EE 559 **Homework - Week 14** Posted: Thur., 4/18/2019
Jenkins Due: Thur., 4/25/2019, 2:00 PM

1. DHS Ch. 3 Problem 2 (The problem starts "Let x have a uniform density…" and you are asked to find the Maximum Likelihood estimate of a parameter of the uniform density.

   Hint: If you find that taking the derivative w.r.t. θ won't give a meaningful answer, try writing the expression for the likelihood and maximizing it by inspection.

2. DHS, Chapter 3, Problem 17 plus additional part (f) given below. This problem starts "The purpose of this problem is to derive the Bayesian classifier for the d-dimensional multivariate Bernoulli case." It walks you through the Bayesian method for parameter estimation in a Bayes minimum error classifier (Approach 2 "Integrating over posterior predictive" in Lecture 25 notes).

   Note that x_i is binary, with $x_i \in \{0,1\}$, and that dataset $\mathcal{D}$ is the same set of samples that we called vector $\underline{z}$ in lecture. Also $\theta_i =$ probability that $x_i = 1$, with $0 \le \theta_i \le 1 \;\; \forall i$.

   Hints: For part (b), the uniform distribution for $p(\theta)$ extends over $0 \le \theta \le 1$; also, use Bayes theorem. For part (d), the integral they refer to is Eq. (2) in Lecture 25 notes (or Eq. (3) with the class distinctions ($|S_i$ and subscripts denoting class) omitted).

   (f) Consider a C-class problem, in which class S_k contains n_k samples denoted as the set $\mathcal{D}_k$ (or the vector $\underline{z}_k$), and these samples sum to $\underline{s}^{(k)} = \left[s_1^{(k)}, \cdots, s_d^{(k)} \right]^T$. Give the Bayes minimum error decision rule based on your estimation technique of parts (a)-(d). Use for estimates of the priors $P(S_k)$, frequency of occurrence of the prototypes.

3. In this problem you will use k-nearest neighbor density estimation. You are given the following data points for a 2-class problem in 1-D feature space:

$$S_1 : \; 1.0, 4.0$$
$$S_2 : \; 3.0, 6.0$$

   For parts (a)-(c) below, use $k = 2$.

(a) Graph the k-nearest neighbors estimates of the density functions $p(x|S_1)$ and $p(x|S_2)$. Be sure to label pertinent values on both axes. Also give the density estimates algebraically, for each region in feature space.

(b) Estimate the a priori probabilities based on frequency of occurrence of the prototypes in each class.

(c) Use the estimates you have developed in (a)-(b) above to find the decision boundaries and regions for a Bayes minimum-error classifier based on k-nearest neighbors.

(d) For this part, use instead the decision rule for a *discriminative* kNN classifier (which doesn't calculate $p(x|S_i)$ explicitly). Let $k = 3$ (= number of samples over all classes that are inside region $\mathcal{R}$), and find the decision boundary and regions. Classify the points 3.25, 3.75. (**Hint:** if you're not sure how to come up with the boundary and regions, try classifying the two points first.)