Problem Set 4 - Gradients and Backpropagation

DS542 - DL4DS

Spring, 2025

Note: Refer to Chapter 7 in *Understanding Deep Learning.*

Problem 4.1 (3 points)

Consider the case where we use the logistic sigmoid function as an activation function, defined as:

$$h = \sigma(z) = \frac{1}{1 + e^{-z}}. \tag{1}$$

Compute the derivative $\frac{\partial h}{\partial z}$. What happens to the derivative when the input takes (i) a large positive value and (ii) a large negative value?

Problem 4.2 (3 points)

Calculate the derivative $\frac{\partial \ell_i}{\partial f[x_i, \phi]}$ for the binary classification loss function:

$$\ell_i = -(1 - y_i) \log[1 - \sigma(f[x_i, \phi])] - y_i \log[\sigma(f[x_i, \phi])], \tag{2}$$

where the function $\sigma(\cdot)$ is the logistic sigmoid, defined as:

$$\sigma(z) = \frac{1}{1 + \exp(-z)}. \tag{3}$$

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$$\frac{\partial h}{\partial z} = \frac{d}{dz}\left(\frac{1}{1+e^{-z}}\right)$$

based on chain rule we get $\frac{\partial h}{\partial z} = \frac{e^{-z}}{(1+e^{-z})^2}$

since $h = \frac{1}{1+e^{-z}}$ and $1-h = \frac{e^{-z}}{1+e^{-z}}$

after derivative $\boxed{\frac{\partial h}{\partial z} = h\cdot(1-h) = \sigma(z)\cdot 1-\sigma(z)}$

(i) $z \to +\infty$

$\sigma(z) \approx 1$ because e^{-z} approaches 0

Thus, $h(1-h) \approx 1\times 0 = 0$ ※

$\sigma(500)\cdot(1-\sigma(500))$

$= \frac{1}{1+e^{-100}} \cdot \frac{e^{-100}}{1+e^{-100}}$

$= 0$

(ii) $z \to -\infty$

$\sigma(z) \approx 0$ because e^{-z} become very large

Thus, $h(1-h) \approx 0\times 1 = 0$ ※

$\sigma(-500)\cdot(1-\sigma(-500))$

$= 0$

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Calculate the derivative $\frac{\partial \ell_i}{\partial f[x_i,\phi]}$ for the binary classification loss function:

$$\ell_i = -(1-y_i)\log[1-\sigma(f[x_i,\phi])] - y_i \log[\sigma(f[x_i,\phi])], \tag{2}$$

where the function $\sigma(\cdot)$ is the logistic sigmoid, defined as:

$$\sigma(z) = \frac{1}{1+\exp(-z)}. \tag{3}$$

$$\log(x)' = \frac{1}{x} \cdot 1$$

define z as $f[x_i, \phi]$

$$\underline{-(1-y_i)\log[1-\sigma(z)]} - \underline{y_i \log[\sigma(z)]}$$

$$(1-y_i) \cdot \frac{1}{1-\sigma(z)} \cdot [-\sigma(z) \cdot (1-\sigma(z))]$$

$$y_i \cdot \frac{1}{\sigma(z)} \cdot [\sigma(z) \cdot (1-(\sigma(z))]$$

$$\boxed{(1-y_i) \cdot \frac{1}{1-\sigma(z)} \cdot [-\sigma(z) \cdot (1-\sigma(z))] - y_i \cdot \frac{1}{\sigma(z)} \cdot [\sigma(z) \cdot (1-(\sigma(z))]} \; \#$$