Adjusting for Unreliability and Longitudinal Noninvariance in Latent Growth Modeling: A Two-Stage Path Analysis Approache

Winnie Wing-Yee Tse[1] & Mark H. C. Lai[1]

[1] Department of Psychology, University of Southern California

Author Note

Winnie Wing-Yee Tse https://orcid.org/0000-0001-5175-6754

Mark H. C. Lai https://orcid.org/0000-0002-9196-7406

Correspondence concerning this article should be addressed to Mark H. C. Lai, University of Southern California, 3620 S McClintock Ave, Los Angeles, CA 90089. E-mail: hokchiol@usc.edu

Adjusting for Unreliability and Longitudinal Noninvariance in Latent Growth Modeling: A Two-Stage Path Analysis Approache

Latent growth modeling is a popular statistical technique in psychological and educational research to examine changes in latent constructs. For latent change estimates to be meaningful, growth models need to adjust for biases due to measurement artifacts of the latent constructs (e.g., unreliability and longitudinal noninvariance). However, such growth models can easily get complex and become challenging to estimate, particularly in small samples. An alternative approach that reduces model complexity for parameter estimation is two-stage path analysis (2S-PA), where factor scores are obtained from measurement models in one stage and used to model latent changes in another stage. This approach has been found to provide better small sample size properties (e.g., higher model convergence rate in complex models; Lai et al., 2022). While the 2S-PA approach has been developed for models not dependent on time (Lai et al., 2022; Lai & Hsiao, 2021), the current paper aims to extend its application to include univariate and bivariate latent growth modeling. We will perform Monte Carlo simulation studies to compare the 2S-PA approach with the JSEM approach, as well as common growth modeling approaches that use factor scores. We will demonstrate the 2S-PA approach to model latent changes with an empirical example.

Longitudinal Invariance

Latent growth modeling examines changes in latent constructs that are measured at multiple occasions. For changes in latent means and variances to be meaningful, the latent constructs need to be measured similarly across measurement occasions—a condition known as measurement invariance, or longitudinal invariance in the context of longitudinal analysis (Cook et al., 2002; Grimm et al., 2016; Horn & McArdle, 1992; Widaman et al., 2010). If longitudinal invariance fails to hold, a condition named *longitudinal noninvariance*, comparisons of latent means and variances may still be permissible in a partial invariance model that correctly adjusts for the bias.

Joint Structural Equation Modeling

The gold standard for modeling changes in latent constructs has been through a second-order growth model, which accounts for measurement unreliability and longitudinal noninvariance. We refer to this approach as joint structural equation modeling (JSEM), which integrates measurement and structural models for the latent constructs across measurement occasions. Despite the flexibility in modeling latent changes, models with the JSEM approach can easily get complex and result in nonconvergence issues, particularly with multiple latent variables and a small sample size (Lai et al., 2022).

Moreover, a long-standing concern with JSEM is that the parameter estimates in the measurement model can be affected by the structural paths (Bollen & Maydeu-Olivares, 2007; Burt, 1976; Levy, 2017). To briefly illustrate, Figure 1 shows an example using the data of Midlife in the United States (MIDUS; Brim et al., 2020; Ryff et al., 2021). Suppose we are interested in predicting participants' perceived purpose in life at Wave 2 by their attitudes toward personal growth at Wave 1. Whereas the factor loadings of the three items of Purpose in Life were 0.50, 0.45, and 0.75 in a one-factor model (Figure 1b), they changed to 0.60, 0.21, and 0.39 after adding a structural path from Personal Growth at Wave 1 to Purpose of Life at Wave 2 (Figure 1c). This example demonstrates a theoretical challenge in using JSEM that adding a structural path in a joint model can change the definition of a latent construct.

Two-Stage Path Analysis

Lai and Hsiao (2021) proposed an alternative approach to JSEM—2S-PA—which separates the measurement and structural models into two stages. In the first stage, measurement invariance testing is performed, and factor scores and their reliability estimates are obtained from the full or partial invariance model. Biases due to noninvariance, if exist, are accounted for in the first stage. In the second stage, structural relations among latent constructs are modeled using the factor scores with constraints on the measurement parameters to control for measurement unreliabilitys. As measurement and structural models are evaluated at different stages, structural paths among latent variables do not change the measurement parameter estimates. In other words, the 2S-PA approach retains the definitions of the latent constructs in the structural model.

The 2S-PA approach has also been shown to overcome computational challenges in estimating complex models in small samples (Lai et al., 2022).

Contributions

The purpose of the current paper is to extend the 2S-PA approach for latent growth modeling and evaluate its performance in modeling latent changes. The 2S-PA approach may have an advantage to parameter estimations in complex growth models that accounts for measurement unreliabilitys and longitudinal noninvariance. The contribution of the paper is three-folded. First, we will provide the mathematical development of the 2S-PA approach for adjusting for measurement unreliability and longitudinal noninvariance when modeling latent changes.

Second, we will perform Monte Carlo simulation studies to evaluate the 2S-PA approach, in comparison with the JSEM approach and factor-score path analysis (FS-PA; i.e., a first-order growth model using factor scores). We aim to evaluate the accuracy, efficiency, and rate of model convergence of each approach in a variety of design conditions, including those with small sample sizes. Figure 2 shows the preliminary simulation results in terms of bias. Overall, the 2S-PA approach performed similarly to the JSEM approach, but the FS-PA approach showed biases in estimating the variance of the intercept factor and the mean and variance of the slope factor. In small sample conditions, whereas the 2S-PA approach showed a larger bias in estimating the mean of the slope factor, the JSEM approach showed a larger bias in estimating the mean of the intercept factor.

Finally, we will demonstrate the 2S-PA approach with the MIDUS data and compare the results with those from the JSEM approach. As an example, we performed longitudinal invariance testing on the Personal Growth scale that has three items. A partial strict invariance model was identified with equality constraints on each item's loadings, intercepts, and unique variances, except for the loading and intercept of Item 1 and the unique varaiance of Item 2 at Wave 1. Based on the partial strict invariance model, we model changes of Personal Growth across three waves of measurement using the JSEM and 2S-PA approach. Table 1 shows that the results of the JSEM and 2S-PA approaches are similar.

References

Bollen, K. A., & Maydeu-Olivares, A. (2007). A polychoric instrumental variable (PIV) estimator for structural equation models with categorical variables. *Psychometrika*, *72*(3), 309–326. https://doi.org/10.1007/s11336-007-9006-3

Brim, O. G., Baltes, P. B., Bumpass, L. L., Cleary, P. D., Featherman, D. L., Hazzard, W. R., Kessler, R. C., Lachman, M. E., Markus, H. R., Marmot, M. G., et al. (2020). *Midlife in the united states (midus 1), 1995-1996*.

Burt, R. S. (1976). Interpretational confounding of unobserved variables in structural equation models. *Sociological Methods & Research*, *5*(1), 3–52. https://doi.org/10.1177/004912417600500101

Cook, T. D., Campbell, D. T., & Shadish, W. (2002). *Experimental and quasi-experimental designs for generalized causal inference*. Houghton Mifflin Boston, MA.

Grimm, K. J., Ram, N., & Estabrook, R. (2016). *Growth modeling: Structural equation and multilevel modeling approaches*. Guilford Publications.

Horn, J. L., & McArdle, J. J. (1992). A practical and theoretical guide to measurement invariance in aging research. *Experimental Aging Research*, *18*(3), 117–144. https://doi.org/10.1080/03610739208253916

Lai, M. H. C., & Hsiao, Y.-Y. (2021). Two-stage path analysis with definition variables: An alternative framework to account for measurement error. *Psychological Methods*. In press. https://doi.org/10.1037/met0000410

Lai, M. H. C., Tse, W. W.-Y., Zhang, G., Li, Y., & Hsiao, Y.-Y. (2022). Correcting for unreliability and partial invariance: A two-stage path analysis approach. *Structural Equation Modeling: A Multidisciplinary Journal*, *0*(0), 1–14. https://doi.org/10.1080/10705511.2022.2125397

Levy, R. (2017). Distinguishing outcomes from indicators via Bayesian modeling. *Psychological Methods*, *22*(4), 632–648. https://doi.org/10.1037/met0000114

Ryff, C. D., Seeman, T., & Weinstein, M. (2021). *Midlife in the United States (MIDUS 2): Biomarker Project, 2004-2009 (ICPSR 29282)*.

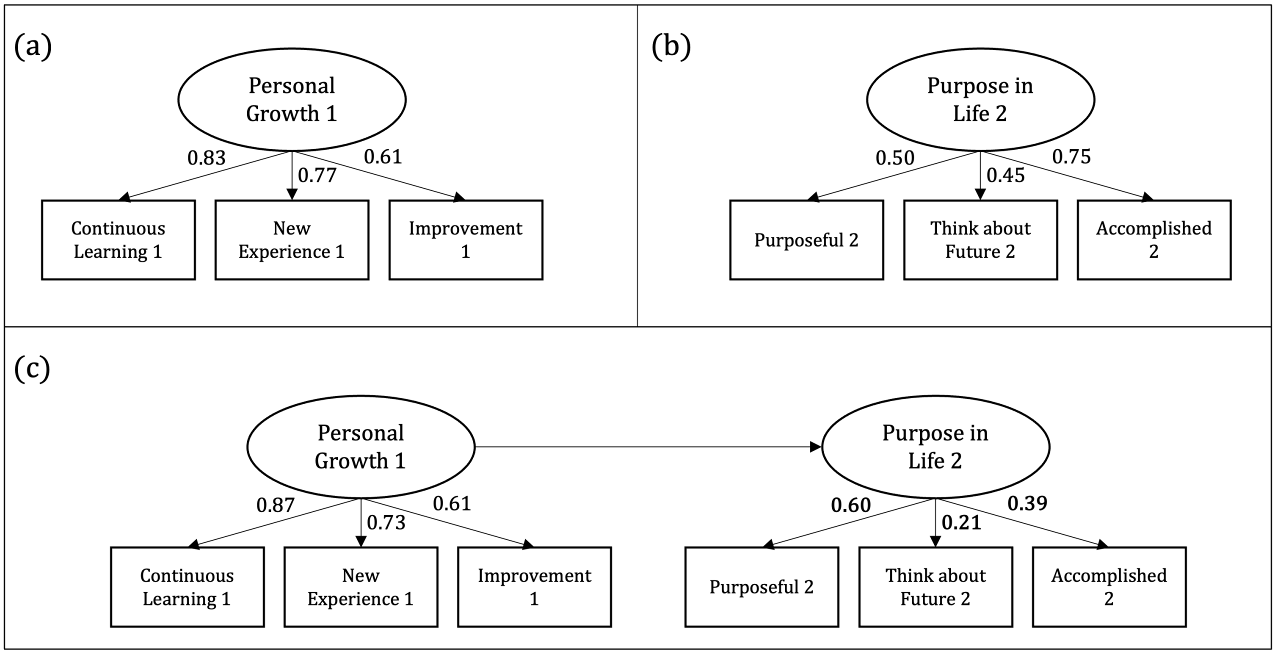
Widaman, K. F., Ferrer, E., & Conger, R. D. (2010). Factorial invariance within longitudinal structural equation models: Measuring the same construct across time. *Child Development Perspectives*, *4*(1), 10–18.

Table 1

Growth Parameter Estimates Using the JSEM and 2S-PA Approaches.

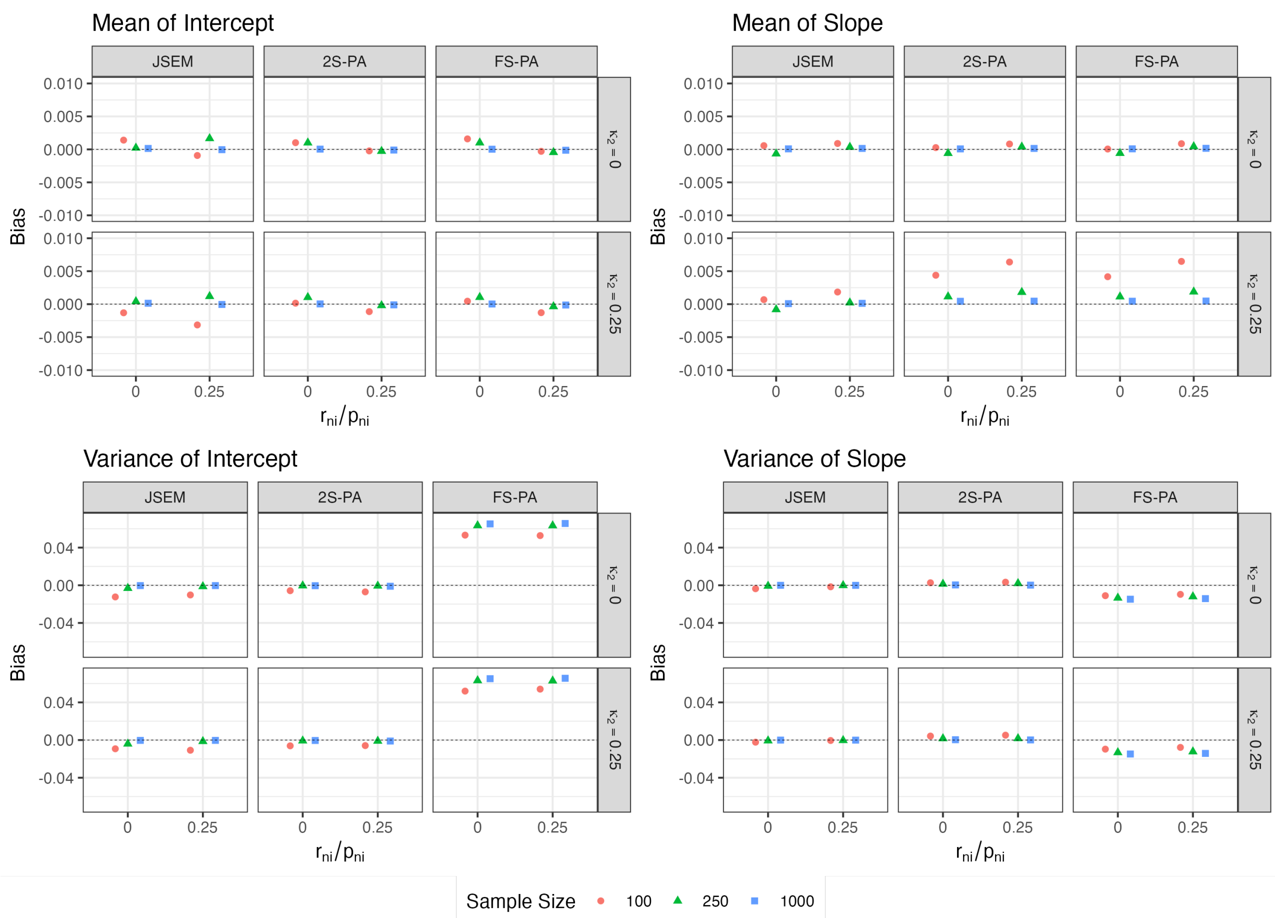
	JSEM	2S-PA
Mean(Intercept)	0.000	-0.070
	[0.000, 0.000]	[-0.145, 0.005]
Mean(Slope)	-0.247	-0.257
	[-0.293, -0.201]	[-0.299, -0.216]
Var(Intercept)	0.759	0.753
	[0.592, 0.927]	[0.597, 0.908]
Var(Slope)	0.086	0.080
	[-0.001, 0.173]	[0.001, 0.159]
Cov(Intercept, Slope)	-0.096	-0.094
	[-0.193, 0.002]	[-0.186, -0.003]
Num.Obs.	833	833

Figure 1



Note. An example based on Personal Growth and Purpose in Life scales from the MIDUS data.

Figure 2



Note. κ_2 = mean of the slope factor. r_{ni}/p_{ni} = proportion of noninvariance. N = sample size. JSEM = joint structural equation model. 2S-PA = two-stage path analysis. FS-PA = factor scores path analysis.