Spatio-temporal analysis

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Toowoomba 21-25 October 2019

Overview

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- Repeated cross-sectional surveys.

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- Analysing reported cases to health facilities.

Geostatistical problems and geostatistical methods

- **Geostatistical problems:** The object of scientific interest is to make inference an spatially continuous surface (e.g. disease risk) or some of its properties (e.g. regional prevalence) in a geographical area of interest.
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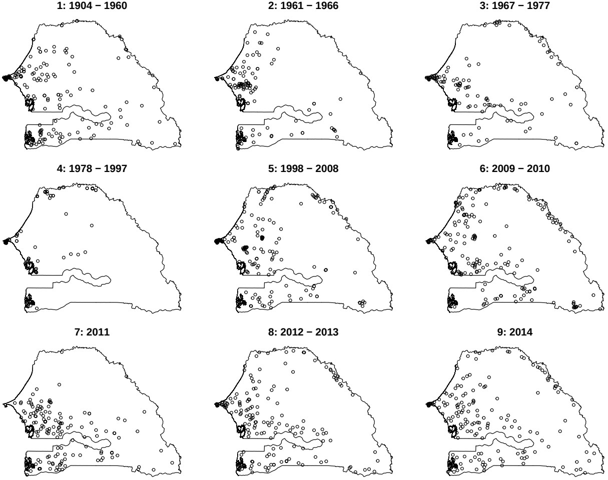
- **Model-based geostatistics:** Likelihood-based methods of inference that use spatially sparse noisy data to make inference on a spatially continuous surface of interest.
- Geostatistical problems are **NOT** dependent on the data-format (e.g. point-level or areal-level data).

Geostatistical problems: some examples

- Spatio-temporal interpolation of disease risk.
- Identification of disease hotspots (i.e. areas where disease risk exceeds policy relevant thresholds).
- Identification of low transmission areas (i.e. areas where disease risk is below policy relevant thresholds).
- Estimating the association between risk factors and disease risk metrics.
- Accounting for the excess of zero-reported cases (spatially structured zero-inflation).
- Combining data from randomised survey and convenience samples.

The data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2: 1961 – 1966
3: 1967 – 1977
4: 1978 – 1997
5: 1998 – 2008
6: 2009 – 2010
7: 2011
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- A model for the data.

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- $E[S(x, t)] = E[Z(x, t)] = 0$; $V[S(x, t)] = \sigma^2$; $V[Z(x, t)] = \tau^2$ and

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- **Question.** What can we do with this model?

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- The theoretical STV

$$\begin{aligned}\gamma(u,v;\theta) &= E[\{W(x,t) - W(x',t')\}^2] \\ &= \tau^2 + \sigma^2[1 - \rho(u,v;\theta)]. \qquad (2)\end{aligned}$$

- The empirical STV

$$\tilde{\gamma}(u,v) = \frac{1}{2|N(u,v)|} \sum_{(i,j)\in N(u,v)} \{\tilde{Z}(x_i,t_i) - \tilde{Z}(x_j,t_j)\}^2. \qquad (3)$$

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  (*iv*) use the resulting B empirical semi-variograms to generate 95% tolerance intervals at each of the pre-defined distance bins.

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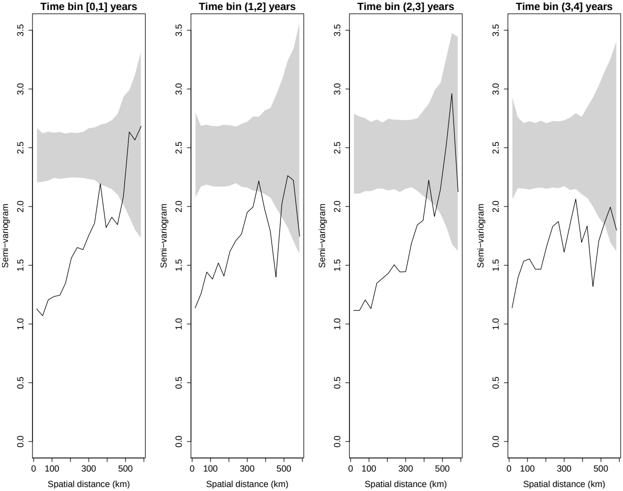

The covariance model

- The Gneiting's spatio-temporal correlation function.

$$\rho(u, v; \theta) = \frac{1}{(1 + v/\psi)^{\delta+1}} \exp\left\{-\frac{u/\phi}{(1 + v/\psi)^{\xi/2}}\right\}. \qquad (4)$$

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- How to choose a covariance model?
- Guiding principle: **parsimony**, e.g. find the simplest model that can explain the variation of the data.

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- Bayesian inference.

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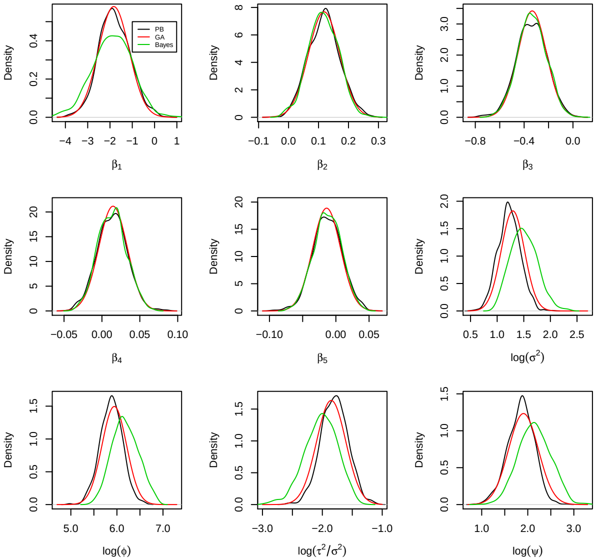
- Prior distributions.
  - $\beta \sim N(0, 10^4)$
  - $\sigma^2 \sim \text{Uniform}(0, 20)$
  - $\phi \sim \text{Uniform}(0, 1000)$
  - $\tau^2/\sigma^2 \sim \text{Uniform}(0, 20)$
  - $\psi \sim \text{Uniform}(0, 20)$

Estimation: Likelihood-based vs Bayesian inference



Validation of the covariance model

- Replace (i) with

  (a) Simulate $W(x_i, t_i)$ at observed locations x_i and times t_i, for $i = 1, \ldots, n$, from its marginal multivariate distribution under the assumed model.

  (b) Conditionally on the simulated values of $W(x_i, t_i)$, simulate binomial data y_i from (1).

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- Test statistics.

$$T = \sum_{k=1}^{K} |N(u_k, t_k)| [\tilde{\gamma}(u_k, v_k) - \gamma(u_k, v_k; \theta)]^2. \tag{5}$$

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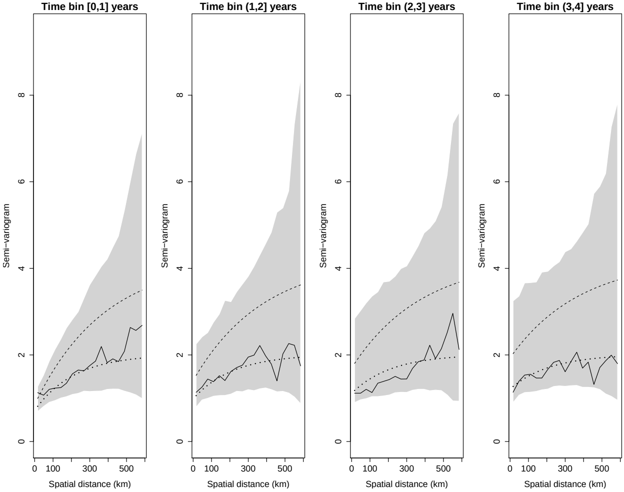

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- Likelihood-based inference (accounting for parameter uncertainty).

$$[W^*|y] = \int \int [\hat{\Lambda}|y][W|y, \hat{\Lambda}][W^*|W, \hat{\Lambda}]\, dW\, d\hat{\Lambda},$$

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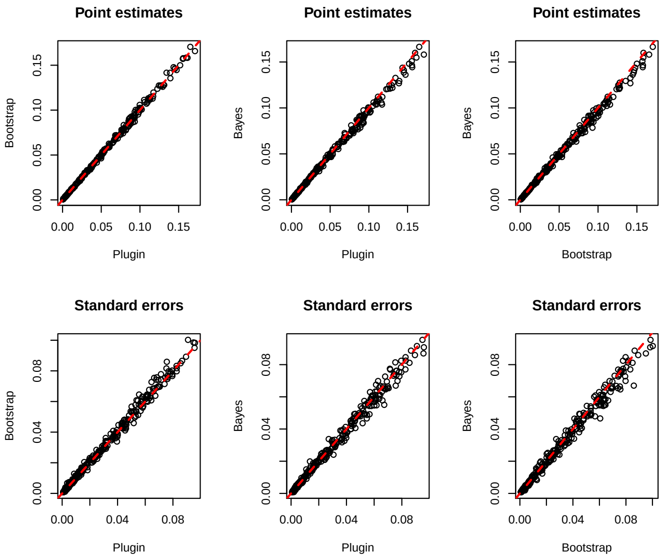
`http://fhm-chicas-apps.lancs.ac.uk/shiny/users/giorgi/mapMalariaSEN/`

Prediction: Likelihood-based vs Bayesian inference



What is a disease outbreak?

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- A disease outbreak is the occurrence of cases of disease in excess **of a policy-relevant risk threshold** in a defined community, geographical area or season.

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- **Population:** WorldPop + 2007 Census Data.
- **Climatic variables:** FLDAS data are available from NASA at daily temporal resolution and 0.1 degree spatial resolution.
  - total rainfall (mm)
  - average weekly temperature ($^\circ$C)
  - average weekly relative humidity (%)
  - average weekly surface barometric pressure (hPa)
  - average weekly saturation vapour pressure deficit (mmHg)

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Objective

Spatio-temporal mapping of exceedance probabilities

$$\mathrm{Prob}\{\exp\{S_{it}\} > l|\text{data}\}$$

at a future week t.

Modelling the residual spatio-temporal variation

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$$S_t = \rho S_{t-1} + W_t.$$

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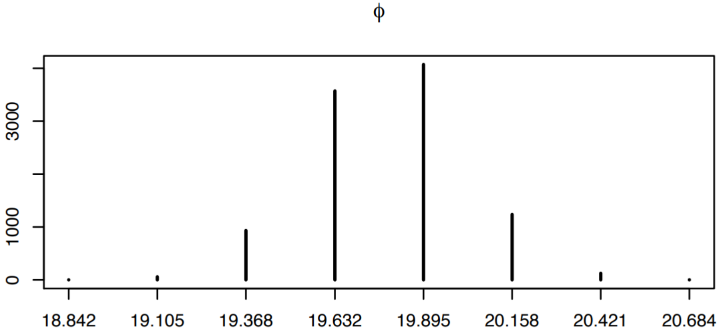
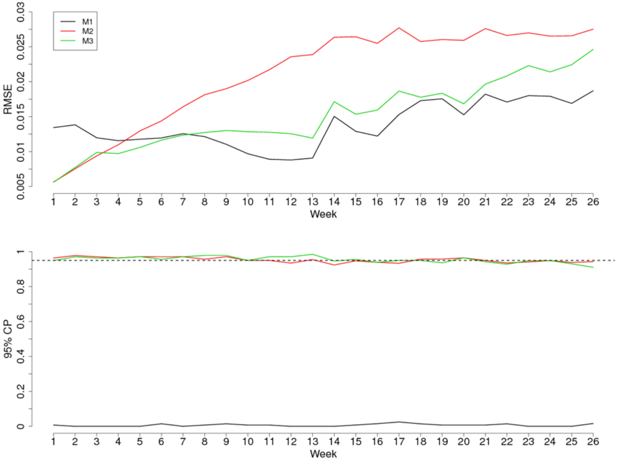
- 95% coverage probabilities (CI)

$$\mathrm{CI}_t = \frac{1}{142}\sum_{i=1}^{142} I(\lambda_{it}^{0.025} < \lambda_{it}^{emp} < \lambda_{it}^{0.975})$$

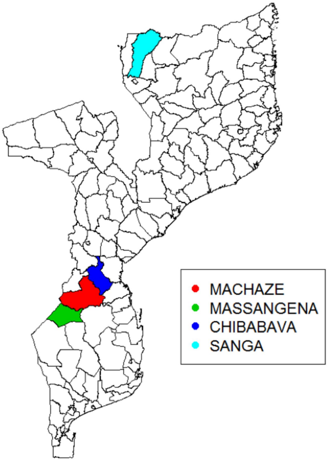
Results: scale of spatial correlation



Results: validation



Results: exceedance probabilities



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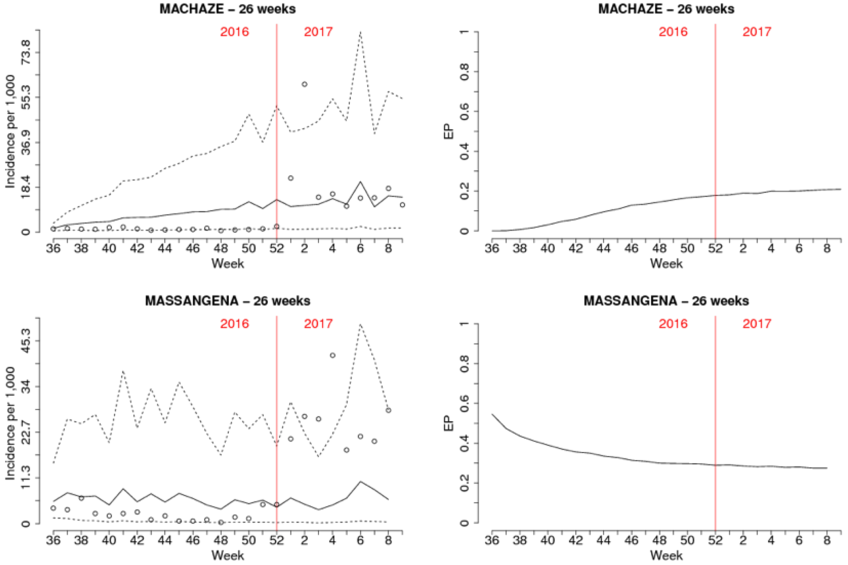

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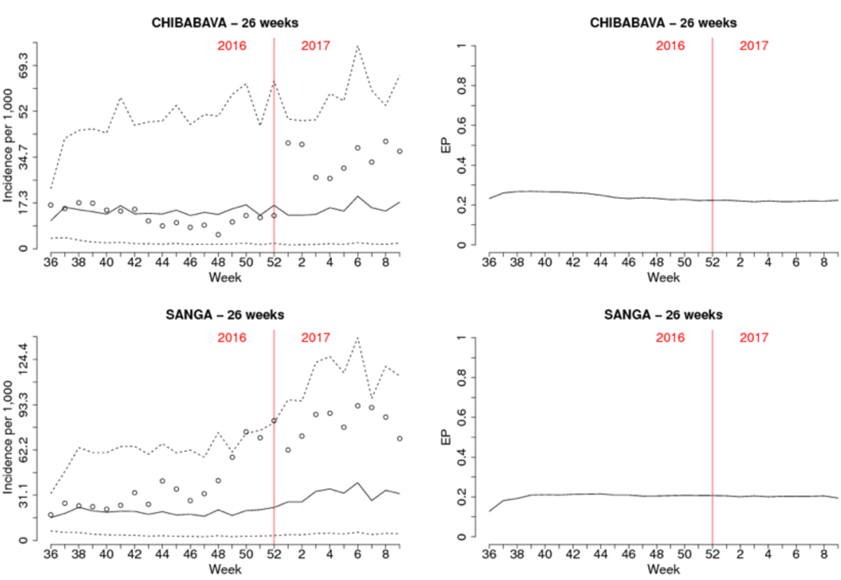

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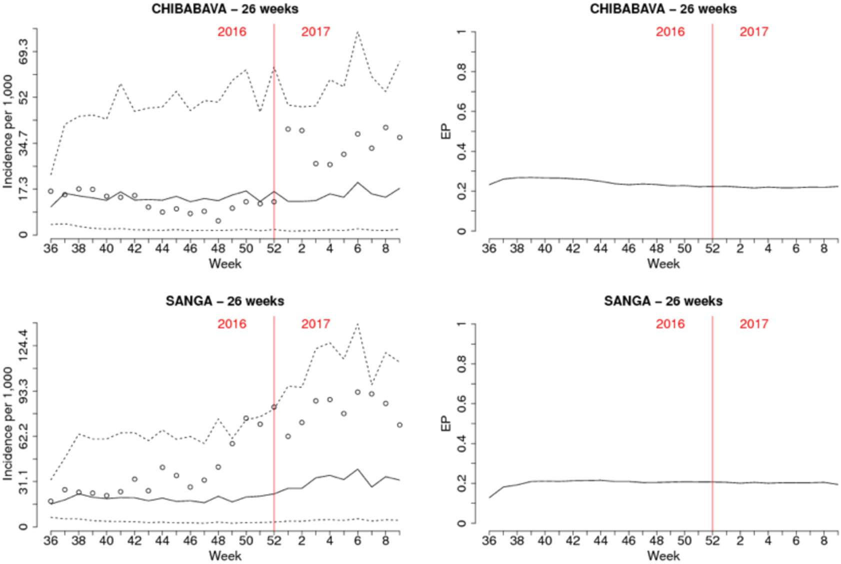
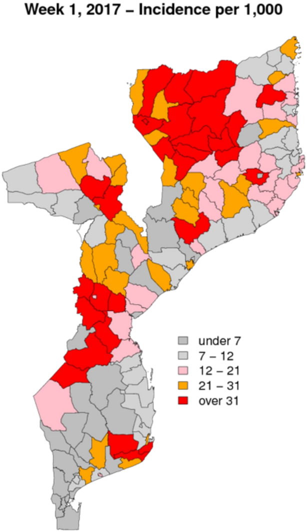
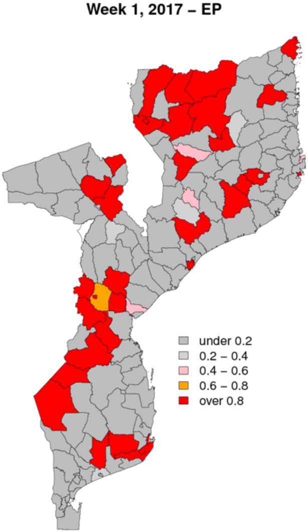


Results: incidence vs exceedance probabilities





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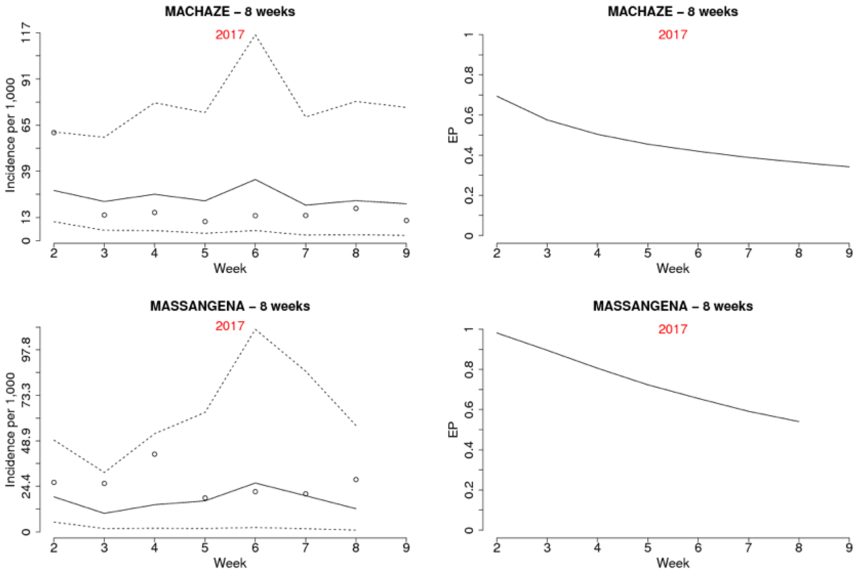

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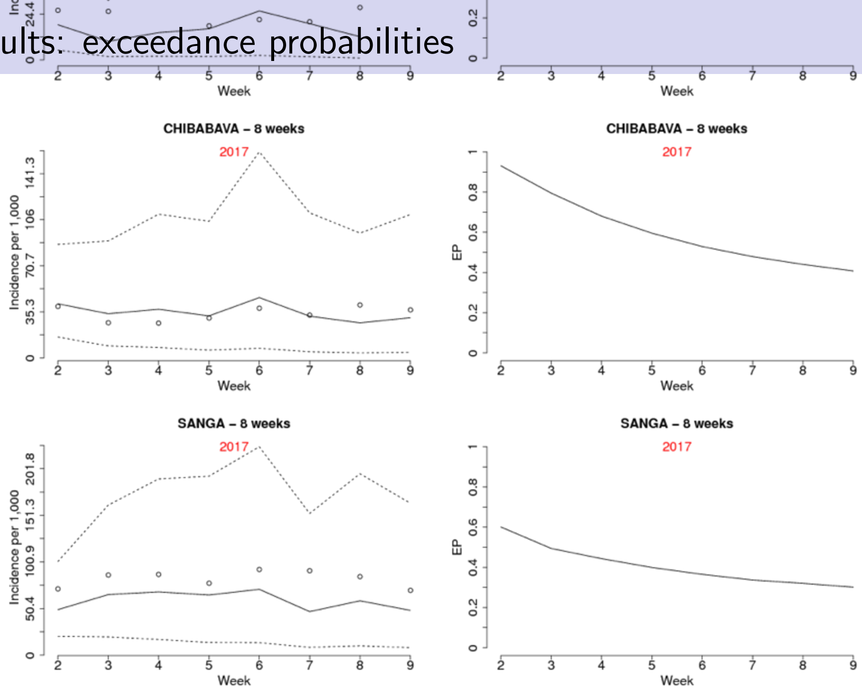

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- Outstanding issue: how to identify relative risk thresholds.
- Web interface: `http://www.lancaster.ac.uk/staff/haleac/malaria/`(by Dr Alison Hale)

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- We need to account for unmeasured risk factors (S_{it}).
- Climatic variables provide information on the long term trends; residual spatio-temporal random effects on short term.
- Exceedance probabilities are a useful tool for detecting outbreaks within at least 4 weeks.
- Outstanding issue: how to identify relative risk thresholds.
- Web interface: `http://www.lancaster.ac.uk/staff/haleac/malaria/`(by Dr Alison Hale)