MATH CIRCLE

SCREENSAVER PATTERNS

KAREN LI & WINNIE WANG

EMORY COLLEGE OF ARTS AND SCIENCES

Institute for Quantitative Theory and Methods

INTRODUCTION

In an animated screensaver, the screensaver dot starts moving at a 45° angle from a corner, constantly bouncing off the sides of the screen. During the process, there is a small chance that the dot will hit exactly the corner of the screen. We believe that both the number of sides the dot hits and the distance it travels before it hits a corner correlate to the dimensions of the screen, $N \times M$. We also believe that there exists a pattern between the ratio $\frac{M}{N}$ and the number of sides it hits and the distance it travels.

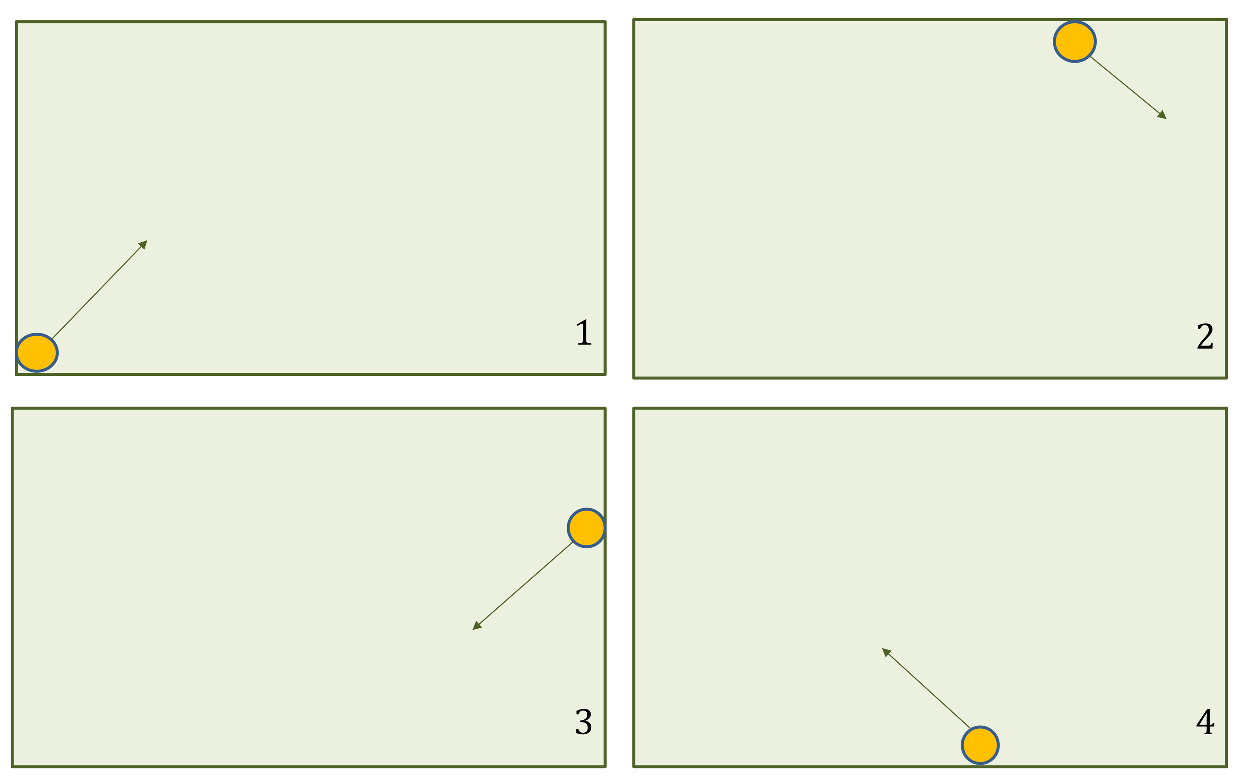

Figure 1: A representation illustrating a screensaver dot launching from the bottom left corner.

EXAMPLE

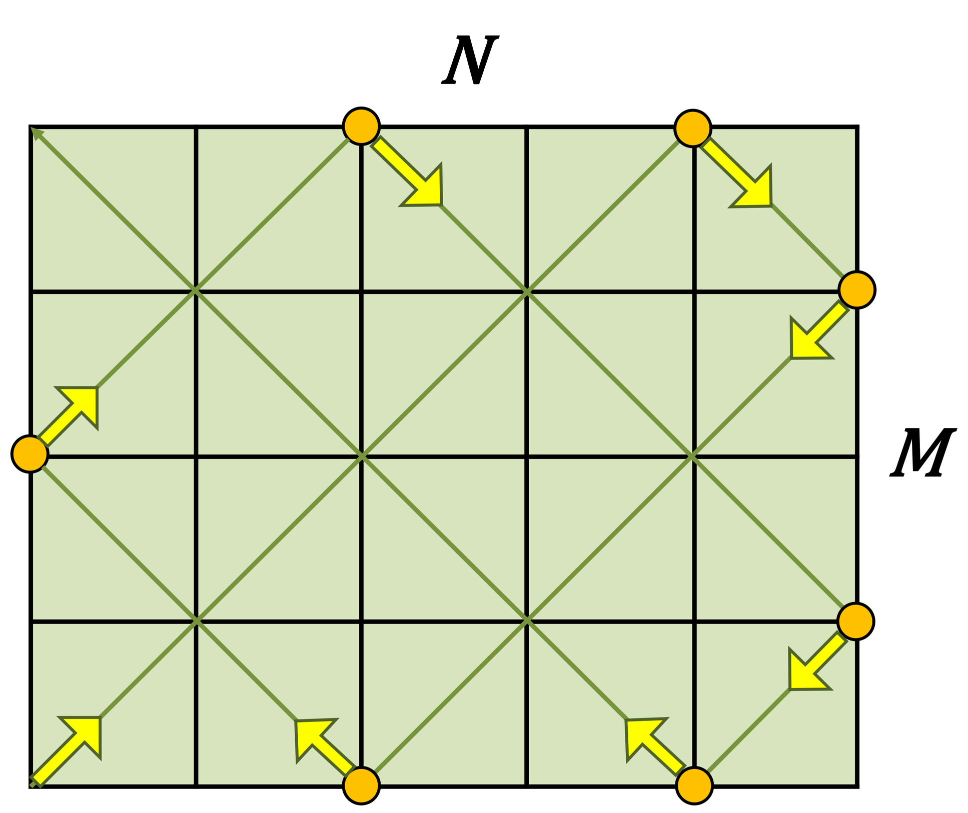

Figure 2: A representation illustrating the path of the dot on a screen with dimensions $N \times M$.

The ratio of $\frac{M}{N}$ is $\frac{\mathbf{4}}{\mathbf{5}}$.

The number of sides the dot hits is 7.

The number of unit-squares it crosses is 20.

The distance (unit) it travels is $20\sqrt{2}$.

METHODOLOGY

After finding a particular case, we continued with more cases, with M and N each denoted as a whole number from 1 to 10.

		N									
		1	2	3	4	5	6	7	8	9	10
M	1	0									
	2	1	0								
	3	2	3	0							
	4	3	1	5	0						
	5	4	5	6	7	0					
	6	5	2	1	3	9	0				
	7	6	7	8	9	10	11	0			
	8	7	3	9	1	11	5	13	0		
	9	8	9	2	11	12	3	14	15	0	
	10	9	4	11	5	1	6	15	7	17	0

* Yellow represents that the GCD of M and N is not 1

Figure 3: A data table listing the number of times the dot bounces off the sides.

The times the dot bounces off the screen (T) has a fixed relationship with the dimension:

$$T = M + N - 2.$$

However, it only works when M and N are relatively prime.

When they have common factors, the time the dot bounces will equal to their reduced ratio's.

Similar to the previous one, we made a table listing the number of times the dot crosses the and the dot's corresponding traveled distance, with M and N each denoted as whole number from 1 to 10.

		N									
		1	2	3	4	5	6	7	8	9	10
M	1	1									
	2	2	2								
	3	3	6	3							
	4	4	4	12	4						
	5	5	10	15	20	5					
	6	6	6	6	12	30	6				
	7	7	14	21	28	35	42	7			
	8	8	8	24	8	40	24	56	8		
	9	9	18	9	36	45	18	63	72	9	
	10	10	10	30	20	10	30	70	40	90	10

* Yellow represents that the GCD of M and N is not 1

Figure 4: A data table listing the number of squares the dot crosses.

We noticed that, when the angle is 45° the number of squares the dot crosses, S, divides the LCM of M and N:

$$S \equiv 0 \text{ mod LCM } (M, N)$$

In fact, there is a particular relationship:

$$S = \text{LCM } (M, N)$$

where the distance is equal to the LCM of M and N.

The total distance D it travels is the sum of the length of the diagonal of each divided unit-square.

$$D = \sqrt{2} \cdot S$$

VARIATION

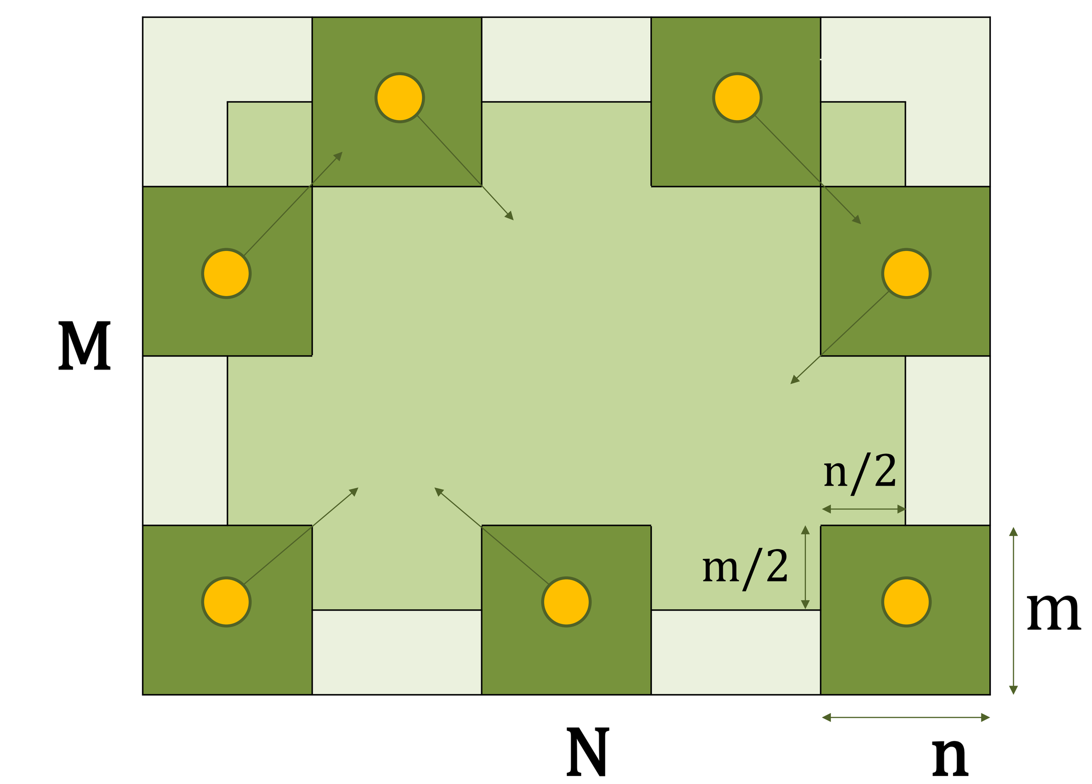

Figure 5: A representation illustrating the path of a rectangle with dimension $n \cdot m$ on a screen with dimensions $N \cdot M$.

One variation of the screensaver pattern is replacing the abstract dot with a rectangle with dimensions $m \cdot n$. Similar to what we did before, we treated the center of the rectangle as the dot, which bounces off the inner screen with dimensions M-m and N-n.

The equation we derived earlier for the number of times the dot bounces off the sides (T) also applies to the rectangle, so that

$$T = (M - m) + (N - n) - 2$$

Likewise, the number of squares the dot crosses, S, divides the LCM of M-m and N-n.

$$S \equiv 0 \text{ mod LCM } (M - m, N - n)$$

Therefore, the number of unit-squares the rectangle crosses is equal to the LCM of the difference of the screen and rectangle's heights and the difference of the screen and rectangle's widths, or

$$S = LCM\ (M - m, N - n)$$

And the same applies to the actual distance traveled:

$$D = \sqrt{2} \cdot S$$

FURTHER RESERCH

- Will the dot ever hit another corner if it does not start from one of the corners of the screen?
- What if the dot starts at an angle other than 45° like 30° or 60°? Are there any requirements of n and m for the dot to hit another corner?

ACKNOWLEDGEMENTS

We would like to thank Math Circle and Emory University for allowing us to have the opportunity to research this challenging mathematical puzzle. In particular we are grateful for the instructors, Mr. Olsen, Mr. Roby, Mr. Elliot, Mr. Sinha, and Bryson Kagy, who were always present and prepared to provide us with helpful information and insight.