A geostatistical framework for combining spatially referenced prevalence data from multiple diagnostics

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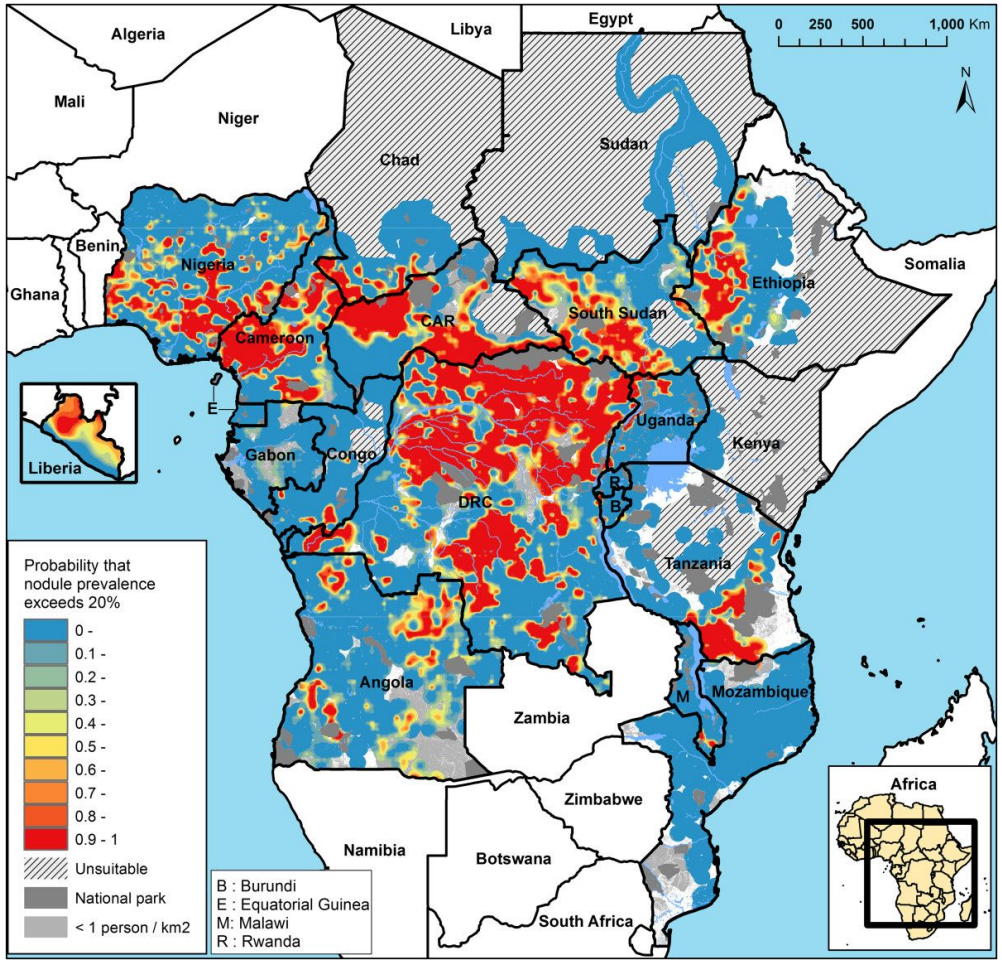
What is geostatistics?

- **Geostatistical problems:** making inference on a spatially continuous surface using noisy spatially discrete data.



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- Objective: to predict $p(x)$ for all $x \in A$

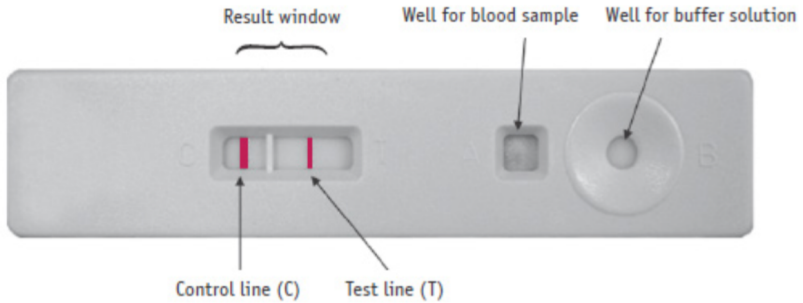
Combining multiple diagnostics

Case I: Predicting a gold-standard diagnostic using a more economic but biased alternative

Combining multiple diagnostics

Case II: Joint predictions of complementary diagnostics

Figure 1. Typical RDT for Malaria Diagnosis



Source: Good practices for selecting and procuring rapid diagnostic tests for malaria. Geneva: WHO; 2011.

A joint modelling framework

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- Crainiceanu, Diggle and Rowlingson (2008), or CDRM

$$\begin{cases} \text{logit}\{p_1(x_i)\} = d^\top(x_i)\beta + S(x_i) \\ \text{logit}\{p_2(x_i)\} = \alpha_0 + \alpha_1 \text{logit}\{p_1(x_i)\} + Z_i, \end{cases} \tag{2}$$

- Limitation: use of a single spatial process in (2) to capture spatial variation in both diagnostics.

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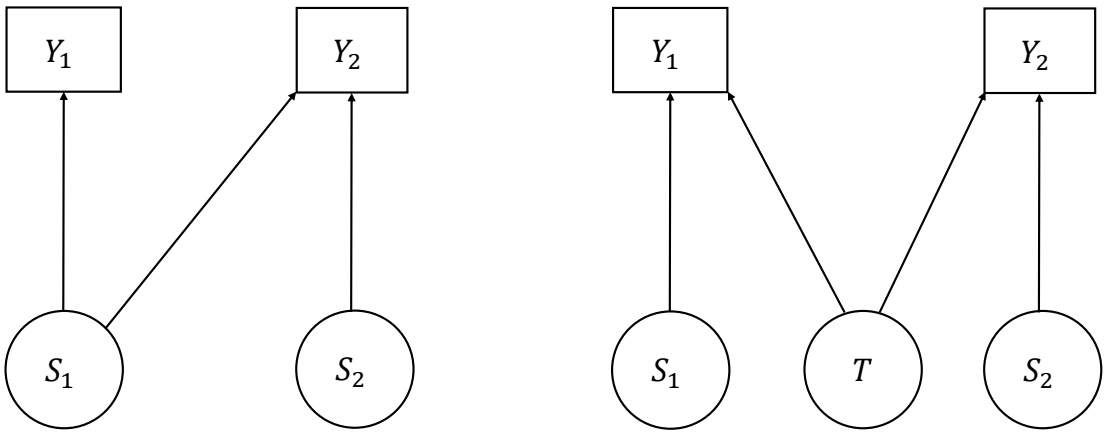

(a) (b)

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- We approximate (5) using MCMC to give

$$L(\theta) \approx L_B(\theta) = \frac{1}{B}\sum_{b=1}^{B} \frac{h(w_{(b)})h(y|w_{(b)})}{h_0(w_{(b)})h_0(y|w_{(b)})}, \tag{6}$$

Model validation

- The empirical variogram for W_k

$$\hat{\gamma}_k(u) = \frac{1}{2|N(u)|} \sum_{(i,j)\in N(u)} \left\{ \hat{W}_k(x_i) - \hat{W}_k(x_j) \right\}^2, \tag{7}$$

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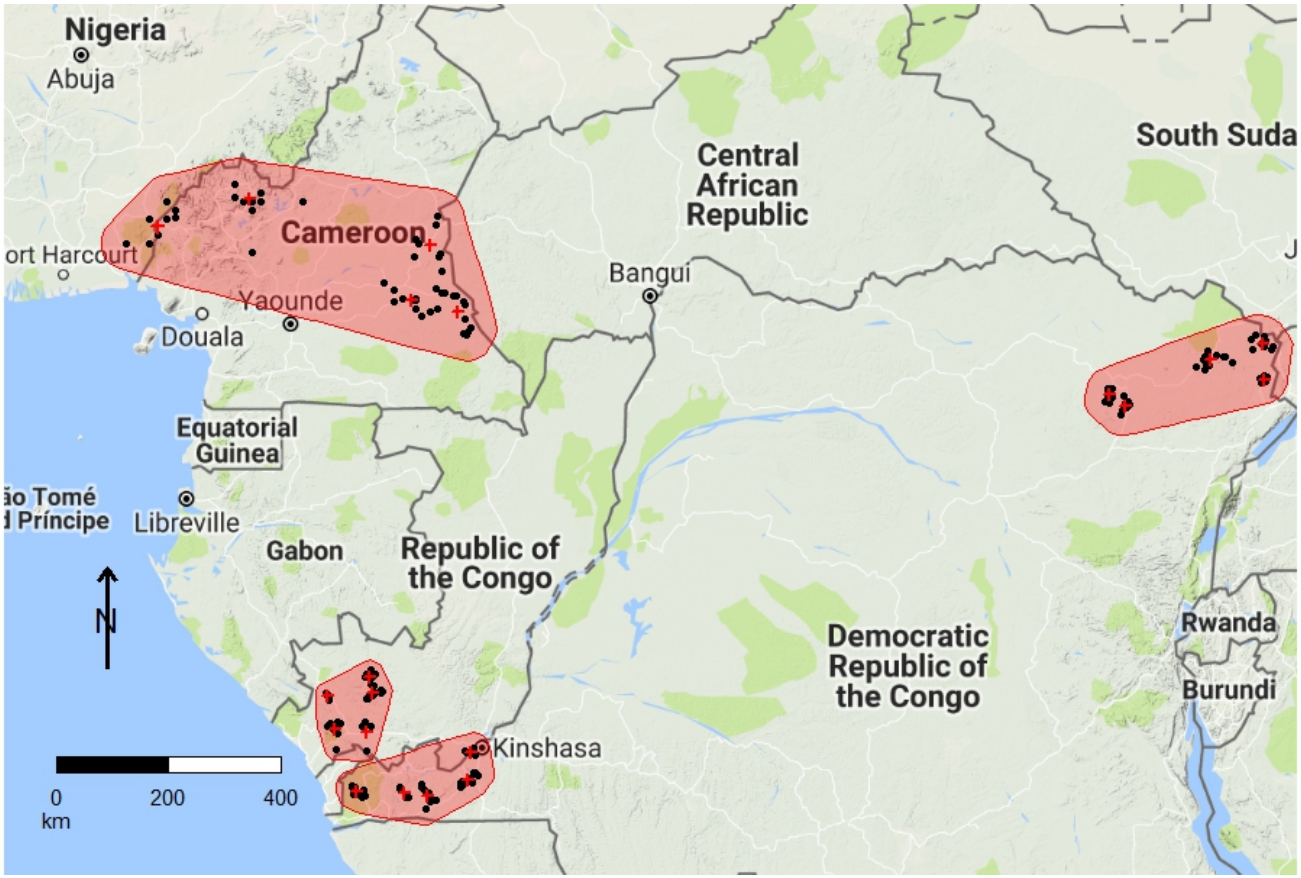
  5. Use the resulting M empirical variograms to generate 95% confidence intervals.

Application: re-analysis of the *Loa loa* data



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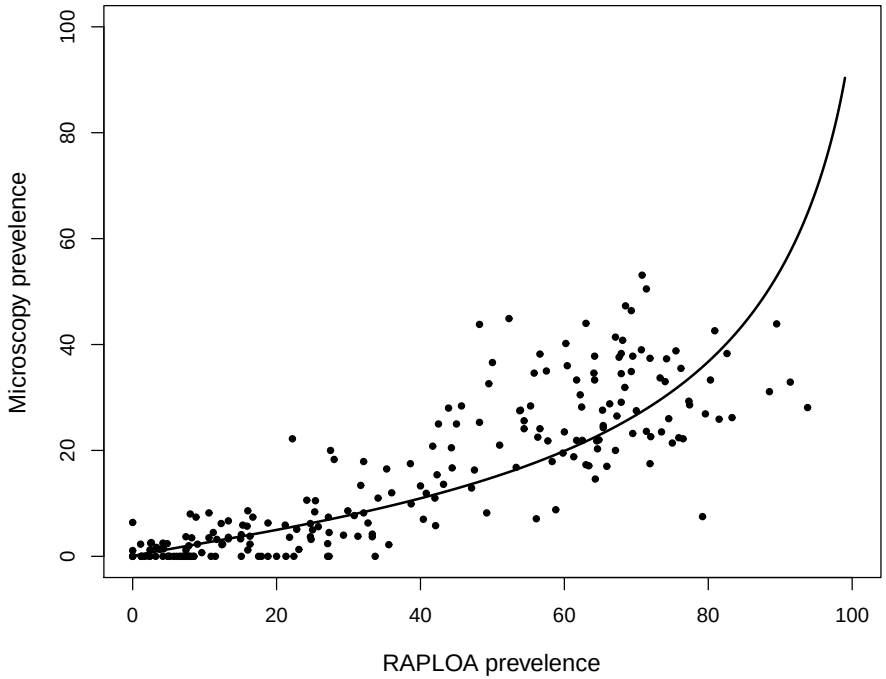
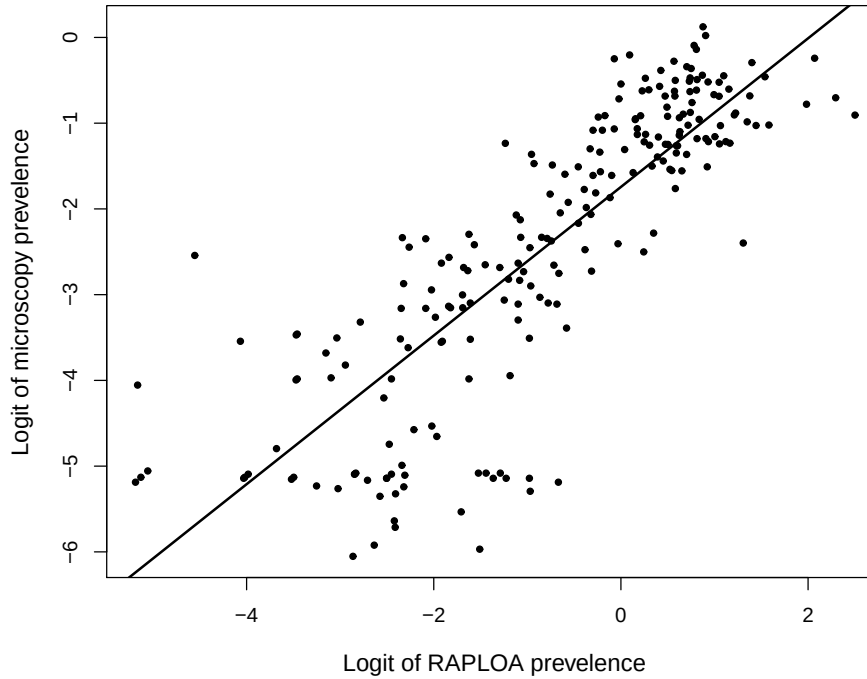


(a)



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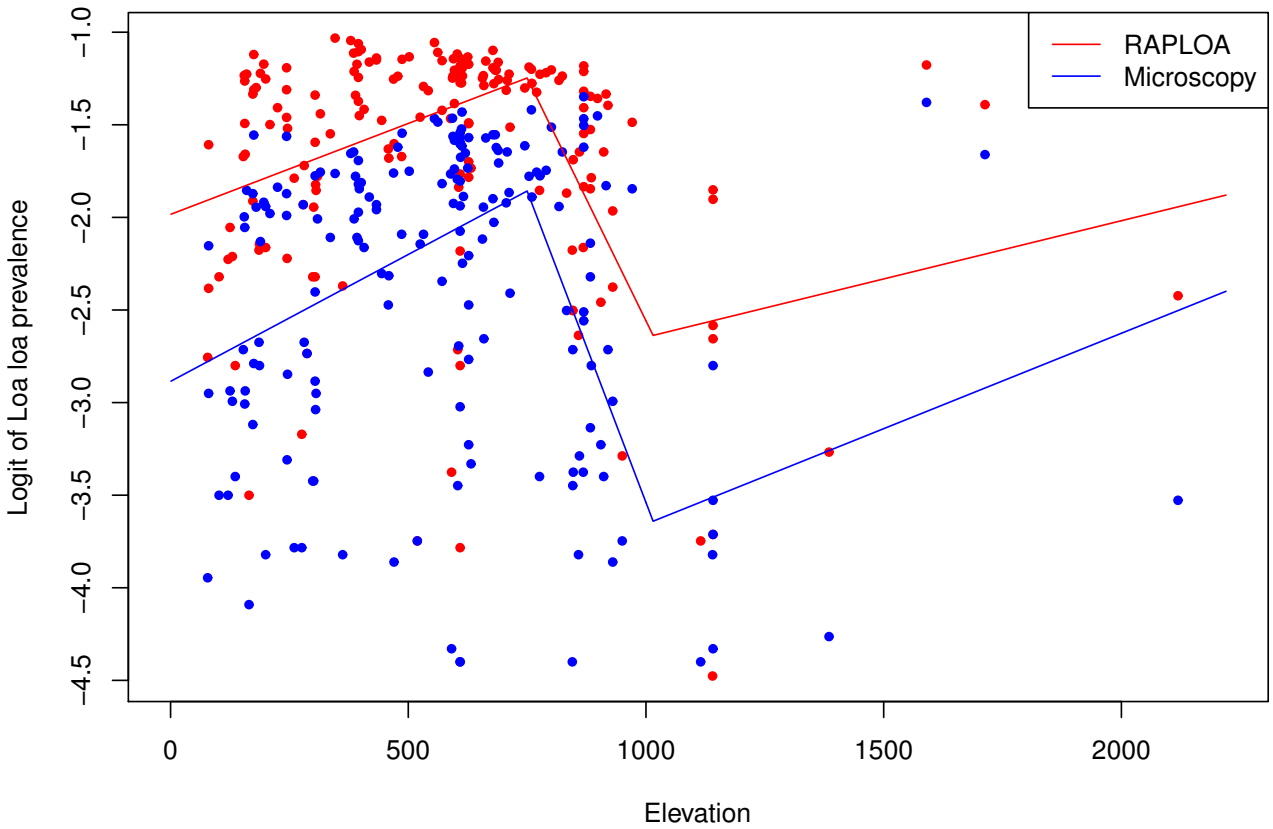

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- Spatial correlation

$$\text{corr}\{S_k(x), S_k(x')\} = \exp\{-\|x - x'\|/\phi_k\}$$

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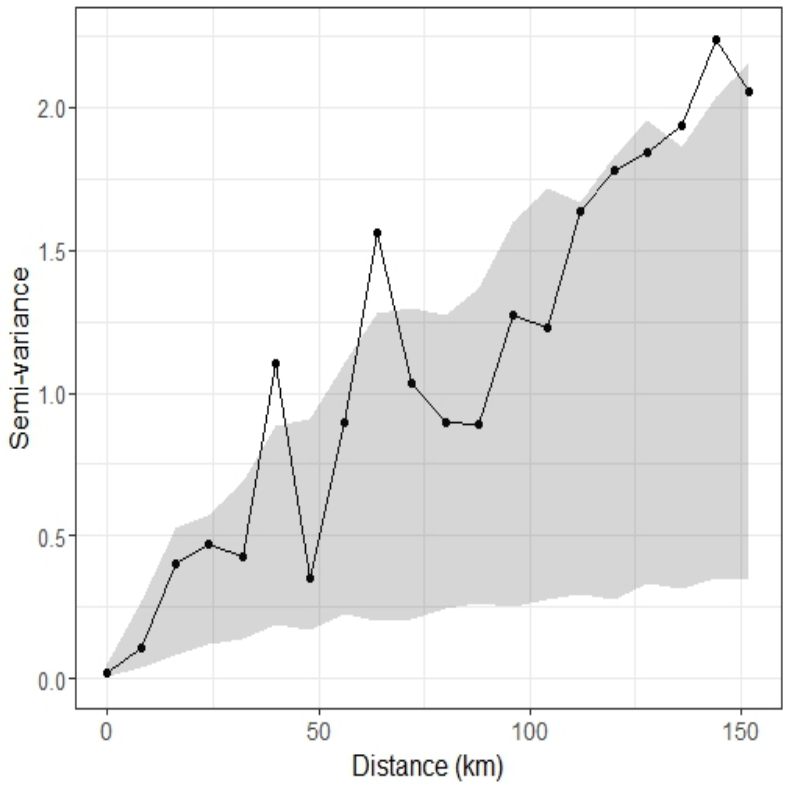
Parameter	Model 1	Model 2
σ_1^2	1.581	1.617
	(0.669, 3.738)	(0.679, 3.851)
σ_2^2	—	0.216
		(0.111, 0.419)
ϕ_1	182.037	187.388
	(64.657, 512.512)	(65.171, 538.807)
ϕ_2	—	23.686
		(6.150, 91.220)
τ_1^2	0.205	0.324
	(0.081, 0.521)	(0.052, 6.229)
τ_2^2	0.324	0.104
	(0.055, 5.873)	(0.018, 5.797)
α	1.005	1.017
	(0.902, 1.107)	(0.939, 1.095)

Application: re-analysis of the *Loa loa* data

Validation of Model 1 (a) and Model 2 (b) Validation *Model 1*



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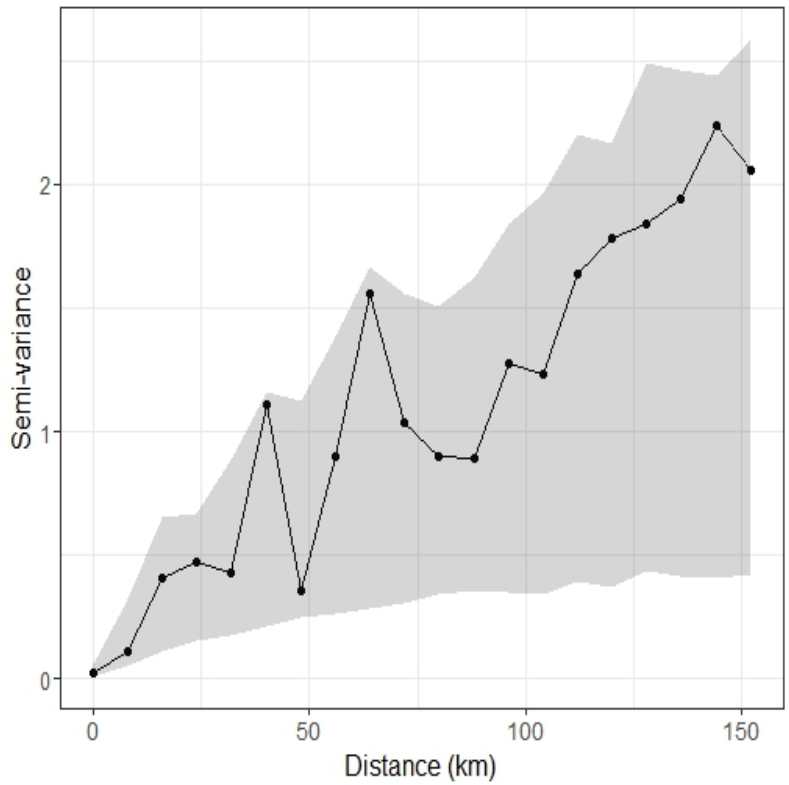

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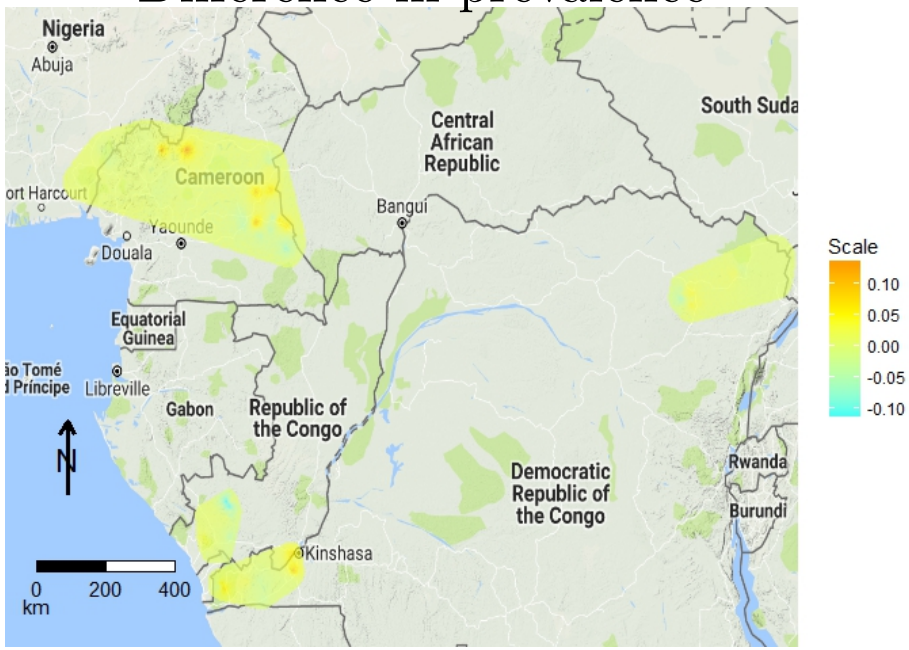
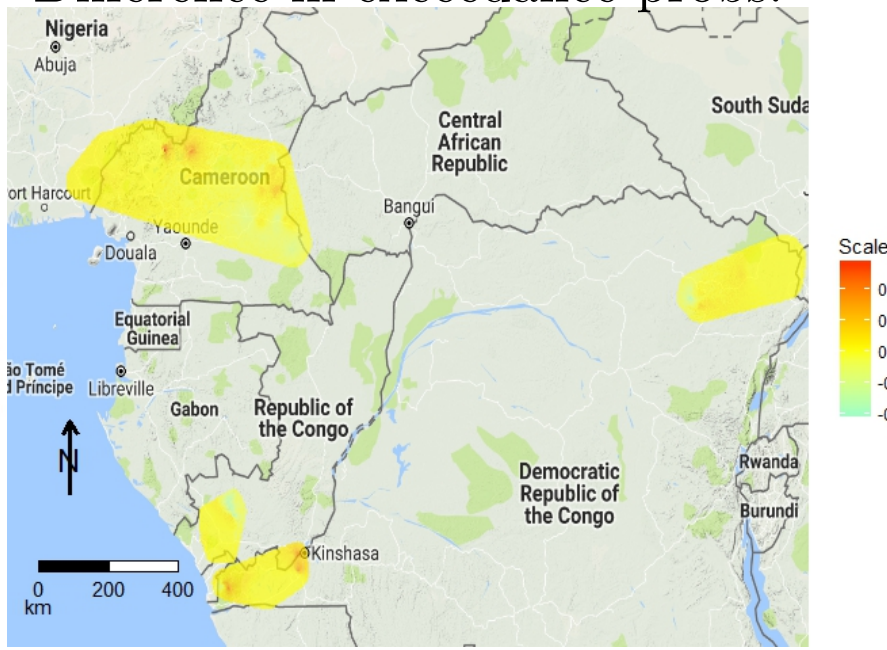
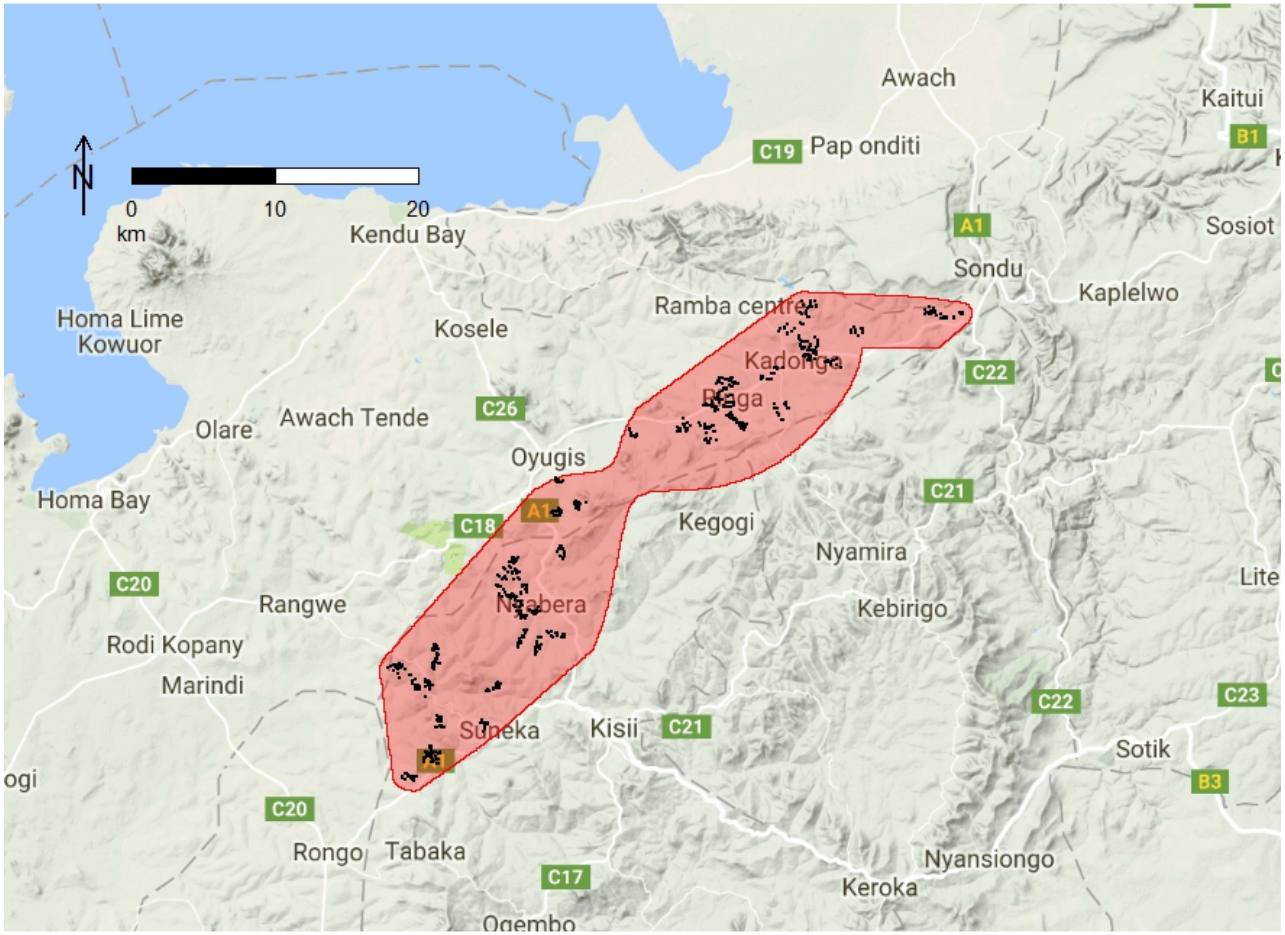
Difference in prevalence



Difference in exceedance probs.



Application: Joint prediction of malaria prevalence using RDT and PCR



Application: Joint prediction of malaria prevalence using RDT and PCR

- Our model for the data

$$\log\left\{\frac{p_{jk}(x_i)}{1-p_{jk}(x_i)}\right\} = \beta_{k,0} + \sum_{l=1}^{3}\beta_{k,l}d_{ij,l} + \nu_k T(x_i) \qquad (10)$$

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- Question 1: what is the most parsimonious and empirically compatible model?
- Question 2: is a joint model an *improvement* over two separate models?

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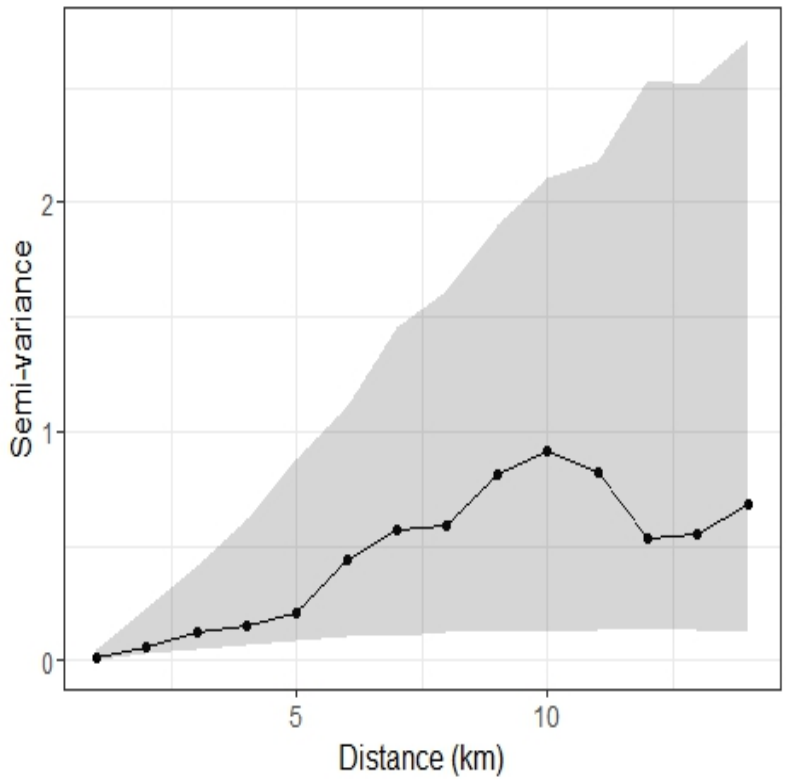

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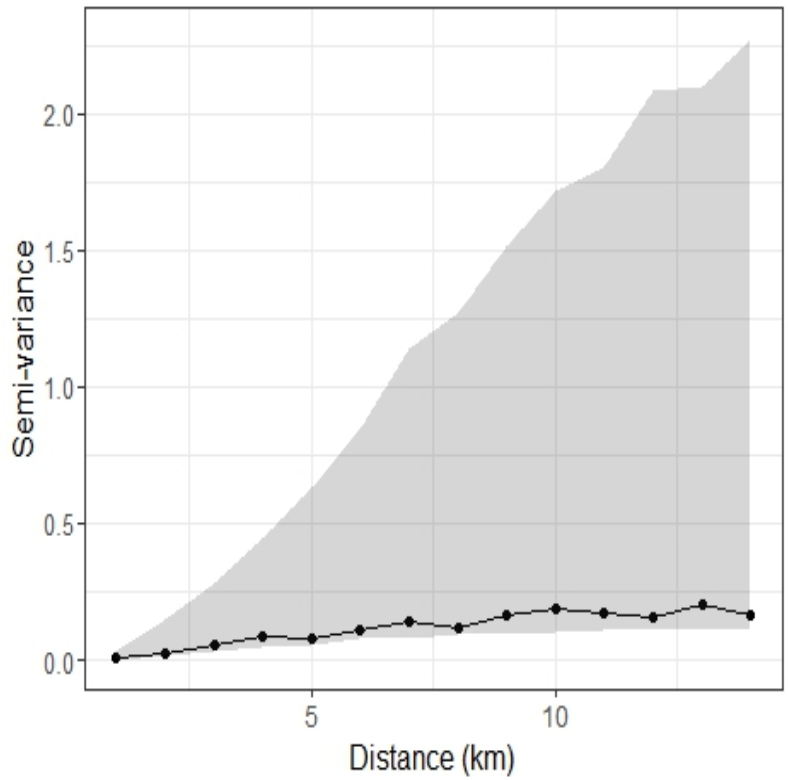

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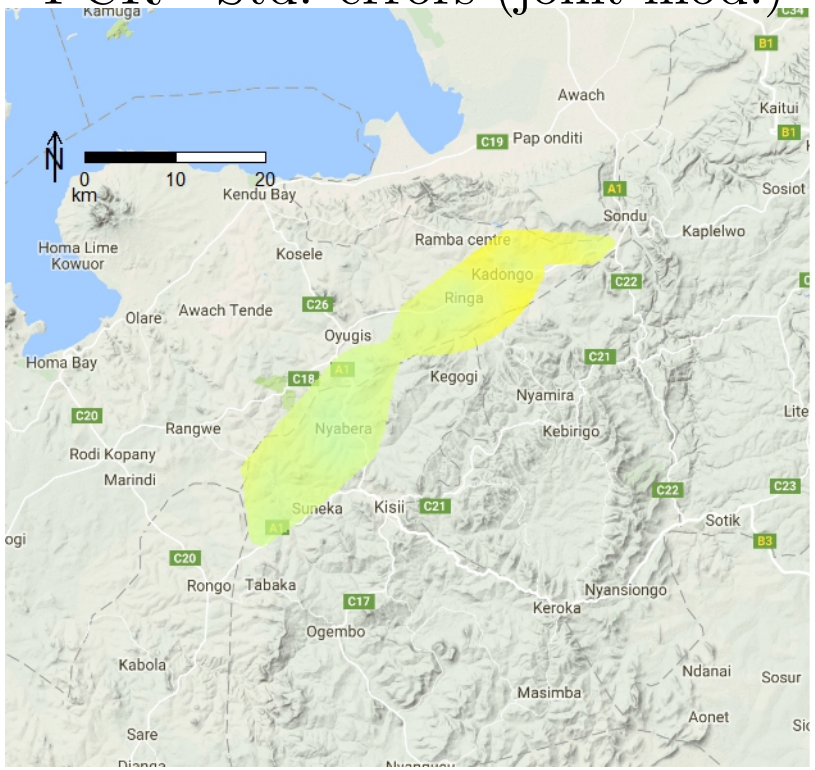
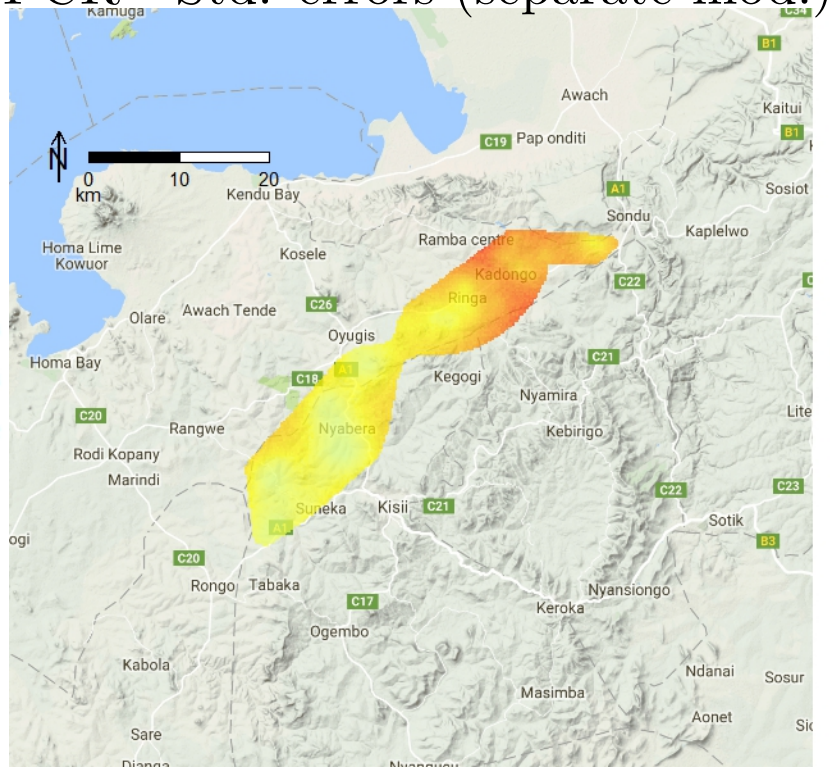
Application: Joint prediction of malaria prevalence using RDT and PCR

PCR - Std. errors (joint mod.)



PCR - Std. errors (separate mod.)



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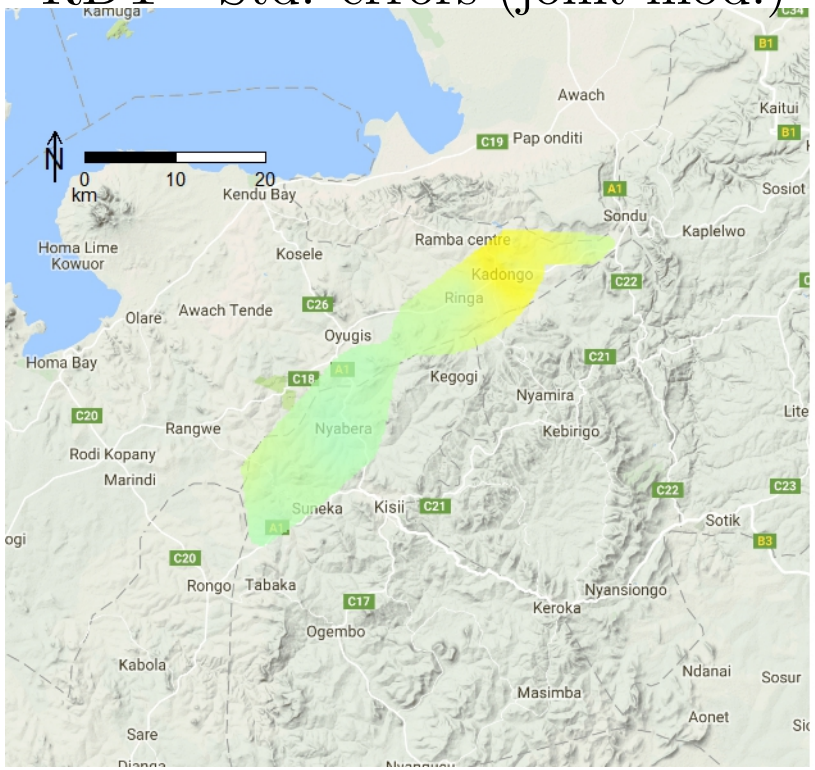

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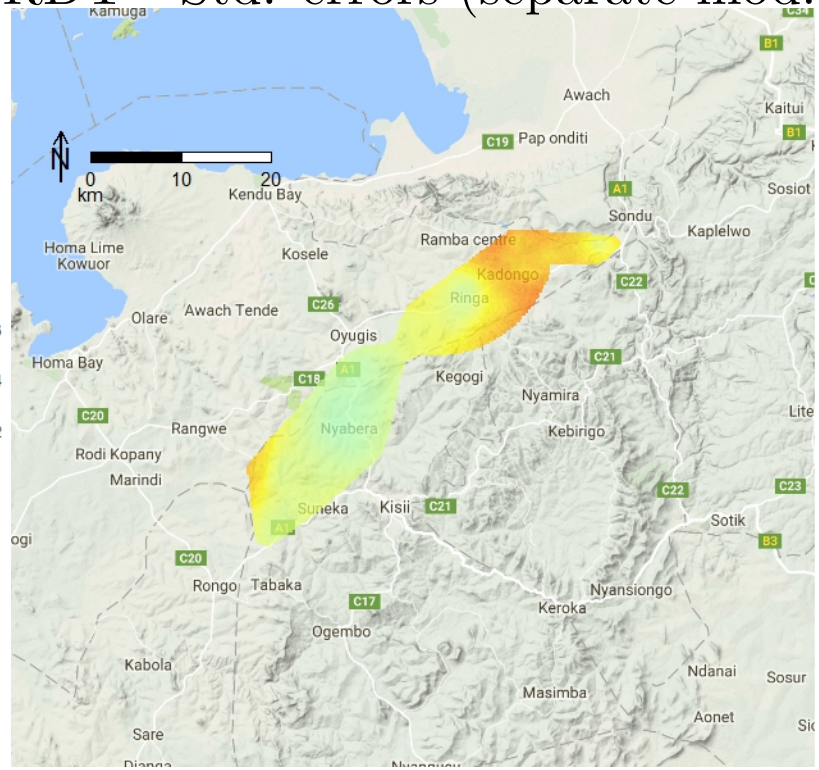

Conclusions

- The problem should inform the modelling approach.
- It is important to develop methods of inference that allow the borrowing of strength of information across multiple diagnostics.
- Ignoring this diagnostic specific spatial variation can lead to unreliably narrow prediction intervals for prevalence.
- Methodology can be easily extended to more than two diagnostics.