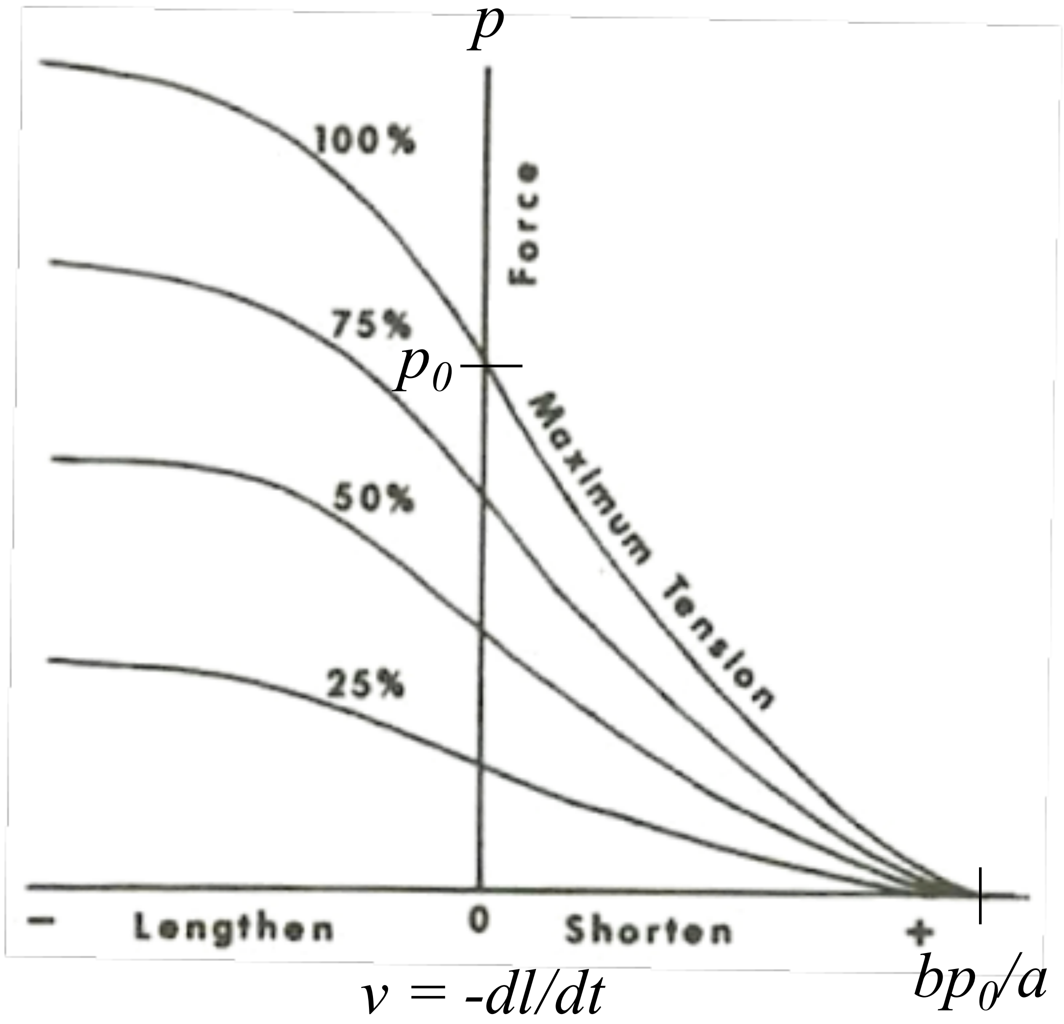
The Basic Hill Model

$$(p+a)v = b(p_0 - p)$$ Force-velocity function of contractile element

$$\frac{dp}{dx} = \alpha$$ Linear series elastic component

$$\frac{dp}{dt} = \alpha\left[\frac{dL}{dt} + \frac{b(p_0 - p)}{p+a}\right]$$





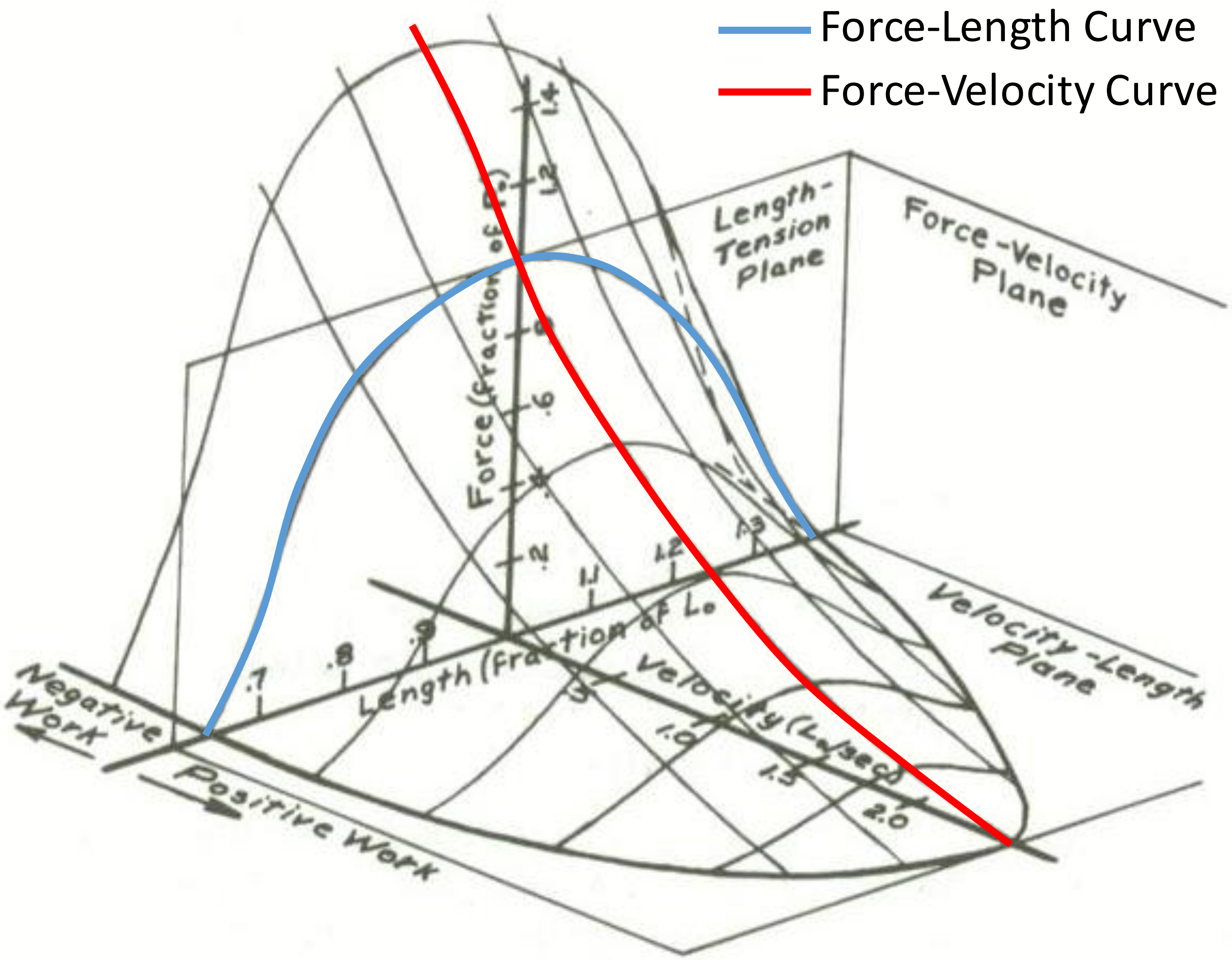
Keener & Sneyd (2001) Mathematical Physiology. Fig. 18.8 and Winter (1990) Fig. 7.12

Some Limitations of the Hill Model

- Only applies if muscle is shortening
- Assumes constant force-length curve
- Ignores excitation-activation dynamics
- Ignores nonlinearities in series elastic component
- Ignores parallel elastic component
- Ignores pennation

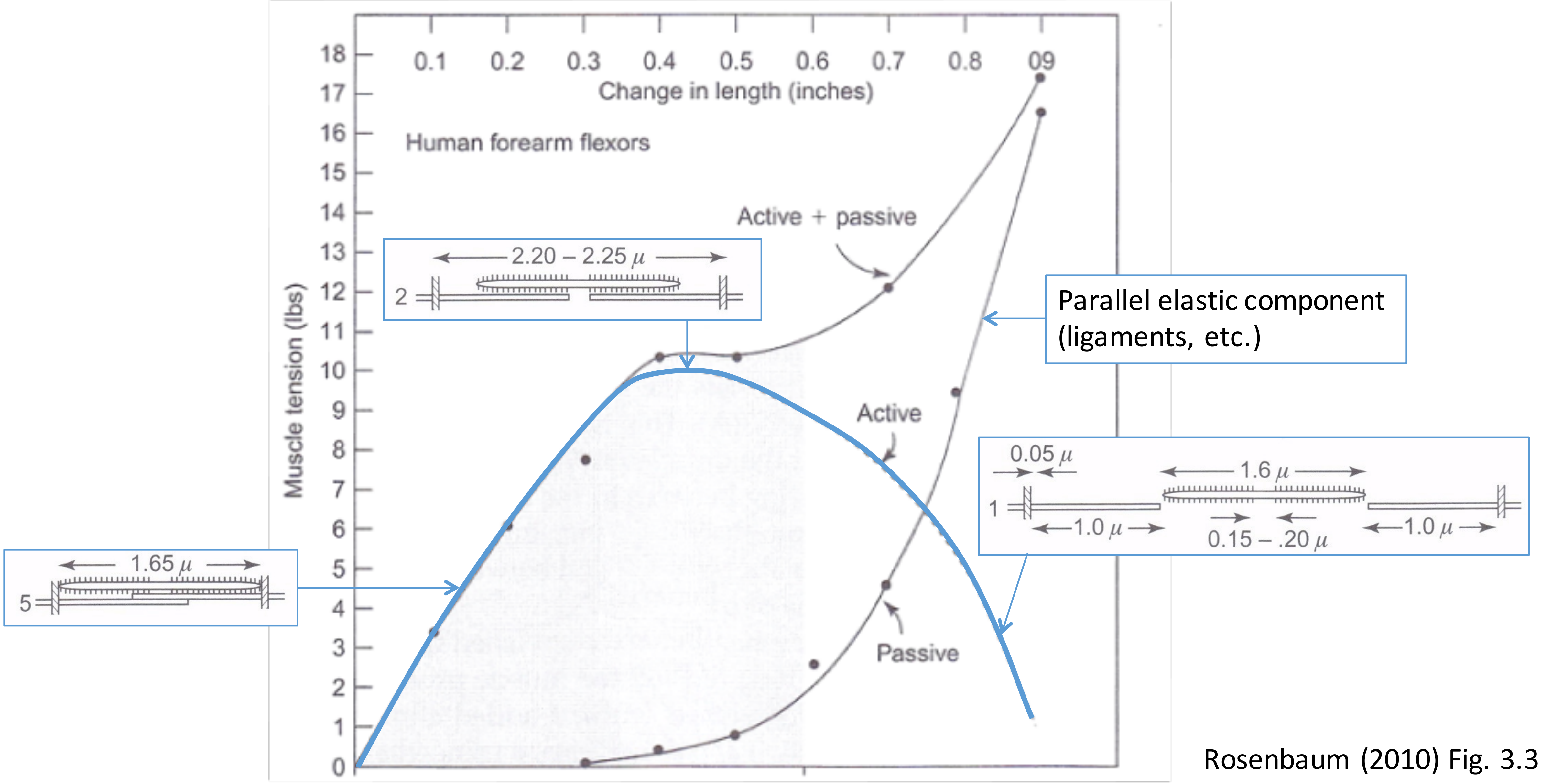
Force-Length and Force-Velocity Curves

- Force-length curve is due to degree of overlap of cross-bridges with binding sites
- Force-velocity curve is due to dynamics of binding and unbinding
  - E.g. at maximum shortening velocity, cross-bridges bend so far before they unbind that they push as much as they pull, so net force is zero



Winter (1990) Biomechanics and Motor Control of Human Movement. Fig. 7.13

Force-Length Curve



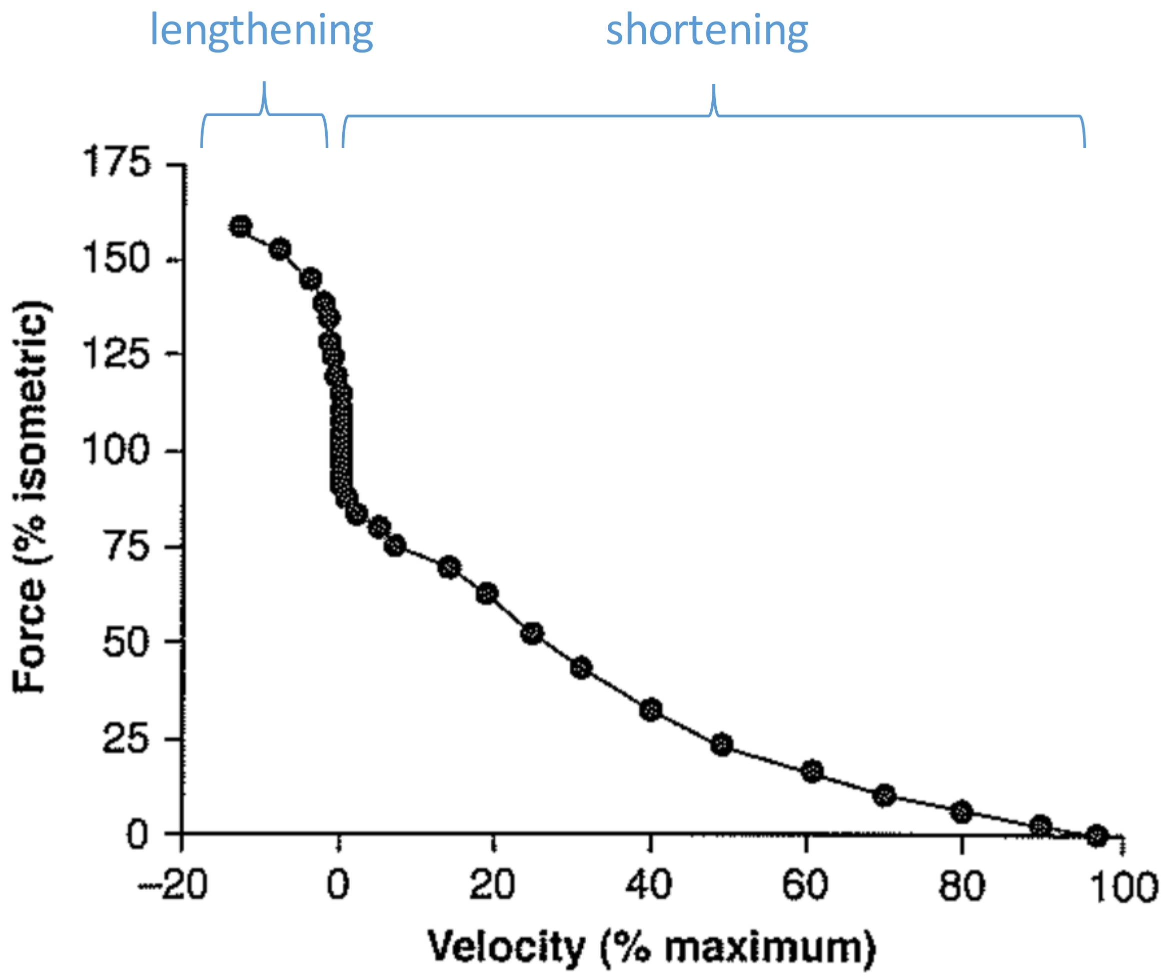
Rosenbaum (2010) Fig. 3.3

Force-Velocity Curve

This is from frog muscle.

The curve roughly resembles the Hill function for shortening velocities.

When lengthening, the maximum force typically increases to between about 1.3x to 1.8x the maximum isometric force and saturates.

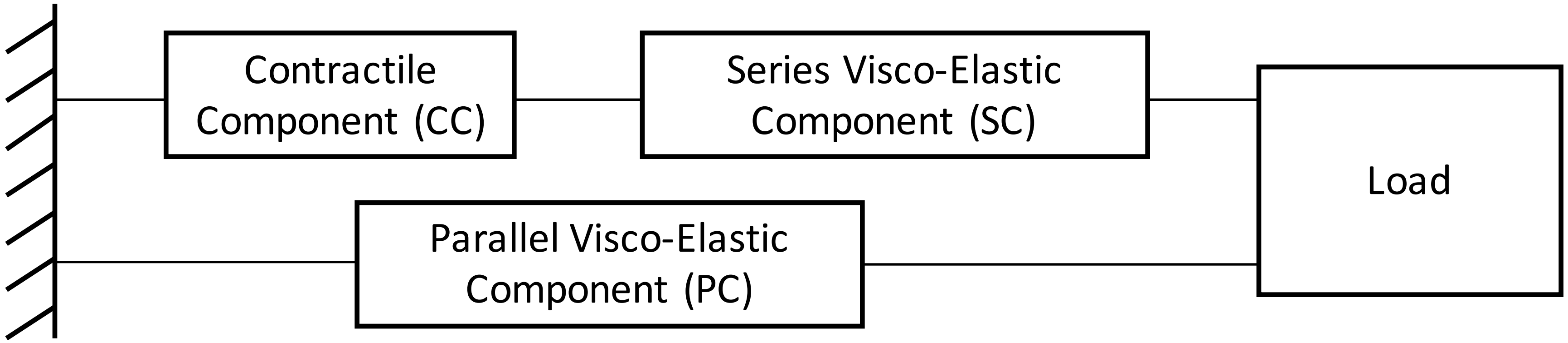


Enoka (2008) *Neuromechanics of Human Movement*

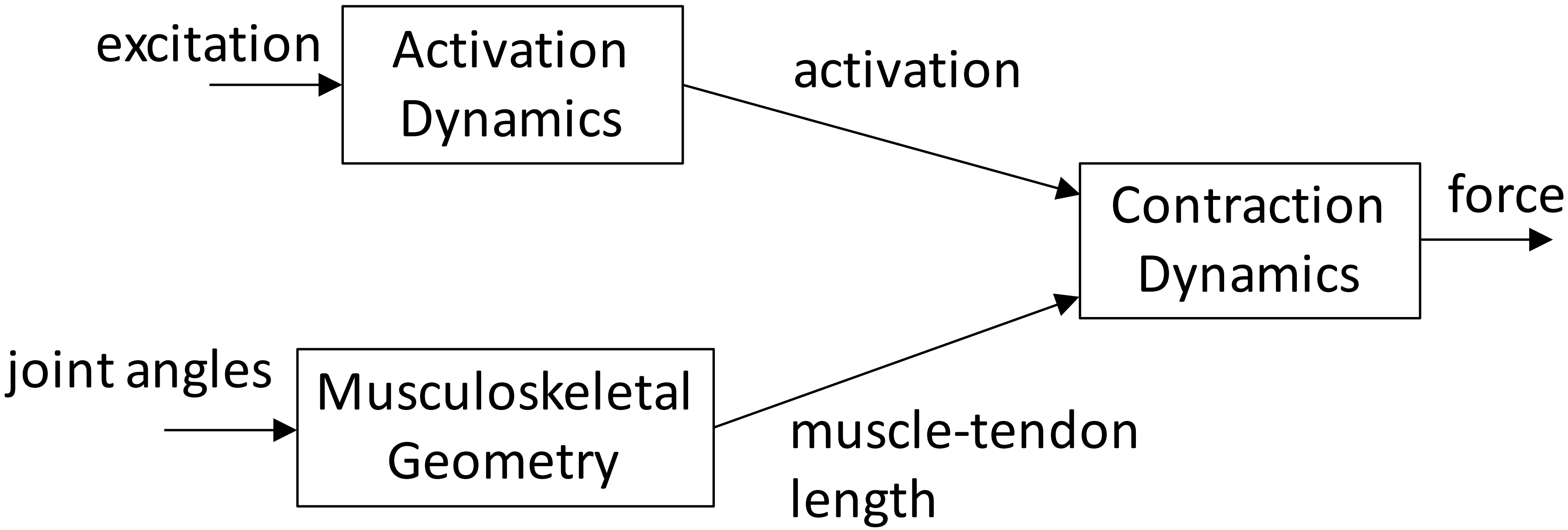
Hill-Type Models of Muscle Dynamics

"Hill-Type" models are widely used for musculoskeletal simulations. They have the structure below, or a similar structure, and they use empirical curves to approximate the force-velocity relationship and other relationships, usually over the full range of possible lengths and velocities.

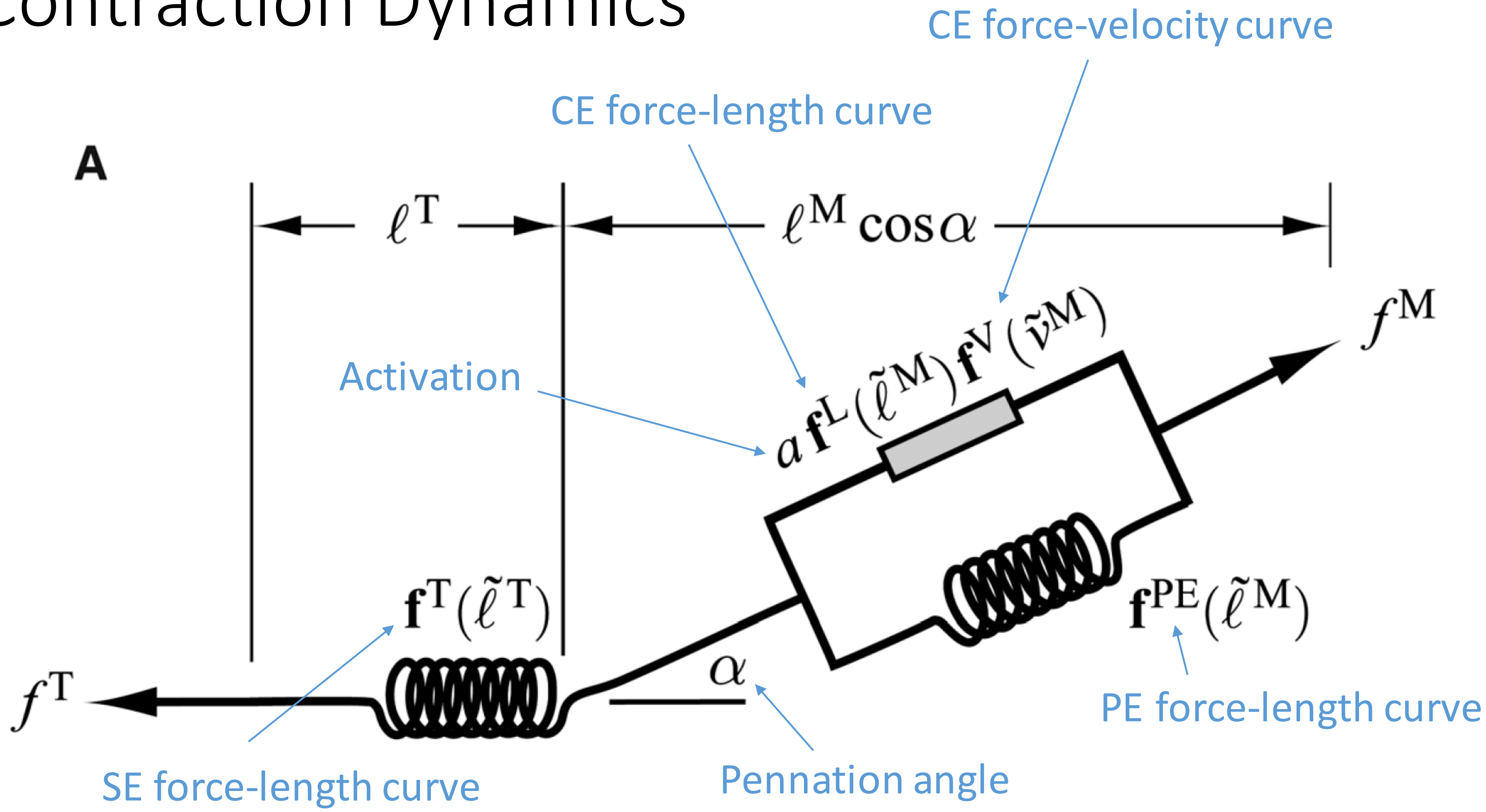
We will focus this week on Hill-type models. Next week we will see the Huxley model, which considers cross-bridge dynamics directly.



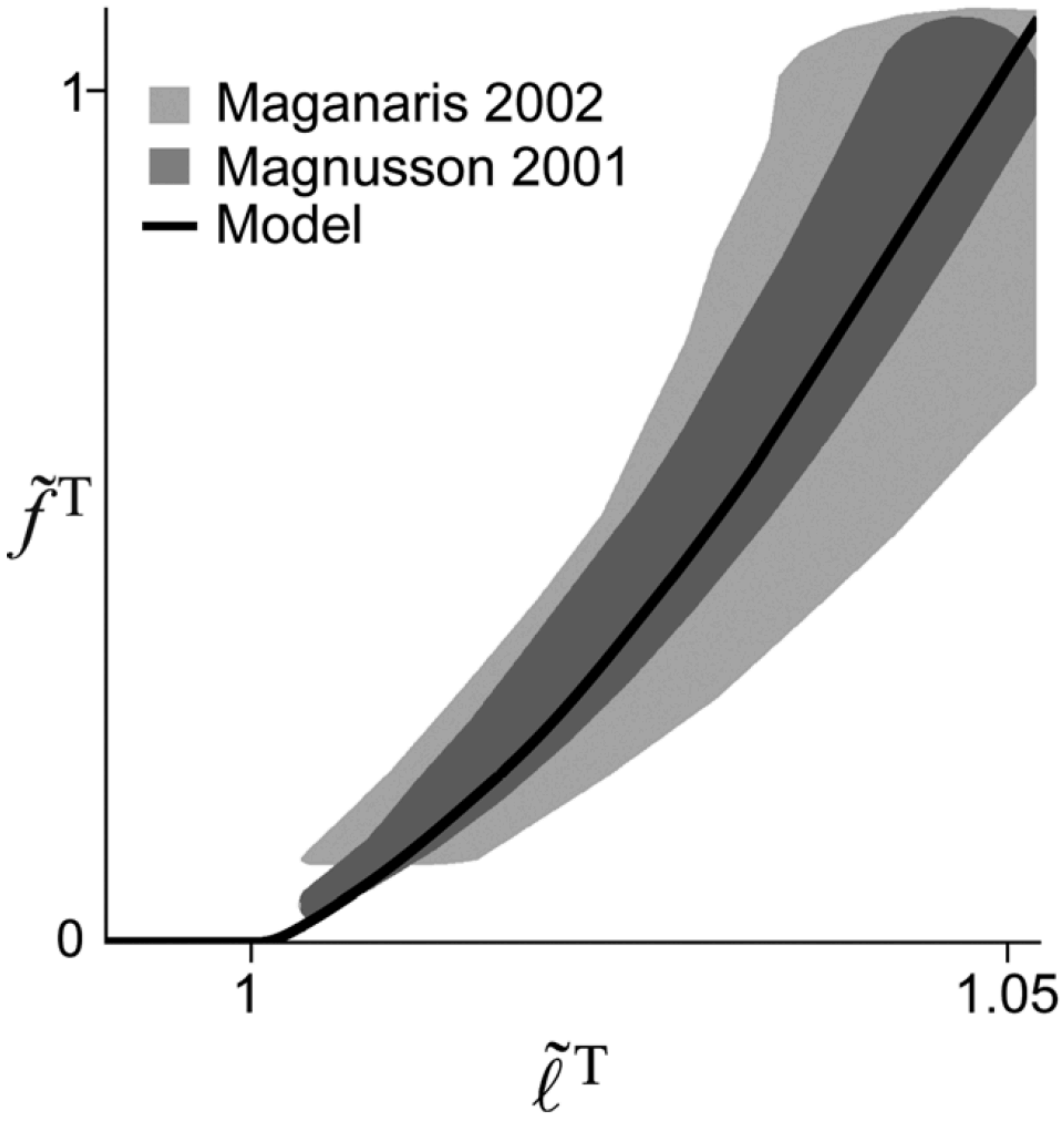
Hill-Type Model Example (Millard et al., 2013)


excitation
Activation Dynamics
activation
Contraction Dynamics
force
joint angles
Musculoskeletal Geometry
muscle-tendon length

Contraction Dynamics

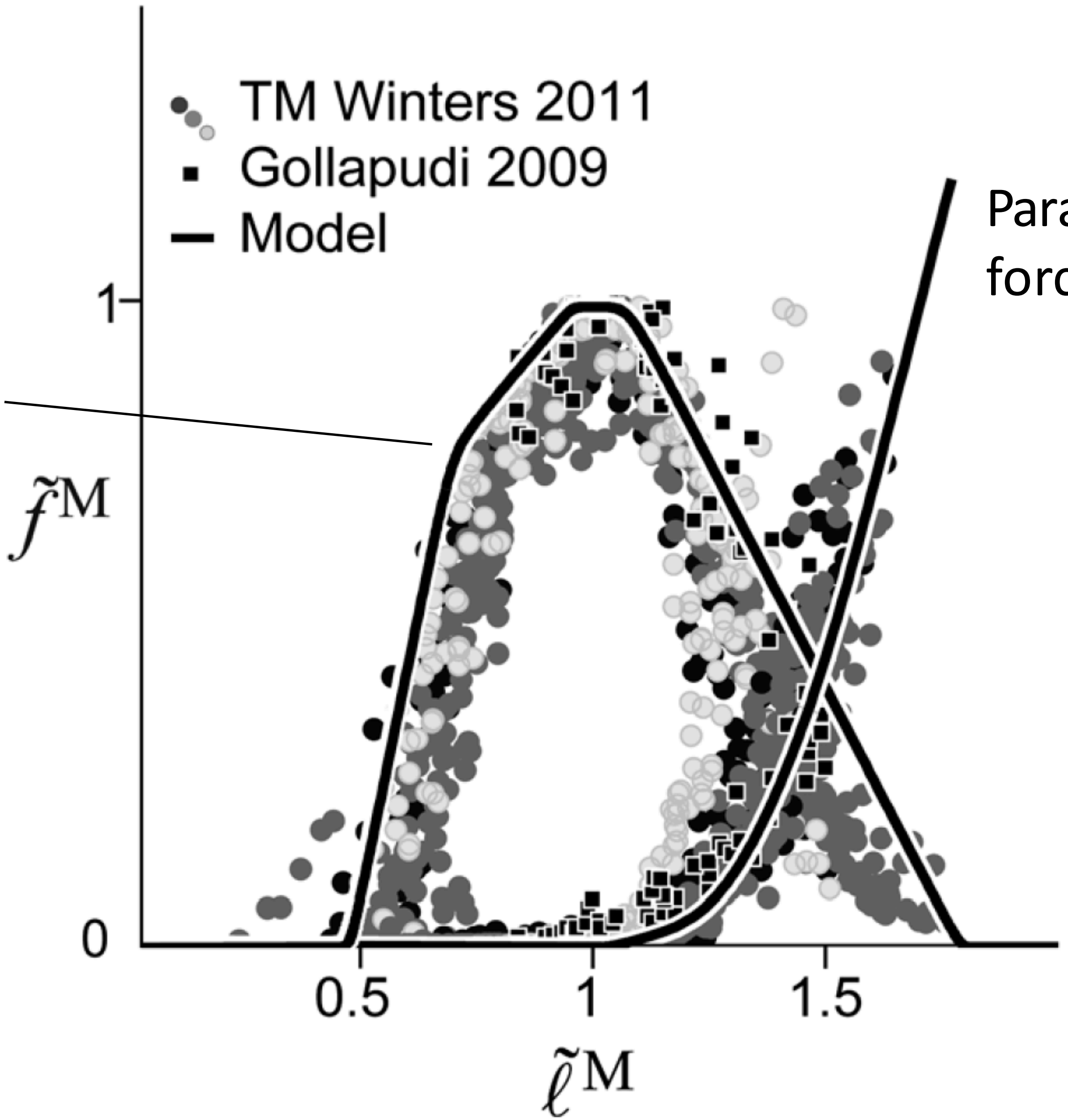


The model is defined in terms of normalized lengths, forces, and velocities. It is rescaled as needed to approximate different muscles.



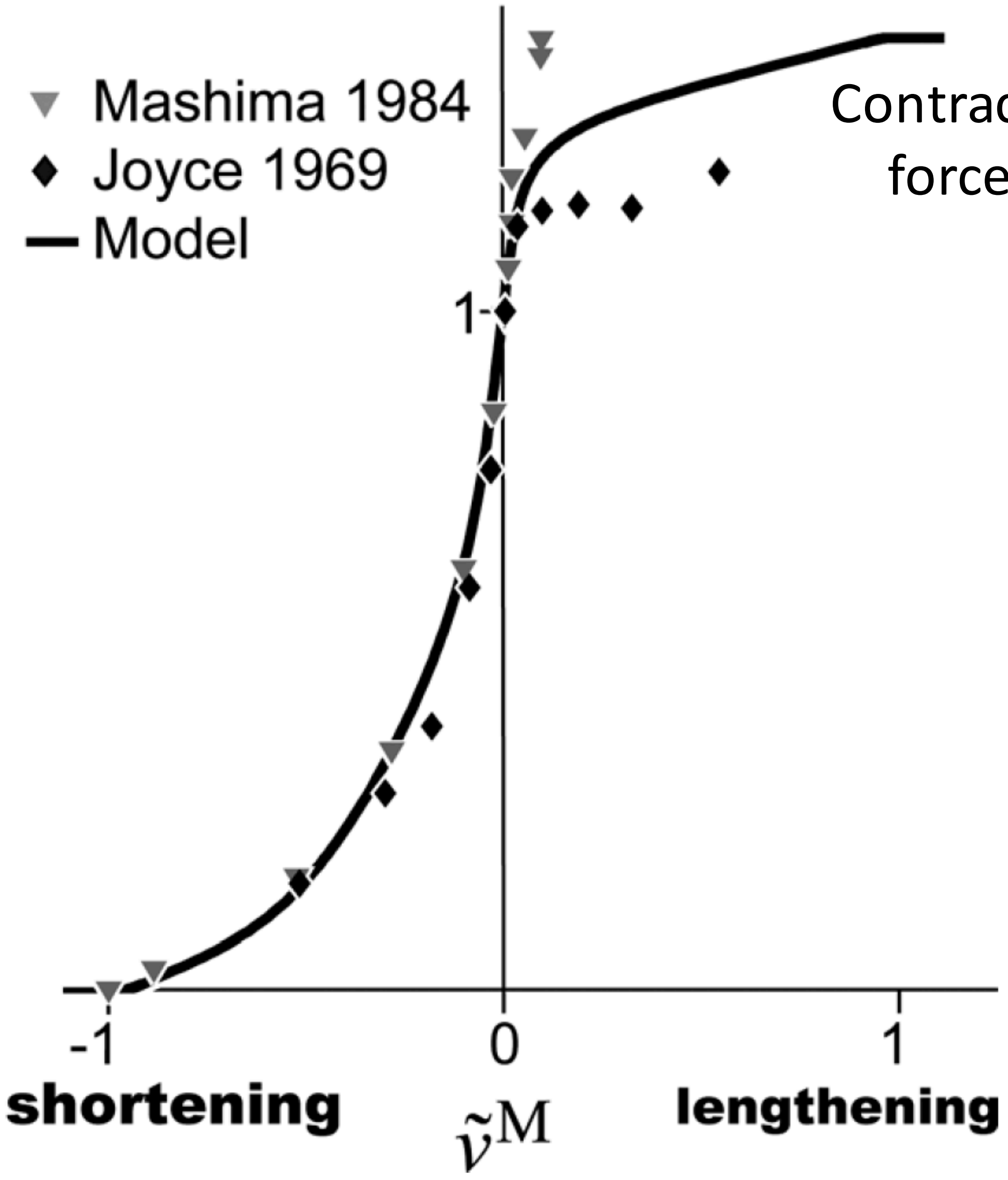
Series elastic (SE) force-length curve

Contractile element (CE) force-length curve



Parallel elastic (PE) force-length curve

Contractile element (CE) force-velocity curve



Approximating the Model Curves

- There are several options for approximating these curves:
  - Interpolation (e.g. splines)
  - Regression (similar to interpolation but approximation doesn’t necessarily pass through data points)
  - Curve fitting (a specified function with parameters fit to data)

Simulating the Model

The mass of the muscle is ignored, so the tension in the SE equals the sum of tension in the CE and PE,

$$f_o^M(af^L(l^M)f^V(v^M) + f^{PE}(l^M))\cos\alpha - f_o^M f^T(l^T) = 0$$

where l^M and l^T are the lengths of the CE and SE, f^L, f^{PE}, and f^T are the force-length functions of the CE, PE, and SE, f^V is the force-velocity function of the CE, a is the activation, α is the pennation angle, and f_o^M is the maximum isometric force. The CE length l^M is a state variable. It is simulated by integrating the CE velocity. The CE velocity is,

$$v^M = f_{inv}^V\left(\frac{\frac{f^T(l^T)}{\cos\alpha} - f^{PE}(l^M)}{af^L(l^M)}\right)$$

Simulating the Model

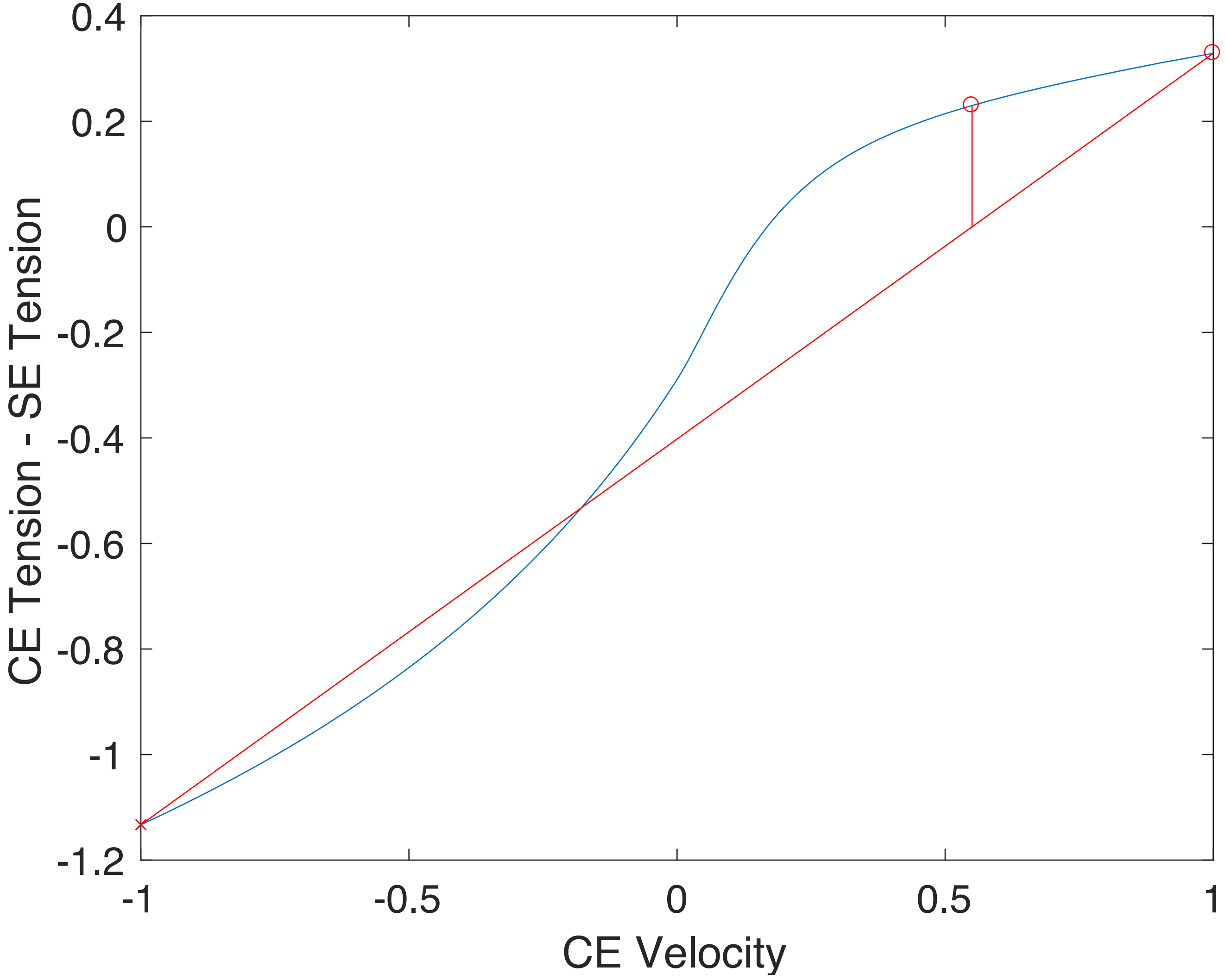
Simulation speed is improved by adding a damping term. This increases the run time of each step, but allows longer average time steps.

In the damped version there is no explicit expression for the velocity v^M, but the velocity is implicitly defined by,

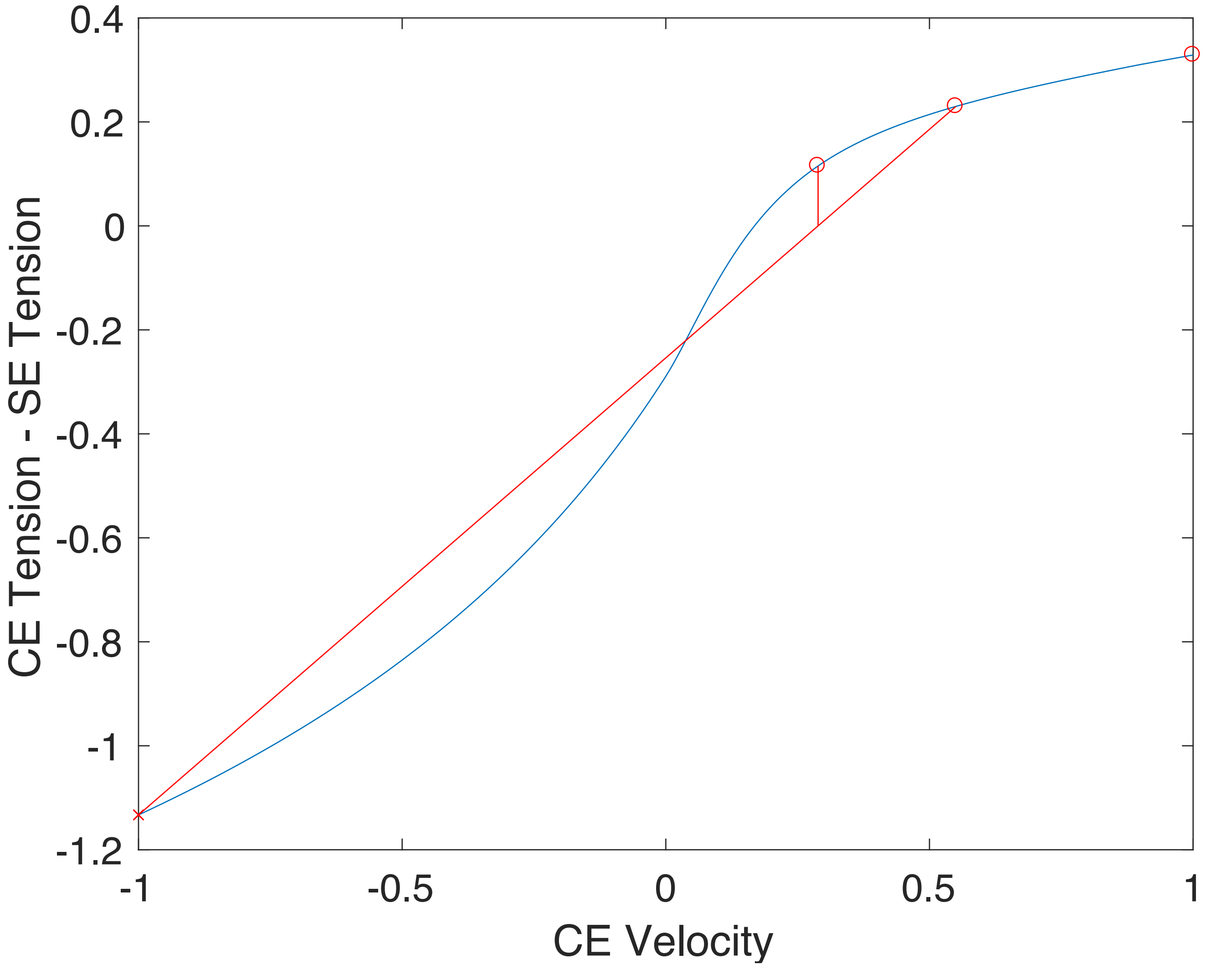
$$f_o^M(af^L(l^M)f^V(v^M) + f^{PE}(l^M) + \beta v^M)\cos\alpha - f_o^M f^T(l^T) = 0$$

where β is the damping coefficient. The velocity is solved in each time step by numerical root finding.

Root Finding with Method of False Position



Root Finding with Method of False Position



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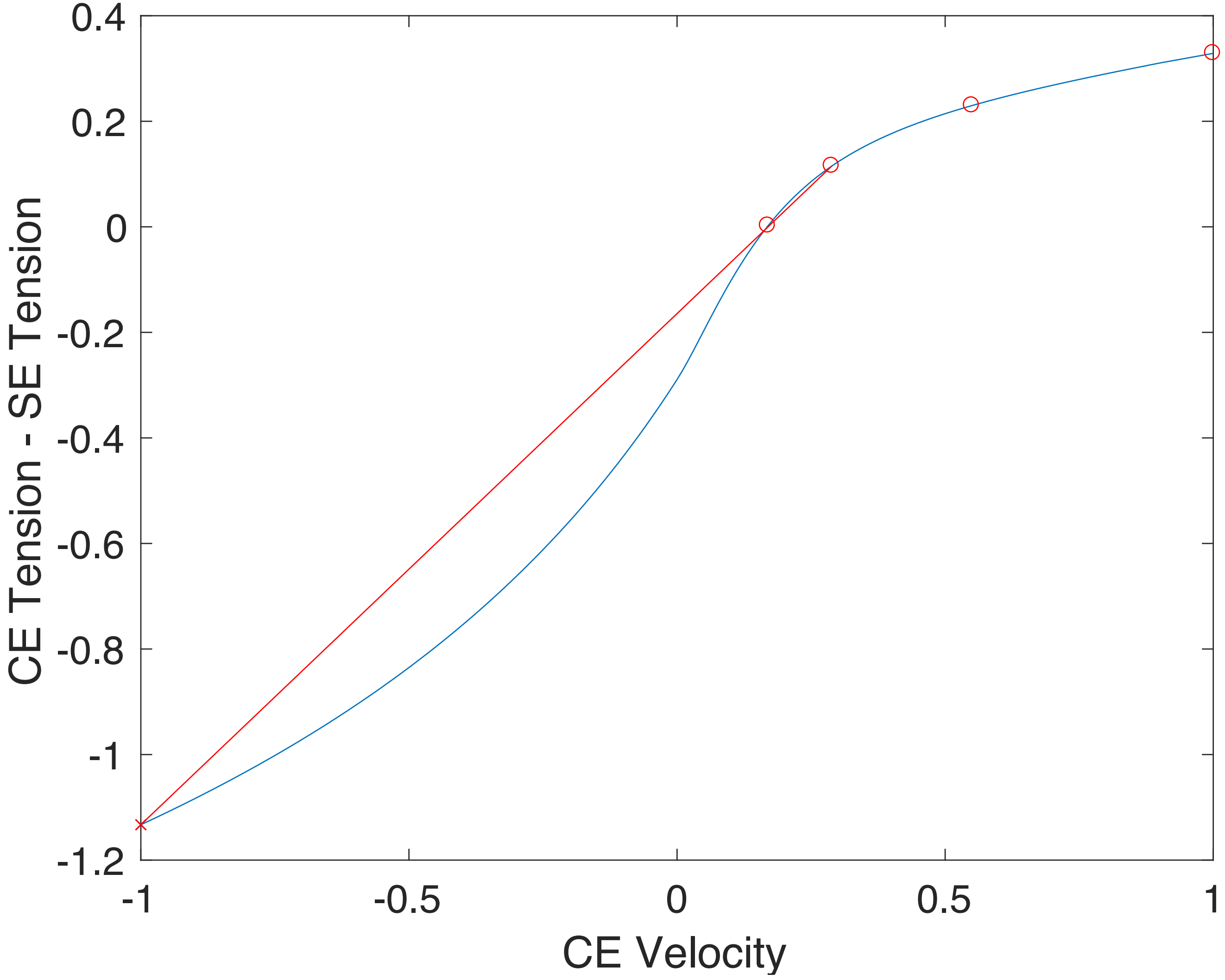
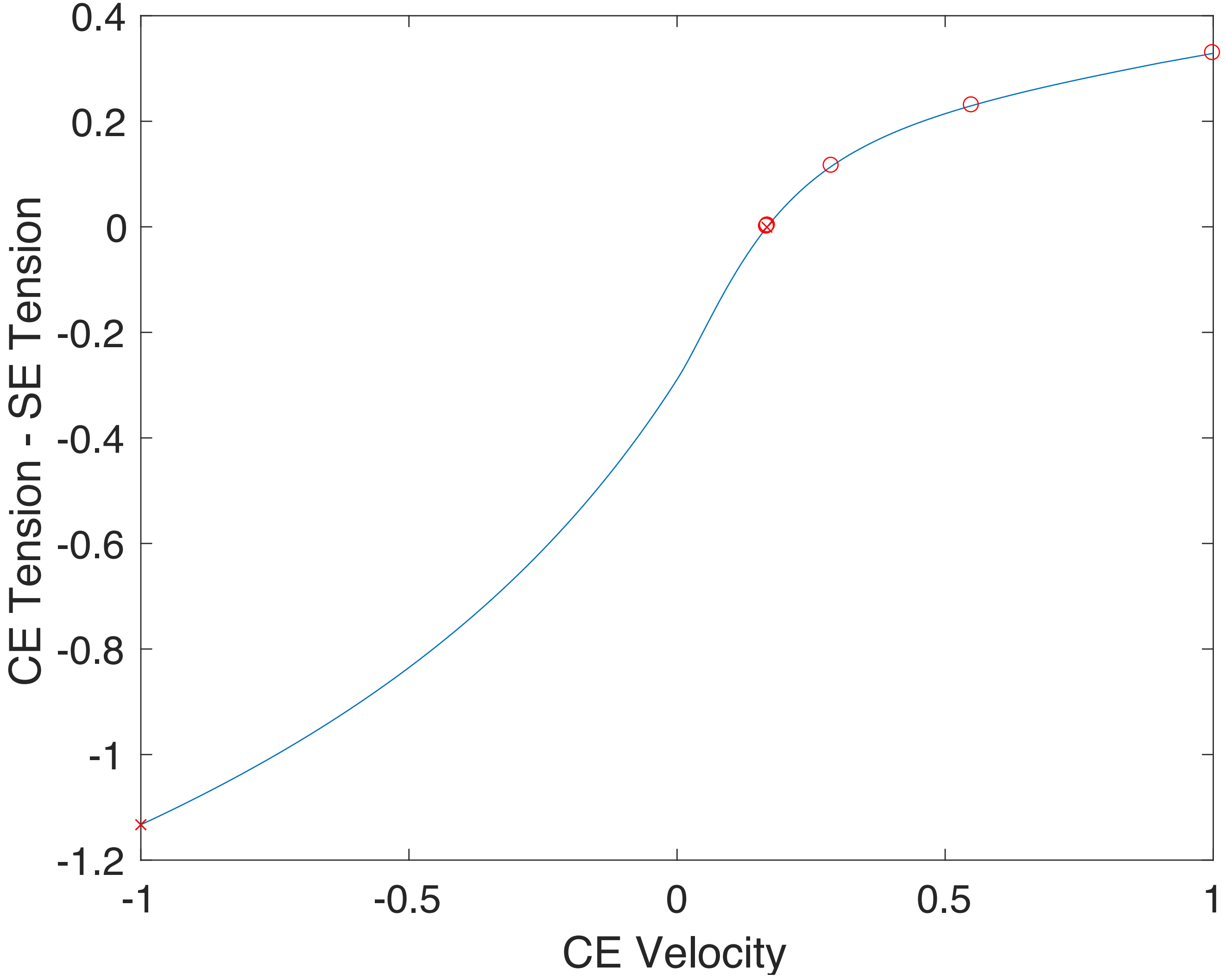


Root Finding with Method of False Position



Simple Systems

Isometric Contraction

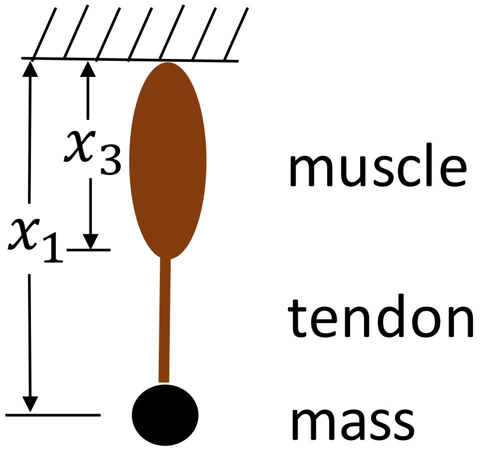
The muscle-tendon length $\mathrm{L} = l^M + l^T$ is constant. There is one state variable, $x = l^M$. Let $v^M(a, l^M, l^T)$ be the CE velocity as a function of activation a, CE length l^M, and SE length l^T. Then,

$$\dot{x} = v^M(a, x, L - x)$$

Suspending a Mass

There are three state variables. Let x_1 be the vertical mass position, x_2 its velocity, and $x_3 = l^M$ the CE length. Note that $l^T = x_1 - x_3$. Finally let $f(l^T)$ be the SE force-length curve. The state equations are,

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = g - f(l^T)/m$$
$$\dot{x}_3 = v^M(a, x_3, x_1 - x_3)$$



where m is the mass and g is the acceleration of gravity.