EE 559 **Homework Week 12** Posted: Thur., 4/4/2019
Jenkins Due: Thur., 4/11/2018, 2:00 PM

1. (a) Two classes are described by normal densities as follows

$$p\left(\underline{x}\middle|S_i\right) = N\left(\underline{x}, \underline{m}_i, \underline{\underline{\Sigma}}_i\right), \quad i = 1, 2$$
$$P\left(S_1\right) = P\left(S_2\right) = 0.5$$

Write an expression for the decision rule, simplified as much as possible. Try to put your answer into this general form:

$$\underline{w}^T \underline{x} \gtrless -w_0$$

and give expressions for $\underline{w}$ and w_0 in terms of given quantities. If it isn't expressible in this form, give the discriminant function in simplest form.

(b) For this part you are also given:

$$\underline{m}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \underline{m}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$
$$\underline{\underline{\Sigma}}_1 = \underline{\underline{\Sigma}}_2 = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 2.25 \end{bmatrix}$$

Solve, algebraically, for the Bayes minimum error classifier (*i.e.*, give the resulting decision rule algebraically). Plot (by hand or by computer) the decision boundary and label the decision regions in 2D nonaugmented feature space.

2. A *Naïve Bayes* classifier is a Bayes classifier in which the features, conditioned on class, are assumed independent; thus:

$$p\left(\underline{x}\middle|S_i\right) = \prod_{j=1}^{D} p\left(x_j\middle|S_i\right) \quad \forall i = 1, \cdots, C.$$

Repeat Problem 1 for a Naïve Bayes classifier; except let the feature variances in part (b) be $\sigma_1^{(i)^2} = 1,\ \sigma_2^{(i)^2} = 2.25$, $i =$ class index $= 1, 2$.

3. A *Bayes minimum risk classifier* uses a different criterion than a Bayes minimum error classifier. This classifier was introduced in Lecture 19, and is covered in DHS Sec. 2.2. In this problem, please use our notation (S_i for class *i* instead of ω_i).

For a Bayes minimum risk classifier with the given information of Problem 1 above, and losses given by:

$$\lambda_{ii} = 0, \;\; i = 1, 2$$
$$\lambda_{12} = 2\lambda_{21} > 0$$

solve for the decision regions and boundary. Plot them in 2D nonaugmented feature space; compare to the plot of Problem 1. Which decision region has grown because of the given losses? For the losses λ_{ij} given above, deciding which class incurs more loss (deciding S_1 or deciding S_2) ?