Institut für Betriebssysteme und Rechnerverbund
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Masterarbeit

Computational Aspects of MaxMin Triangulations

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for my daughter—in the hope that she will understand sometime

Erklärung

Ich versichere, die vorliegende Arbeit selbstständig und nur unter Benutzung der angegebenen Hilfsmittel angefertigt zu haben. Bei den Experimenten sind keine unbeteiligten Dreiecke zu Schaden gekommen.

Braunschweig, den 11. Juli 2013

Abstract

Maxmin length triangulations are just awesome.

Zusammenfassung

Maxmin Triangulationen sind einfach super.

Contents

1. Introduction

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1 pages

Triangulations, that is subdividing the plane (or a polygon) into triangles, are a popular topic in computational geometry — not only because of their connections to other problems but also due to their practical applications. They can be helpful as a preprocessing in other algorithms or as a tool in geometric proofs. One popular example is the artgallery problem . Another area where triangulations are widely used is mesh generation and approximation of complex geometric structures.

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For different use cases the objectives for a triangulation vary. For example, the widely known Delaunay triangulation [2, Section 9.2] tends to avoid '"skinny'" triangles and is therefore useful for meshes. One can image many different properties to be optimized: Edge lengths, triangle area, angle, and degree in a vertex are some of them. Many have already been looked into, but for some of them no application is known by now — so they remain theoretical problems. In chapter 3 we will have a brief overview of different kinds of triangulations.

This thesis will focus on the MaxMin Length Triangulation (MMLT). Stated an open problem in 1991 [16], it has been proven to be NP-complete in 2012 [18]. However our assumption is that the hard instances are rare and that random instances can be solved in polynomial time on average. Therefore we provide an algorithmic idea in chapter 4 and its implementation in chapter 5.

Even though there seems to be no known application of MMLT yet, it is Greg's [31] preferred kind of triangulation.

2. Integer Programming

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Definition 2.1 ((Linear) Integer Program (IP))
An IP is a system of integer variables $x \in \mathbb{Z}^n$ with a set of constraints on them and often an objective function $c^T x$. We consider only the case where the constraints are linear: $Ax \leq b$ or $Ax \geq b$.

glue text

Problem 2.2 (IP in canonical form [36])

$$\begin{aligned} \text{maximize} \quad & c^T x \\ \text{subject to} \quad & Ax \leq b \\ & x \geq 0 \\ & x \in \mathbb{Z} \end{aligned}$$

or

$$\begin{aligned} \text{minimize} \quad & c^T x \\ \text{subject to} \quad & Ax \geq b \\ & x \leq 0 \\ & x \in \mathbb{Z} \end{aligned}$$

Theorem 2.3
Solving IPs is NP-hard.

Proof:
Even the special case where there is no objective function, only binary variables, and only equality constraints is NP-complete [27]. Therefore the more general problem is NP-hard.

2.1. SAT

glue text

Problem 2.4 (3-SAT)

Given: Set of boolean literals $x_i \in X$, a formula in conjunctive normal form consisting of clauses $c \in C$ each involving exactly three literals (or their negations)

Sought: An assignment for X which lets all clauses in C evaluate to "true"

glue text

Theorem 2.5 (NP-completeness of 3-SAT)
Problem 2.4 is NP-complete. [27, Satisfiability with at most 3 Literals per Clause]

glue text

Problem 2.6 (IP formulation of 3-SAT)

$$\begin{array}{lll} \text{(minimize} & & 1) \\ \text{subject to} & \forall c \in C: & \sum_{x_i \in c} x_i + \sum_{\neg x_i \in c} (1 - x_i) = 1 \\ & \forall x_i \in X: & x_i \in \{0, 1\} \end{array}$$

glue text

Problem 2.7 (Planar 3-SAT)
An instance of the 3-SAT problem with literals X and clauses C which can be represented by a planar graph $G = (V, E)$ such that

$$V = \{v : v \in X \cup C\}$$
$$E = \{\{x, c\} : x \in X,\ c \in C,\ (x \in c) \vee (\neg x \in c)\}$$

glue text

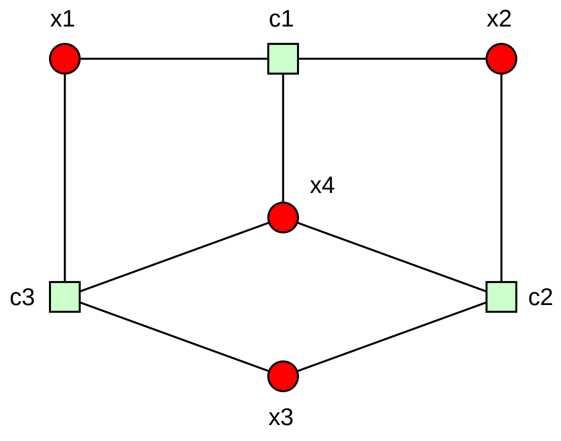

Figure 2.1.: Example of a Planar 3SAT instance which represents the term $\underbrace{(x_1 \vee x_2 \vee x_4)}_{c_1} \wedge \underbrace{(\neg x_2 \vee x_3 \vee \neg x_4)}_{c_2} \wedge \underbrace{(x_1 \vee \neg x_3 \vee \neg x_4)}_{c_3}$

glue text

Theorem 2.8 (NP-completeness of Planar 3-SAT)
Problem 2.7 is still NP-complete. [29]

2.2. Vertex Cover

glue text

Definition 2.9 (Vertex Cover)
Given an undirected graph $G = (V, E)$, a vertex cover $V_{\text{cover}} \subseteq V$ for G is a vertex set such that every edge $e \in E$ is incident to at least one vertex $v \in V_{\text{cover}}$:

$$\forall e \in E : \exists v \in V_{\text{cover}} : v \in e$$

glue text

Definition 2.10 (Minimal Vertex Cover)
A vertex cover V_{cover} for an undirected graph $G = (V, E)$ is minimal if there is no vertex $v \in V_{\text{cover}}$ such that $V_{\text{cover}} \setminus \{v\}$ remains a vertex cover.

glue text

Definition 2.11 (Minimum Vertex Cover)
A vertex cover V_{cover} for an undirected graph $G = (V, E)$ is minimum if there is no other vertex cover V'_{cover} for G which has fewer vertices:

$$\forall\ V'_{\text{cover}} \text{ vertex cover} : |V_{\text{cover}}| \leq |V'_{\text{cover}}|$$

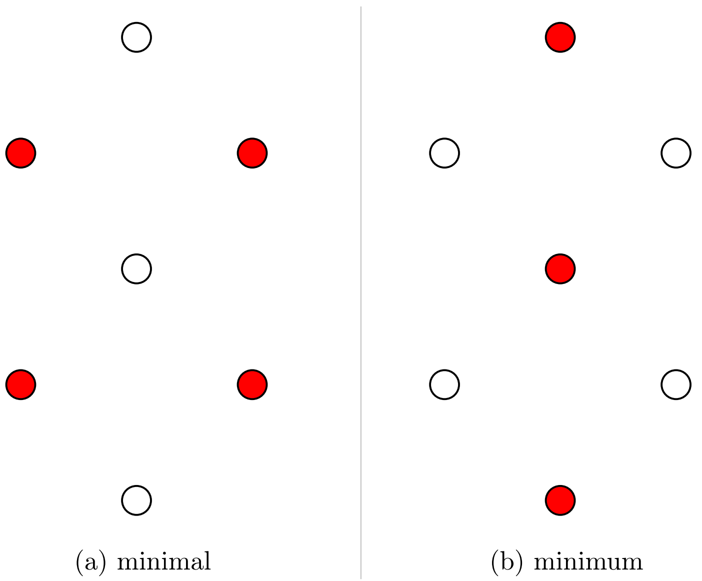

Figure 2.2.: Example of minimal vs. minimum vertex cover

glue text

Problem 2.12 (IP Formulation of Minimum Vertex Cover)

$$\begin{aligned}
\text{minimize} \quad & \sum_{v \in V} x_v \\
\text{subject to} \quad \forall \{v, w\} \in E : \quad & x_v + x_w \geq 1 \\
\forall v \in V : \quad & x_v \in \{0, 1\}
\end{aligned}$$

glue text

Theorem 2.13
Vertex Cover is NP-complete. [27]

glue text

Definition 2.14 (*Well-covered* Graph)
An undirected graph $G = (V, E)$ is *well-covered* iff every minimal vertex cover for G is also a minimum vertex cover for G. [37]

glue text

Theorem 2.15
In a *well-covered* graph, all minimal vertex covers have the same cardinality. [37]

2.3. Independent Set

glue text

Definition 2.16 (Independent Set)
Given an undirected graph $G = (V, E)$, an independent set $V_{\mathrm{IS}} \subseteq V$ is a vertex set such that no two vertices $v, w \in V_{\mathrm{IS}}$ are incident to the same edge $\{v, w\} \in E$:

$$\forall v \in V_{\mathrm{IS}} : \forall \{v, w\} \in E : w \notin V_{\mathrm{IS}}$$

glue text

Definition 2.17 (Maximal/Maximum Independent Set)
For an undirected graph $G = (V, E)$, an independent set $V_{\mathrm{IS}} \subseteq V$ is maximal if there is no vertex $v \in V \setminus V_{\mathrm{IS}}$ such that $V_{\mathrm{IS}} \cup \{v\}$ remains an independent set. It is maximum if there is no independent set ${V_{\mathrm{IS}}}'$ with larger cardinality.

glue text

Problem 2.18 (IP Formulation of Maximum Independent Set)

$$\begin{aligned}
\text{maximize} \quad & \sum_{v \in V} x_v \\
\text{subject to} \quad \forall \{v,w\} \in E: \quad & x_v + x_w \le 1 \\
\forall v \in V: \quad & x_v \in \{0,1\}
\end{aligned}$$

glue text

Theorem 2.19 (Independent Set and Vertex Cover)
For an undirected graph $G = (V, E)$, $V_{\text{IS}} \subseteq V$ is a maximum independent set iff $V_{\text{cover}} = V \setminus V_{\text{IS}}$ is a minimum vertex cover.

Proof:
Let V_{cover} be a vertex cover for G.

$$\begin{aligned}
\forall e \in E : \exists v \in V_{\text{cover}} : v \in e \iff & \forall \{v,w\} \in E : v \in V_{\text{cover}} \lor w \in V_{\text{cover}} \\
\iff & \forall \{v,w\} \in E : \neg (v \notin V_{\text{cover}} \land w \notin V_{\text{cover}}) \\
\iff & \forall \{v,w\} \in E : \neg (v \in (V \setminus V_{\text{cover}}) \land w \in (V \setminus V_{\text{cover}})) \\
\iff & \forall v \in (V \setminus V_{\text{cover}}) : \forall \{v,w\} \in E : w \notin (V \setminus V_{\text{cover}}) \\
\iff & (V \setminus V_{\text{cover}}) \text{ independent set}
\end{aligned}$$

Assume V_{cover} is a minimum vertex cover for G and the independent set $V_{\text{IS}} = V \setminus V_{\text{cover}}$ is not maximum. Then there is an independent $V'_{\text{IS}} \subseteq V$ with $|V_{\text{IS}}| < |V'_{\text{IS}}|$. But then for the vertex cover $V'_{\text{cover}} = V \setminus V'_{\text{IS}}$ the following holds: $|V'_{\text{cover}}| < |V_{\text{cover}}|$— which is a contradiction to V_{cover} being minimum. The same argumentation applies in the other direction.

glue text

Theorem 2.20 (Independent Set in *well-covered* Graphs)
For a *well-covered* graph $G = (V, E)$, every maximal independent set has the same size and is therefore maximum.

Proof:
Theorem 2.20 follows directly from definition 2.14 and theorems 2.15 and 2.19.

examples: fig. 2.2

glue text

Algorithmus 2.1 : Greedy algorithm for independent set

Input : Undirected graph $G = (V, E)$
Output : Maximal independent set $V_{\mathrm{IS}} \subseteq V$ for G

```
1 Set V_IS = ∅
2 foreach v ∈ V do
3     if ∀{v, w} ∈ E : (w ∉ V_IS) then
4         Set V_IS = V_IS ∪ {v}
5 return V_IS
```

glue text

Theorem 2.21 (Correctness and Complexity of Algorithm 2.1)
Algorithm 2.1 always finds a maximal independent set in $O(|E|)$ time.

Proof:
Because the vertices are processed sequentially, for every edge $\{v, w\} \in E$ at most one of v and w is added to V_{IS}. Therefore V_{IS} is an independent set. Additionally every vertex $v \in V$ is processed and if there is no $w \in V_{\mathrm{IS}}$ with $\{v, w\} \in E$ then $v \in V_{\mathrm{IS}}$. So V_{IS} is maximal.

The for-loop runs $|V|$ times but the if-statement is only executed twice for every edge $e \in E$. Every other statement runs in $O(1)$ time. Thus algorithm 2.1 needs $O(|E|)$ time.

glue text

Theorem 2.22 (Algorithm 2.1 in *well-covered* Graphs)
For a *well-covered* graph algorithm 2.1 always finds a maximum independent set in $O(|E|)$ time.

Proof:
Theorem 2.22 follows directly from theorems 2.20 and 2.21.

3. Triangulations

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glue text

Definition 3.1 (Complete Graph)
Given a vertex set V, the complete graph $K_V = (V, E)$ for V contains all possibles undirected edges between two vertices of V:

$$E = \{e = \{v, w\} : v, w \in V \wedge v \neq w\})$$

glue text

Definition 3.2 (Conflict Graph)
The conflict graph $G_{\text{conf}}(E, X) = (E, X)$ for a set of edges E and a set of edge conflicts

$$X \subseteq \{\{e_i, e_j\} : e_i, e_j \in E \wedge e_i \neq e_j\}$$

is an undirected graph with E as vertices and X as edges.

glue text

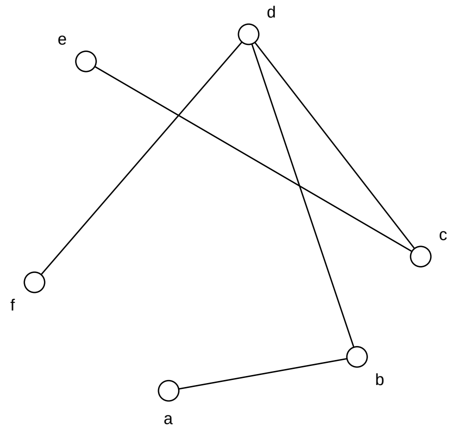
Definition 3.3 (Triangulation)
Given the complete graph $K_V = (V, E)$ for a vertex set V and a set of edge conflicts

$$X \subseteq \{\{e_i, e_j\} : e_i, e_j \in E \wedge e_i \neq e_j\}$$

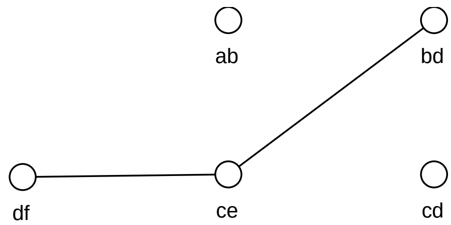
such that the conflict graph $G_{\text{conf}}(E, X)$ is *well-covered*. A *triangulation* $T(V, X) \subseteq E$ of V with respect to X is a maximum set of non-conflicting edges:

$$e_i \in T(V, X) \iff e_i \in E \wedge \forall e_j \in T(V, X) : \{e_i, e_j\} \notin X$$

glue text



(a) segments



(b) conflict graph

Figure 3.1.: Example of the conflict graph for a given set of segments and the conflicts being all intersections

glue text

Theorem 3.4 (Equality of Triangulation and Maximum Independent Set)
Every triangulation $T(V, X)$ of a vertex set V with respect to conflicts X is a maximum independent set for the conflict graph $G_{\text{conf}}(E, X)$ and vice versa. Hereby E are the edges of the complete graph K_V.

Proof:
Theorem 3.4 follows directly from definition 2.17 and theorem 2.20.

Theorem 3.5
From theorems 2.22 and 3.4 follows that finding the triangulation $T(V, X)$ of a vertex set V with respect to conflicts X can be calculated in $O(|X|)$ time.

glue text

Definition 3.6 (Triangulation with Forbidden Edges)
A *triangulation with forbidden edges* $T(V, X, F)$ is a triangulation of the vertex set V with respect to conflicts X which does not contain any of the edges in F:

$$\forall e \in F : e \notin T(V, X, F)$$

glue text

Theorem 3.7 (NP-completeness of Triangulation with Forbidden Edges)
The decision problem whether a triangulation with forbidden edges $T(V, X, F)$ exists is NP-complete. [30, triangulation existence problem]

glue text

Definition 3.8 (Constrained Triangulation)
A *constrained triangulation* $T(V, X, C)$ is a triangulation of the vertex set V with respect to conflicts X which contains the edge constrains C:

$$\forall e \in C : e \in T(V, X, C)$$

3.1. Point Set Triangulations

glue text

Definition 3.9 (Planar Points)
A *planar set of points* or *set of points in the plane* P is a set of points with two coordinates:

$$P \subseteq \{p = (p_x, p_y) : p_x, p_y \in F\}$$

We do not make any assumptions on the coordinates besides F being a field, e.g. the real numbers $\mathbb{R}$. Furthermore, because P is a set, no duplicate points $p = (p_x, p_y) \in P$ and $p' = (p'_x, p'_y) \in P$ with $p_x = p'_x$ and $p_y = p'_y$ are allowed. Note however that we do not require the points in P to be in general position as that would forbid some interesting instances.

do we need this definition at all?

glue text

Definition 3.10 (Line Segments)
A *line segment* $s = (p, q)$ is determined by its endpoints $p, q \in P$ with P being a point set. For compatibility with other definitions, s is directed from p to q, i.e. $(p, q) \neq (q, p)$ and contains all points "between" p and q:

$$m \in s \iff \exists a \in [0, 1] : m = p + a \cdot (q - p)$$

glue text

Definition 3.11 (Crossing)
Two line segments $s_i = (p_i, q_i)$ and $s_j = (p_j, q_j)$ with different slope are *crossing*, if their intersection is not empty and not an endpoint, i.e.

$$s_i, s_j \ \textit{crossing} \iff (p = s_i \cap s_j) \wedge (|s_i \cap s_j| = 1) \wedge (p \notin \{p_i, q_i, p_j, q_j\})$$

Two segments s_i and s_j are *non-crossing* if they are not *crossing*. A set S of segments is *crossing* if at least two segments $s_i, s_j \in S$ are *crossing*. It is *non-crossing* if each pair $s_i, s_j \in S$ is *non-crossing*.

glue text

Definition 3.12 (Overlapping Segments)
Given a point set P and a line segment $s = (p, q)$ with $p, q \in P$. s is *overlapping* in P iff there is a point $p' \in P$ which lies in its interior:

$$s \text{ } overlapping \iff \exists p' \in P : (p' \in s) \wedge (p' \notin \{p, q\})$$

s is *non-overlapping* if it is not *overlapping*.

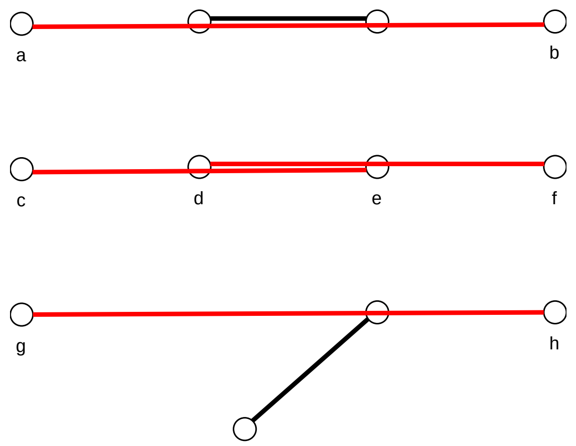


Figure 3.2.: Examples of overlapping segments ($\{a, b\}, \{c, e\}, \{d, f\}, \{g, h\}$)

glue text

Definition 3.13 (Topological Representation)
A vertex set $V(P)$ represents a point set P iff there is exactly one vertex $v_p \in V(P)$ for each point $p \in P$ and v_p can be identified by p and vice versa.

remove comment

Definition 3.14 (Point Set Triangulation)
The triangulation $T(P)$ of a point set P is a triangulation $T(P) = T(V, X)$ where the vertex set $V = V(P)$ represents P, the conflicts X are all *crossing* segments, and which contains no *overlapping* segments:

$$\begin{aligned} (p,q),(p',q') \ \textit{crossing} &\iff \big\{\{v_p, v_q\}, \{v_{p'}, v_{q'}\}\big\} \in X \\ (p,q) \ \textit{overlapping} &\implies \{v_p, v_q\} \notin T(P) \end{aligned}$$

For convenience we define $s = (p,q) \equiv e = \{v_p, v_q\}$ such that $s \in T(P) \iff e \in T(P)$. For a slightly different yet equivalent definition of a point set triangulation see also [2, Section 9.1].

glue text

Theorem 3.15 (Time for Planar Point Set Triangulation)
A point set $P \subseteq \mathbb{R}$ can be triangulated in $O(n \log n)$ time with $n = |P|$. [2, Theorem 9.12]

glue text

Definition 3.16 (Constrained Point Set Triangulation)
Analogous to the Constrained Triangulation (definition 3.8), a *constrained point set triangulation* $T(P, C)$ of a point set P with line segment constraints C is a point set triangulation $T(P, C) = T(P)$ such that $C \subseteq T(P, C)$.

glue text

Theorem 3.17
A constrained point set triangulation $T(P, C)$ for a point set $P \subseteq R$ and line segment constraints $C \subseteq P^2$ can be calculated in $O(n \log n)$ time. [6]

3.2. Intersection Graph

glue text

Definition 3.18 (Intersection Graph)
For a set of line segments S the *intersection graph* $G_{\text{cross}}(S) = (V_S, X)$ consists of a vertex $v_s \in V_S$ for every line segment $s \in S$ and an edge $\{v_{s_i}, v_{s_j}\} \in X$ for every pair of *crossing* segments $s_i, s_j \in S$. It is the geometric equivalent of the conflict graph (definition 3.2).

glue text

Theorem 3.19 (Computational Complexity of Intersection Graph)
Given a set of line segments S the intersection graph $G_{\text{cross}}(S) = (V_S, X)$ can be calculated in $O(m \log m + i \log m)$ time where $m = |V_S| = |S|$ and $i = |X|$ is the number of intersections in S. [2, Lemma 2.3]

glue text

Theorem 3.20 (Complexity of Point Set Intersections)
For a point set P with n points the set of all line segments S with endpoints in P has $\Theta(n^4)$ intersections. [33] Thus calculating the intersection graph for S takes $\Omega(n^4)$ time.

glue text

Theorem 3.21 (Non-Optimality of Sweep Algorithm for Point Sets)
The sweep algorithm presented in [2, Section 2.1] with the time complexity of theorem 3.19 is not optimal for calculating the intersections of all line segments with endpoints in a given point set P as it takes $O(n^4 \log n)$ time.

Algorithmus 3.1 : Naive Intersection Algorithm

Input : Set of segments S

Output : Intersection graph $G_{\text{cross}}(S) = (V_S, X)$

Set $V_S = \{v_s : s \in S\}$

Set $X = \emptyset$

foreach $s \in S$ **do**

  foreach $s_\times \in S$ *with* $|s| \leq |s_\times|$ **do**

    if s *and* $s_\times$ *are crossing* **then**

      Add $\{v_s, v_{s_\times}\}$ to X

return $G_{\text{cross}}(S) = (V_S, X)$

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glue text

Theorem 3.22 (Complexity and Correctness of Algorithm 3.1)
Algorithm 3.1 finds all intersecting line segments in $O(m^2)$ time with $m = |S|$.

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Theorem 3.23 (Optimality of Algorithm 3.1 for Point Set)
Algorithm 3.1 is asymptotically optimal for finding the intersections of all line segments with endpoints in a given point set P as it takes $O(n^4)$ time for $n = |P|$.

3.3. Polygon Triangulations

glue text

Definition 3.24 (Polygon Triangulation)
A triangulation $T(P)$ of a polygon P bounded by line segments are all boundary and interior edges of P in a constrained point set triangulation of the polygon vertices with the polygon boundary as constraints.

glue text

Theorem 3.25 (Generalization of Point Set Triangulation)
Given a point set P, a triangulation $T(\text{conv}(P) \cup P)$ of the polygon bounded by the convex hull $\text{conv}(P)$ of P and containing all inner points of P is a triangulation of P.

3.4. Edge Flipping

glue text

Definition 3.26 (Edge Flip)
Given a triangulation $T(V, X)$ for a vertex set V with respect to a set of conflicts X, (e, f) with $e \in T(V, X)$ and $f \notin T(V, X)$ is an *edge flip* iff $T(V, X) \setminus \{e\} \cup \{f\}$ is a triangulation for V with respect to X.

glue text

Theorem 3.27 (Edge Flips are Conflicts)
Given a triangulation $T(V, X)$ for a vertex set V with respect to a set of conflicts X, every edge flip (e, f) is a conflict: $\{e, f\} \in X$.

Proof:
Assume $\{e, f\} \notin X$. Further assume that

$$\neg\exists e' \in T(V, X) \setminus \{e\} : \{e', f\} \in X.$$

Then f can be added to $T(V, X)$ (without removing e) and therefore $T(V, X)$ is no triangulation—which is a contradiction. Now let $e' \in T(V, X) \setminus \{e\}$ such that $\{e', f\} \in X$. Then $e' \in T(V, X) \setminus \{e\} \cup f$ —which contradicts that $T(V, X) \setminus \{e\} \cup f$ is a triangulation. Therefore every edge flip (e, f) is a conflict.

glue text

Definition 3.28 (Flip Graph)
The flip graph $G_{\text{flip}}(V, X) = (V_T, E_{\text{flip}})$ for a vertex set V and edge conflicts X contains a vertex $v \in V_T$ for every triangulation of V with respect to X and edges $e \in E_{\text{flip}}$ for every possible edge flip.

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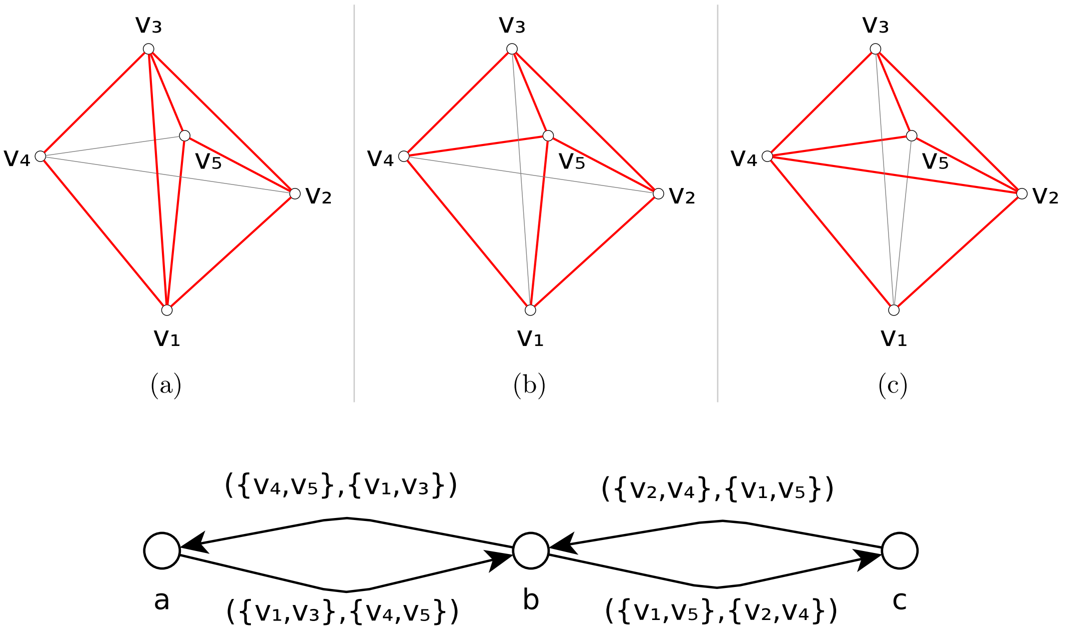

Figure 3.3.: Example triangulations and their flip graph

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Theorem 3.29 (Connectivity of the Flip Graph)
The flip graph $G_{\text{flip}}(V, X)$ is connected in two dimensions [45, Behauptung 4] and has a diameter of at most $6n - 30$ for $n = |V|$. [4] Therefore every triangulation $T(V, X)$ of a vertex set V with respect to edge conflicts X can be transformed into any other triangulation of V with respect to X in $O(n)$ time.

For three dimensions, it is still an open problem whether the flip graph is connected. [13]

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- local vs. global optimal
- hint to edge flipping paper
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notes

3.5. Related Work

After class was over, Greg asked Mrs. Minerva if there are different kinds of triangulations. She replied that the problem of triangulating has kept researchers busy for over 100 years already [22] and that people have found different aspects in that a triangulation can be optimal.

The most famous class is the Delaunay triangulation [2, Section 9.2]. It forces every circumcircle of a triangle to be empty of other points and therefore maximizes the minimum angle [2, Theorem 9.9]. There is an edge flipping algorithm which calculates it in $O(n \log n)$ expected time using $O(n)$ space [2, Theorem 9.12].

There are several other triangulations which can be computed in polynomial time. Minimizing the maximum edge length in $O(n^2)$ times was one of the first results [16]. The counterpart of a Delaunay triangulation, minimizing the maximum angle, takes $O(n^2 \log n)$ time and $O(n)$ space [3]. The same approach can also produce triangulations which maximize the minimum height of a triangle. Finally, the same reference shows also that minimizing the maximum slope and minimizing the maximum eccentricity can both be done in $O(n^3)$ time and $O(n^2)$ space. A triangulation which minimizes or maximizes the area of triangles can be computed in $O(n^2 \log n)$ time with $O(n^2)$ space. [44]

Other triangulations have been proven NP-hard or NP-complete. One of them is to minimize the edge length sum (also known as the minimum weight triangulation) which is NP-hard [35]. Maximizing the minimum edge length was stated an open problem [16] but 20 years later it has been shown that it is NP-complete [18]. The latter one remains NP-hard for polygons with holes and interior points [7] but can be solved in $O(n^3)$ time for simple polygons and even in linear time for convex polygons [23].

4. MaxMin Length Triangulation

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Problem 4.1 (MaxMin Length Triangulation (MMLT))

Given: Set of points P, length function $|s|$ for each line segment s with endpoints in P (e.g. Euclidean distance of the endpoints)

Sought: Point Set Triangulation $T_{\text{opt}} = T(P)$ of P which maximizes $\min\limits_{s \in T_{\text{opt}}} |s|$

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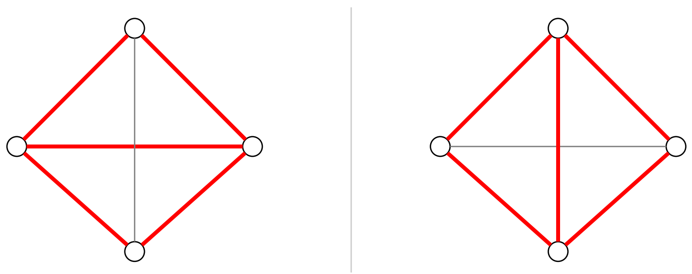


Figure 4.1.: Example of different optimal solutions for the same point set

4.1. Complexity

glue text [18]

Problem 4.2 (Covering by Disjoint Segments (CDS))

Given: Set of line segments S, set of target points $T \subseteq \{s_i \cap s_j : s_i, s_j \in S\}$

Sought: Set of non-intersecting line segments $S_{\text{opt}} \subseteq S$ such that T is covered:

$$\forall p \in T : \exists s \in S_{\text{opt}} : p \in s$$

glue text

Theorem 4.3
Problem 4.2 is NP-complete.

Proof:
CDS can be reduced to Planar 3-SAT (problem 2.7) and vice-versa. For further details of the proof refer to [18].

glue text

Theorem 4.4 (NP-hardness of Problem 4.1)
Problem 4.1 (MMLT) is NP-hard—even for the case where P is planar.

Proof:
...

proof

4.2. Geometric Approaches

```
- sweep line does not work (non-local segments)
- flip does not work (locally non-optimal flip necessary)
```

notes

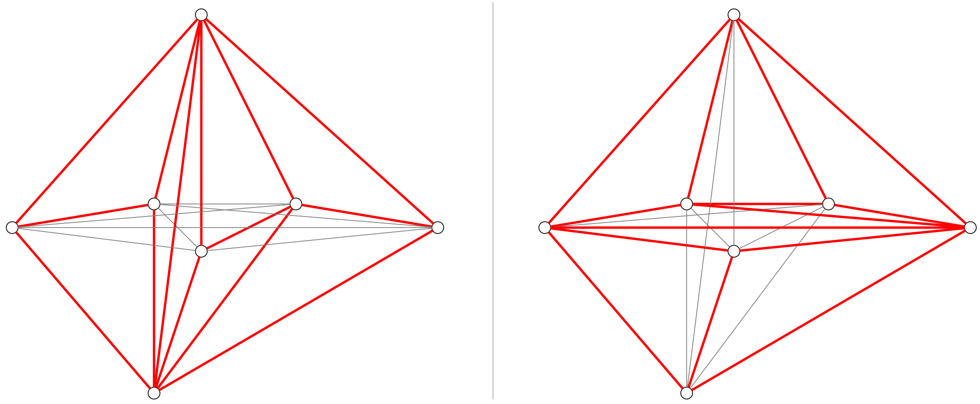


Figure 4.2.: Example of necessary locally non–optimal flips: The shortest edge needs to be flipped in before the optimal solution can be achieved.

4.3. Combinatorial Approach

glue text

Definition 4.5 (Edge Length Order)
Given a set of edges E representing a set of line segments S and two edges $e_{s_i}, e_{s_j} \in E$ representing two line segments $s_i, s_j \in S$, respectively. Let $|s|$ for $s \in S$ be the segment length and $s_i < s_j$ the lexicographical order of $s_i, s_j \in S$. The edge length order is defined as

$$e_{s_i} < e_{s_j} \iff |s_i| < |s_j| \vee ((|s_i| = |s_j|) \wedge (s_i < s_j)).$$

glue text

Definition 4.6 (Edge Length Index)
Given a set of edges E representing a set of line segments S and an edge $e \in E$. The *edge length index* $\mathrm{idx}(e)$ is the index of e in E sorted by edge length order.

glue text

Theorem 4.7 (Uniqueness of Edge Length Index)
For a set of edges E representing line segments S, there can be no two edges $e, e' \in E$ with the same edge length index.

Proof:
For two two segments $s, s' \in S$ the lexicographical order is unique—i.e. either $s < s'$ or $s > s'$ but not both. The same holds for the edge length order.

glue text

Problem 4.8 (MaxMin Edge Length Index Triangulation (MELT))

Given: Set of points P

Sought: Point Set Triangulation $T_{\text{opt}} = T(P)$ of P which maximizes the minimum edge length index $\min\limits_{e \in T_{\text{opt}}} \mathrm{idx}(e)$.

glue text

Theorem 4.9 (Uniqueness of Optimal MELT Solution)
From Theorem 4.7 follows directly that problem 4.8 has a unique optimal solution.

glue text

Theorem 4.10 (Equality of MELT and MMLT)
Every optimal MELT solution is an optimal MMLT solution.

Proof:
Let T_{opt} be the optimal MELT solution for a point set P and assume that that there is a MMLT solution T with $\min\limits_{s \in T} |s| < \min\limits_{s \in T_{\text{opt}}} |s|$. By definitions 4.5 and 4.6 $\text{idx}(\arg\min\limits_{s \in T} |s|) < \text{idx}(\arg\min\limits_{s \in T_{\text{opt}}} |s|)$—which contradicts T_{opt} being optimal.

glue text

Theorem 4.11 (NP-hardness of MELT)
From theorem 4.10 follows that MELT is NP-hard.

glue text

Problem 4.12 (Integer Program (IP) Formulation of MELT)

$$
\begin{aligned}
E &: \textit{non-overlapping} \text{ line segments with endpoints in } P \\
X &: \text{pairs of } \textit{crossing} \text{ line segments} \\
T &: \text{MELT solution}
\end{aligned}
$$

$$
\begin{aligned}
\text{maximize} \quad & \min_{e \in T} x_e \cdot \text{idx}(e) \\
\text{subject to } \forall \{e_i, e_j\} \in X : \quad & x_{e_i} + x_{e_j} \leq 1 \\
\forall e_i \in E : x_{e_i} + & \sum_{\{e_i, e_j\} \in X} x_{e_j} \geq 1 \\
\forall e \in E : \quad & x_e \in \{0, 1\}
\end{aligned}
$$

```
IP:
- maximize min. edge index
- no conflicting edges
- for every edge: either edge or crossing edge picked
- n^2 variables, O(n^4) restrictions
```

notes

4.4. Separators

glue text

Definition 4.13 (Short Edges)
Short edges within a set of edges E are all edges with an edge length index smaller than a certain threshold:

$$E_{\text{short}}(E, \ell) := \{e \in E : \text{idx}(e) < \ell\}$$

glue text

Definition 4.14 (Separators)
Given a set of edges E and edge conflicts $X \subseteq E^2$. The set of separators $E_{\text{sep}}(E, X, e)$ for an edge $e \in E$ are all edges that improve the MELT solution, i.e. all e_{sep} with $\{e, e_{\text{sep}}\} \in X$ which have a higher edge length index:

$$E_{\text{sep}}(E, X, e) := \{e_{\text{sep}} \in E : \text{idx}(e) < \text{idx}(e_{\text{sep}}) \land \{e, e_{\text{sep}}\} \in X\}$$

glue text

Theorem 4.15 (upper bound)
Given the optimal MELT solution T_{opt} for a point set P, let the edge set E represent all *non-overlapping* line segments S with endpoints in P and X be all pairs of *crossing* line segment in S. Every edge $e \in E$ without separators is an upper bound for T_{opt}:

$$\forall e \in E,\ E_{\text{sep}}(E, X, e) = \emptyset : \min_{e_{\min} \in T_{\text{opt}}} \text{idx}(e_{\min}) \leq \text{idx}(e)$$

which is equivalent to

$$\begin{aligned} \forall e \in E : \neg \exists e_{\text{sep}} \in E : \text{idx}(e) < \text{idx}(e_{\text{sep}}) \wedge \ \{e, e_{\text{sep}}\} \in X \\ \implies \min_{e_{\min} \in T_{\text{opt}}} \text{idx}(e_{\min}) \leq \text{idx}(e) \end{aligned}$$

Proof:
Assume

$$\exists e \in E,\ E_{\text{sep}}(E, X, e) = \emptyset : \min_{e_{\min} \in T_{\text{opt}}} \text{idx}(e_{\min}) > \text{idx}(e)$$

This implies $e \notin T_{\text{opt}}$ and

$$\forall e' \in E : \text{idx}(e') < \text{idx}(e) \implies e' \notin T_{\text{opt}}.$$

With $E_{\text{sep}}(E, X, e) = \emptyset$ it follows that $\forall \{e, e_\times\} \in X : e_\times \notin T_{\text{opt}}$. This means that for T_{opt} to be a triangulation e has to be in T_{opt}—which is a contradiction.

glue text

Definition 4.16 (Shortest Non-separable Edge)
Given a set of edges E and edge conflicts $X \subseteq E^2$. The *shortest non-separable edge* e_{nose} is the edge with the smallest edge length index which has no separators:

$$e_{\text{nose}} := \operatorname*{arg\,min}_{e \in E : E_{\text{sep}}(E, X, e) = \emptyset} \text{idx}(e)$$

glue text

glue text

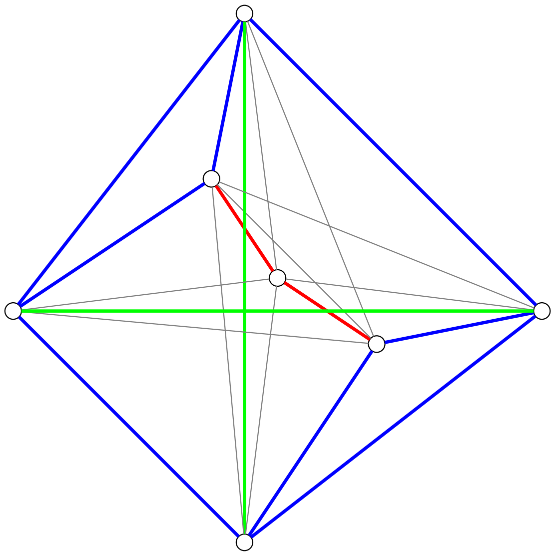


Figure 4.3.: Example where the upper bound from theorem 4.15 is not tight. One of the short segments (red) is part of an optimal MMLT solution because their separators (green) cross. Thus the shortest segment in the solution is shorter than the shortest one of all segments without separators (blue).

Problem 4.17 (Non-Conflicting Separators (NOCS))

Given: Set of short edges E and their separators E_{sep}, edge conflicts $X \subseteq (E \cup E_{\text{sep}})^2$

Sought: Set of non-conflicting edges

$$E_{\text{opt}} \subseteq E \cup \bigcup_{e \in E} E_{\text{sep}}(e)$$

which contains for every edge $e \in E$ either e itself or at least one separator $e_{\text{sep}} \in E_{\text{sep}}(e)$ and maximizes the smallest edge length index $\min_{e \in E_{\text{opt}}} \text{idx}(e)$

glue text

Theorem 4.18 (Equality of MELT and NOCS)
Given a point set P, edges E representing all *non-overlapping* line segments S with endpoints in P, and all pairs of *crossing* line segments X in S, let e_{nose} be the shortest non-separable edge in E. The optimal MELT solution T_{opt} for P and the optimal NOCS solution S_{opt} for $E_{\text{short}}(E, \text{idx}(e_{\text{nose}}))$ have the same value:

$$\min_{e \in T_{\text{opt}}} \text{idx}(e) = \min_{e \in S_{\text{opt}}} \text{idx}(e)$$

Proof:
...

proof

glue text

Theorem 4.19 (NP-hardness of NOCS)
For $E_{\text{short}}(E, \text{idx}(e_{\text{nose}})) \cup e_{\text{nose}} = E$, NOCS and MELT are the same problem. Therefore NOCS is also NP-hard.

glue text

Problem 4.20 (IP Formulation of NOCS)

$$\begin{aligned} E &: \text{short edges} \\ E_{\text{sep}} &: \text{separators} \\ X &: \text{edge conflicts} \\ S &: \text{NOCS solution} \end{aligned}$$

$$\begin{array}{lrr} \text{maximize} & \min_{e \in S} x_e \cdot \text{idx}(e) & \\ \text{subject to} & \forall \{e_i, e_j\} \in X : & x_{e_i} + x_{e_j} \leq 1 \\ & \forall e_i \in E : x_{e_i} + & \sum_{e_j \in E_{\text{sep}}(e_i)} x_{e_j} \geq 1 \\ & \forall e \in E \cup \bigcup_{e \in E} E_{\text{sep}}(e) : & x_e \in \{0, 1\} \end{array}$$

```
IP:
- maximize min. edge index
- no conflicting edges
- for every short edge: either edge or separator picked
- worst case: n^2 variables, O(n^4) restrictions
- random points: O(n) variables, < O(n^3) restrictions?
```

notes

glue text

Definition 4.21 (Completion of NOCS)
Given an optimal NOCS solution T_{opt} for a set of short edges E, their separators E_{sep}, and conflicts X, the completion of T_{opt} is

$$T := T_{\text{opt}} \cup \left\{ e_{\text{sep}} \in \bigcup_{e \in E} E_{\text{sep}}(e) : \neg \exists e \in T : \{e, e_{\text{sep}}\} \in X \right\}.$$

If $T = T_{\text{opt}}$, T_{opt} is *complete*.

glue text

Theorem 4.22 (Optimality of NOCS Completion)
Given an optimal NOCS solution T_{opt} the completion T of T_{opt} is also an optimal NOCS solution.

Proof:
...

proof

Theorem 4.23 (Connection of NOCS and Independent Set)
Every complete optimal NOCS solution T_{opt} for a set of short edges E, their separators E_{sep}, and conflicts $X \subseteq (E \cup E_{\text{sep}})^2$ is a maximal independent set (definition 2.17) for the undirected graph $G = (E \cup E_{\text{sep}}, X)$.

Proof:
...

proof

4.5. Algorithm

...

glue text

update notation in algorithm

use subroutines?

Algorithmus 4.1 : MMLT algorithm

Input : Planar point set P
Output : An optimal MMLT solution for P

Generate the induced segments S_P for P
Find the shortest non-separable segment e_{nose} and calculate intersections for $E_{\text{short}}(|e_{\text{nose}}|)$ and their separators
Set the lower bound $\mathsf{lb} = 0$
Set the upper bound ub such that $S_P[\mathsf{ub}] = e_{\text{nose}}$
Set $\mathsf{last} = \emptyset$
while $\mathsf{lb} < \mathsf{ub}$ **do**
    Set $\mathsf{mid} = \left\lceil \frac{\mathsf{lb}+\mathsf{ub}}{2} \right\rceil$
    Find optimal NOCS solution S_{opt} for $E_{\text{short}}(P, |S_P[\mathsf{ub}|)]$ without using any segment from $E_{\text{short}}(P, |S_P[\mathsf{mid}|)]$
    if *there is a solution* S_{opt} **then**
        Set $\mathsf{last} = S_{\text{opt}}$
        Maximize lb such that $|S_P[\mathsf{lb}]| = \min_{s \in S_{\text{opt}}} |s|$
        if $\mathsf{lb} > \mathsf{ub}$ **then**
            $\mathsf{lb} = \mathsf{ub}$
    else
        if $\mathsf{lb} < \mathsf{mid}$ **then**
            Set $\mathsf{ub} = \mathsf{mid} - 1$
        else
            Set $\mathsf{ub} = \mathsf{lb}$
return *constrained triangulation* T_{opt} *with* last *as constraints*

5. Implementation

In this chapter, we will briefly describe the program components—roughly split into the geometry and the optimization part. To let the program run close to the hardware layer (which usually leads to fast execution times), the code was written in C++. First attempts to use Python instead (for the sake of clarity and better readability) stumbled over the non-readiness of the CGAL bindings and the lack of good alternatives. For the technical documentation, please refer to appendix A.

5.1. Geometry

The geometry part consists of number, point, and segment types, data structures for triangulation, convex hull, and bounding box, and it also handles the segment intersection. For most of it Winfried made extensive use of CGAL, which will be introduced in section 5.1.1.

5.1.1. CGAL

CGAL [5] is an Open Source library (mainly) for computational geometry written in C++. It includes most of the common algorithms in the field and also offers efficient data structures. Through the use of C++ templates it is flexible and extendable: For example it is common to adjust the underlying number types to the application.

5.1.2. Kernel

A kernel in CGAL is something like a computational geometry operating system: It holds the basic type definitions like numbers, points, lines, and line segments. Basic operations such as intersection, angle calculations, or comparisons are also part of it.

In our application we use the built-in `Exact_predicates_inexact_constructions_kernel`[1] [17], which uses double as a number type and is not capable of constructing new geometric objects from existing ones accurately. Both properties lead to faster execution time yet do not produce wrong results in our case.

On top of the CGAL kernel there are two modifications: One is for printing points and segments without the need to use streams, the other one to output them to SVG (see also sections 5.3.2 and 5.3.6). Additionally segments are indexed by length and have the information whether they overlap with other segments attached to them.

[1] Thanks to Michael Hemmer for making me aware that I should use it!

5.1.3. Triangulation

CGAL brings along a constrained triangulation already [9] which triangulates a point set with respect to a given mandatory set of (non-crossing) edges. For this application the class was extended to be drawable to SVG and to find the shortest edge which is part of the triangulation.

5.1.4. Convex Hull

This class directly calls the `convex_hull_2` function of CGAL [10] which itself defaults to the algorithm of Akl & Toussaint [1]. The only extension is that the class serves as container which holds the output points and contains a function to find the shortest segment within.

For the algorithm this class is not necessary as its bound is worse than the one of the SAT solution. It is left in the implementation though as a measure of quality for the other bound.

5.1.5. Intersection Algorithm

To find all pairs of intersecting segments we use algorithm 3.1. For the intersection check itself we make use of the CGAL function `do_intersect` [14]. In contrast to the `intersecttion` function [25], it does not actually compute the intersection and therefore performs much better.[2]

5.1.6. Intersection Graph

The intersection graph (as defined in definition 3.18) stores for every segment the indices of all intersecting segments. This graph data structure (adjacency list) performs well for few edges (in this case intersections) per vertex (in this case line segment)—which we assume here. It may however in future versions of the implementation be replaced by a sparse adjacency matrix from the section 5.3.1 library.

5.2. Optimization

The optimization part itself consists mainly of data structures for SAT problems and solutions, an interface for SAT solvers and the solvers themselves (currently only CPLEX).

Only segment indices and intersections are passed to the SAT problem. This is because geometry does not influence the problem, but only topology. Also it is easier that way to keep track of which segments take part in the restrictions.

5.2.1. SAT problem

This class serves two purposes: To grant an interface to the relevant data for the SAT (i.e. segments and intersections) and to set the short segment range.

[2] Thanks again to Michael Hemmer!

5.2.2. SAT solution

The SAT solution class mainly just stores the segment indices derived from solving the SAT problem — which can be none if no feasible solution is found. Additionally it offers methods for drawing short segments and separators and for finding the shortest segment of the solution.

5.2.3. SAT solver

To unify the way solving the SAT problem is done, there are three interfaces: The base SAT solver and two derived interfaces for decision and optimization problems. They all share methods for adding forbidden segments, intersection and separation restrictions and for running the actual solving. Furthermore there is a method for binding short segments to the objective function for optimization problems.

5.2.4. CPLEX

IBM ILOG CPLEX [24] is a commercial optimization suite written in C. It contains a standalone tool for solving optimization problems and also includes libraries for being used in other programs or even other programming languages. According to [34] CPLEX is one of the two fastest MILP solvers.

In our application we use the Concert API for C++ [8] to access CPLEX. It allows for adding variables and restrictions to a model, extracting them to more efficient data structures and then running several solving algorithms on it.

5.3. Remains

And then there is the rest: A controller class for the whole algorithm which combines all the other components and utility classes for reading input files, debug output and assertions, test case generation, and drawing certain states of the algorithm to SVG.

5.3.1. Boost

Boost [12] is a collection of free (as in freedom) C++ libraries containing tools for various tasks. It makes extensive use of the C++ pre-processor (mostly templates) and aims to extend the C++ standard library (STL).

In our implementation, we use the Spirit [21] library for parsing the JSON input files (see section 5.3.3), and the Program Options library [38] for parsing command line arguments and configuration files. Later versions of the implementation may as well use the Boost Graph Library [43] for the intersection graph, and the Geometry library [19] for creating SVG images (see section 5.3.6).

5.3.2. Qt

Qt [41] is a framework for cross-platform application development written in C++. It features some enhancements to the C++ standard library including an own string class `QString` [40] which supports a different formatting syntax than the `std::string`, offers a UI description format with integration into a even processing framework, and also comes with a simplified built system qmake which wraps platform dependent tools such as GNU make.

We make use of `QString` for our logger class (section 5.3.4), generate SVG images through the `QPainter` class, and build our applications using qmake (which also allows for integration in the C++ IDE Qtcreator).

5.3.3. JSON Parser

JSON Spirit [46] uses the Boost Spirit library [21] for parsing the input point files in JSON (JavaScript Object Notation) [26] format. As of writing, the built-in JSON support of Qt unfortunately has a bug [32] such that it is not compatible with the C++ standard library (and therefore also neither with CGAL nor with Boost). After parsing the input file, all points are stored in a sorted set to allow for fast lookup.

5.3.4. Logger

Our logger class supports different levels of verbosity (debug, info, error, print) and adds the current timestamp to each of them. Additionally, there are shortcut methods for output of measured times, and the current status of the algorithm. Debug output is only included in the programs if they are compiled in debug mode. For convenience, all methods accept `QString`.

5.3.5. Point Generator

For testing and analyzing the algorithm, we generated different instances of point sets and stored them for repeated runs. We hereby rely on the `Random_points_in_square_2` class [42] in combination with the `Creator_uniform_2` class [11]—both part of CGAL (section 5.1.1).

5.3.6. SVG Painter

Mainly for debugging purposes and to visualize different steps of the algorithm, we included the possibility to draw certain data structures to SVG using the `QPainter` class [39].

6. Results

...

3 pages

some very nice graphs

6.1. Technical Details

- Intel® Core™ 2 Duo CPU E6850
- 2 GB RAM
- CGAL version

6.2. Segment Lengths

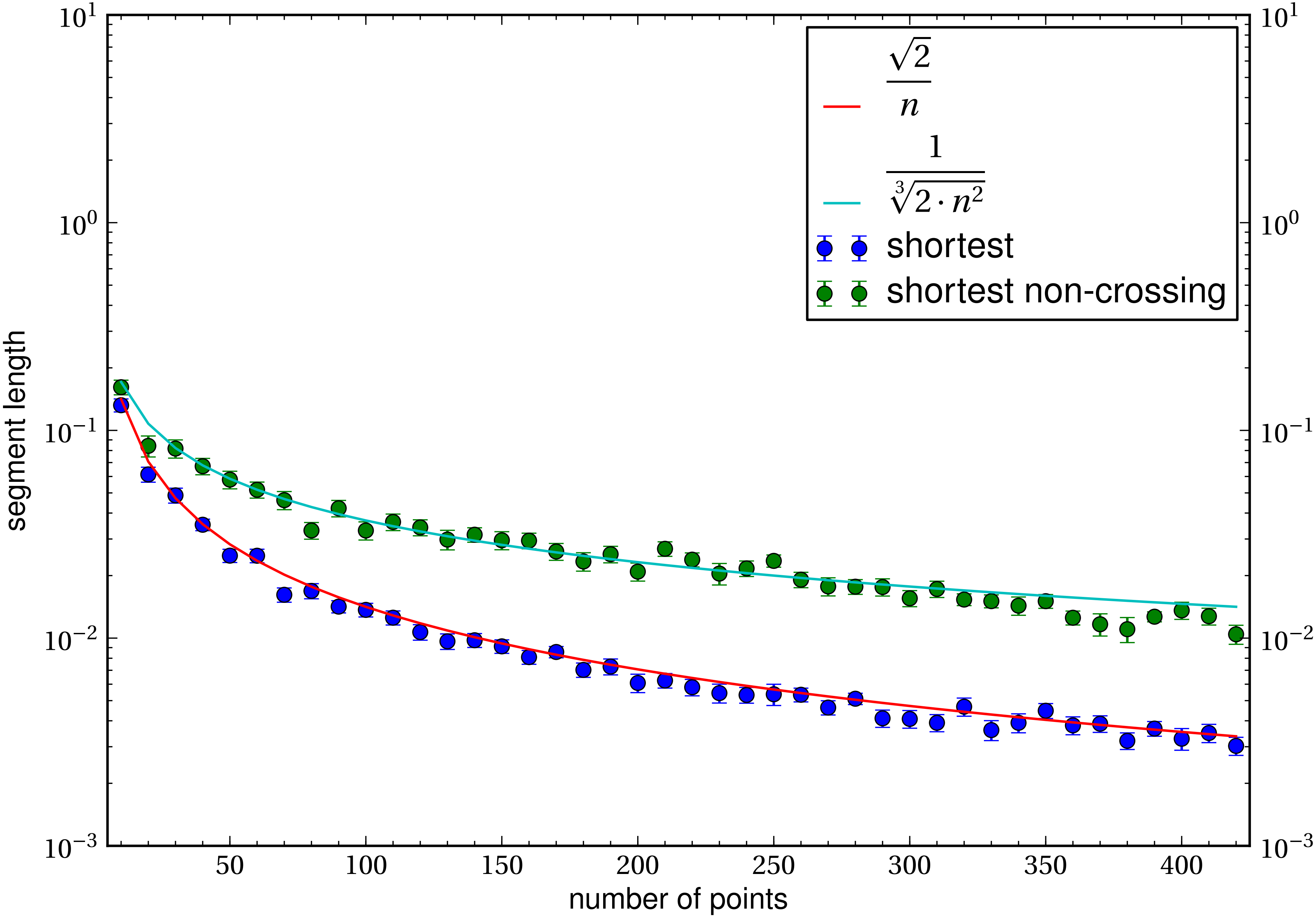
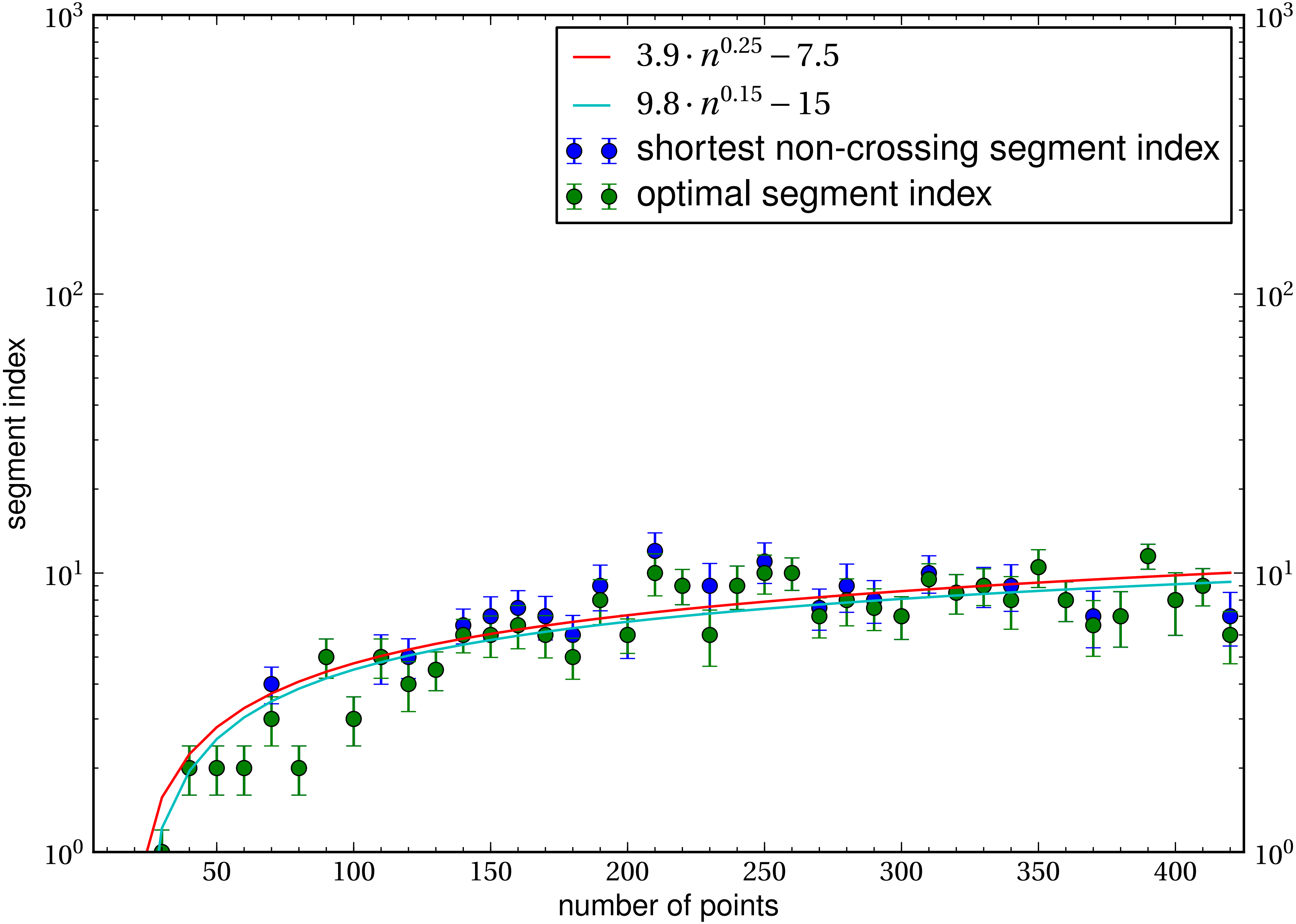


Figure 6.1.: Comparison of segment lengths



$3.9 \cdot n^{0.25} - 7.5$
$9.8 \cdot n^{0.15} - 15$
shortest non-crossing segment index
optimal segment index
segment index
number of points
10^0
10^1
10^2
10^3
50
100
150
200
250
300
350
400

6.3. Execution Time

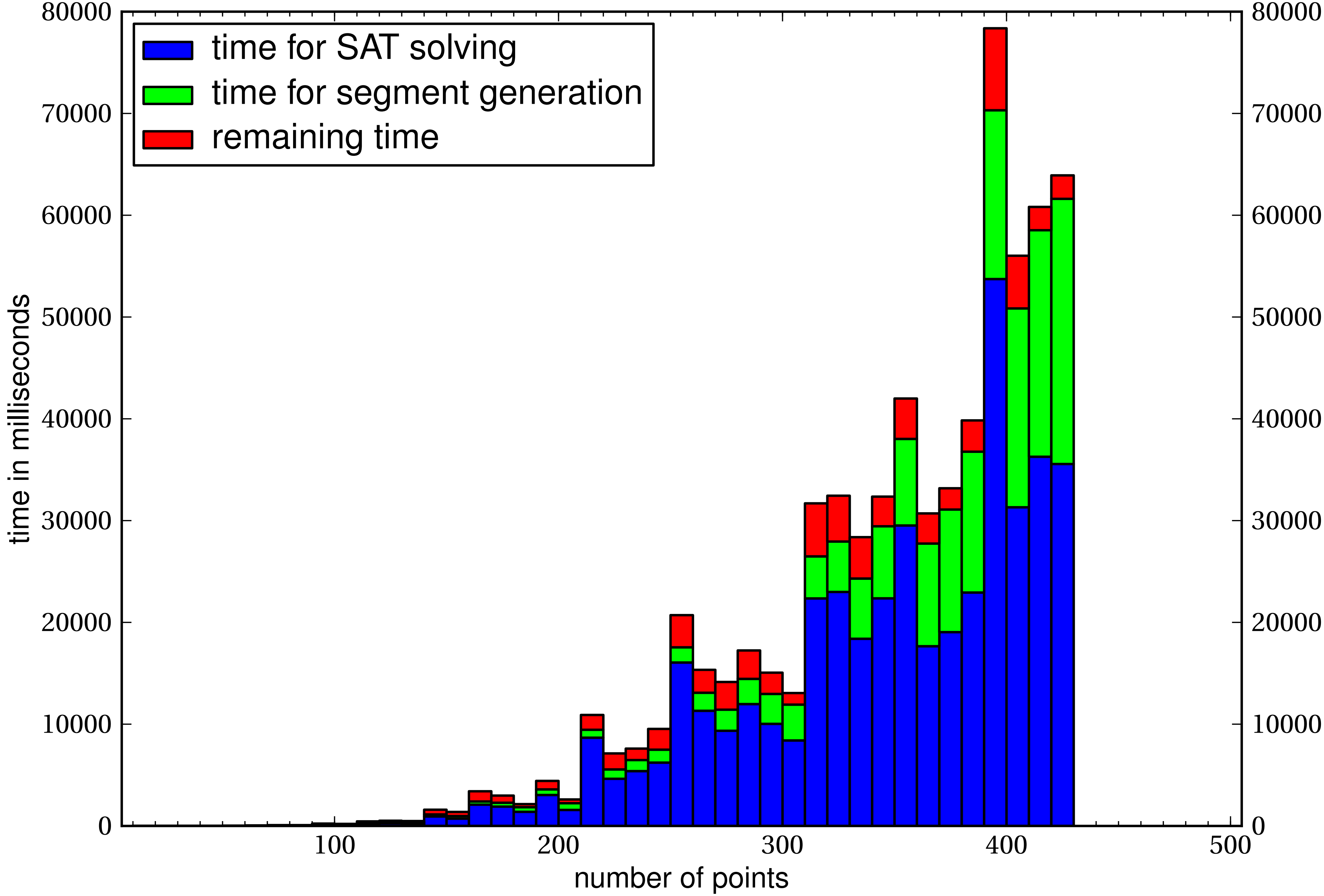


Figure 6.3.: Comparison of execution times

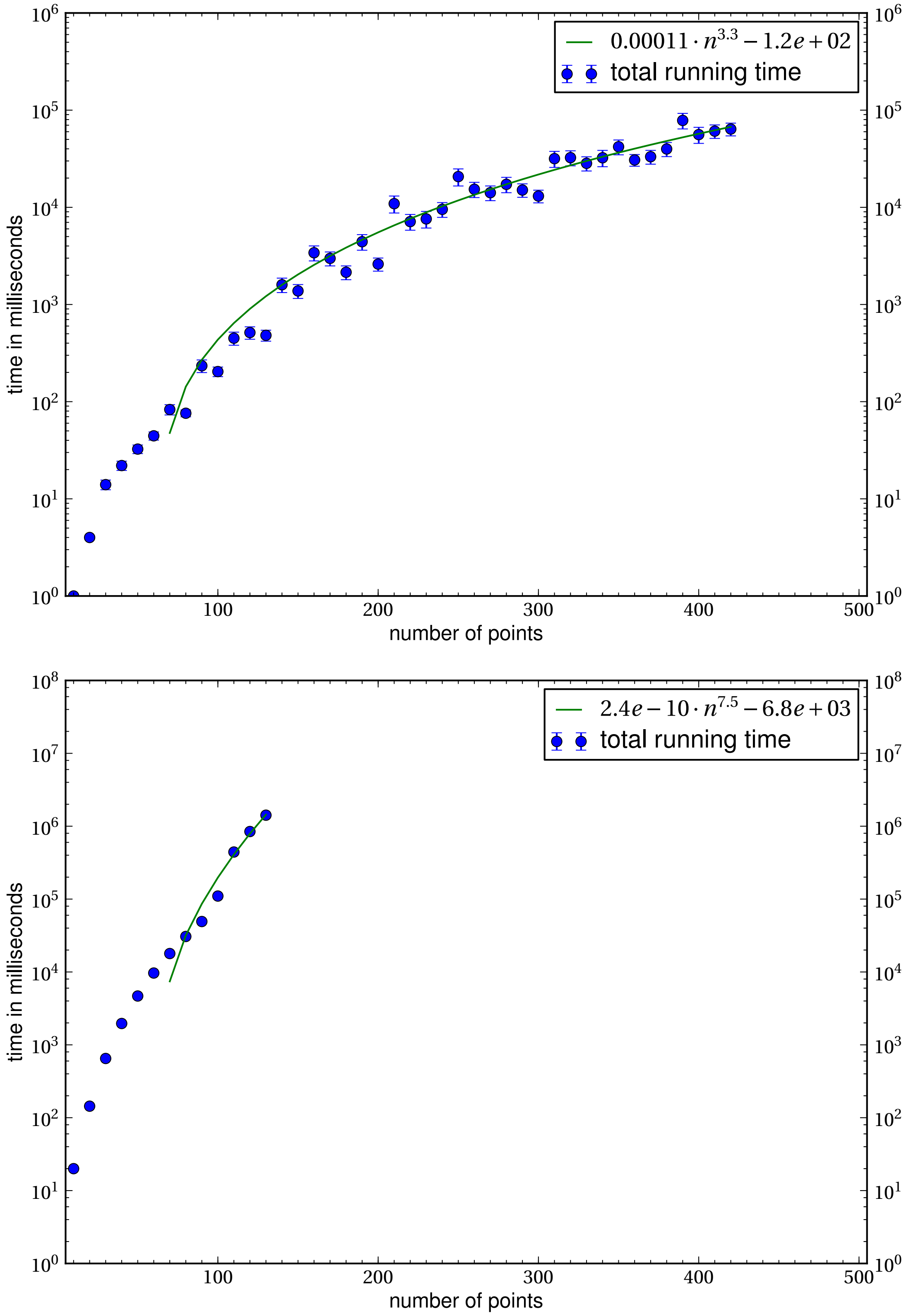


Figure 6.4.: Total execution time

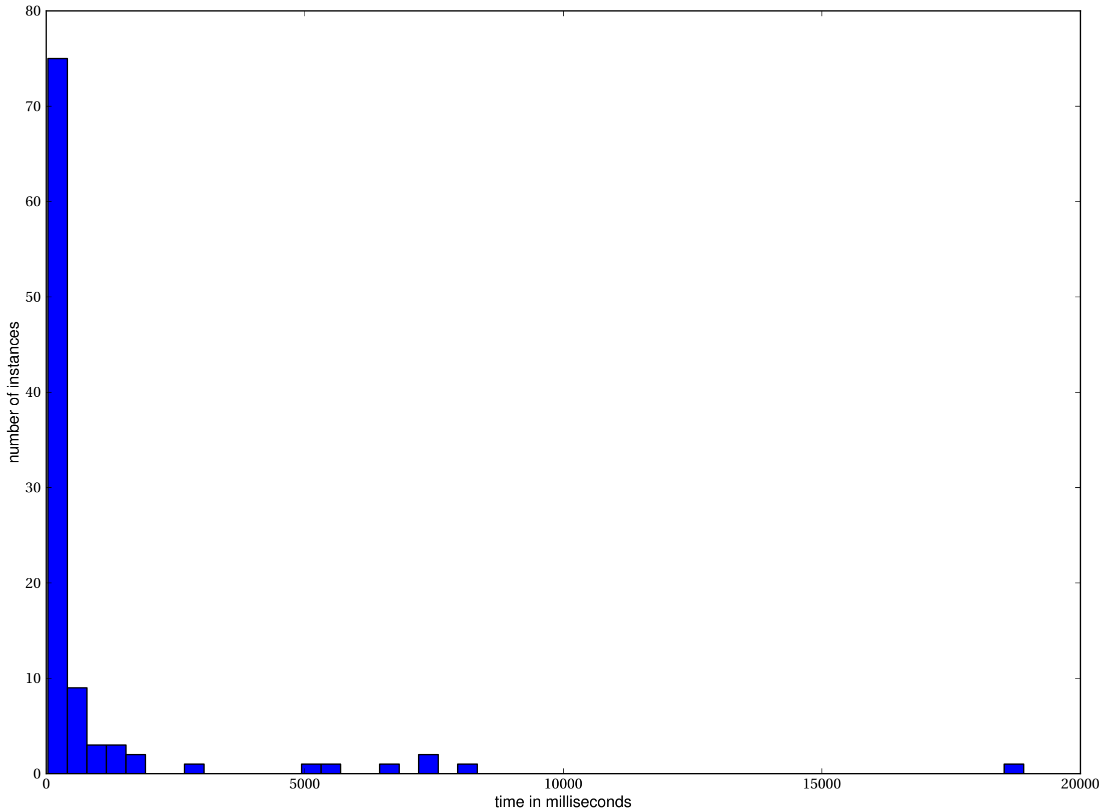


Figure 6.5.: Histogram of execution times for 70 points

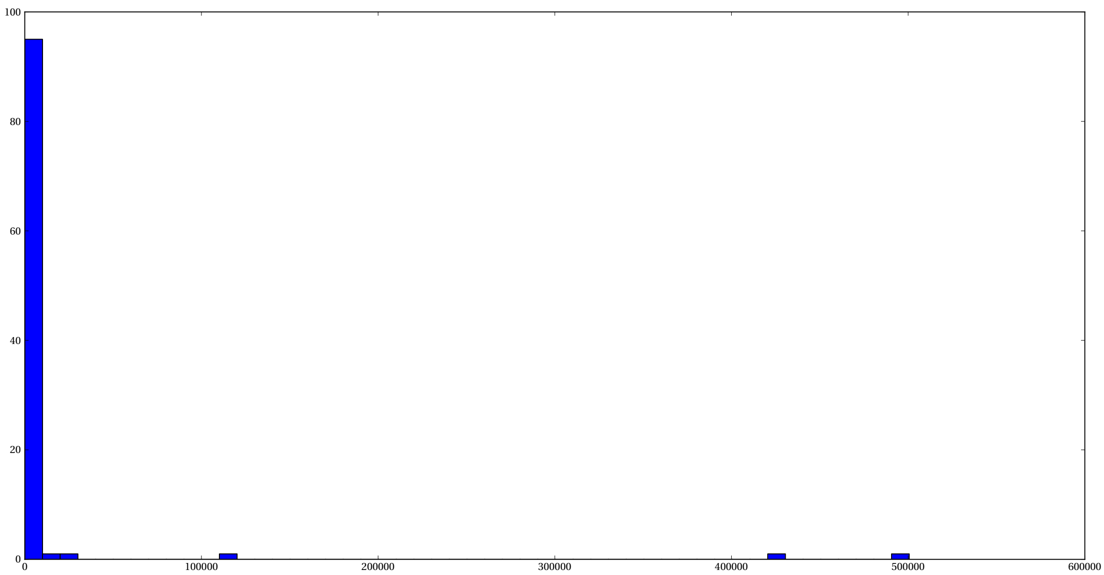


Figure 6.6.: Histogram of execution times for 80 points

6.4. Aborted instances

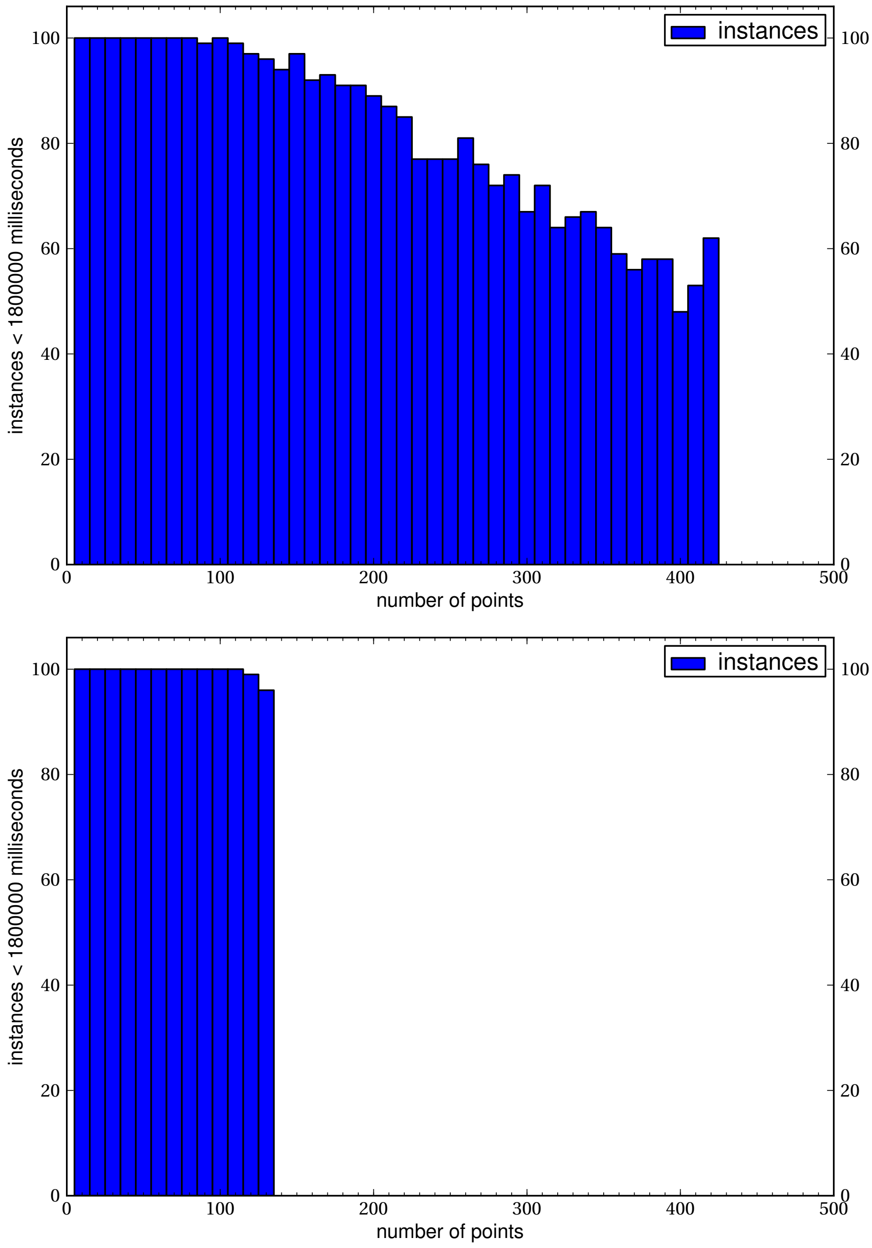


Figure 6.7.: Histogram of finished instances (complete SAT)

7. Conclusion

...

2 pages

With the MaxMin Length Triangulation (MMLT) algorithm at hand Greg made his way to world domination.

be conclusive

```
- other distributions
  * larger grids / true uniform
  * gaussian
  * cluster
- find patterns / graph classes
  * longest edge of each clique?
- polygons
- greedy independent set as lower bound
```

notes

A. Documentation

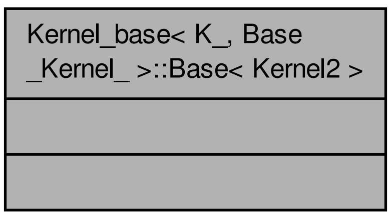
The following is the technical documentation of the program, including class structure and interfaces. It was generated using Doxygen [15]. For an overview of the program components see chapter 5. The source code itself is hosted at: `https://bitbucket.org/winniehell/mmlt`

Contents

1 Class Documentation

1.1 Kernel_base< K_, Base_Kernel_ >::Base< Kernel2 > Struct Template Reference

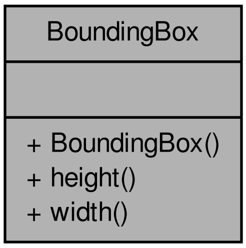
Collaboration diagram for Kernel_base< K_, Base_Kernel_ >::Base< Kernel2 >:



1.2 BoundingBox Class Reference

```
#include <bounding_box.h>
```

Collaboration diagram for BoundingBox:



Public Member Functions

- BoundingBox (const PointSet &points)
- Number height () const
- Number width () const

1.2.1 Constructor & Destructor Documentation

1.2.1.1 BoundingBox::BoundingBox (const PointSet & *points*) `[inline]`

1.2.2 Member Function Documentation

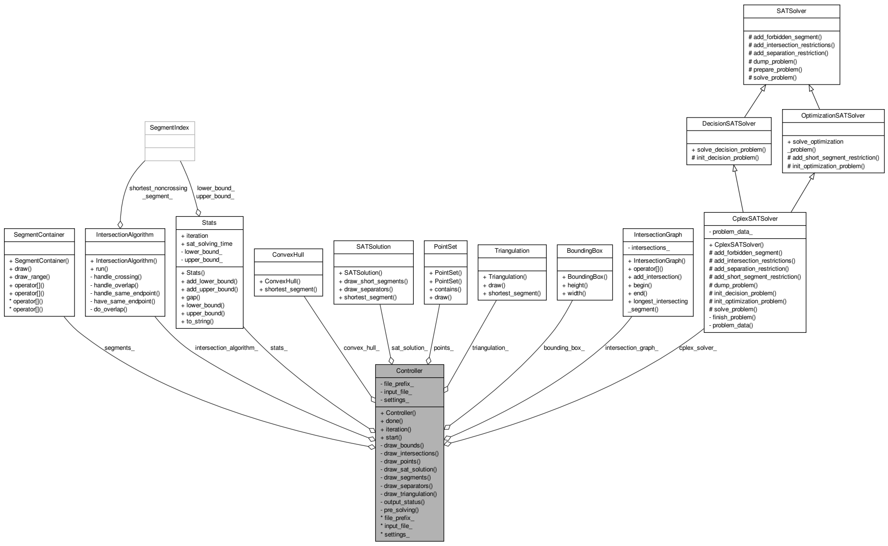
1.2.2.1 Number BoundingBox::height ( ) const `[inline]`

1.2.2.2 Number BoundingBox::width ( ) const `[inline]`

1.3 Controller Class Reference

```
#include <controller.h>
```

Collaboration diagram for Controller:



Public Member Functions

- Controller (const QString &file_prefix, QFile &input_file, const QSettings &settings)
- void done ()
- bool iteration ()
- bool start ()

Private Member Functions

- void draw_bounds () const
- void draw_intersections () const
- void draw_points (SVGPainter &painter) const
- void draw_sat_solution () const
- void draw_segments (SVGPainter &painter) const
- void draw_separators () const
- void draw_triangulation () const
- void output_status () const
- void pre_solving ()

Private Attributes

independent members

- CplexSATSolver cplex_solver_
- IntersectionAlgorithm intersection_algorithm_
- SATSolution sat_solution_
- Stats stats_

input parameters

- const QString & file_prefix_
- QFile & input_file_
- const QSettings & settings_

dependent on input parameter

- const PointSet points_

dependent on input points

- const BoundingBox bounding_box_
- const ConvexHull convex_hull_
- SegmentContainer segments_
- Triangulation triangulation_

dependent on segments

- IntersectionGraph intersection_graph_

1.3.1 Constructor & Destructor Documentation

1.3.1.1 Controller::Controller (const QString & *file_prefix*, QFile & *input_file*, const QSettings & *settings*)

1.3.2 Member Function Documentation

1.3.2.1 void Controller::done ( )

called after the algorithm finished

1.3.2.2 void Controller::draw_bounds ( ) const [private]

1.3.2.3 void Controller::draw_intersections ( ) const [private]

1.3.2.4 void Controller::draw_points (SVGPainter & *painter*) const [private]

1.3.2.5 void Controller::draw_sat_solution ( ) const [private]

1.3.2.6 void Controller::draw_segments (SVGPainter & *painter*) const [private]

1.3.2.7 void Controller::draw_separators ( ) const [private]

1.3.2.8 void Controller::draw_triangulation ( ) const [private]

1.3.2.9 bool Controller::iteration ( )

run next iteration

Returns

true if next iteration should be triggered

1.3.2.10 void Controller::output_status ( ) const [private]

dumps the current algorithm status

1.3.2.11 void Controller::pre_solving ( ) [private]

does some pre-processing

1.3.2.12 bool Controller::start ( )

start the algorithm

Returns

true if iteration should be triggered

1.3.3 Member Data Documentation

1.3.3.1 const BoundingBox Controller::bounding_box_ [private]

1.3.3.2 const ConvexHull Controller::convex_hull_ [private]

1.3.3.3 CplexSATSolver Controller::cplex_solver_ [private]

1.3.3.4 const QString& Controller::file_prefix_ [private]

1.3.3.5 QFile& Controller::input_file_ [private]

1.3.3.6 IntersectionAlgorithm Controller::intersection_algorithm_ [private]

1.3.3.7 IntersectionGraph Controller::intersection_graph_ [private]

1.3.3.8 const PointSet Controller::points_ [private]

1.3.3.9 SATSolution Controller::sat_solution_ [private]

1.3.3.10 SegmentContainer Controller::segments_ [private]

1.3.3.11 const QSettings& Controller::settings_ [private]

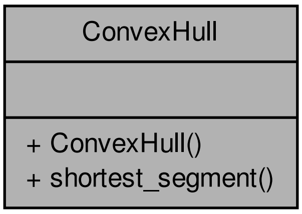
1.3.3.12 Stats Controller::stats_ [private]

1.3.3.13 Triangulation Controller::triangulation_ [private]

1.4 ConvexHull Class Reference

#include <convex_hull.h>

Collaboration diagram for ConvexHull:



Public Member Functions

- ConvexHull (const PointSet &points)
- const SegmentIndex & shortest_segment (const SegmentContainer &segments) const

1.4.1 Constructor & Destructor Documentation

1.4.1.1 ConvexHull::ConvexHull (const PointSet & *points*)

compute convex hull of given point set

1.4.2 Member Function Documentation

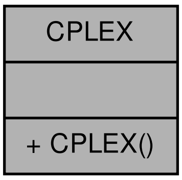
1.4.2.1 const SegmentIndex & ConvexHull::shortest_segment (const SegmentContainer & *segments*) const

find the convex hull segment with minimum length

1.5 CPLEX Class Reference

`#include <concert.h>`

Collaboration diagram for CPLEX:



Public Member Functions

- CPLEX ()

1.5.1 Detailed Description

ugly CPLEX code is not our fault helper class for CPLEX concert API

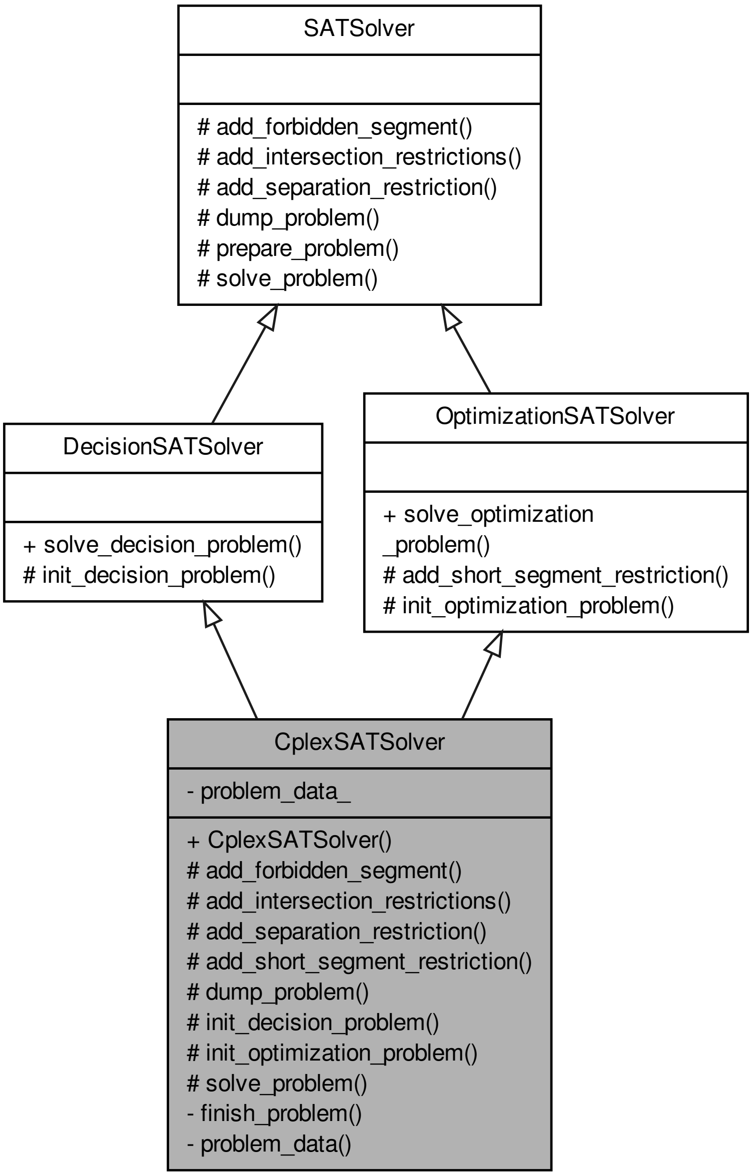
1.5.2 Constructor & Destructor Documentation

1.5.2.1 CPLEX::CPLEX ( ) [inline]

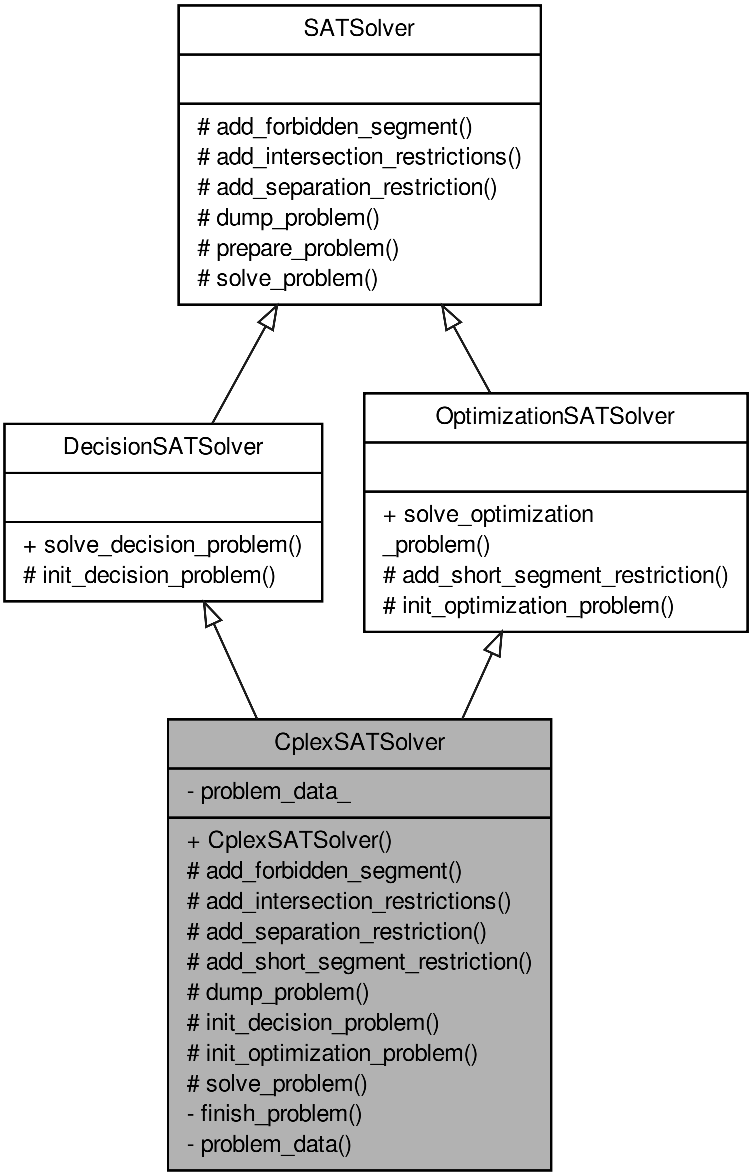
1.6 CplexSATSolver Class Reference

`#include <cplex_sat_solver.h>`

Inheritance diagram for CplexSATSolver:



Collaboration diagram for CplexSATSolver:



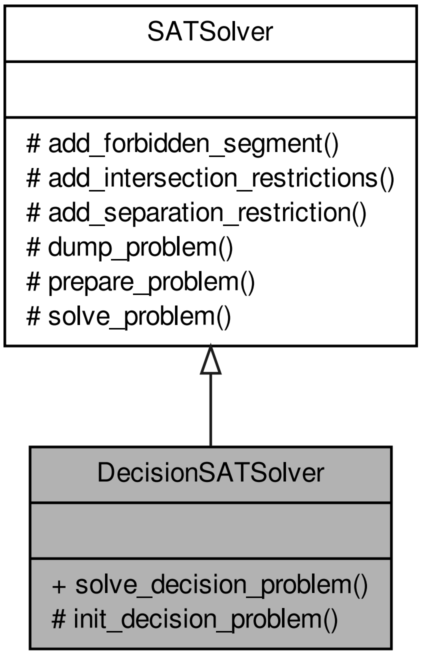
Classes

- struct ProblemData

Public Member Functions

- CplexSATSolver ()

Collaboration diagram for DecisionSATSolver:



Public Member Functions

- void solve_decision_problem (const QSettings &settings, const QString &file_prefix, const SATProblem &problem, SATSolution &solution)

Protected Member Functions

- virtual void init_decision_problem (const SATProblem ∗problem)=0

1.6.1 Detailed Description

interface for decision SAT solvers

1.6.2 Member Function Documentation

1.6.2.1 virtual void DecisionSATSolver::init_decision_problem (const SATProblem ∗ *problem*) [protected], [pure virtual]

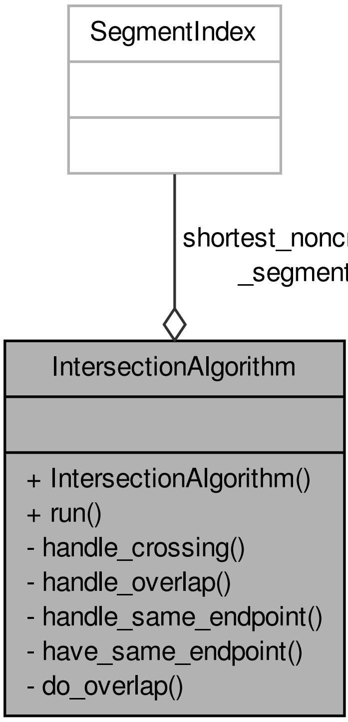
Implemented in CplexSATSolver.

1.6.2.2 void DecisionSATSolver::solve_decision_problem (const QSettings & *settings,* const QString & *file_prefix,* const SATProblem & *problem,* SATSolution & *solution*)

1.7 IntersectionAlgorithm Class Reference

```
#include <intersection_algorithm.h>
```

Collaboration diagram for IntersectionAlgorithm:



Public Member Functions

- IntersectionAlgorithm ()
- void run (IntersectionGraph &igraph, SegmentContainer &segments)

Public Attributes

- SegmentIndex shortest_noncrossing_segment_

Private Member Functions

- void handle_crossing (IntersectionGraph &igraph, const Segment &s1, const Segment &s2)
- void handle_overlap (IntersectionGraph &igraph, const Segment &s1, const Segment &s2)
- void handle_same_endpoint (const Segment &s1, const Segment &s2) const
- bool have_same_endpoint (const Segment &s1, const Segment &s2) const
- bool do_overlap (Segment &s1, Segment &s2) const

1.7.1 Constructor & Destructor Documentation

1.7.1.1 IntersectionAlgorithm::IntersectionAlgorithm ( )

1.7.2 Member Function Documentation

1.7.2.1 bool IntersectionAlgorithm::do_overlap (Segment & *s1*, Segment & *s2*) const [private]

checks if two segments overlap

Returns

the outer segment

1.7.2.2 void IntersectionAlgorithm::handle_crossing (IntersectionGraph & *igraph*, const Segment & *s1*, const Segment & *s2*) [private]

segments cross

1.7.2.3 void IntersectionAlgorithm::handle_overlap (IntersectionGraph & *igraph*, const Segment & *s1*, const Segment & *s2*) [private]

segments intersect but do not cross

1.7.2.4 void IntersectionAlgorithm::handle_same_endpoint (const Segment & *s1*, const Segment & *s2*) const [private]

segments have the same end point

1.7.2.5 bool IntersectionAlgorithm::have_same_endpoint (const Segment & *s1*, const Segment & *s2*) const [private]

checks if two segments share an endpoint

Returns

the endpoint

1.7.2.6 void IntersectionAlgorithm::run (IntersectionGraph & *igraph*, SegmentContainer & *segments*)

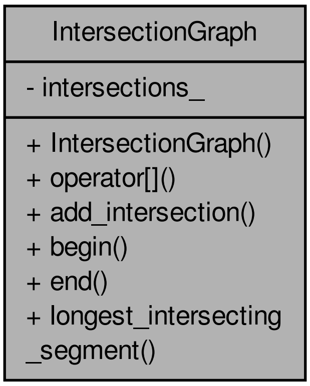
1.7.3 Member Data Documentation

1.7.3.1 SegmentIndex IntersectionAlgorithm::shortest_noncrossing_segment_

1.8 IntersectionGraph Class Reference

```
#include <intersection_graph.h>
```

Collaboration diagram for IntersectionGraph:



Public Member Functions

- IntersectionGraph (const SegmentIndex &size)
- const Intersections & operator[] (const SegmentIndex &index) const
- void add_intersection (const Segment &s1, const Segment &s2)
- IntersectionsVector::const_iterator begin () const
- IntersectionsVector::const_iterator end () const
- const SegmentIndex & longest_intersecting_segment (const SegmentIndex &index) const

Private Attributes

- IntersectionsVector intersections_

1.8.1 Constructor & Destructor Documentation

1.8.1.1 IntersectionGraph::IntersectionGraph (const SegmentIndex & *size*)

default constructor

1.8.2 Member Function Documentation

1.8.2.1 void IntersectionGraph::add_intersection (const Segment & *s1*, const Segment & *s2*)

add two intersecting segments to the graph

1.8.2.2 IntersectionsVector::const_iterator IntersectionGraph::begin ( ) const [inline]

1.8.2.3 IntersectionsVector::const_iterator IntersectionGraph::end ( ) const [inline]

1.8.2.4 const SegmentIndex & IntersectionGraph::longest_intersecting_segment (const SegmentIndex & *index*) const

1.8.2.5 const Intersections& IntersectionGraph::operator[] (const SegmentIndex & *index*) const [inline]

Returns

all intersecting segments for a segment

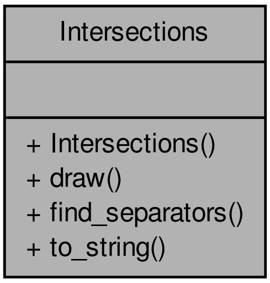
1.8.3 Member Data Documentation

1.8.3.1 IntersectionsVector IntersectionGraph::intersections_ [private]

1.9 Intersections Class Reference

`#include <intersections.h>`

Collaboration diagram for Intersections:



Public Member Functions

- Intersections ()
- void draw (QPainter &painter, const SegmentContainer &segments) const
- void find_separators (const SegmentIndex &segment_index, const SegmentContainer &segments, std::vector< SegmentIndex > &separators) const
- QString to_string (const SegmentContainer &segments) const

1.9.1 Detailed Description

sorted set of intersecting segments

1.9.2 Constructor & Destructor Documentation

1.9.2.1 Intersections::Intersections ( ) [inline]

default constructor

1.9.3 Member Function Documentation

1.9.3.1 void Intersections::draw (QPainter & *painter*, const SegmentContainer & *segments*) const

draws intersections using QPainter

1.9.3.2 void Intersections::find_separators (const SegmentIndex & *segment_index*, const SegmentContainer & *segments*, std::vector< SegmentIndex > & *separators*) const

finds all separators for a given segment and stores them in the passed container

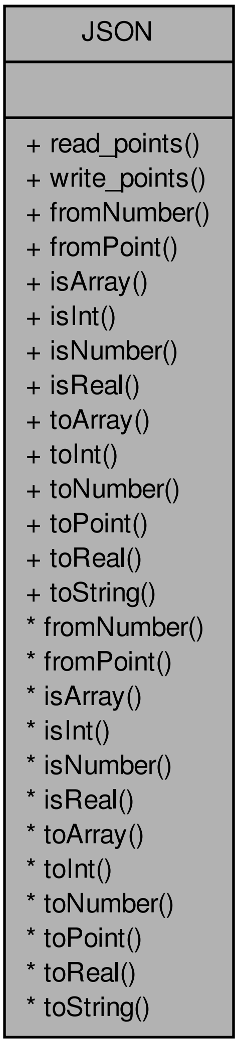
1.9.3.3 QString Intersections::to_string (const SegmentContainer & *segments*) const

output intersections to QString

1.10 JSON Class Reference

`#include <json.h>`

Collaboration diagram for JSON:



Static Public Member Functions

- template<typename OutputIterator >
  static bool read_points (QFile &file, OutputIterator output)
- template<typename Container >
  static bool write_points (const std::string &file_name, Container points)

helper functions

- static JSONValue fromNumber (const Number &value)
- static JSONArray fromPoint (const Point &point)
- static bool isArray (const JSONValue &value)
- static bool isInt (const JSONValue &value)
- static bool isNumber (const JSONValue &value)
- static bool isReal (const JSONValue &value)
- static const JSONArray & toArray (const JSONValue &value)
- static int toInt (const JSONValue &value)
- static Number toNumber (const JSONValue &value)
- static Point toPoint (const JSONValue &value)
- static double toReal (const JSONValue &value)
- static const std::string & toString (const JSONValue &value)

1.10.1 Member Function Documentation

1.10.1.1 JSON::JSONValue JSON::fromNumber (const Number & *value*) [static]

1.10.1.2 JSON::JSONArray JSON::fromPoint (const Point & *point*) [static]

1.10.1.3 bool JSON::isArray (const JSONValue & *value*) [static]

1.10.1.4 bool JSON::isInt (const JSONValue & *value*) [static]

1.10.1.5 bool JSON::isNumber (const JSONValue & *value*) [static]

1.10.1.6 bool JSON::isReal (const JSONValue & *value*) [static]

1.10.1.7 template<typename OutputIterator > static bool JSON::read_points (QFile & *file,* OutputIterator *output*) [inline], [static]

1.10.1.8 const JSON::JSONArray & JSON::toArray (const JSONValue & *value*) [static]

1.10.1.9 int JSON::toInt (const JSONValue & *value*) [static]

1.10.1.10 Number JSON::toNumber (const JSONValue & *value*) [static]

1.10.1.11 Point JSON::toPoint (const JSONValue & *value*) [static]

1.10.1.12 double JSON::toReal (const JSONValue & *value*) [static]

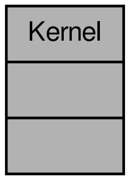
1.10.1.13 const std::string & JSON::toString (const JSONValue & *value*) [static]

1.10.1.14 template<typename Container > static bool JSON::write_points (const std::string & *file_name,* Container *points*) [inline], [static]

1.11 Kernel Struct Reference

```
#include <kernel.h>
```

Collaboration diagram for Kernel:



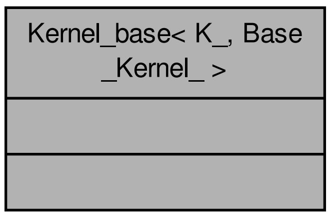
1.11.1 Detailed Description

customized kernel

1.12 Kernel_base< K_, Base_Kernel_ > Class Template Reference

```
#include <kernel.h>
```

Collaboration diagram for Kernel_base< K_, Base_Kernel_ >:



Classes

- struct Base

1.12.1 Detailed Description

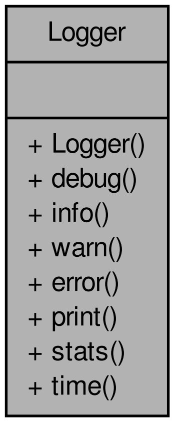
template< typename K_, typename Base_Kernel_ >class Kernel_base< K_, Base_Kernel_ >

kernel base with customized PointC2 and SegmentC2

1.13 Logger Class Reference

```
#include <logger.h>
```

Collaboration diagram for Logger:



Public Member Functions

- Logger ()
- void debug (const QString &message) const
- void info (const QString &message) const
- void warn (const QString &message) const
- void error (const QString &message) const
- void print (const QString &message) const
- void stats (const Stats &stats) const
- void time (const QString &identifier, int milliseconds) const

1.13.1 Constructor & Destructor Documentation

1.13.1.1 Logger::Logger ( )

1.13.2 Member Function Documentation

1.13.2.1 void Logger::debug (const QString & *message*) const

1.13.2.2 void Logger::error (const QString & *message*) const

1.13.2.3 void Logger::info (const QString & *message*) const

1.13.2.4 void Logger::print (const QString & *message*) const

1.13.2.5 void Logger::stats (const Stats & *stats*) const

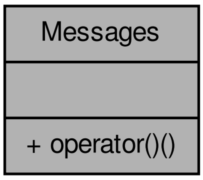
1.13.2.6 void Logger::time (const QString & *identifier,* int *milliseconds*) const

1.13.2.7 void Logger::warn (const QString & *message*) const

1.14 Messages Class Reference

```
#include <logger.h>
```

Collaboration diagram for Messages:



Public Member Functions

- QString operator() (const char ∗text)

1.14.1 Detailed Description

helper class for string literals

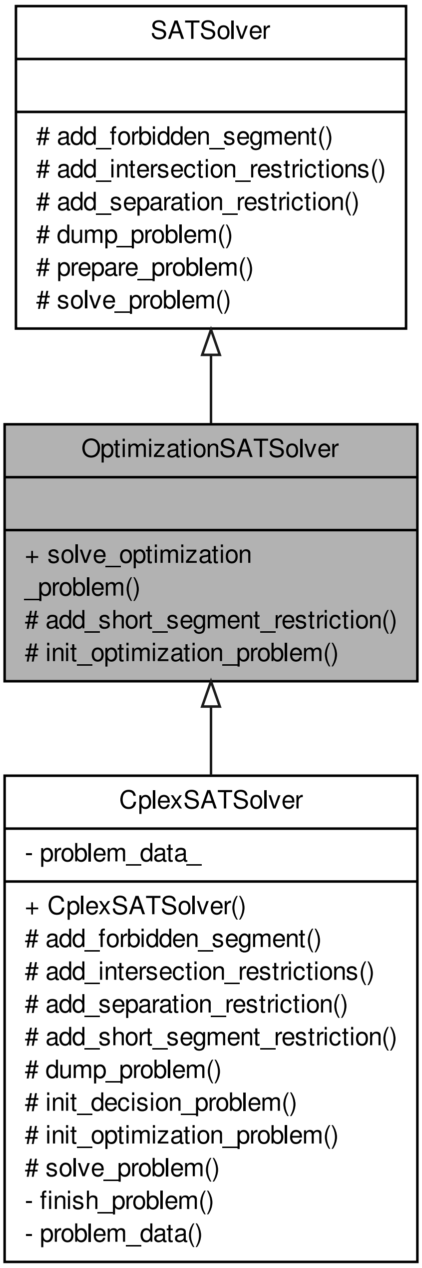
1.14.2 Member Function Documentation

1.14.2.1 QString Messages::operator() (const char ∗ *text*) [inline]

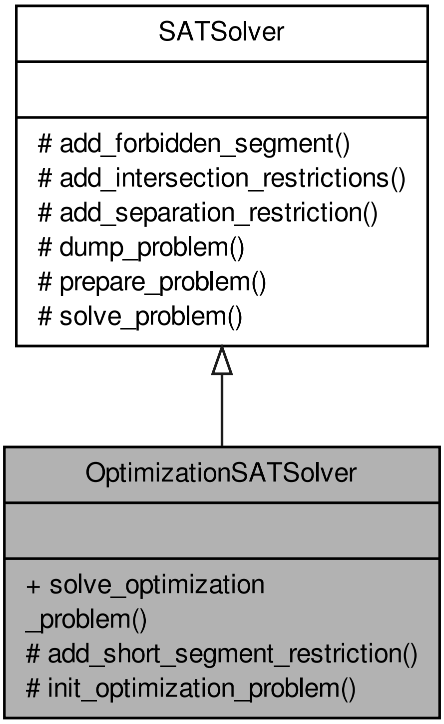
1.15 OptimizationSATSolver Class Reference

```
#include <sat_solver.h>
```

Inheritance diagram for OptimizationSATSolver:



Collaboration diagram for OptimizationSATSolver:



Public Member Functions

- void solve_optimization_problem (const QSettings &settings, const QString &file_prefix, const SATProblem &problem, SATSolution &solution)

Protected Member Functions

- virtual void add_short_segment_restriction (const SATProblem ∗problem, const SegmentIndex &index)=0
- virtual void init_optimization_problem (const SATProblem ∗problem)=0

1.15.1 Detailed Description

interface for optimization SAT solvers

1.15.2 Member Function Documentation

1.15.2.1 virtual void OptimizationSATSolver::add_short_segment_restriction (const SATProblem ∗ *problem*, const SegmentIndex & *index*) [protected], [pure virtual]

Implemented in CplexSATSolver.

1.15.2.2 virtual void OptimizationSATSolver::init_optimization_problem (const SATProblem * *problem*) [protected], [pure virtual]

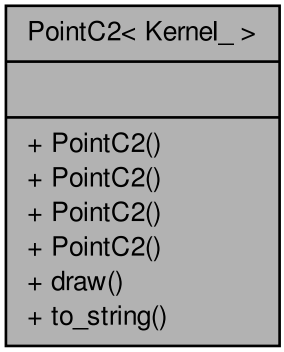
Implemented in CplexSATSolver.

1.15.2.3 void OptimizationSATSolver::solve_optimization_problem (const QSettings & *settings,* const QString & *file_prefix,* const SATProblem & *problem,* SATSolution & *solution*)

1.16 PointC2< Kernel_ > Class Template Reference

```
#include <point.h>
```

Collaboration diagram for PointC2< Kernel_ >:



Public Member Functions

- PointC2 ()
- PointC2 (const CGAL::Origin &origin)
- PointC2 (const FT &x, const FT &y)
- PointC2 (const FT &hx, const FT &hy, const FT &hw)
- void draw (QPainter &painter) const
- QString to_string () const

Private Types

- typedef Kernel_::FT FT
- typedef CGAL::PointC2< Kernel_ > PointBase

1.16.1 Detailed Description

template<class Kernel_>class PointC2< Kernel_ >

customized point type

1.16.2 Member Typedef Documentation

1.16.2.1 template<class Kernel_ > typedef Kernel_::FT **PointC2**< Kernel_ >::**FT** `[private]`

1.16.2.2 template<class Kernel_ > typedef CGAL::PointC2<Kernel_> **PointC2**< Kernel_ >::**PointBase** `[private]`

1.16.3 Constructor & Destructor Documentation

1.16.3.1 template<class Kernel_ > **PointC2**< Kernel_ >::PointC2 ( ) `[inline]`

empty constructor

1.16.3.2 template<class Kernel_ > **PointC2**< Kernel_ >::PointC2 (const CGAL::Origin & *origin*) `[inline]`

origin constructor

1.16.3.3 template<class Kernel_ > **PointC2**< Kernel_ >::PointC2 (const **FT** & *x*, const **FT** & *y*) `[inline]`

Cartesian constructor

1.16.3.4 template<class Kernel_ > **PointC2**< Kernel_ >::PointC2 (const **FT** & *hx*, const **FT** & *hy*, const **FT** & *hw*) `[inline]`

homogeneous constructor

1.16.4 Member Function Documentation

1.16.4.1 template<class Kernel_ > void **PointC2**< Kernel_ >::draw (QPainter & *painter*) const

draw segment using given QPainter

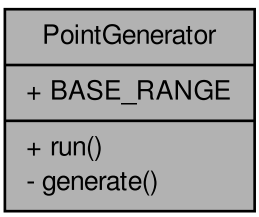
1.16.4.2 template<class Kernel_ > QString **PointC2**< Kernel_ >::to_string ( ) const

dump point to QString

1.17 PointGenerator Class Reference

```
#include <point_generator.h>
```

Collaboration diagram for PointGenerator:



Static Public Member Functions

- static void run (const QSettings &settings)

Static Public Attributes

- static const double BASE_RANGE = 100.0

Static Private Member Functions

- template<typename GeneratorType >
  static void generate (const QString &base_name, std::size_t num_points, std::size_t num_iterations)

1.17.1 Member Function Documentation

1.17.1.1 template<typename GeneratorType > static void PointGenerator::generate (const QString & *base_name*, std::size_t *num_points*, std::size_t *num_iterations*) [inline], [static], [private]

1.17.1.2 static void PointGenerator::run (const QSettings & *settings*) [inline], [static]

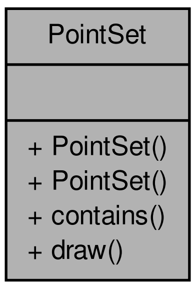
1.17.2 Member Data Documentation

1.17.2.1 const double PointGenerator::BASE_RANGE = 100.0 [static]

1.18 PointSet Class Reference

#include <point_set.h>

Collaboration diagram for PointSet:



Public Member Functions

- PointSet ()
- PointSet (QFile &input_file)
- bool contains (const Point &point) const
- void draw (QPainter &painter) const

1.18.1 Detailed Description

(sorted) set of points

1.18.2 Constructor & Destructor Documentation

1.18.2.1 PointSet::PointSet ( )

empty set

1.18.2.2 PointSet::PointSet (QFile & *input_file*)

read points from file

1.18.3 Member Function Documentation

1.18.3.1 bool PointSet::contains (const Point & *point*) const [inline]

shortcut for STL count()

Returns

true if point is in set

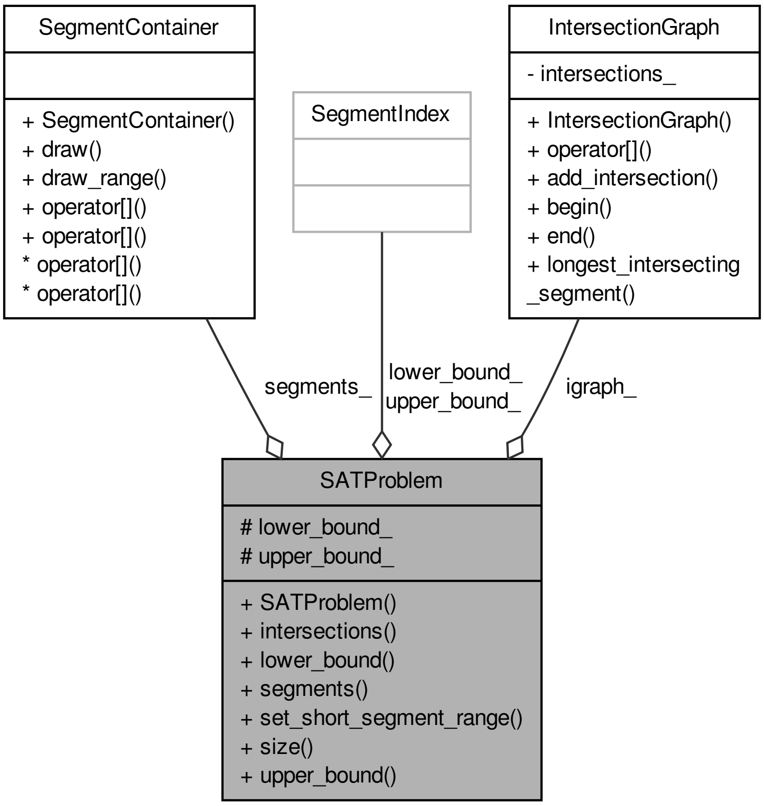
1.18.3.2 void PointSet::draw (QPainter & *painter*) const

output point set using QPainter

1.19 SATProblem Class Reference

```
#include <sat_problem.h>
```

Collaboration diagram for SATProblem:



Public Member Functions

- SATProblem (const IntersectionGraph &igraph, const SegmentContainer &segments)
- const Intersections & intersections (const SegmentIndex &index) const
- const SegmentIndex & lower_bound () const
- const SegmentContainer & segments () const
- void set_short_segment_range (const SegmentIndex &lower_bound, const SegmentIndex &upper_bound)
- SegmentIndex size () const
- const SegmentIndex & upper_bound () const

Protected Attributes

- const IntersectionGraph & igraph_
- const SegmentContainer & segments_
- SegmentIndex lower_bound_
- SegmentIndex upper_bound_

1.19.1 Constructor & Destructor Documentation

1.19.1.1 SATProblem::SATProblem (const IntersectionGraph & *igraph*, const SegmentContainer & *segments*)

default constructor

1.19.2 Member Function Documentation

1.19.2.1 const Intersections& SATProblem::intersections (const SegmentIndex & *index*) const [inline]

1.19.2.2 const SegmentIndex& SATProblem::lower_bound ( ) const [inline]

1.19.2.3 const SegmentContainer& SATProblem::segments ( ) const [inline]

1.19.2.4 void SATProblem::set_short_segment_range (const SegmentIndex & *lower_bound,* const SegmentIndex & *upper_bound*)

set range of segments to consider

1.19.2.5 SegmentIndex SATProblem::size ( ) const [inline]

1.19.2.6 const SegmentIndex& SATProblem::upper_bound ( ) const [inline]

1.19.3 Member Data Documentation

1.19.3.1 const IntersectionGraph& SATProblem::igraph_ [protected]

1.19.3.2 SegmentIndex SATProblem::lower_bound_ [protected]

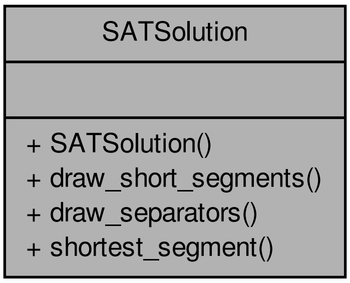
1.19.3.3 const SegmentContainer& SATProblem::segments_ [protected]

1.19.3.4 SegmentIndex SATProblem::upper_bound_ [protected]

1.20 SATSolution Class Reference

`#include <sat_solution.h>`

Collaboration diagram for SATSolution:



Public Member Functions

- SATSolution ()
- void draw_short_segments (QPainter &painter, const SegmentIndex &num_short_segments, const Segment-Container &segments) const
- void draw_separators (QPainter &painter, const SegmentIndex &num_short_segments, const Segment-Container &segments) const
- const SegmentIndex & shortest_segment () const

1.20.1 Constructor & Destructor Documentation

1.20.1.1 SATSolution::SATSolution ( ) [inline]

1.20.2 Member Function Documentation

1.20.2.1 void SATSolution::draw_separators (QPainter & *painter*, const SegmentIndex & *num_short_segments*, const SegmentContainer & *segments*) const

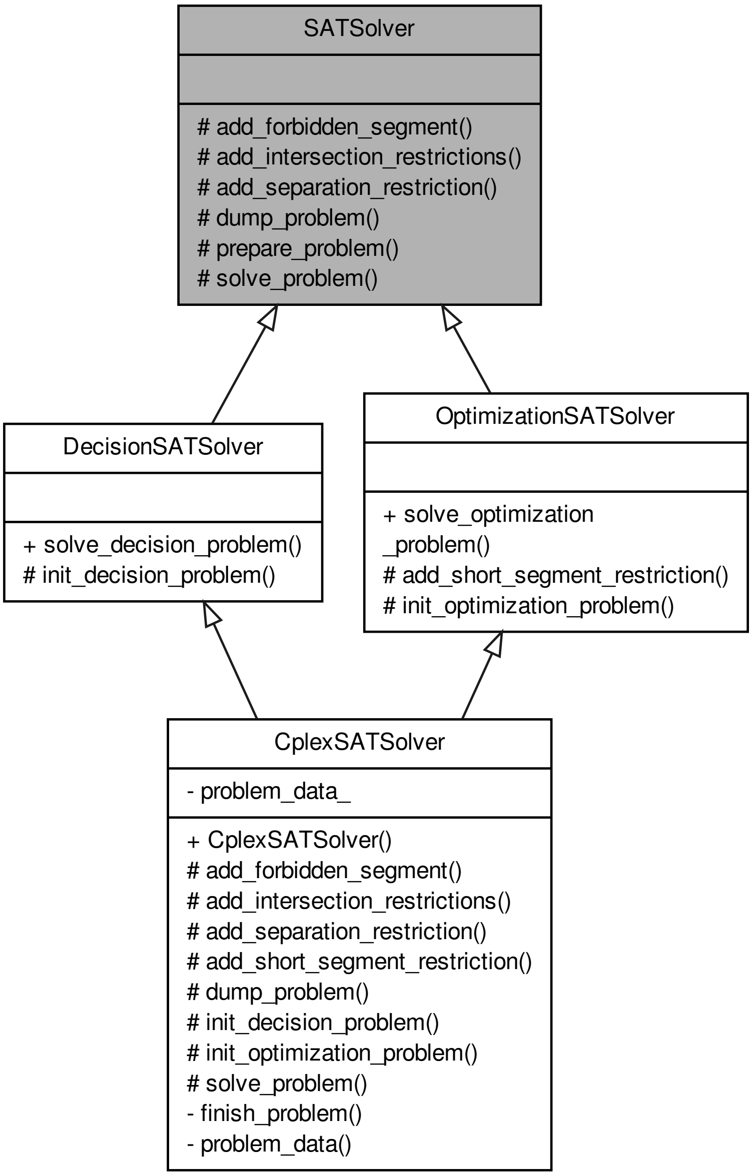
1.20.2.2 void SATSolution::draw_short_segments (QPainter & *painter*, const SegmentIndex & *num_short_segments*, const SegmentContainer & *segments*) const

1.20.2.3 const SegmentIndex & SATSolution::shortest_segment ( ) const

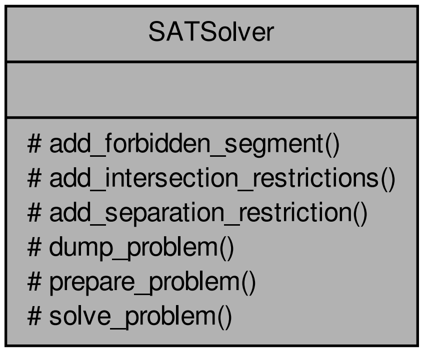
1.21 SATSolver Class Reference

```
#include <sat_solver.h>
```

Inheritance diagram for SATSolver:



Collaboration diagram for SATSolver:



Protected Member Functions

- virtual void add_forbidden_segment (const SATProblem ∗problem, const SegmentIndex &index)=0
- virtual void add_intersection_restrictions (const SATProblem ∗problem, const SegmentIndex &index, const Intersections &igroup)=0
- virtual void add_separation_restriction (const SATProblem ∗problem, const SegmentIndex &index, const std::vector< SegmentIndex > &separators)=0
- virtual void dump_problem (const QString &file_prefix, const SATProblem ∗problem)=0
- void prepare_problem (const SATProblem &problem)
- virtual void solve_problem (const SATProblem ∗problem, SATSolution &solution)=0

1.21.1 Detailed Description

interface for SAT solvers

1.21.2 Member Function Documentation

1.21.2.1 virtual void SATSolver::add_forbidden_segment (const SATProblem ∗ *problem*, const SegmentIndex & *index*) [protected], [pure virtual]

Implemented in CplexSATSolver.

1.21.2.2 virtual void SATSolver::add_intersection_restrictions (const SATProblem ∗ *problem*, const SegmentIndex & *index*, const Intersections & *igroup*) [protected], [pure virtual]

Implemented in CplexSATSolver.

1.21.2.3 virtual void SATSolver::add_separation_restriction (const SATProblem ∗ *problem*, const SegmentIndex & *index*, const std::vector< SegmentIndex > & *separators*) [protected], [pure virtual]

Implemented in CplexSATSolver.

1.21.2.4 virtual void SATSolver::dump_problem (const QString & *file_prefix*, const SATProblem ∗ *problem*) [protected], [pure virtual]

Implemented in CplexSATSolver.

1.21.2.5 void SATSolver::prepare_problem (const SATProblem & *problem*) [protected]

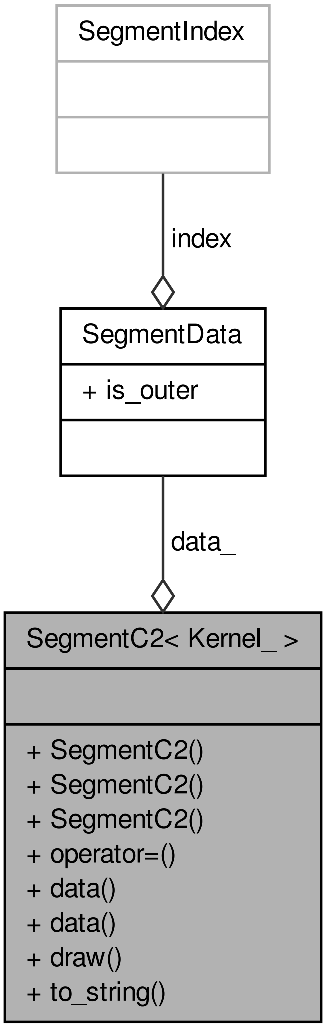
1.21.2.6 virtual void SATSolver::solve_problem (const SATProblem * *problem,* SATSolution & *solution*) [protected], [pure virtual]

Implemented in CplexSATSolver.

1.22 SegmentC2< Kernel_ > Class Template Reference

`#include <segment.h>`

Collaboration diagram for SegmentC2< Kernel_ >:



Public Member Functions

- SegmentC2 ()
- SegmentC2 (const Point_2 &source, const Point_2 &target)

- SegmentC2 (const SegmentC2 &other)
- SegmentC2 & operator= (const SegmentC2 &other)
- SegmentData & data ()
- const SegmentData & data () const
- void draw (QPainter &painter) const
- QString to_string () const

Private Attributes

- SegmentData data_

1.22.1 Detailed Description

template<class Kernel_>class SegmentC2< Kernel_ >

customized segment type

1.22.2 Constructor & Destructor Documentation

1.22.2.1 template<class Kernel_ > SegmentC2< Kernel_ >::SegmentC2 ( ) [inline]

empty constructor

1.22.2.2 template<class Kernel_ > SegmentC2< Kernel_ >::SegmentC2 (const Point_2 & *source*, const Point_2 & *target*) [inline]

base constructor

1.22.2.3 template<class Kernel_ > SegmentC2< Kernel_ >::SegmentC2 (const SegmentC2< Kernel_ > & *other*) [inline]

copy constructor

1.22.3 Member Function Documentation

1.22.3.1 template<class Kernel_ > SegmentData& SegmentC2< Kernel_ >::data ( ) [inline]

getter for attached data

1.22.3.2 template<class Kernel_ > const SegmentData& SegmentC2< Kernel_ >::data ( ) const [inline]

constant getter for attached data

1.22.3.3 template<class Kernel_ > void SegmentC2< Kernel_ >::draw (QPainter & *painter*) const

draw segment using given QPainter

1.22.3.4 template<class Kernel_ > SegmentC2& SegmentC2< Kernel_ >::operator= (const SegmentC2< Kernel_ > & *other*) [inline]

assignment operator

1.22.3.5 template<class Kernel_ > QString SegmentC2< Kernel_ >::to_string ( ) const

dump segment to QString

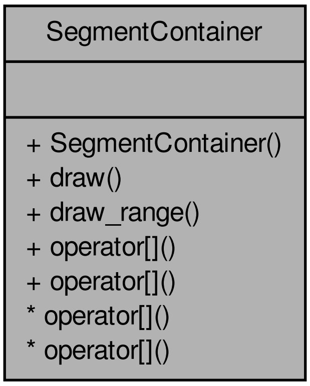
1.22.4 Member Data Documentation

1.22.4.1 template<class Kernel_ > SegmentData SegmentC2< Kernel_ >::data_ [private]

1.23 SegmentContainer Class Reference

`#include <segment_container.h>`

Collaboration diagram for SegmentContainer:



Public Member Functions

- SegmentContainer (const PointSet &points)
- void draw (QPainter &painter) const
- void draw_range (QPainter &painter, const SegmentIndex &lower_bound, const SegmentIndex &upper_-bound) const

access i-th shortest segment

these operators assume that the segment set is not changed after construction

- Segment & operator[] (const SegmentIndex &index)
- const Segment & operator[] (const SegmentIndex &index) const

1.23.1 Detailed Description

container of segments sorted by length

1.23.2 Constructor & Destructor Documentation

1.23.2.1 SegmentContainer::SegmentContainer (const PointSet & *points*)

construct segments for all point pairs from set

1.23.3 Member Function Documentation

1.23.3.1 void SegmentContainer::draw (QPainter & *painter*) const

draws all segments

1.23.3.2 void SegmentContainer::draw_range (QPainter & *painter*, const SegmentIndex & *lower_bound*, const SegmentIndex & *upper_bound*) const

draw a range of segments

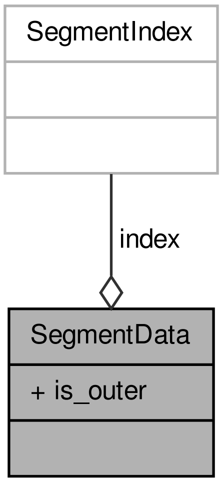
1.23.3.3 Segment & SegmentContainer::operator[] (const SegmentIndex & *index*)

1.23.3.4 const Segment & SegmentContainer::operator[] (const SegmentIndex & *index*) const

1.24 SegmentData Struct Reference

`#include <segment.h>`

Collaboration diagram for SegmentData:



Public Attributes

- SegmentIndex index
- bool is_outer

1.24.1 Detailed Description

data attached to a segment

1.24.2 Member Data Documentation

1.24.2.1 SegmentIndex SegmentData::index

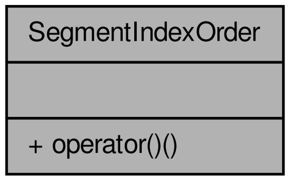
1.24.2.2 bool SegmentData::is_outer

true if the segment includes another

1.25 SegmentIndexOrder Struct Reference

`#include <orders.h>`

Collaboration diagram for SegmentIndexOrder:



Public Member Functions

- CGAL::Comparison_result operator() (const Segment &s, const Segment &t) const

1.25.1 Detailed Description

CGAL order for Segment by index

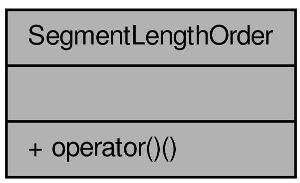
1.25.2 Member Function Documentation

1.25.2.1 CGAL::Comparison_result SegmentIndexOrder::operator() (const Segment & *s*, const Segment & *t*) const

1.26 SegmentLengthOrder Struct Reference

`#include <orders.h>`

Collaboration diagram for SegmentLengthOrder:



Public Member Functions

- CGAL::Comparison_result operator() (const Segment &s, const Segment &t) const

1.26.1 Detailed Description

CGAL order for Segment by length

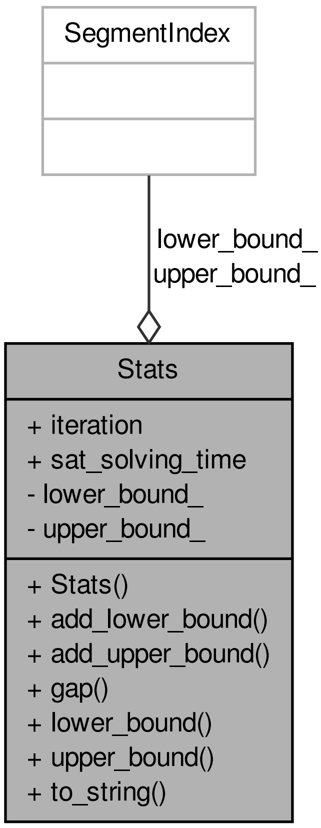
1.26.2 Member Function Documentation

1.26.2.1 CGAL::Comparison_result SegmentLengthOrder::operator() (const Segment & *s*, const Segment & *t*) const

1.27 Stats Class Reference

`#include <stats.h>`

Collaboration diagram for Stats:



Public Member Functions

- Stats ()
- void add_lower_bound (const SegmentIndex &bound)

- void add_upper_bound (const SegmentIndex &bound)
- SegmentIndex gap () const
- const SegmentIndex & lower_bound () const
- const SegmentIndex & upper_bound () const
- QString to_string () const

Public Attributes

- size_t iteration
- quint64 sat_solving_time

Private Attributes

- SegmentIndex lower_bound_
- SegmentIndex upper_bound_

1.27.1 Constructor & Destructor Documentation

1.27.1.1 Stats::Stats ( ) [inline]

1.27.2 Member Function Documentation

1.27.2.1 void Stats::add_lower_bound (const SegmentIndex & *bound*) [inline]

1.27.2.2 void Stats::add_upper_bound (const SegmentIndex & *bound*) [inline]

1.27.2.3 SegmentIndex Stats::gap ( ) const [inline]

1.27.2.4 const SegmentIndex& Stats::lower_bound ( ) const [inline]

1.27.2.5 QString Stats::to_string ( ) const [inline]

1.27.2.6 const SegmentIndex& Stats::upper_bound ( ) const [inline]

1.27.3 Member Data Documentation

1.27.3.1 size_t Stats::iteration

1.27.3.2 SegmentIndex Stats::lower_bound_ [private]

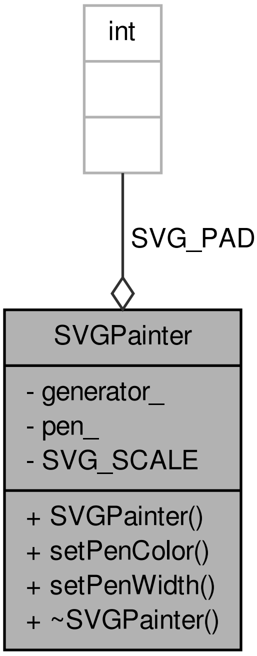
1.27.3.3 quint64 Stats::sat_solving_time

1.27.3.4 SegmentIndex Stats::upper_bound_ [private]

1.28 SVGPainter Class Reference

```
#include <svg_painter.h>
```

Collaboration diagram for SVGPainter:



Public Member Functions

- SVGPainter (const QString &file_prefix, const QString &file_name, const BoundingBox &bbox)
- void setPenColor (const QColor &color)
- void setPenWidth (int width)
- ∼SVGPainter ()

Private Attributes

- QSvgGenerator generator_
- QPen pen_

Static Private Attributes

- static const int SVG_PADDING = 10
- static const double SVG_SCALE = 4.0

1.28.1 Constructor & Destructor Documentation

1.28.1.1 SVGPainter::SVGPainter (const QString & *file_prefix*, const QString & *file_name*, const **BoundingBox** & *bbox*)

1.28.1.2 SVGPainter::∼SVGPainter ( )

1.28.2 Member Function Documentation

1.28.2.1 void SVGPainter::setPenColor (const QColor & *color*)

1.28.2.2 void SVGPainter::setPenWidth (int *width*)

1.28.3 Member Data Documentation

1.28.3.1 QSvgGenerator SVGPainter::generator_ [private]

1.28.3.2 QPen SVGPainter::pen_ [private]

1.28.3.3 const int SVGPainter::SVG_PADDING = 10 [static], [private]

padding for SVG images

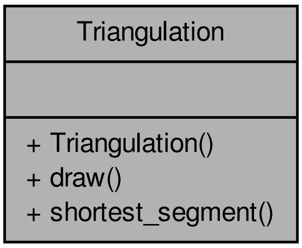
1.28.3.4 const double SVGPainter::SVG_SCALE = 4.0 [static], [private]

scale for SVG images

1.29 Triangulation Class Reference

`#include <triangulation.h>`

Collaboration diagram for Triangulation:



Public Member Functions

- Triangulation (const PointSet &points)
- void draw (QPainter &painter) const
- const SegmentIndex & shortest_segment (const SegmentContainer &segments) const

1.29.1 Constructor & Destructor Documentation

1.29.1.1 Triangulation::Triangulation (const PointSet & *points*)

default constructor

1.29.2 Member Function Documentation

1.29.2.1 void Triangulation::draw (QPainter & *painter*) const

draw triangulation segments using given QPainter

1.29.2.2 const SegmentIndex & Triangulation::shortest_segment (const SegmentContainer & *segments*) const

Glossary

Glossary

e_{nose} . . . . shortest non-separable edge

e_{sep} . . . . separating edge

$s_\times$ . . . . crossing segment

CDS . . . . Covering by Disjoint Segments

IP . . . . Integer Program

MELT . . . . MaxMin Edge Length Index Triangulation

MMLT . . . . MaxMin Length Triangulation

NOCS . . . . Non-Conflicting Separators

Properties

crossing . . . . see definition 3.11

non-crossing . . . . see definition 3.11

non-overlapping . . . . see definition 3.12

overlapping . . . . see definition 3.12

well-covered . . . . see definition 2.14

Sets

E_{flip} . . . . set of edge flips (see definition 3.26)

E_{sep} . . . . set of separators (see definition 4.14)

E_{short} . . . . set of short edges (see definition 4.13)

G_{conf} . . . . conflict graph (see definition 3.2)

G_{cross} . . . . intersection graph (see definition 3.18)

G_{flip} . . . . flip graph (see definition 3.28)

S_{opt} . . . . an optimal set of segments

T_{opt} . . . . an optimal triangulation

V_{IS} . . . . independent set (see definition 2.16)

V_{cover} . . . . vertex cover (see definition 2.9)

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Liste der noch zu erledigenden Punkte